Asymptotics of hard diffraction

Stéphane Munier

Centre de physique théorique
École polytechnique, CNRS, IP Paris

Based on work with **Anh Dung Le** and Al Mueller
arXiv:2010.15546, PRD and arXiv:2103.10088, submitted

CPHT Centre de Physique Théorique

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his PhD thesis includes the material of this talk!

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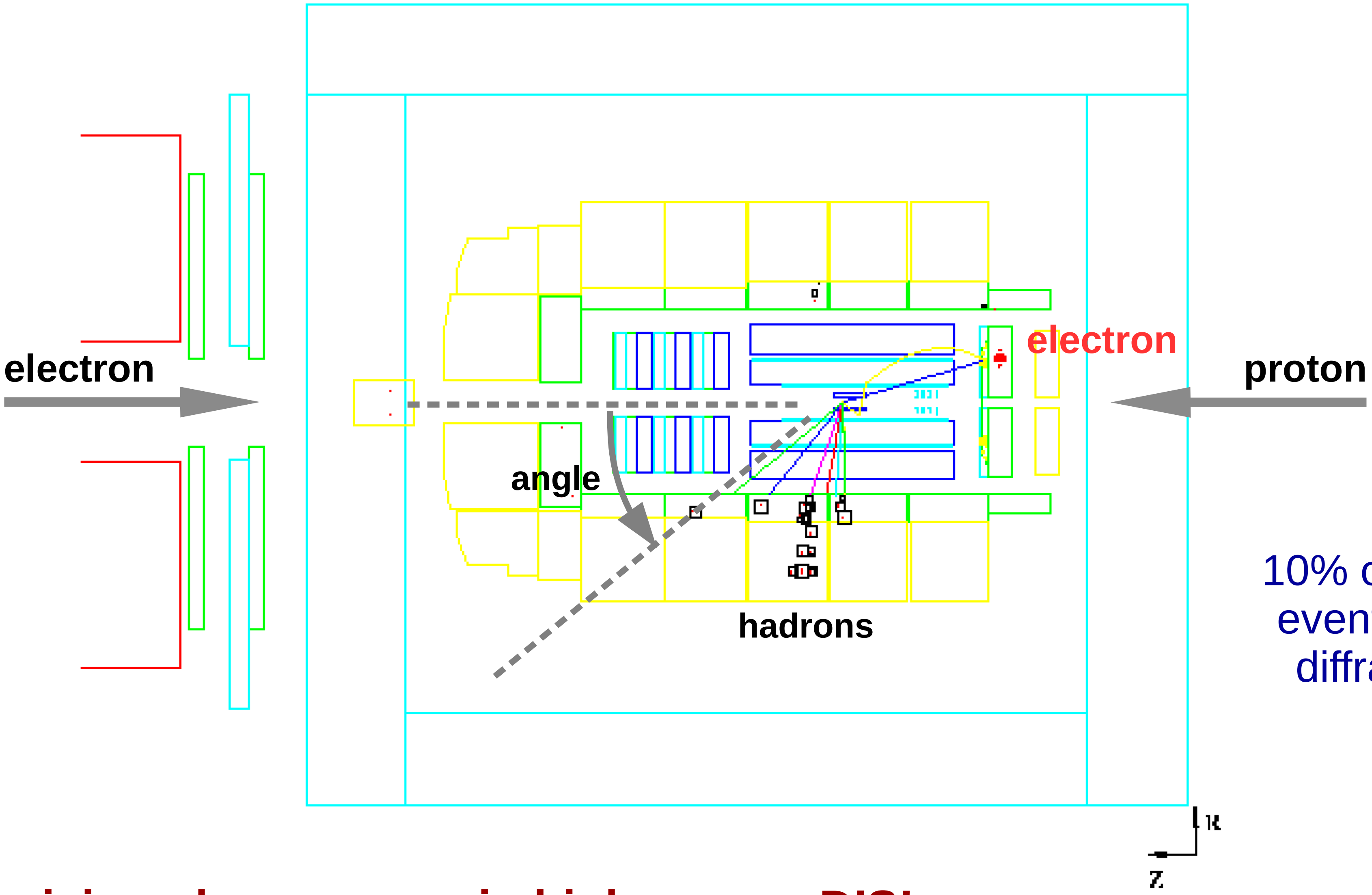
Diffraction in deep-inelastic scattering

A major highlight of HERA!



