

# Forward trijet production and saturation

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*presented at*

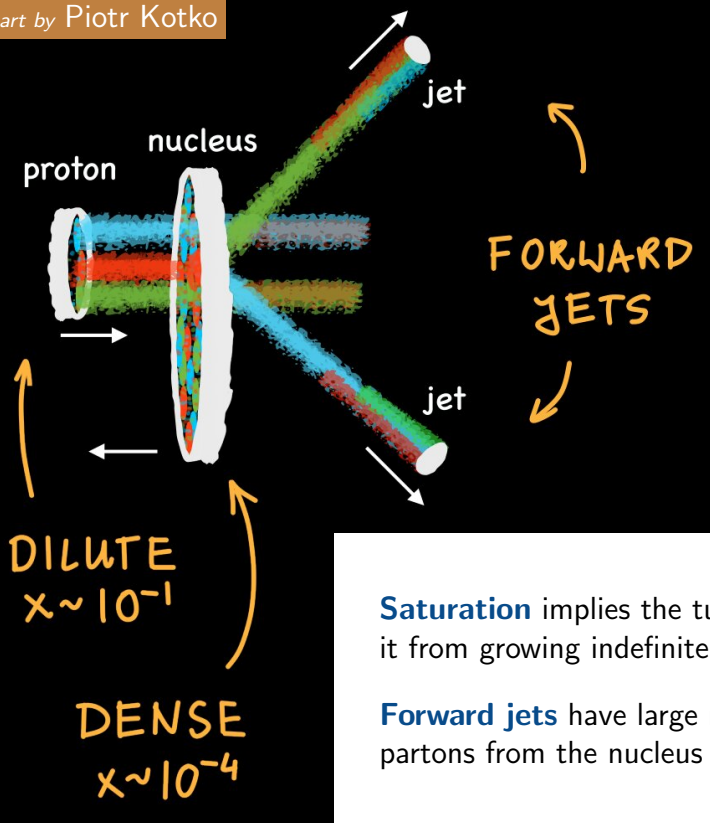
DIS 2021, Stony Brook, NY

13-04-2021

This research was supported by grant agreement No. 824093 with **STRONG-2020**

# QCD evolution, dilute vs. dense, forward jets

art by Piotr Kotko



A **dilute** system carries a few **high- $x$**  partons contributing to the hard scattering.

A **dense** system carries many **low- $x$**  partons.

At high density, gluons are imagined to undergo recombination, and to saturate.

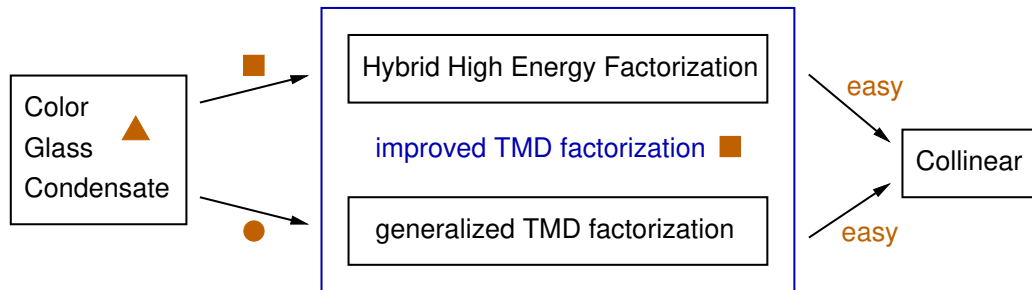
This is modeled with non-linear evolution equations, involving explicit **non-vanishing  $k_T$** .

