Dijet production at EIC

I. Scimemi, based on 2008.07531 , M. G. Echevarria, R. Fernandez del Castillo, Y. Makris

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The gluon TMD quest

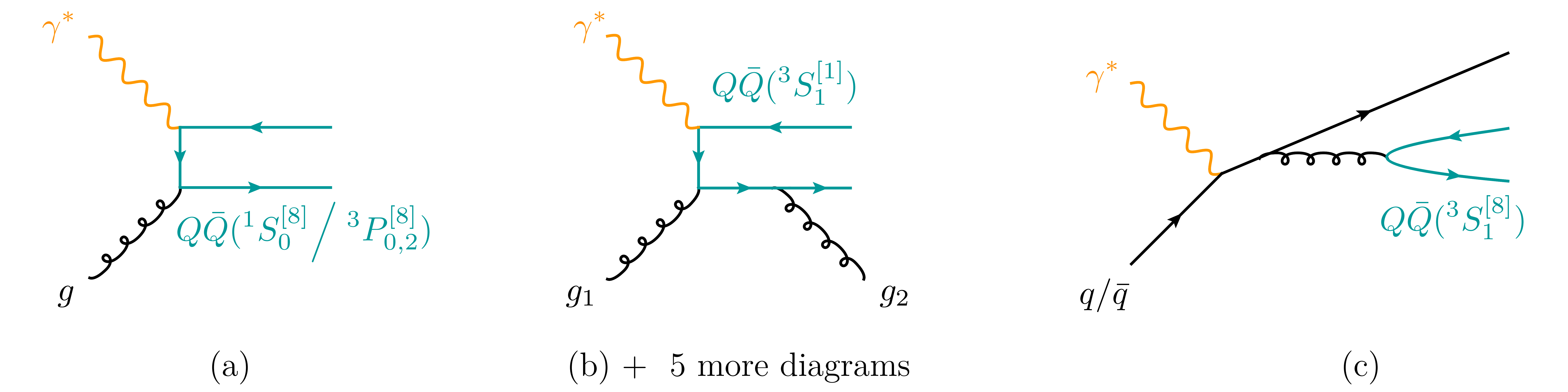
Gluon Polarization

Nucleon Polarization

GLUONS	*unpolarized*	*circular*	*linear*
U	f_1^g		$h_1^{\perp g}$
L		g_{1L}^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	h_{1T}^g, $h_{1T}^{\perp g}$

- Much effort in using unpolarized targets, unpolarized and linearly polarized gluons appear together
- The only color neutral particle available for this search is the Higgs
- Perturbative calculations at NNLO for unpolarized nucleon distributions, D. Gutierrez-Reyes, S. Leal-Gomez, et al. JHEP 1911 (2019) 121, M.-X Luo, et al. JHEP 2001 (2020) 040

Quarkonia



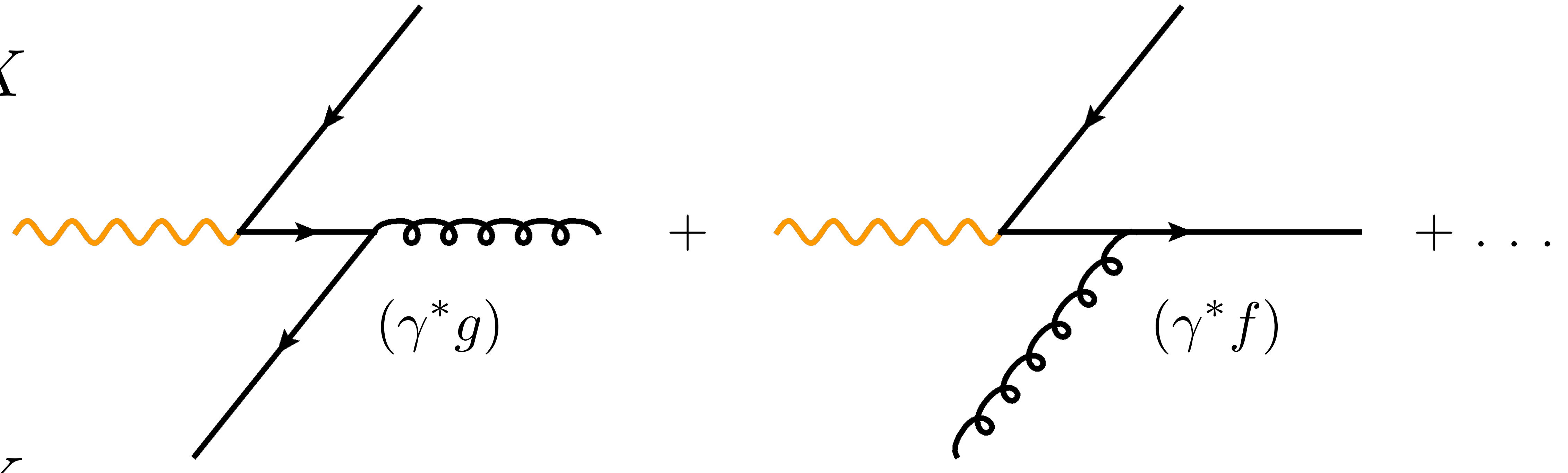
M.G. Echevarría, Y. Makris, I.S. 2007.05547