10% of HERA events were diffractive!

Surprising phenomenon in high-energy DIS!

Its existence seems almost contradictory with the parton model...

Its observation boosted saturation physics!

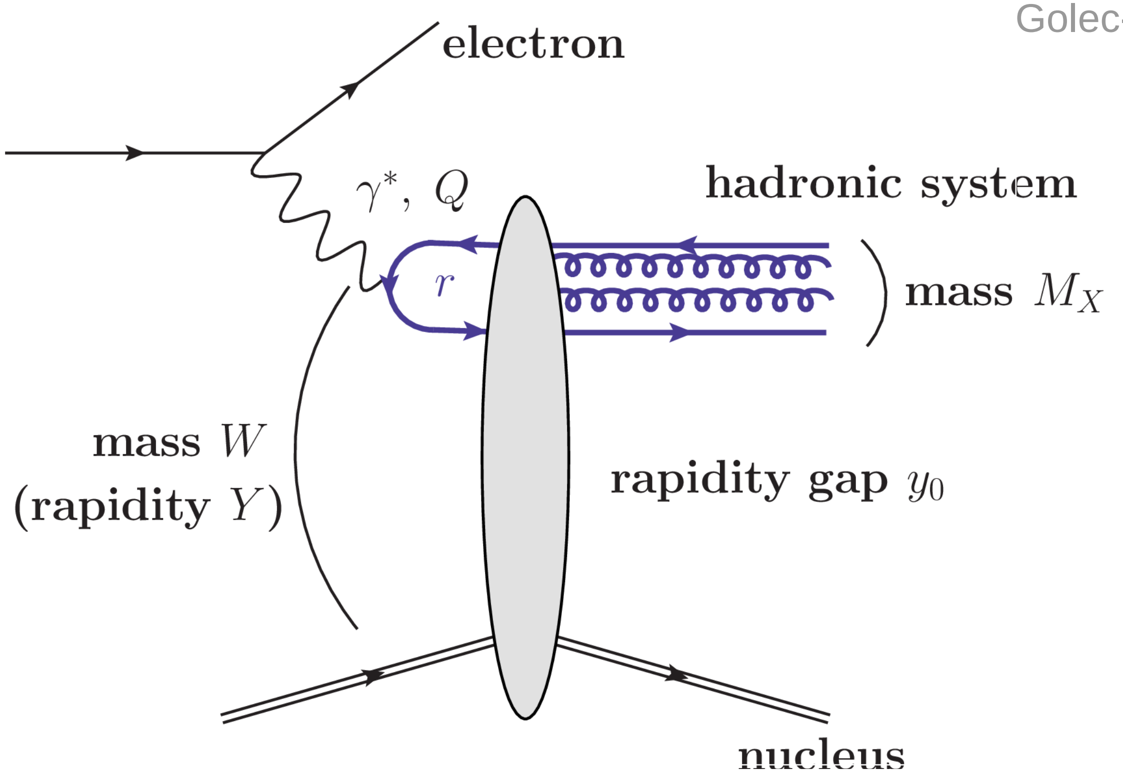
[Golec-Biernat, Wüsthoff 1998]

Diffraction in deep-inelastic scattering

This talk: Diffractive dissociative electron-***nucleus*** scattering

~ $q\bar{q}$ dipole-nucleus, size r~1/Q

[Nikolaev,Zakharov '90
Golec-Biernat, Wüsthoff 1998
etc...]



