Gluon Saturation in Proton and its Contribution to Single Semi-hard Gluon Production in High Energy pA Collisions

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Ming Li and Vladimir Skokov, arXiv: 2102.01594, 2104.01879

Outline

- **Introduction.** (*Saturation correction to single gluon production*)
- **Classical Yang-Mills Equations in the Dilute-Dense Regime and the LSZ Reduction Formula**. (*Perturbative solutions, order-*g^3 *gluon fields*)
- **First Saturation Correction to Single Inclusive Semi-Hard Gluon Production.**
- **Summary and Outlook.**

First Saturation Correction to Single Gluon Production

Single Gluon Production Amplitude

$$M = M_{(1)} + M_{(3)} + M_{(5)} + \dots$$

$g\rho_P$ $\quad$ $g^3\rho_P^2$ $\quad$ $g^5\rho_P^3$

Amplitude Squared

$$|M|^2 = |M_{(1)}|^2 + M_{(1)}M^*_{(3)} + M^*_{(1)}M_{(3)} + M^*_{(3)}M_{(3)} + M^*_{(1)}M_{(5)} + M_{(1)}M^*_{(5)} + \dots$$

Leading Order $g^2\rho_P^2$

$g^4\rho_P^3$

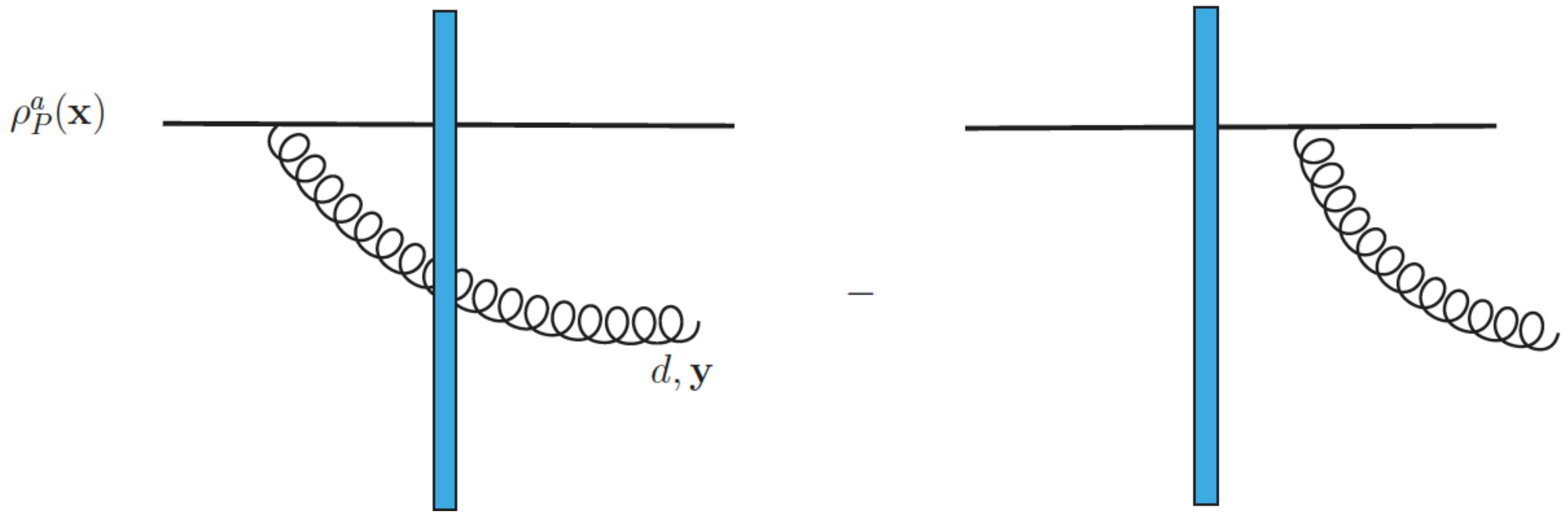
First Saturation Correction $g^6\rho_P^4$

In MV model $\langle g^4\rho_P^2\rangle \sim g^4\mu_P^2 \sim Q^2_{s,P}$ $\quad$ $\langle \rho_P^3\rangle = 0$

- **Gluon Production Amplitude** $M_{(1)}$

Order-$g\rho_P$

Perturbatively at order g . No interactions among color charges in the projectile.



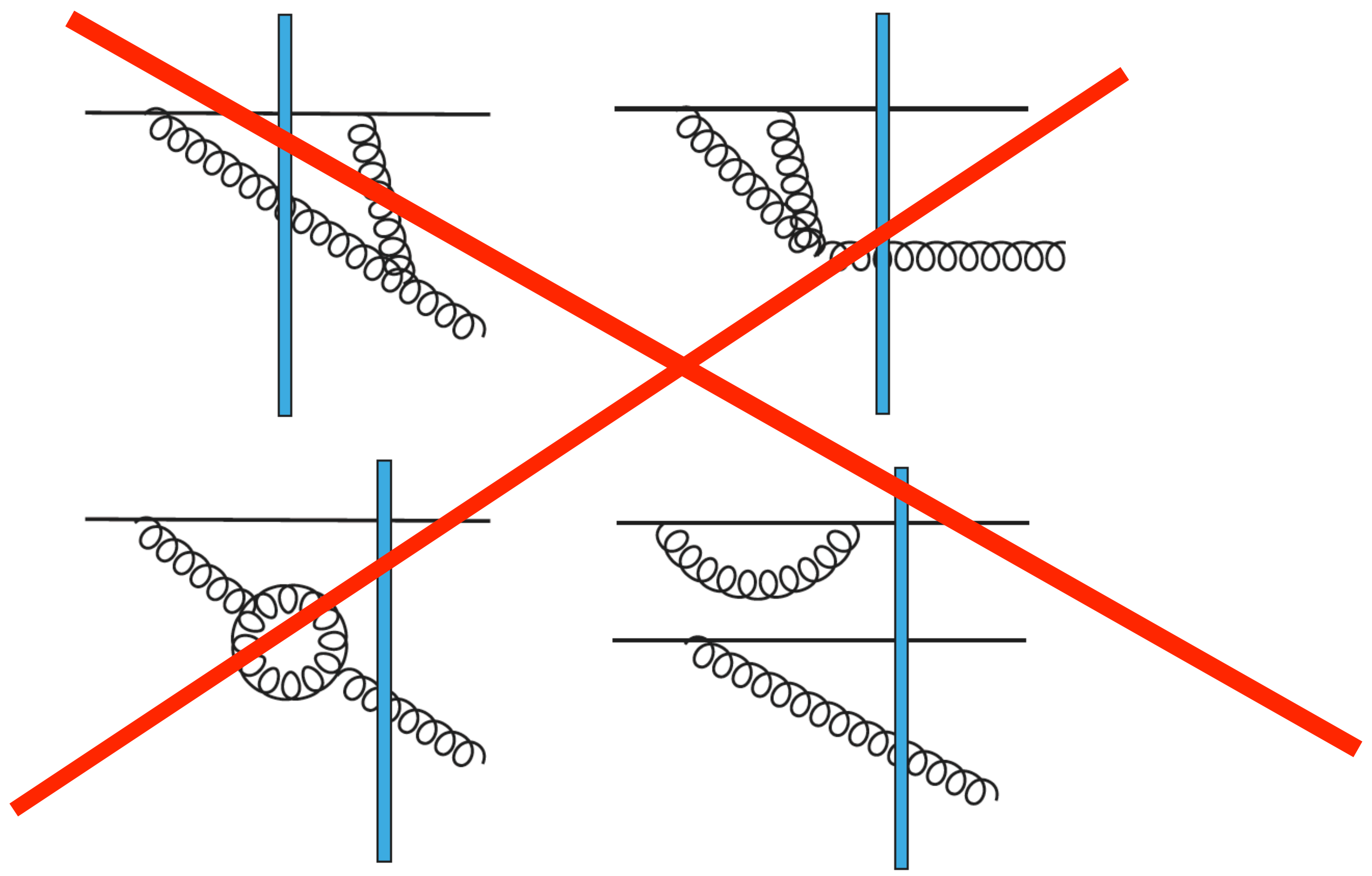
$$\frac{dN}{d^2\mathbf{k}} = \left\langle \frac{dN}{d^2\mathbf{k}}(\rho_P, \rho_T)\right\rangle_{\rho_P,\rho_T} = \frac{1}{\alpha_s}\sum_{n=1}^{\infty}\left(\frac{Q^2_{s,P}}{k_\perp^2}\right)^n f_n\left(\frac{Q^2_{s,T}}{k_\perp^2}\right) \qquad Q^2_{s,P} < k_\perp^2 < Q^2_{s,T}$$