- Interference of gluon and quark TMD
- Mixing of quarkonia spin states due to soft gluon emissions

The di-jet and heavy meson pair cases

$\ell + h \to \ell' + J_1 + J_2 + X$

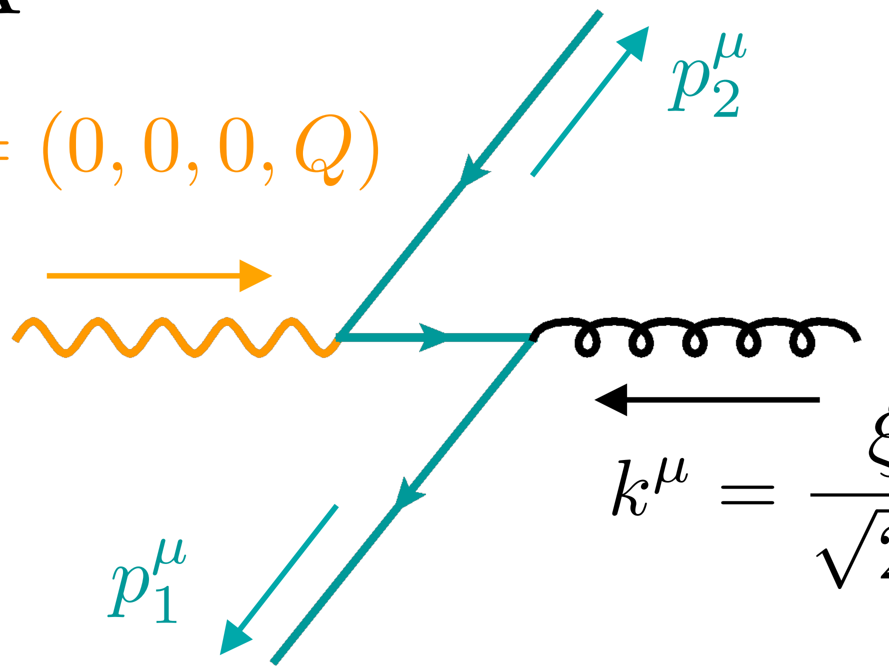
dijet LO process:



$\ell + h \to \ell' + H + \bar{H} + X$

$$q^\mu = \frac{Q}{\sqrt{2}}(n^\mu - \bar{n}^\mu) = (0, 0, 0, Q)$$

heavy meson pair at LO:



