

**DIJET PRODUCTION AT EIC
AND INTERPLAY OF SUDAKOV AND SATURATION EFFECTS
IN WEIZSACKER-WILLIAMS TMD GLUON DISTRIBUTION**

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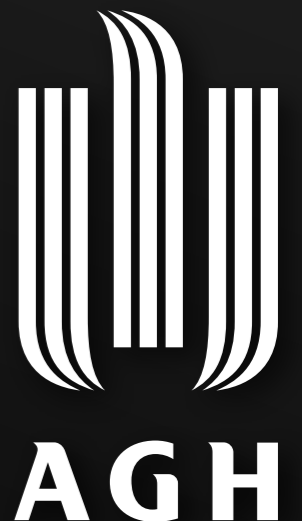
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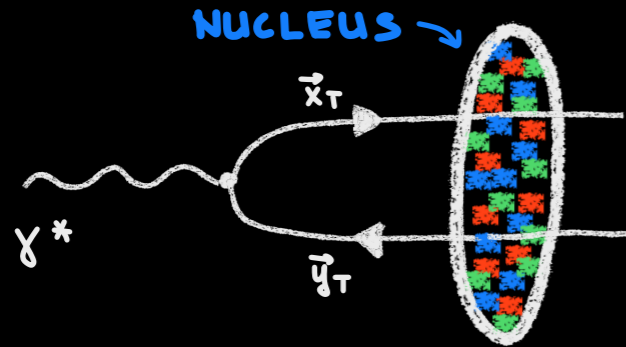
NCN GRANT DEC-2018/31/D/ST2/02731



INTRODUCTION

Jet/hadron production in γA collisions at small x

Dipole scattering off a color field of nucleus within the Color Glass Condensate (CGC)



[F. Dominguez, C. Marquet, B-W. Xiao, F. Yuan, 2011]

$$\frac{d\sigma_{\gamma A \rightarrow 2j}}{d^3p_1 d^3p_2} \sim \int \frac{d^2x}{(2\pi)^2} \frac{d^2x'}{(2\pi)^2} \frac{d^2y}{(2\pi)^2} \frac{d^2y'}{(2\pi)^2} e^{-i\vec{p}_{T1} \cdot (\vec{x}_T - \vec{x}'_T)} e^{-i\vec{p}_{T2} \cdot (\vec{y}_T - \vec{y}'_T)} \times \psi_z^* (\vec{x}'_T - \vec{y}'_T) \psi_z (\vec{x}_T - \vec{y}_T) \times \left\{ 1 + S_x^{(4)} (\vec{x}_T, \vec{y}_T; \vec{y}'_T, \vec{x}'_T) - S_x^{(2)} (\vec{x}_T, \vec{y}_T) - S_x^{(2)} (\vec{y}'_T, \vec{x}'_T) \right\}$$

$$U(\vec{x}_T) = \mathcal{P} \exp \left\{ ig \int_{-\infty}^{+\infty} dx^+ A_a^- (x^+, \vec{x}_T) t^a \right\}$$

$$S_x^{(2)} \sim \langle \text{Tr} U(\vec{y}_T) U^\dagger(\vec{x}_T) \rangle_x$$

(dipole)



dipole TMD gluon distribution

$$\mathcal{F}_{qg}^{(1)}(x, k_T)$$

$$S_x^{(4)} \sim \langle \text{Tr} U(\vec{x}_T) U^\dagger(\vec{x}'_T) U(\vec{y}'_T) U^\dagger(\vec{y}_T) \rangle_x$$

(quadrupole)



Weizsäcker-Williams TMD gluon distribution

$$\mathcal{F}_{gg}^{(3)}(x, k_T)$$

accessible in
DIJET PRODUCTION

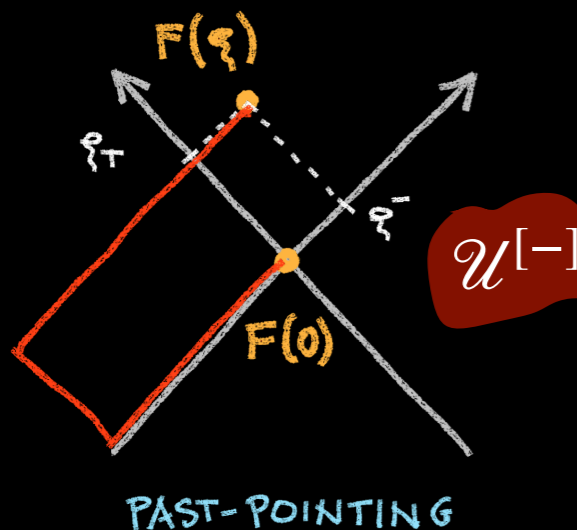
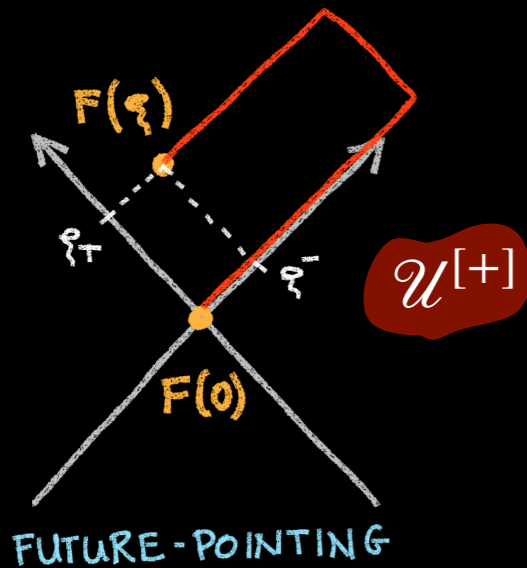
Generic operator definition

$$\mathcal{F}_g(x, k_T) = 2 \int \frac{d\xi^+ d^2\xi_T}{(2\pi)^3 P^-} e^{ixP^- \xi^+ - i\vec{k}_T \cdot \vec{\xi}_T} \langle P | \text{Tr} \left[\hat{F}^{i-}(\xi^+, \vec{\xi}_T, \xi^- = 0) \mathcal{U}_{C_1} \hat{F}^{i-}(0) \mathcal{U}_{C_2} \right] | P \rangle$$

GLUON FIELD
 $\hat{F} = F_a t^a$

GAUGE LINKS
 in fundamental color representation

(unpolarized)



Gauge links $\mathcal{U}_{C_1}, \mathcal{U}_{C_2}$ depend on the color structure of the hard process. They are built from two basic Wilson lines:

$$\begin{aligned} \mathcal{U}^{[\pm]} = & [0, (\pm\infty, \vec{0}_T, 0)] \\ & \times [(\pm\infty, \vec{0}_T, 0), (\pm\infty, \vec{\xi}_T, 0)] \\ & \times [(\pm\infty, \vec{\xi}_T, 0), (\xi^+, \vec{\xi}_T, 0)] \end{aligned}$$

[C. Bomhof, P. Mulders, F. Pijlman, 2004]

$$[x, y] = \mathcal{P} \exp \left\{ ig \int_{\overline{xy}} dz_\mu A_a^\mu(z) t^a \right\}$$