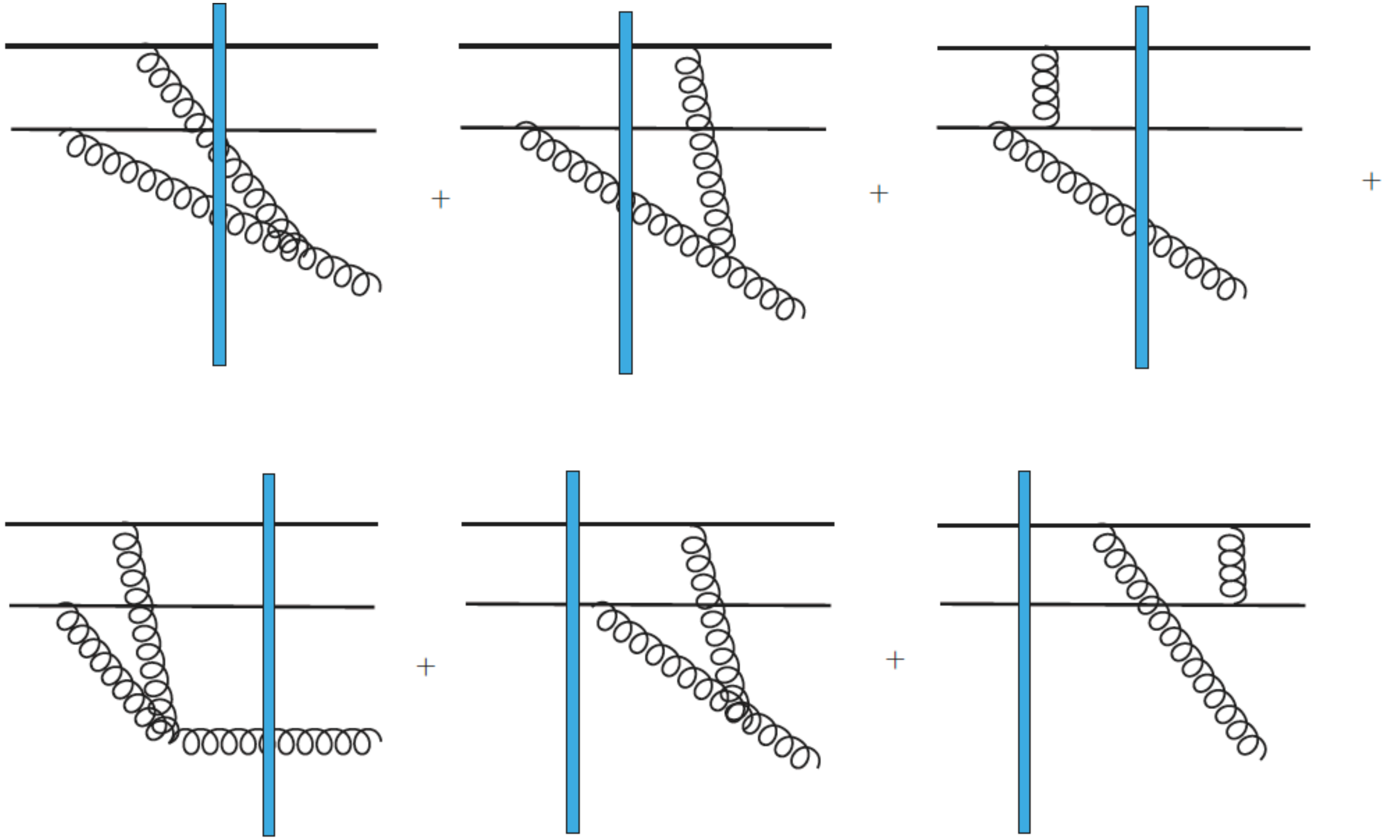
First Saturation Correction to Single Gluon Production

- **Gluon Production Amplitude** $M_{(3)}$

Order-$g^3\rho_P^2$

Perturbatively at order g^3 but it involves interactions of ***two color charges*** in the projectile



