Transverse momentum dependent splitting functions in the Parton Branching method

DIS 2021

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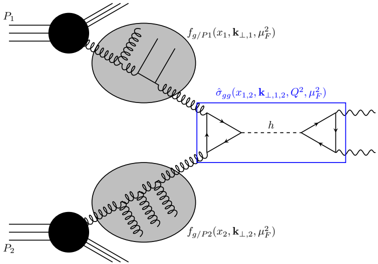
Universiteit Antwerpen

Motivation



- Collinear factorization is commonly used to describe hard production processes at LHC
- Some classes of processes require more general scheme
- Factorization in partonic cross-section and transverse momentum dependent PDFs (TMDs)

- TMDs much less known than PDFs at present $\longrightarrow$ future experimental programs
- TMDs from Parton Branching (PB) method: Can be used in Monte Carlo events generators $\longrightarrow$ Widely applicable

Parton Branching Method

- $P^R_{ab}(z)$: (real emission part of) Splitting functions: Probabilty that a branching will happen
  b: incoming parton, a: outgoing parton, z momentum fraction of parton a to b
- Sudakov form factor:
  $\Delta_a(\mu^2) = \exp\left(-\sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int_0^{z_M} dz\, z\, P^R_{ba}(z)\right)$
  Interpretation: probability of an evolution without any resolvable branchings

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Iterative form of the PB evolution equations: [Hautmann, Jung, Lelek, Radescu, Zlebcik JHEP 01 (2018) 070, 1708.03279]

$$\tilde{A}_a(x, \mathbf{k}_\perp, \mu^2) = \Delta_a(\mu^2)\tilde{A}_a(x, \mathbf{k}_\perp, \mu_0^2) + \sum_b \int \frac{d^2\mu'_\perp}{\pi\mu'^2}\Theta(\mu^2 - \mu'^2)\Theta(\mu'^2 - \mu_0^2)\frac{\Delta_a(\mu^2)}{\Delta_a(\mu'^2)} \times$$
$$\times \int_x^{z_M} dz P^R_{ab}(z)\Delta_b(\mu'^2)\tilde{A}_b\left(\frac{x}{z}, \mathbf{k}_\perp + (1-z)\mu'_\perp, \mu_0^2\right) + \ldots$$

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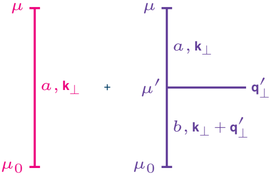

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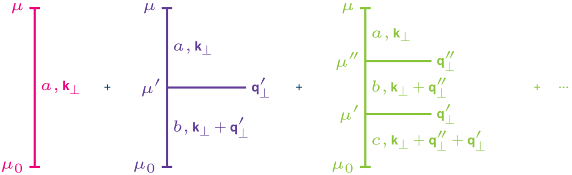

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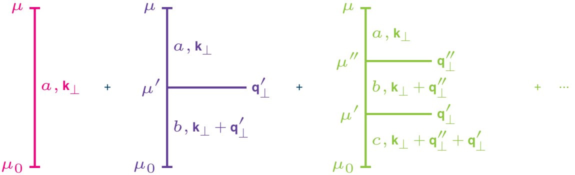

$\mathbf{q}'_\perp = (1-z')\mu'$
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PB calculates $\mathbf{k}_\perp$ from every branching:
$\mathbf{k}_\perp = \mathbf{k}_{\perp,0} - \sum_i \mathbf{q}_{\perp,i}$

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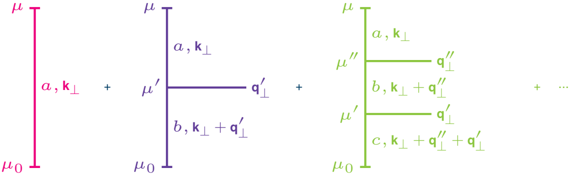

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$\mathbf{k}_\perp = \mathbf{k}_{\perp,0} - \sum_i \mathbf{q}_{\perp,i}$

Can be solved with MC methods.

