Searching for parton entanglement

Double Λ hyperon polarization

Kong Tu

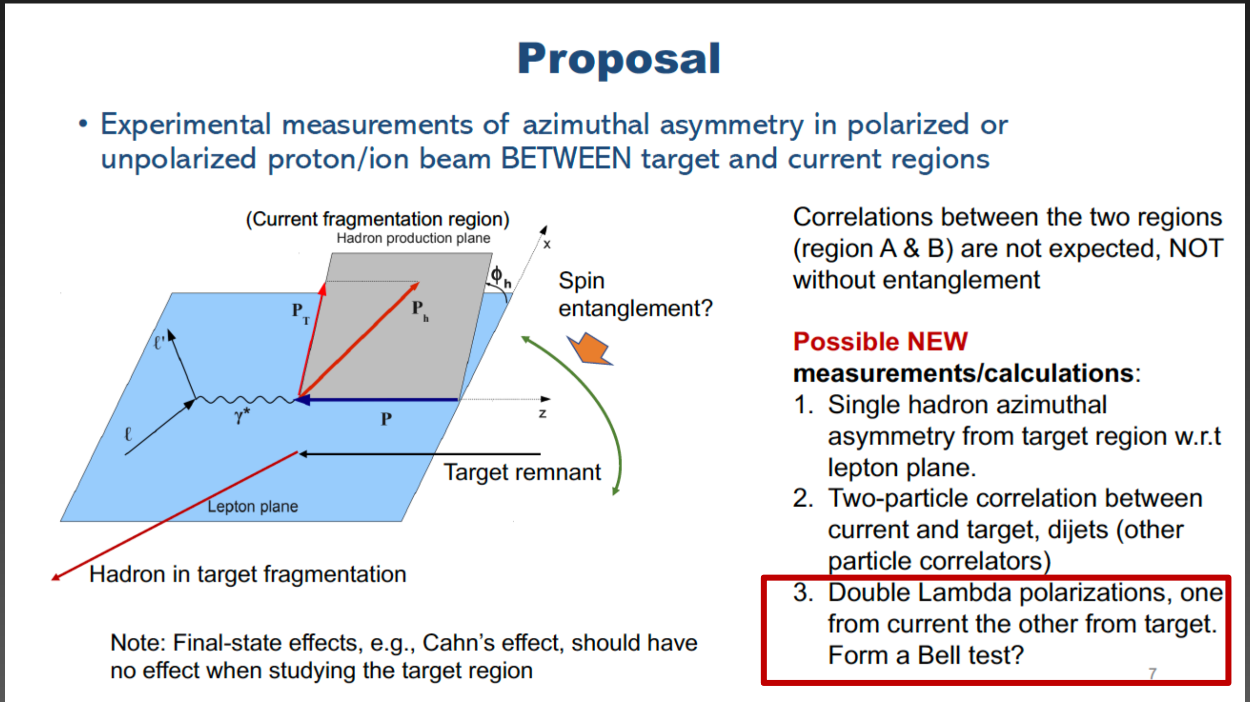
BNL

10.29.2020

In collaboration with W.Gong, G. Parida, R. Venugopalan, manuscript in progress.

Motivation

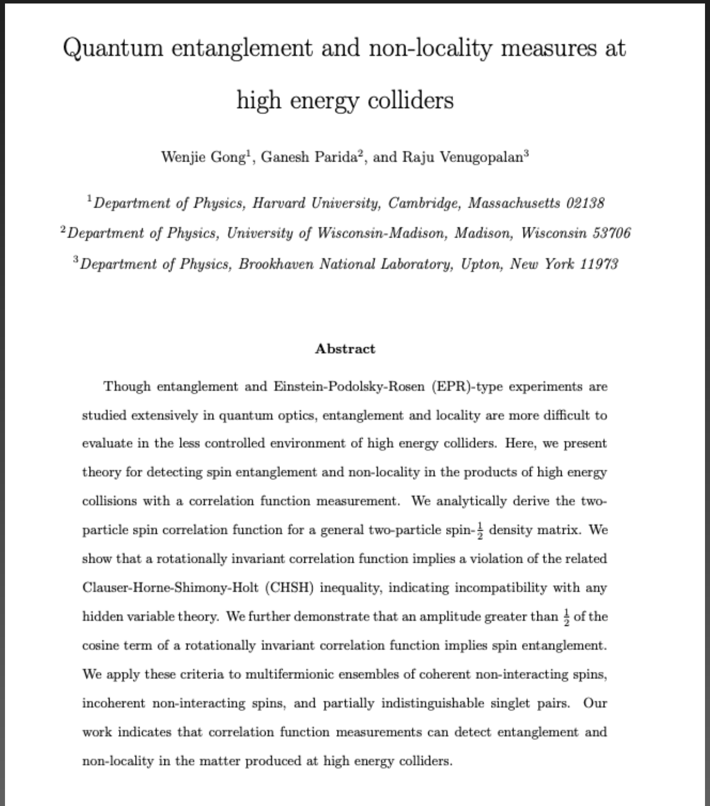
Z.Tu's LDRD FY21 proposal (didn't fly)



Listed as one of my interesting direction for a new proposal

- Double lambda polarizations
- Bell test?

W. Gong, G. Parida, and R. Venugopalan



Quantum entanglement and non-locality measures at high energy colliders

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Abstract

Though entanglement and Einstein-Podolsky-Rosen (EPR)-type experiments are studied extensively in quantum optics, entanglement and locality are more difficult to evaluate in the less controlled environment of high energy colliders. Here, we present theory for detecting spin entanglement and non-locality in the products of high energy collisions with a correlation function measurement. We analytically derive the two-particle spin correlation function for a general two-particle spin-$\frac{1}{2}$ density matrix. We show that a rotationally invariant correlation function implies a violation of the related Clauser-Horne-Shimony-Holt (CHSH) inequality, indicating incompatibility with any hidden variable theory. We further demonstrate that an amplitude greater than $\frac{1}{3}$ of the cosine term of a rotationally invariant correlation function implies spin entanglement. We apply these criteria to multifermionic ensembles of coherent non-interacting spins, incoherent non-interacting spins, and partially indistinguishable singlet pairs. Our work indicates that correlation function measurements can detect entanglement and non-locality in the matter produced at high energy colliders.

They had been working on a theory problem during a summer program

Motivation

- Very naturally and smoothly, these two projects combined
- This paper has both the theory and experiment of spin entanglement, with suggestion to measure at EIC and at RHIC.
- (Hopefully) this will help us understand more of spin physics, e.g., TMDs.