$$k^\mu = \frac{\xi}{\sqrt{2}x} Q \bar{n}^\mu = \frac{\xi}{2x}(Q, 0, 0, -Q)$$

EIC: $p_T \in [5, 40]$, central rapidity

Observables

$$\frac{d\sigma}{dx\,d\eta_1\,d\eta_2\,dp_T\,d\boldsymbol{r}_T}$$

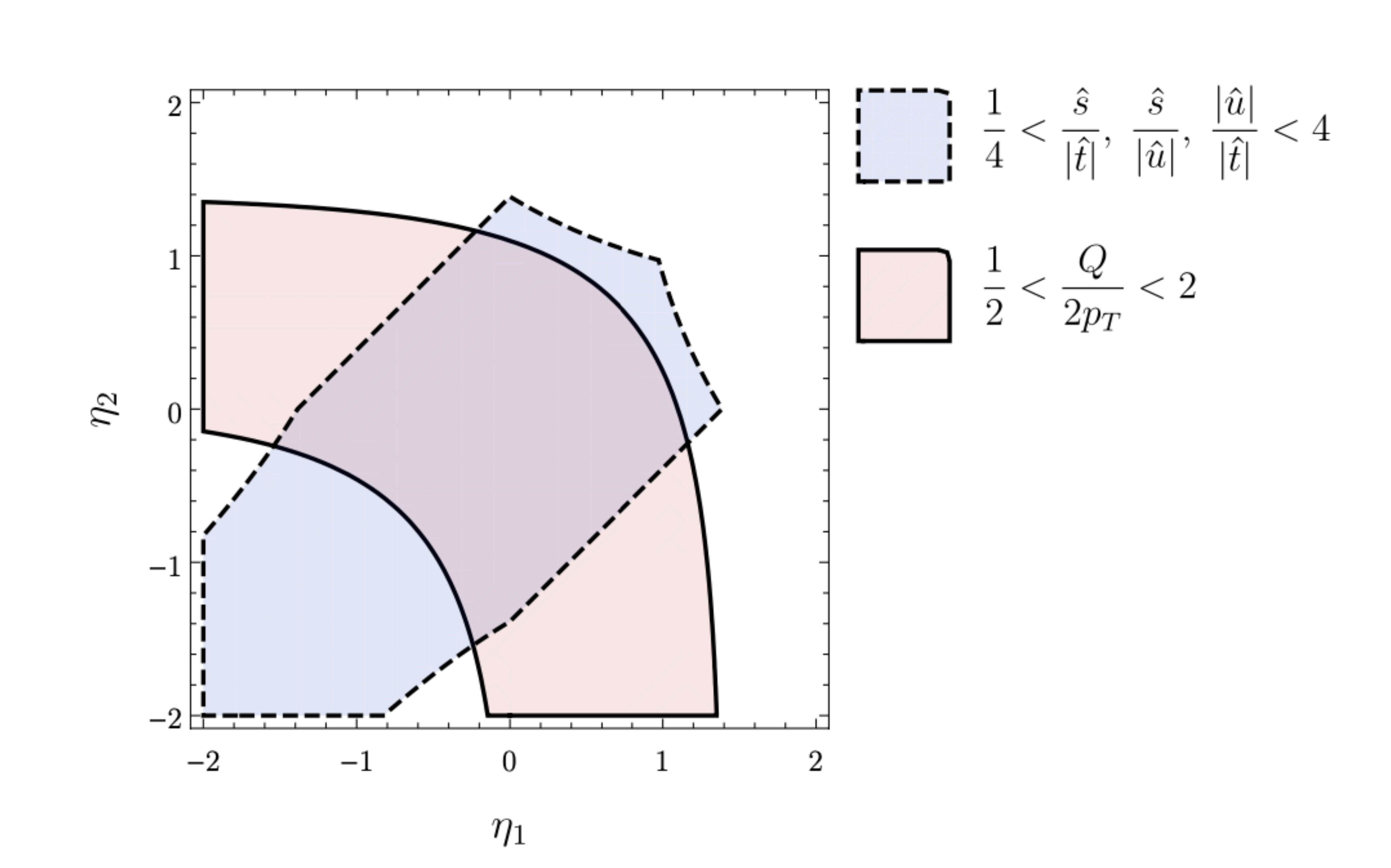
All transverse quantities are referred to the beam axis in the Breit frame

$$\boldsymbol{r}_T = \boldsymbol{p}_{1T} + \boldsymbol{p}_{2T}, \qquad p_T = \frac{|\boldsymbol{p}_{1T}| + |\boldsymbol{p}_{2T}|}{2}$$