TMD Splitting functions

- PB uses DGLAP splitting functions, but those are not valid for small-x
- In PB $k_\perp$ is known at every branching

$\Rightarrow$ Goal of this work: extend PB by including TMD splitting functions

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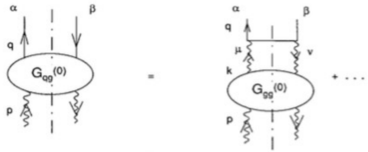
- Concept from high-energy factorization [Catani, Hautmann NPB427 (1994) 475524, hep-ph/9405388]
- Goal of TMD Splitting Functions:
  - Resummation in α_s ln $\frac{1}{x}$
  - Exact kinematics in both $k_\perp$ and x



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  - Resummation in $\alpha_s \ln \frac{1}{x}$
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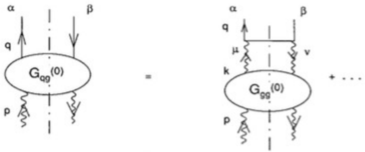
- $\tilde{P}_{qg}(z, \mathbf{k}_\perp, \mathbf{q}_\perp)$ originally calculated



- Recently other splitting functions calculated [Gituliar, Hentschinski, Kutak JHEP 01 (2016) 181, 1511.08439], [Hentschinski, Kusina, Kutak, Serino EPJC 78 (2018) 174, 1711.04587]

TMD Splitting functions in PB

TMD Splitting functions: $\tilde{P}^{R}_{ab}(z, \mathbf{k}'_{\perp}, \mu'_{\perp}) = \frac{d^2\tilde{\mathbf{k}}_{\perp}}{\tilde{\mathbf{k}}^2_{\perp}} \frac{{\mu'}^2_{\perp}}{d^2\mu'_{\perp}} \tilde{P}^{R}_{ab}(z, \mathbf{k}'_{\perp}, \tilde{\mathbf{k}}_{\perp})$[1]

- Defined within 2GI-Kernel, Jacobian included in redefinition
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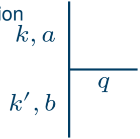

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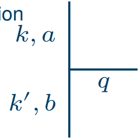

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In limit $\mathbf{k}'_\perp \to 0$

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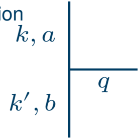

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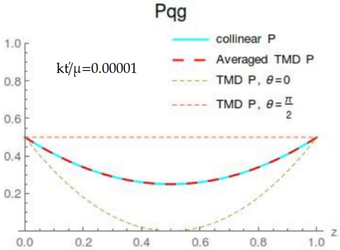

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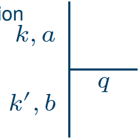

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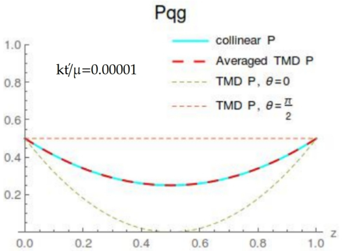

All $\tilde{P}^R_{ab}(z, \mathbf{k}'_\perp, \mu'_\perp)$ are positive definite

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$k_\perp$-dependent Sudakov form factor

Resolvable branchings:

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Sudakov form factor: probability of an evolution without any resolvable branchings

- Definition of $\mathbf{k}_\perp$-dependent Sudakov form factor
- Should sum over all possible splittings $\to$ Angular averaged splitting functions

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$$\Delta_a(\mu^2, \mathbf{k}_\perp^2) = \exp\left(-\sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int_0^{z_M} dz\, z\, \bar{P}^R_{ba}(z, k_\perp, \mu')\right)$$

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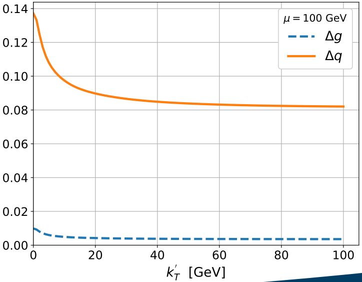