What is the distribution of the rapidity gap y_0?

$$\left|\frac{1}{\sigma_{tot}}\frac{d\sigma_{diff}}{dy_0}\right|$$

Diffraction in deep-inelastic scattering

Formulation in terms of evolution equations

Forward elastic S-matrix element for dipole-nucleus scattering:

$$S(r,y=0)=e^{-\frac{r^2 Q_A^2}{4}}$$

(Q_A = nuclear saturation momentum)

[McLerran, Venugopalan, 1993
Golec-Biernat, Wüsthoff]

$$\partial_y S(r,y)=\bar{\alpha}\int\frac{d^2r'}{2\pi}\frac{r^2}{r'^2(r-r')^2}\left[S(r',y)S(r-r',y)-S(r,y)\right]$$

[Balitsky, Kovchegov
1996-1999]

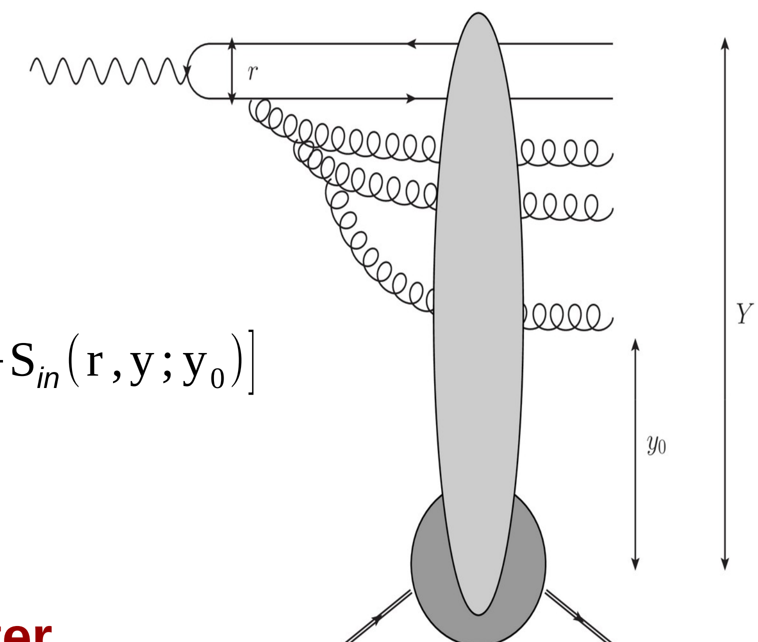
$$\Rightarrow\quad \sigma_{tot}=2[1-S(r,Y)]$$

Define S_{in} at $y=y_0$ as $\quad S_{in}(r,y_0;y_0)=[S(r,y_0)]^2$

and for $y>y_0$: $\quad \partial_y S_{in}(r,y;y_0)=\bar{\alpha}\int\frac{d^2r'}{2\pi}\frac{r^2}{r'^2(r-r')^2}$

$$\times\left[S_{in}(r',y;y_0)S_{in}(r-r',y;y_0)-S_{in}(r,y;y_0)\right]$$

$$\Rightarrow\quad \frac{d\sigma_{diff}}{dy_0}=-\frac{\partial}{\partial y_0}S_{in}(r,Y;y_0)$$



The solution to these equations is what we are after, but they are difficult to solve!

[**Kovchegov, Levin** 2000
numerical solution by Levin, Lublinsky 2001
See also Anh Dung Le's talk]

Outline

- Probabilistic picture
- Nuclear scattering configurations of onia
- Analytical asymptotics for the gap distribution

