RECENT ADVANCES ON SPIN AND 3D NUCLEON STRUCTURE

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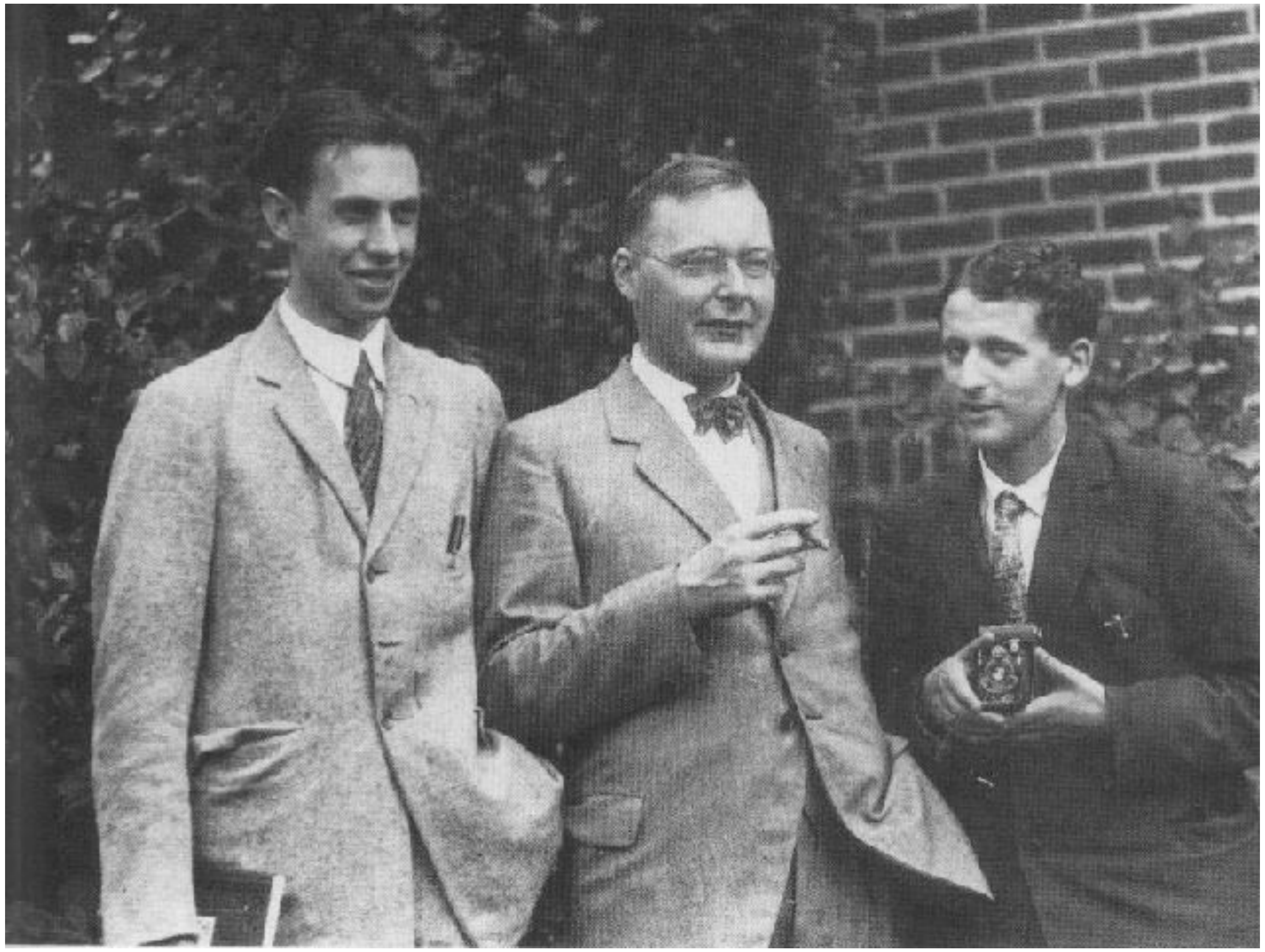
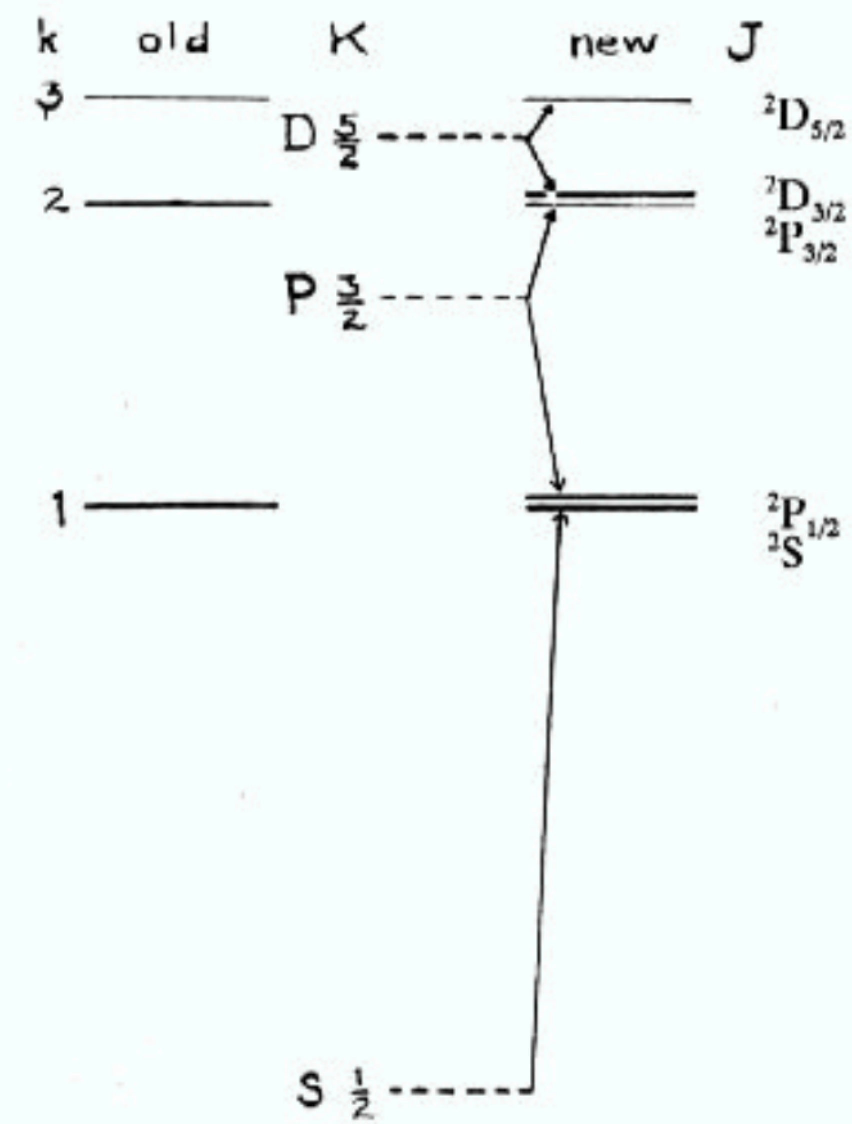
Goudsmith and Uhlenbeck 1925: "self-rotating" electron





"This is a good idea. Your idea may be wrong, but since both of you are so young without any reputation, you would not lose anything by making a stupid mistake" (Ehrenfest)

G.E. Uhlenbeck and S. Goudsmith, Naturwissenschaften 47, 953 (1925); Nature 117, 264 (1926)

"If Kronig had not left Leiden and had stayed with Ehrenfest, then things would have taken another course. Ehrenfest would have encouraged him and said: "That you ought publish". With Pauli, of course, it was entirely different….. "(Goudsmith, April 1971)

25 March 1926.

Dear Goudsmit

....

I think you and Uhlenbeck have been very lucky to get your spinning electron published and talked about it before Pauli heard of it. It appears that more than a year ago Kronig believed in the spinning electron and worked out something; the first person he showed it to was Pauli. Pauli ridiculed the whole thing so much that the first person became also the last and no one else heard anything of it. Which all goes to show that the infallibility of the Deity does not extend to his self-styled vicar on earth.

Part of a letter by L.A. Thomas to Goudsmith, 1926: Goudsmith archive of the AIP Center for History of Physics

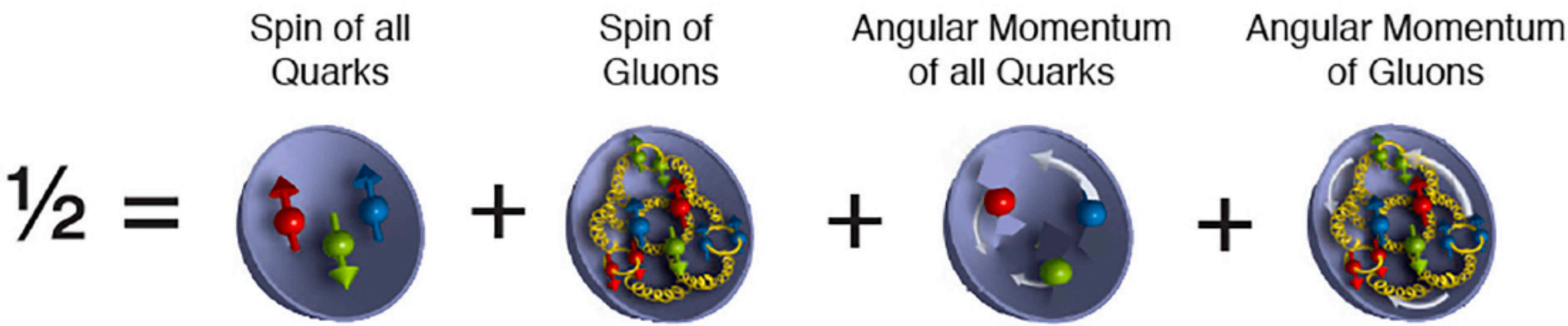
After almost 100 years,
what did we learn about the spin of the proton?

The most important lesson we have learned is that
QCD has a very sophisticated way to build up the proton structure

How can we explore the origin of the nucleon spin?

- Sum Rules: use decompositions of the nucleon spin in various physical parts and relate them to experimental observables
- Lattice QCD: ab initio calculations of spin
- 3D nucleon structure: assessments of spin and orbital angular momentum from multidimensional map of the nucleon

Spin Sum Rules



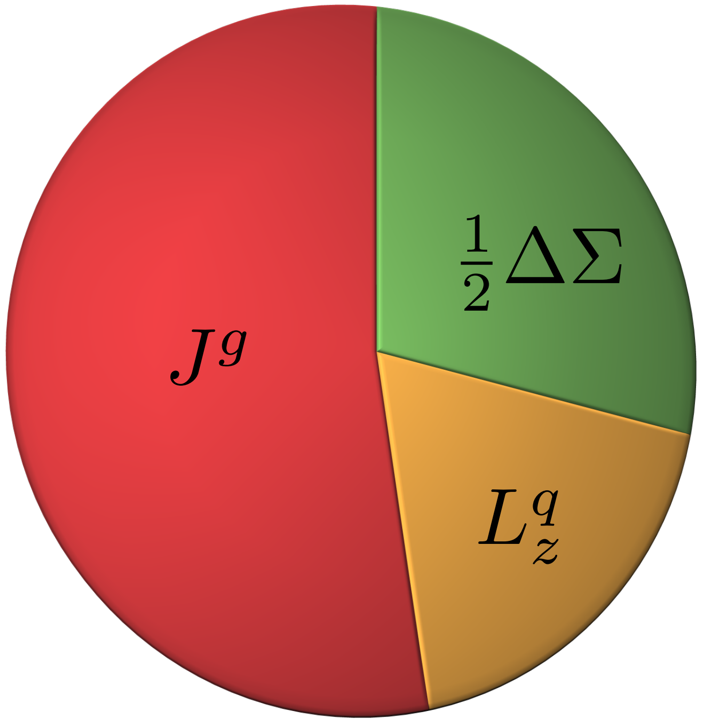
Each individual parts is renormalisation scale dependent

There exist more than one way to split the Angular Momentum Operator

We look for a spin sum rule with the following properties:

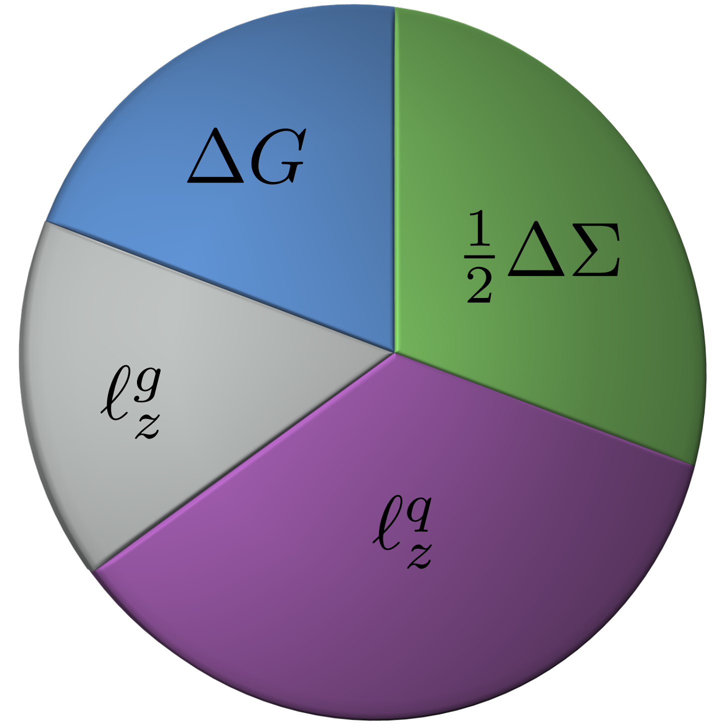
- Experimental measurability: HERMES, COMPASS, polarized RHIC, JLab12, EIC, EICc, ….
- Intuitive and clear partonic interpretation

Ji (covariant) Sum Rule	Jaffe-Manohar (canonical) Sum Rule



$$\frac{1}{2} = \underbrace{\frac{1}{2}\Delta\Sigma(\mu) + L_z^q(\mu)}_{J^q} + J^g(\mu)$$

- each term is gauge invariant
- frame independent
- it works also for the transverse AM in the infinite momentum frame
- J^q and J^g can be obtained from moments of GPDs



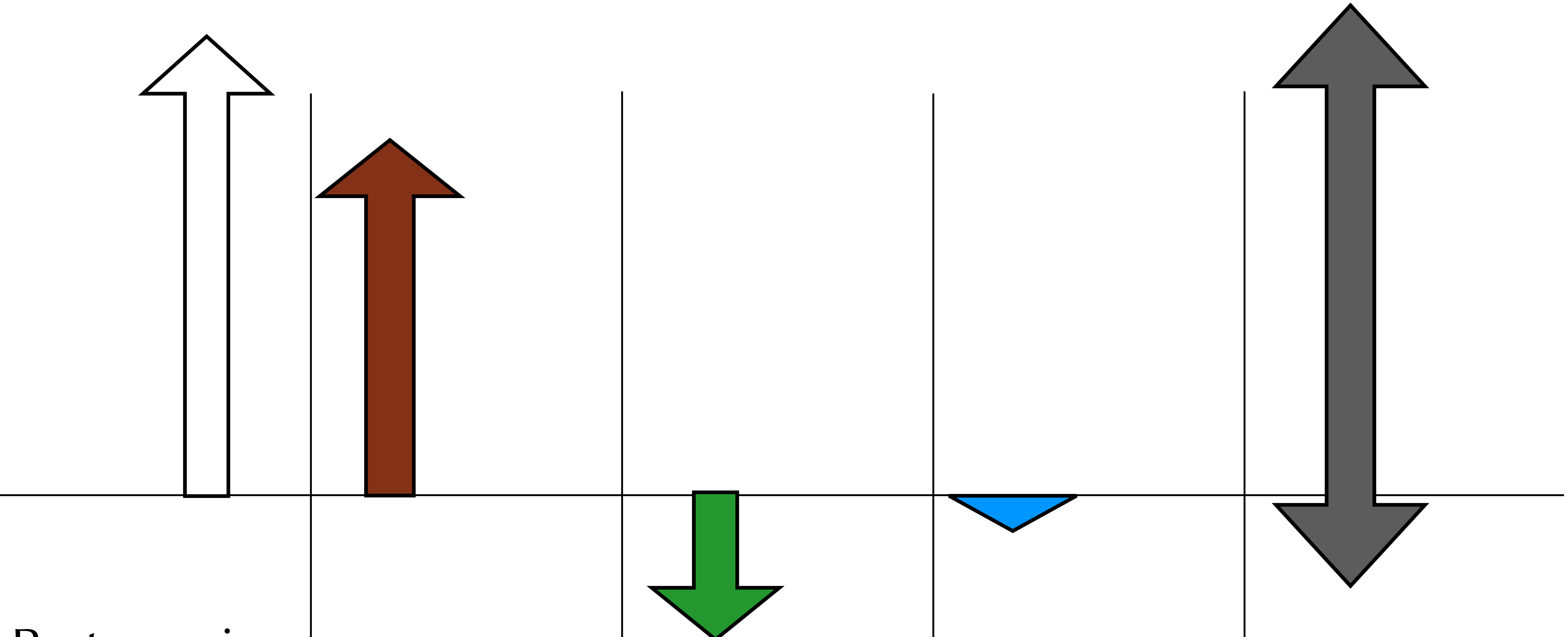
$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma(\mu) + \ell_z^q(\mu) + \ell_z^g(\mu) + \Delta G(\mu)$$

- ℓ_z^q, ℓ_z^g, ΔG are gauge dependent, BUT measurable
- simple partonic interpretation in the IMF
- $\Delta\Sigma$, ΔG can be obtained from pol. PDFs
- ℓ_z^q, ℓ_z^g can be obtained from twist-3 GPDs and Wigner distributions

$$\ell_z^q = L_z^q + l_z^{q,\mathrm{pot}} \longrightarrow \ell_z^g + \Delta G + l_z^{q,\mathrm{pot}} = J^g$$

Y. Guo, X. Ji, K. Shiells, arXiv: 2101.05243; X. Ji, F. Yuan, Y. Zhao, Nature Rev. Phys. 3 (2021) 1

Status of Spin Sum Rule



Proton spin
(Q^2=10 GeV^2)

$u+\bar{u}$		$d+\bar{d}$		$s+\bar{s}$		g		
spin (%)	OAM	spin (%)	OAM	spin (%)	OAM	spin (%)	OAM	
81 ± 1		-43 ± 1		-11 ± 2		40 ± 60		DSSV14
79 ± 5		-45 ± 5		12 ± 8		3 ± 324		NNPDF