New paper

Searching for parton spin entanglement via double Λ hyperon polarization and the CHSH inequality test in high energy collisions

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(Dated: October 27, 2020)

After 50 years of discovery of quarks, one of the most fundamental questions in modern physics is the spin structure of the proton. More than three decades have passed since the first and surprising result on the decomposition of the proton spin, but the question of how partons collectively contribute to the proton spin remains a question to-date. Hereby, we focus on a new perspective of the nucleon spin structure in terms of parton spin entanglement inside of proton. We propose to study the spin entanglement by measuring the Λ polarization with respect to another Λ particle in high energy collisions, where these two Λ particles can be close or far away in space-time in the same event. In order to unambiguously observe spin entanglement, we devise a Clauser-Horne-Shimony-Holt (CHSH) inequality test to investigate the correlation between the two Λ particles. In the Monte Carlo simulations, we find that any nonzero polarization signal between the two Λ particles would result in a cosine modulation in the proposed two-particle correlation function, which is found to be an evidence of violation of CHSH inequality. The observation of such violation implies a quantum entangled system of proton constituents and would provide great insights into the nucleon spin structure.

Keywords: quantum entanglement, polarization, proton spin structure, CHSH inequality, Electron-Ion Collider

I. INTRODUCTION

In high energy collisions, the Λ polarization has been observed across a wide range of collision systems, going

or anti-aligned with the target polarization, or alternatively, the strange quark simply does not have a spin correlation with the target polarization. The question ~~of~~ whether the strange quark has intrinsic transversity

(draft in progress, stay tuned!)

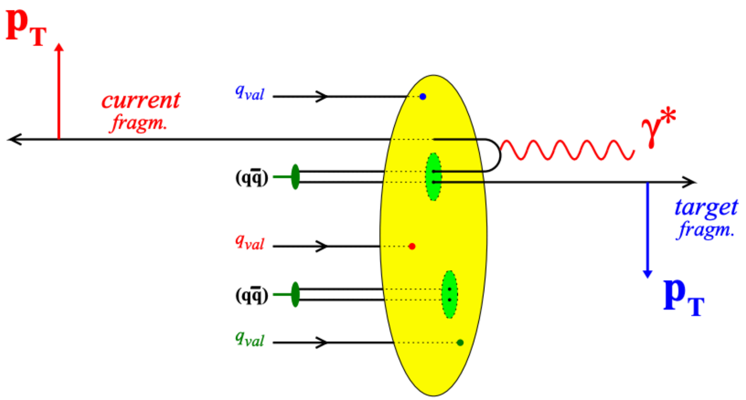
Physics questions

Peter Schweitzer

- Probing parton correlations with current and target fragmentation

target fragmentation

- correlations in current ↔ target fragmentation
  when correlated $q\bar{q}$ pair "split up"!
  $Q^2 \sim$ few GeV^2 not too large
  (to avoid loss of correlation
  due to gluon radiation)



Interesting idea on probing qqbar pair in DIS and form correlation between current and target fragmentation

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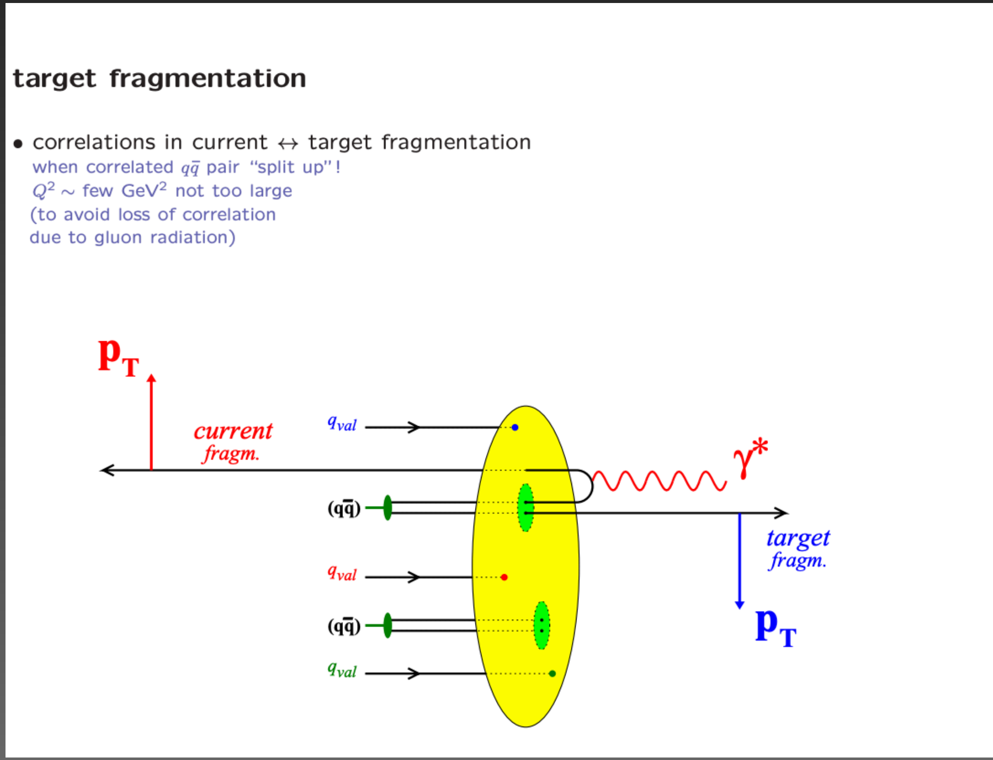

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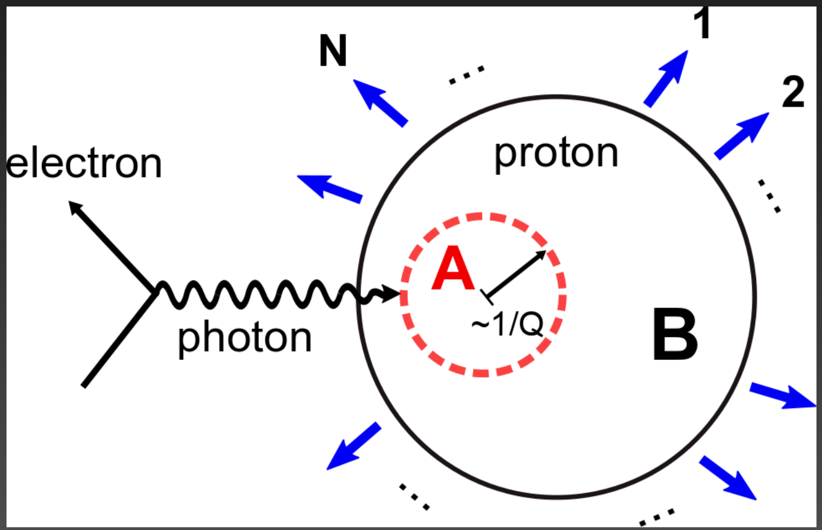
Idea based on entanglement



(ZT, D.Kharzeev, T.Ullrich PRL 2020)

- The nature of these two correlations are different, one general question:

How to tell the physics is entanglement?