**Saturation** implies the turnover of the gluon density, stopping it from growing indefinitely for small  $x$ .

**Forward jets** have large rapidities, and trigger events in which partons from the nucleus have small  $x$ .

# ITMD Factorization

For forward dijet production  
in dilute-dense hadronic collisions



▲ McLerran, Venugopalan 1994, Iancu, Venugopalan 2003

■ Kotko, Kutak, Marquet, Petreska, Sapeta, AvH 2015, Altinoluk, Boussarie, Kotko 2019

● Dominguez, Marquet, Xiao, Yuan 2011

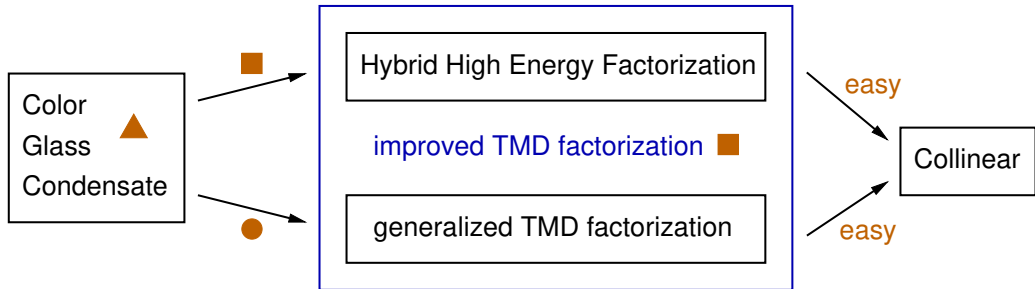
Model interpolating between hybrid High Energy Factorization and Generalized TMD factorization and valid for kinematical regions with **hard scale**  $\gtrsim k_T \gtrsim$  **saturation scale**.

Partonic cross section  $d\hat{\sigma}_{gb}^{(i)}$  **depends on color-structure  $i$** ,  
and is calculated with **space-like initial-state gluons**.

$$d\sigma_{AB \rightarrow X} = \int dk_T^2 \int d\mathbf{x}_A \sum_i \int d\mathbf{x}_B \sum_y \Phi_{gy}^{(i)}(\mathbf{x}_A, k_T, \mu) f_y(\mathbf{x}_B, \mu) d\hat{\sigma}_{gy \rightarrow X}^{(i)}(\mathbf{x}_A, \mathbf{x}_B, k_T, \mu)$$

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Model interpolating between hybrid High Energy Factorization and Generalized TMD factorization and valid for kinematical regions with  $\text{hard scale} \gtrsim k_T \gtrsim \text{saturation scale}$ .

ITMD formalism is fully obtained from the CGC formalism, by neglecting certain twist corrections. Antinoluk, Boussarie, Kotko 2019

$$d\sigma_{AB \rightarrow X} = \int dk_T^2 \int d\mathbf{x}_A \sum_i \int d\mathbf{x}_B \sum_y \Phi_{gy}^{(i)}(\mathbf{x}_A, k_T, \mu) f_y(\mathbf{x}_B, \mu) d\hat{\sigma}_{gy \rightarrow X}^{(i)}(\mathbf{x}_A, \mathbf{x}_B, k_T, \mu)$$

# ITMD\* factorization for more than 2 jets

We want to establish a similar factorization for more than 2 jets.

However, the ITMD formalism does not account for linearly polarized gluons in unpolarized target.

Such a contribution is absent for massless 2-particle production in CGC theory, but does appear in heavy quark production (Marquet, Roiesnes, Taels 2018, Altinoluk, Marquet, Taels 2021), in the correlation limit for 3-parton final-states (Altinoluk, Boussarie, Marquet, Taels 2020), and can be concluded to be present from 3-jet formulae in CGC (Iancu, Mulian 2019).