STRAIGHT LINE SEGMENT

Light-cone basis:

$$v^\pm = v^\mu n_\mu^\pm, \quad n^\pm = (1, 0, 0, \mp 1)$$

$$v^\mu = \frac{1}{2} v^+ n^- + \frac{1}{2} v^- n^+ + v_T^\mu$$

All possible operators

$$\mathcal{F}_{qg}^{(1)} \sim \langle P | \text{Tr} \left[\hat{F}^{i-}(\xi) \mathcal{U}^{[-]\dagger} \hat{F}^{i-}(0) \mathcal{U}^{[+]} \right] | P \rangle \quad (\text{DIPOLE})$$

$$\mathcal{F}_{qg}^{(2)} \sim \langle P | \frac{\text{Tr} \mathcal{U}^{[\square]}}{N_c} \text{Tr} \left[\hat{F}^{i-}(\xi) \mathcal{U}^{[+]\dagger} \hat{F}^{i-}(0) \mathcal{U}^{[+]} \right] | P \rangle$$

$$\mathcal{F}_{qg}^{(3)} \sim \langle P | \text{Tr} \left[\hat{F}^{i-}(\xi) \mathcal{U}^{[+]\dagger} \hat{F}^{i-}(0) \mathcal{U}^{[\square]} \mathcal{U}^{[+]} \right] | P \rangle$$

$$\mathcal{F}_{gg}^{(1)} \sim \langle P | \frac{\text{Tr} \mathcal{U}^{[\square]\dagger}}{N_c} \text{Tr} \left[\hat{F}^{i-}(\xi) \mathcal{U}^{[-]\dagger} \hat{F}^{i-}(0) \mathcal{U}^{[+]} \right] | P \rangle$$

$$\mathcal{F}_{gg}^{(2)} \sim \langle P | \text{Tr} \left[\hat{F}^{i-}(\xi) \mathcal{U}^{[\square]\dagger} \right] \text{Tr} \left[\hat{F}^{i-}(0) \mathcal{U}^{[\square]} \right] | P \rangle$$

$$\mathcal{F}_{gg}^{(3)} \sim \langle P | \text{Tr} \left[\hat{F}^{i-}(\xi) \mathcal{U}^{[+]\dagger} \hat{F}^{i-}(0) \mathcal{U}^{[+]} \right] | P \rangle \quad (\text{WEIZSACKER-WILLIAMS})$$

$$\mathcal{F}_{gg}^{(4)} \sim \langle P | \text{Tr} \left[\hat{F}^{i-}(\xi) \mathcal{U}^{[-]\dagger} \hat{F}^{i-}(0) \mathcal{U}^{[-]} \right] | P \rangle$$

$$\mathcal{F}_{gg}^{(5)} \sim \langle P | \text{Tr} \left[\hat{F}^{i-}(\xi) \mathcal{U}^{[\square]\dagger} \mathcal{U}^{[+]\dagger} \hat{F}^{i-}(0) \mathcal{U}^{[\square]} \mathcal{U}^{[+]} \right] | P \rangle$$

$$\mathcal{F}_{gg}^{(6)} \sim \langle P | \frac{\text{Tr} \mathcal{U}^{[\square]}}{N_c} \frac{\text{Tr} \mathcal{U}^{[\square]\dagger}}{N_c} \text{Tr} \left[\hat{F}^{i-}(\xi) \mathcal{U}^{[+]\dagger} \hat{F}^{i-}(0) \mathcal{U}^{[+]} \right] | P \rangle$$

$$\mathcal{F}_{gg}^{(7)} \sim \langle P | \frac{\text{Tr} \mathcal{U}^{[\square]}}{N_c} \text{Tr} \left[\hat{F}^{i-}(\xi) \mathcal{U}^{[\square]\dagger} \mathcal{U}^{[+]\dagger} \hat{F}^{i-}(0) \mathcal{U}^{[+]} \right] | P \rangle$$

WILSON LOOP $\rightarrow \mathcal{U}^{[\square]} = \mathcal{U}^{[+]} \mathcal{U}^{[-]\dagger}$

(DIPOLE)

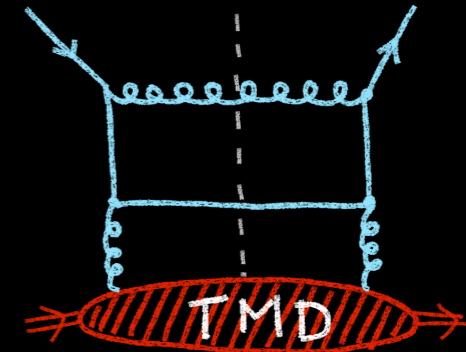
↑
in the
small-x
limit



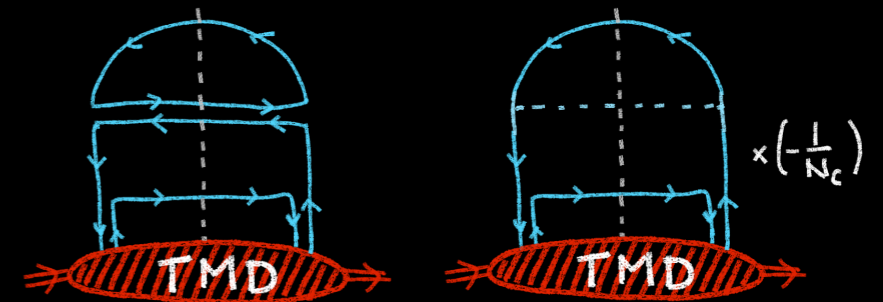
(WEIZSACKER-WILLIAMS)

Example

TMD gluon distribution for the following process:



Two independent color flows:



$$\rightsquigarrow \frac{N_c}{2C_F} \mathcal{F}_{qg}^{(2)} - \frac{1}{2N_c C_F} \mathcal{F}_{qg}^{(1)}$$

Gluon TMD for any multiparticle process is given by a linear combination of these "basis" TMDs.

FRAMEWORK

Small-x improved TMD Factorization (ITMD*)

Factorization for $P_T \gg Q_s$

average P_T of jets \uparrow
saturation scale \uparrow

- resummation of kinematic twists
- only leading genuine twist (no hard MPIs)
- no linearly polarized gluons (assume $Q^2/P_T^2 \ll 1$)

[PK, K. Kutak, C. Marquet, E. Petreska, S. Sapeta, A. van Hameren, 2015]
[A. van Hameren, PK, K. Kutak, C. Marquet, E. Petreska, S. Sapeta, 2016]
[PK, K. Kutak, S. Sapeta, A. Stasto, M. Strikman, 2017]
[T. Altinoluk, R. Boussarie, PK, 2019]

see also:

[T. Altinoluk, R. Boussarie, 2019]
[H. Fujii, C. Marquet, K. Wanatabe, 2020]
[T. Altinoluk, C. Marquet, P. Taels, 2021]

$$d\sigma_{\gamma^*A \rightarrow 2j+X} \sim \int \frac{dx}{x} \int d^2k_T \mathcal{F}_{gg}^{(3)}(x, k_T, \mu) d\sigma_{\gamma^*g^* \rightarrow j_1j_2}(x, k_T, \mu)$$

Weizsacker-Williams TMD

with Sudakov resummation

off-shell matrix elements

Implemented in KaTie Monte Carlo
[A. van Hameren]

Related studies for dijet/dihadron at EIC

Back-to-back regime using MV model + Sudakov

[L. Zheng, E.C. Aschenauer, J.H. Lee, B-W. Xiao, 2014]

Full CGC calculations (no Sudakov)

[A. Dumitru, V. Skokov, 2018]

[H. Mantysaari, N. Mueller, F. Salazar, B. Schenke, 2019]

[F. Salazar, B. Schenke, 2020]

1. Data driven dipole TMD

Balitsky-Kovchegov type equation with kinematic constraint,
DGLAP correction and running coupling:

[J. Kwieciński, A. Martin, A. Stasto, 1997]