Non-locality and CHSH test

Now assumes two spin ½ particles – spin singlet
(like the famous EPR paradox)

When measure these two particles spin, the expectation value is:

$$E(\hat{a}, \hat{b}) = \langle\psi| \hat{a} \cdot \boldsymbol{\sigma_1} \hat{b} \cdot \boldsymbol{\sigma_2} |\psi\rangle$$

where $\hat{a}, \hat{b}$ are the two arbitrary direction one wants to measure

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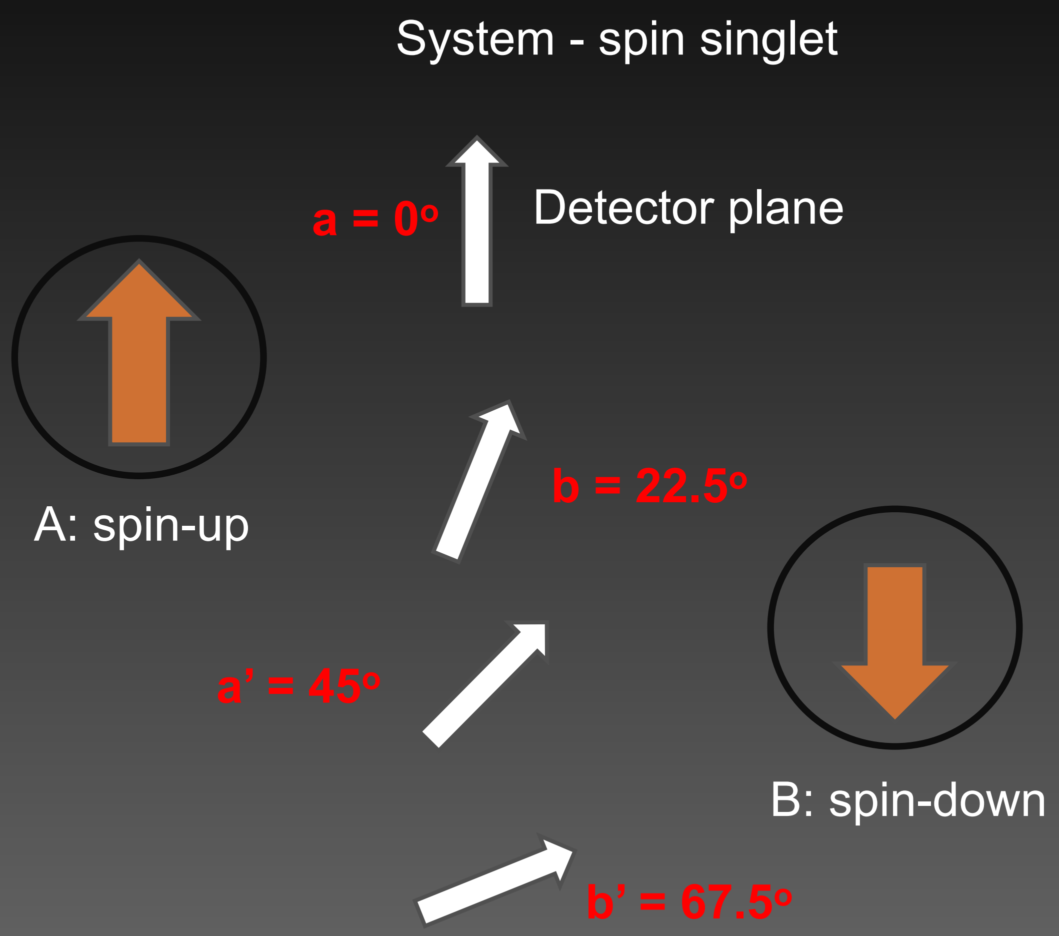
- The CHSH inequality test, a test based on the original Bell's inequality is to test non-locality

$$|E(\hat{a},\hat{b}) - E(\hat{a},\hat{b}')| + |E(\hat{a}',\hat{b}') + E(\hat{a}',\hat{b})| \leq 2.$$

$\hat{a},\hat{b}$ and with 's they are different angles, see wiki:
https://en.wikipedia.org/wiki/CHSH_inequality



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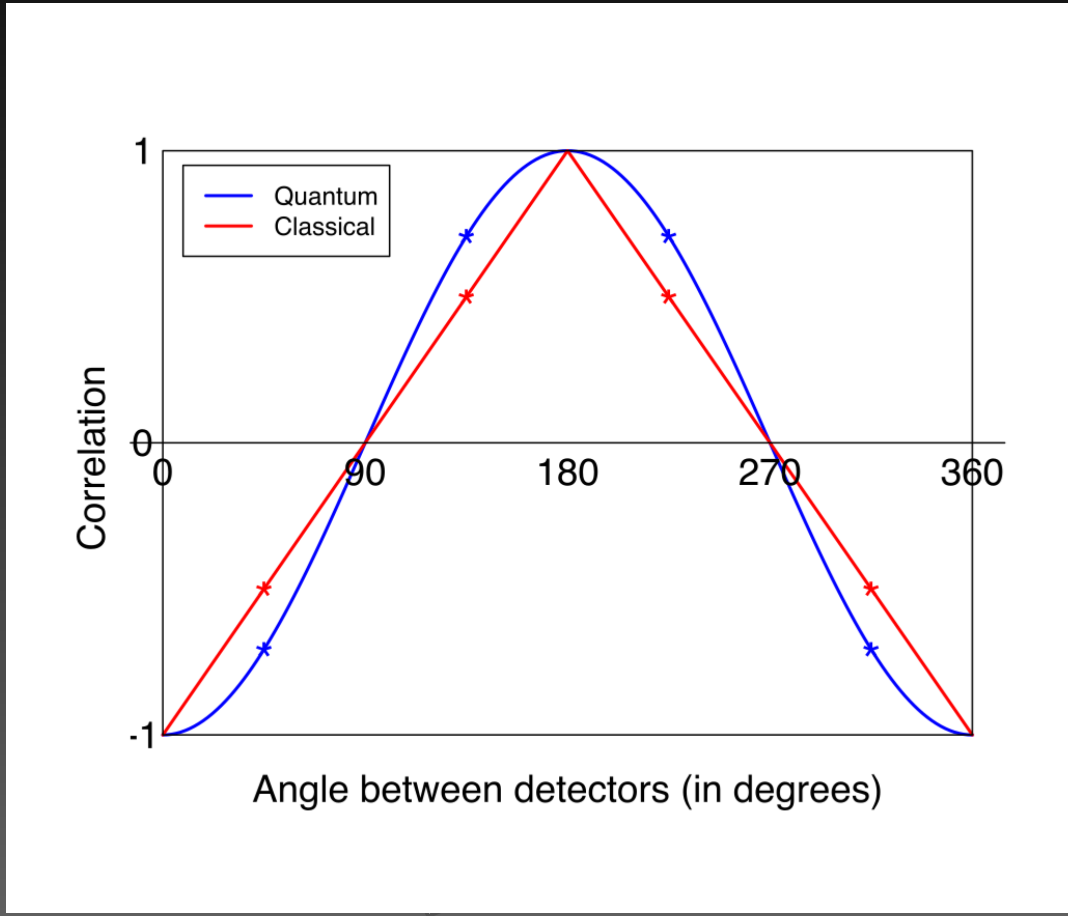