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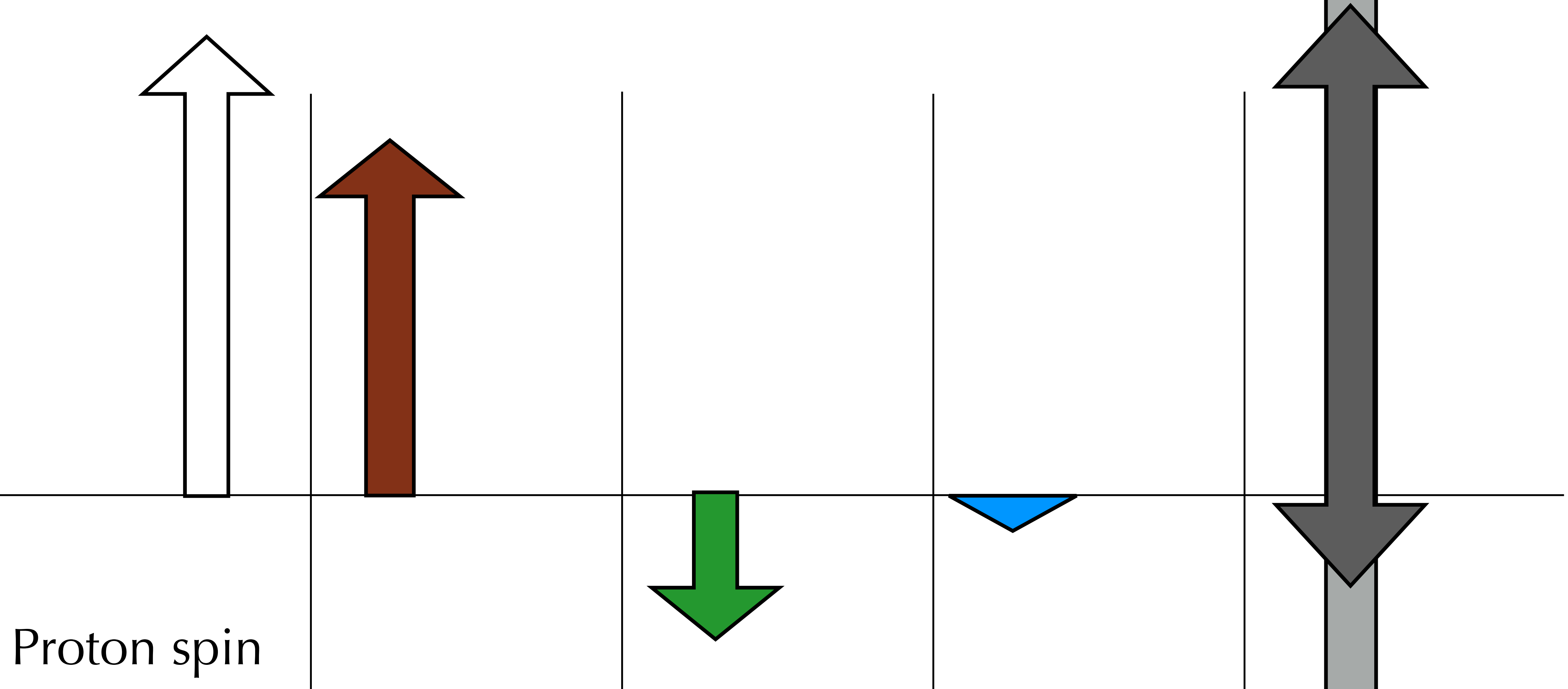

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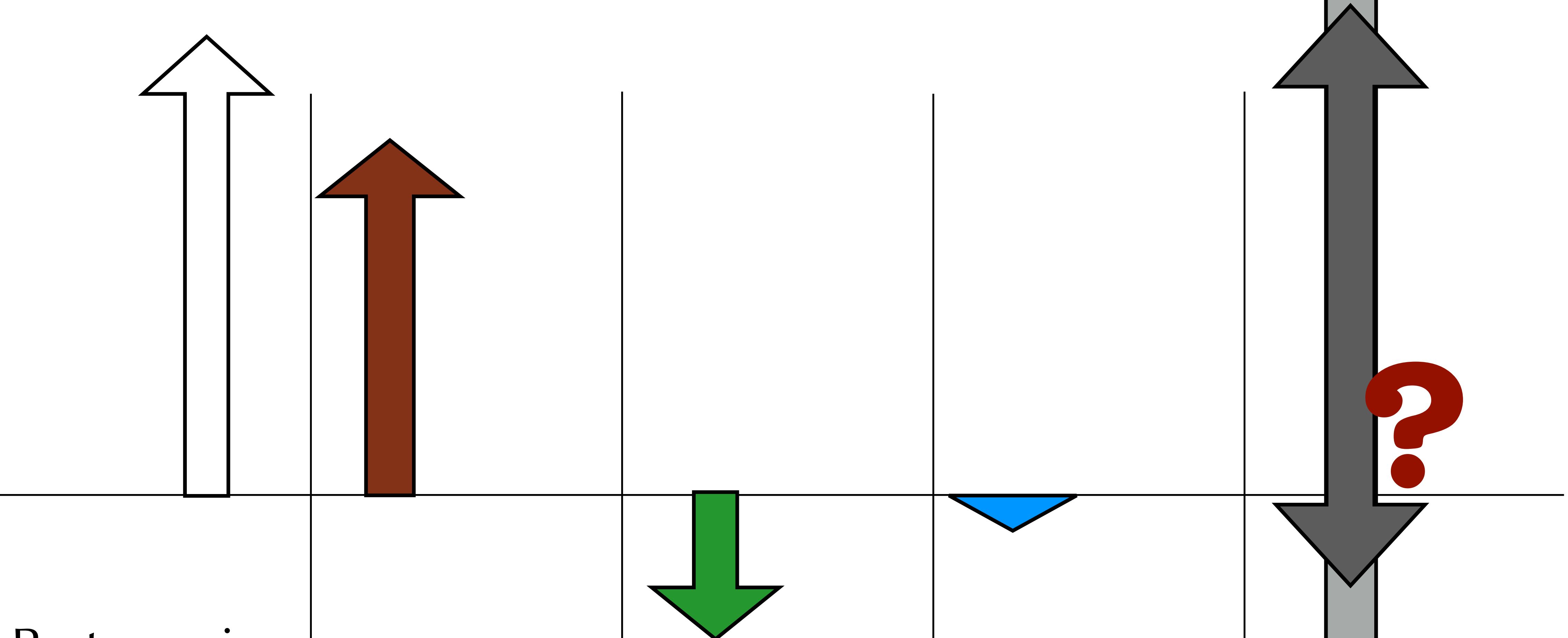

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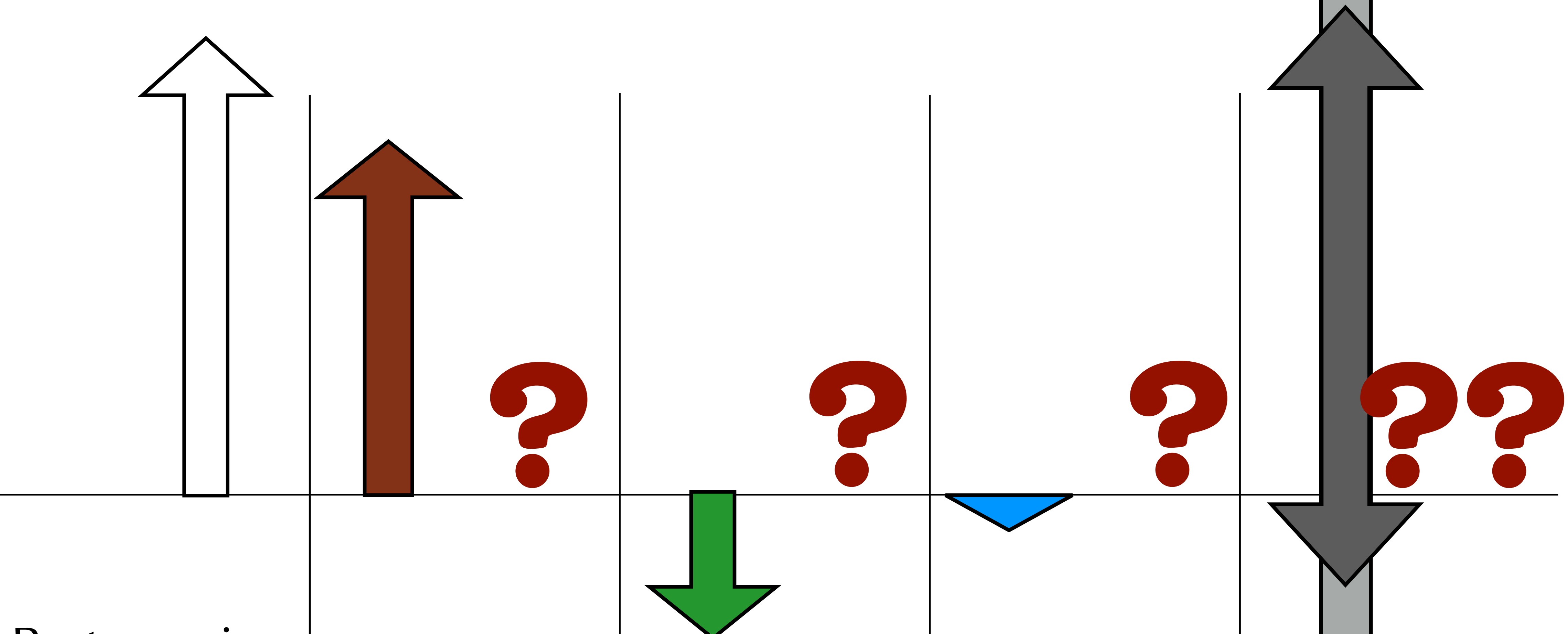

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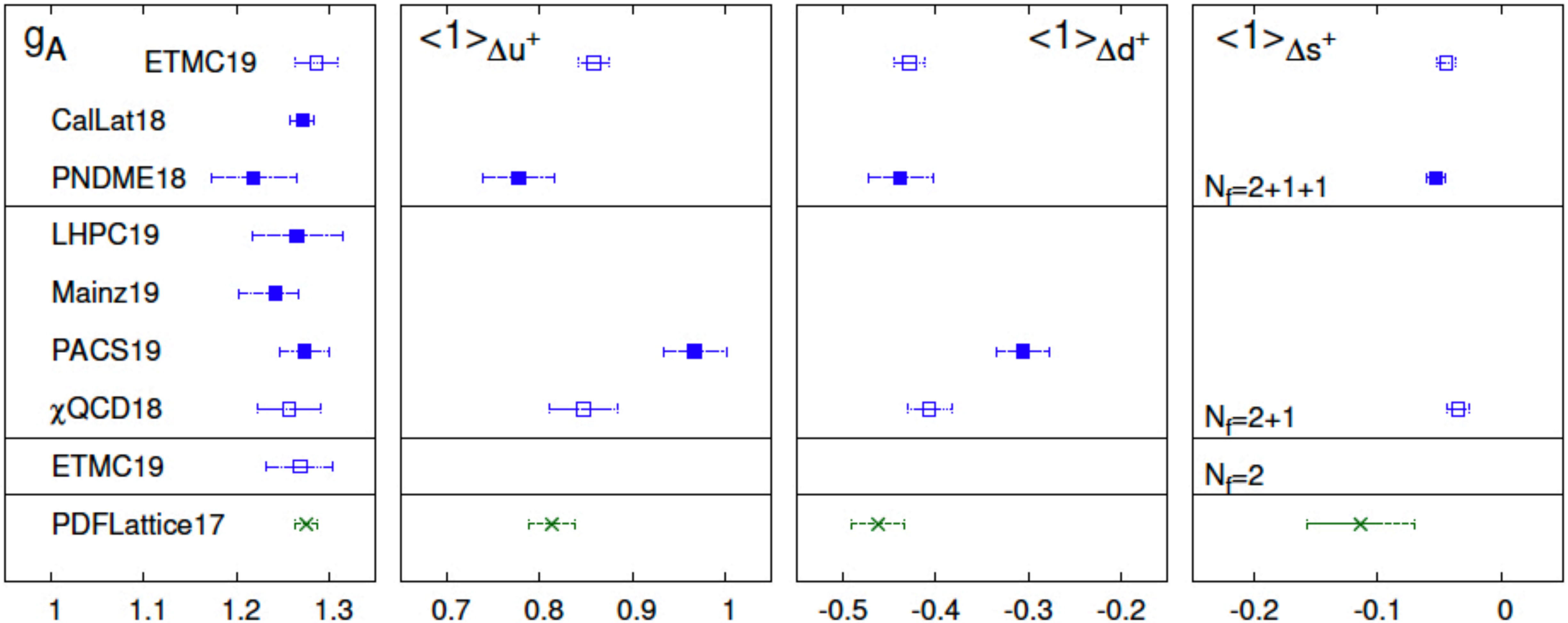
Comparison with Lattice calculations

Quark spin $Q^2 = 4\ \mathrm{GeV}^2$



PDFLattice17: average of phenomenological extractions: JAM15, NNPDFpol1.1, DSSV08 from the community white paper, Prog.Part.Nucl.Phys. 100 (2018) 107

Overall fair agreement between lattice calculations and phenomenological fits

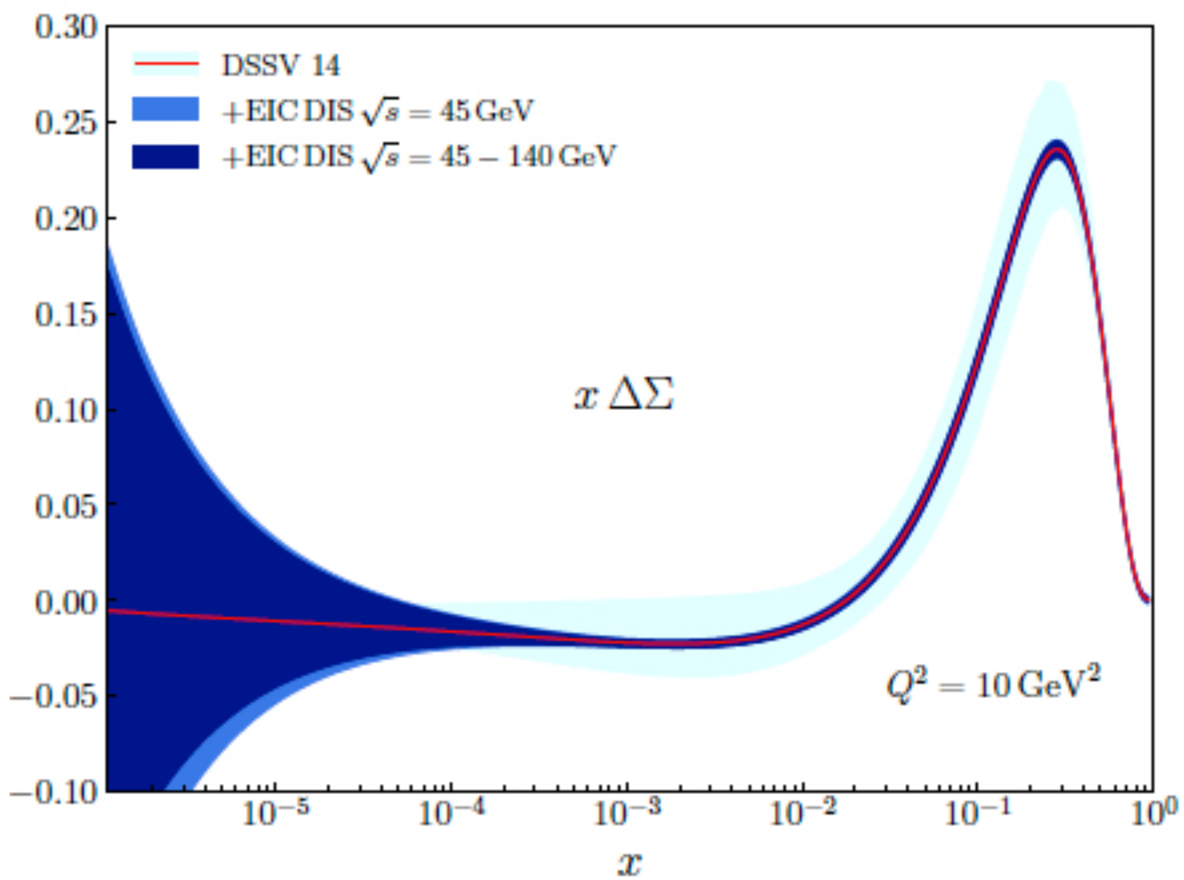
The uncertainties of the two have comparable size

Lattice QCD results could provide useful inputs to global fits of polarized PDFs

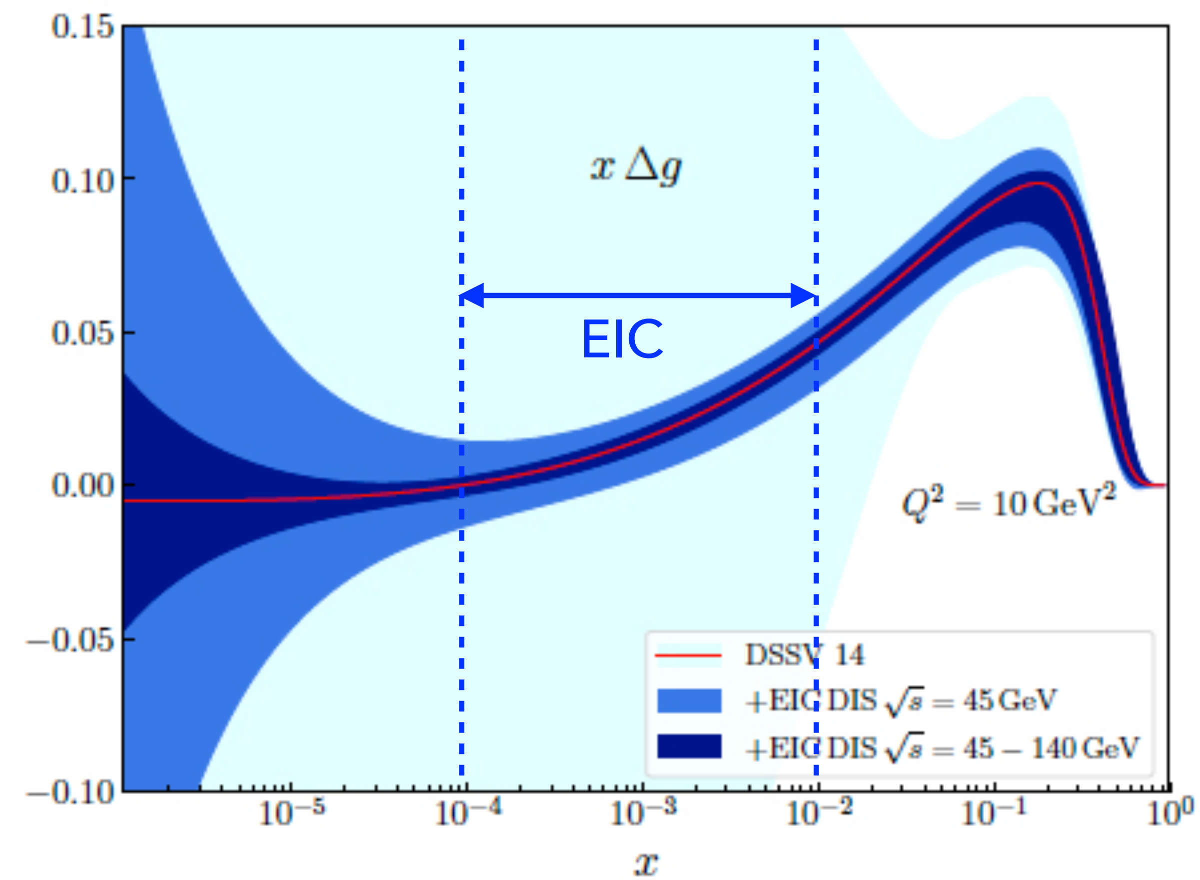
→ *Talk of N. Sato, Spin session, Thur.*

Impact of future EIC DIS measurements for quark and gluon spin contributions

Quark Spin



Gluon Spin



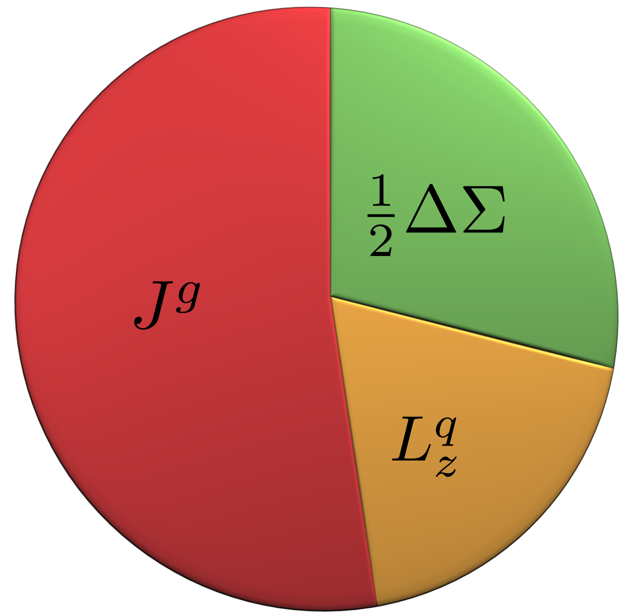
EIC Yellow Report: arXiv: 2103.05419

We are constantly improving the knowledge of the contributions to the spin of the nucleon

What about a direct measurement of orbital angular momentum?