The Sudakov form factor decrease with higher $k_\perp$

Studies on TMD P in evolution equations

New evolution equation:

$$\tilde{\mathcal{A}}_a(x,\mathbf{k}_\perp,\mu^2) = \Delta_a(\mu^2,\mathbf{k}_\perp)\tilde{\mathcal{A}}_a(x,\mathbf{k}_\perp,\mu_0^2) + \sum_b \int \frac{d^2\mu'_\perp}{\pi\mu'^2}\frac{\Delta_a(\mu^2,\mathbf{k}_\perp)}{\Delta_a(\mu'^2,\mathbf{k}_\perp)}\Theta(\mu^2-\mu'^2)\Theta(\mu'^2-\mu_0^2)\times$$
$$\times\int_x^{z_M} dz \tilde{P}^R_{ab}(z,\mathbf{k}_\perp+(1-z)\mu'_\perp,\mu'_\perp)\tilde{\mathcal{A}}_b(\frac{x}{z},\mathbf{k}_\perp+(1-z)\mu'_\perp,\mu'^2)$$

Studies on evolution using same starting distribution:

$$\tilde{\mathcal{A}}_a(x,\mathbf{k}_{\perp,0},\mu_0^2) = \tilde{f}_a(x,\mu_0^2)\times\frac{1}{q_s^2\pi}\,\text{exp}\left(-\frac{\mathbf{k}_{\perp,0}^2}{q_s^2}\right),$$

$\tilde{f}_a(x,\mu_0^2)$:HERAPDF20_LO_EIG,
μ_0 =1.3784 GeV, q_s=0.5GeV

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$\tilde{f}_a(x,\mu_0^2)$:HERAPDF20_LO_EIG,
μ_0 =1.3784 GeV, q_s=0.5GeV

Momentum of proton is **conserved** in equation with **TMD P and TMD Sudakov**
(and for col. P with col. Sudakov: the regular PB method):

Studies on TMD P in evolution equations

New evolution equation:

$$\tilde{\mathcal{A}}_a(x,\mathbf{k}_\perp,\mu^2) = \Delta_a(\mu^2,\mathbf{k}_\perp)\tilde{\mathcal{A}}_a(x,\mathbf{k}_\perp,\mu_0^2) + \sum_b \int \frac{d^2\mu'_\perp}{\pi\mu'^2}\frac{\Delta_a(\mu^2,\mathbf{k}_\perp)}{\Delta_a(\mu'^2,\mathbf{k}_\perp)}\Theta(\mu^2-\mu'^2)\Theta(\mu'^2-\mu_0^2)\times$$
$$\times\int_x^{z_M} dz \tilde{P}^R_{ab}(z,\mathbf{k}_\perp+(1-z)\mu'_\perp,\mu'_\perp)\tilde{\mathcal{A}}_b(\frac{x}{z},\mathbf{k}_\perp+(1-z)\mu'_\perp,\mu'^2)$$

Studies on evolution using same starting distribution:

$$\tilde{\mathcal{A}}_a(x,\mathbf{k}_{\perp,0},\mu_0^2) = \tilde{f}_a(x,\mu_0^2)\times\frac{1}{q_s^2\pi}\,\text{exp}\left(-\frac{\mathbf{k}^2_{\perp,0}}{q_s^2}\right),$$

$\tilde{f}_a(x,\mu_0^2)$:HERAPDF20_LO_EIG, μ_0 =1.3784 GeV, q_s=0.5GeV

Momentum of proton is **conserved** in equation with **TMD P and TMD Sudakov** (and for col. P with col. Sudakov: the regular PB method):

$\sum_a \int_{10^{-5}}^1 dx \tilde{f}_a(\mu^2,x)$:

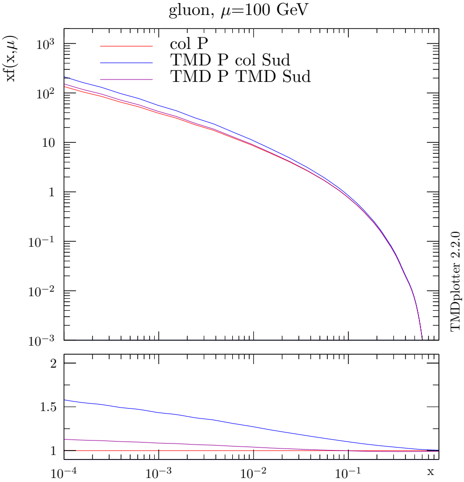
μ (GeV)	col. P	TMD P col. Sud.	TMD P TMD Sud.
100	0.992	1.129	0.991

Collinear PDFs or integrated TMD: $\tilde{f}_a(x,\mu^2) = \int d^2\mathbf{k}_\perp \tilde{\mathcal{A}}_a(x,\mathbf{k}_\perp,\mu^2)$

Integrated TMDs

We studied effects of TMD Splitting functions on the evolution.
No fits has been done yet:

- Not yet ready for phenomenology



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- PB with TMD Splitting functions in resolvable branchings, but collinear Sudakov
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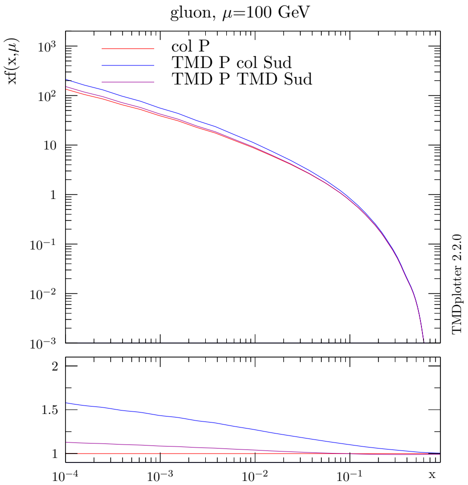

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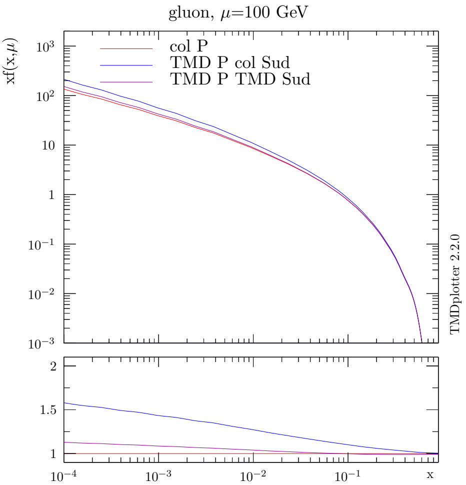
- Effects are small for large x $\rightarrow$ reasonable since $\tilde{P}_{ab}(z, \mathbf{k}_\perp, \mathbf{q}_\perp) \xrightarrow[k_\perp \to 0]{} P_{ab}(z)$
- TMD Sudakov suppression compared to collinear Sudakov
- PB is capable of handling TMD P and TMD sudakov



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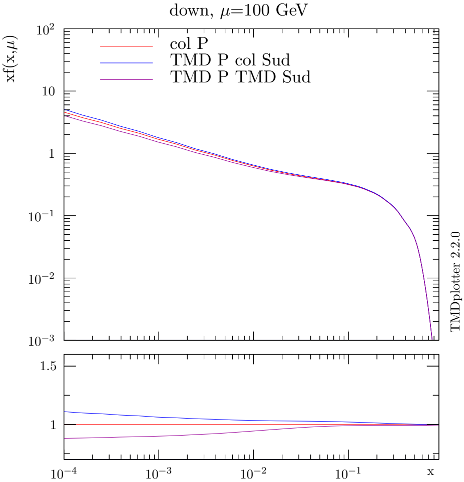

