

Toward full result for NLO dijet production in proton-nucleus collisions

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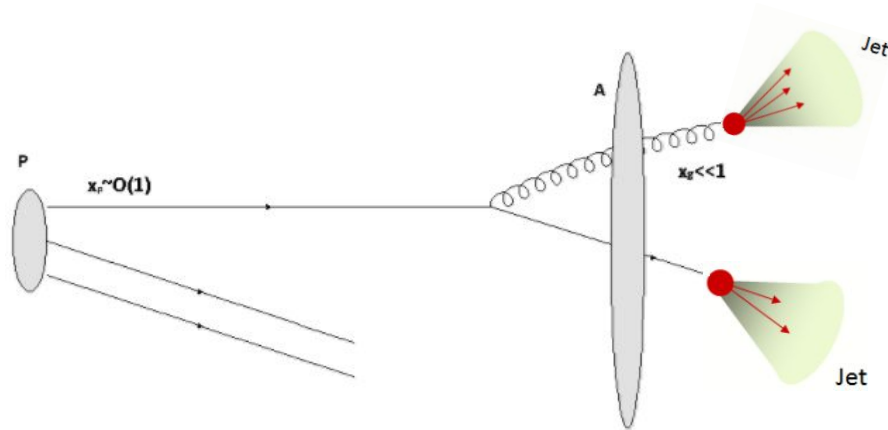
Based on hep-ph/1809.05526,
hep-ph/[2009.11930](#) (with E. Iancu)



Forward Jet Production

The basic setup: a large- x parton from the proton scatters off the small- x gluon distribution in the target nucleus. The large- x parton is most likely a quark. We adopt the formalism of the LC outgoing state, using the CGC effective theory together with the hybrid factorization.

The LO result appears in hep-ph/0708.0231 (C. Marquet).



Quark fragmentation in the presence of a shockwave.

The Outgoing State Formalism

The time evolution of the initial (bare) quark state is given by:

$$|q_\lambda^\alpha(q^+, \mathbf{q})\rangle_{\text{in}} \equiv U(0, -\infty) |q_\lambda^\alpha(q^+, \mathbf{q})\rangle$$

Where U denotes the unitary operator, defined as

$$U(t, t_0) = \text{T exp} \left\{ -i \int_{t_0}^t dt_1 H_I(t_1) \right\}$$

The quark outgoing state is given by:

$$|q_\lambda^\alpha(q^+, \mathbf{w})\rangle_{\text{out}} \equiv U(\infty, 0) \hat{S} U(0, -\infty) |q_\lambda^\alpha(q^+, \mathbf{w})\rangle$$

This state encodes the information both on the **time evolution** and **interaction with the target nucleus** of the incoming quark state.

The expectation values of operators are directly related to the outgoing state:

$$\langle \hat{O} \rangle = \left\langle {}_{\text{out}} \langle q_\lambda^\alpha(q^+, \mathbf{w}) | \hat{O} | q_\lambda^\alpha(q^+, \mathbf{w}) \rangle_{\text{out}} \right\rangle_{\text{egc}}$$

The LO Forward Dijet Cross Section

From the production state we can pass easily to the quark-gluon dijet cross section:

$$\frac{d\sigma_{\text{LO}}^{qA \rightarrow qg+X}}{d^3k d^3p} \equiv \frac{1}{2N_c L} \text{out} \langle q_\lambda^\alpha(q^+, \mathbf{q}) | \hat{N}_q(p) \hat{N}_g(k) | q_\lambda^\alpha(q^+, \mathbf{q}) \rangle_{\text{out}}$$

The following number density operators were introduced:

$$\hat{N}_q(p) \equiv \frac{1}{(2\pi)^3} b_\lambda^{\alpha\dagger}(p) b_\lambda^\alpha(p) \quad \hat{N}_g(k) \equiv \frac{1}{(2\pi)^3} a_i^{a\dagger}(k) a_i^a(k)$$

Then the result for the leading-order dijet cross section is given by (at large N_c):