Ji's Sum Rule

X. Ji, PRL 78 (1997) 610



$\frac{1}{2} = J^q + J^g$ $\qquad$ $L_z^q = J^q - \frac{1}{2}\Delta\Sigma$

$$J^{q,g} = \frac{1}{2}\int_{-1}^{1} dx\, x\, \left(H^{q,g}(x,\xi,0) + E^{q,g}(x,\xi,0)\right)$$

at $\xi = 0$ unpolarized PDF $\qquad$ not directly accessible

- $H(x,\xi,t), E(x,\xi,t)$: twist-2 GPDs

X. Ji, F. Yuan, Y. Zhao, Nature Rev. Phys. 3 (2021) 1

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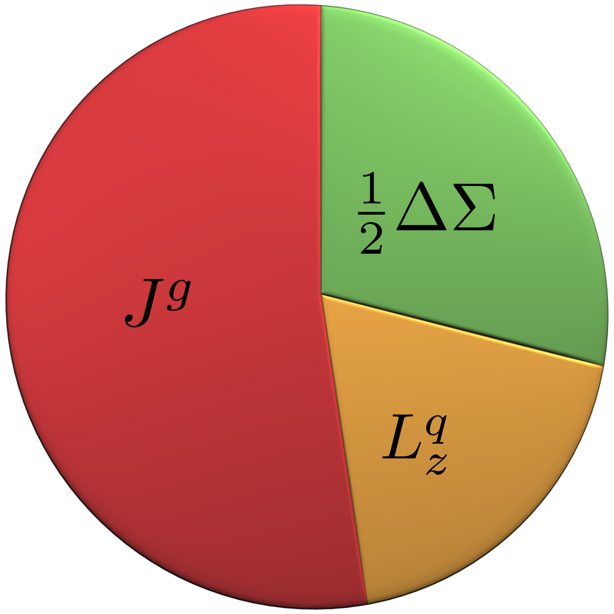

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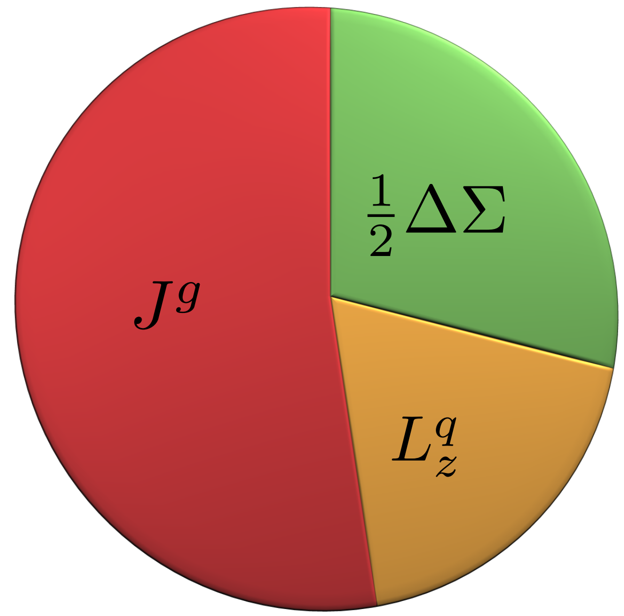

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- Requires spanning x at fixed values of ξ ($\xi = 0$ is the most convenient)

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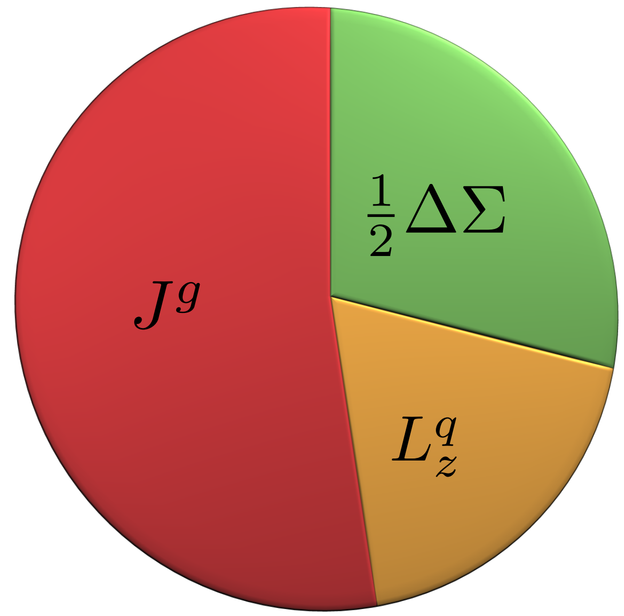

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at $\xi = 0$ unpolarized PDF — not directly accessible

- $H(x,\xi,t), E(x,\xi,t)$: twist-2 GPDs
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- $J^{q,g}(x) \neq \frac{1}{2}[xH^{q,g}(x,0,0) + E^{q,g}(x,0,0)] \longrightarrow$ not angular momentum density

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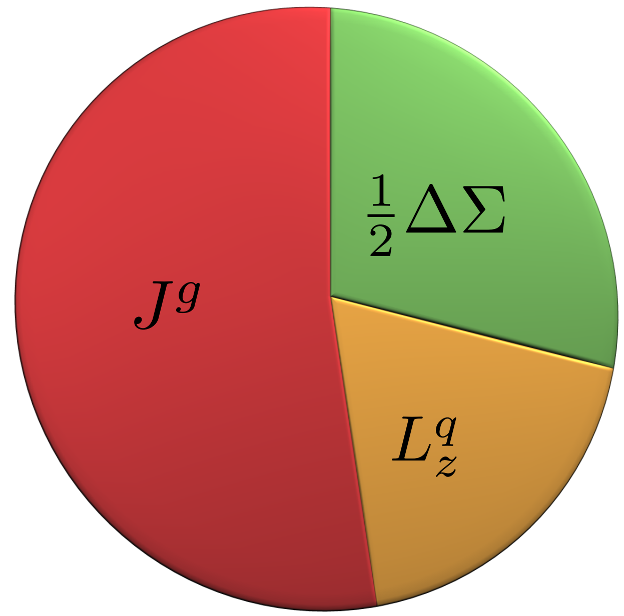

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- $J^{q,g}(x) \neq \frac{1}{2}[xH^{q,g}(x,0,0) + E^{q,g}(x,0,0)] \longrightarrow$ not angular momentum density
- OAM can be related to twist-3 GPDs: not simple partonic interpretation, but definition of a gauge-invariant covariant OAM density

X. Ji, F. Yuan, Y. Zhao, Nature Rev. Phys. 3 (2021) 1

Orbital Angular momentum of the proton from available GPD measurements

$$J^{q,g} = \frac{1}{2}\int_{-1}^{1} dx\, x\, \left(H^{q,g}(x,\xi,0) + E^{q,g}(x,\xi,0)\right) \qquad L^q = J^q - \frac{1}{2}\Delta\Sigma$$

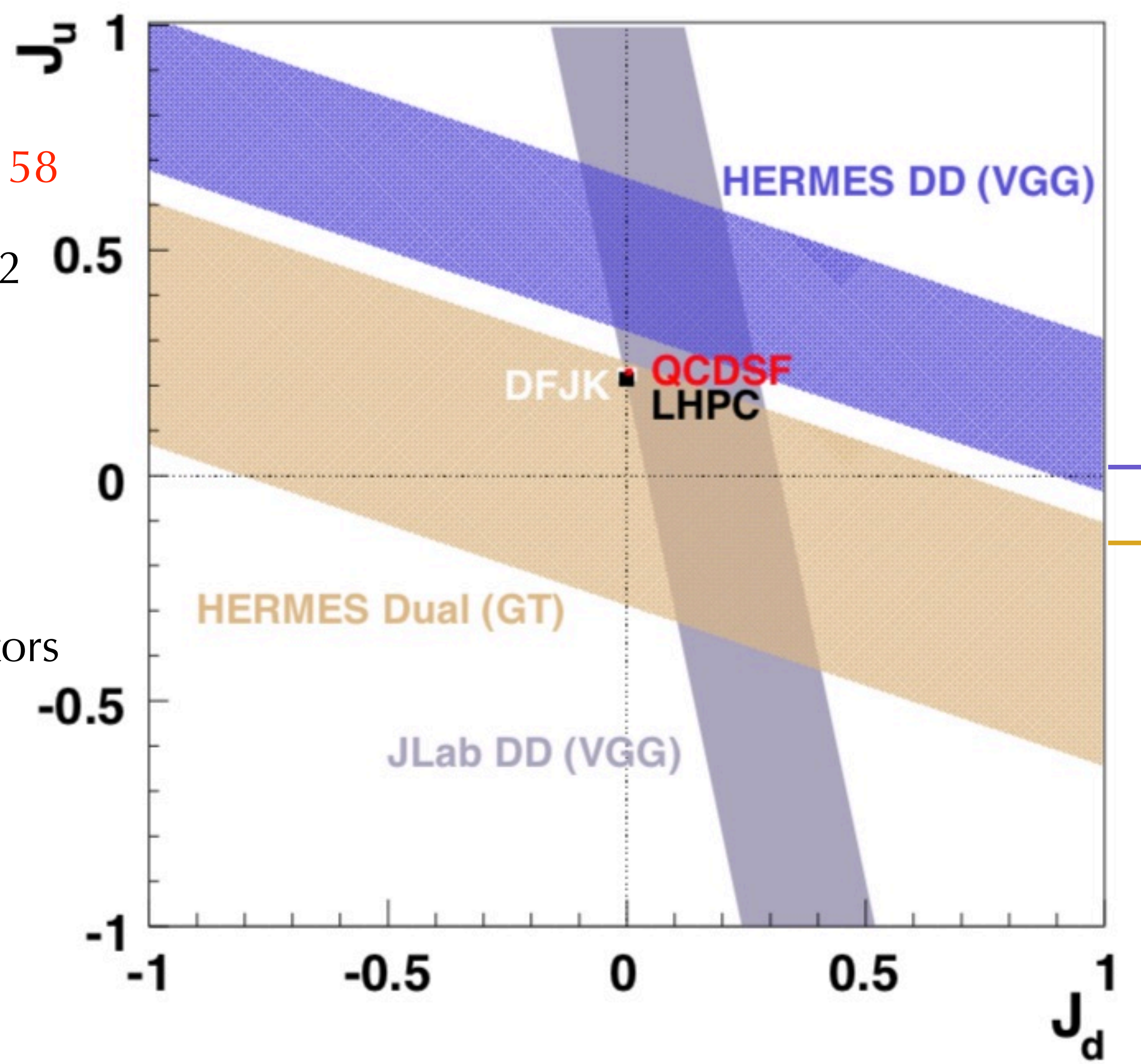
Lattice results

QCDSF: PoS (Lattice 2007) 158

LHPC: PRD77 (2008) 094502

GPDs extracted from form factors

DFJK, EPJC39 (2005) 1



extractions from HERMES data using two different models

JLab Hall A, Phys. Rev. Lett. 99 (2007) 242501

Hermes Coll., JHEP 06 (2008) 066

Improved accuracy with JLab12 and future EIC measurements!

Orbital angular momentum of the proton from Wigner functions

$$l_z^q = \int \mathrm{d}x\mathrm{d}^2\vec{k}_\perp\mathrm{d}^2\vec{b}_\perp(\vec{b}_\perp \times \vec{k}_\perp)\rho_{LU}^{q,\mathcal{W}}(\vec{b}_\perp,\vec{k}_\perp,x)$$

$$l_z^q = \int \mathrm{d}^2\vec{b}_\perp\vec{b}_\perp \times \langle\vec{k}_\perp^q\rangle \longrightarrow \langle\vec{k}_\perp(\vec{b}_\perp)\rangle = \int \mathrm{d}x\mathrm{d}\vec{k}_\perp\,\vec{k}_\perp\,\rho_{LU}^{q,\mathcal{W}}(\vec{b}_\perp,\vec{k}_\perp,x)$$

Lorcé, BP, PRD 84 (2011) 014015

Hatta, PLB 708 (2012) 186

Lorcé, BP, Xiong, Yuan, PRD 85 (2012) 114006

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- intuitive definition of OAM
- mutually orthogonal components of quark position and momentum
  ⟶ no conflict with uncertainty principle
- the integrand $l_z^q(x)$ represents the OAM density
- same equation for both Jaffe-Manohar (staple-like link) and Ji (straight link) OAM
- equation holds also for gluon OAM
- it can be calculated in LQCD *Engelhardt, PRD95 (2017) 094505*

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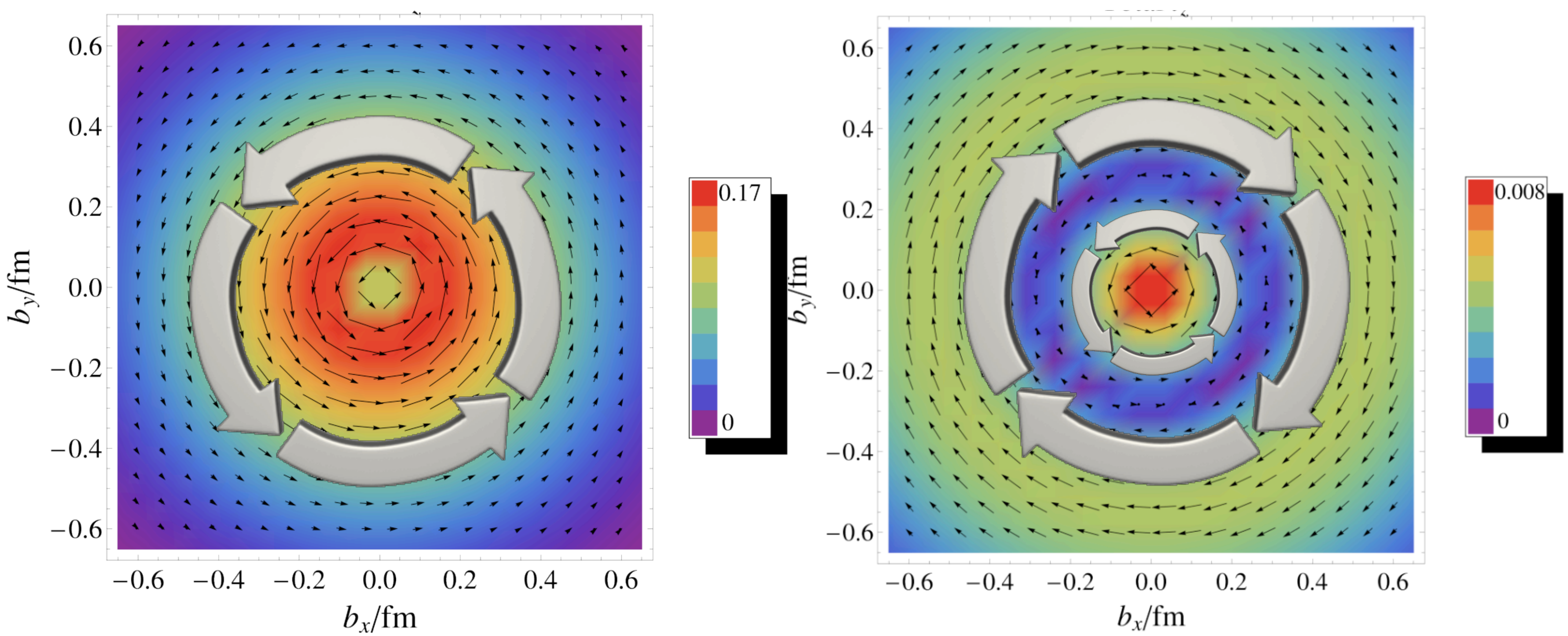


Proton spin

u-quark OAM

d-quark OAM

Lorcé, BP, PRD 84 (2011) 014015

Hatta, PLB 708 (2012) 186

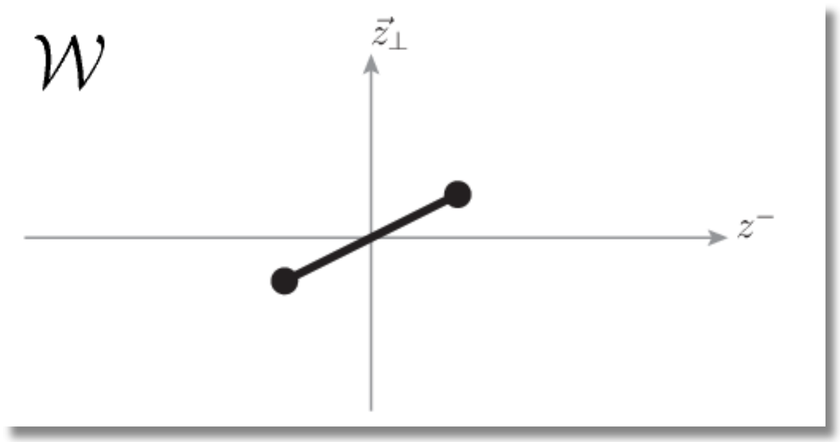
Lorcé, BP, Xiong, Yuan, PRD 85 (2012) 114006

$$\boxed{l_z^q = \int \mathrm{d}x\mathrm{d}^2\vec{k}_\perp \mathrm{d}^2\vec{b}_\perp (\vec{b}_\perp \times \vec{k}_\perp)\rho_{LU}^{q,\mathcal{W}}(\vec{b}_\perp, \vec{k}_\perp, x)}$$