Prediction if CHSH angles are continuous

Non-locality and CHSH test

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Write the two particles spin wavefunction in the Bell basis, construct their density matrix

$$|B_1\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$
$$|B_2\rangle = \frac{|00\rangle - |11\rangle}{\sqrt{2}}$$
$$|B_3\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}}$$
$$|B_4\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}.$$

Density matrix

$$\rho_{ab} = \sum_{i=1}^{4}\sum_{j=1}^{4}\lambda_{ij}\,|B_i\rangle\,\langle B_j|\,,$$

Non-locality and CHSH test

(sorry, I skip a few steps obviously)

Then, it comes to a relation between the correlation function C_{ab} and relative angle θ_{ab}

$$C_{ab} = \frac{P(\hat{\mathbf{a}}, \hat{\mathbf{b}})}{P(\hat{\mathbf{a}})P(\hat{\mathbf{b}})} = 1 + (\lambda_{11} - \lambda_{44})\cos(\theta_{ab}).$$

- P($\hat{a}, \hat{b}$) is the probability of particle A & B are both spin align with $\hat{a}, \hat{b}$
- P($\hat{a}$) = P($\hat{b}$) = 50%

Non-locality and CHSH test

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Two important conclusions:

- If the correlation function satisfy this relation that has a cosine modulation - CHSH inequality is violated and it is nonlocal
- To prove it is "entanglement", it needs to satisfy the so-called "entanglement fidelity" where the coefficient is > ½
  (sufficient but not necessary condition)

This is a general conclusion for spin singlet for setting up CHSH inequality test

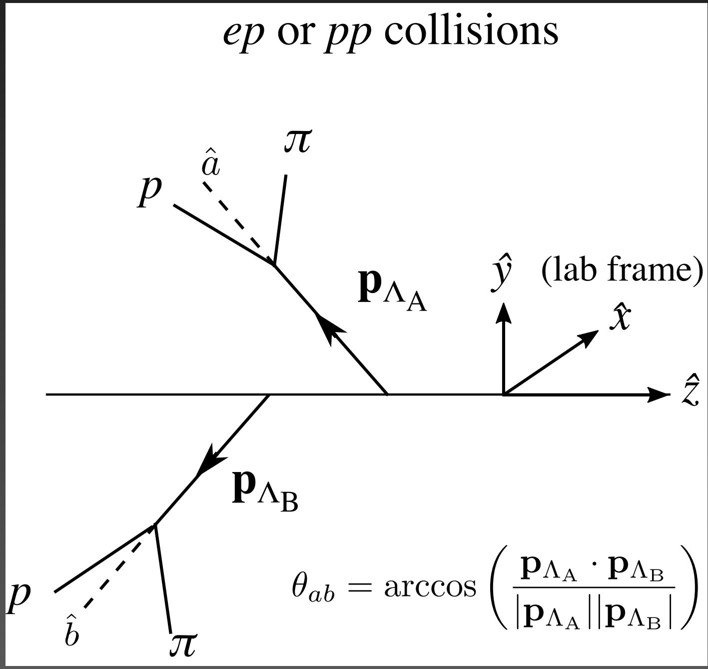
Measure particle spin in HEP

- In nuclear and particle physics, one of the most important and popular ways of measuring spin is by Λ polarizations
- Due to Λ's weak decay, the spin of the Λ (or strange quark most people believe) can be determined thru Λ's decay product.
- Many examples in our field, e.g., vorticity in HIC.
- What about making use of Λ's and by measuring double Λ polarizations?

$$\frac{dN}{d\cos(\theta^*)} \propto 1 + \alpha P_{\Lambda,\Lambda} \cos(\theta^*),$$

α ~ 0.642 weak decay constant

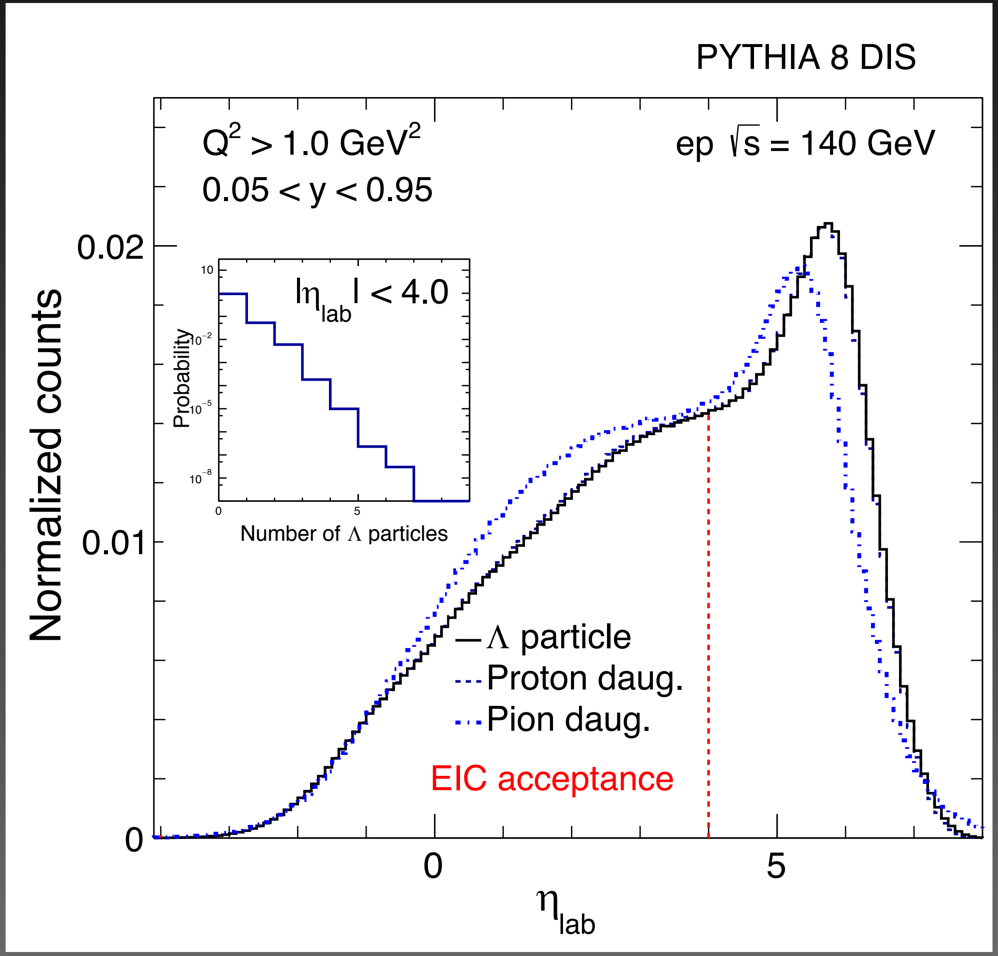
CHSH inequality test



"Nature choose detector planes"
(unlike normal quantum experiments)