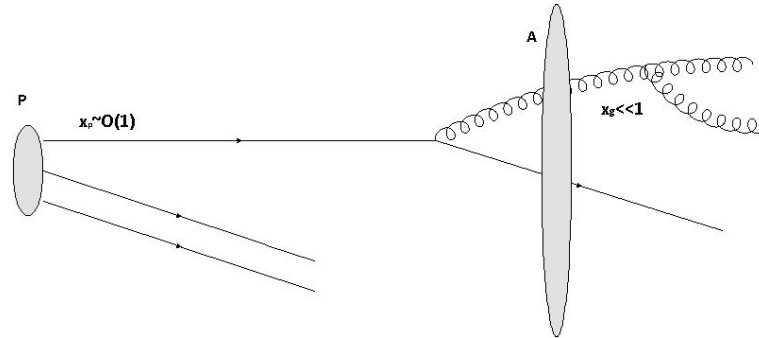
$$\begin{aligned} \frac{d\sigma_{\text{LO}}^{qA \rightarrow qg+X}}{dk^+ d^2\mathbf{k} dp^+ d^2\mathbf{p}} &= \frac{2\alpha_s C_F}{(2\pi)^6 q^+} \frac{(1 + (1 - \vartheta)^2)}{\vartheta} \delta(q^+ - k^+ - p^+) \\ &\times \int_{\mathbf{x}, \bar{\mathbf{x}}, \mathbf{z}, \bar{\mathbf{z}}} \frac{\mathbf{X} \cdot \bar{\mathbf{X}}}{\mathbf{X}^2 \bar{\mathbf{X}}^2} e^{-i\mathbf{p} \cdot (\mathbf{x} - \bar{\mathbf{x}}) - i\mathbf{k} \cdot (\mathbf{z} - \bar{\mathbf{z}})} \\ &\times [Q(\mathbf{x}, \mathbf{z}, \bar{\mathbf{z}}, \bar{\mathbf{x}}) \mathcal{S}(\mathbf{z}, \bar{\mathbf{z}}) - \mathcal{S}(\mathbf{x}, \mathbf{z}) \mathcal{S}(\mathbf{z}, \bar{\mathbf{w}}) - \mathcal{S}(\mathbf{w}, \bar{\mathbf{z}}) \mathcal{S}(\bar{\mathbf{z}}, \bar{\mathbf{x}}) + \mathcal{S}(\mathbf{w}, \bar{\mathbf{w}})] \end{aligned}$$

The Trijet Setup

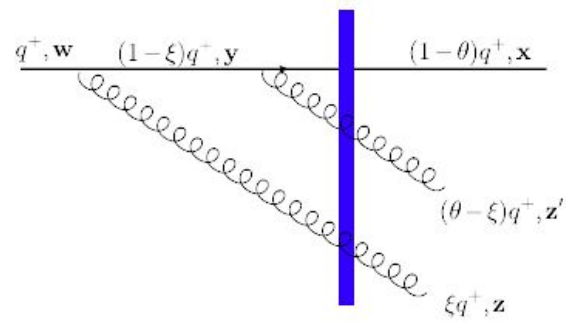
Two possible configurations of 3 particles in the final state:

- a) **Quark, quark and anti-quark,**
- b) **Quark together with two gluons.**

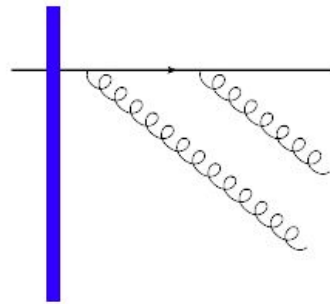
The production of these configurations happen via two successive parton splittings (in the light-cone formalism, there are also 1- \rightarrow 3 instantaneous vertices).