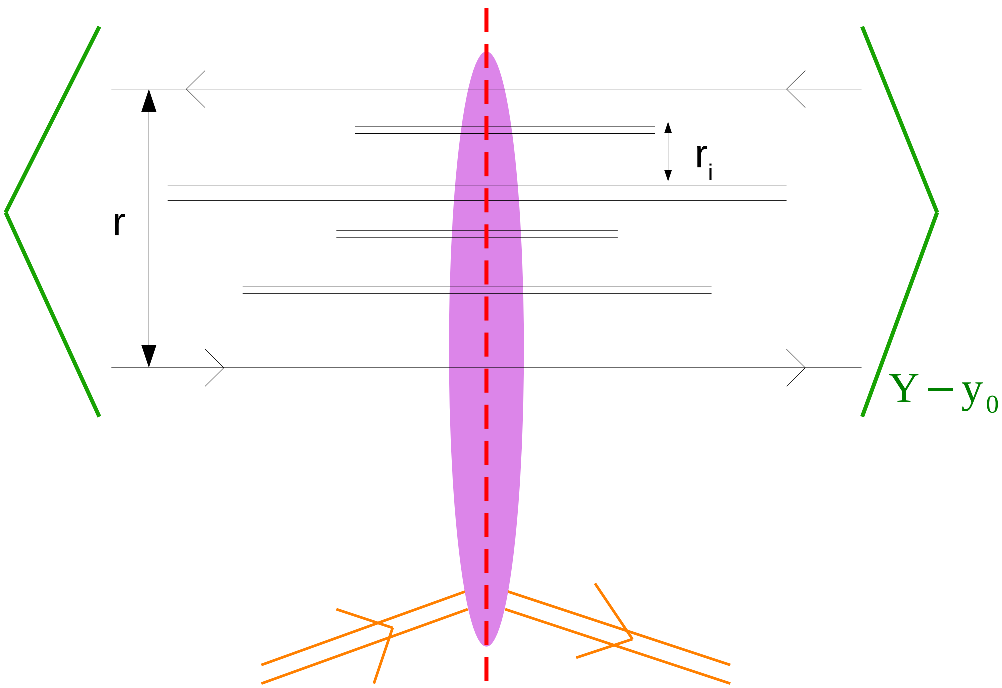
Probabilistic picture

Cross sections from S-matrix elements

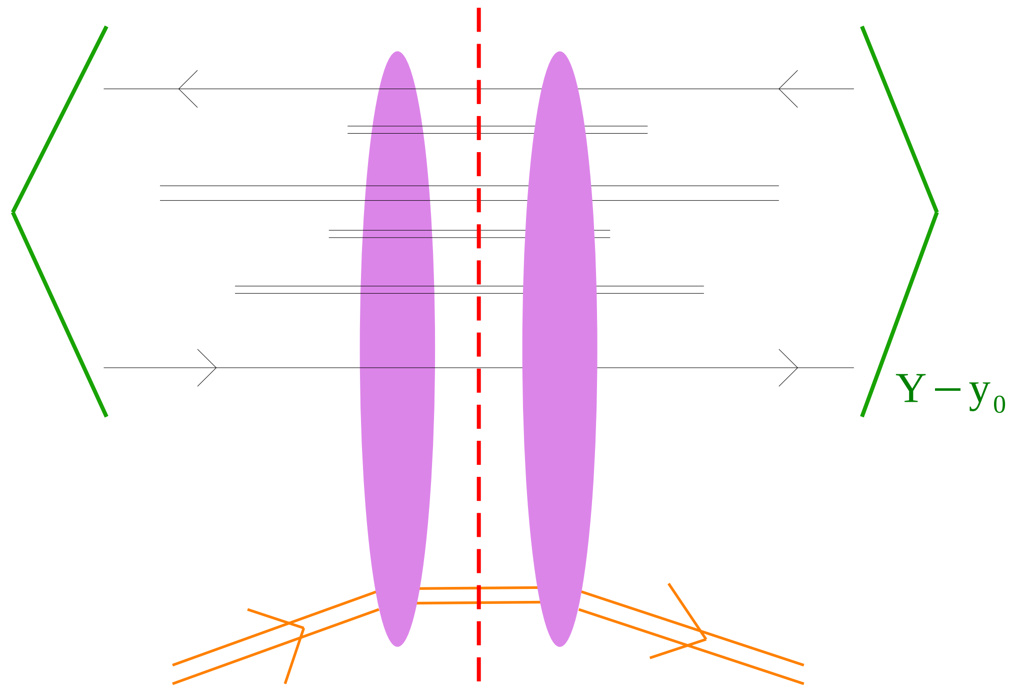
Onium evolved to rapidity Y-y_0 = collection of color dipoles

(we represent one realization of the modulus² of the wave function)

[Mueller 1993]