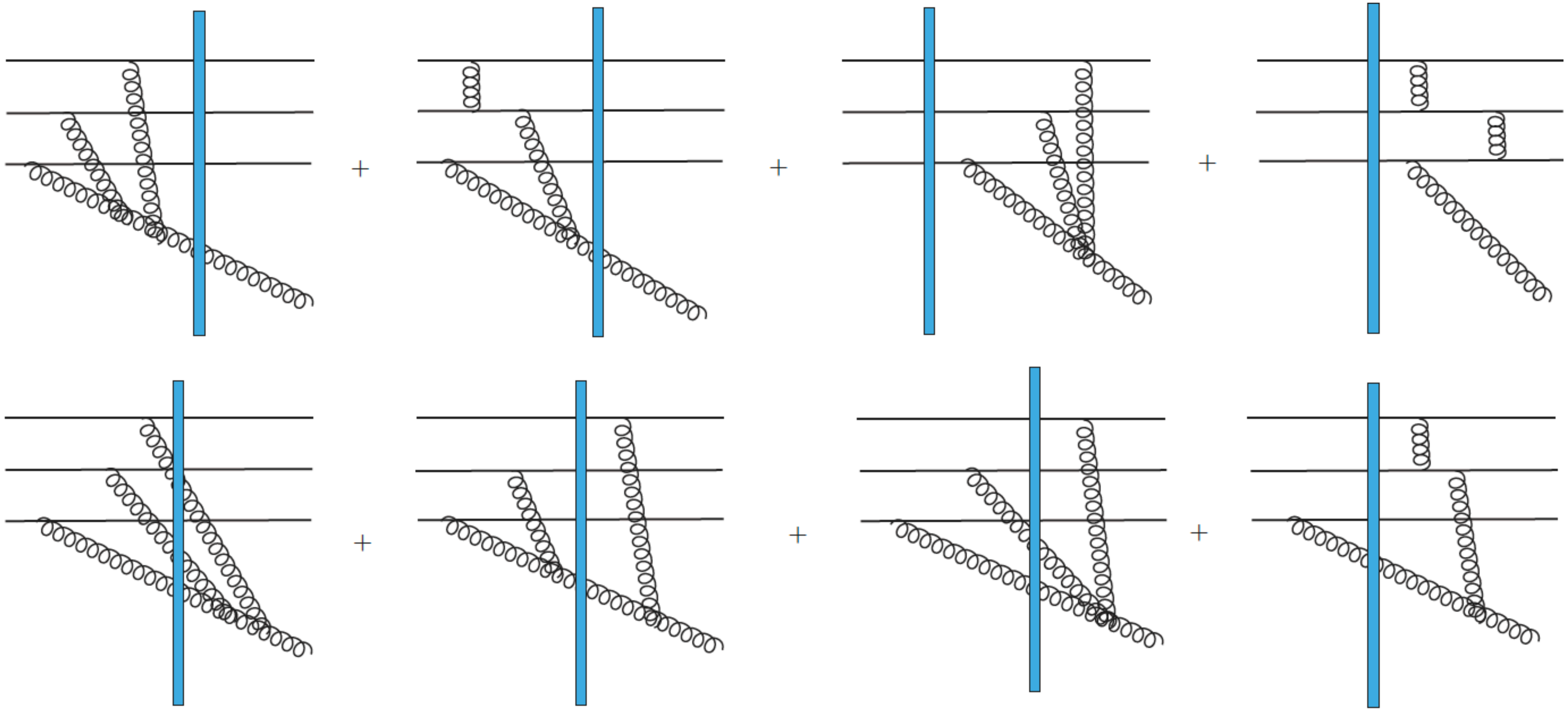


First Saturation Correction to Single Gluon Production

- **Gluon Production Amplitude** $M_{(5)}$

Order-$g^5\rho_P^3$

Perturbatively at order g^5 but it involves interactions of ***three color charges*** in the projectile



Defining the Gluon Spectrum

- **CGC Approach: solve the classical Yang-Mills equations in the dilute-dense regime and use the LSZ reduction formula**

Schenke, Schlichting, Venugopalan (2015)

McLerran, Skokov (2017)

Define Asymptotic Creation Operators:

$$\hat{a}^{a\dagger}_{\eta}(\mathbf{k}) = -\, i\tau\sqrt{\frac{\pi}{4}}\left(H_1^{(2)}(k_\perp\tau)\overleftrightarrow{\partial_\tau}\,\tilde{\beta}^a(\tau,\mathbf{k})\right)\Big|_{\tau=+\infty},$$

$$\hat{a}^{a\dagger}_{\perp}(\mathbf{k}) = -\, i\tau\sqrt{\frac{\pi}{4}}\left(H_0^{(2)}(k_\perp\tau)\overleftrightarrow{\partial_\tau}\,\beta^a_\perp(\tau,\mathbf{k})\right)\Big|_{\tau=+\infty}$$

Projection on asymptotic free particle state at infinite time through the Hankel functions

$$H_1^{(2)}(k_\perp\tau) \sim \sqrt{\frac{2}{\pi k_\perp \tau}}e^{-i(k_\perp\tau-\frac{3}{4}\pi)}, \quad H_0^{(2)}(k_\perp\tau) \sim \sqrt{\frac{2}{\pi k_\perp \tau}}e^{-i(k_\perp\tau-\frac{1}{4}\pi)}$$

Single Gluon Production:

$$\frac{dN}{d^2\mathbf{k}} = \frac{1}{(2\pi)^2}\left(\hat{a}^\dagger_\eta(\mathbf{k})\hat{a}_\eta(\mathbf{k}) + \hat{a}^\dagger_\perp(\mathbf{k})\hat{a}_\perp(\mathbf{k})\right)$$

For first saturation correction, creation operators at order-g^3 and order-g^5 are needed.

$$\beta^{(1)}(\tau,\mathbf{k}) = b_\eta(\mathbf{k})\frac{J_1(k_\perp\tau)}{k_\perp\tau},$$

$$\beta^{(1)}_i(\tau,\mathbf{k}) = \frac{-i\epsilon_{il}\mathbf{k}_l}{k_\perp^2}b_\perp(\mathbf{k})J_0(k_\perp\tau).$$

$$\tilde{\beta}^{(1)}(\tau,\mathbf{k}),\beta^{(1)}_\perp(\tau,\mathbf{k})$$

$$\tilde{\beta}^{(3)}(\tau,\mathbf{k}),\beta^{(3)}_\perp(\tau,\mathbf{k})$$

Classical Yang-Mills Equations and Initial Conditions

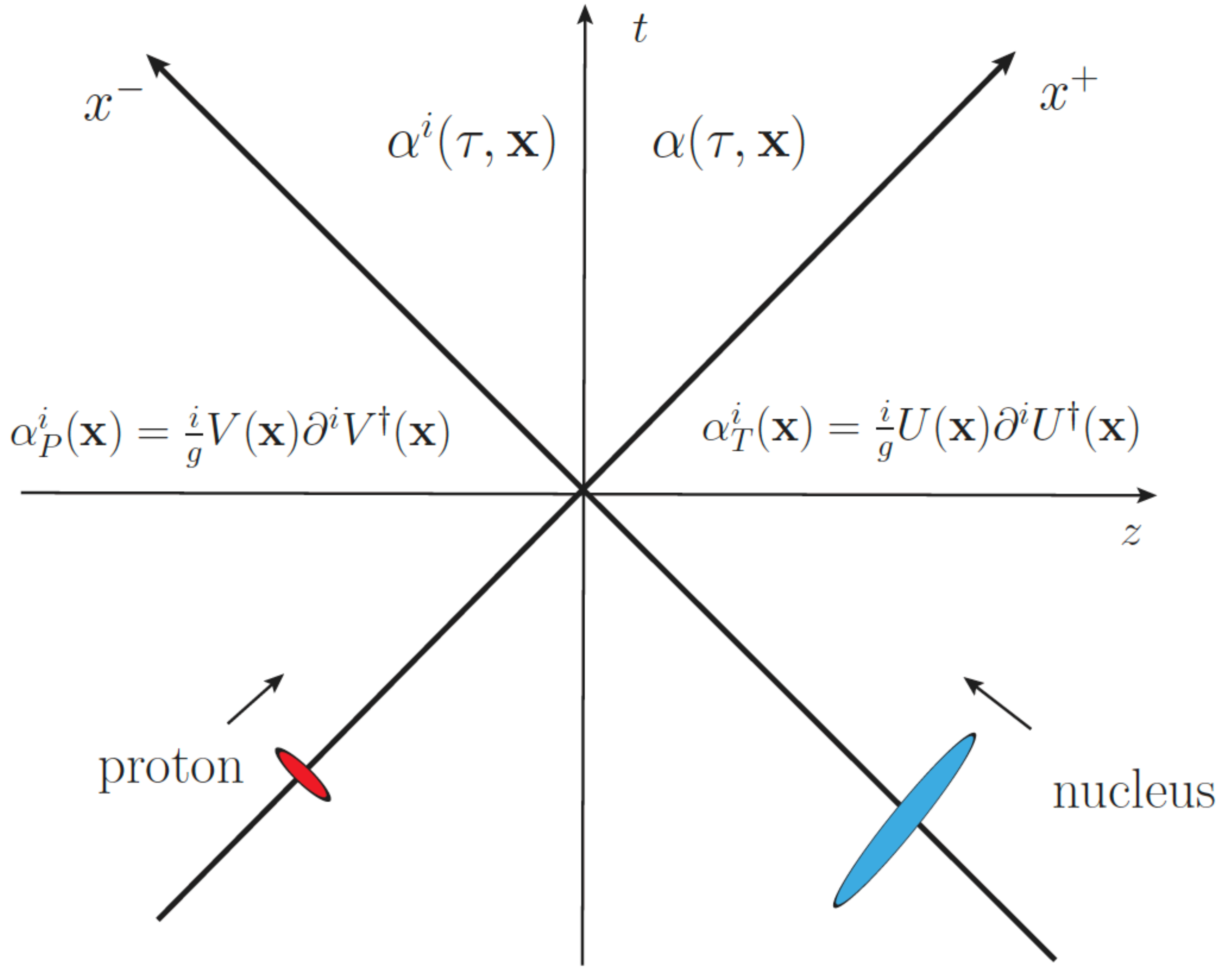
Classical Yang-Mills equations in the Fock-Schwinger gauge, assuming boost-invariance