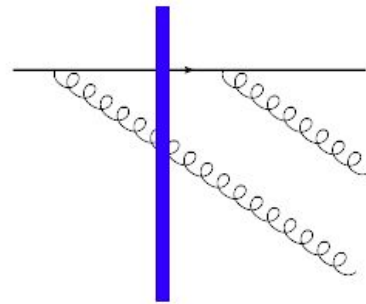
An example for a contribution with 3 particles in the final state



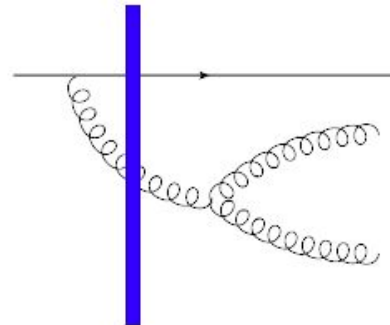
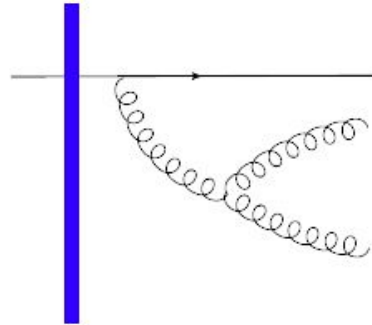
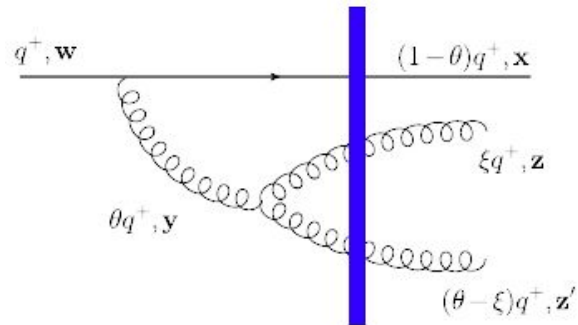
(a)



(b)



(c)



From Trijet to “real” NLO Dijet

The two contributions to the cross section are:

$$\frac{d\sigma^{qA \rightarrow qq\bar{q}+X}}{d^3q_1 d^3q_2 d^3q_3} \equiv \frac{1}{2N_c L} \text{out} \langle q_\lambda^\alpha(q^+, \mathbf{q}) | \hat{\mathcal{N}}_q(q_1) \hat{\mathcal{N}}_q(q_2) \hat{\mathcal{N}}_{\bar{q}}(q_3) | q_\lambda^\alpha(q^+, \mathbf{q}) \rangle_{\text{out}}$$

$$\frac{d\sigma^{qA \rightarrow qgg+X}}{d^3q_1 d^3q_2 d^3q_3} \equiv \frac{1}{2N_c L} \text{out} \langle q_\lambda^\alpha(q^+, \mathbf{q}) | \hat{\mathcal{N}}_q(q_1) \hat{\mathcal{N}}_g(q_2) \hat{\mathcal{N}}_g(q_3) | q_\lambda^\alpha(q^+, \mathbf{q}) \rangle_{\text{out}}$$

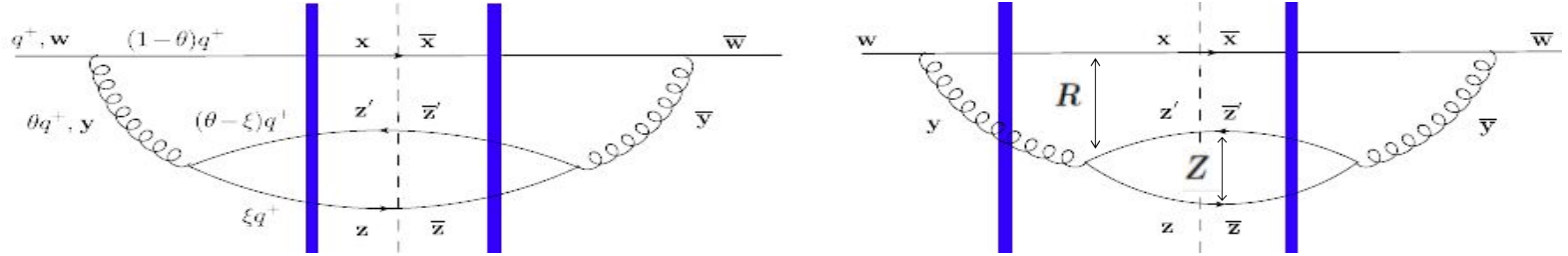
The trijet cross section is given by their sum:

$$\frac{d\sigma^{pA \rightarrow 3jet+X}}{d^3q_1 d^3q_2 d^3q_3} = \int dx_p q(x_p, \mu^2) \left(\frac{d\sigma^{qA \rightarrow qgg+X}}{d^3q_1 d^3q_2 d^3q_3} + \frac{d\sigma^{qA \rightarrow qq\bar{q}+X}}{d^3q_1 d^3q_2 d^3q_3} \right)$$