TMDs vs $\mathbf{k}_\perp$

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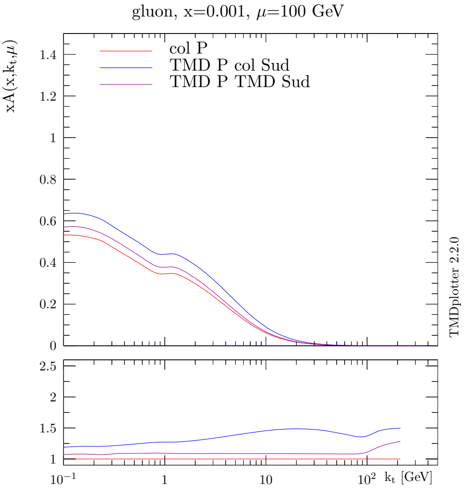
- Whole $\mathbf{k}_\perp$-region is affected
- Suppression from TMD Sudakov in whole $k_\perp$-region



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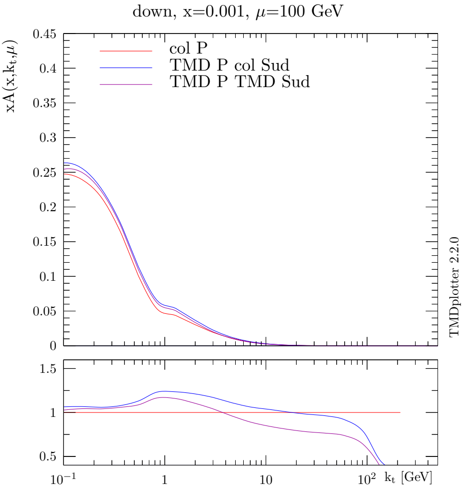

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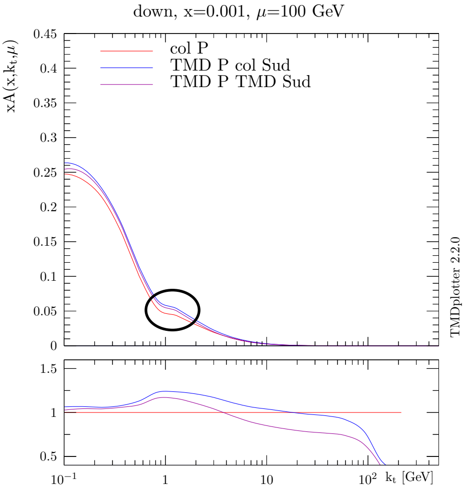
Kink distributions $\rightarrow$ non-perturbative input



Non-perturbative input

Dynamical resolution scale:

[Hautmann, Keersmaekers, Lelek, van Kampen NuclPhysB

(2019) 114795,1908.08524]

$z_M = 1 - \frac{q_0}{\mu'} \to q_\perp > q_0$

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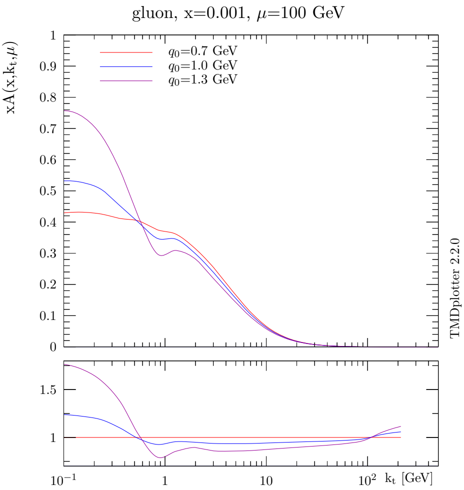
Kink due to matching intrinsic $k_{\perp,0}$ and evolution

- $\mathbf{k}_\perp = \mathbf{k}_{\perp,0} - \sum_i \mathbf{q}_{\perp,i} \rightarrow$ $k_\perp$ from evolution starts to build around q_0
- Many branchings smooth out kinks