Nucleus at rapidity y_0



$$\sigma_{\rm tot}=2\left\langle 1-S(\{r_i\},y_0)\right\rangle_{Y-y_0}$$

Total cross section

$$\sigma_{\rm diff}=\left\langle [1-S(\{r_i\},y_0)]^2\right\rangle_{Y-y_0}$$

Diffractive dissociation cross section
conditioned to a minimum gap y_0

Probabilistic picture

S-matrix elements in the "y_0-frame"

$$S(\{r_i\}, y_0) = \prod_i S(r_i, y_0)$$

independence of scatterings

each factor solves the BK equation

introduce $x \equiv \ln[1/(r^2 Q_A^2)]$ and number density n(x)

$$= \prod_{x'} [S(x', y_0)]^{n(x')dx'} \underset{dx' \to 0}{=} e^{-\int dx' n(x') \ln[1/S(x', y_0)]}$$

assume $S(x', y_0)$ close to 1

$$\simeq e^{-\int dx' n(x')[1 - S(x', y_0)]} \equiv e^{-I}$$

I = "overlap" of dipole density and dipole scattering amplitude

Cross sections as probabilities

$$\sigma_{tot} = 2\langle 1 - S(\{r_i\}, y_0)\rangle_{Y-y_0}$$
$$= 2\langle 1 - e^{-I}\rangle_{Y-y_0}$$
$$= 2\langle (e^I - 1) e^{-I}\rangle_{Y-y_0}$$
$$= 2\sum_{k=1}^{\infty} \left\langle \frac{I^k}{k!} e^{-I} \right\rangle_{Y-y_0}$$

$$\sigma_{tot} \equiv 2\sum_{k=1}^{\infty} w_k$$

Interpretation:
probability that k dipoles effectively scatter

$$\sigma_{diff} = \langle [1 - S(\{r_i\}, y_0)]^2 \rangle_{Y-y_0}$$

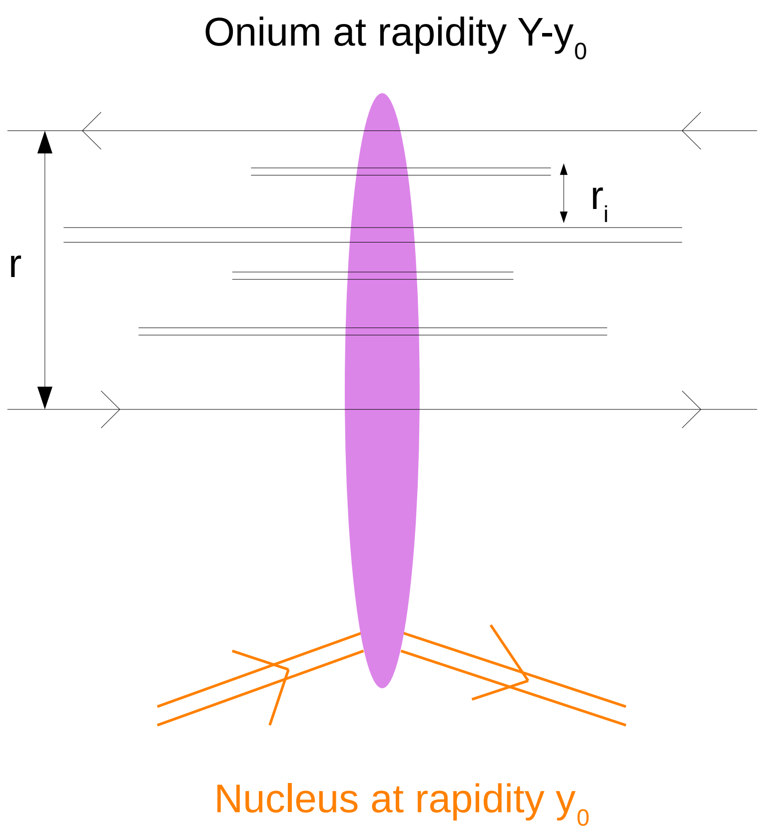
$$\sigma_{diff} = \sum_{k \ \text{even} \geq 2} w_k$$