[Lorcé, BP (2011)]
[Lorcé, BP, Xiong, Yuan(2011)]

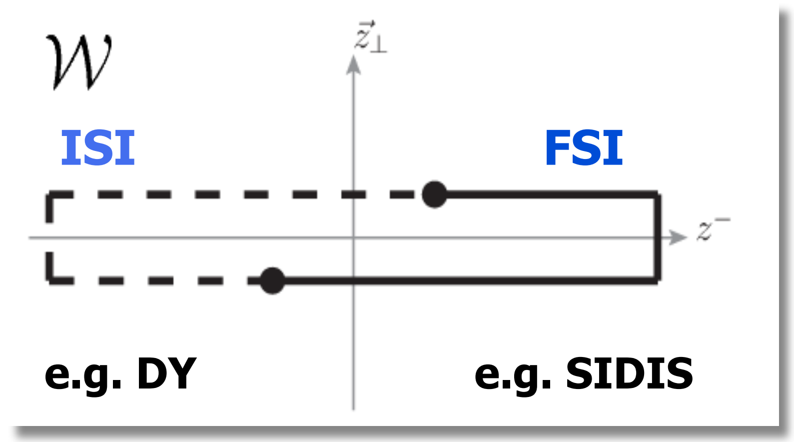
same equation for both Ji (straight link) OAM and Jaffe-Manohar (staple-like link)

Ji $\qquad L_z^q \qquad$ *[Ji (1997)]*



[Ji, Xiong, Yuan (2012)]
[Burkardt (2012)]

JM $\qquad \ell_z^q \qquad$ *[Jaffe, Manohar (1990)]*



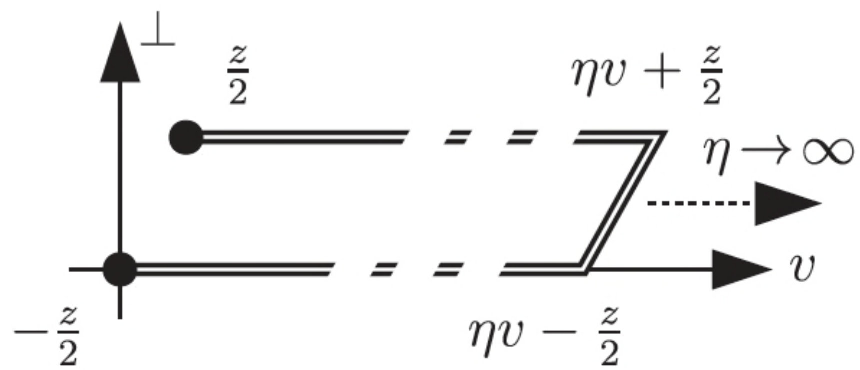
$$\ell_z^{\mathrm{DY}} = \ell_z^{\mathrm{SIDIS}}$$

[Hatta (2012)]

$$\ell_z^q - L_z^q = l_z^{q,\mathrm{pot}}$$

difference between the two definitions can be interpreted as
the change in the quark OAM as the quark leaves the target in a DIS experiment
[M. Burkardt (2013)]

Pioneer Lattice calculation of OAM



Continuous interpolation between the Ji limit $\eta = 0$ and the Jaffe-Manohar limit $\eta \to \infty$

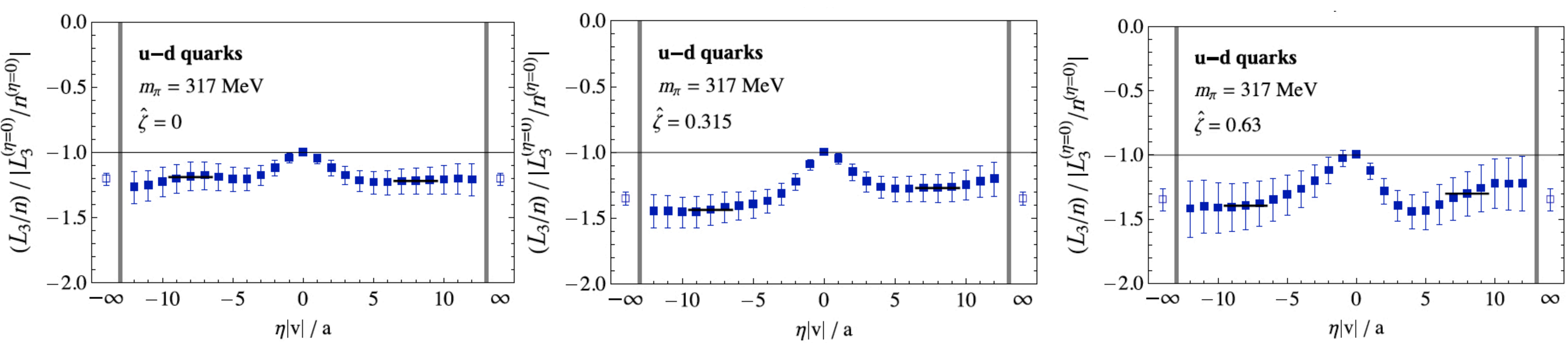
Staple direction off the light-cone

light-cone limit for $\hat{\zeta} = \frac{v \cdot P}{\sqrt{|v^2|}\sqrt{|P^2|}} \to \infty$

M. Engelhardt, Phys. Rev. D95, 094505 (2017)

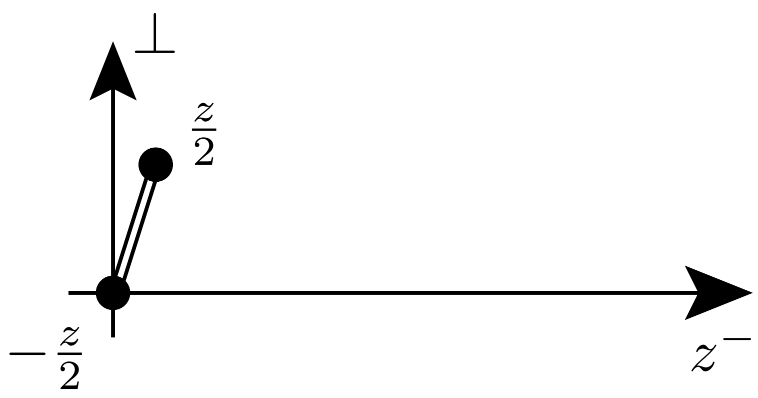
M. Engelhardt et al., PRD102, 074505 (2020)

Jaffe-Manohar OAM ← **Ji OAM** ↓ → **Jaffe-Manohar OAM**



nucleon rapidity →

Pioneer Lattice calculation of OAM



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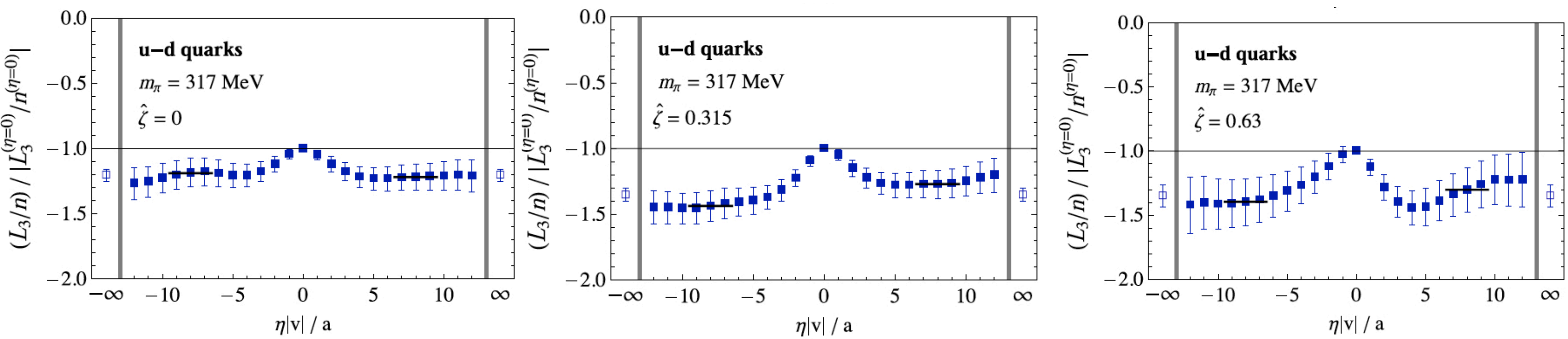

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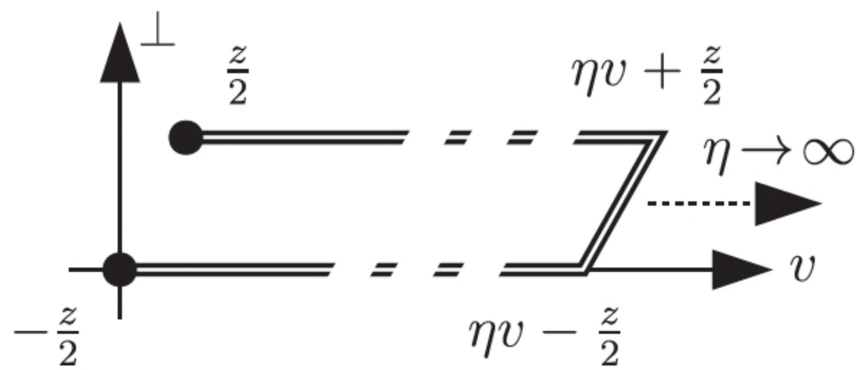

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M. Engelhardt, Phys. Rev. D95, 094505 (2017)

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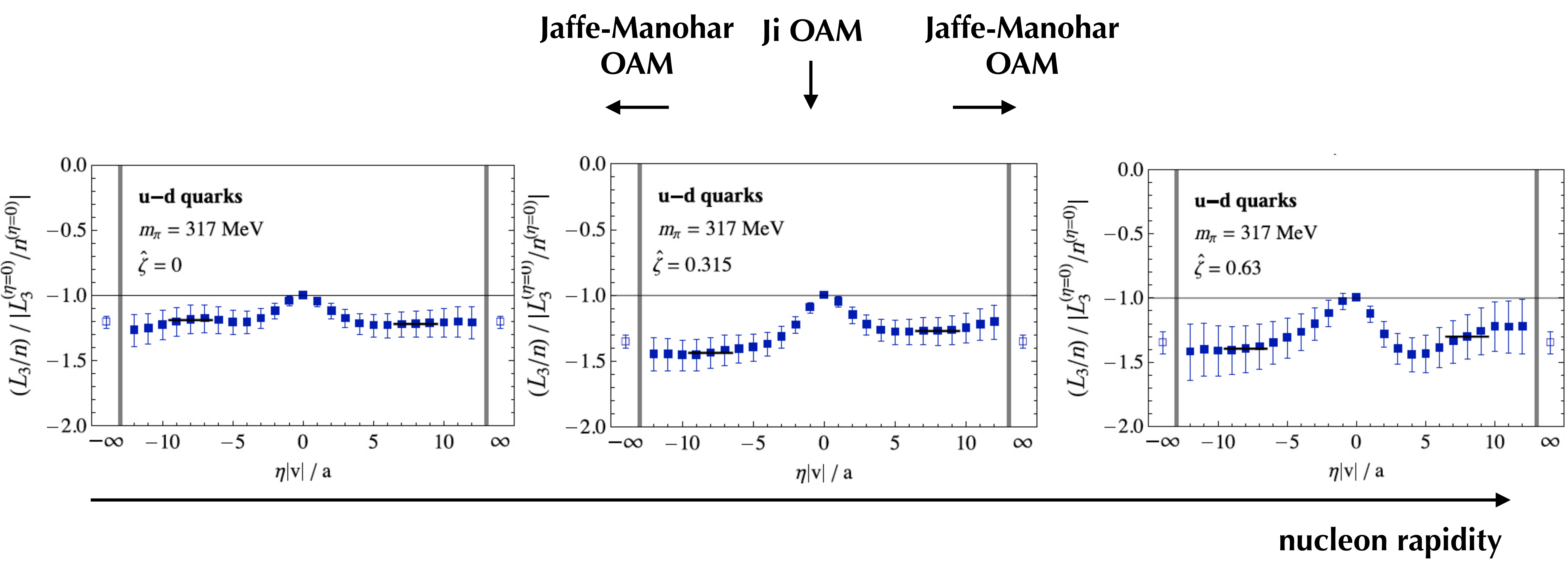
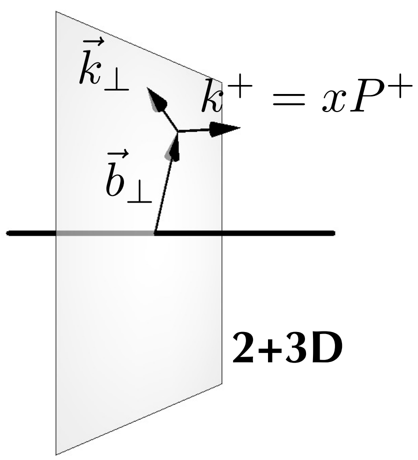


Mapping the nucleon in multidimensional space



quark polarization

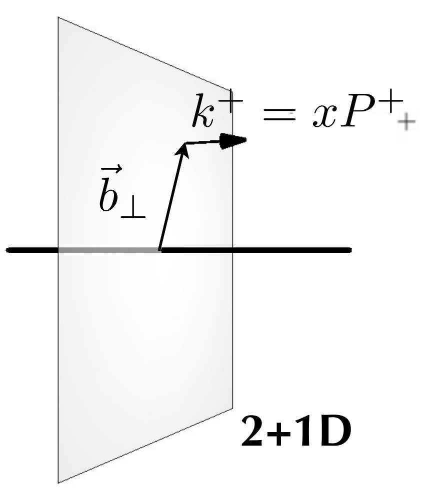
nucleon polarization

$\mathcal{W}_X$	U	L	T_x	T_y
U	$\langle 1 \rangle$	$\langle S_L^q \ell_L^q \rangle$	$\langle S_x^q \ell_x^q \rangle$	$\langle S_y^q \ell_y^q \rangle$
L	$\langle S_L \ell_L^q \rangle$	$\langle S_L S_L^q \rangle$	$\langle S_L \ell_L^q S_x^q \ell_x^q \rangle$	$\langle S_L \ell_L^q S_y^q \ell_y^q \rangle$
T_x	$\langle S_x \ell_x^q \rangle$	$\langle S_x \ell_x^q S_L^q \ell_L^q \rangle$	$\langle S_x S_x^q \rangle$	$\langle S_x \ell_x^q S_y^q \ell_y^q \rangle$
T_y	$\langle S_y \ell_y^q \rangle$	$\langle S_y \ell_y^q S_L^q \ell_L^q \rangle$	$\langle S_y \ell_y^q S_x^q \ell_x^q \rangle$	$\langle S_y S_y^q \rangle$

$\boxed{\xi = 0}$

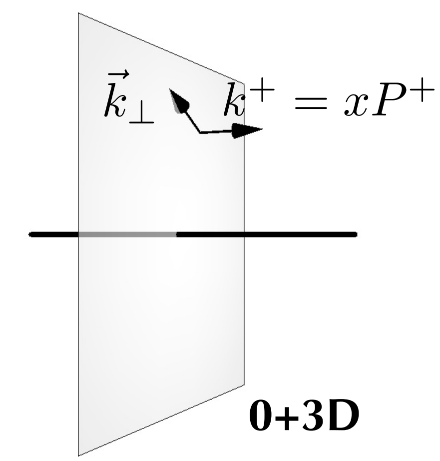
$\int d^2\vec{k}_\perp$

$\int d^2\vec{b}_\perp$



GPD	U	L	T
U	H		$\mathcal{E}_T$
L		$\tilde{H}$	$\tilde{E}_T$
T	E	$\tilde{E}$	$H_T,\ \tilde{H}_T$

TMD	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$



each distribution contains unique information

the distributions in **red** vanish if there is no quark orbital angular momentum

the distributions in **black** survive in the collinear limit

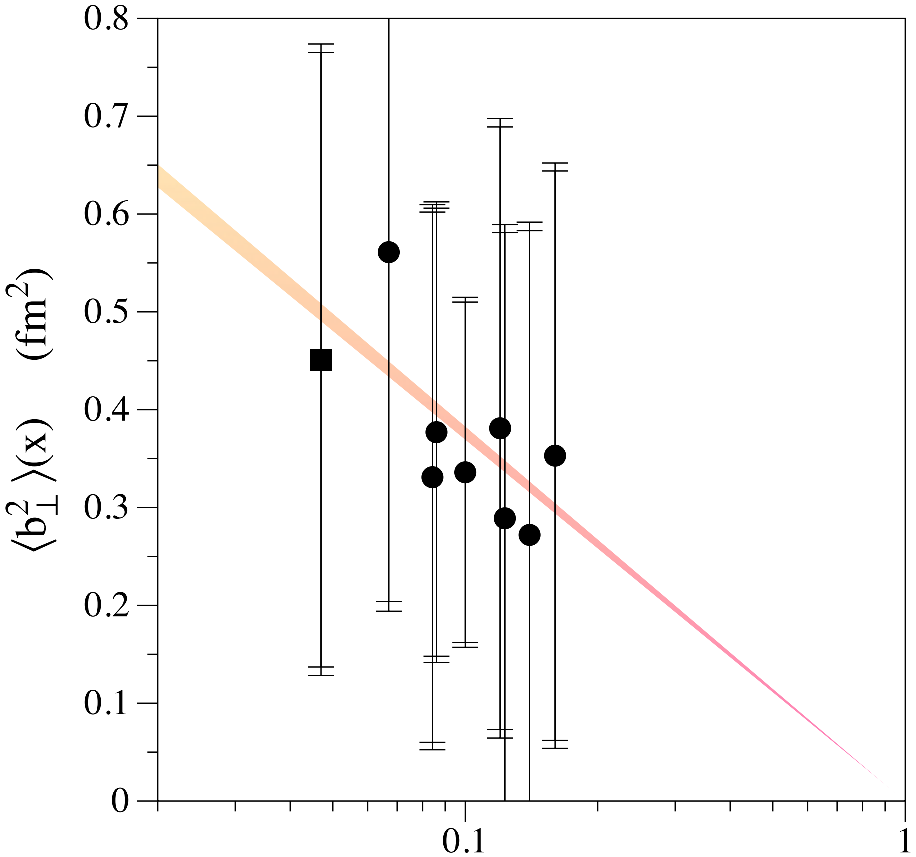
x-dependent transverse squared charge radius

$$H(x,0,\vec{b}_\perp) = \int_{-\infty}^{+\infty} \mathrm{d}^2\vec{\Delta}_\perp\, H(x,0,t)\, e^{-i\vec{\Delta}_\perp\cdot\vec{b}_\perp} \longrightarrow \langle \vec{b}_\perp^{\,2}(x)\rangle = \frac{\int \mathrm{d}^2\vec{b}_\perp\, \vec{b}_\perp^{\,2}\, H(x,0,b_\perp)}{\int \mathrm{d}^2\vec{b}_\perp H(x,0,b_\perp)}$$