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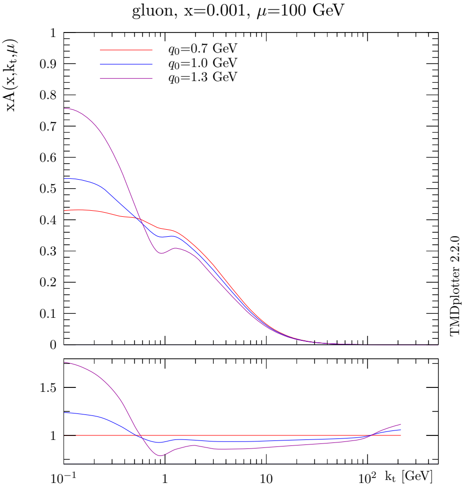

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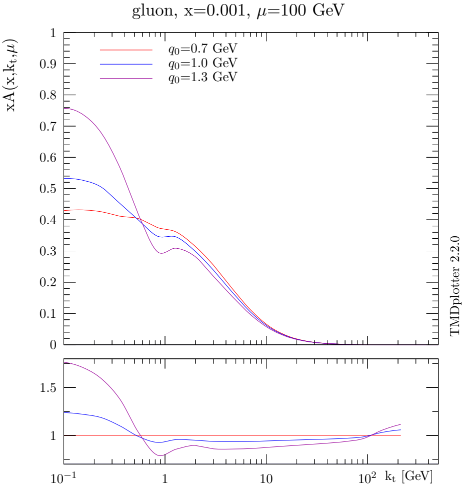
TMD P: similar results



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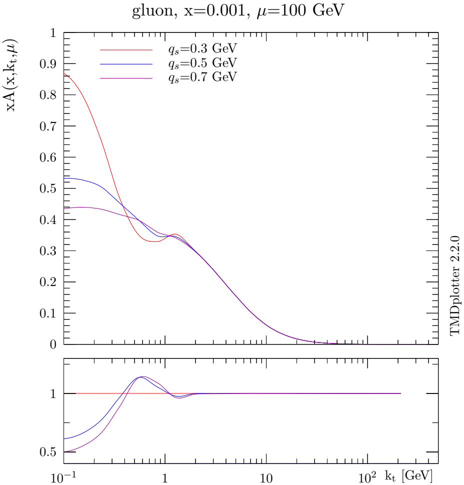
When q_s close to q_0: smoother

Large $k_\perp$-region unaffected by q_s when $\mu \gg \mu_0$



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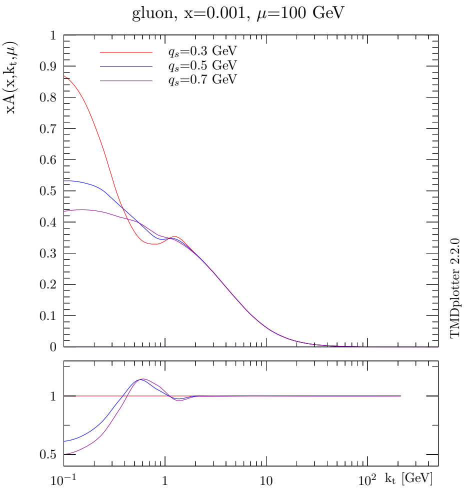

Summary and outlook

- I presented a parton branching algorithm for space-like parton evolution with $k_\perp$-dependent splitting functions
- The splitting functions are a (positive-definite) $k_T \neq 0$ continuation of the LO DGLAP splitting functions originally obtained from high-energy factorization
- $k_\perp$-dependent splittings affect both real emission and Sudakov form factors
- Our evolution equations conserve the momentum of the proton
- New code is working and produces both collinear and TMD parton distributions - paper in preparation
- Prospects to do phenomenology:
  - Perform fits to DIS and DY data to determine nonperturbative TMDs
  - Use them to make PB-TMD predictions for LHC and EIC processes including for the first time the effects of TMD splittings

Thank you!