Small transverse momentum condition on the imbalance momentum

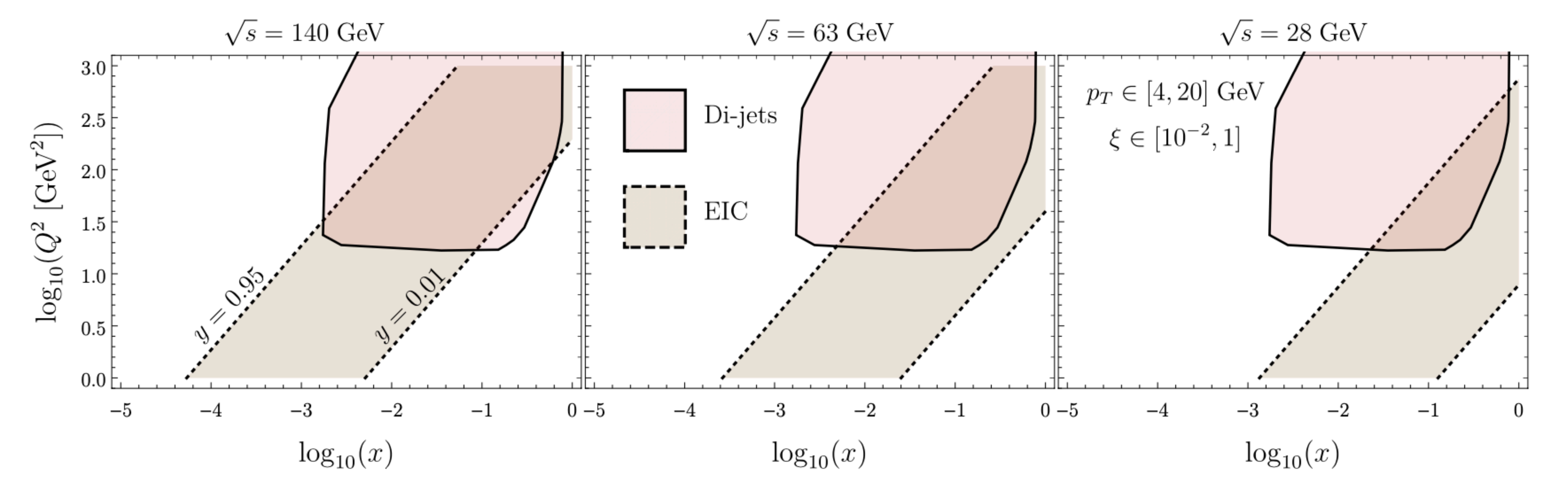
$$|\boldsymbol{r}_T| \ll p_T$$

Phase space at EIC

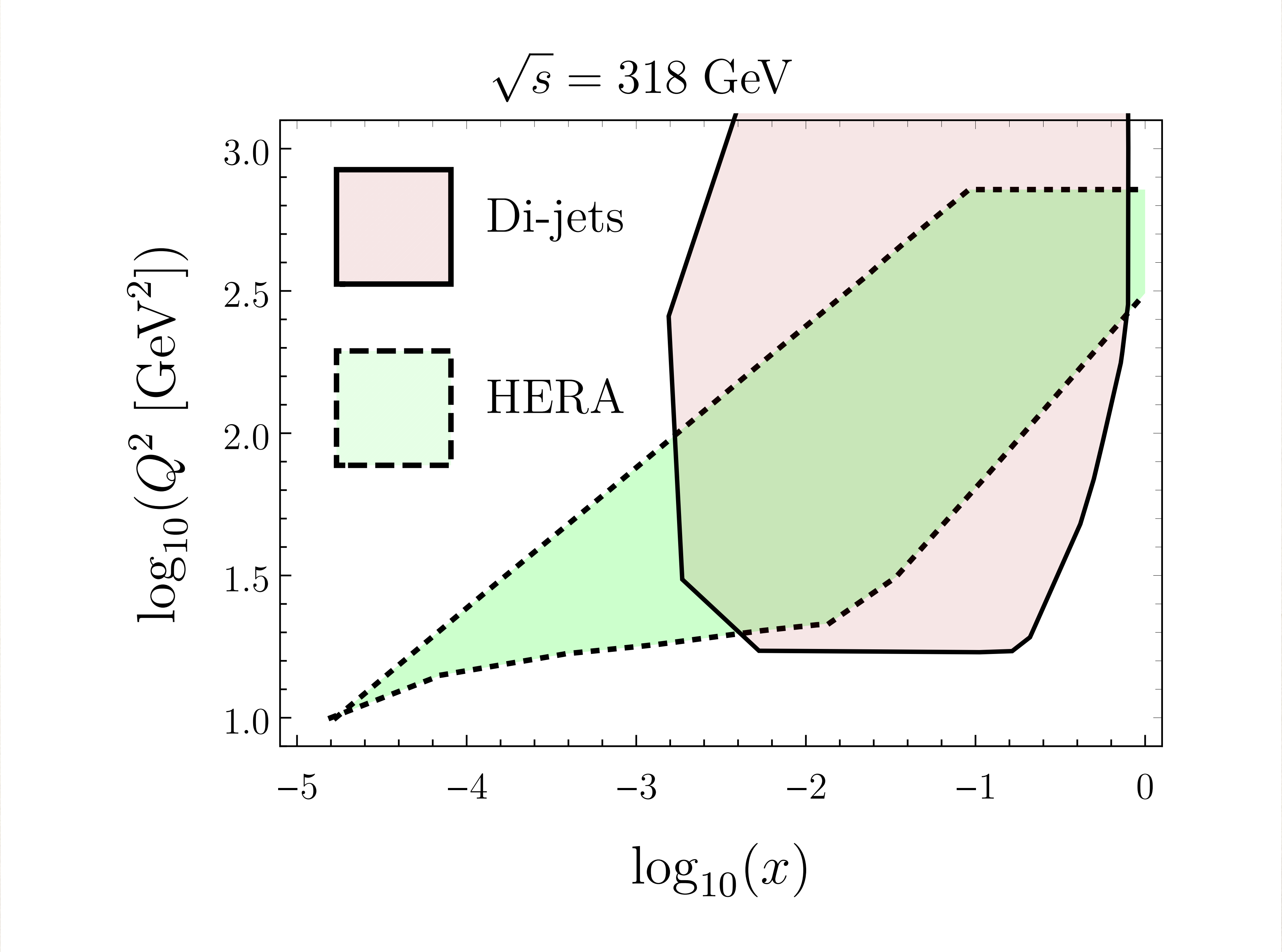


Typical expected
Conditions for our case

Phase space at EIC…



Phase space at HERA (Zeus+H1)



Factorization

$$\frac{d\sigma(\gamma^* g)}{dx d\eta_1 d\eta_2 dp_T d\boldsymbol{r}_T} = \sum_{P=U,L} \sigma_0^{gP} \sum_f H^P_{\gamma^* g\to f\bar{f}}(\hat{s},\hat{t},\hat{u},\mu) \int \frac{d^2\boldsymbol{b}}{(2\pi)^2} \exp(i\boldsymbol{b}\cdot\boldsymbol{r}_T)\, F_g^P(\xi,\boldsymbol{b},\mu,\zeta_1)$$