The real dijet cross section is related to trijet cross section by the integration over the unmeasured parton:

$$\frac{d\sigma_R^{qA \rightarrow 2jet+X}}{d^3q_1 d^3q_2} = \int d^3q_3 \frac{d\sigma^{qA \rightarrow 3jet+X}}{d^3q_1 d^3q_2 d^3q_3}$$

The quark quark anti-quark Trijet



$$\begin{aligned}
 \frac{d\sigma^{qA \rightarrow qq\bar{q}+X}}{dk_1^+ d^2k_1 dk_2^+ d^2k_2 dk_3^+ d^2k_3} &= \frac{\alpha_s^2 C_F N_f}{2(2\pi)^{10} (q^+)^2} \delta(q^+ - k_1^+ - k_2^+ - k_3^+) \\
 &\times \int_{\bar{x}, \bar{z}, \bar{z}', x, z, z'} e^{-ik_1 \cdot (x - \bar{x}) - ik_2 \cdot (z - \bar{z}) - ik_3 \cdot (z' - \bar{z}')} \frac{R^i Z^j \bar{R}^m \bar{Z}^n}{Z^2 \bar{Z}^2} \\
 &\times \left[\mathcal{K}_0^{ijmn}(x, z, z', \bar{x}, \bar{z}, \bar{z}', \vartheta, \xi) \mathcal{W}_0(x, z, z', \bar{x}, \bar{z}, \bar{z}') \right. \\
 &\left. - (z, z' \rightarrow y) - (\bar{z}, \bar{z}' \rightarrow \bar{y}) + (z, z' \rightarrow y \ \& \ \bar{z}, \bar{z}' \rightarrow \bar{y}) \right] + (k_1^+ \leftrightarrow k_2^+, k_1 \leftrightarrow k_2).
 \end{aligned}$$

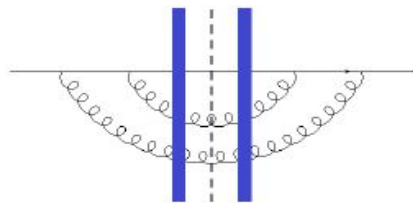
At large Nc:

$$\mathcal{W}_0(x, z, z', \bar{x}, \bar{z}, \bar{z}') \simeq Q(x, z', \bar{z}', \bar{x}) S(z, \bar{z}) - S(z, \bar{w}) S(x, z') - S(w, \bar{z}) S(\bar{z}', \bar{x}) + S(w, \bar{w})$$

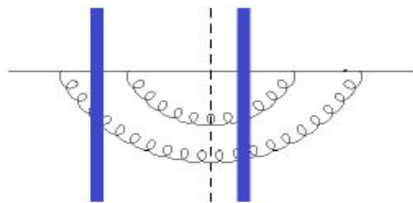
$$\mathcal{K}_0^{ijmn}(x, z, z', \bar{x}, \bar{z}, \bar{z}', \vartheta, \xi) \equiv \frac{\Phi_{\lambda_3 \lambda_2 \lambda_1 \lambda}^{ij}(x, z, z', \vartheta, \xi) \Phi_{\lambda_3 \lambda_2 \lambda_1 \lambda}^{mn*}(\bar{x}, \bar{z}, \bar{z}', \vartheta, \xi)}{[\vartheta^2(1 - \vartheta)R^2 + \xi(\vartheta - \xi)Z^2] [\vartheta^2(1 - \vartheta)\bar{R}^2 + \xi(\vartheta - \xi)\bar{Z}^2]}$$

The effective vertex contain the information about both the reg. and inst. interactions.