This contribution cannot straightforwardly be formulated in terms of gauge-invariant off-shell hard scattering amplitudes

$$\sum_{i,j} \mathcal{M}_i^* \left( \frac{\mathbf{k}_T^{(i)} \mathbf{k}_T^{(j)}}{2|\vec{\mathbf{k}}_T|^2} (\mathcal{F} + \mathcal{H}) + \frac{\mathbf{q}_T^{(i)} \mathbf{q}_T^{(j)}}{2|\vec{\mathbf{q}}_T|^2} (\mathcal{F} - \mathcal{H}) \right) \mathcal{M}_j \quad , \quad \vec{\mathbf{q}}_T \cdot \vec{\mathbf{k}}_T = 0$$

$\sum_i \mathcal{M}_i \mathbf{k}_T^{(i)}$  is gauge invariant while  $\sum_i \mathcal{M}_i \mathbf{q}_T^{(i)}$  is not. For dijets, it happens that  $\mathcal{F} = \mathcal{H}$ .

In the following only the manifestly gauge-invariant contribution is included, hence the designation ITMD\*.

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shell

$\sum_i \mathcal{M}_i$

In the

designation ITMD\*.

Using the axial gauge with gluon propagator

$$\frac{-i}{K^2} \left( g^{\mu\nu} - \frac{P^\mu K^\nu + K^\mu P^\nu}{P \cdot K} \right) \quad P^\mu \text{ hadron momentum}$$

the amplitude  $\mathcal{M}$  for a process involving an off-shell gluon with momentum  $xP^\mu + k_T^\mu$  can be written as

$$\mathcal{M} = k_T^\mu \mathcal{M}_\mu = - \sum_{i=1}^2 k_T^{(i)} \mathcal{M}_i$$

where  $\mathcal{M}_\mu$  is obtained from the usual Feynman graphs indeed with one gluon simply left “off-shell”. The role of “polarization vector” is played by  $k_T^\mu$ .

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uet, Tael  
ian 2019).

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# ITMD\* factorization for more than 2 jets

Schematic hybrid (non-ITMD) factorization formula

$$d\sigma = \sum_{y=g,u,d,\dots} \int dx_1 d^2k_T \int dx_2 d\Phi_{g^*y \rightarrow n} \frac{1}{\text{flux}_{gy}} \mathcal{F}_g(x_1, k_T, \mu) f_y(x_2, \mu) \sum_{\text{color}} \left| \mathcal{M}_{g^*y \rightarrow n}^{(\text{color})} \right|^2$$

$$\mathcal{F}_g \sum_{\text{color}} \left| \mathcal{M}^{(\text{color})} \right|^2 = \mathcal{F}_g \sum_{i_1, i_2, \dots, i_{n+2}} \sum_{j_1, j_2, \dots, j_{n+2}} \left( \tilde{\mathcal{M}}_{j_1 j_2 \dots j_{n+2}}^{i_1 i_2 \dots i_{n+2}} \right)^* \left( \tilde{\mathcal{M}}_{j_1 j_2 \dots j_{n+2}}^{i_1 i_2 \dots i_{n+2}} \right)$$



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with (Bomhof, Mulders, Pijlman 2006; Bury, Kotko, Kutak 2018)

$$\begin{aligned} & (N_c^2 - 1) \sum_{i_1, \dots, i_n} \sum_{j_1, \dots, j_{n+2}} \sum_{\bar{i}_1, \dots, \bar{i}_{n+2}} \sum_{\bar{j}_1, \dots, \bar{j}_{n+2}} \left( \tilde{\mathcal{M}}_{j_1 j_2 \dots j_{n+2}}^{i_1 i_2 \dots i_{n+2}} \right)^* \left( \tilde{\mathcal{M}}_{\bar{j}_1 \bar{j}_2 \dots \bar{j}_{n+2}}^{\bar{i}_1 \bar{i}_2 \dots \bar{i}_{n+2}} \right) \\ & \times 2 \int \frac{d^4 \xi}{(2\pi)^3 P^+} \delta(\xi_+) e^{ik \cdot \xi} \left\langle P \left| \left( \hat{F}^+(\xi) \right)_{i_1}^{j_1} \left( \hat{F}^+(0) \right)_{\bar{i}_1}^{\bar{j}_1} \left( u^{[\lambda_2]} \right)_{i_2 \bar{i}_2} \left( u^{[\lambda_2] \dagger} \right)^{\bar{j}_2 \bar{j}_2} \dots \right. \right. \\ & \left. \left. \dots \left( u^{[\lambda_{n+2}]} \right)_{i_{n+2} \bar{i}_{n+2}} \left( u^{[\lambda_{n+2} \dagger]} \right)^{\bar{j}_{n+2} \bar{j}_{n+2}} \right| P \right\rangle \end{aligned}$$