$$\times K_P S_{\gamma g}(\boldsymbol{b},\eta_1,\eta_2,\mu,\zeta_2)(\mathcal{C}_f(\boldsymbol{b},R,\mu)J_f(p_T,R,\mu))\big(\mathcal{C}_{\bar{f}}(\boldsymbol{b},R,\mu)J_{\bar{f}}(p_T,R,\mu)\big)$$

$$\frac{d\sigma(\gamma^* f)}{dx d\eta_1 d\eta_2 dp_T d\boldsymbol{r}_T} = \sigma_0^{f} \sum_f H^U_{\gamma^* f\to g\bar{f}}(\hat{s},\hat{t},\hat{u},\mu) \int \frac{d^2\boldsymbol{b}}{(2\pi)^2} \exp(i\boldsymbol{b}\cdot\boldsymbol{r}_T)\, F_g^U(\xi,\boldsymbol{b},\mu,\zeta_1)$$

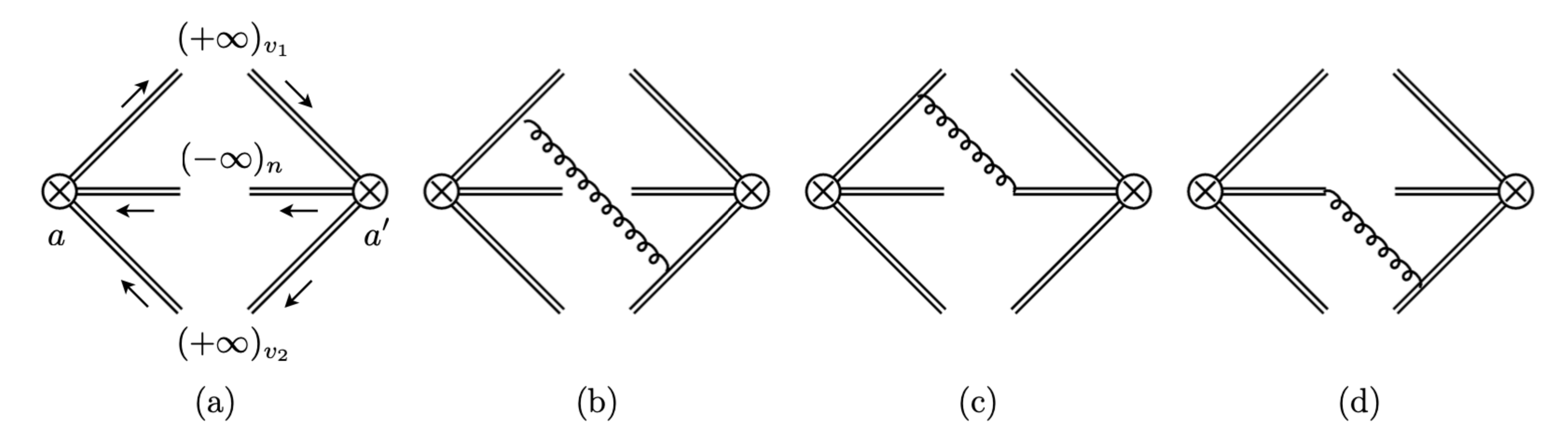
$$\times S_{\gamma f}(\boldsymbol{b},\eta_1,\eta_2,\mu,\zeta_2)(\mathcal{C}_g(\boldsymbol{b},R,\mu)J_g(p_T,R,\mu))\big(\mathcal{C}_{\bar{f}}(\boldsymbol{b},R,\mu)J_{\bar{f}}(p_T,R,\mu)\big)$$