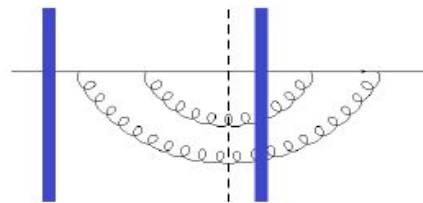
Gluons Contributions to Trijet Production



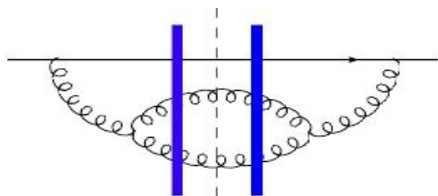
(a)



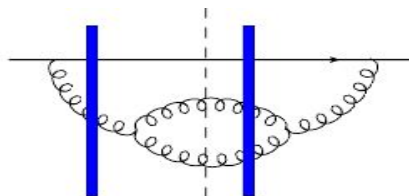
(b)



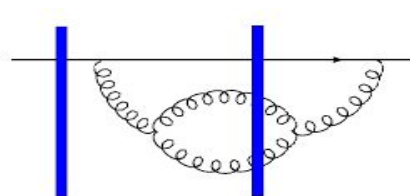
(c)



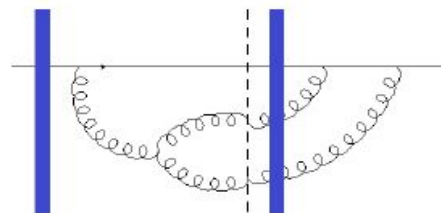
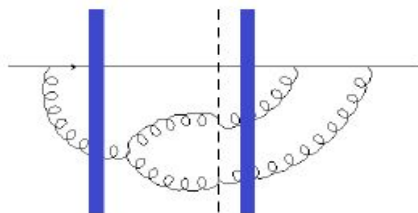
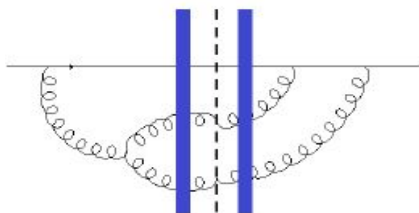
(a)



(b)