where  $P$  is the light-like momentum of the hadron (with  $P^- = 0$ ), and  $k^\mu = xP^\mu + k_T^\mu$ ,

where  $\hat{F}$  is the field strength,

and  $U^\pm$  is a Wilson line from 0 to  $\xi$ , via a “staple-like detour” to  $\pm\infty$  depending on the type and state (initial/final) of parton.

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where  $P$  is  
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and  $u^\pm$  is  
type and st

$$\tilde{\mathcal{M}}_{j_1 j_2 \dots j_{n+2}}^{i_1 i_2 \dots i_{n+2}} = \sum_{\sigma \in S_{n+2}} \delta_{j_{\sigma(1)}}^{i_1} \delta_{j_{\sigma(2)}}^{i_2} \dots \delta_{j_{\sigma(n+2)}}^{i_{n+2}} \mathcal{A}_\sigma$$

Kanaki, Papadopoulos 2000; Maltoni, Paul, Stelzer, Willenbrock 2003

$p^\mu + k_T^\mu$ ,

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with “TMD-valued color matrix”

$$(N_c^2 - 1) \sum_{\sigma \in S_{n+2}} \sum_{\tau \in S_{n+2}} \mathcal{A}_\sigma^* \tilde{\mathcal{C}}_{\sigma\tau}(x, |k_T|) \mathcal{A}_\tau \quad , \quad \tilde{\mathcal{C}}_{\sigma\tau}(x, |k_T|) = N_c^{\bar{\lambda}(\sigma,\tau)} \tilde{\mathcal{F}}_{\sigma\tau}(x, |k_T|)$$

where each function  $\tilde{\mathcal{F}}_{\sigma\tau}$  is one of 10 functions

$$\mathcal{F}_{qg}^{(1)} \quad , \quad \mathcal{F}_{qg}^{(2)} \quad , \quad \mathcal{F}_{qg}^{(3)} \\ \mathcal{F}_{gg}^{(1)} \quad , \quad \mathcal{F}_{gg}^{(2)} \quad , \quad \mathcal{F}_{gg}^{(3)} \quad , \quad \mathcal{F}_{gg}^{(4)} \quad , \quad \mathcal{F}_{gg}^{(5)} \quad , \quad \mathcal{F}_{gg}^{(6)} \quad , \quad \mathcal{F}_{gg}^{(7)}$$

# ITMD\* factorization for more than 2 jets

$$\mathcal{F}_{qg}^{(1)}(x, k_T) = \left\langle \text{Tr} \left[ \hat{F}^{i+}(\xi) u^{[-]\dagger} \hat{F}^{i+}(0) u^{[+]} \right] \right\rangle, \quad \langle \dots \rangle = 2 \int \frac{d^4 \xi \delta(\xi_+)}{(2\pi)^3 P^+} e^{ik \cdot \xi} \langle P | \dots | P \rangle$$

$$\mathcal{F}_{qg}^{(2)}(x, k_T) = \left\langle \frac{\text{Tr} [u^{[\square]}]}{N_c} \text{Tr} \left[ \hat{F}^{i+}(\xi) u^{[+]\dagger} \hat{F}^{i+}(0) u^{[+]} \right] \right\rangle$$

$$\mathcal{F}_{qg}^{(3)}(x, k_T) = \left\langle \text{Tr} \left[ \hat{F}^{i+}(\xi) u^{[+]\dagger} \hat{F}^{i+}(0) u^{[\square]} u^{[+]} \right] \right\rangle$$