[K. Kutak, J. Kwieciński, 2003]

$$\mathcal{F}_{qg}^{(1)}(x, k_T^2) = \mathcal{F}_0(x, k_T^2) + \frac{\alpha_s N_c}{\pi} \int_x^1 \frac{dz}{z} \int_{k_{T0}^2}^{\infty} \frac{dq_T^2}{q_T^2} \left\{ \frac{q_T^2 \mathcal{F}\left(\frac{x}{z}, q_T^2\right) \theta\left(\frac{k_T^2}{z} - q_T^2\right) - k_T^2 \mathcal{F}\left(\frac{x}{z}, k_T^2\right)}{|q_T^2 - k_T^2|} + \frac{k_T^2 \mathcal{F}\left(\frac{x}{z}, k_T^2\right)}{\sqrt{4q_T^4 + k_T^4}} \right\}$$

$$+ \frac{\alpha_s}{2\pi k_T^2} \int_x^1 dz \left\{ \left(P_{gg}(z) - \frac{2N_c}{z} \right) \int_{k_{T0}^2}^{k_T^2} dq_T^2 \mathcal{F}\left(\frac{x}{z}, q_T^2\right) + z P_{gq}(z) \Sigma\left(\frac{x}{z}, k_T^2\right) \right\}$$

$$- \frac{2\alpha_s^2}{R^2} \left\{ \left[\int_{k_T^2}^{\infty} \frac{dq_T^2}{q_T^2} \mathcal{F}(x, q_T^2) \right]^2 + \mathcal{F}(x, k_T^2) \int_{k_T^2}^{\infty} \frac{dq_T^2}{q_T^2} \ln\left(\frac{q_T^2}{k_T^2}\right) \mathcal{F}(x, q_T^2) \right\}$$

fitted to
DIS HERA
data

[K. Kutak, S. Sapeta, 2012]

for NUCLEUS: $R_A = A^{1/3} R_p$ ↑

2. Weizsacker-Williams TMD with Sudakov resummation

At large N_c limit and Gaussian approximation:

[A. Mueller, B-W. Xiao, F. Yuan, 2013]

$$\mathcal{F}_{gg}^{(3)}(x, k_T, \mu) = \frac{C_F}{2\pi^4 \alpha_s} \int d^2b \int \frac{d^2r}{r_T^2} e^{-i\vec{k}_T \cdot \vec{r}_T} [1 - S_F^2(x, r_T)] e^{\mathbf{S}(\mu, r_T)}$$

$$S_F(x, r) = \frac{2\pi^2 \alpha_s}{N_c S_{\perp}} \int \frac{d^2k_T}{k_T^2} e^{i\vec{k}_T \cdot \vec{r}_T} \mathcal{F}_{qg}^{(1)}(x, k_T)$$

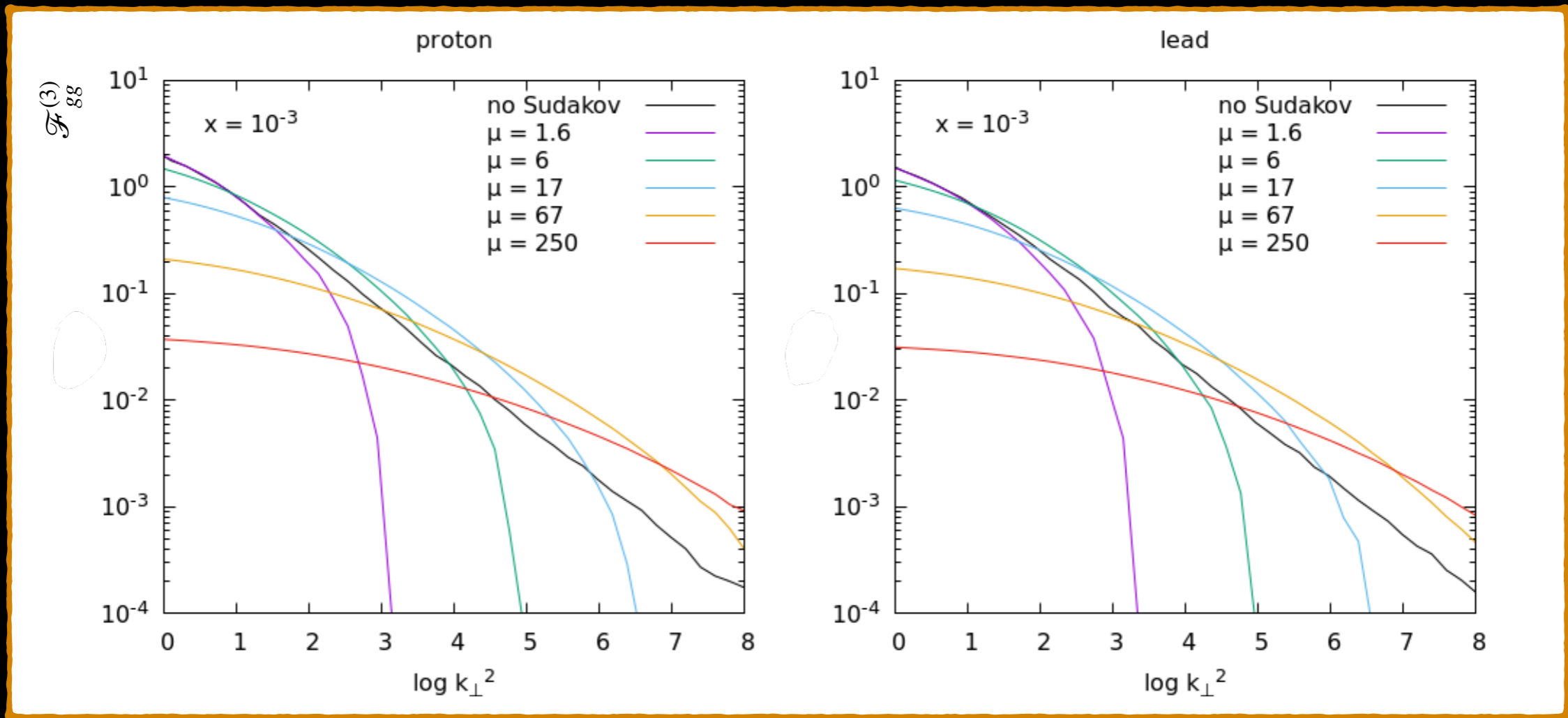
dipole

Sudakov
factor

$$\mathbf{S}(\mu, r_T) = -\frac{\alpha_s N_c}{4\pi} \ln^2(A\mu^2 r_T^2)$$

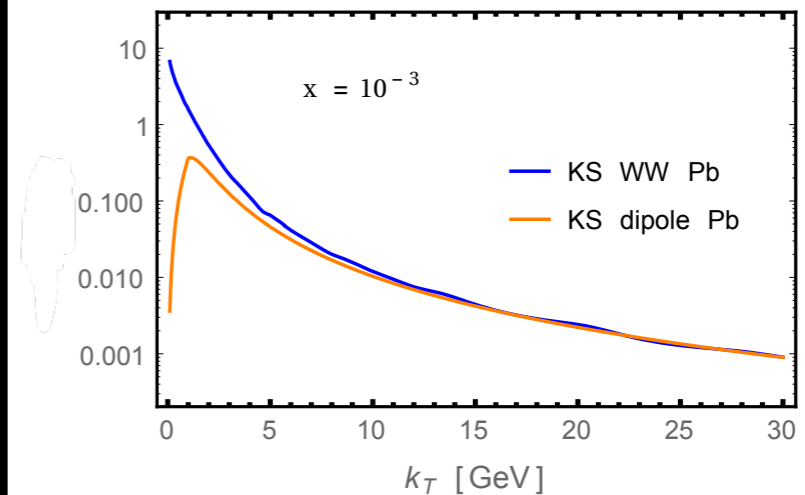
RESULTS

The Weizsacker-Williams TMD with Sudakov resummation



Weizsacker-Williams vs dipole gluon TMD distribution

different behaviour
at small k_T
(convergence at large k_T)