$$A^+ = x^+\alpha,\;\; A^- = -x^-\alpha,\;\; A^i = \alpha^i$$

$$\partial_\tau^2\alpha + \frac{3}{\tau}\partial_\tau\alpha - [D_i,[D_i,\alpha]] = 0\,,$$

$$-ig[\alpha,\tau\partial_\tau\alpha] + [D_i,\frac{1}{\tau}\partial_\tau\alpha_i] = 0\,,$$

$$\frac{1}{\tau}\partial_\tau\alpha_i + \partial_\tau^2\alpha_i - ig\tau^2[\alpha,[D_i,\alpha] - D_jF_{ji} = 0.$$



Initial Conditions

$$\alpha(\tau=0,\mathbf{x}) = \frac{ig}{2}[\alpha_P^i(\mathbf{x}),\alpha_T^i(\mathbf{x})],$$

$$\alpha^i(\tau=0,\mathbf{x}) = \alpha_P^i(\mathbf{x}) + \alpha_T^i(\mathbf{x}).$$

Additional gauge transformation by $W(x)$ to ensure $\partial^i\beta^i(\tau=0,\mathbf{x}) = 0$

$$\beta(\tau=0,\mathbf{x}) = \frac{1}{2}\mathcal{W}^\dagger\Big(\partial^i(U^\dagger\alpha_P^iU) - U^\dagger\partial^i\alpha_P^iU\Big)\mathcal{W},$$

$$\beta^i(\tau=0,\mathbf{x}) = \mathcal{W}^\dagger U^\dagger\alpha_P^iU\mathcal{W} + \frac{i}{g}\mathcal{W}^\dagger\partial^i\mathcal{W}.$$

In the dilute-dense regime, both the equations and the initial conditions are expanded as power series of coupling constant g.

$$\alpha(\tau,\mathbf{x}) = \sum_{n=0}^\infty g^n\alpha^{(n)}(\tau,\mathbf{x}),\;\; \alpha_i(\tau,\mathbf{x}) = \sum_{n=0}^\infty g^n\alpha_i^{(n)}(\tau,\mathbf{x})$$

Results: Classical Gluon Fields at Next to Leading Order

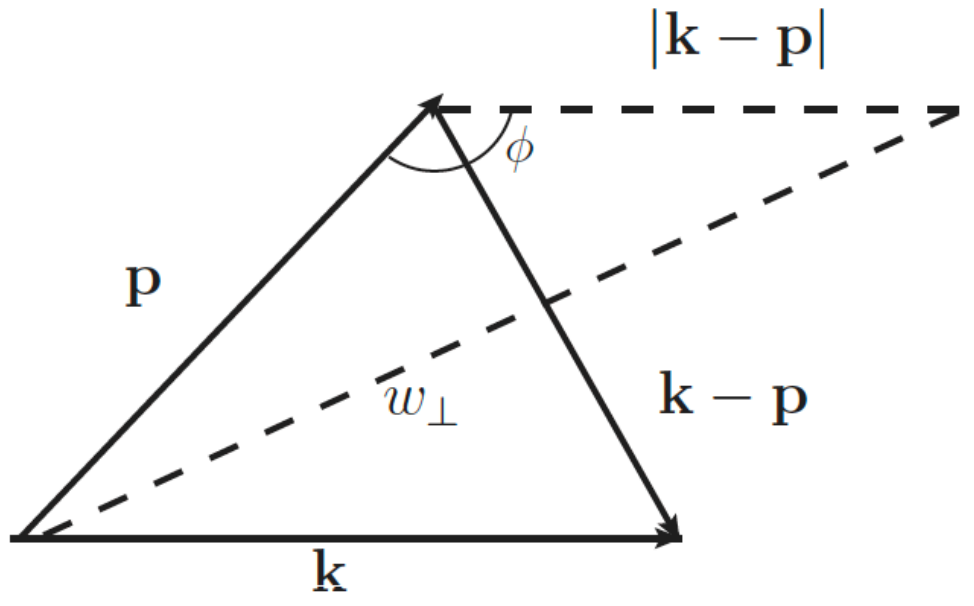
The Solutions:

$$\beta^{(3)}(\tau,\mathbf{k}) = 2\beta^{(3)}(\tau=0,\mathbf{k})\frac{J_1(k_\perp\tau)}{k_\perp\tau} - i\int\frac{d^2\mathbf{p}}{(2\pi)}\left[b_\perp(\mathbf{p}), b_\eta(\mathbf{k}-\mathbf{p})\right]\frac{\mathbf{k}\times\mathbf{p}}{p_\perp^2|\mathbf{k}-\mathbf{p}|^2}\int_{-\pi}^{\pi}\frac{d\phi}{2\pi}\left(1+\frac{2\mathbf{k}\cdot(\mathbf{k}-\mathbf{p})}{w_\perp^2-k_\perp^2}\right)\left(\frac{J_1(w_\perp\tau)}{w_\perp\tau}-\frac{J_1(k_\perp\tau)}{k_\perp\tau}\right)$$