$(t = -\vec{\Delta}_\perp^2)$ $\quad\xi = 0$ extrapolation from data

x-dependent transverse squared radius

CLAS and HERMES data



x-range of data: $0.05 \le x \le 0.2$

The errors are large,
but slowly we are getting some 3D information

Dupré et al., PRD95(2017)011501

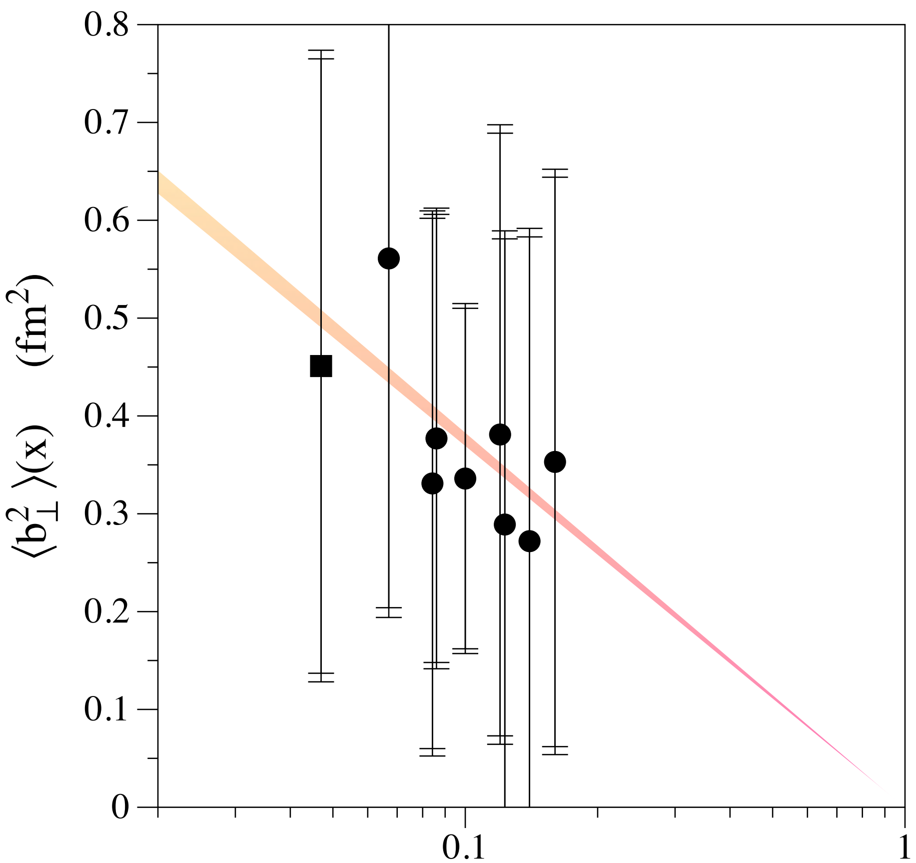
x-dependent transverse squared charge radius

$$H(x,0,\vec{b}_\perp) = \int_{-\infty}^{+\infty} \mathrm{d}^2\vec{\Delta}_\perp\, H(x,0,t)\, e^{-i\vec{\Delta}_\perp\cdot\vec{b}_\perp} \longrightarrow \langle \vec{b}_\perp^{\,2}(x)\rangle = \frac{\int \mathrm{d}^2\vec{b}_\perp\, \vec{b}_\perp^{\,2}\, H(x,0,b_\perp)}{\int \mathrm{d}^2\vec{b}_\perp H(x,0,b_\perp)}$$

$(t = -\vec{\Delta}_\perp^2)$ $\quad \xi = 0$ extrapolation from data

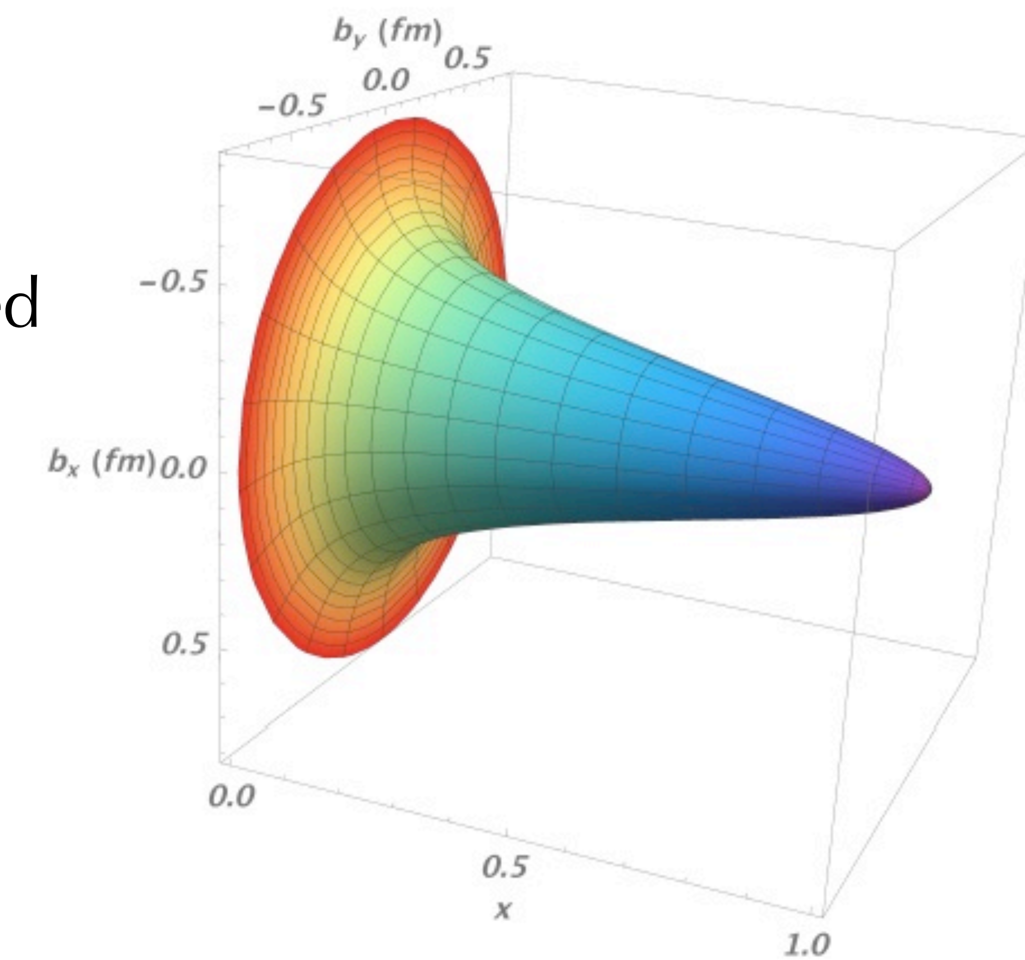
x-dependent transverse squared radius

CLAS and HERMES data



x-range of data: $0.05 \leq x \leq 0.2$

extrapolating
in the unmeasured
x-range

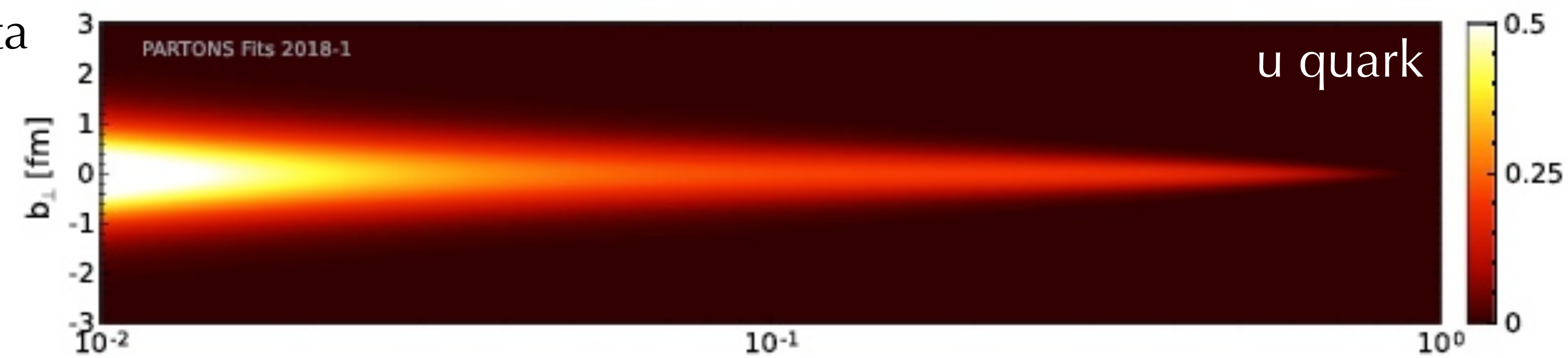


As x $\longrightarrow$ 1, the active parton carries all the momentum
and represents the centre of momentum

Dupré et al., PRD95(2017)011501

New parametrization based on DRs: reduce problems related to the extrapolation to $\xi = 0$

CLAS and HERMES data

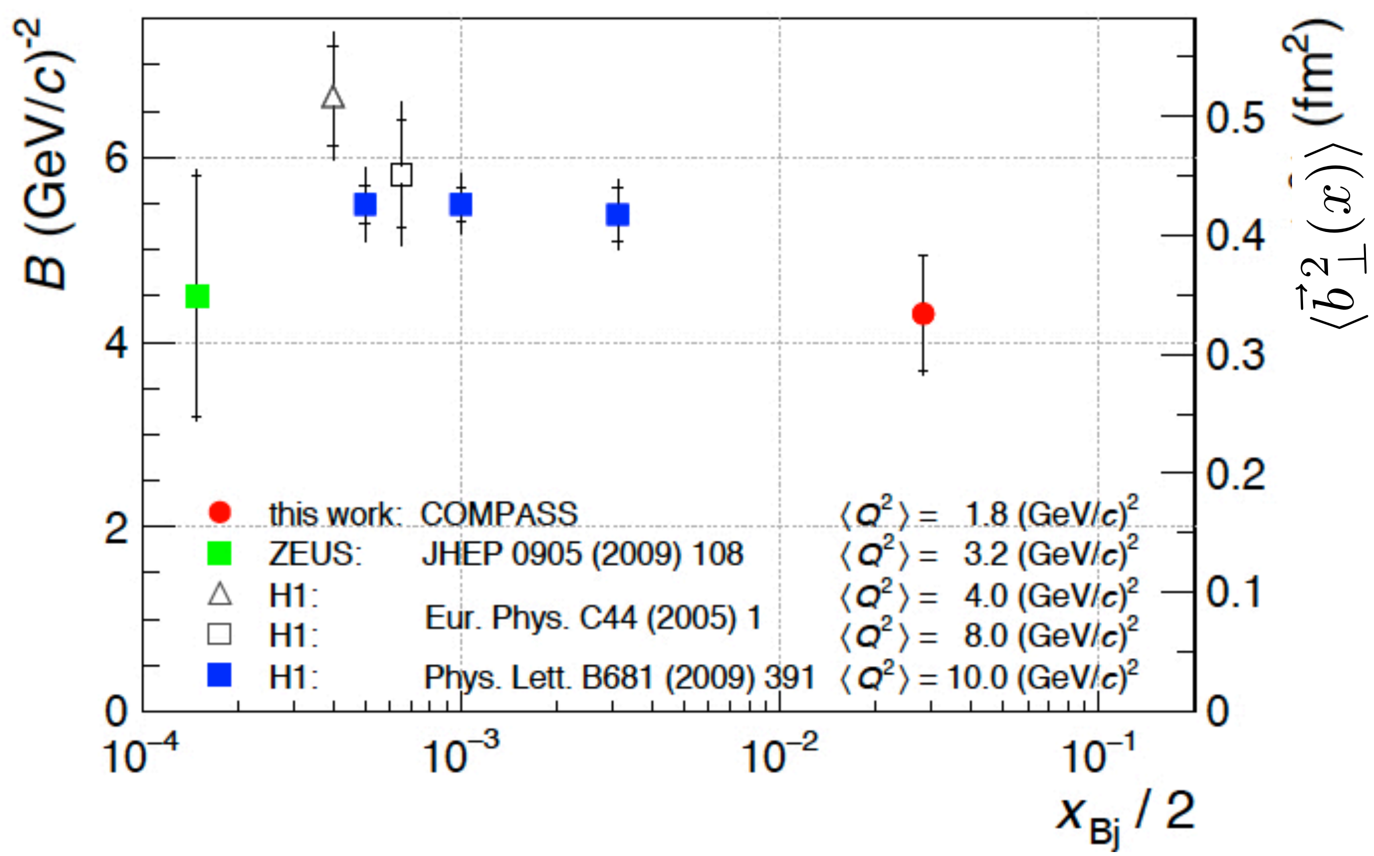


Moutarde et al., EPJC78(2018)890

New results from COMPASS Coll.: Phys. Lett. B793 (2019) 188

$$\frac{d\sigma}{dt} \approx e^{-B(x)|t|}$$

$$\langle \vec{b}_\perp^{\,2}(x) \rangle = 2\langle B(x) \rangle$$



Model dependence can not be avoided, but different fit methods and parametrizations can help to constraint the theoretical uncertainties

Form Factors of Energy Momentum Tensor

Energy Density Momentum Density

$$T^{\mu\nu} = \begin{pmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{pmatrix}$$

shear forces

pressure

Energy Flux Momentum Flux

$$\langle p|\, T^{Q,G}_{\mu\nu}\, |p'\rangle = \bar{u}(p')\left[M_2^{Q,G}(t)\frac{P_\mu P_\nu}{M_N} + J^{Q,G}(t)\frac{i(P_\mu \sigma_{\nu\rho} + P_\nu \sigma_{\mu\rho})\Delta^\rho}{2M_N} + d_1^{Q,G}(t)\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{5M_N} \pm \bar{c}(t) g_{\mu\nu}\right] u(p)$$

Relation with second-moments of GPDs:

$$\sum_q \int \mathrm{d}x\, x\, H^q(x,\xi,t) = M_2^Q(t) + \frac{4}{5} d_1^Q(t)\xi^2$$

$$\sum_q \int \mathrm{d}x\, x\, E^q(x,\xi,t) = 2J^Q(t) - M_2^Q(t) - \frac{4}{5} d_1^Q(t)\xi^2$$

"Charges" of the EMT Form Factors at t=0

$M_2(0)$ nucleon momentum carried by parton

$J(0)$ angular momentum of partons

$d_1(0)$ D-term ("stability" of the nucleon)

Form Factors of Energy Momentum Tensor

Energy Density Momentum Density

$$T^{\mu\nu} = \begin{pmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{pmatrix}$$

shear forces

pressure

Energy Flux Momentum Flux

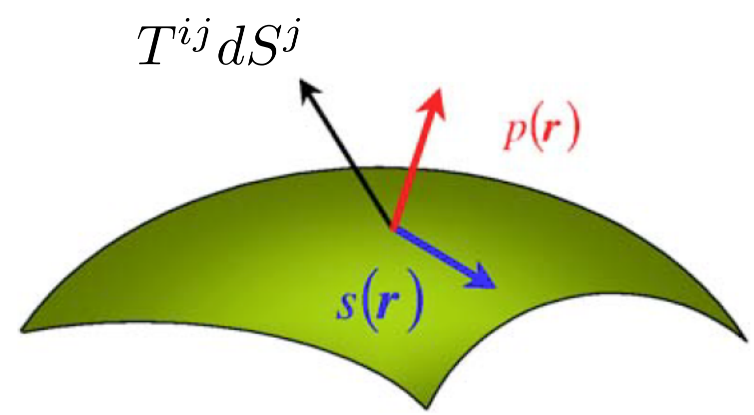
➡ Fourier transform in coordinate space

$$T_{ij}^{Q}(\vec{r}) = s(\vec{r})\left(\frac{r_i r_j}{r^2} - \frac{1}{3}\delta_{ij}\right) + p(\vec{r})\,\delta_{ij}$$