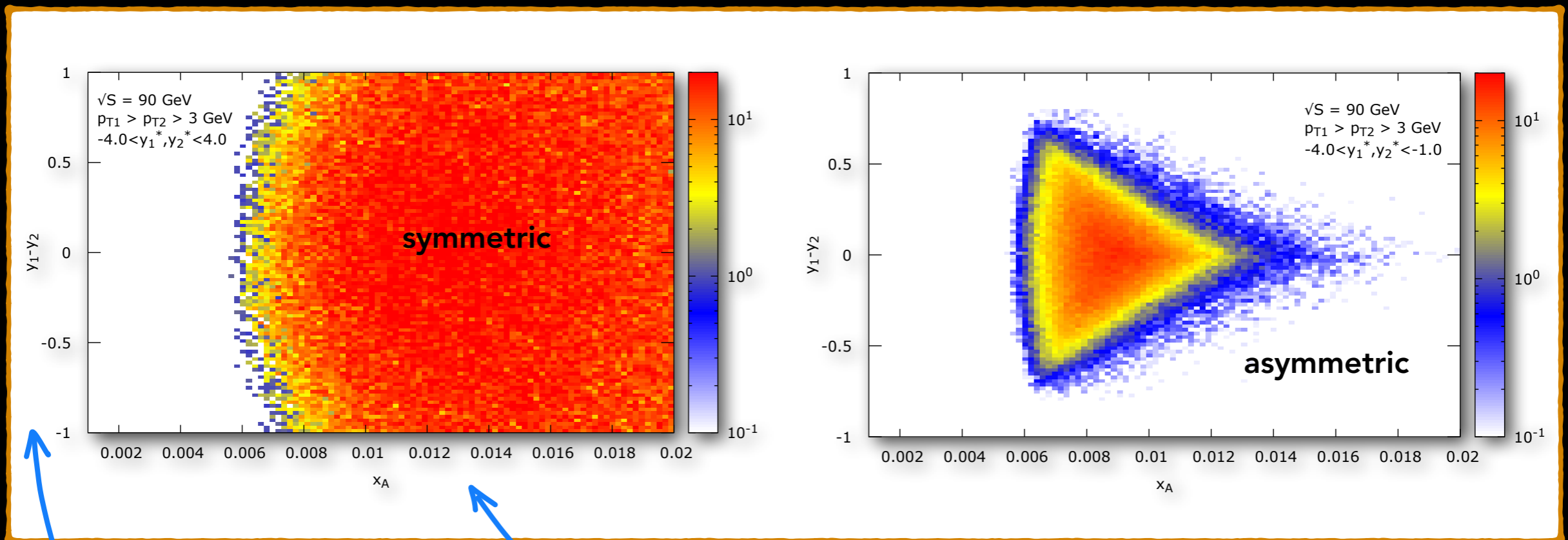
RESULTS

Kinematic setup

Cuts

- CM energy: $\sqrt{s} = 90 \text{ GeV}$ (for e-p and e-Pb)
- inelasticity: $0.1 < \nu < 0.85$
- virtuality: $Q^2 > 1 \text{ GeV}^2$
- jet transverse momenta: $p_{T1} > p_{T2} > 3 \text{ GeV}$ (Breit frame)
- jet radius: $\Delta R > 1$ (Breit frame)