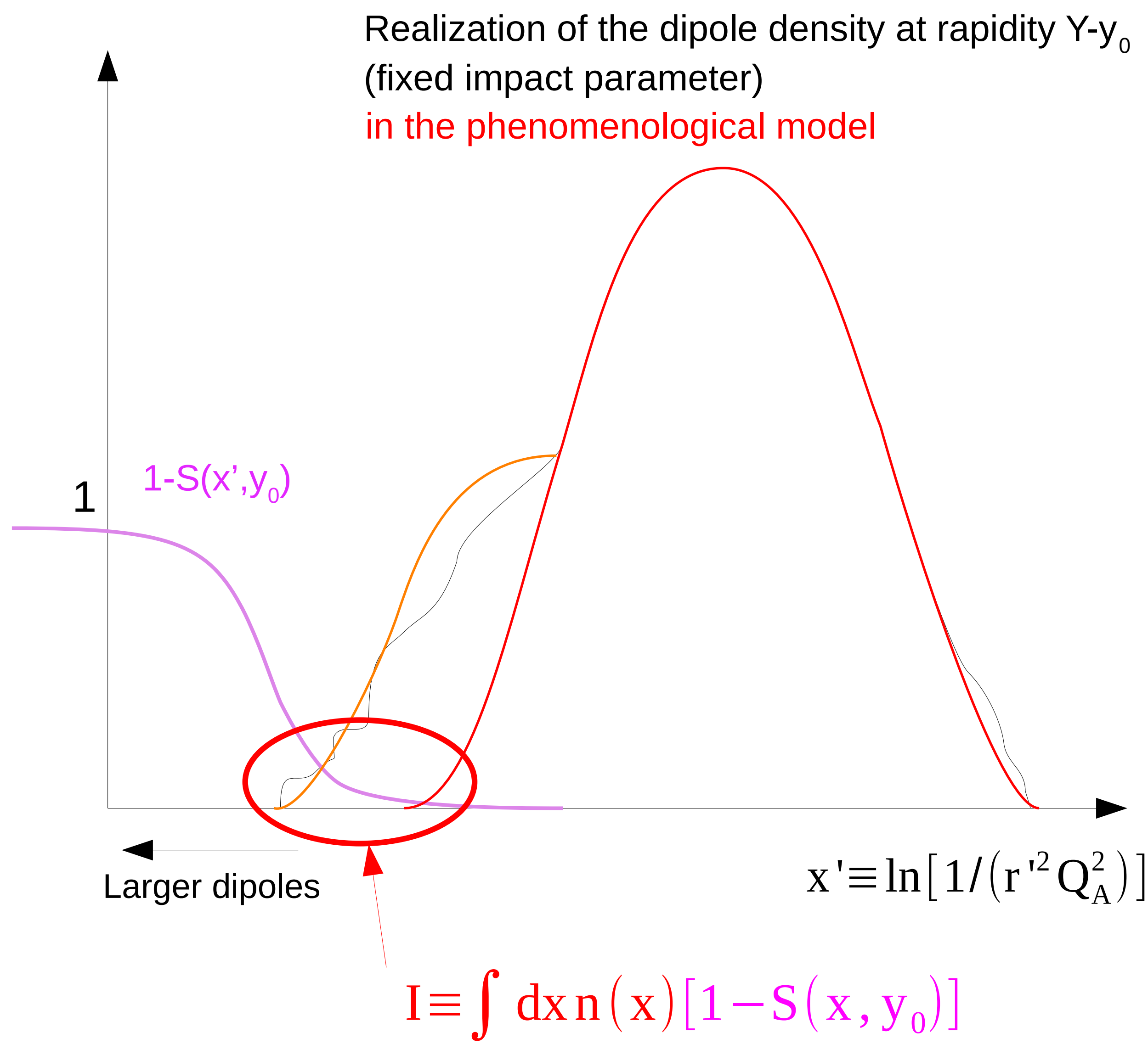
Diffractive cross section
=
Probability of an even number of participating dipoles

Need to understand the scattering configurations of onia!

Nuclear scattering configurations of onia

“y_0-frame”





Phenomenological model for dipole density = “mean-field” evolution + 1 single fluctuation

Nuclear scattering configurations of onia

Phenomenological model

[Mueller, SM, 2014
Le, Mueller, SM, 2020]

The "mean-field" dipole density is obtained by solving the BFKL equation for the mean dipole density in a dipole of initial size r, with an absorptive boundary effectively restoring the discreteness of the dipoles

$$n(r') = \text{const}_1 \times \ln\frac{R(Y-y_0)}{r'} \times \left(\frac{R(Y-y_0)}{r'}\right)^{2\gamma_0} e^{-\ln^2[r'^2/[R(Y-y_0)]^2]/[2\chi''(\gamma_0)\bar{\alpha}(Y-y_0)]}$$