$$\beta_\perp^{(3)}(\tau,\mathbf{k}) = \beta_\perp^{(3)}(\tau=0,\mathbf{k})J_0(k_\perp\tau) + \frac{i}{k_\perp}\int\frac{d^2\mathbf{p}}{(2\pi)^2}\left[b_\eta(\mathbf{p}), b_\eta(\mathbf{k}-\mathbf{p})\right]\frac{\mathbf{k}\times\mathbf{p}}{2p_\perp^2|\mathbf{k}-\mathbf{p}|^2}\int_{-\pi}^{\pi}\frac{d\phi}{2\pi}\left(1+\frac{2\mathbf{p}\cdot(\mathbf{k}-\mathbf{p})}{w_\perp^2-k_\perp^2}\right)(J_0(w_\perp\tau)-J_0(k_\perp\tau))$$
$$-\frac{i}{k_\perp}\int\frac{d^2\mathbf{p}}{(2\pi)^2}\left[b_\perp(\mathbf{p}), b_\perp(\mathbf{k}-\mathbf{p})\right]\frac{(\mathbf{k}\times\mathbf{p})(-\mathbf{p}\cdot\mathbf{k}+p_\perp^2+k_\perp^2)}{p_\perp^2|\mathbf{k}-\mathbf{p}|^2}\int_{-\pi}^{\pi}\frac{d\phi}{2\pi}\frac{1}{w_\perp^2-k_\perp^2}(J_0(w_\perp\tau)-J_0(k_\perp\tau))$$

$$\Lambda^{(3)}(\tau,\mathbf{k}) = -\frac{i}{k_\perp^2}\int\frac{d^2\mathbf{p}}{(2\pi)^2}\left[b_\eta(\mathbf{p}), b_\eta(\mathbf{k}-\mathbf{p})\right]\frac{\mathbf{k}\cdot(\mathbf{k}-2\mathbf{p})}{4p_\perp^2|\mathbf{k}-\mathbf{p}|^2}\int_{-\pi}^{\pi}\frac{d\phi}{2\pi}\left(1-\frac{p_\perp^2+|\mathbf{k}-\mathbf{p}|^2}{w_\perp^2}\right)(1-J_0(w_\perp\tau))$$
$$-\frac{i}{k_\perp^2}\int\frac{d^2\mathbf{p}}{(2\pi)^2}\left[b_\perp(\mathbf{p}), b_\perp(\mathbf{k}-\mathbf{p})\right]\frac{\mathbf{k}\cdot(\mathbf{k}-2\mathbf{p})\mathbf{p}\cdot(\mathbf{k}-\mathbf{p})}{2p_\perp^2|\mathbf{k}-\mathbf{p}|^2}\int_{-\pi}^{\pi}\frac{d\phi}{2\pi}\frac{1}{w_\perp^2}(1-J_0(w_\perp\tau))$$

$$\beta_i^{(3)}(\tau,\mathbf{k}) = \frac{-i\epsilon^{ij}\mathbf{k}_j}{k_\perp}\beta_\perp^{(3)}(\tau,\mathbf{k}) - i\mathbf{k}_i\Lambda^{(3)}(\tau,\mathbf{k})$$



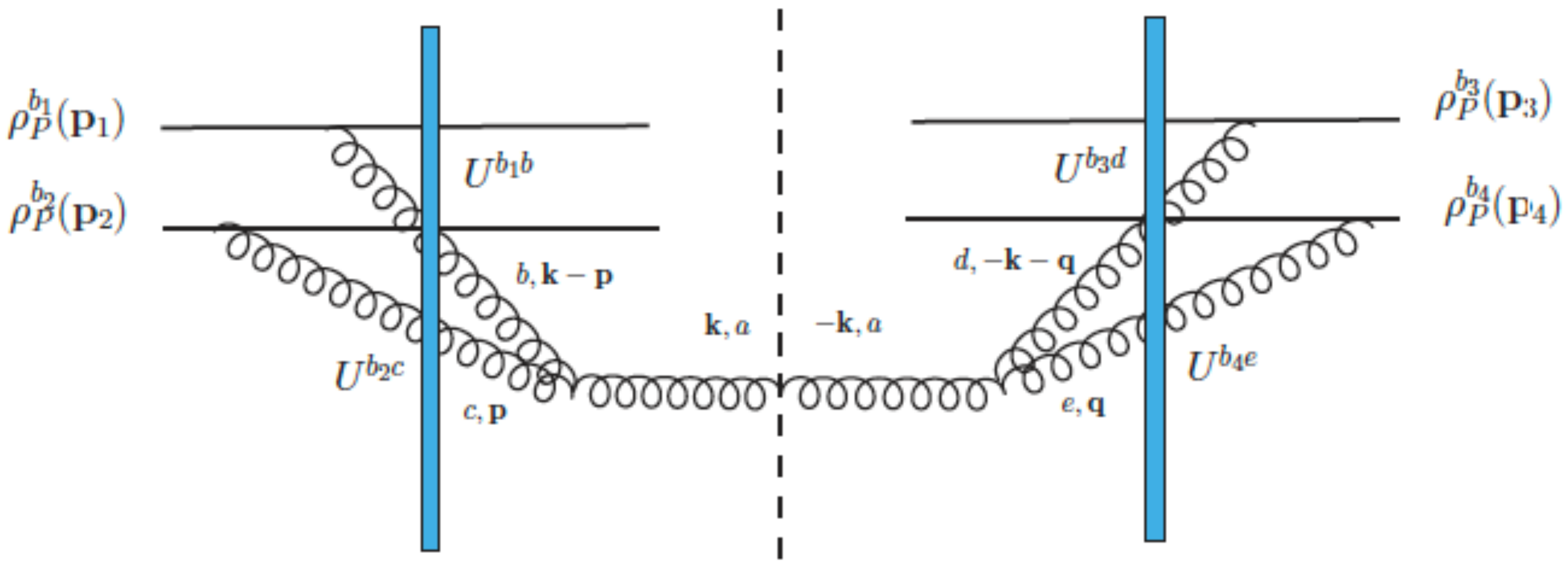
$$w_\perp = \sqrt{p_\perp^2+|\mathbf{k}-\mathbf{p}|^2-2p_\perp|\mathbf{k}-\mathbf{p}|\cos\phi}$$