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$$\mathcal{F}_{gg}^{(2)}(x, k_T) = \frac{1}{N_c} \left\langle \text{Tr} \left[ \hat{F}^{i+}(\xi) u^{[\square]\dagger} \right] \text{Tr} \left[ \hat{F}^{i+}(0) u^{[\square]} \right] \right\rangle$$

$$\mathcal{F}_{gg}^{(3)}(x, k_T) = \left\langle \text{Tr} \left[ \hat{F}^{i+}(\xi) u^{[+]\dagger} \hat{F}^{i+}(0) u^{[+]} \right] \right\rangle$$

$$\mathcal{F}_{gg}^{(4)}(x, k_T) = \left\langle \text{Tr} \left[ \hat{F}^{i+}(\xi) u^{[-]\dagger} \hat{F}^{i+}(0) u^{[-]} \right] \right\rangle$$

$$\mathcal{F}_{gg}^{(5)}(x, k_T) = \left\langle \text{Tr} \left[ \hat{F}^{i+}(\xi) u^{[\square]\dagger} u^{[+]\dagger} \hat{F}^{i+}(0) u^{[\square]} u^{[+]} \right] \right\rangle$$

$$\mathcal{F}_{gg}^{(6)}(x, k_T) = \left\langle \frac{\text{Tr} [u^{[\square]}]}{N_c} \frac{\text{Tr} [u^{[\square]\dagger}]}{N_c} \text{Tr} \left[ \hat{F}^{i+}(\xi) u^{[+]\dagger} \hat{F}^{i+}(0) u^{[+]} \right] \right\rangle$$

$$\mathcal{F}_{gg}^{(7)}(x, k_T) = \left\langle \frac{\text{Tr} [u^{[\square]}]}{N_c} \text{Tr} \left[ \hat{F}^{i+}(\xi) u^{[\square]\dagger} u^{[+]\dagger} \hat{F}^{i+}(0) u^{[+]} \right] \right\rangle$$

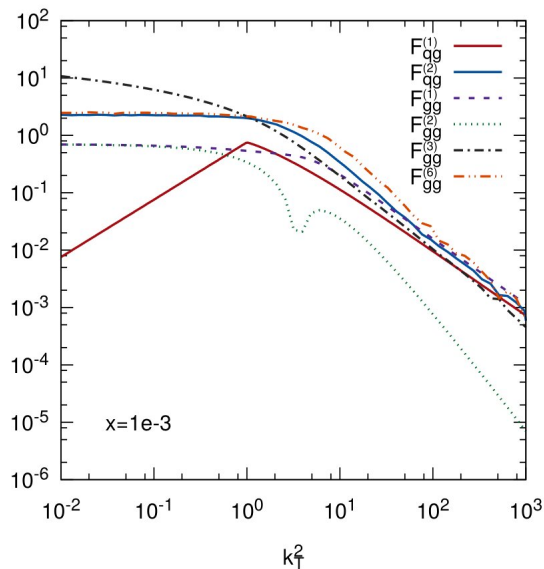
# ITMD gluons

Start with dipole distribution  $\mathcal{F}_{qg}^{(1)}(x, k_T) = \langle \text{Tr} [\hat{F}^{i+}(\xi) \mathcal{U}^{[-]\dagger} \hat{F}^{i+}(0) \mathcal{U}^{[+]}] \rangle$  evolved via the BK equation formulated in momentum space supplemented with subleading corrections and fitted to  $F_2$  data (Kutak, Sapeta 2012)

