Nuclear PDFs and EIC

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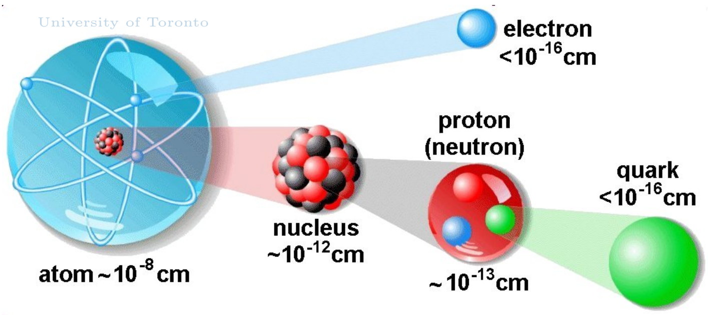
Outline

- Introduction
- Proton PDFs and global QCD analysis
- Nuclear PDFs
- nCTEQ results
- Summary

Structure of matter

- Structure of matter depends on the resolution scale at which it is observed

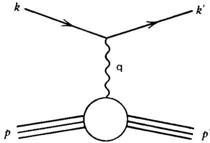
J.J. Thomson 1897

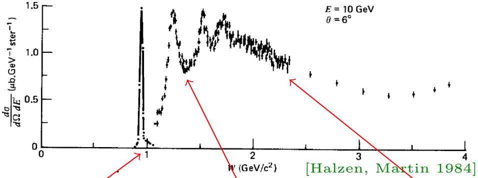


J. Dalton 1808 R. Brown 1827	E. Rutherford H. Geiger E. Marsden 1909	E. Rutherford 1917	SLAC 1968 quark model: Gell-Mann, Zweig 1964 parton model: R. Feynman 1969

Structure of matter

- Until 1960's protons and neutrons were regarded as elementary particles and basic constituents of matter.
- The first evidence for their inner structure came from inelastic collisions.





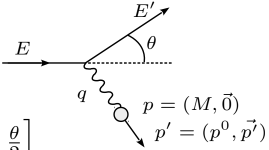
Elastic peak | Resonance region | Inelastic region

Elastic scattering (spin-1/2 relativistic point-like particle):

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}} \frac{E'}{E} \left[\cos^2 \frac{\theta}{2} - \frac{q^2}{2M^2} \sin^2 \frac{\theta}{2}\right]$$

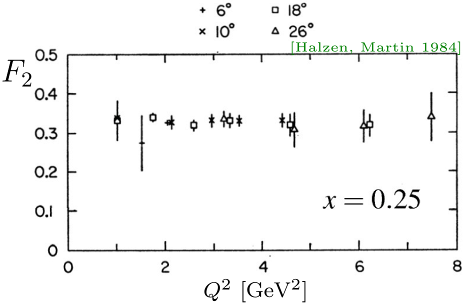
Deep Inelastic Scattering (DIS):

$$\frac{d\sigma}{dE' d\Omega} = \frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}} \left[\frac{F_2(x,Q^2)}{E-E'} \cos^2 \frac{\theta}{2} + \frac{2F_1(x,Q^2)}{M} \sin^2 \frac{\theta}{2}\right]$$



Structure of matter

- Until 1960's protons and neutrons were regarded as elementary particles and basic constituents of matter.
- The first evidence for their inner structure came from inelastic collisions.
- From the observation of **Bjorken scaling** (cross section becomes approximately independent on the energy scale if proton is made up from point-like particles) in SLAC in late 60's.

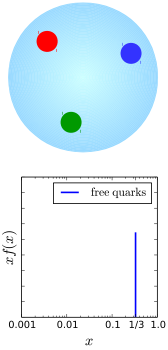


- Explanation given by Feynman in **parton model** (partons later identified with quarks from Gell-Mann model).

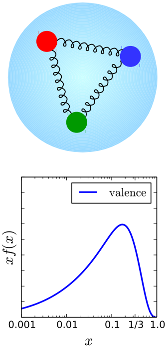
Parton Distribution Functions (PDFs)

PDF $[f_{a/p}(x,\mu)]$: probability that a parton a carries fraction x of proton's momentum (valid at leading-order of QCD).

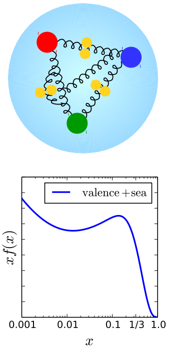
Free quarks



Bound quarks



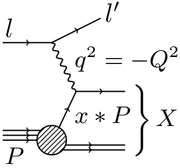
Bound quarks + QCD effects



$$x = \frac{\text{longitudinal parton momentum}}{\text{longitudinal nucleon momentum}} = \frac{p^+_{\text{parton}}}{p^+_{\text{nucleon}}}, \quad \text{where} \quad p^\pm = (p^0 \pm p^3)/\sqrt{2}$$

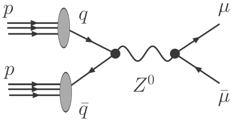
PDFs and QCD Factorization

- **Factorization** in case of **Deep Inelastic Scattering** (DIS)



$$\frac{d^2\sigma}{dx dQ^2} = \sum_{i=q,\bar{q},g} \int_x^1 \frac{dz}{z} f_i(z,\mu) d\hat{\sigma}_{il\to l'X}\left(\frac{x}{z}, \frac{Q}{\mu}\right) + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{Q^2}\right)$$

- **Factorization** in case of **Drell-Yan lepton pair production** (DY)



$$\sigma_{pp\to l\bar{l}X} = \sum_{i,j=q,\bar{q},g} \int_{x_1}^1 dz_1 \int_{x_2}^1 dz_2$$
$$\times\, f_i(z_1,\mu) f_j(z_2,\mu) \hat{\sigma}_{ij\to l\bar{l}X}\left(\frac{x_1}{z_1}, \frac{x_2}{z_2}, \frac{Q}{\mu}\right) + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{Q^2}\right)$$

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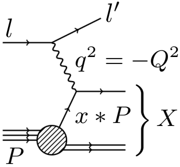

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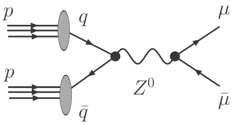

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- $f_i(z,\mu)$ – proton PDFs of parton i (**non-perturbative**).

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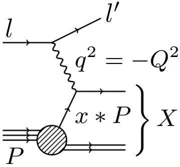

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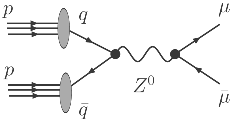

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PDFs are **UNIVERSAL** – do not depend on the process!!!

PDFs and QCD Factorization

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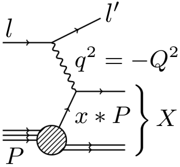

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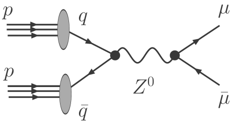

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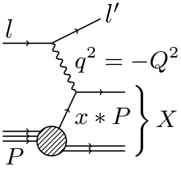

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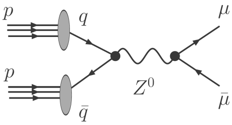

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- $f_i(z,\mu)$ – proton PDFs of parton i (**non-perturbative**).
- $\hat{\sigma}$ – parton level matrix element (**calculable in pQCD**).
- $\mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{Q^2}\right)$ – non-leading terms defining accuracy of factorization formula.

PDFs and QCD Factorization

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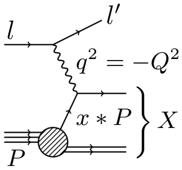

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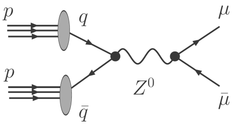

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- Q – characteristic energy scale (DIS: 4-momentum transfer $-q^2$, DY: mass of produced Z/γ boson).

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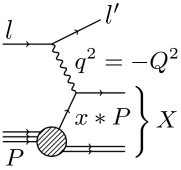

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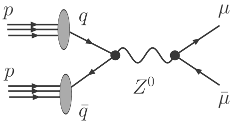

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- Q – characteristic energy scale (DIS: 4-momentum transfer $-q^2$, DY: mass of produced Z/γ boson).
- μ – factorization scale, naturally set to be of order $\sim Q$ (set to be the same as renormalization scale).