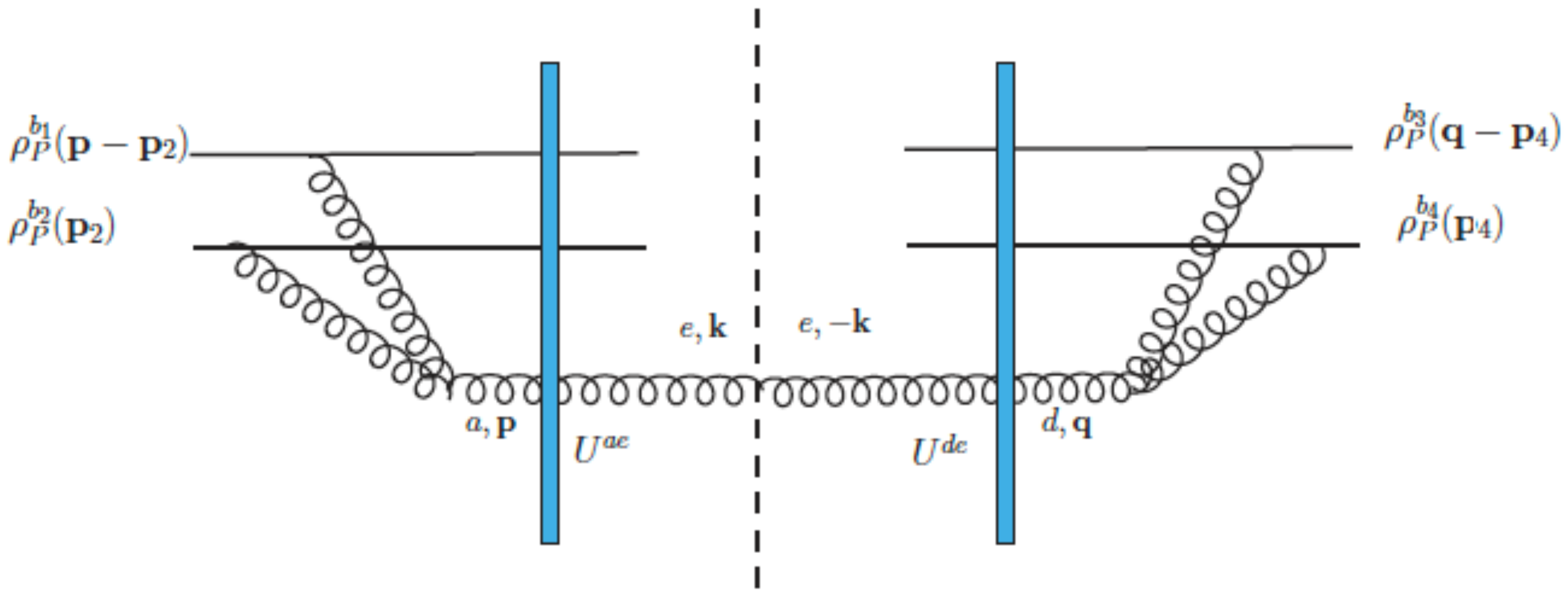
$$|p_\perp - |\mathbf{k}-\mathbf{p}|| \le w_\perp \le p_\perp + |\mathbf{k}-\mathbf{p}|$$

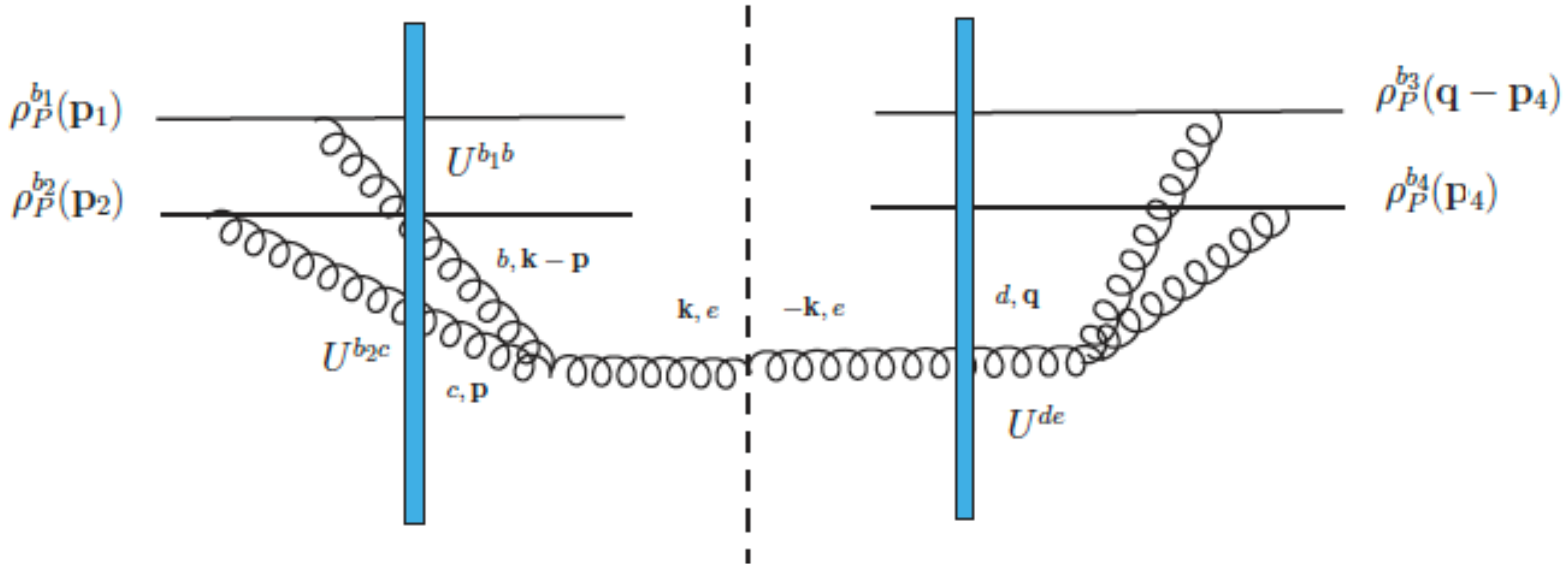
Results: First Saturation Correction

$$\frac{d^2N}{d^2\mathbf{k}}\bigg|_{\mathrm{FSC}} = |M_{(3)}|^2 + M_{(1)}M^*_{(5)} + M^*_{(1)}M_{(5)}$$

$$\begin{aligned}
&M_{(3)}(\mathbf{k})M^*_{(3)}(\mathbf{k})\\
=&\frac{1}{\pi}\int_{\mathbf{p},\mathbf{q},\mathbf{p}_2,\mathbf{p}_4}\mathcal{H}_1(\mathbf{p},\mathbf{q},\mathbf{p}_2,\mathbf{p}_4)f^{ab_1b_2}f^{db_3b_4}\rho_P^{b_1}(\mathbf{p}-\mathbf{p}_2)\rho_P^{b_2}(\mathbf{p}_2)\rho_P^{b_3}(\mathbf{q}-\mathbf{p}_4)\rho_P^{b_4}(\mathbf{p}_4)\\
&\qquad\times U^{ae}(\mathbf{k}-\mathbf{p})U^{de}(-\mathbf{k}-\mathbf{q})\\
&+\frac{1}{\pi}\int_{\mathbf{p},\mathbf{q},\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_4}\mathcal{H}_2(\mathbf{p},\mathbf{q},\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_4,\mathbf{k})f^{db_3b_4}f^{ebc}\rho_P^{b_1}(\mathbf{p}_1)\rho_P^{b_2}(\mathbf{p}_2)\rho_P^{b_3}(\mathbf{q}-\mathbf{p}_4)\rho_P^{b_4}(\mathbf{p}_4)\\
&\qquad\times U^{b_1b}(\mathbf{k}-\mathbf{p}-\mathbf{p}_1)U^{b_2c}(\mathbf{p}-\mathbf{p}_2)U^{de}(-\mathbf{k}-\mathbf{q})\\
&+\frac{1}{\pi}\int_{\mathbf{p},\mathbf{q},\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3,\mathbf{p}_4}\mathcal{H}_3(\mathbf{p},\mathbf{q},\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3,\mathbf{p}_4,\mathbf{k})f^{abc}f^{ade}\rho_P^{b_1}(\mathbf{p}_1)\rho_P^{b_2}(\mathbf{p}_2)\rho_P^{b_3}(\mathbf{p}_3)\rho_P^{b_4}(\mathbf{p}_4)\\
&\qquad\times U^{b_1b}(\mathbf{k}-\mathbf{p}-\mathbf{p}_1)U^{b_2c}(\mathbf{p}-\mathbf{p}_2)U^{b_3d}(-\mathbf{k}-\mathbf{q}-\mathbf{p}_3)U^{b_4e}(\mathbf{q}-\mathbf{p}_4)\\
&+\mathrm{c.c.}\,.
\end{aligned}$$