- symmetric rapidity: $-4 < y_1^*, y_2^* < 4$ (CM frame)
- asymmetric rapidity: $-4 < y_1^*, y_2^* < -1$

Symmetric vs asymmetric rapidity window



rapidity difference

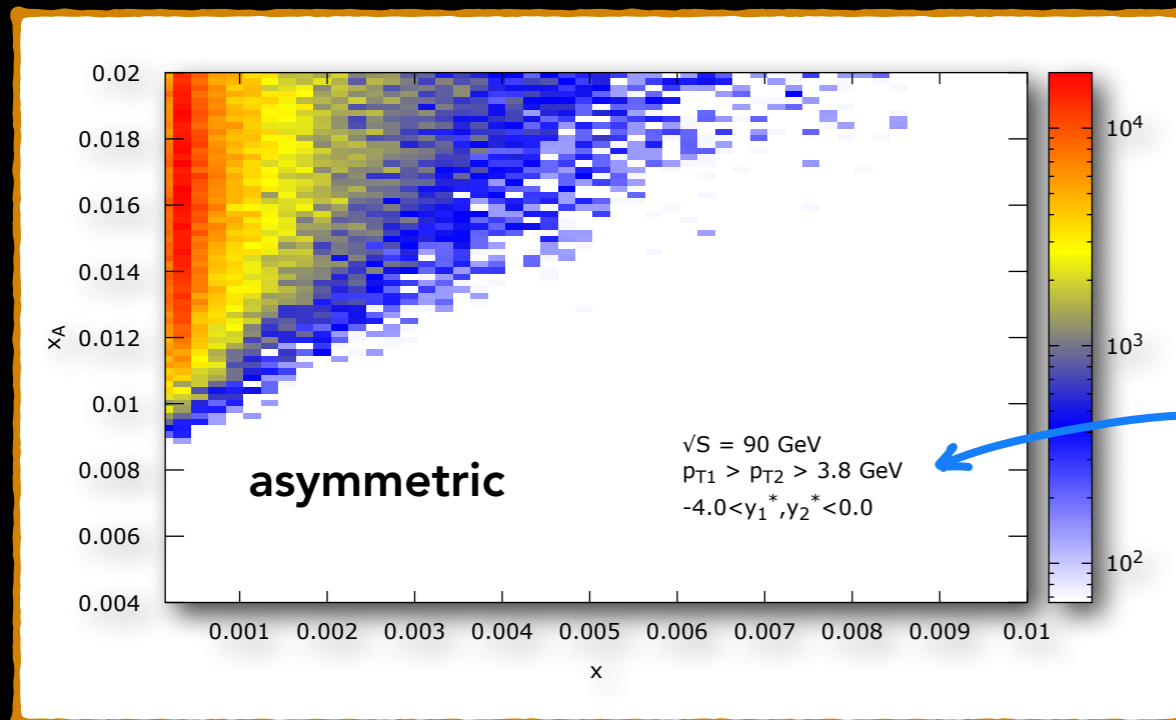
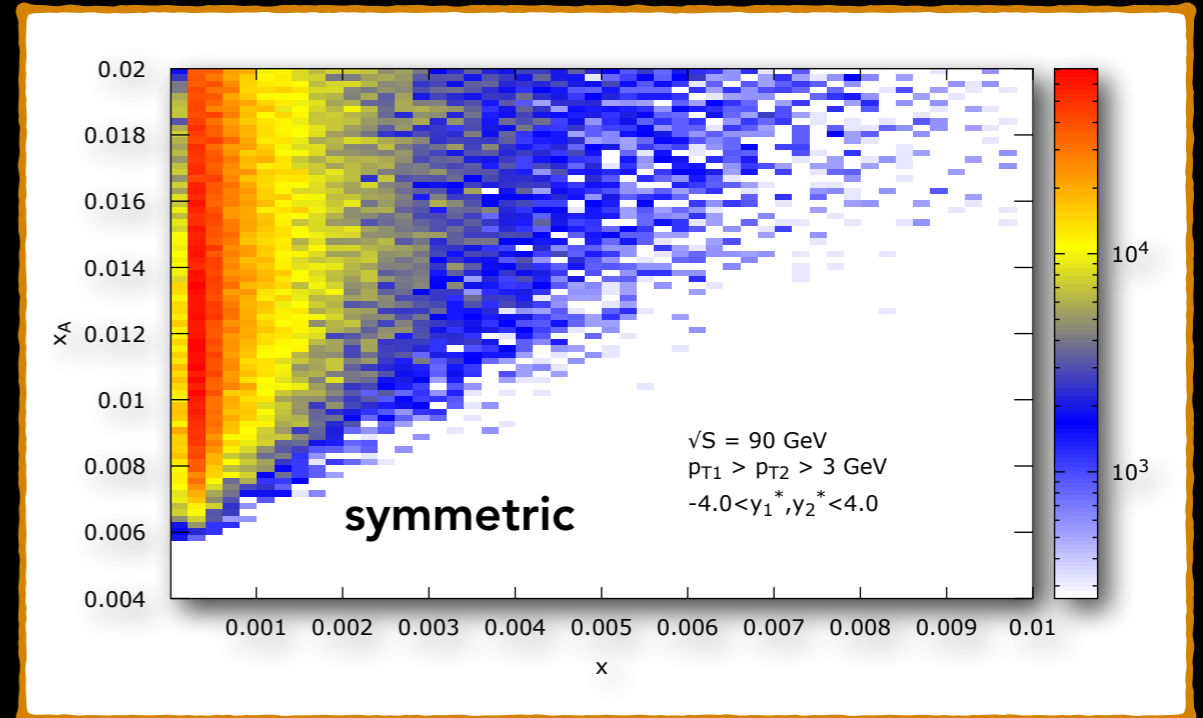
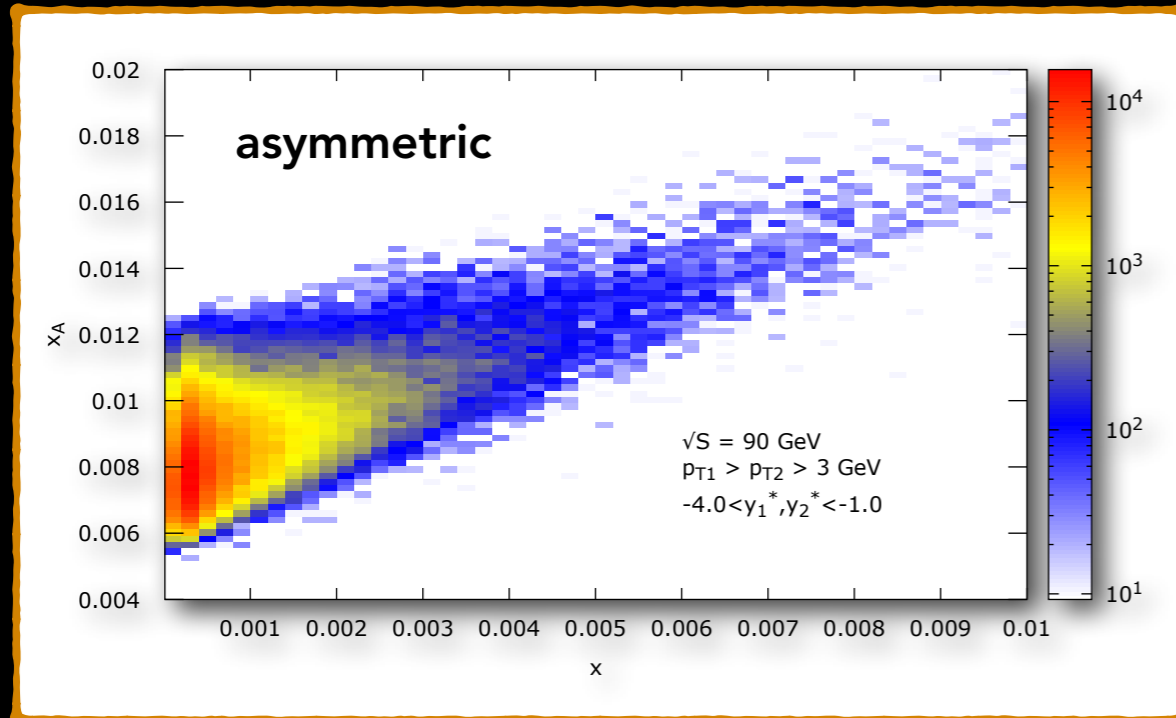
x of gluon
 $x_A \neq x_{Bj}$