A new soft function appear

A new soft function, properties, resummation

Only formally similar to R. Zhu, P. Sun, F. Yuan arXiv:1309.0780

$$\hat{S}_{\gamma g}(\boldsymbol{b}) = \frac{1}{C_F C_A} \langle 0 | \mathcal{S}_n^\dagger(\boldsymbol{b}, -\infty)_{ca'} \mathrm{Tr}\left[S_{v_2}(+\infty, \boldsymbol{b}) T^{a'} S_{v_1}^\dagger(+\infty, \boldsymbol{b}) S_{v_1}(+\infty, 0) T^a S_{v_2}^\dagger(+\infty, 0) \right] \mathcal{S}_n(0, -\infty)_{ac} | 0 \rangle .$$



Only one rapidity regulator needed because we measure only momenta transverse to *n*

Re-organization, zero-bin, ..

The zero-bin subtracted TMD $\hat{B}_i(\xi, \boldsymbol{b}, \mu, k^-\delta_+) = \dfrac{B_i^{\text{un.}}(\xi, \boldsymbol{b}, \mu, k^-/\delta^-)}{S(\boldsymbol{b}, \mu, \sqrt{\delta^+\delta^-})}$

Factorization of soft-function $S(\boldsymbol{b}, \mu, \sqrt{\delta^+\delta^-}) = S^{\frac{1}{2}}(\boldsymbol{b}, \mu, \delta^+\nu)S^{\frac{1}{2}}(\boldsymbol{b}, \mu, \delta^-/\nu)$

Re-organization of the product of beam function and soft function in TMD and subtracted soft-function

$$\hat{B}_i(\xi, \boldsymbol{b}, \mu, k^-\delta_+)\hat{S}_{\gamma i}(\boldsymbol{b}, \mu, \sqrt{A_n}\,\delta_+) = F_i(\xi, \boldsymbol{b}, \mu, \zeta_1)\, S_{\gamma i}(\boldsymbol{b}, \mu, \zeta_2)$$

$$F_i(\xi, \boldsymbol{b}, \mu, \zeta_1) = \left.\frac{B_i^{\text{un.}}(\xi, \boldsymbol{b}, \mu, k^-/\delta^-)}{S^{\frac{1}{2}}(\boldsymbol{b}, \mu, \delta^-/\nu)}\right|_{\sqrt{2}\,k^-/\nu \to \sqrt{\zeta_1}} \qquad S_{\gamma i}(\boldsymbol{b}, \mu, \zeta_2) = \left.\frac{\hat{S}_{\gamma i}(\boldsymbol{b}, \mu, \sqrt{A_n}\,\delta^+)}{S^{\frac{1}{2}}(\boldsymbol{b}, \mu, \delta^+\nu)}\right|_{\nu/\sqrt{2A_n} \to \sqrt{\zeta_2}}$$

Scales and consistency check

$$\zeta_1\,\zeta_2 = \frac{(k^-)^2}{A_n} = \frac{\hat{u}\,\hat{t}}{\hat{s}} = p_T^2 \to \zeta_1 = p_T^2;\ \zeta_2 = 1$$

Direct check at NLO $\quad \frac{d}{d\ln\mu} G(\mu) = \gamma_G(\mu)\, G(\mu)$

$(\gamma^* g)$-channel $\quad \gamma_{H_{\gamma g}} + \gamma_{S_{\gamma g}} + \gamma_{F_g} + 2\gamma_{J_f} + \gamma_{\mathcal{C}_1} + \gamma_{\mathcal{C}_2} + \gamma_\alpha = 0$