Properties of PDFs

- **Number sum rules** – connect partons to quarks from SU(3) flavour symmetry of hadrons; proton (uud), neutron (udd). For protons:

$$\int_0^1 dx \underbrace{[f_u(x) - f_{\bar{u}}(x)]}_{u-\text{valence distr.}} = 2 \qquad \int_0^1 dx \underbrace{[f_d(x) - f_{\bar{d}}(x)]}_{d-\text{valence distr.}} = 1$$

$$\int_0^1 dx[f_s(x) - f_{\bar{s}}(x)] = \int_0^1 dx[f_c(x) - f_{\bar{c}}(x)] = 0$$

- **Momentum sum rule** – momentum conservation connecting all flavours

$$\sum_{i=q,\bar{q},g} \int_0^1 dx\, x f_i(x) = 1$$

Momentum carried by up and down quarks is only around half of the total proton momentum the rest of the momentum is carried by gluons and small amount by sea quarks. In case of CT14NLO PDFs ($\mu = 1.3$ GeV):

$$\int_0^1 dx\, x[f_u(x) + f_d(x)] \simeq 0.51$$

$$\int_0^1 dx\, x f_g(x) \simeq 0.40$$

Scale dependence of PDFs $f_i(x, \mu)$

- x-**dependence** of PDFs is NOT calculable in pQCD
- μ^2-**dependence** is calculable in pQCD – given by **DGLAP** (Dokshitzer-Gribov-Lipatov-Altarelli-Parisi) evolution equations

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DGLAP evolution equations

$$\frac{df_q(x,\mu^2)}{d\log\mu^2} = \frac{\alpha_S(\mu^2)}{2\pi}\int_x^1 \frac{dy}{y}\left[P_{qq}\left(\frac{x}{y}\right)f_q(y,\mu^2) + P_{qg}\left(\frac{x}{y}\right)f_g(y,\mu^2)\right]$$

$$\frac{df_g(x,\mu^2)}{d\log\mu^2} = \frac{\alpha_S(\mu^2)}{2\pi}\int_x^1 \frac{dy}{y}\left[P_{gg}\left(\frac{x}{y}\right)f_g(y,\mu^2) + P_{gq}\left(\frac{x}{y}\right)f_q(y,\mu^2)\right]$$

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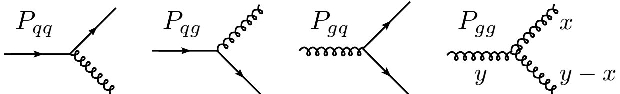
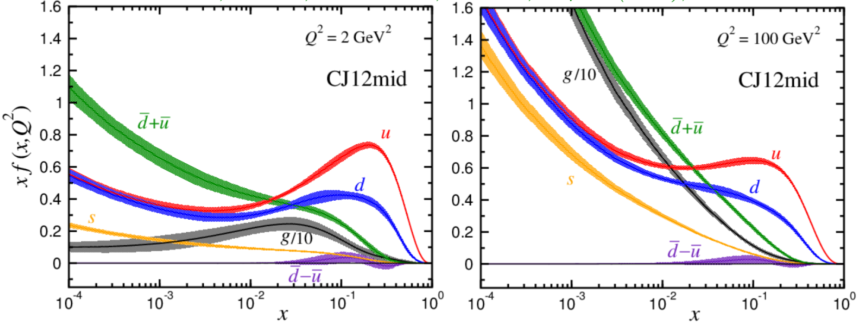
- Different PDFs mix – set of $(2n_f + 1)$ coupled integro-differential equations.
- Initial conditions obtained from fitting experimental data.
- Splitting functions are calculable in pQCD
  $P_{ij}(z) = P_{ij}^{(0)}(z) + \frac{\alpha_S}{2\pi} P_{ij}^{(1)}(z) + \cdots$
  they have interpretation as probabilities of parton splittings:



Scale dependence of PDFs

Owens, Accardi, Melnitchouk, PRD 87, 094012 (2013), `arXiv:1212.1702`



- u and d have a characteristic valence bump at $x \sim 1/3$; they dominate at large x and $u > d$; at low x, $u \simeq d$.
- g dominates at low-x and falls steeply at large x.
- Gluon radiation and creation of $q\bar{q}$ pairs – QCD evolution – transfer of momentum from large to small $x \to$ gluon and quark distributions get steeper at small x.

Determination of PDFs

- **Problem:**
  In order to calculate any high-energy cross-section in pQCD we need PDFs but we don't know how to compute them.
  [Recently there is a significant progress of Lattice QCD computations, see e.g. Prog.Part.Nucl.Phys. 100 (2018) 107; Adv.High Energy Phys. 2019 3036904]

- **Solution:**
  We will determine initial conditions for DGLAP equations by fitting experimental data in a process of *global QCD analysis*.

Schematics of Global Analysis

1. Choose experimental data (e.g. DIS, DY, inclusive jet prod., etc.)
2. Parametrize PDFs at low initial scale $\mu = Q_0 = 1.3\text{GeV}$:

$$f_i(x, Q_0) = f_i(x; a_0, a_1, \dots) = a_0 x^{a_1}(1-x)^{a_2} P(x; a_3, \dots)$$

3. Use DGLAP equation to evolve $f_i(x, \mu)$ from $\mu = Q_0$ to $\mu = Q_{\max}$.
4. Calculate theory predictions corresponding to the data (σ_{DIS}, σ_{DY}, etc.).
5. Calculate appropriate χ^2 function – compare data and theory

$$\chi^2(\{a_i\}) = \sum_{\text{experiments}} w_n \chi_n^2(\{a_i\})$$

$$\chi_n^2(\{a_i\}) = \sum_{\text{data points}} \left(\frac{\text{data} - \text{theory}(\{a_i\})}{\text{uncertainty}}\right)^2$$

(by default $w_n = 1$)

6. Minimize χ^2 function with respect to parameters $a_0, a_1, \dots$

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3. Use DGLAP equation to evolve $f_i(x, \mu)$ from $\mu = Q_0$ to $\mu = Q_{\max}$.
4. Calculate theory predictions corresponding to the data (σ_{DIS}, σ_{DY}, etc.).
5. Calculate appropriate χ^2 function – compare data and theory

$$\chi^2(\{a_i\}) = \sum_{\text{experiments}} w_n \chi^2_n(\{a_i\})$$

$$\chi^2_n(\{a_i\}) = \sum_{\text{data points}} \left(\frac{\text{data} - \text{theory}(\{a_i\})}{\text{uncertainty}} \right)^2$$

(by default $w_n = 1$)

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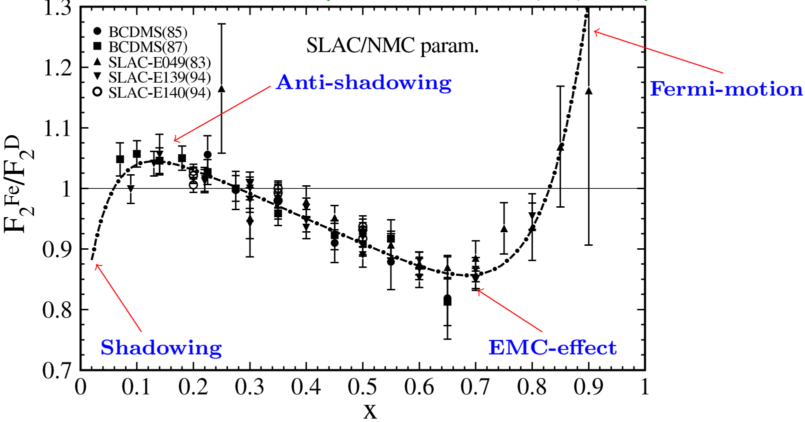
Nuclear PDFs

Motivation

- Cross-sections in nuclear collisions are modified

$$F_2^A(x) \neq Z F_2^p(x) + N F_2^n(x)$$

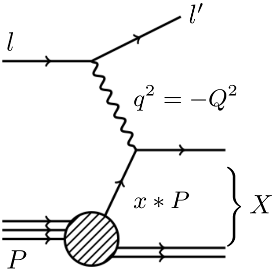
[Schienbein *et al.* PRD 77 (2008) 054013]



- Can we translate this modifications into **universal nuclear PDFs**?

Motivation

- Factorization in case of deep inelastic scattering (DIS)

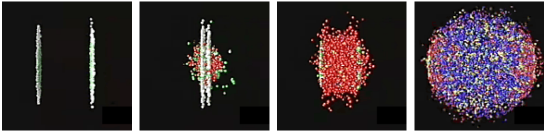


$$\frac{d^2\sigma}{dxdQ^2} = \sum_i f_i(x, Q^2) \otimes d\hat{\sigma}_{il \to l' X}$$

- We assume that the nuclear effects can be absorbed into the universal nPDFs.
- Do not consider any cold nuclear matter effects (e.g. energy loss).

Motivations: Why do we need nuclear PDFs?

- Information on the structure of nucleus.
- Description of high-energy heavy ion collisions at the **LHC** and **RHIC**.



Key ingredient to use perturbative probes of QGP (separate non QGP effects in pA, AA collisions).

- Computation of prompt atmospheric neutrino flux.
- Differentiate flavors in free-proton PDFs (e.g. strange)

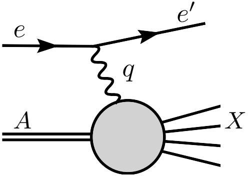
charged lepton DIS

$$F_2^{l^\pm} \sim \left(\tfrac{1}{3}\right)^2 [d+s] + \left(\tfrac{2}{3}\right)^2 [u+c]$$

neutrino DIS

$$F_2^{\nu} \sim \left[d+s+\bar{u}+\bar{c}\right]$$
$$F_2^{\bar{\nu}} \sim \left[\bar{d}+\bar{s}+u+c\right]$$
$$F_3^{\nu} \sim 2\left[d+s-\bar{u}-\bar{c}\right]$$
$$F_3^{\bar{\nu}} \sim 2\left[u+c-\bar{d}-\bar{s}\right]$$

Variables: DIS of nuclear target $eA \to e'X$



- DIS variables in case on nucleons in nucleus $\begin{cases} Q^2 \equiv -q^2 \\ x_A \equiv \frac{Q^2}{2\,p_A \cdot q} \end{cases}$
  - p^A – nucleus momentum
  - $x_A \in (0, 1)$ – analog of Bjorken variable (fraction of the nucleus momentum carried by a nucleon)
- Analogue variables for partons:
  - $p_N = \frac{p_A}{A}$ – *average* nucleon momentum
  - $x_N \equiv \frac{Q^2}{2\,p_N \cdot q} = A\,x_A$ – parton momentum fraction with respect to the average nucleon momentum p_N
  - $x_N \in (0, A)$ – parton can carry more than the average nucleon momentum p_N.