good focus on
small x_A

RESULTS

Kinematic setup

Bjorken x vs Gluon x_A



larger p_T
cut

for asymmetric
window

$$x_{Bj} \sim 10^{-4}$$

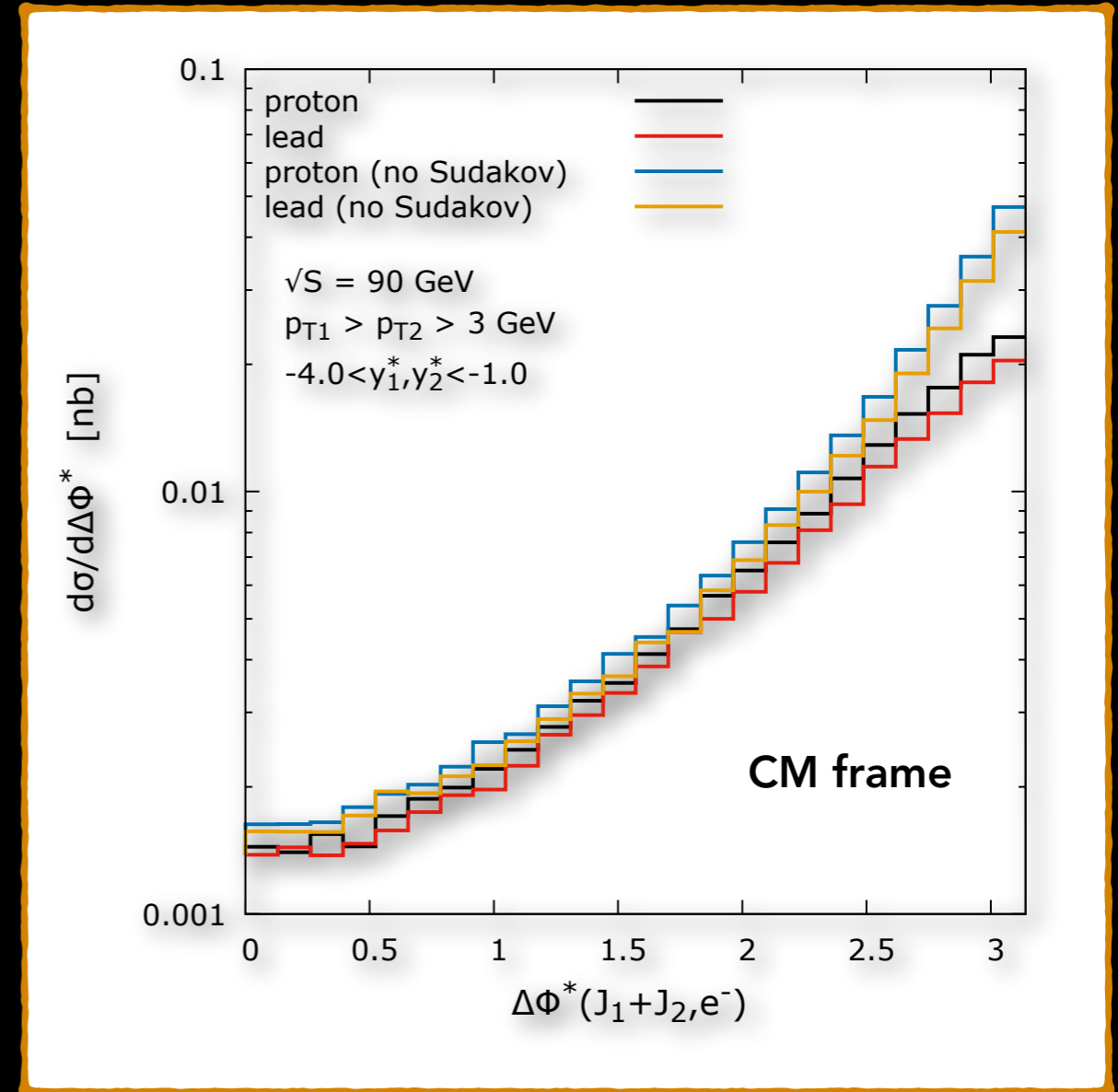
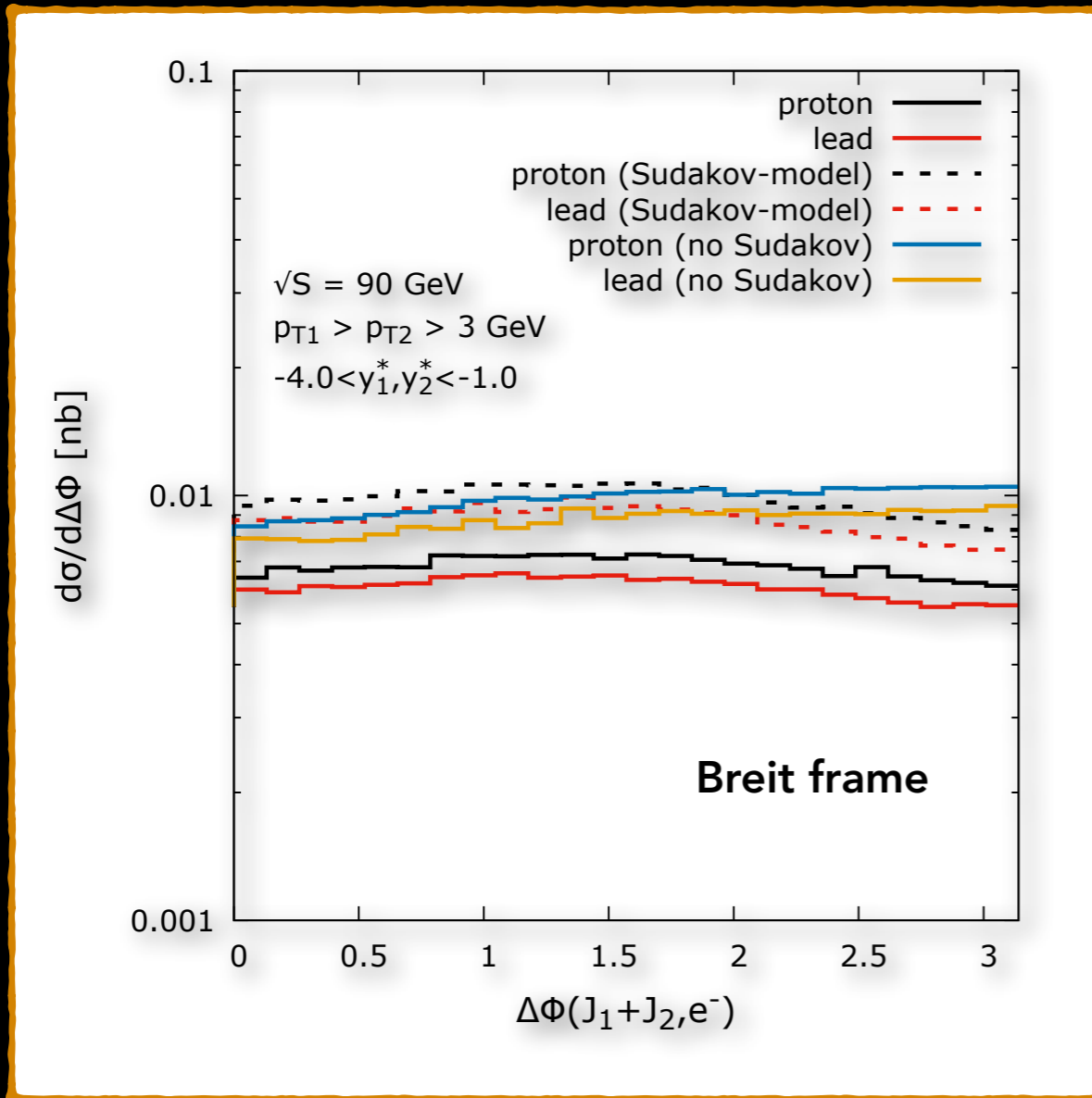


$$x_A \sim 6 \times 10^{-3}$$

RESULTS

Azimuthal correlations

Azimuthal angle between jet plane and electron $\Delta\Phi(J_1 + J_2, e^-)$



*Sudakov-model: resummation done by reweighting MC generated events via the probability of survival given by the Sudakov factor.

[A. van Hameren, PK, K. Kutak, S. Sapeta, 2014]