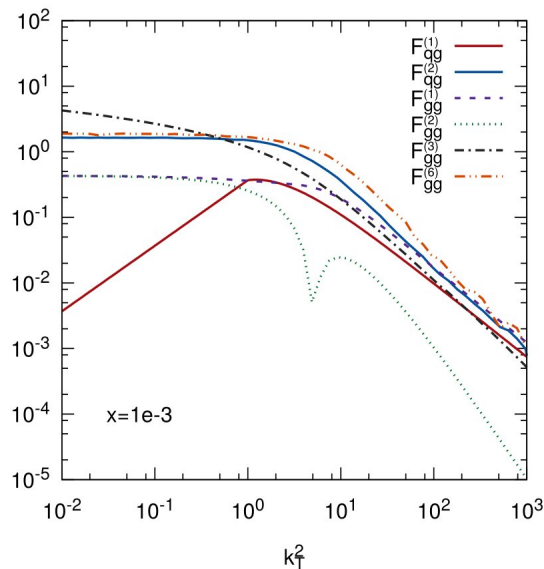
All other distribution appearing in dijet production,  $\mathcal{F}_{qg}^{(2)}, \mathcal{F}_{gg}^{(1)}, \mathcal{F}_{gg}^{(2)}, \mathcal{F}_{gg}^{(6)}$ , in the mean-field approximation (AvH, Marquet, Kotko, Kutak, Sapeta, Petreska 2016).

This is, at leading order in  $1/N_c$ . In this approximation, the same distributions suffice for trijets.

KS gluon TMDs in proton

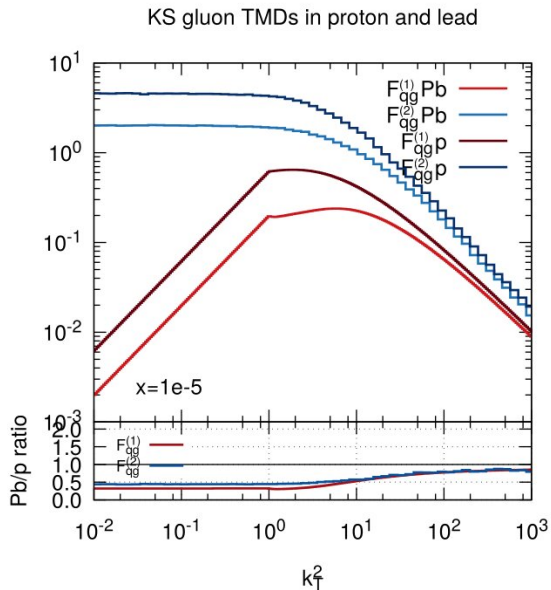
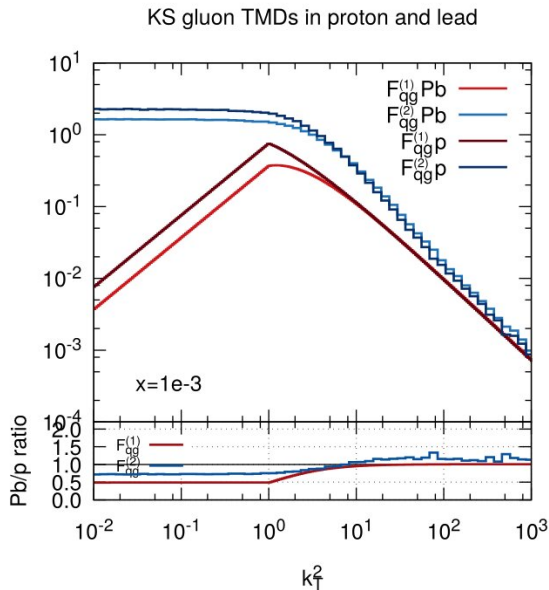


KS gluon TMDs in lead



Dependence of  $\mathcal{F}_{qg}^{(1)}$  on  $k_T$  below 1GeV approximated by power-like fall-off. For higher values of  $|k_T|$  it is a solution to the BK equation.

TMDs decrease as  $1/|k_T|$  for increasing  $|k_T|$ , except  $\mathcal{F}_{gg}^{(2)}$ , which decreases faster (even becomes negative, absolute value shown here).



Ratio Pb/p is smaller than 1 for small  $x$ ,  
but can become larger than 1 for moderate  $x$  and large  $|k_T|$ .