Assumptions entering the nuclear PDF analysis

1. **Factorization** & DGLAP evolution
   - allow for definition of **universal PDFs**
   - make the formalism **predictive**
   - needed even if it is broken
2. Isospin symmetry $\begin{cases} u^{n/A}(x) = d^{p/A}(x) \\ d^{n/A}(x) = u^{p/A}(x) \end{cases}$ $\quad f_i^{(A,Z)} = \frac{Z}{A} f_i^{p/A} + \frac{A-Z}{A} f_i^{n/A}$
3. The *bound proton* PDFs have the *same evolution equations* and sum rules as the free proton PDFs *provided we neglect any contributions from the region* $x > 1$ (which is expected to have negligible contribution [PRC 73, 045206 (2006), arXiv:hep-ph/0509241])

Then observables $\mathcal{O}^A$ can be calculated as:

$$\mathcal{O}^A = Z\,\mathcal{O}^{p/A} + (A-Z)\,\mathcal{O}^{n/A}$$

With the above assumptions we can use the free proton framework to analyze nuclear data

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Uncertainties in global analysis

- **Experimental errors** (included in PDFs error analysis)
- **Theoretical uncertainties**
  (e.g. MHO, HF schemes; not included; EPJC 79 (2019) 931)
- **"Details"** of Global Fits
  (e.g. parametrization; not included)

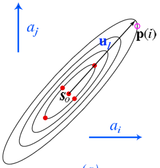
Propagating experimental errors to PDFs:

- Hessian Method
  - Eigenvector PDFs
  - Quadratic approximation
  - Simple computation of correlations
- Lagrange Multipliers
- Monte Carlo Methods
  - generate N data samples by varying data within errors;
  - perform N fits to the samples $\rightarrow$ PDF replicas
  - estimate uncertainty by calculating moments of PDF replicas

Hessian method

[JHEP 07 (2002) 012, `arXiv:hep-ph/0201195`]

- **Expand** χ^2 function around minimum, $\{a_i^0\}$,
  $$\chi^2 = \chi_0^2 + \sum_{ij} \frac{1}{2}(a_i - a_i^0)(a_j - a_j^0)\left(\frac{\partial^2\chi^2}{\partial a_i \partial a_j}\right)_0$$



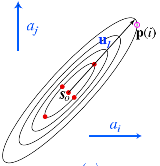
(a)
Original parameter basis

Hessian method

[JHEP 07 (2002) 012, `arXiv:hep-ph/0201195`]

- **Expand** χ^2 function around minimum, $\{a_i^0\}$, and **diagonalize**

$$\chi^2 = \chi_0^2 + \sum_{ij} \frac{1}{2}(a_i - a_i^0)(a_j - a_j^0)\left(\frac{\partial^2\chi^2}{\partial a_i \partial a_j}\right)_0 = \chi_0^2 + \sum_i \lambda_i z_i^2$$



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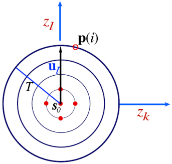

(b)
Orthonormal eigenvector basis

Hessian method

[JHEP 07 (2002) 012, arXiv:hep-ph/0201195]

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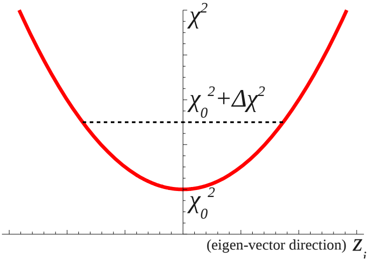
- Choose tolerance criteria $\Delta\chi^2 = \chi^2 - \chi_0^2$ value (defining 1-σ uncertainty),
  - ideal case $\Delta\chi^2 = 1$
  - realistic global analysis $\Delta\chi^2 \sim 1 - 100$



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[JHEP 07 (2002) 012, arXiv:hep-ph/0201195]

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  - ideal case $\Delta\chi^2 = 1$
  - realistic global analysis $\Delta\chi^2 \sim 1 - 100$
- Construct error PDFs corresponding to each eigenvector direction:

  $$f_i^\pm = f(\{z_i\}) = f(0, \ldots, z_i = \pm\sqrt{\Delta\chi^2}, \ldots, 0)$$
  $$z_i = \pm\sqrt{\Delta\chi^2}$$

- Calculate errors of observable X:

  $$\Delta X = \sqrt{\sum_i \left(\frac{\partial X}{\partial z_i} \times \delta z_i\right)^2} \simeq \boxed{\frac{1}{2}\sqrt{\sum_i \left[X(f_i^+) - X(f_i^-)\right]^2}}$$

Differences with the free-proton PDFs

- Theoretical status of Factorization
- Parametrization – more parameters to model A-dependence
- Different data sets – much less data:
  - Less data → less constraining power → more assumptions (fixing) about a_i parameters
  - **Assumptions limit/replace uncertainties!**

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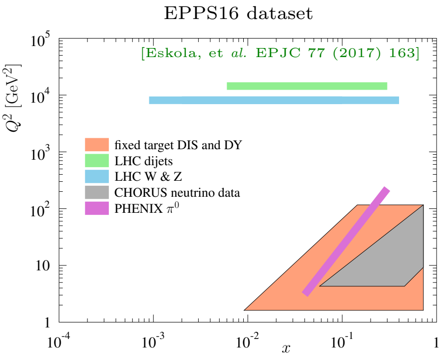

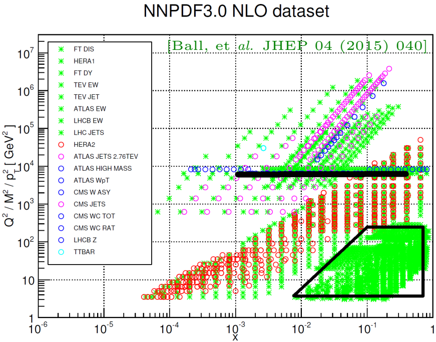

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Summary of available nuclear PDFs

Available nuclear PDFs

- **Multiplicative nuclear correction factors**

$$f_i^{p/A}(x_N, \mu_0) = R_i(x_N, \mu_0, A)\ f_i^{free\ proton}(x_N, \mu_0)$$

  - **HKN**: Hirai, Kumano, Nagai
    [PRC 76, 065207 (2007), arXiv:0709.3038]
  - **DSSZ**: de Florian, Sassot, Stratmann, Zurita
    [PRD 85, 074028 (2012), arXiv:1112.6324]
    EPPS16: Eskola, Paakkinen, Paukkunen, Salgado
    [EPJC 77 (2017) 163, arXiv:1612.05741]
  - **KT16**: H.Khanpour, S.A.Tehrani
    [PRD 93, 014026 (2016), arXiv:1601.00939]

- **Native nuclear PDFs**

$$f_i^{p/A}(x_N, \mu_0) = f_i(x_N, A, \mu_0), \quad f_i(x_N, A=1, \mu_0) \equiv f_i^{free\ proton}(x_N, \mu_0)$$

  - **nCTEQ15(WZ)**: Jezo, Kovarik, Kusina, Olness, Schienbein, et al.
    [PRD 93, 085037 (2016), arXiv:1509.00792; EPJC 80 (2020) 968]
  - **TUJU19**: M.Walt, I.Helenius, W.Vogelsang
    [PRD 100, 096015 (2019), arXiv:1908.03355]
  - **nNNPDF2.0**: Khalek, Ethier, Rojo, van Weelden
    [JHEP 09 (2020) 183, arXiv:2006.14629]

nPDF framework

- **Parametrization**
  - PDF of nucleus (A - mass, Z - charge)

$$f_i^{(A,Z)}(x,Q) = \frac{Z}{A} f_i^{p/A}(x,Q) + \frac{A-Z}{A} f_i^{n/A}(x,Q)$$

  - bound neutron PDFs, $f_i^{n/A}$, constructed assuming iso-spin symmetry
    (Note that $f_i^{p/A}$, $f_i^{n/A}$ are not physical objects just convenient way of parameterizing f_i^A)
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nCTEQ15 [arXiv:1509.00792]

$$x f_i^{p/A}(x,Q_0) = x^{c_1}(1-x)^{c_2} e^{c_3 x}(1+e^{c_4}x)^{c_5}$$

$$c_k \to c_k(A) \equiv c_{k,0} + c_{k,1}\left(1 - A^{-c_{k,2}}\right)$$

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EPPS16 [arXiv:1612.05741]

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$$R_i^A(x,Q_0) = \begin{cases} a_0 + a_1(x-x_a)^2 & x \le x_a \\ b_0 + b_1 x^\alpha + b_2 x^{2\alpha} + b_3 x^{3\alpha} & x_a \le x \le \\ c_0 + (c_1 - c_2 x)(1-x)^{-\beta} & x_e \le x \le \end{cases}$$

$$d_i \to d_i(A) = d_i(A_{\rm ref})\left(\frac{A}{A_{\rm ref}}\right)^{\gamma_i[d_i(A_{\rm ref})-1]},$$

with $d_i = a_i, b_i, \ldots$ and $A_{\rm ref} = 12$

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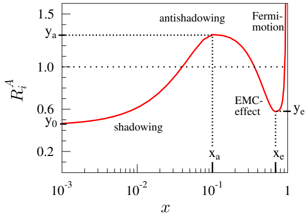

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Comparison of available nPDFs