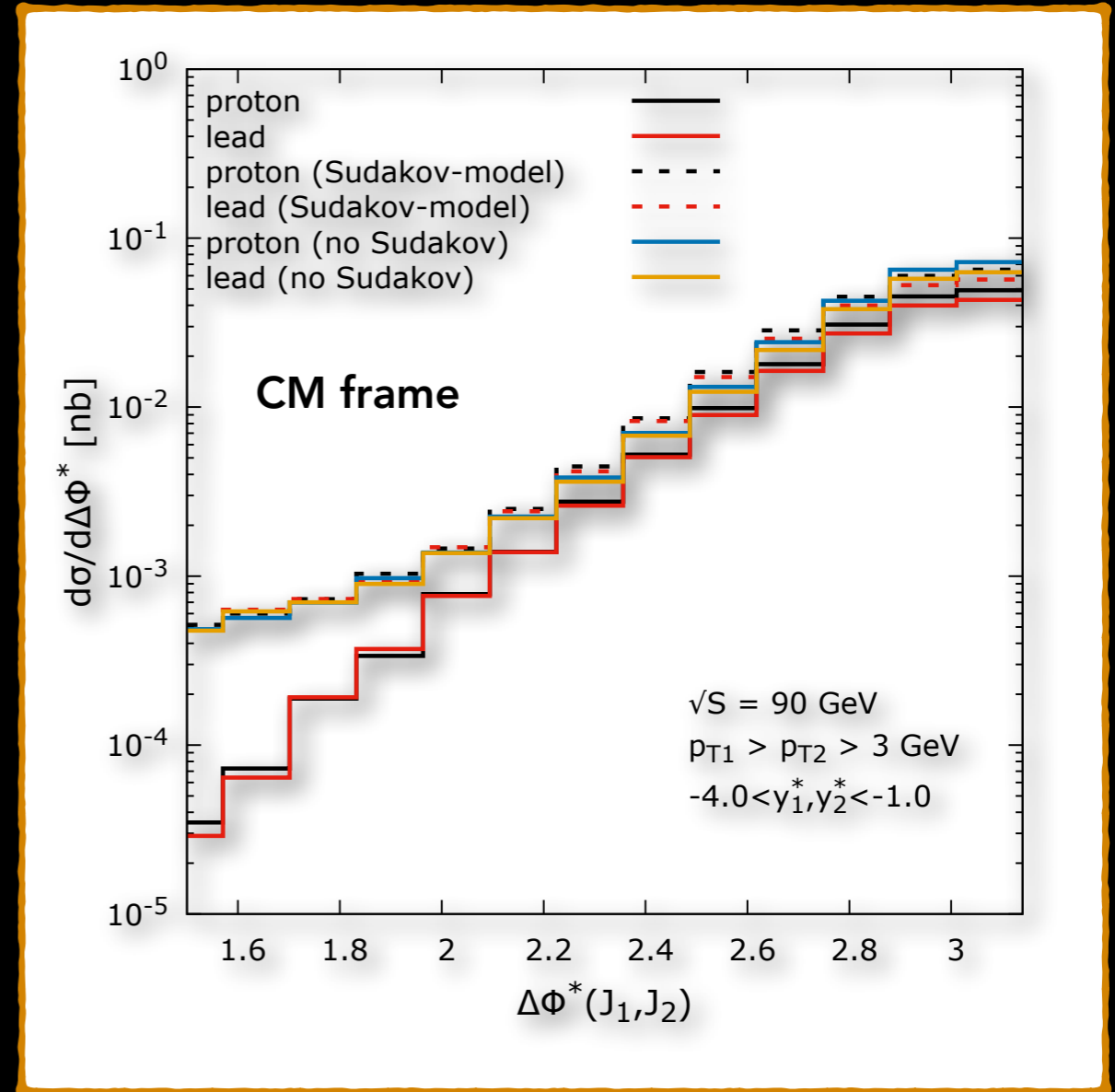
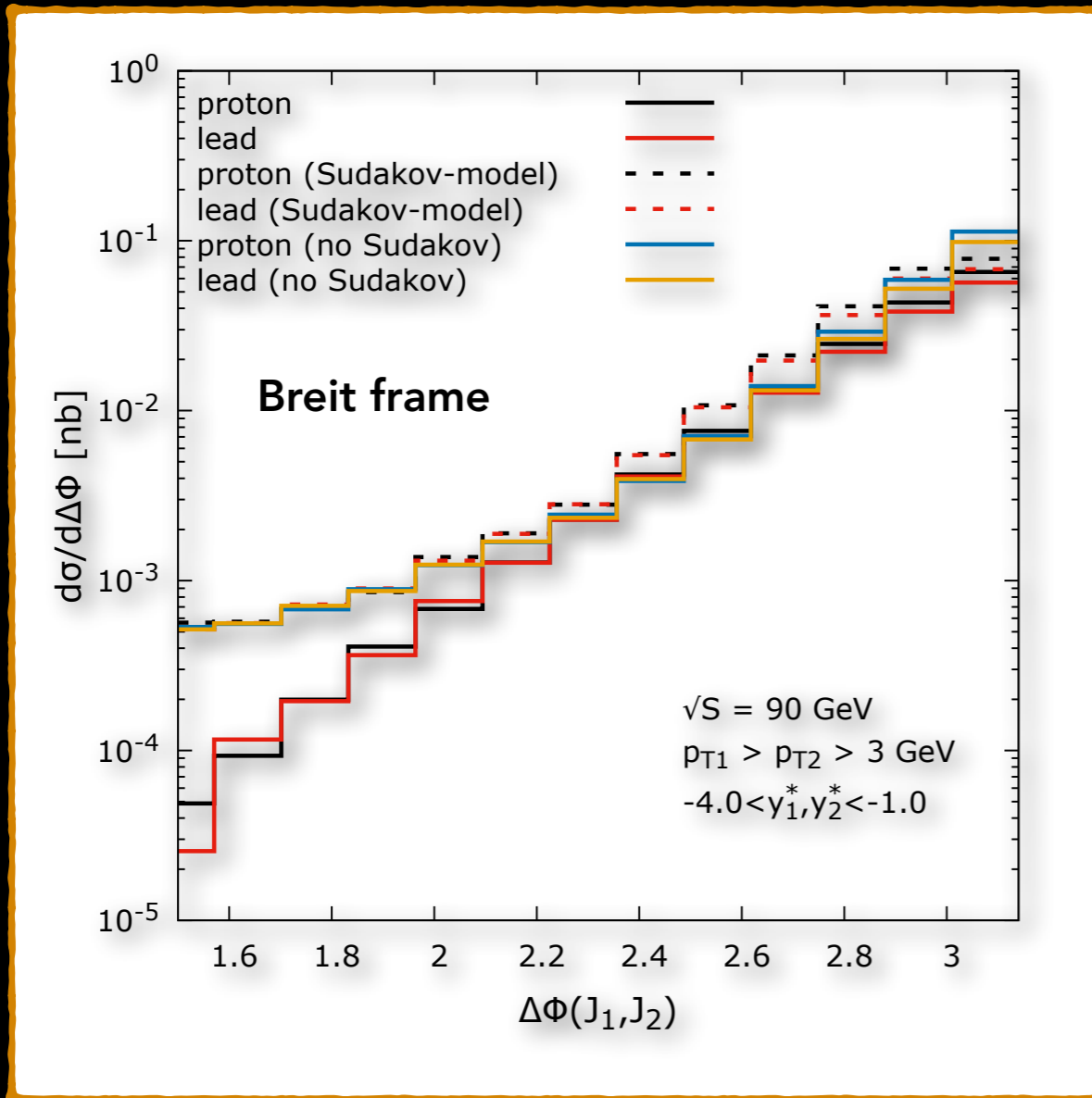
* Large Sudakov effects.

* Saturation effects: up to 15% suppression.

RESULTS

Azimuthal correlations

Azimuthal angle between the jets $\Delta\Phi(J_1, J_2)$



* Very steep distributions.
* Sudakov-model has little effect.

* Sudakov effects significant for smaller $\Delta\phi$.

SUMMARY

- New data-driven calculation for the Weizsacker-Williams TMD gluon distribution with the Sudakov resummation.
- New calculation of dijet production in the small- x region at EIC within ITMD* approach, in particular for azimuthal correlations:
 - between dijet plane and electron
 - between the jets
- Various rapidity windows and frames have been studied.
- Small p_T of jets seems to be required to access gluon $x < 10^{-3}$.
- Asymmetric rapidity window is important to focus on the small gluon x .
- Sudakov resummation changes results significantly.
- Saturation effects are up to 15% for $p_T > 3$ GeV.
- Next natural step: full NLO computation...

BACKUP

BACKUP

Limiting cases of CGC in dilute-dense collisions

CGC
dilute-dense

three scales:

$Q_s \gg \Lambda_{\text{QCD}}$ — saturation scale

k_T — jet transverse momentum imbalance

P_T — jet average transverse momentum

$$P_T \gg k_T \sim Q_s$$



TMD
GENERALIZED
FACTORIZATION

leading twist

[F. Dominguez, C. Marquet, B. Xiao, F. Yuan, 2011]
[C. Marquet, E. Petreska, C. Roiesnel, 2016]
[C. Marquet, C. Roiesnel, P. Taels, 2018]
[T. Altinoluk, R. Boussarie, C. Marquet, P. Taels, 2019]
[T. Altinoluk, R. Boussarie, C. Marquet, P. Taels, 2020]

$$P_T \sim k_T \gg Q_s$$



DILUTE

k_T -FACTORIZATION
BFKL dynamics

[S. Catani, M. Ciafaloni, F. Hautmann, 1991]
[M. Deak, F. Hautmann, H. Jung, K. Kutak, 2009]
[E. Iancu, J. Leidet, 2013]

$$P_T \gg Q_s$$



ITMD
"IMPROVED"
TMD factorization

all kinematic twists

[PK, K. Kutak, C. Marquet, E. Petreska, S. Sapeta, A. van Hameren, 2015]
[A. van Hameren, PK, K. Kutak, C. Marquet, E. Petreska, S. Sapeta, 2016]
[T. Altinoluk, R. Boussarie, PK, 2019]



Small-x limit of TMD gluon distributions

$$\int \frac{d\xi^+ d^2\xi_T}{(2\pi)^3 P^-} e^{ixP^- \xi^+ - i\vec{k}_T \cdot \vec{\xi}_T} \langle P | \text{Tr} \left[\hat{F}^{i-} \left(\xi^+, \vec{\xi}_T, \xi^- = 0 \right) \mathcal{U}_{C_1} \hat{F}^{i-} (0) \mathcal{U}_{C_2} \right] | P \rangle$$