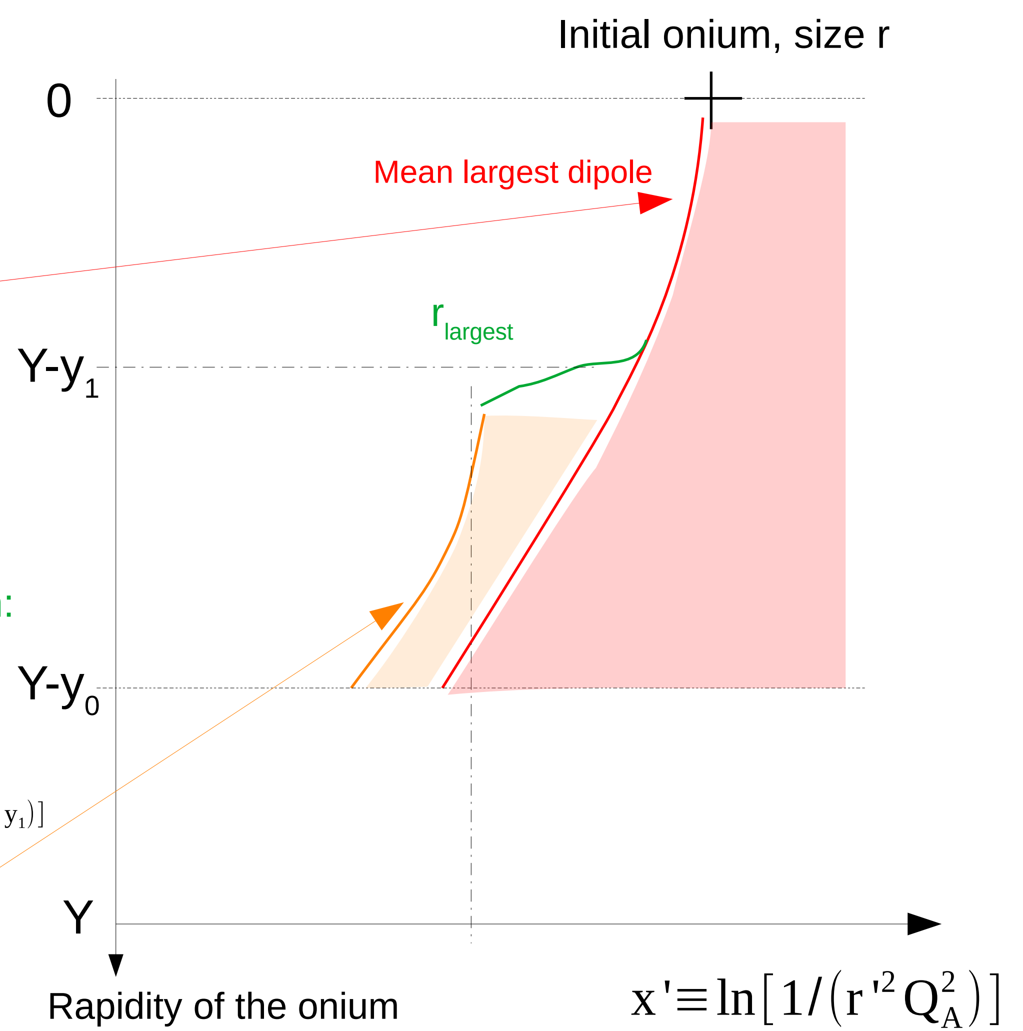
$$[R(\tilde{y})]^2 = r^2 e^{\bar{\alpha}\chi'(\gamma_0)\tilde{y} - \frac{3}{2\gamma_0}\ln\tilde{y}}$$

$$[\chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma),\quad \gamma_0 = 0.63\ldots]$$

The single fluctuation is a very large dipole produced at some rapidity Y-y_1.
One sets its probability distribution to be that of the largest dipole at y_1, which solves exactly the BK equation:

$$\text{proba}(r_{\text{largest}}) = \text{const}_2 \times \ln\frac{r_{\text{largest}}}{R(Y-y_1)} \left(\frac{R(Y-y_1)}{r_{\text{largest}}}\right)^{2\gamma_0} e^{-\ln^2[r_{\text{largest}}^2/[R(Y-y_1)]^2][2\chi''(\gamma_0)\bar{\alpha}(Y-y_1)]}$$