	EPPS16	TUJU19	nCTEQ15	nCTEQ15WZ	nNNPDF2.0
FT NC DIS	✓	✓	✓	✓	✓
FT CC DIS	✓	✓	✗#	✗	✓
FT Drell-Yan	✓	✗	✓	✓	✗
RHIC π^0	✓	✗	✓	✓	✗
LHC W/Z	✓	✗	✗*	✓	✓
LHC dijet	✓	✗	✗	✗	✗
QCD order	NLO	NLO & NNLO	NLO	NLO	NLO
Kinematic cuts	$Q > 1.3\text{GeV}$	$Q^2 > 3.5\text{GeV}^2$ $W^2 > 12\text{GeV}^2$ $x < 0.7$	$Q > 2\text{GeV}$ $W > 3.5\text{GeV}$	$Q > 2\text{GeV}$ $W > 3.5\text{GeV}$	$Q^2 > 3.5\text{GeV}^2$ $W^2 > 12.5\text{GeV}^2$
No data points	1811	2336	740	860	1467
No free param.	20	16	16	19	NN
χ^2/dof	1.00	0.89(0.86)	0.81	0.91	0.98
Error analysis	Hessian	Hessian	Hessian	Hessian	Monte Carlo
Tolerance $\Delta\chi^2$	52	50	35	35	–
Proton baseline	CT14NLO	HERAPDF2.0	CTEQ6.1	CTEQ6.1	NNPDF3.1
Heavy-quark eff.	✓	✓	✓	✓	✓
Flavour sep.	✓	✗	✓	✓	✓
Reference	[1612.05741]	[1908.03355]	[1509.00792]	[2007.09100]	[2006.14629]

In a separate dediacated analysis [PRL106, 122301, (2011), 1012.0286; PRD80, 094004, (2009), 0907.2357]
* See a reweighting study [EPJC77, 488 (2017), 1610.02925]

Comparison of available nPDFs

	EPPS16	**TUJU19**	**nCTEQ15**	**nCTEQ15WZ**	**nNNPDF2.0**
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FT CC DIS	✓	✓	✗#	✗	✓
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RHIC π^0	✓	✗	✓	✓	✗
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Error analysis	Hessian	Hessian	Hessian	Hessian	Monte Carlo
Tolerance $\Delta\chi^2$	52	50	35	35	–
Proton baseline	CT14NLO	HERAPDF2.0	CTEQ6.1	CTEQ6.1	NNPDF3.1
Heavy-quark eff.	✓	✓	✓	✓	✓
Flavour sep.	✓	✗	✓	✓	✓
Reference	[1612.05741]	[1908.03355]	[1509.00792]	[2007.09100]	[2006.14629]

In a separate dediacated analysis [PRL106, 122301, (2011), 1012.0286; PRD80, 094004, (2009), 0907.2357]

* See a reweighting study [EPJC77, 488 (2017), 1610.02925]

Comparison of available nPDFs

	EPPS16	**TUJU19**	**nCTEQ15**	**nCTEQ15WZ**	**nNNPDF2.0**
FT NC DIS	✓	✓	✓	✓	✓
FT CC DIS	✓	✓	✗#	✗	✓
FT Drell-Yan	✓	✗	✓	✓	✗
RHIC π^0	✓	✗	✓	✓	✗
LHC W/Z	✓	✗	✗*	✓	✓
LHC dijet	✓	✗	✗	✗	✗
QCD order	NLO	NLO & NNLO	NLO	NLO	NLO
Kinematic cuts	$Q > 1.3\text{GeV}$	$Q^2 > 3.5\text{GeV}^2$ $W^2 > 12\text{GeV}^2$ $x < 0.7$	$Q > 2\text{GeV}$ $W > 3.5\text{GeV}$	$Q > 2\text{GeV}$ $W > 3.5\text{GeV}$	$Q^2 > 3.5\text{GeV}^2$ $W^2 > 12.5\text{GeV}^2$
No data points	1811	2336	740	860	1467
No free param.	20	16	16	19	NN
χ^2/dof	1.00	0.89(0.86)	0.81	0.91	0.98
Error analysis	Hessian	Hessian	Hessian	Hessian	Monte Carlo
Tolerance $\Delta\chi^2$	52	50	35	35	–
Proton baseline	CT14NLO	HERAPDF2.0	CTEQ6.1	CTEQ6.1	NNPDF3.1
Heavy-quark eff.	✓	✓	✓	✓	✓
Flavour sep.	✓	✗	✓	✓	✓
Reference	[1612.05741]	[1908.03355]	[1509.00792]	[2007.09100]	[2006.14629]

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LHC W/Z	✓	✗	✗*	✓	✓
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QCD order	NLO	NLO & NNLO	NLO	NLO	NLO
Kinematic cuts	$Q > 1.3\text{GeV}$	$Q^2 > 3.5\text{GeV}^2$ $W^2 > 12\text{GeV}^2$ $x < 0.7$	$Q > 2\text{GeV}$ $W > 3.5\text{GeV}$	$Q > 2\text{GeV}$ $W > 3.5\text{GeV}$	$Q^2 > 3.5\text{GeV}^2$ $W^2 > 12.5\text{GeV}^2$
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Heavy-quark eff.	✓	✓	✓	✓	✓
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Reference	[1612.05741]	[1908.03355]	[1509.00792]	[2007.09100]	[2006.14629]

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* See a reweighting study [EPJC77, 488 (2017), 1610.02925]

nCTEQ15 fit details [Kovarik, et *al.* PRD 93 (2016) 085037]

Fit properties:

- fit @NLO
- $Q_0 = 1.3\text{GeV}$
- using ACOT heavy quark scheme
- kinematic cuts:
  $Q > 2\text{GeV}$, $W > 3.5\text{GeV}$
  $p_T > 1.7$ GeV
- 708 (DIS & DY) + 32 (single π^0) = 740 data points after cuts
- 16+2 free parameters
  - 7 gluon
  - 7 valence
  - 2 sea
  - 2 pion data normalizations
- $\chi^2 = 587$, giving $\chi^2/\text{dof} = 0.81$

Error analysis:

- use Hessian method

$$\chi^2 = \chi_0^2 + \frac{1}{2} H_{ij}(a_i - a_i^0)(a_j - a_j^0)$$

$$H_{ij} = \frac{\partial^2 \chi^2}{\partial a_i \partial a_j}$$

- tolerance $\Delta\chi^2 = 35$ (every nuclear target within 90% C.L.)
- eigenvalues span 10 orders of magnitude $\rightarrow$ require numerical precision
- use noise reducing derivatives

nCTEQ15 fit details [Kovarik, et *al.* PRD 93 (2016) 085037]

Kinematic cuts

nCTEQ:

$$\begin{cases} Q > 2 \text{ GeV} \\ W > 3.5 \text{ GeV} \end{cases}$$

EPS: $Q > 1.3$ GeV

HKN: $Q > 1$ GeV

DSSZ: $Q > 1$ GeV

Fit properties:

- fit @NLO
- $Q_0 = 1.3$
- using AC
- kinematic cuts: $Q > 2\text{GeV}$, $W > 3.5\text{GeV}$ $p_T > 1.7$ GeV
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$- a_i^0)(a_j - a_j^0)$

$$H_{ij} = \frac{\partial^2 \chi^2}{\partial a_i \partial a_j}$$

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EPS: $Q > 1.3$ GeV

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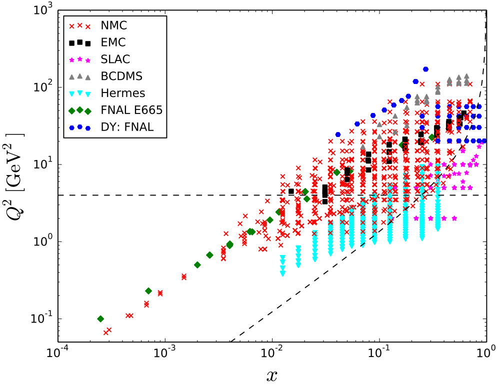

nCTEQ: 740 data points

EPS09: 929 data points

nCTEQ15 fit details [Kovarik, et *al.* PRD 93 (2016) 085037]

Hessian method

- choice of tolerance: $T = 35$ [PRD65 (2001) 014012, arXiv:hep-ph/0101051]
- quadratic approximation

$p_T > 1.7$ GeV

- 708 (DIS & DY) + 32 (single π^0) = 740

Error analysis:

- use Hessian method

$$\chi^2 = \chi_0^2 + \frac{1}{2} H_{ij}(a_i - a_i^0)(a_j - a_j^0)$$

$$H_{ij} = \frac{\partial^2 \chi^2}{\partial a_i \partial a_j}$$

- tolerance $\Delta\chi^2 = 35$ (every nuclear target within 90% C.L.)