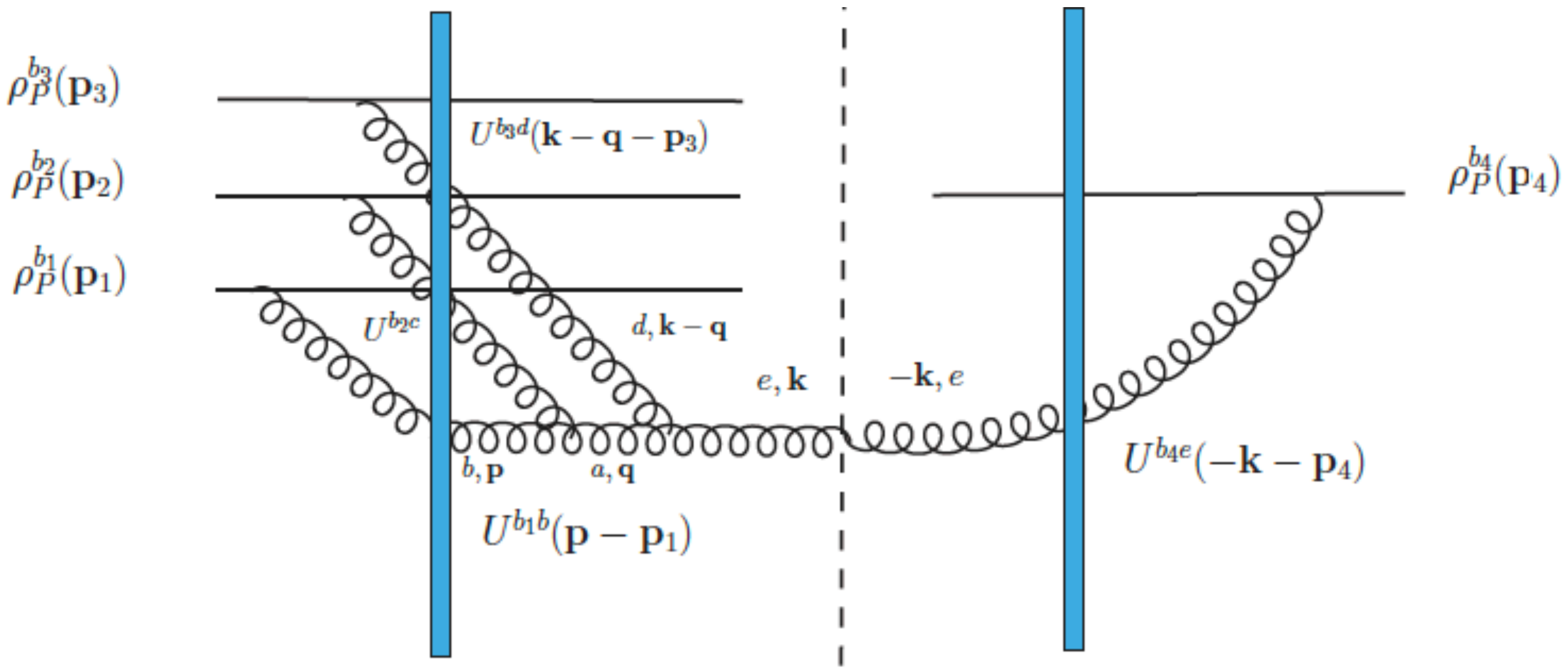


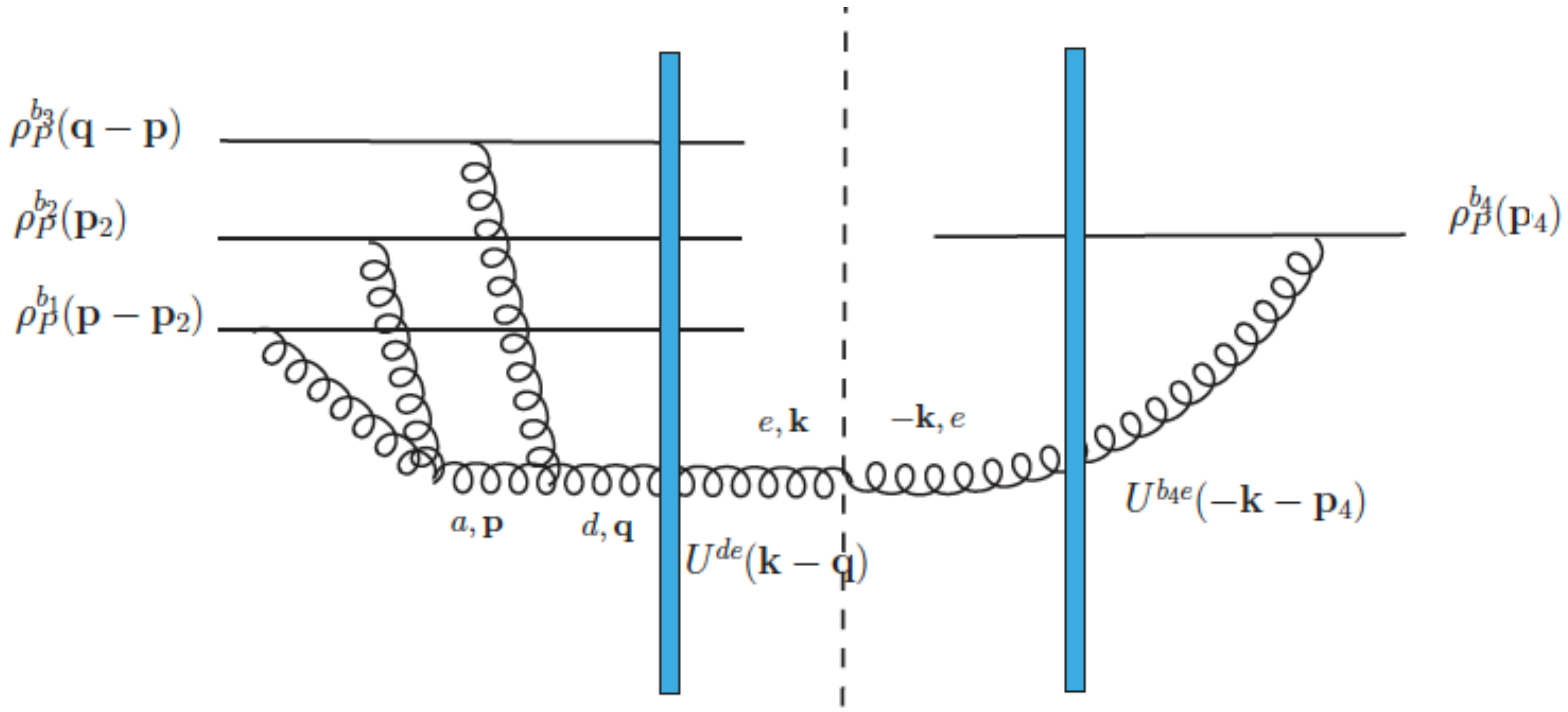


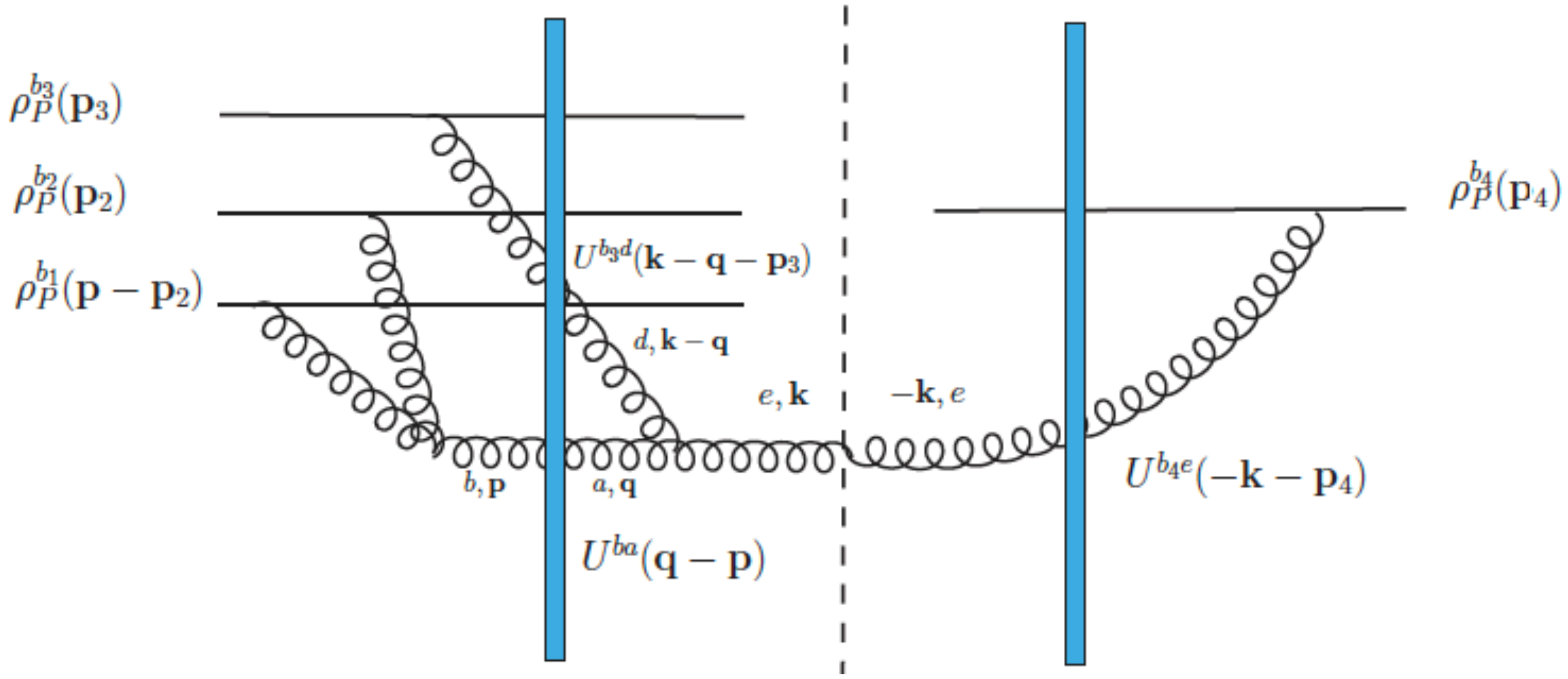
Results: First Saturation Correction

$$\left.\frac{d^2N}{d^2\mathbf{k}}\right|_{\mathrm{FSC}} = |M_{(3)}|^2 + M_{(1)}M^*_{(5)} + M^*_{(1)}M_{(5)}$$

$$
\begin{aligned}
&M_{(1)}M^*_{(5)} + M^*_{(1)}M_{(5)} \\
=&\frac{1}{\pi}\int_{\mathbf{p},\mathbf{q},\mathbf{p}_2,\mathbf{p}_4} \mathcal{F}_1(\mathbf{p},\mathbf{q},\mathbf{p}_2,\mathbf{p}_4,\mathbf{k}) f^{dab_3} f^{ab_1b_2} \rho_P^{b_1}(\mathbf{p}-\mathbf{p}_2)\rho_P^{b_2}(\mathbf{p}_2)\rho_P^{b_3}(\mathbf{q}-\mathbf{p})\rho_P^{b_4}(\mathbf{p}_4) \\
&\qquad \times U^{de}(\mathbf{k}-\mathbf{q})U^{b_4e}(-\mathbf{k}-\mathbf{p}_4) \\
&+\frac{1}{\pi}\int_{\mathbf{p},\mathbf{q},\mathbf{p}_2,\mathbf{p}_3,\mathbf{p}_4} \mathcal{F}_2(\mathbf{p},\mathbf{q},\mathbf{p}_2,\mathbf{p}_3,\mathbf{p}_4,\mathbf{k}) f^{bb_1b_2} f^{ade} \rho_P^{b_1}(\mathbf{p}-\mathbf{p}_2)\rho_P^{b_2}(\mathbf{p}_2)\rho_P^{b_3}(\mathbf{p}_3)\rho_P^{b_4}(\mathbf{p}_4) \\
&\qquad \times U^{ba}(\mathbf{q}-\mathbf{p})U^{b_3d}(\mathbf{k}-\mathbf{q}-\mathbf{p}_3)U^{b_4e}(-\mathbf{k}-\mathbf{p}_4) \\