The dipole further evolves deterministically



Analytical asymptotics for the gap distribution

from the phenomenological model, in the "scaling region"

[Le, Mueller, SM, 2021]

$$w_k = \left\langle \frac{I^k}{k!} e^{-I} \right\rangle_{Y-y_0} \qquad \sigma_{\text{tot}} = 2\sum_{k=1}^{\infty} w_k \qquad \sigma_{\text{diff}} = \sum_{k\ \text{even}\geq 2}^{\infty} w_k$$

Probabilistic weights of the number k of participating dipoles

(= number of exchanged "Pomerons" between the onium and the nucleus):

$$w_1 = \text{const} \times 2 \ln \frac{1}{r Q_s(Y)} [r Q_s(Y)]^{2\gamma_0}$$

$$w_k \underset{Y\to\infty}{=} \frac{\text{const}}{\gamma_0} \frac{1}{k(k-1)} \left(1 + \sqrt{\frac{2}{\pi \chi''(\gamma_0)}} \frac{1}{\sqrt{\bar{\alpha} y_0}} \ln \frac{1}{r^2 Q_s^2(Y)} \right) [r Q_s(Y)]^{2\gamma_0}$$

$$Q_s^2(y) = Q_A^2 e^{\bar{\alpha}\chi'(\gamma_0) y - \frac{3}{2\gamma_0} \ln y}$$

$$1 \ll \ln[r Q_s(Y)] \leq \sqrt{\bar{\alpha} Y}$$

Interesting feature: w_k decays slowly with k:
Many-exchange events are typical!

$$\frac{w_k}{w_2} = \frac{2}{k(k-1)}$$

[see also Salam; Mueller, Salam, 1995-1996]

Gap distribution: *NB: derivative of w_k with respect to y_0 requires a careful calculation (to keep finite Y terms)*

$$\left| \frac{1}{\sigma_{\text{tot}}} \frac{d\sigma_{\text{diff}}}{dy_0} \right| = \frac{\sum_{k\ \text{even}} \partial w_k / \partial y_0}{2\left(w_1 + \sum_{k\geq 2} w_k\right)} = \frac{\ln 2}{\gamma_0 \sqrt{2\pi\chi''(\gamma_0)}} \frac{1}{\sqrt{\bar{\alpha}}} \left[\frac{Y}{y_0(Y-y_0)} \right]^{3/2} \exp\left(-\frac{\ln^2[r^2 Q_s^2(Y)]}{2\chi''(\gamma_0)\bar{\alpha}(Y-y_0)} \right)$$

NEW

[Mueller, SM, 2018;
see also Contreras, Levin, Meneses, Potashnikova, 2018]

Summary

- We have predicted the rapidity gap distribution in diffractive dissociation of a virtual photon off a nucleus, ***including the overall constant***:

$$\left|\frac{1}{\sigma_{tot}}\frac{d\sigma_{diff}}{dy_0}\right| = \frac{\ln 2}{\gamma_0\sqrt{2\pi\chi''(\gamma_0)}}\frac{1}{\sqrt{\bar{\alpha}}}\left[\frac{Y}{y_0(Y-y_0)}\right]^{3/2}\exp\left(-\frac{\ln^2[r^2 Q_s^2(Y)]}{2\chi''(\gamma_0)\bar{\alpha}(Y-y_0)}\right)$$

NB: We have successful numerical checks of this formula!

- Partonic interpretation: the rapidity gap is due to a **large fluctuation** (unusually low-transverse momentum gluon) in the course of the QCD evolution of the onium Fock state

  Once this fluctuation has occurred, typically many exchanges occur!

Outlook

- Phenomenology: DIS at EIC/LHeC, convolute with virtual photon wave function; study phenomenologically subasymptotic corrections (finite-Y, running coupling etc...) . **Detailed study: *see Anh Dung Le's talk!***
- Analytical study of the running coupling
- Aim at a systematic calculation of finite-Y corrections
  → *requires to understand better general branching processes!*