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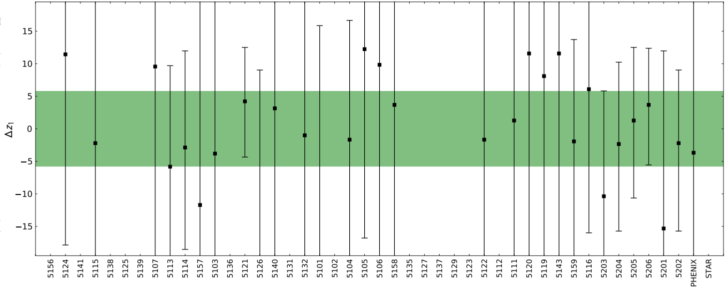

nCTEQ15 fit details [Kovarik, et *al.* PRD 93 (2016) 085037]

Hessian method

- choice of tolerance: $T = 35$
- quadratic approximation

F

- kinematic cuts:
  $Q > 2\text{GeV}$, $W > 3.5\text{GeV}$
  $p_T > 1.7$ GeV
- 708 (DIS & DY) + 32 (single π^0) = 740 data points after cuts
- 16+2 free parameters
  - 7 gluon
  - 7 valence
  - 2 sea
  - 2 pion data normalizations
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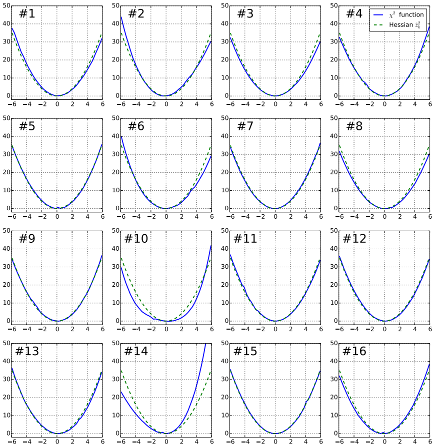
Error analysis:

- use Hessian method



nCTEQ15 fit details [Kovarik, et *al.* PRD 93 (2016) 085037]

F

Fit quality

- $\chi^2/dof = 0.81$

- using ACOT heavy quark sche
- kinematic cuts:
  $Q > 2\text{GeV}$, $W > 3.5\text{GeV}$
  $p_T > 1.7$ GeV
- 708 (DIS & DY) + 32 (single π
  data points after cuts
- 16+2 free parameters
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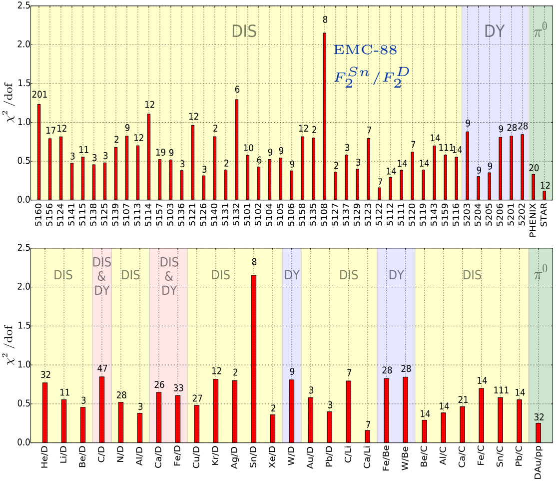
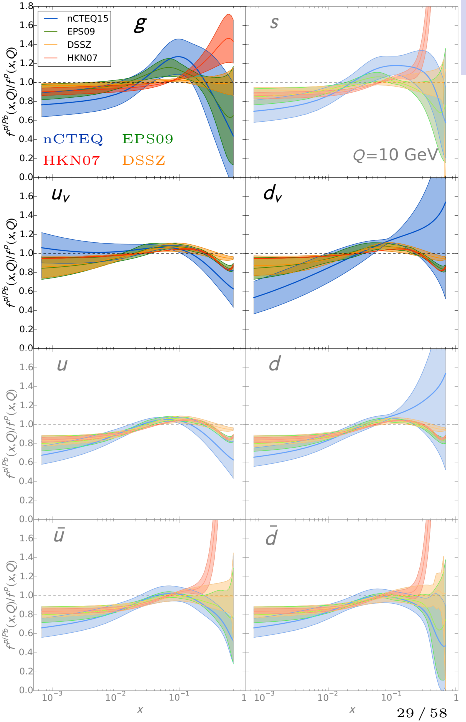


nCTEQ results

Nuclear correction factors ($Q = 10$ GeV)

$$R_i^{\rm Pb} = \frac{f_i^{p/{\rm Pb}}(x,Q)}{f_i^p(x,Q)}$$

- different solution for d-valence & u-valence compared to EPS09 & DSSZ
- sea quark nuclear correction factors similar to EPS09
- nuclear correction factors depend largely on underlying proton baseline

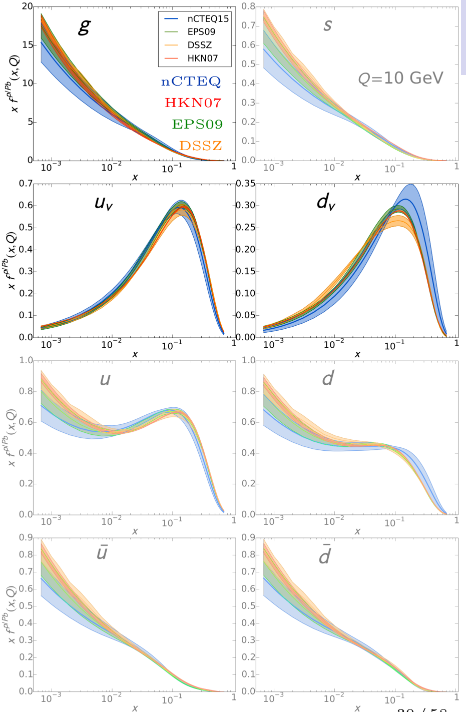


nCTEQ results

Bound proton PDFs ($Q = 10\text{GeV}$)

$$x\, f_i^{p/\mathrm{Pb}}(x, Q)$$

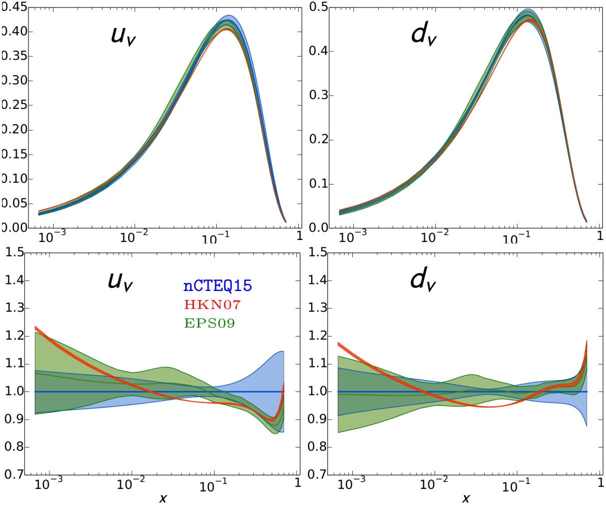
- nCTEQ features larger uncertainties than previous nPDFs
- better agreement between different groups (nPDFs don't depend on proton baseline)



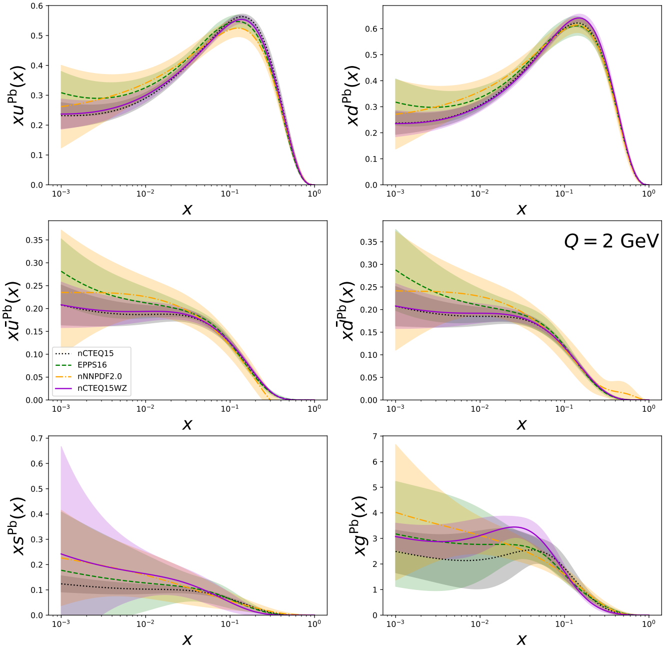
Valence nuclear distributions

Full lead nucleus distribution:

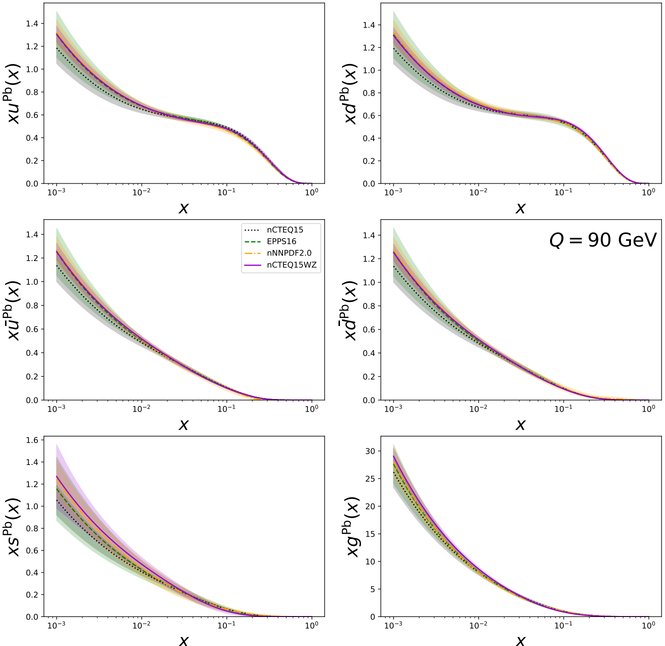
$$f^{\mathrm{Pb}} = \frac{82}{208} f^{p/\mathrm{Pb}} + \frac{208-82}{208} f^{n/\mathrm{Pb}}$$



Recent nPDFs [Kusina, et *al.* EPJC 80 (2020) 968]



Recent nPDFs [Kusina, et *al.* EPJC 80 (2020) 968]



Current status

The picture coming from different nPDF analyses is rather *consistent* but we still don't know a lot of things:

- Compatibility of NC DIS and CC DIS $\rightarrow$ *universality*.
- Better information on *strange* and *gluon* nPDFs.
- Small-x (and onset of non-linear regime).
- Large-x (lack of good data, HT corrections, $x > 1$ region).
- A-dependence.