&+\frac{1}{\pi}\int_{\mathbf{p},\mathbf{q},\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3,\mathbf{p}_4} \mathcal{F}_3(\mathbf{p},\mathbf{q},\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3,\mathbf{p}_4,\mathbf{k}) f^{abc} f^{ade} \rho_P^{b_1}(\mathbf{p}_1)\rho_P^{b_2}(\mathbf{p}_2)\rho_P^{b_3}(\mathbf{p}_3)\rho_P^{b_4}(\mathbf{p}_4) \\
&\qquad \times U^{b_1b}(\mathbf{p}-\mathbf{p}_1)U^{b_2c}(\mathbf{q}-\mathbf{p}-\mathbf{p}_2)U^{b_3d}(\mathbf{k}-\mathbf{q}-\mathbf{p}_3)U^{b_4e}(-\mathbf{k}-\mathbf{p}_4) \\
&+ \text{c.c.}
\end{aligned}
$$







Summary and Outlooks

- We presented results for the complete first saturation correction of the proton in high energy proton-nucleus collisions, especially its contribution to single inclusive semi-hard gluon production.
- Our results are expressed as a functional of the color charge densities of the projectile and the target (through the Wilson lines), these results are useful for computing double/multiple gluon productions.
- We need to average over the color charge configurations to obtain the event-averaged results, this will be the topic of a follow-up paper, as well as phenomenological predictions.