$(\gamma^* f)$-channel $\quad \gamma_{H_{\gamma f}} + \gamma_{S_{\gamma f}} + \gamma_{F_f} + \gamma_{J_f} + \gamma_{J_g} + \gamma_{\mathcal{C}_f} + \gamma_{\mathcal{C}_g} + \gamma_\alpha = 0.$

Heavy di-hadron production $\ell + p \to \ell + H + \bar{H} + X$

In principle all very similar (we consider only the charm case for EIC)

Momentum imbalance
$$\boldsymbol{r}_T = \boldsymbol{p}_T^H + \boldsymbol{p}_T^{\bar{H}}$$

Di-hadron Momentum
$$p_T = \frac{|\boldsymbol{p}_T^H| + |\boldsymbol{p}_T^{\bar{H}}|}{2}$$

$$|\boldsymbol{r}_T|,\ m_H \ll p_T^{H,\bar{H}}$$

$$\frac{d\sigma(\gamma^* g)}{dx d\eta_H d\eta_{\bar{H}} dp_T d\boldsymbol{r}_T} = H^{\mu\nu}_{\gamma^* g \to Q\bar{Q}}(\hat{s}, \hat{t}, \hat{u}, \mu) \int \frac{d\boldsymbol{b}}{(2\pi)^2} \exp(i\boldsymbol{b}\cdot\boldsymbol{r}_T)\, F_{g,\mu\nu}(\xi, \boldsymbol{b}, \mu, \zeta_1)$$
$$\times\, S_{\gamma g}(\boldsymbol{b}, \mu, \zeta_2)\, J_{Q\to H}(\boldsymbol{b}, p_T, m_Q, \mu)\, J_{\bar{Q}\to\bar{H}}(\boldsymbol{b}, p_T, m_Q, \mu)$$

Only gluons in initial state
$$F_g^{\mu\nu}(\xi, \boldsymbol{b}) = f_1(\xi, \boldsymbol{b}) \frac{g_T^{\mu\nu}}{d-2} + h_1^{\perp}(\xi, \boldsymbol{b}) \left(\frac{g_T^{\mu\nu}}{d-2} + \frac{b^\mu b^\nu}{\boldsymbol{b}^2} \right)$$

Scales and bHQET

New scales appear, whose logs should be resummed $\mu_+ = m_Q; \quad \mu_{\mathcal{J}} = m_Q \frac{r_T}{p_T}$

bHQET

$$p_Q^\mu\Big|_{\text{rest frame}} = m_Q \beta^\mu + k_s^\mu \,, \qquad k_s^\mu \sim \Lambda_{\text{QCD}}(1,1,1)_v, \quad \Longrightarrow$$

$$p_Q^\mu\Big|_{\text{boosted frame}} \simeq \left(2E_H, \frac{m_H^2}{2E_H}, \Lambda_{\text{QCD}}\right)_v, \qquad k_{uc}^\mu \sim \Lambda_{\text{QCD}}\left(\frac{2E_H}{m_H}, \frac{m_H}{2E_H}, 1\right).$$

Typical scaling $r_T \sim \Lambda_{\text{QCD}} \frac{2p_T}{m_H}$

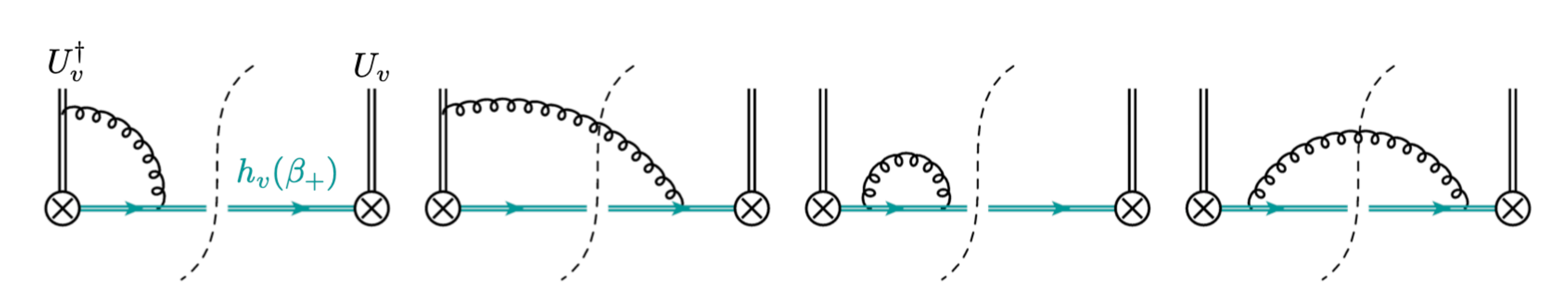
Matching bHQET to massive SCET

βμ is the collinear velocity of the heavy hadron and vμ is the lightlike vector along the direction of the boosted quark.