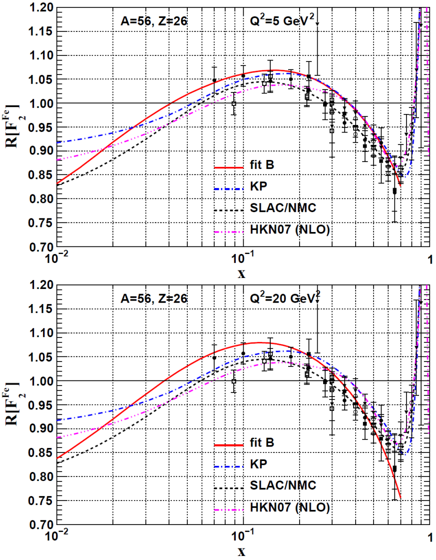
We can try to answer to some of these questions using current data but for some we need to wait for *new experiments*.

Selected results

- Compatibility of NC & CC DIS
- Strange nPDF
- Small-x gluon
- Large-x
- EIC and future constraints

Compatibility of NC DIS and CC DIS [PRD 80 (2009) 094004]

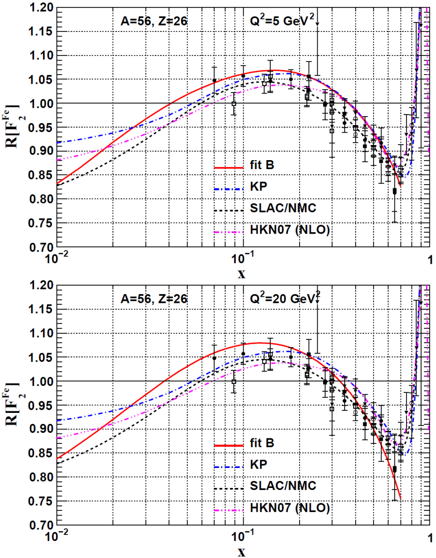
$F_2^{\ell^\pm \mathrm{Fe}}/F_2^{\ell^\pm \mathrm{D}}$



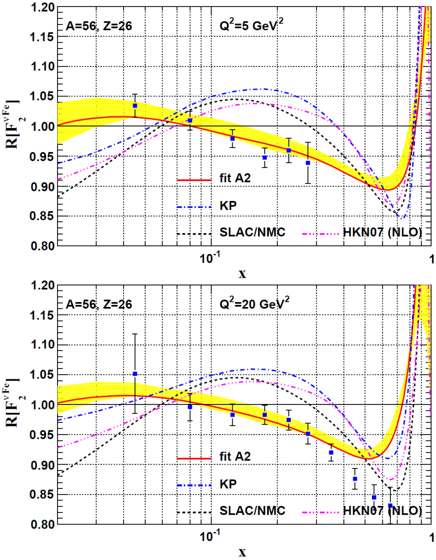
$F_2^{\nu \mathrm{Fe}}/F_2^{\nu \mathrm{D}}$ [PRD 77 (2008) 054013]

Compatibility of NC DIS and CC DIS [PRD 80 (2009) 094004]

$F_2^{\ell^\pm \mathrm{Fe}} / F_2^{\ell^\pm \mathrm{D}}$



$F_2^{\nu \mathrm{Fe}} / F_2^{\nu \mathrm{D}}$ [PRD 77 (2008) 054013]



Selected results

- Compatibility of NC & CC DIS
- Strange nPDF
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- EIC and future constraints

Extracting s-quark PDF

[PRD 85, 094028 (2012)]

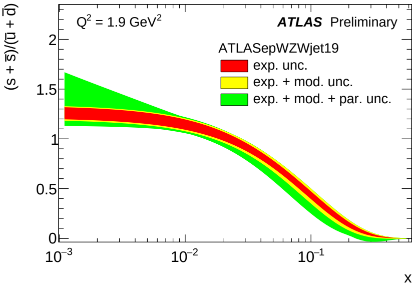
Predominant information on strange used to come from difference of NC and CC DIS F_2 structure function (at LO neglecting charm)

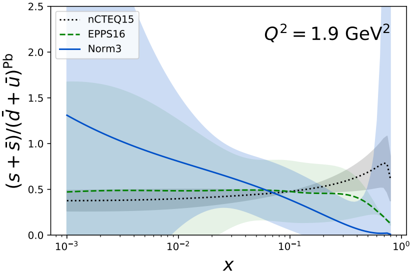
$$\Delta F_2 = \frac{5}{18} F_2^{CC} - F_2^{NC} \sim \frac{x}{6}[s(x) + \bar{s}(x)]$$

s is small compared to u and d PDFs → large uncertainties → it was assumed (CTEQ6.1, CTEQ6.5)

$$s(x) = \bar{s}(x) \sim \kappa \, \frac{\bar{u}(x) + \bar{d}(x)}{2}, \qquad \kappa = \frac{1}{2}$$

→ underestimation of s PDF uncertainty, as $\bar{u}$, $\bar{d}$ are much better constrained.





- Need to do a combined analysis of pPb W/Z LHC data and *neutrino* data.

Possible future constraints of s-PDF

- **Lattice QCD** see e.g. [arXiv:2006.08636]
  - not too long future use **PDF-moments** calculated from Lattice;
  - less established more powerful: **quasi/pseudo-PDFs** (give full x-dependence);
  - especially viable for *helicity PDFs* which are less constraint.
- **EIC**
  - e.g. using charm jets [arXiv:2006.12520]
  - big advantage: will provide data for **protons** but also for a **range of nuclei**.

Selected results

- Compatibility of NC & CC DIS
- Strange nPDF
- Small-x gluon
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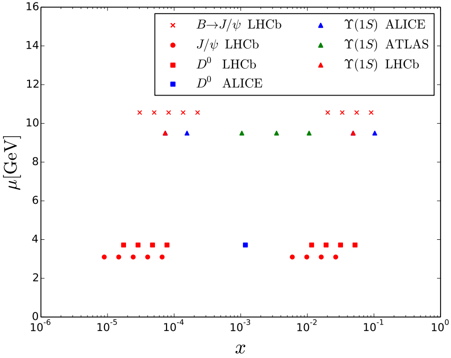
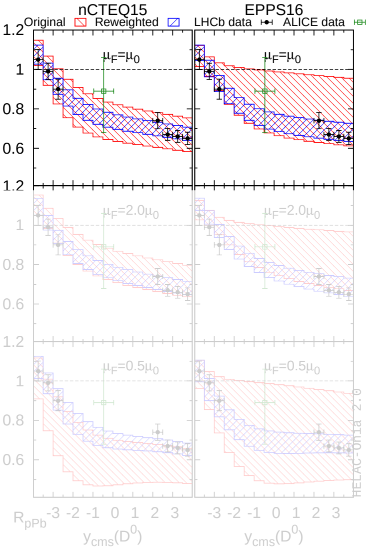
Small-x gluon from pPb LHC heavy-flavour data

[PRL 121, 052004 (2018)]

	D^0	J/ψ	$B \to J/\psi$	$\Upsilon(1S)$
μ_0	$\sqrt{4M_{D^0}^2 + P_{T,D^0}^2}$	$\sqrt{M_{J/\psi}^2 + P_{T,J/\psi}^2}$	$\sqrt{4M_B^2 + \left(\frac{M_B}{M_{J/\psi}} P_{T,J/\psi}\right)^2}$	$\sqrt{M_{\Upsilon(1S)}^2 + P_{T,\Upsilon(1S)}^2}$
p+p data	LHCb [1]	LHCb [2,3]	LHCb [2,3]	ALICE [4], ATLAS [5], CMS [6], LHCb [7,8]
R_{pPb} data	ALICE [9], LHCb [15]	ALICE [10,11], LHCb [16,12]	LHCb [12]	ALICE [13], ATLAS [14], LHCb [17]



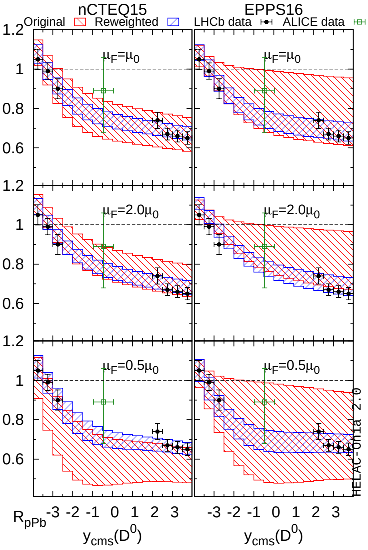
Reweighting with D^0 data



LHCb [JHEP 1710 (2017) 090, 1707.02750]
ALICE [PRL113, 232301 (2014), 1405.3452]

- Initial description of data is good for both nCTEQ15 and EPPS16.
- Substantial reduction of uncertainty especially for EPPS16.