shear forces pressure

$$\boxed{d_1^Q(0) = 5\pi M_N \int_0^\infty \mathrm{d}r\, r^4\, p(r)}$$

"mechanical properties" of nucleon

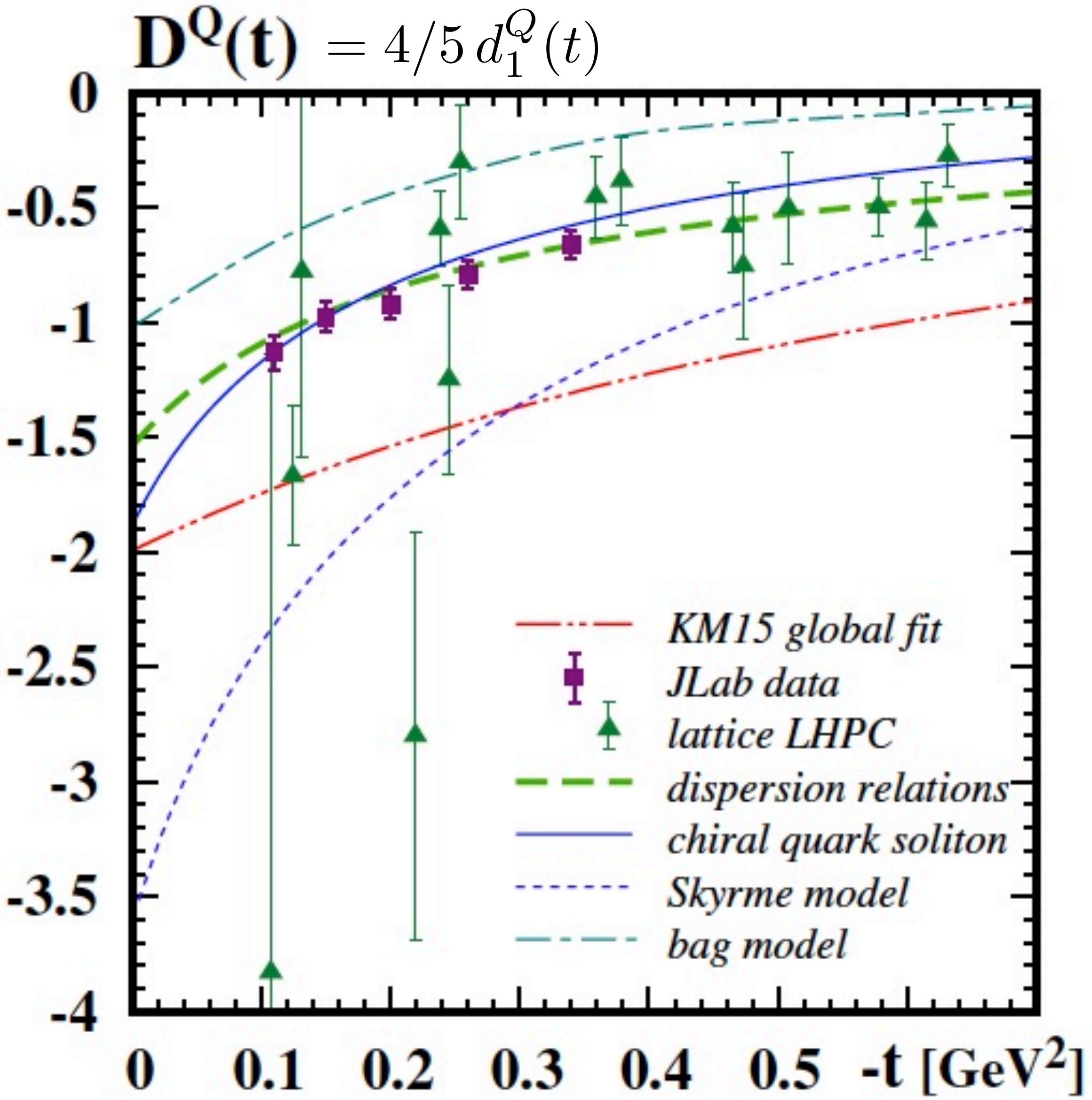


M. Polyakov, PLB 555 (2003) 57

→ *Spin session, Thur. (Metz, Sznajder, Nair, Tezgin, Focillon, Rodini)*

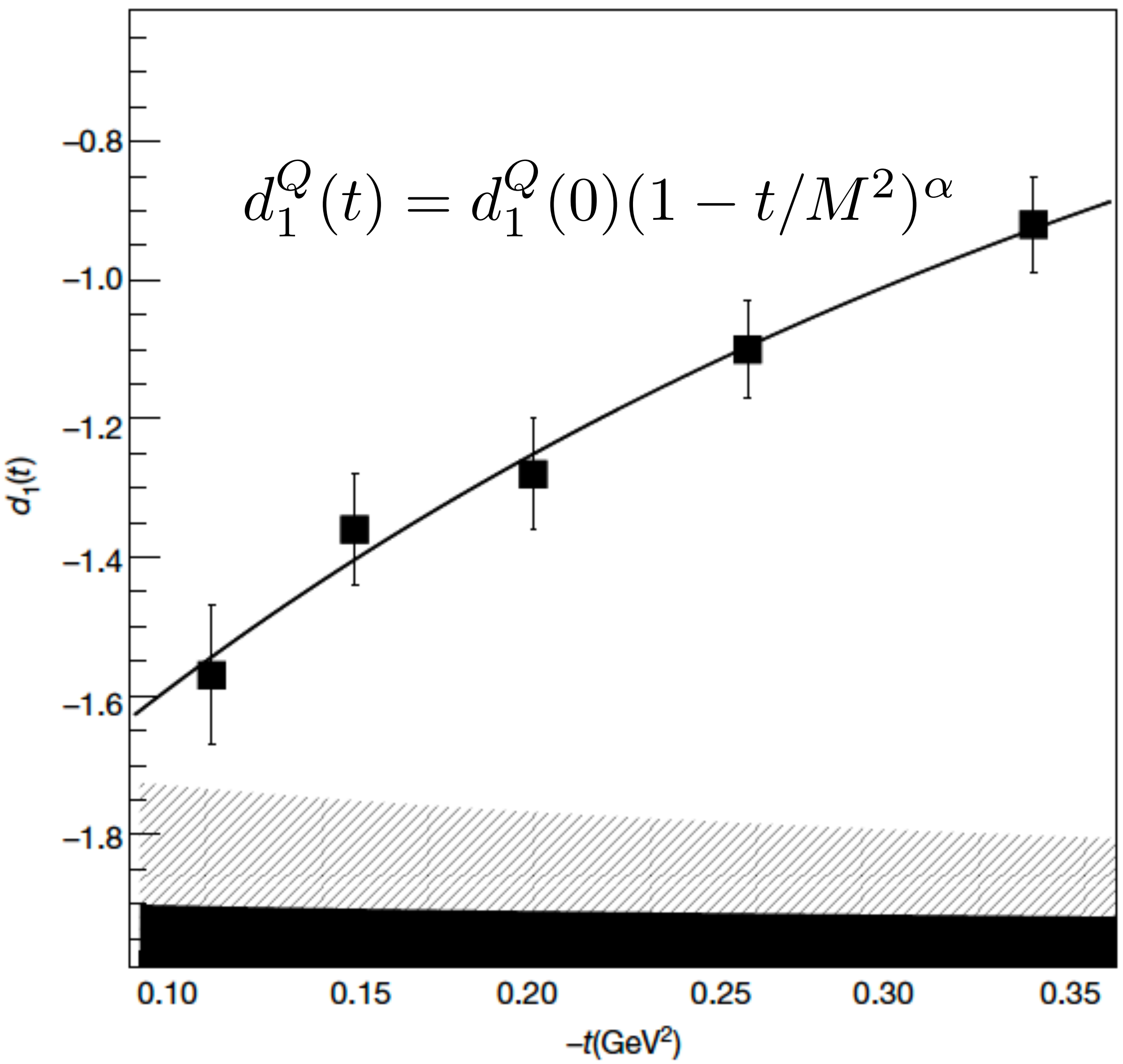
D(t) form factor from data

Girod, Elouadrhiri,Burkert, Nature 557 (2018) 7705
and arXiv: 2104.02031;
CLAS 6GeV data



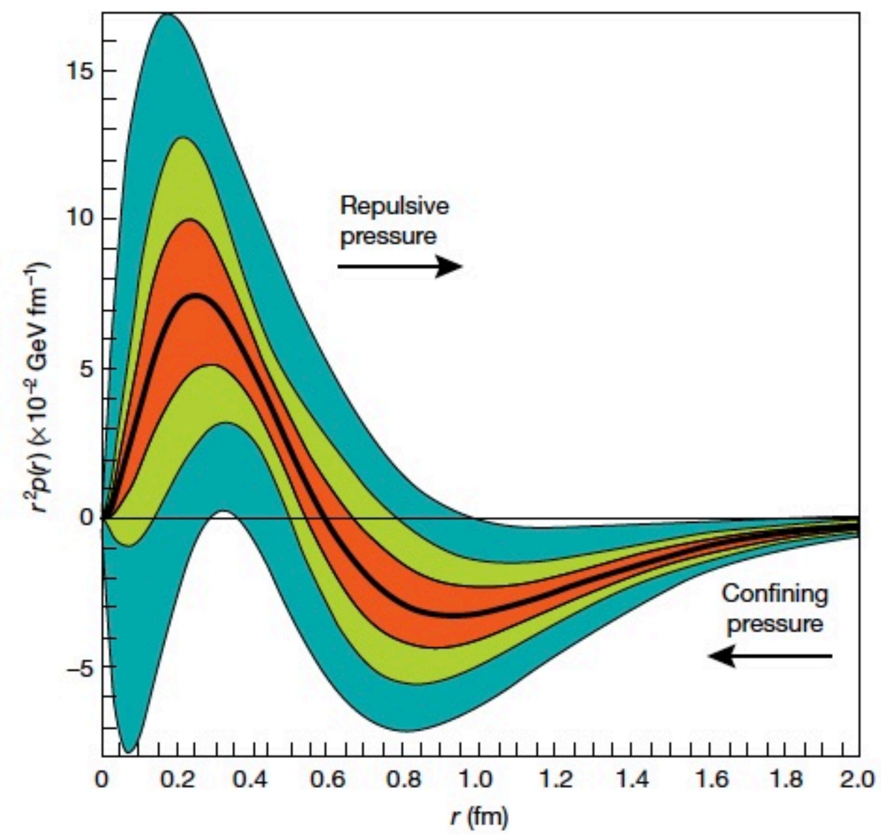
D(t) form factor from data

Girod, Elouadrhiri,Burkert, Nature 557 (2018) 7705
and arXiv: 2104.02031;
CLAS 6GeV data

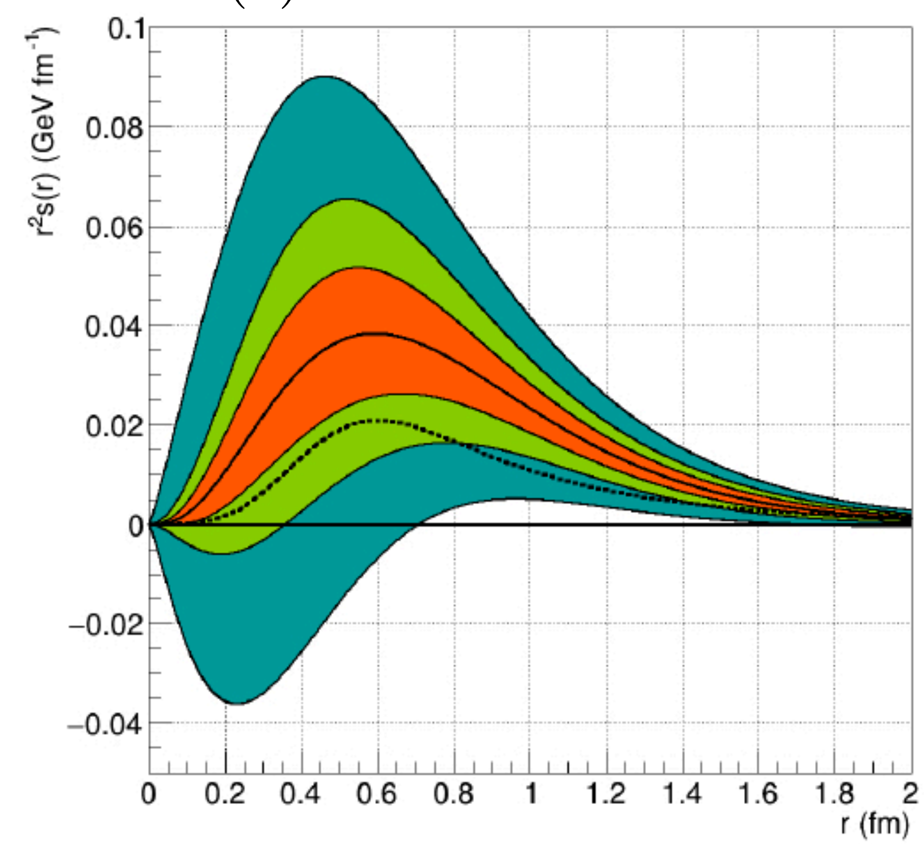


FT

$r^2p(r)$ radial pressure distribution



$r^2s(r)$ shear forces distribution



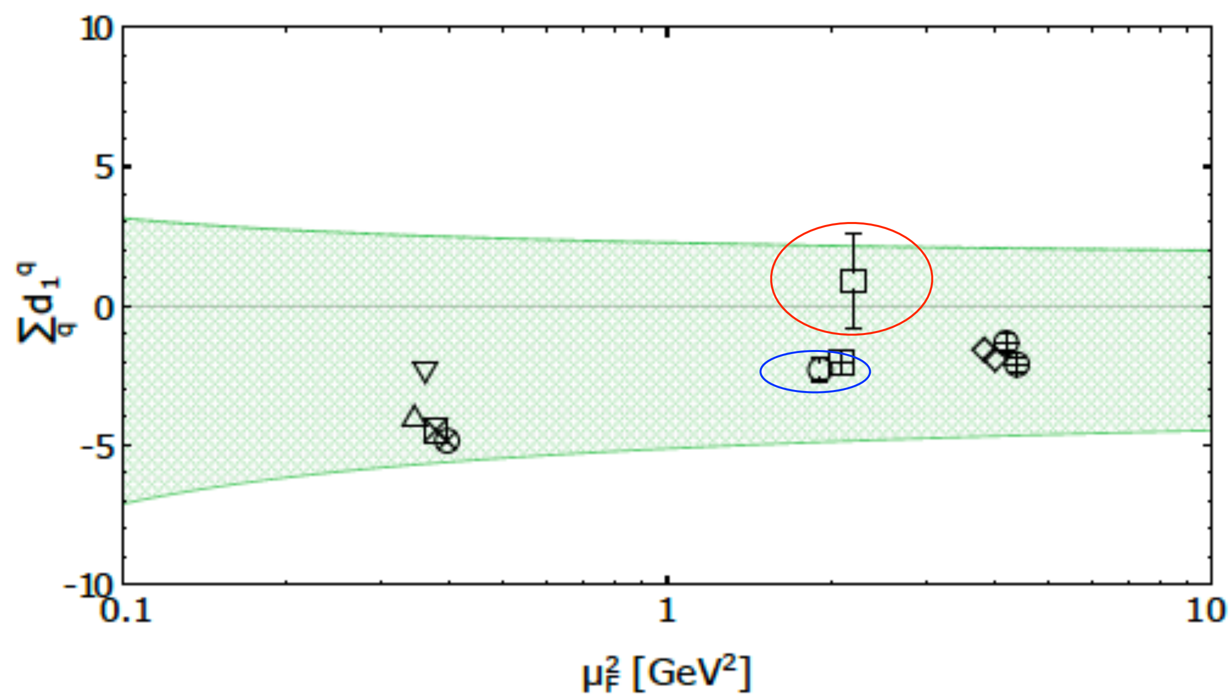
data before 6 GeV JLab
6 GeV JLab data
projected 12 GeV JLab data

Necessary to verify model assumptions in the exp extraction
with more data coming from JLab, COMPASS and the future EIC, EIcC

Kumericki, Nature 570 (2019) 7759; Dutriex et al, arXiv: 2101.03855

Talk of P. Sznajder, Spin session, Thursday

global fit to DVCS data
with artificial neural networks



CLAS data, with fixed param.,
Girod et al.

CLAS data, with neural networks
Kumericki

$$\sum_q d_1^q < 0$$

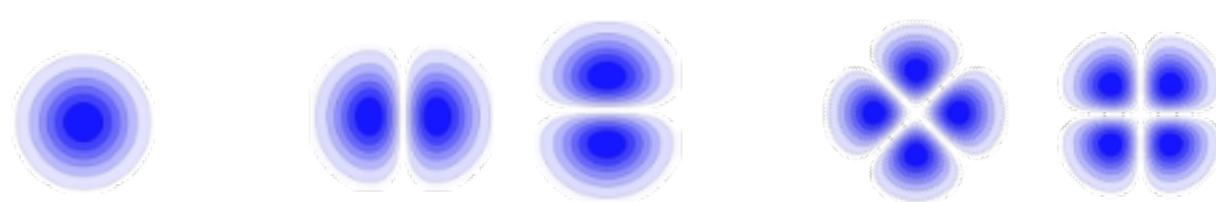
in all model calculations
for a stable proton

Marker in Fig. 3	$\sum_q d_1^q(\mu_F^2)$	μ_F^2 in GeV^2	# of flavours	Type
○	$-2.30 \pm 0.16 \pm 0.37$	2.0	3	from experimental data
□	0.88 ± 1.69	2.2	2	from experimental data
◇	-1.59	4	2	t-channel saturated model
	-1.92	4	2	t-channel saturated model
△	-4	0.36	3	χQSM
▽	-2.35	0.36	2	χQSM
⊠	-4.48	0.36	2	Skyrme model
⊞	-2.02	2	3	LFWF model
⊗	-4.85	0.36	2	χQSM
⊕	-1.34 ± 0.31	4	2	lattice QCD ($\overline{\mathrm{MS}}$)
	-2.11 ± 0.27	4	2	lattice QCD ($\overline{\mathrm{MS}}$)

Key information from Transverse Momentum Dependent PDFs

- Complete momentum spectrum of single particle
- Transverse momentum size as function of x (3D map)
- Spin-Spin and Spin-Orbit Correlations of partons
- Information on parton orbital angular momentum
  (no direct model-independent relation)

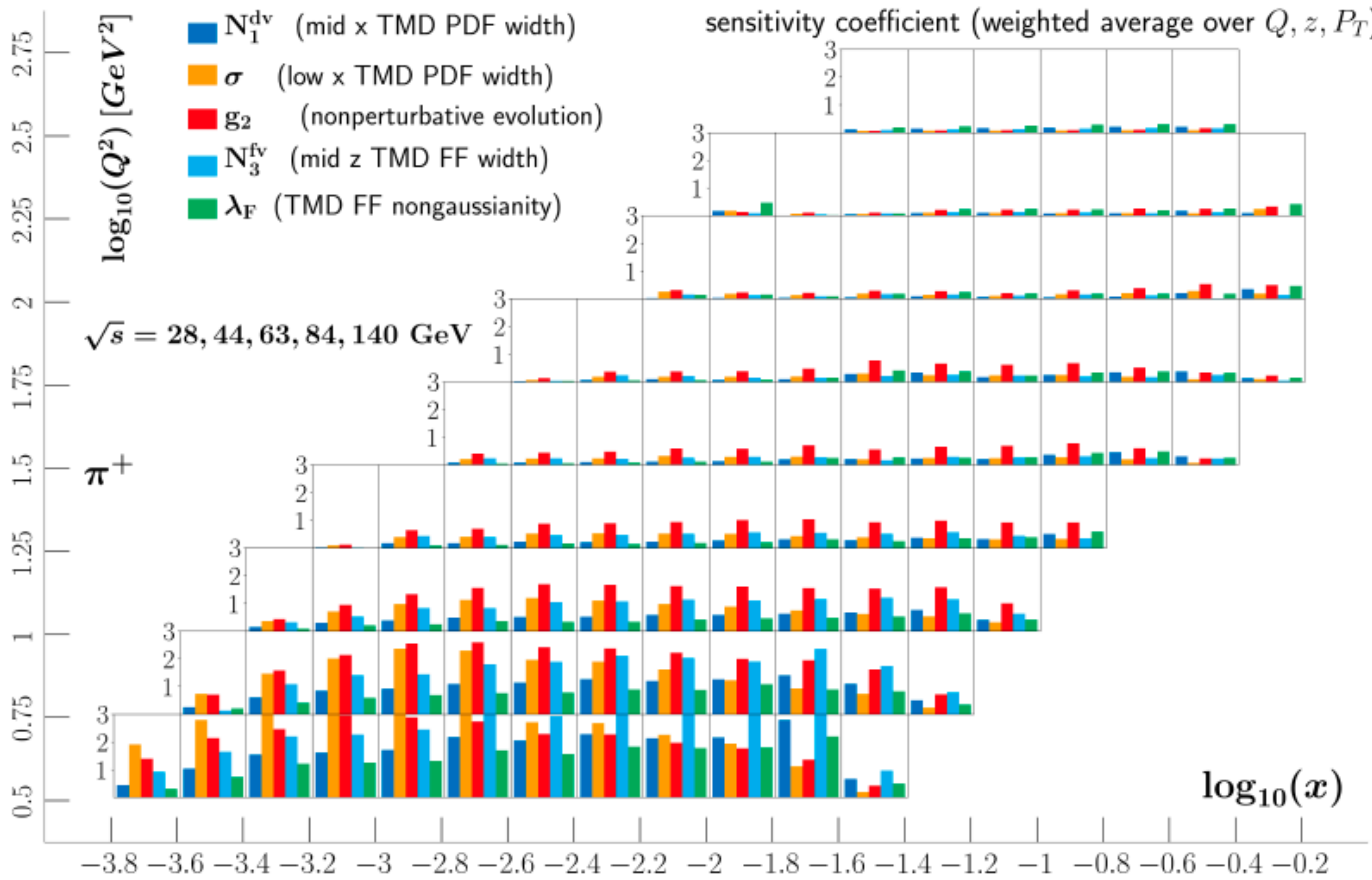
TMD	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1, \ h_{1T}^\perp$



unpolarized
target and partons

deformation due to
spin-spin and spin-orbit correlations

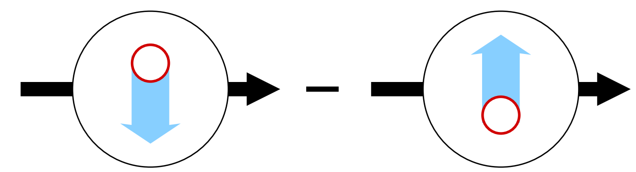
Foreseen EIC impact on unp. TMDs: SIDIS measurements