# Set up

We consider p-p and p-Pb collisions at 5.02 TeV producing at least 3 jets with forward rapidities  $3.2 < |y_1^*, y_2^*, y_3^*| < 4.9$  in the CM frame.

Jet definition:  $\Delta R > 0.5$ ,  $p_T > 20 \text{ GeV}$

renormalization/factorization scale:  $(p_{T1} + p_{T2} + p_{T3})/3$

Collinear PDFs: CTEQ10NLO from LHAPDF6

Include all partonic processes with 5 light flavors with an (off-shell) gluon and a quark or gluon in the initial state.

observables:

$\Delta\phi_{12}$  (angle between 2 hardest jets),

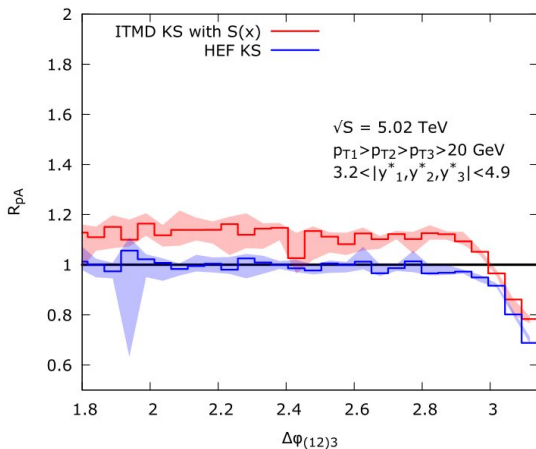
$\Delta\phi_{13}$  (angle between hardest jet and 3<sup>rd</sup> hardest jet),

$\Delta\phi_{(12)3}$  (angle between the sum of the two hardest and the 3<sup>rd</sup> hardest jet. Is sensitive to momentum imbalance)

Nuclear modification ratio  $R_{pA} = \frac{1}{A} \frac{d\sigma^{pPb}/d\mathcal{O}}{d\sigma^{pp}/d\mathcal{O}}$  where  $A$  is the number of nucleons

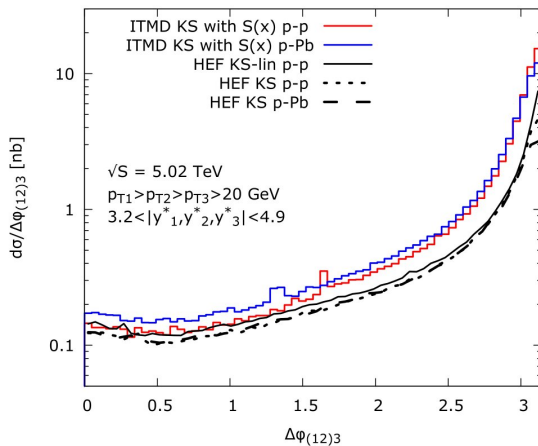
Calculations performed independently with LxJet (Kotko) and KaTIE (AvH 2018)





$S(x)$  refers to the  $x$ -dependent treatment of the nuclear target area, guaranteeing unitarity.

Saturation effect for  $\Delta\phi_{(12)3} \approx \pi$ , enhancement of pPb result for  $\Delta\phi_{(12)3} < \pi$  due to broadening of the TMD distributions.



ITMD\* normalization significantly larger than HEF, due to different shape and normalization of the extra TMDs present in ITMD\* but not in HEF.

# Summary

- small- $x$  Improved TMD factorization allows to consistently include saturation effects in calculations for forward dijets
- we extended ITMD factorization to ITMD\* for more than 2 jets, and performed explicit calculations for 3 jets
- we observe significant saturation effects in the nuclear modification factor for momentum imbalance-sensitive observable
- we observe significant differences between results from ITMD\* and  $k_T$ /high-energy factorization, implying strong discriminating potential

Thank you for your attention.