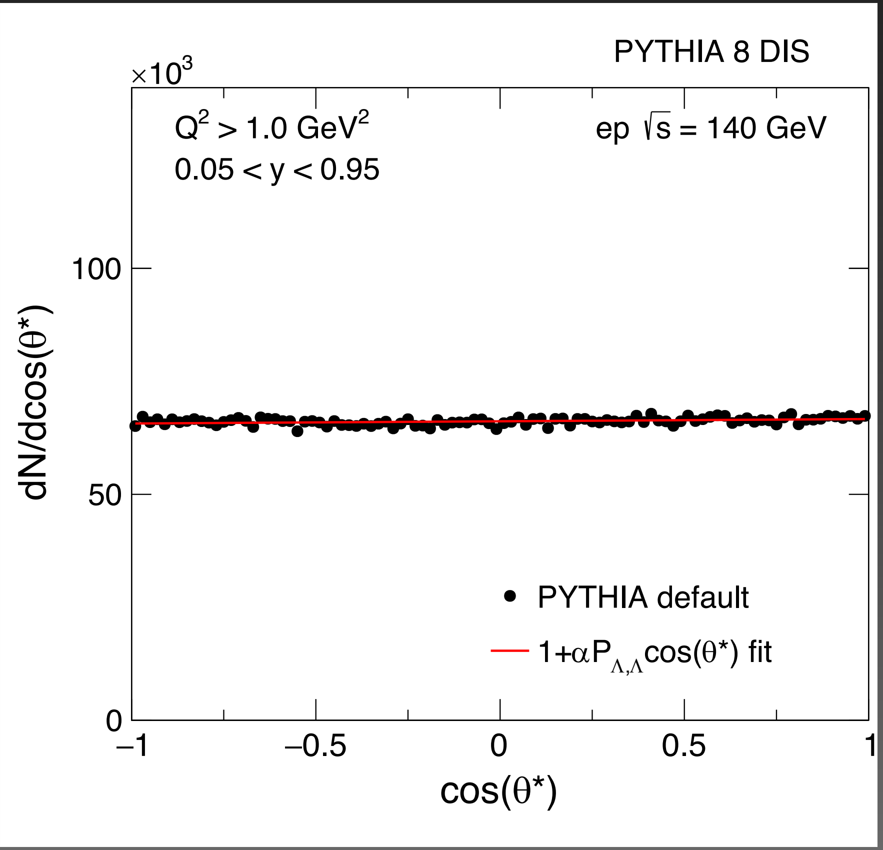
Monte Carlo – PYTHIA 8

- ep DIS sample with 400M events, while pp collisions would be just fine as well
- $1 < Q^2 < 100$ GeV^2 , $0.05 < y < 0.95$ – standard kinematic phase space.
- No correlation/polarization was built in PYTHIA 8 between two Λ particles – we did confirm that.
- Λ and daughter particles kinematic distributions are checked, with an EIC main detector acceptance illustrated.
- Small panel – number of Λ distributions.
- Λ selections:
  - $p_T > 0.15$ GeV for both Λ and daughters
  - η_{lab} is within (-4,+4) for both Λ and daughters.



First observable - $P_{\Lambda,\Lambda}$

θ^* is the polar angle btw the proton daughter of Λ_A with respect to $\Lambda_{B,}$ in Λ_A rest frame



No polarization is observed in default PYTHIA 8. Good!

First observable - $P_{\Lambda,\Lambda}$

Reweighting events according to this,

$$\frac{dN}{d\cos(\theta^*)} \propto 1 + \alpha P_{\Lambda,\Lambda} \cos(\theta^*),$$