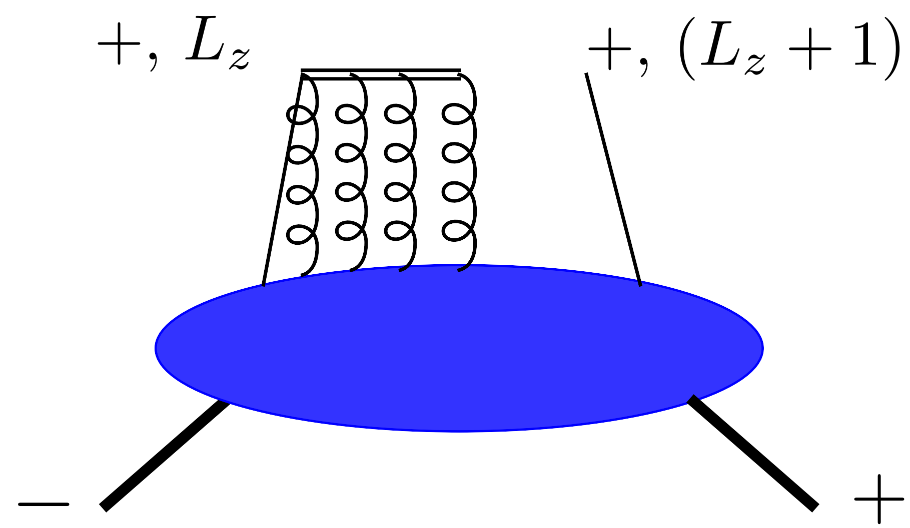
Sensitivity coefficients: measure of the correlation between fit parameters and measurable quantities at EIC

EIC Yellow Report: arXiv: 2103.05419
based on Bacchetta, et al., JHEP 07 (2020) 117 → *Talk of C. Bissolotti, Spin session, Thur.*

Sivers function

$f_{1T}^{\perp} =$ 

unpolarized quarks in $\perp$ pol. nucleon



the helicity mismatch requires orbital angular momentum (OAM)

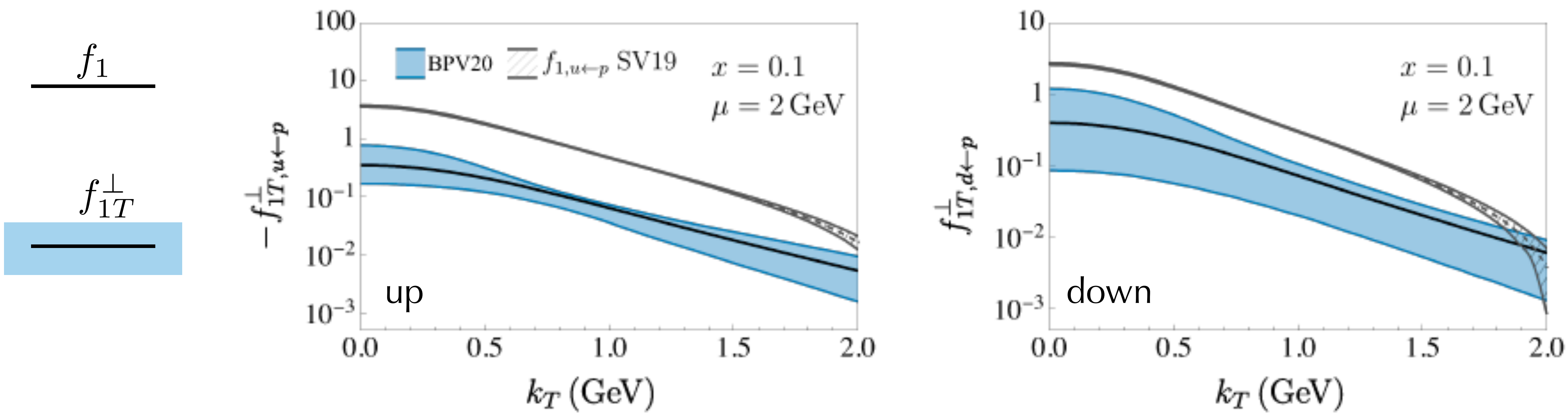
non trivial correlation between quark OAM and nucleon transverse spin

no counterpart in GPD and PDF case

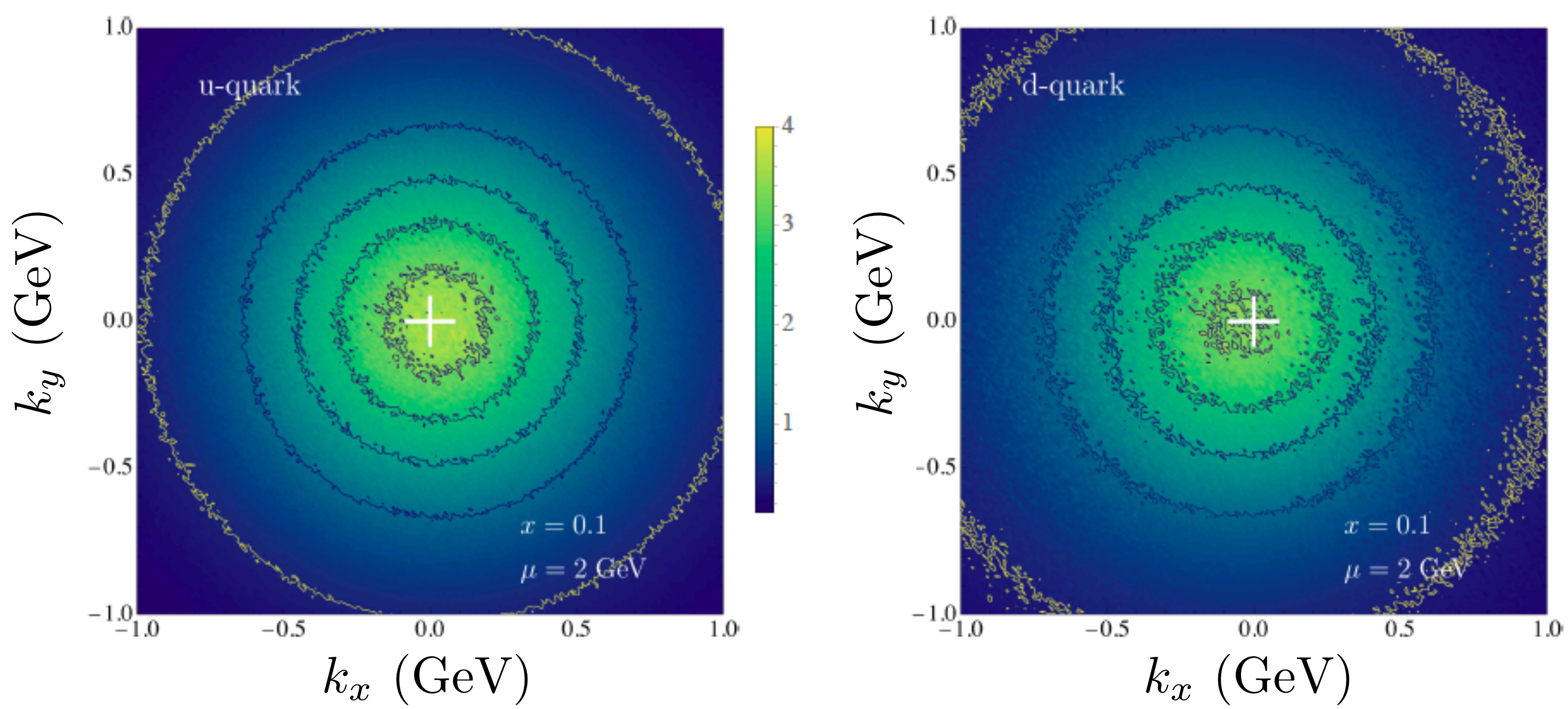
non-zero ONLY with final-state interaction

$$f_{1T}^{\mathrm{SIDIS}}(x, k_\perp) = -f_{1T}^{\mathrm{DY}}(x, k_\perp)$$

Global fit to SIDIS, DY, W$^{\pm}$/Z boson production



$$\rho_{UT_y}(x, \vec{k}_\perp, S_y) = f_1(x, k_\perp) - \frac{k_x}{M} f_{1T}^{\perp}(x, k_\perp)$$



M. Bury, A. Prokudin, A. Vladimirov, arXiv: 2103.03270

→ *more on Sivers extractions and gluon Sivers data from RHIC, Tue. Spin session*

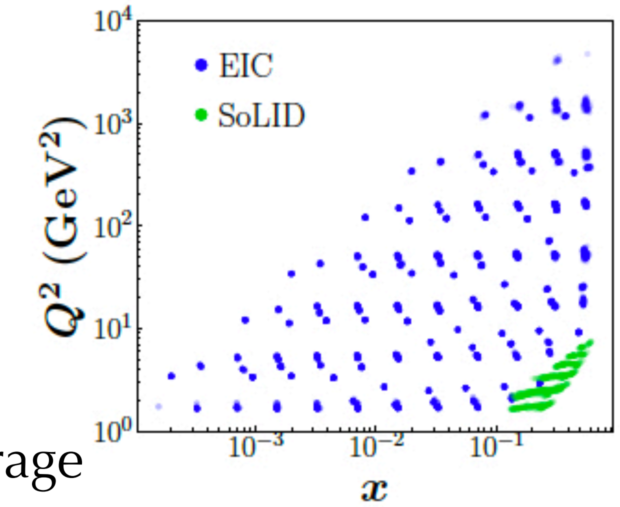
Transversity and tensor charge from SOLID@JLab and EIC

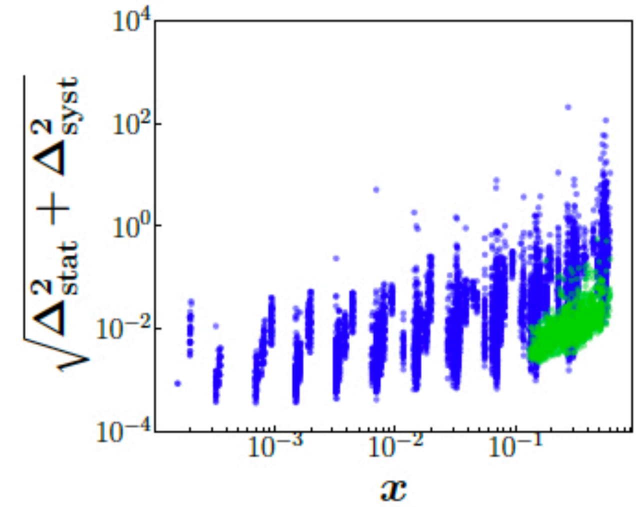
SOLID and EIC
kinematic coverage

$$e + N^{\uparrow} \to e + \pi^{\pm} + X$$

$$N^{\uparrow} = p^{\uparrow},\ {}^{3}He^{\uparrow}$$

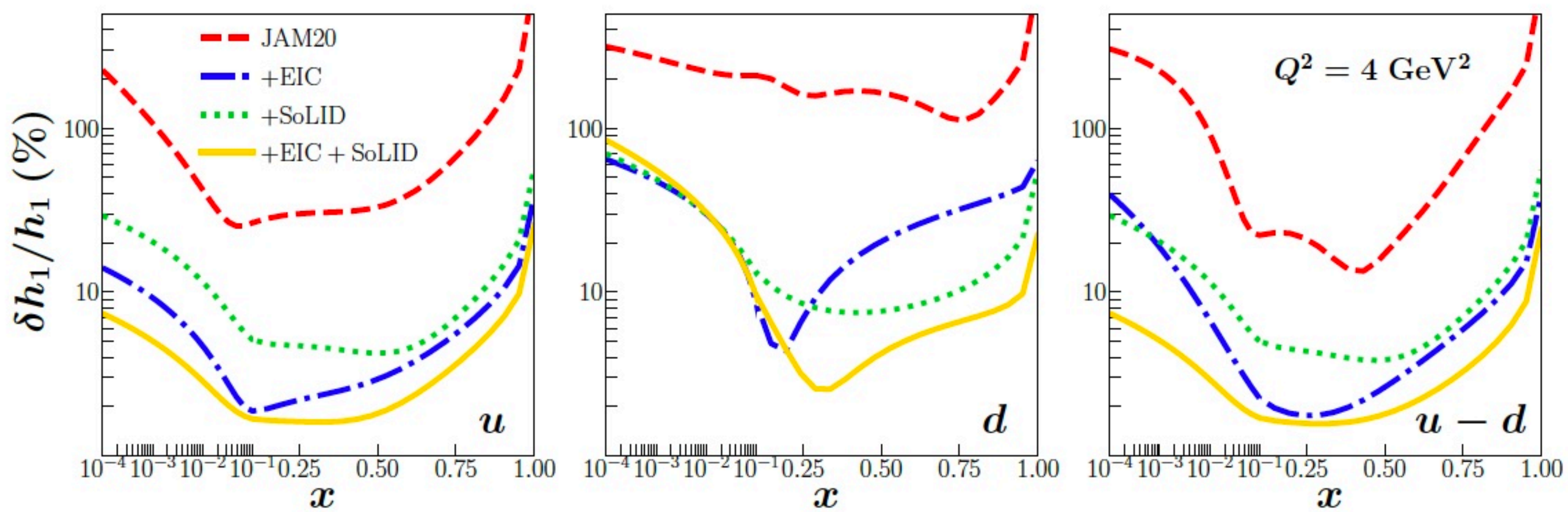
current SIDIS kinematic coverage
$0.02 \leq x \leq 0.3$





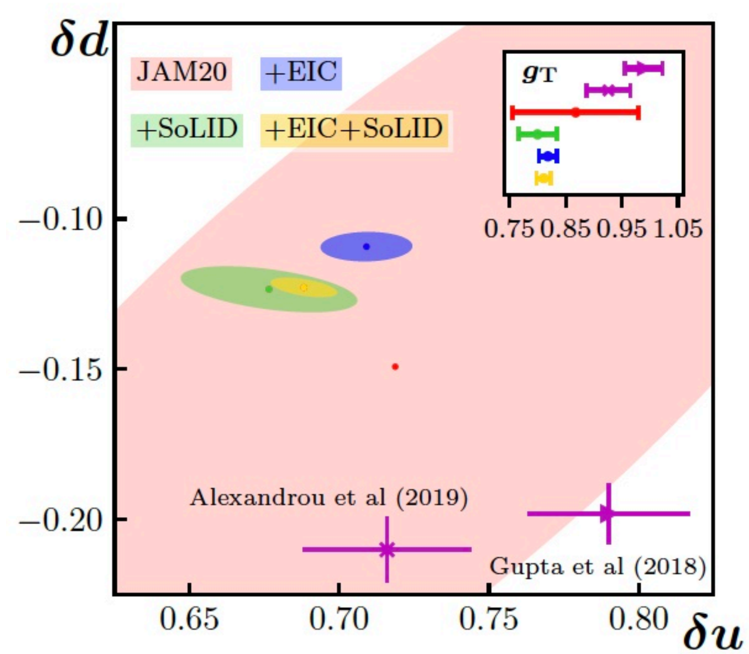
SOLID and EIC
quadrature
of syst. and stat. Errors