$x \rightarrow 0$

Dependence on x is only via the small- x evolution equations:

- BFKL (Balitsky-Fadin-Kuraev-Lipatov).
- BK (Balitsky-Kovchegov) and modifications
- JIMWLK (Balitsky-Jalilian-Marian-Iancu-McLerran-Weigert-Leonidov-Kovner)

Correspondence to CGC

Example:

$$\mathcal{F}_{qg}^{(1)} \sim \int \frac{d^2x_T d^2y_T}{(2\pi)^4} k_T^2 e^{-i\vec{k}_T \cdot (\vec{x}_T - \vec{y}_T)} \langle \text{Tr} [U(\vec{x}_T) U^\dagger(\vec{y}_T)] \rangle_x$$

DIPOLE
GLUON DISTRIBUTION

WILSON LINES

AVERAGE OVER
CGC COLOR SOURCES

$$U(\vec{x}_T) = \mathcal{P} \exp \left\{ ig \int_{-\infty}^{+\infty} dx^+ A_a^-(x^+, \vec{x}_T) t^a \right\}$$

$$\langle \dots \rangle_x \rightarrow \frac{\langle \text{Tr} \dots \text{Tr} \rangle}{\langle \text{Tr} \mathbb{1} \rangle}$$

Intensively studied:

- [D. Kharzeev, Y. Kovchegov, K. Tuchin, 2003]
- [B. Xiao, F. Yuan, 2010]
- [F. Dominguez, C. Marquet, B. Xiao, F. Yuan, 2011]
- [A. Metz, J. Zhou, 2011]
- [E. Akcakaya, A. Schafer, J. Zhou, 2012]
- [C. Marquet, E. Petreska, C. Roiesnel, 2016]
- [I. Balitsky, A. Tarasov, 2015, 2016]
- [D. Boer, P. Mulders, J. Zhou, Y. Zhou, 2017]
- [C. Marquet, C. Roiesnel, P. Taels, 2018]
- [Y. Kovchegov, D. Pitonyak, M. Sievert, 2017, 2018]
- [T. Altinoluk, R. Boussarie, 2019]
- [R. Boussarie, Y. Mehtar-Tani, 2020]

BACKUP

Dijet correlations in pA collisions in ITMD

Measurement of dijet azimuthal correlations in p+p and p+Pb. [ATLAS, Phys. Rev. C100 (2019)]

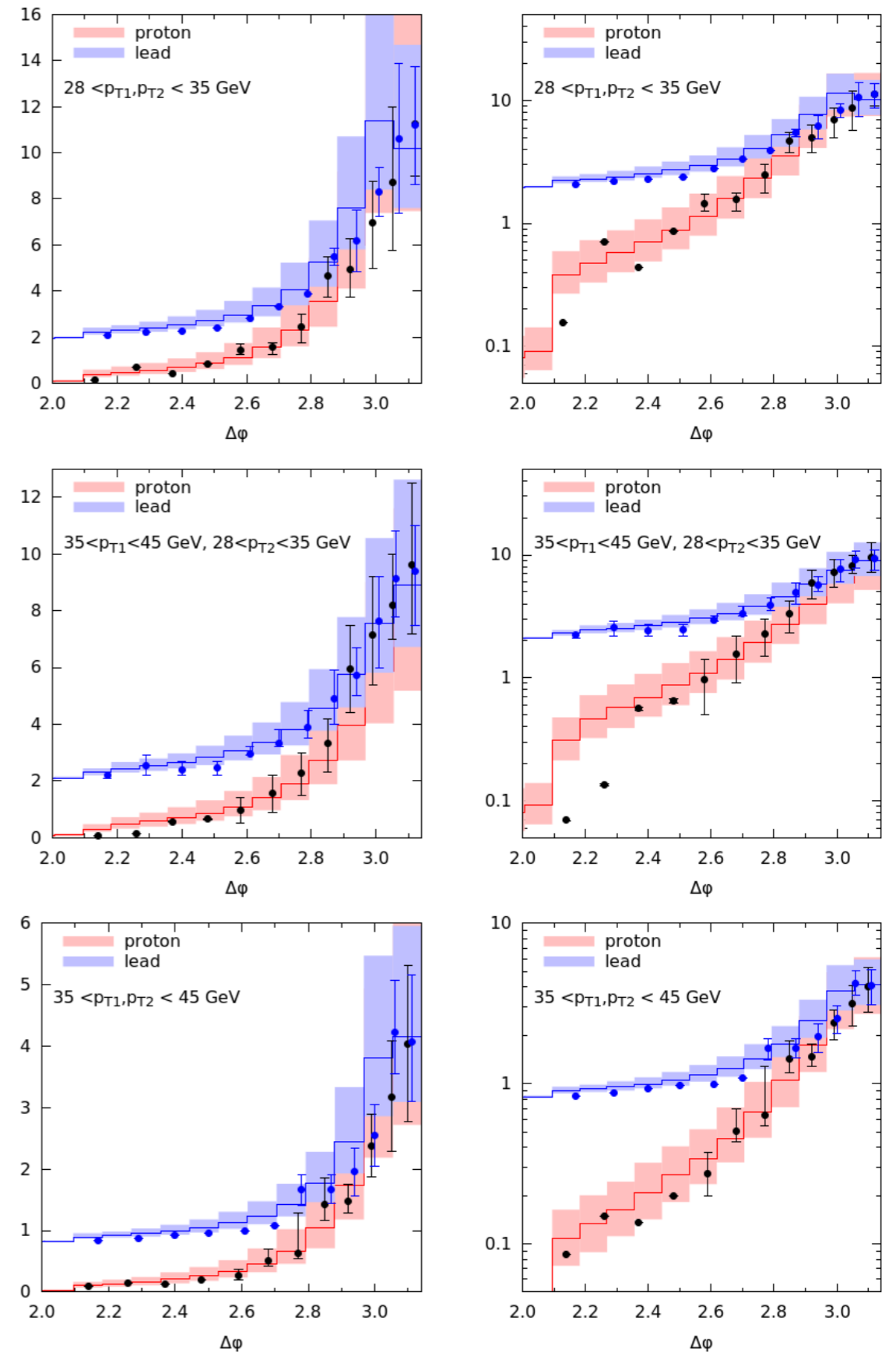
$\sqrt{s} = 5.02 \text{ TeV}$ rapidity: $2.7 < y_1, y_2 < 4.5$

$$C_{12} = \frac{1}{N_1} \frac{dN_{12}}{d\Delta\phi}$$

NUMBER OF DIJETS
AZIMUTHAL ANGLE BETWEEN JETS
NUMBER OF LEADING JETS

We study an interplay of saturation and Sudakov resummation vs the **shape** of C_{12} .

Good description of the broadening effects



A. Van Hameren, P. Kotko, K. Kutak, S. Sapeta, Phys. Lett. B795 (2019) 511

KATIE

<https://bitbucket.org/hameren/katie>

- parton level event generator, like ALPGEN, HELAC, MADGRAPH, etc.
- arbitrary processes within the standard model (including effective Higgs-gluon coupling) with several final-state particles.
- 0, 1, or 2 off-shell initial states.
- produces (partially un)weighted event files, for example in the LHEF format.
- requires LHAPDF. TMD PDFs can be provided as files containing rectangular grids, or with TMDlib.
- a calculation is steered by a single input file.
- employs an optimization stage in which the pre-samplers for all channels are optimized.
- during the generation stage several event files can be created in parallel.
- event files can be processed further by parton-shower program like CASCADE.
- (evaluation of) matrix elements now separately available, including C++ interface.