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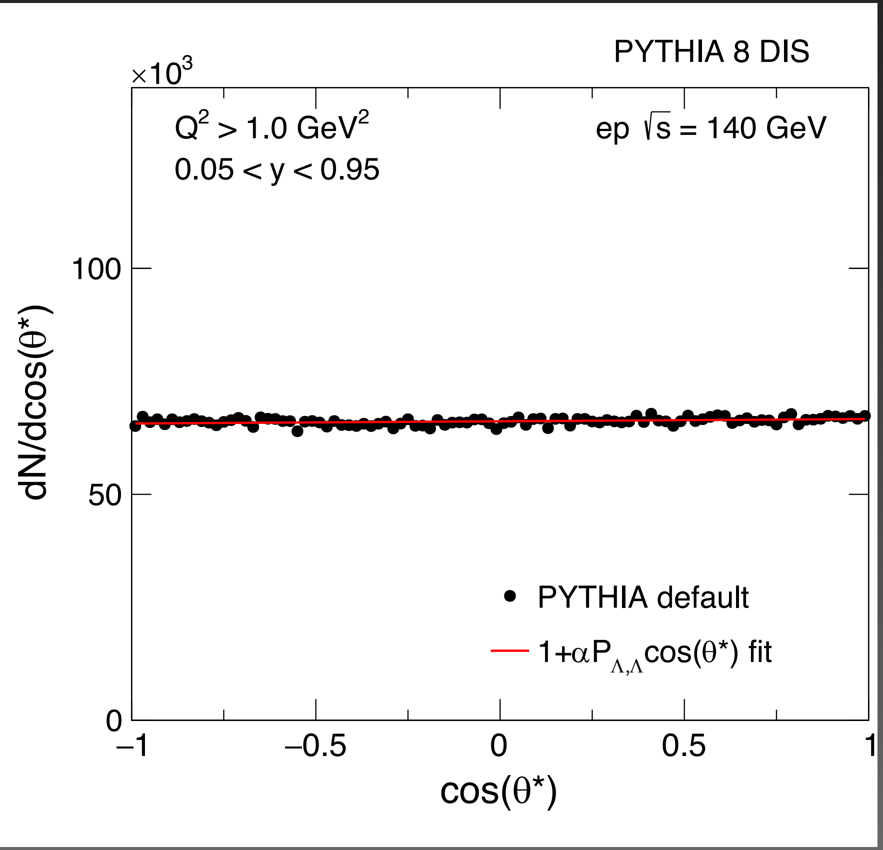
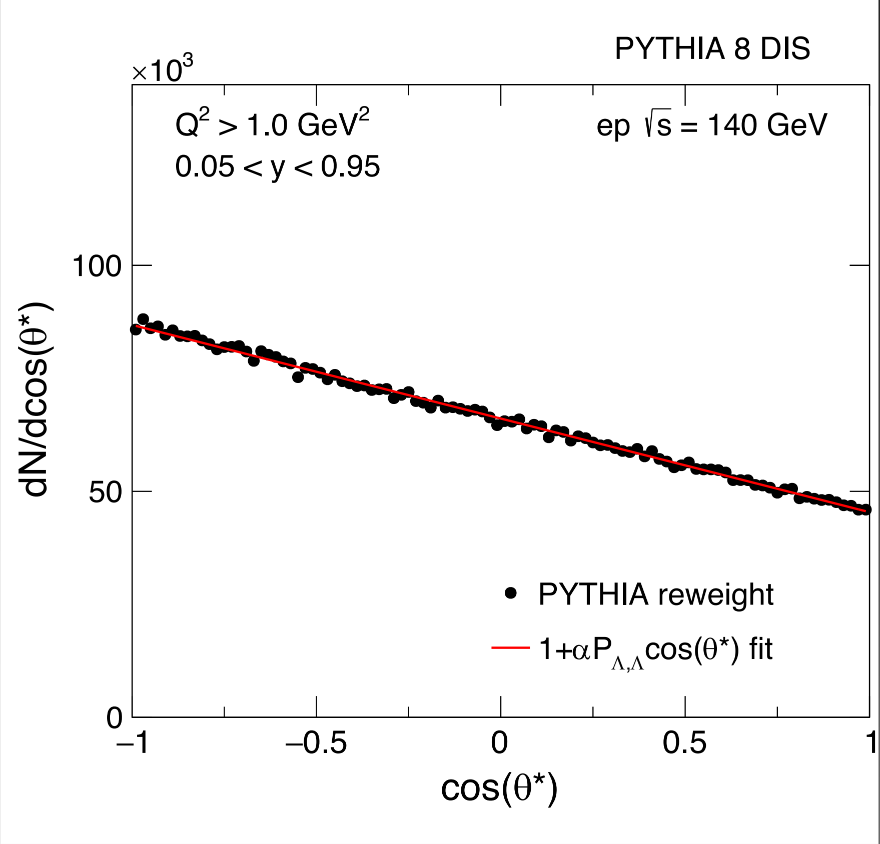




No polarization is observed in default PYTHIA 8. Good!

Of course, now this looks like finite polarization with our input

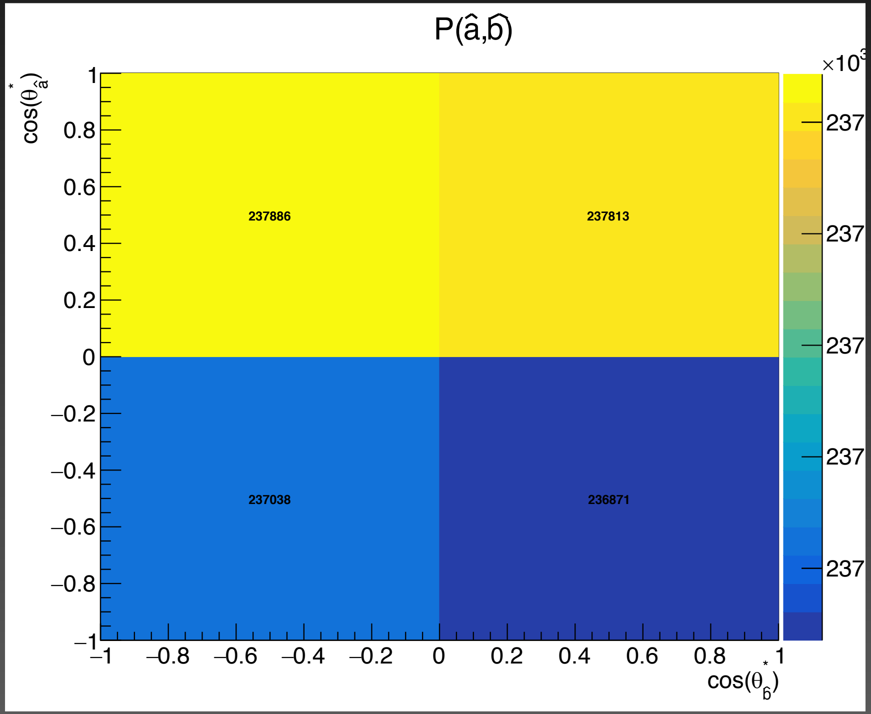
Second observable – C_{ab}

$$C_{ab} = \frac{P(\hat{\mathbf{a}}, \hat{\mathbf{b}})}{P(\hat{\mathbf{a}})P(\hat{\mathbf{b}})} = 1 + (\lambda_{11} - \lambda_{44}) \cos(\theta_{ab}).$$

- θ_{ab} is the relative angle between two Λs – serves as CHSH angles
- P($\hat{a}, \hat{b}$) is the joint probability of two Λs spin with respect to their **own** axis.
- P($\hat{a}$) = P($\hat{b}$) = 50% → a constant

(think of the EPR paradox for any single spin measurement)

(color is misleading, but they are flat)