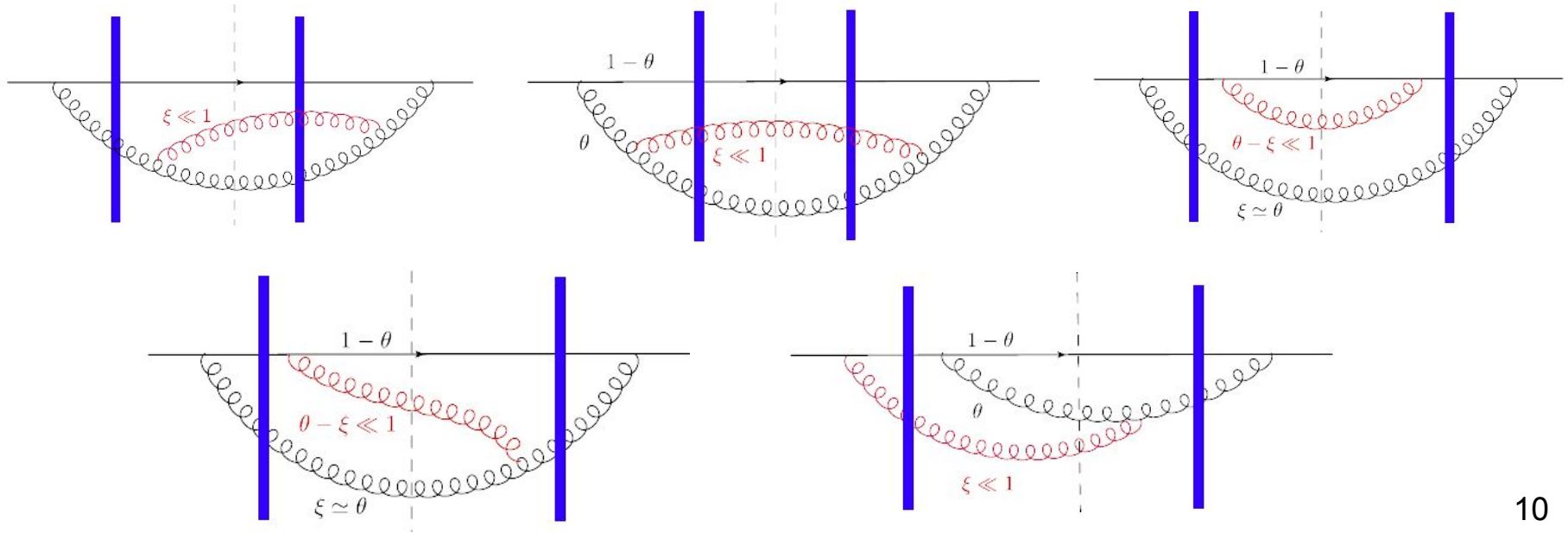


(c)



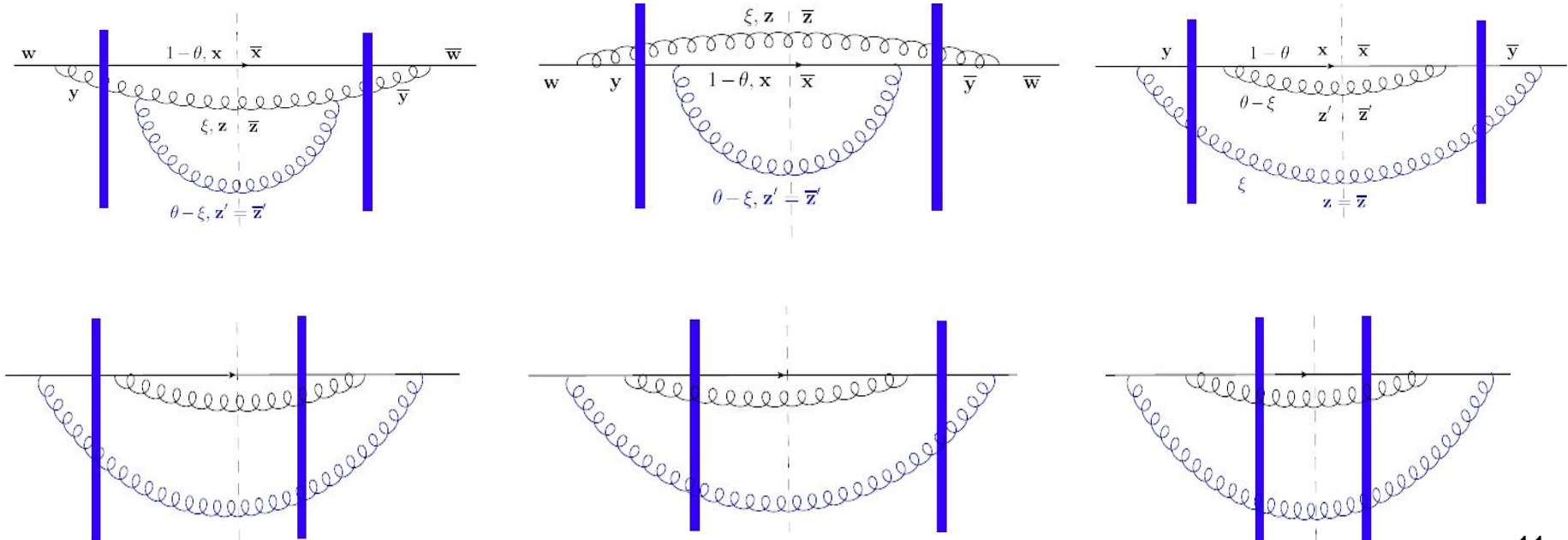
Recovering the B-JIMWLK Evolution

In the limit when one of the gluons become soft (eikonal emission vertex = no recoil of the emitter), the general NLO result has to reduce to one step in the real part of B-JIMWLK evolution of the leading order dijet production result.



Recovering the Real DGLAP Evolution

In the collinear limit, when the separation between partons become arbitrarily large, we recover the DGLAP evolution of the initial / final quark state distribution.



Summary

- 1) We computed the NLO outgoing state and dijet production cross section of an incoming quark.
- 2) Full match has been established between the eikonal limit of the NLO result and the action of JIMWLK on the LO cross section.
- 3) Full match has been established with DGLAP evolution in the collinear limit.