Reweighting with D^0 data



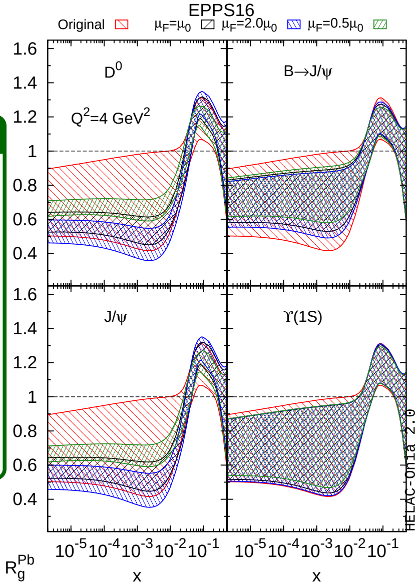
LHCb [JHEP 1710 (2017) 090, 1707.02750]
ALICE [PRL113, 232301 (2014), 1405.3452]

- Initial description of data is good for both nCTEQ15 and EPPS16.
- Substantial reduction of uncertainty especially for EPPS16.
- If we include factorization scale uncertainty errors increase and it can become the dominant uncertainty.

Reweighting results: $R_g^{\mathrm{Pb}} = f_g^{\mathrm{Pb}}/f_g^p$

Comments

- First clear experimental observation of gluon SHADOWING at low x.
- The scale ambiguity for D^0 and J/ψ production is now the dominant uncertainty.
- Non-prompt $B \to J/\psi$ are really promising if improved data can be obtained.



Consistency with other data

- The results of the [PRL 121 (2018), 052004] study were successfully used e.g. to describe data at RHIC.

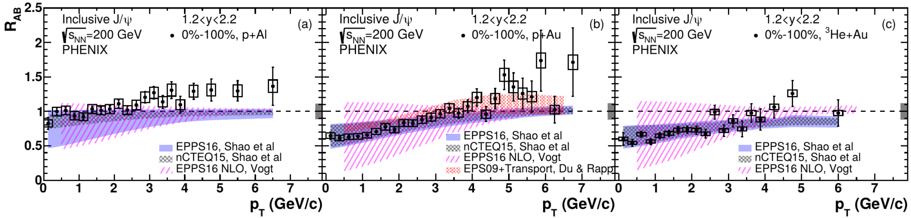


FIG. 10. Nuclear modification factor of inclusive J/ψ as a function of p_T at forward rapidity ($p/^3$He-going direction) for 0%–100% p+Al, p+Au, and ^{3}He+Au collisions. Bars (boxes) around data points represent point-to-point uncorrelated (correlated) uncertainties. The theory bands are discussed in the text.

arXiv:1910.14487
see also: K. Smith, Quark Matter 2019

Selected results

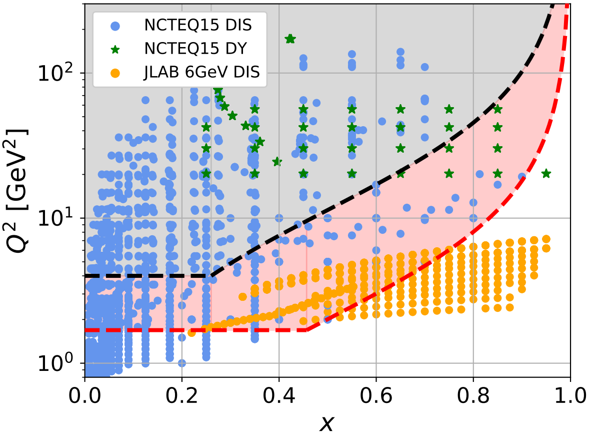
- Compatibility of NC & CC DIS
- Strange nPDF
- Small-x gluon
- Large-x
- EIC and future constraints

Large-x data from JLAB

In (n)PDF analyses we use kinematic cuts to exclude data that are

- in non-perturbative region
- have significant higher-twist corrections.

This is typically done by cuts on Q^2 and $W^2 = Q^2(1-x)/x + M_N^2$.



$Q > 2$ GeV
$W > 3.5$ GeV

$Q > 1.3$ GeV
$W > 1.7$ GeV

Large-x data from JLAB

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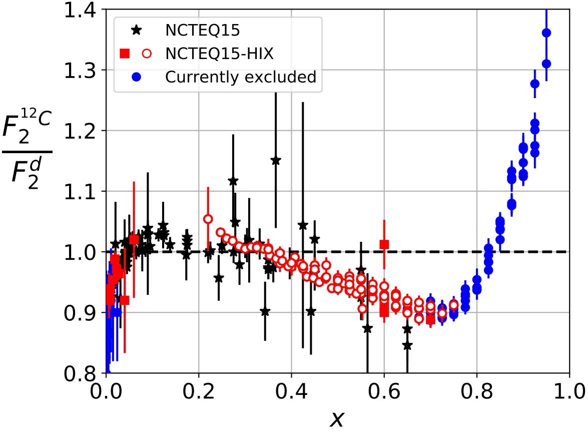


$Q > 2$ GeV
$W > 3.5$ GeV

$Q > 1.3$ GeV
$W > 1.7$ GeV

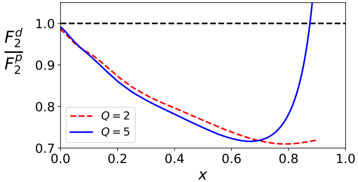
Large-x data from JLAB

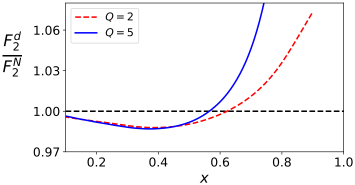
Target	Experiment	ID	Ref.	# data	#data after cuts
^{208}Pb/D	CLAS	9976	[11]	25	24
^{56}Fe/D	CLAS	9977	[11]	25	24
^{27}Al/D	CLAS	9978	[11]	25	24
^{12}C/D	CLAS	9979	[11]	25	24
^{4}He/D	Hall C	9980	[12]	25	17
		9981	[12]	26	16
^{3}He/D	Hall C	9982	[12]	25	17
		9983	[12]	26	16
^{64}Cu/D	Hall C	9984	[12]	25	17
		9985	[12]	26	16
^{9}Be/D	Hall C	9986	[12]	25	17
		9987	[12]	26	16
^{197}Au/D	Hall C	9988	[12]	24	17
		9989	[12]	26	16
^{12}C/D	Hall C	9990	[12]	25	17
		9991	[12]	17	7
		9992	[12]	26	16
		9993	[12]	18	6
		9994	[12]	17	7
		9995	[12]	15	2
		9996	[12]	19	7
		9997	[12]	16	2
		9998	[12]	21	8
		9999	[12]	18	3
Total				546	428

Large-x data from JLAB

Effects we include:

- *Target-mass corrections* (OPP) & *dynamic higher-twist* effects $\rightarrow$ to good extent cancel in ratio.
- *Deuteron corrections*

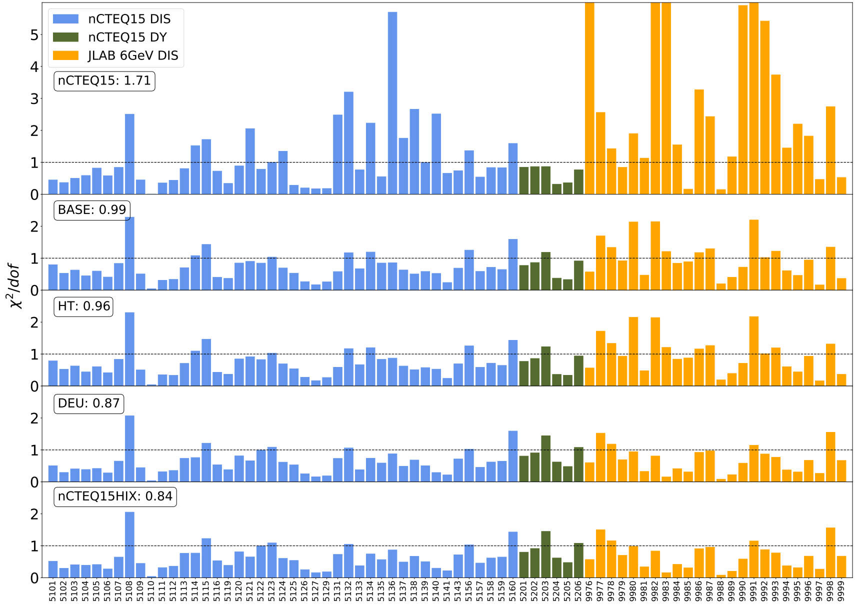




Effects that would need to be included when allowing for even higher-x (lower W):

- Non-vanishing structure functions at $x > 1$ and corresponding extension of DGLAP evolution.
- Threshold resummation.

Large-x data from JLAB



Selected results

- Compatibility of NC & CC DIS
- Strange nPDF
- Small-x gluon
- Large-x
- EIC and future constraints

A-dependence

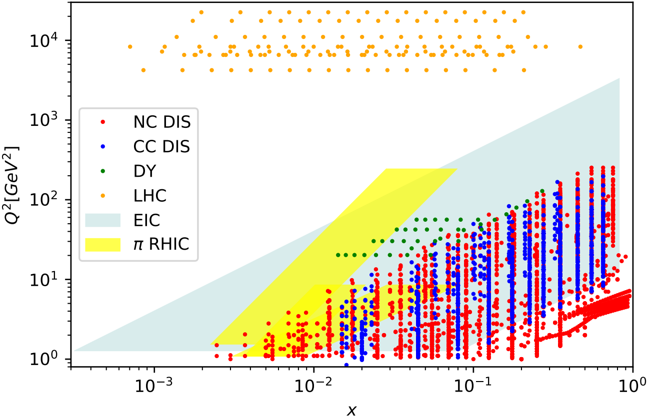
- At the moment the A-dependence is predominantly assumed in certain form, e.g.

$$c_k(A) \equiv c_{k,0} + c_{k,1}\left(1 - A^{-c_{k,2}}\right) \quad \text{or} \quad d_i(A) = d_i(A_{\text{ref}})\left(\frac{A}{A_{\text{ref}}}\right)^{\gamma_i[d_i(A_{\text{ref}})-1]}$$

- There is not enough data to really constrain the A-dependence and check if such assumption is justified.
- Hopefully data from EIC will allow to answer this.