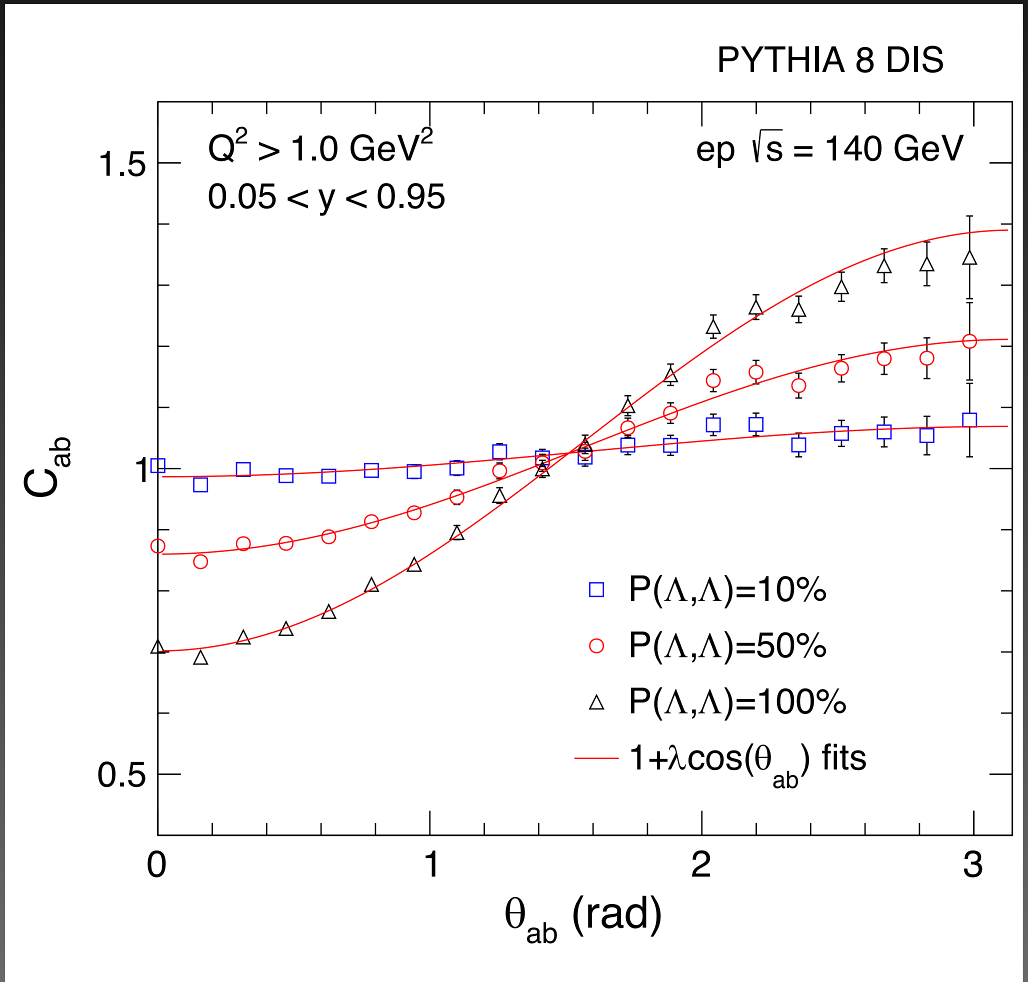


- $\cos(\theta_a^*)$ – Λ_A polarization w.r.t $\hat{a}$, which is Λ_A momentum itself.
- Similar for Λ_B

CHSH inequality test in DIS

- Event-by-event, we reweigh using the factor based on the first observable, $\mathbf{P}_{\Lambda,\Lambda}$

(just a reminder, $\mathbf{P}_{\Lambda,\Lambda}$ is polarization particle A w.r.t B)



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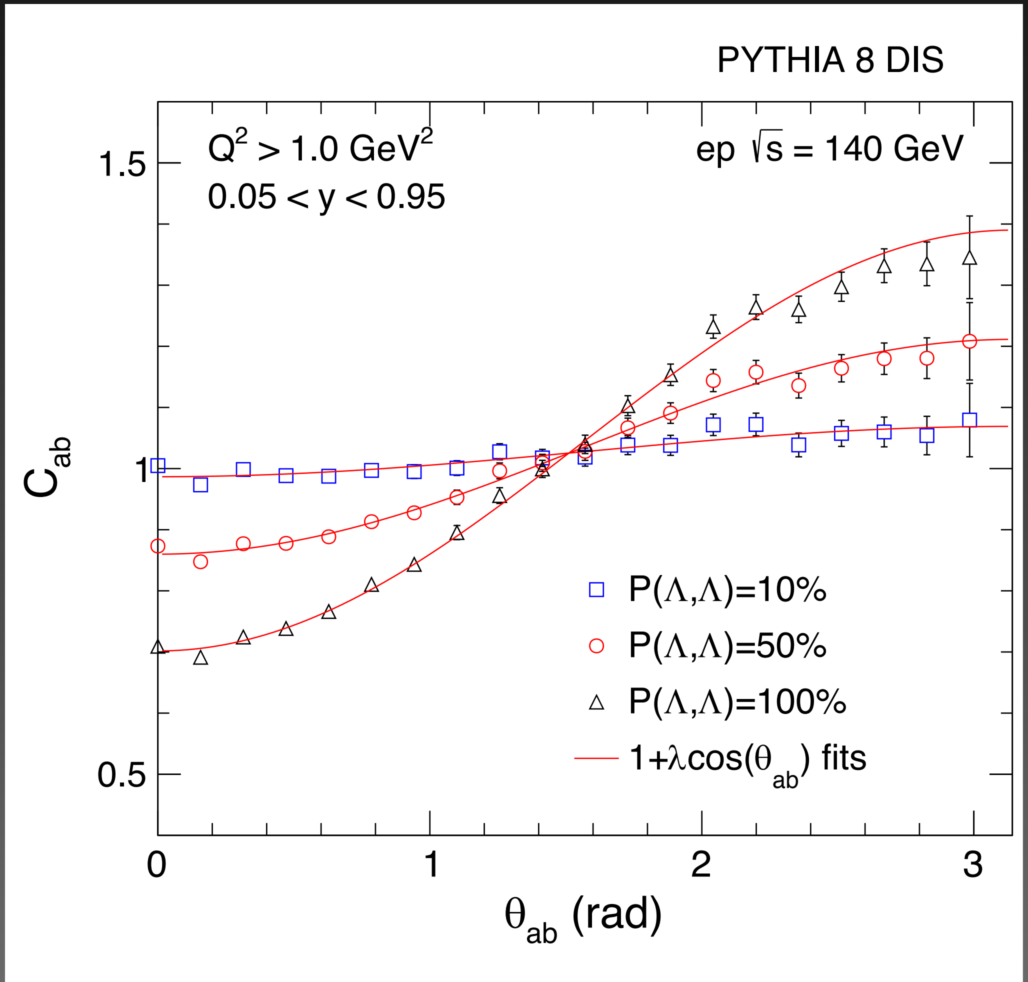
- Observation – cosine modulation in terms of CHSH angles.

- Based on the theory, it automatically violates CHSH inequality, proving it is nonlocal effect.