relative error of
transversity

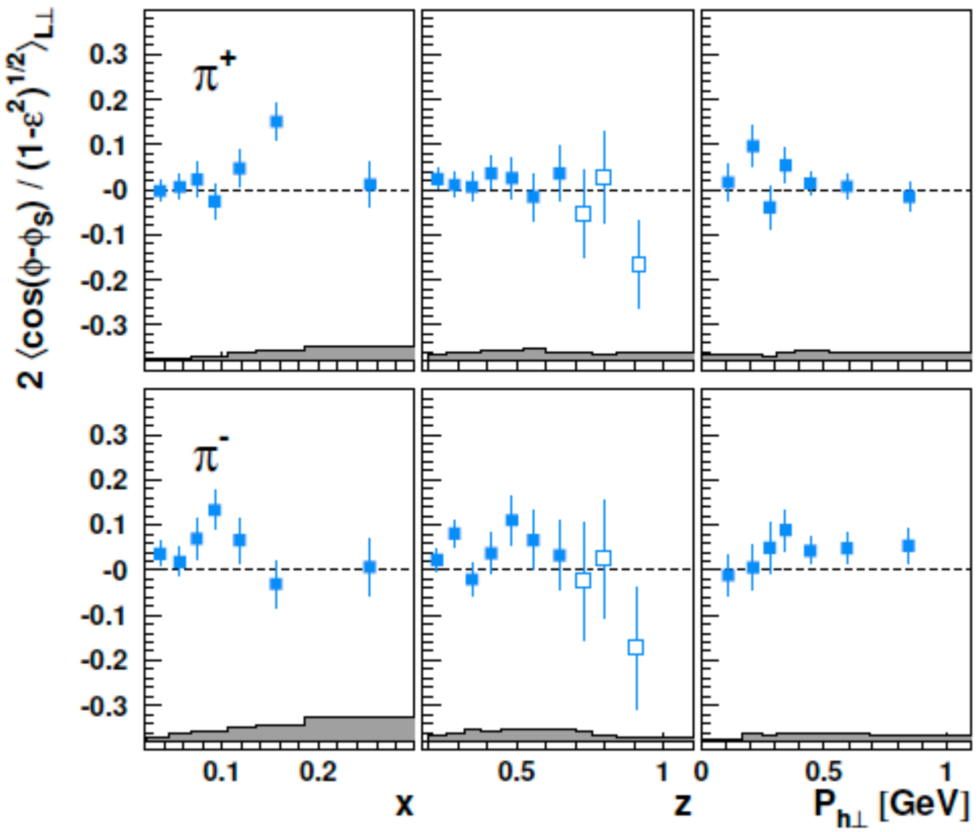


$$\delta q = \int_0^1 h_1^q(x)\mathrm{d}x$$

current phen. extraction

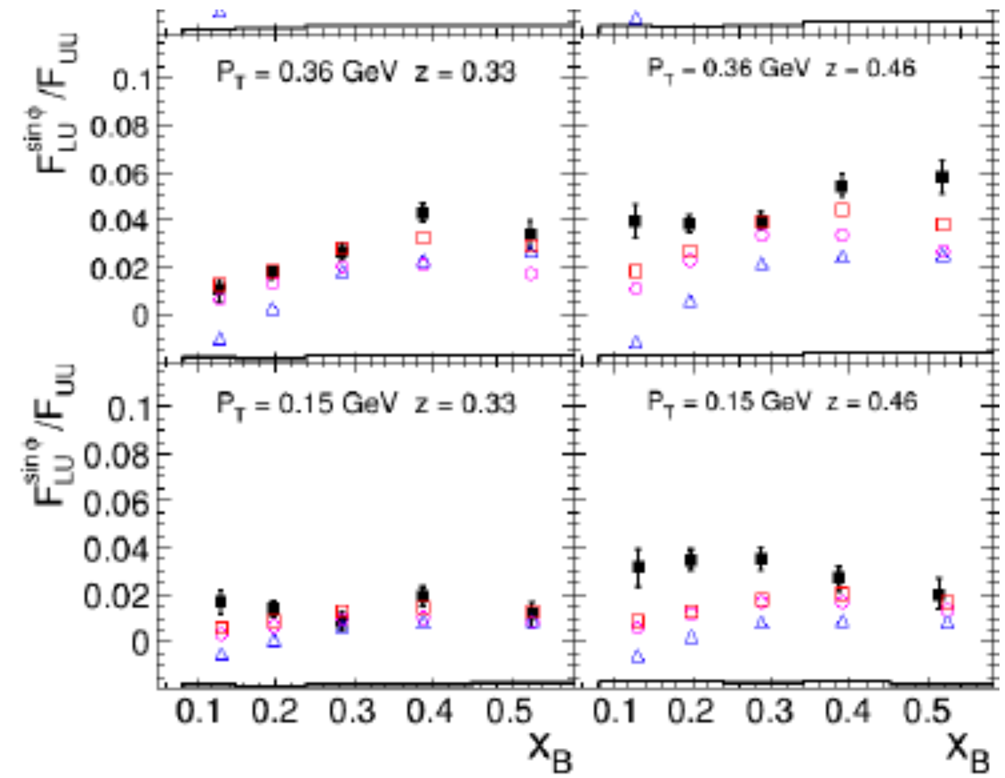


Gamberg, Kang, Pitonyak, Prokudin, Sato, Seidl, arXiv:2101.062700 → *more on transversity extractions on Wed. Spin session*



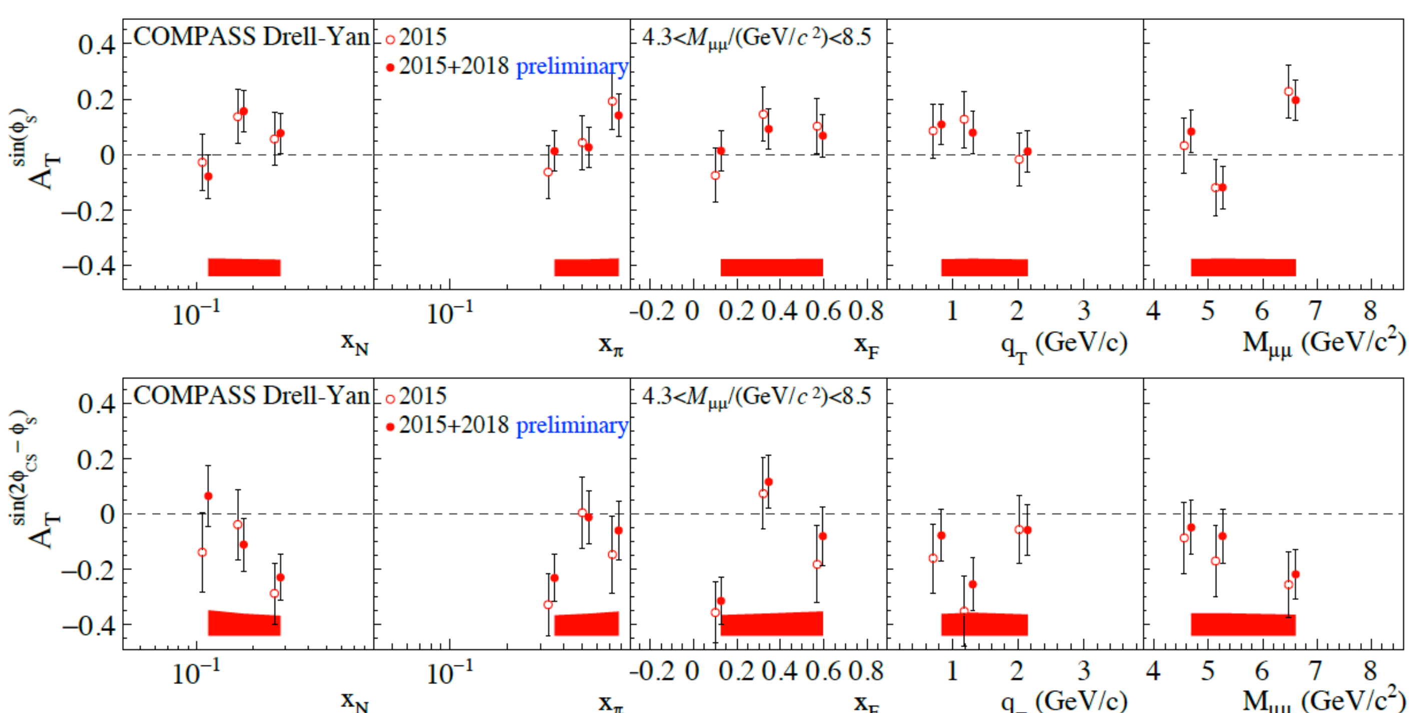
→ *Talk of L. Pappalardo, Tue. Spin session*

New SIDIS analysis: JHEP 12 (2020) 010

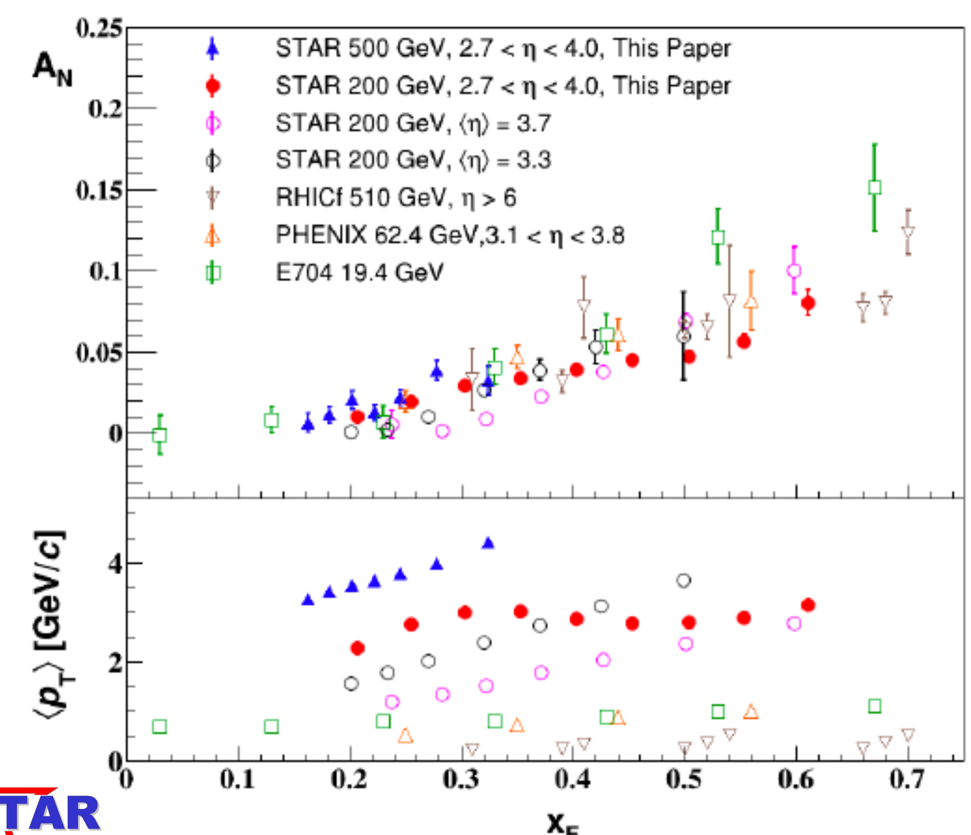


→ *Talk of S. Diehl, Thu. Spin session*

New SIDIS data: JHEP 12 (2020) 010



→ *Talk of B. Parsamyan, Tue. Spin session*



→ *3 talks on Tue., spin session*

Conclusions

- The field of ``Spin and 3D structure" is continuously advancing and is prominently included in the strategic plans worldwide

- Our knowledge of GPD and TMD keeps increasing, but still many challenging questions

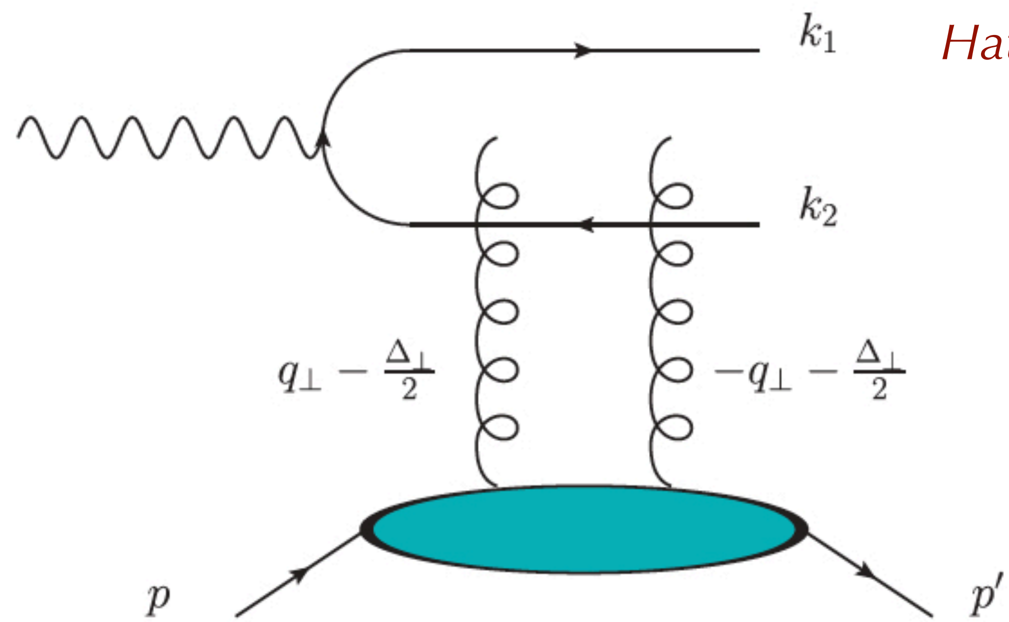
- PDFs, GPDs, TMDs, Wigner distributions are sensitive to various aspects of angular momentum: putting together direct, indirect, and model inspired information from different sides it is our best hope to make quantitative assessments

- Exciting program of talks at this conference!!!

Observables for GTMDs and Wigner functions

Diffractive Exclusive back-to-back dijet production in ℓN / ℓA collisions

Hatta, Xiao, Yuan, PRL 116 (2016) 202301



$$\vec{\Delta}_\perp \approx -(\vec{k}_{\perp,1} + \vec{k}_{\perp,2}) \qquad \vec{k}_\perp \sim \vec{P}_\perp = \frac{(\vec{k}_{\perp,1} - \vec{k}_{\perp,2})}{2} \qquad |\vec{P}_\perp| \gg |\vec{k}_{\perp,1} + \vec{k}_{\perp,2}|$$

- Reconstruction of full dijet kinematics and measure the azimuthal modulations in the angle between $\vec{\Delta}_\perp$ and $\vec{P}_\perp$

- At small x: sensitivity to gluon GTMDs

- Estimates in the CGC effective field theory suggest that modulations are maximum some tens of percent level *Mäntysaari, Mueller, Schenke, PRD99 (2019) 074004*

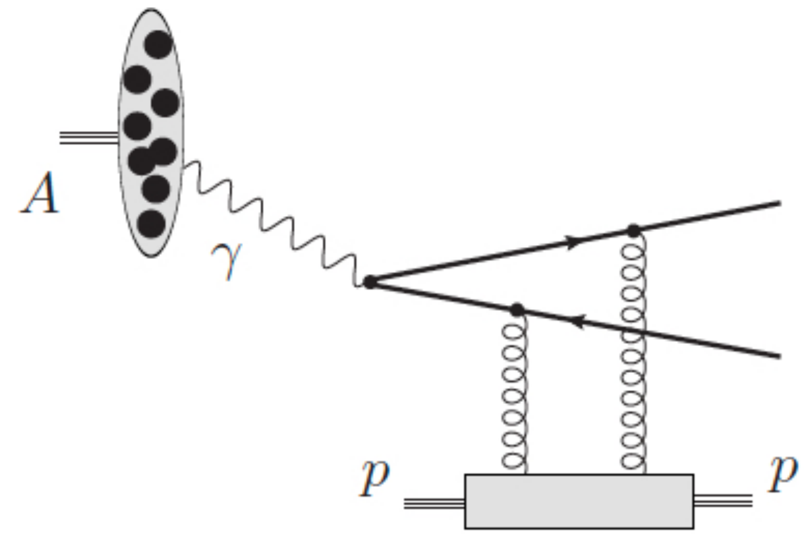
- With proton polarization one may access $F^g_{1,4}$

Hatta, Nakagawa, Xiao, Yuan, Zhao, PRD 95 (2017) 114032; Ji, Yuan, Zhao, PRL 118 (2017) 192004

Observables for GTMDs and Wigner functions

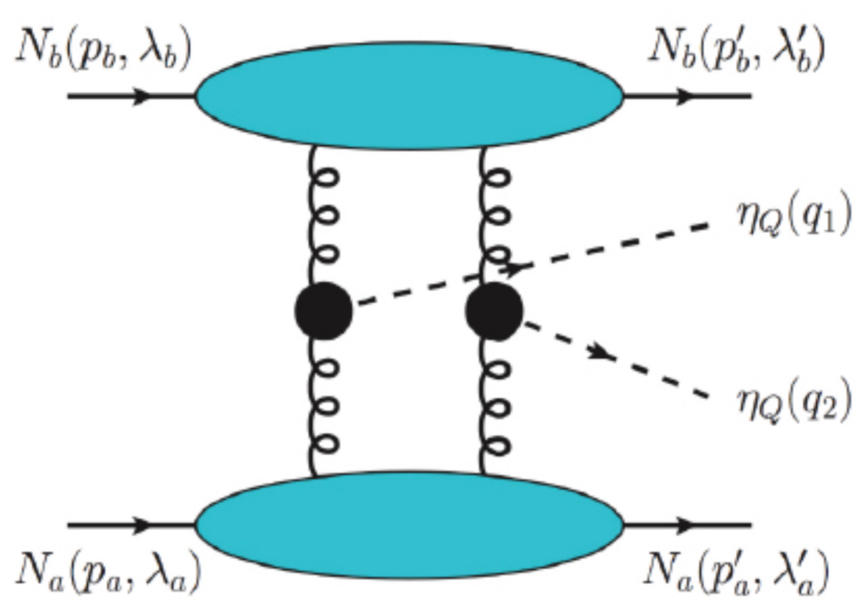
Exclusive dijet production in pA UPC (gluon GTMDs)

Hagiwara, Hatta, Pasechnik, Tasevsky, Teryaev, PRD 96 (2016) 034009



Exclusive double quarkonia production in hadronic collisions (gluon GTMDs)

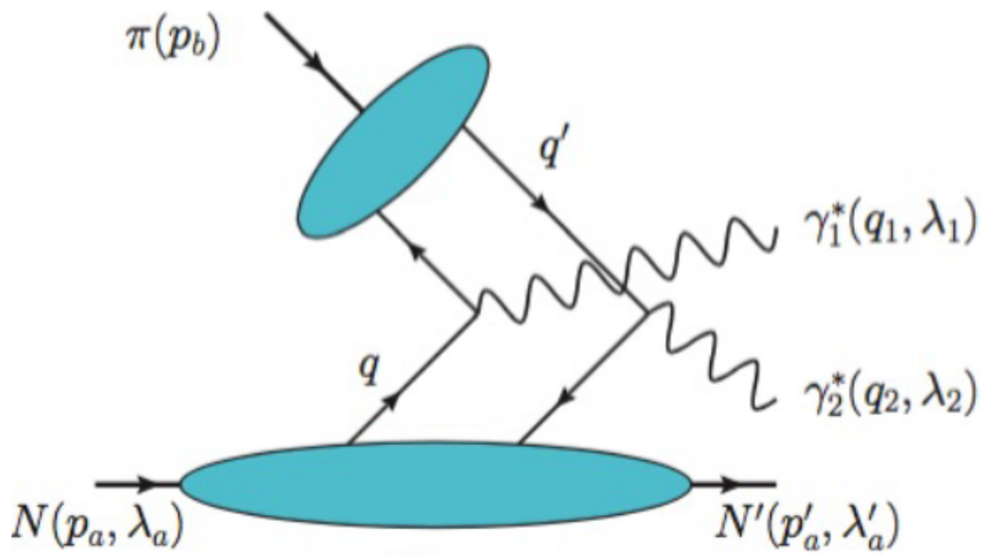
Bhattacharya, Metz, Ojha, Tsai, Zhou, arXiv:1802.10550 *Boussarie, Hatta, Xiao, Yuan, PRD 98 (2015) 074015*



Observables for GTMDs and Wigner functions

Exclusive pion-nucleon double Drell-Yan (quark GTMDs)

Bhattacharya, Metz, Zhou, PLB 771 (2017) 396



- At present, the only known process that is sensitive to quark GTMDs

- In leading order is sensitive to ERBL region only

- Low count rate (amplitude $T \sim \alpha_{\mathrm{em}}^2$)

A worldwide challenge