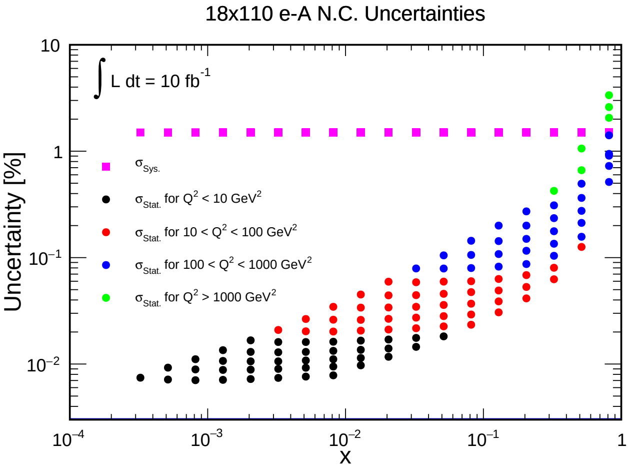
Electron-Ion Collider oportunities for nPDFs

- Range of nuclei: Au, Cu, Fe, C, He, ...
- CM energy $\sqrt{s} \sim 40 - 140\sqrt{Z/A}$ GeV
- Very large luminosity $\sim 10^{33} - 10^{34}$ cm^{-2} s^{-1} ($\sim 100 - 1000$ times higher than HERA)
- Wide kinematic coverage

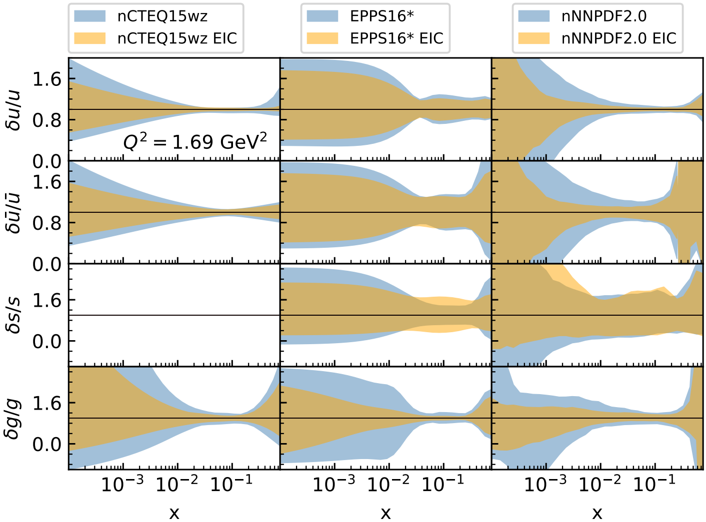


Electron-Ion Collider oportunities for nPDFs

- Small uncertainties



- Great prospects for understanding nuclear structure in particular nPDFs



Where do you get the PDFs/nPDFs from?

LHAPDF 6.2.1

LHAPDF Documentation

Introduction

LHAPDF is a general purpose C++ interpolator, used for evaluating PDFs from discretised data files. Previous versions of LHAPDF were written in Fortran 77/90 and are documented at http://lhapdf.hepforge.org/lhapdf5/.

LHAPDF6 vastly reduces the memory overhead of the Fortran LHAPDF (from gigabytes to megabytes!), entirely removes restrictions on numbers of concurrent PDFs, allows access to single PDF members without needing to load whole sets, and separates a new standardised PDF data format from the code library so that new PDF sets may be created and released easier and faster. The C++ LHAPDF6 also permits arbitrary parton contents via the standard PDG ID code scheme, is computationally more efficient (particularly if only one or two flavours are required at each phase space point, as in PDF reweighting), and uses a flexible metadata system which fixes many fundamental metadata and concurrency bugs in LHAPDF5.

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- Introduction
- Installation
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- Usage
  - Building against LHAPDF
  - Runtime symbol resolution
  - Trick to remove unwanted PDF members
  - Trick to use zipped data files
- Authors
- Support and bug reporting
- For developers

Compatibility routines are provided as standard for existing C++ and Fortran codes using the LHAPDF5 and PDFLIB legacy interfaces, so you can keep using your existing codes. But the new interface is much more powerful and pleasant to work with, so we think you'll want to switch once you've used it!

LHAPDF6 is documented in more detail in http://arxiv.org/abs/1412.7420

Installation

The source files can be downloaded from https://www.hepforge.org/downloads/lhapdf

Summary

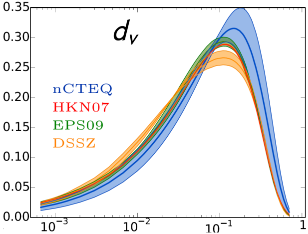
Summary

- Nuclear PDFs are determined by fitting experimental data following the road paved by the proton PDF global analyses.

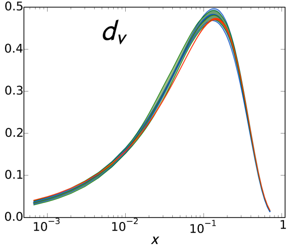
- There are several sets of nPDFs available
  - EPPS16
  - nCTEQ15
  - ...

- Bound proton PDFs, $f_i^{p/A}$, are only **effective** means of parameterizing the full nPDFs f_i^A

$d_v^{p/\mathrm{Pb}}$

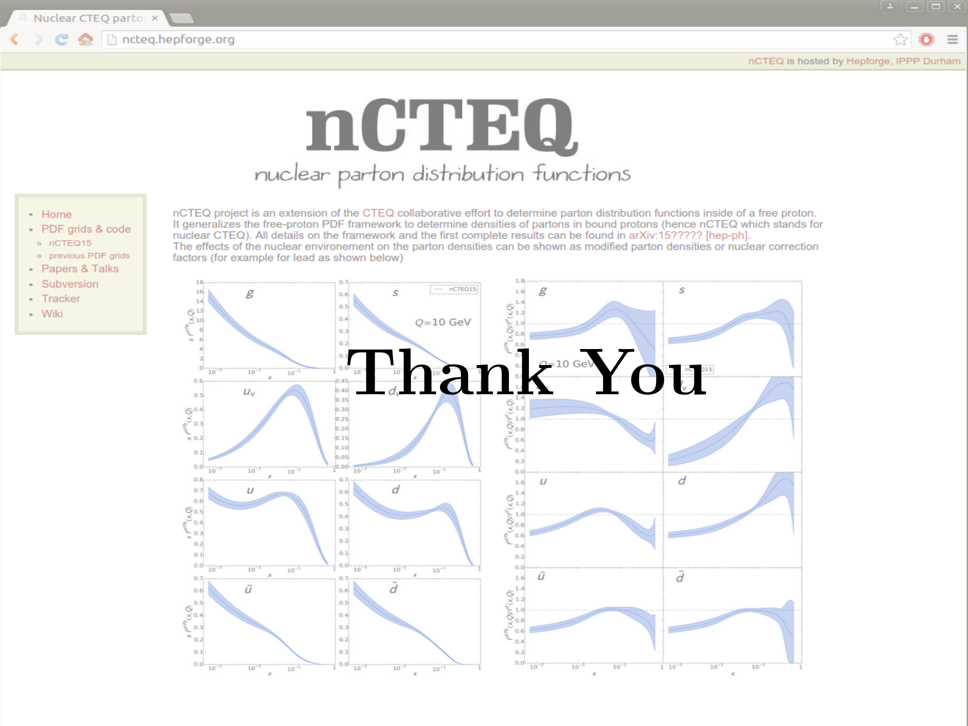


$d_v^{\mathrm{Pb}} = \frac{82}{208} d_v^{p/\mathrm{Pb}} + \frac{208-82}{208} d_v^{n/\mathrm{Pb}}$



Summary

- The current nPDFs are generally compatible with each other but we still don't have satisfactory knowledge of all flavours especially at small and large x.
- The currently available data (LHC, JLAB) can be utilize to further constrain nPDFs, however, only to a certain extent.
- On the other hand the planed EIC will bring the nPDFs to a precise era, and will allow to study not only the collinear distributions but also more complicted objects like TMDs, GPDs.


Nuclear CTEQ parto
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nCTEQ is hosted by Hepforge, IPPP Durham
nCTEQ
nuclear parton distribution functions
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PDF grids & code
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previous PDF grids
Papers & Talks
Subversion
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Wiki
nCTEQ project is an extension of the CTEQ collaborative effort to determine parton distribution functions inside of a free proton. It generalizes the free-proton PDF framework to determine densities of partons in bound protons (hence nCTEQ which stands for nuclear CTEQ). All details on the framework and the first complete results can be found in arXiv:15????? [hep-ph].
The effects of the nuclear environement on the parton densities can be shown as modified parton densities or nuclear correction factors (for example for lead as shown below)
nCTEQ15
Q=10 GeV

Thank You