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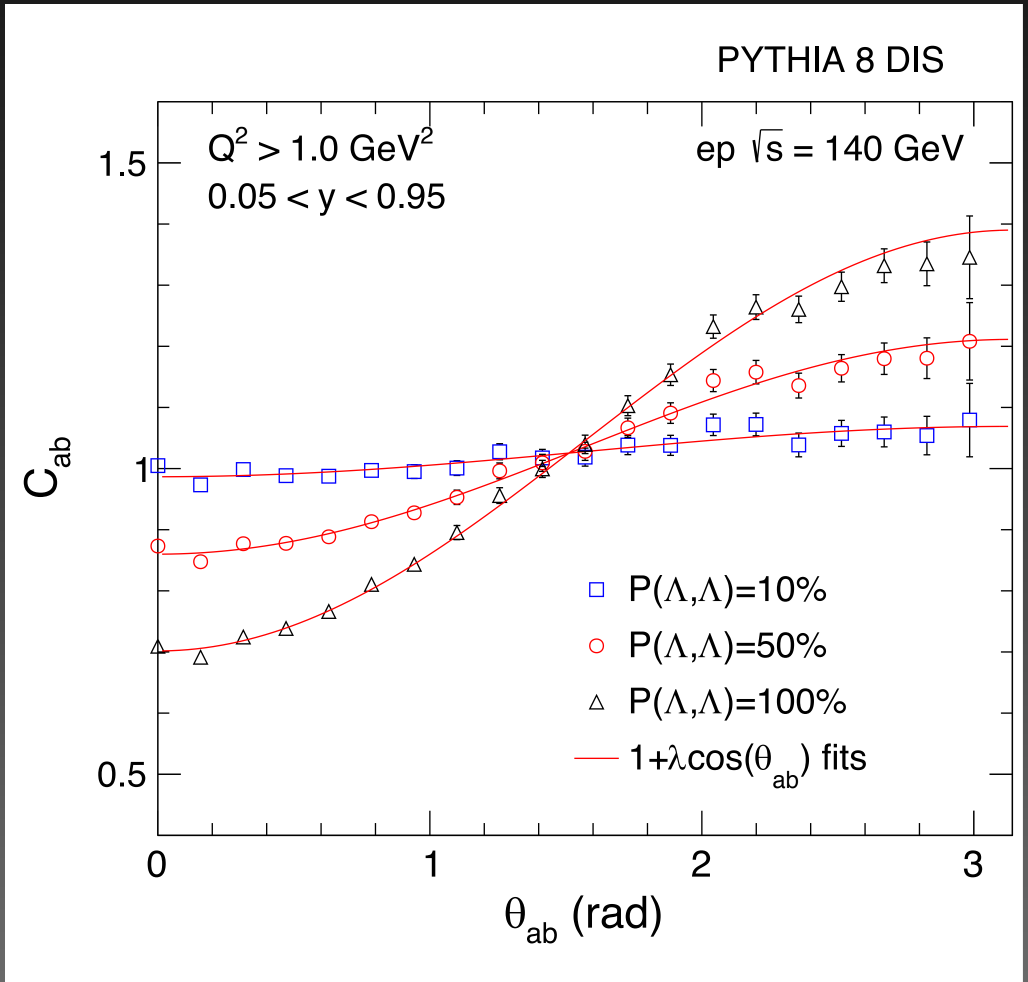
- Based on the theory, it automatically violates CHSH inequality, proving it is nonlocal effect.

- How about "entanglement fidelity"?
  - $\lambda = 0.067$ +/- 0.009 (**P**$_{\Lambda,\Lambda}$ = 10%)
  - $\lambda = 0.273$ +/- 0.009 (**P**$_{\Lambda,\Lambda}$ = 50%)
  - $\lambda = 0.523$ +/- 0.008 (**P**$_{\Lambda,\Lambda}$ = 100%)



Only **P**$_{\Lambda,\Lambda}$ = 100% fulfill entanglement fidelity, where $\lambda > 1/2$?

CHSH inequality test in DIS

- It turns out the “entanglement fidelity” > ½ was based on polarization = 100%;

- However, in Λ decays, the absolute maximum bounded by 0.642, the weak decay constant.

$$\frac{dN}{d\cos(\theta^*)} \propto 1 + \alpha P_{\Lambda,\Lambda} \cos(\theta^*),$$

- Therefore, we need a “entanglement fidelity” as a function of polarization, where qualitatively, it should be less strict than ½ .

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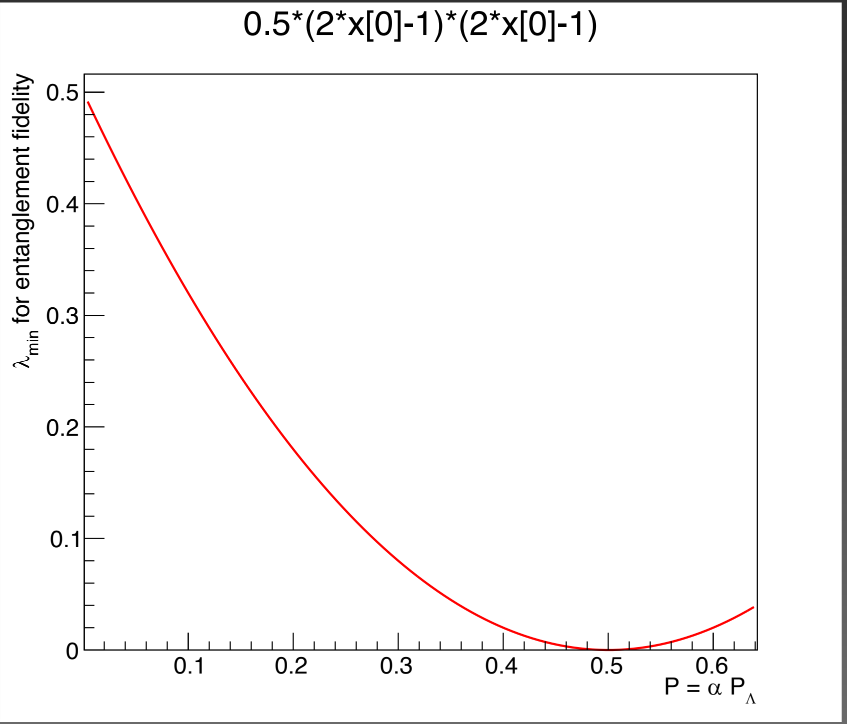
For a quick look, Wenjie calculates this

$$\frac{P(|\hat{n}_1\rangle, |\hat{n}_2\rangle)}{P(|\hat{n}_1\rangle)P(|\hat{n}_2\rangle)} = 1 + (2a-1)(2b-1)(\lambda_{11} - \lambda_{44})\cos(\theta_2 - \theta_1)$$

a and b are the assumed polarization for each Λ



Qualitatively it makes sense!

Summary and outlook

- This is a first-time study utilizing double Λ polarization to define an unambiguous CHSH test in high energy hadron collisions.
- A generic two spin-½ particles are derived in terms of CHSH
- Applications to ep DIS and pp collisions
- Outlook:
    - What if proton is transversely polarized? We could compare polarized vs unpolarized, it will provide even more information!
    - Can look at this in pp collisions, the method is not DIS only.

  (e.g., long range vs short range…)

Can we connect this to TMDs? Future direction.