$$W_v^\dagger \xi_v \to C_+(m_Q,\mu) W_v^\dagger h_{v\beta_+}$$

$$J_{Q\to H}(\boldsymbol{b}, p_T, m_Q, \mu) = |C_+(m_Q,\mu)|^2 \mathcal{J}_{Q\to H}\left(\boldsymbol{b}, \frac{m_Q}{p_T}, \mu\right)$$

Shape function in b-space



Fragmentation Shape function/bHQET Jet function connection

$$\mathcal{J}_{Q\to H}\left(\boldsymbol{b},\frac{m_Q}{p_T},\mu\right)=\int d\boldsymbol{r}\,\exp(i\boldsymbol{b}\cdot\boldsymbol{r})\mathcal{J}_{Q\to H}(\boldsymbol{r})$$

$$\mathcal{J}_{Q\to H}(\boldsymbol{r})=\frac{1}{2\,p_H^-\,N_C}\sum_X\langle 0|\delta^{(2)}\left(\boldsymbol{r}-i\boldsymbol{v}\left(\bar{v}\cdot\partial\right)\right)W_v^\dagger h_{v\beta_+}|XH\rangle\langle XH|\bar{h}_{v,\beta_+}W_v\,\bar{\not{v}}|0\rangle$$

$$S_{Q\to H}(\omega)=\frac{1}{2N_c}\sum_X\langle 0|\delta(\omega-i\sqrt{2}\,\bar{v}\cdot\partial)W_v^\dagger h_{v\beta_+}|H_\beta X\rangle\langle H_\beta X|\bar{h}_{v,\beta_+}W_v\frac{\bar{\not{v}}}{\sqrt{2}}|0\rangle$$

$$\tilde{S}_{Q\to H}(\tau)=\int d\omega\,\exp(i\omega\tau)S_{Q\to H}(\omega)$$

$$\mathcal{J}_{Q\to H}(\boldsymbol{b})=\frac{m_H}{\sqrt{2}\,p_H^-}\tilde{S}_{Q\to H}\left(\tau\to\frac{\boldsymbol{v}\cdot\boldsymbol{b}}{\sqrt{2}}\right)$$

Soft function AD up to 3-loops

$$\gamma_{S_{\gamma g}} = -\left(\gamma_{H_{\gamma g}} + \gamma_{F_g} + \gamma_\alpha + \gamma_{\mathcal{J}}(\boldsymbol{v}_1) + \gamma_{\mathcal{J}}(\boldsymbol{v}_2) + 2\gamma_+\right)$$

$$\gamma_{S_{\gamma g}} = \gamma_{\text{cusp}}\left[2C_F \ln\left(\frac{B\mu^2 e^{2\gamma_E} 4p_T^2 c_{\boldsymbol{b}}^2}{\hat{s}}\right) - C_A \ln\zeta_2\right] + \delta\gamma_{S_{\gamma g}}$$

$$\delta\gamma^{[1]}_{S_{\gamma g}} = 0$$

$$\delta\gamma^{[2]}_{S_{\gamma g}} = C_F\left[C_A\left(\frac{1616}{27} - \frac{22}{9}\pi^2 - 56\,\zeta_3\right) + n_f T_F\left(-\frac{448}{27} + \frac{8}{9}\pi^2\right)\right]$$

$$\delta\gamma^{[3]}_{S_{\gamma g}} = ..$$

New AD property

$$\gamma^{[1]}_{S_{\gamma g}} = 4\left\{-C_A \ln\zeta_2 + 2C_F\left[\ln(B\mu^2\, e^{2\gamma_E}) - \ln\hat{s} + \ln p_T^2 + \ln(4c_{\boldsymbol{b}}^2)\right]\right\}$$

$$\gamma^{[1]}_{S_{\gamma f}} = 4\left\{(C_F + C_A)\left[\ln(B\mu^2 e^{2\gamma_E}) - \ln\hat{s} + \ln p_T^2 + \ln(4c_{\boldsymbol{b}}^2)\right] + (C_F - C_A)\left[\ln\left(\frac{\hat{t}}{\hat{u}}\right) - i\pi\text{sign}(c_{\boldsymbol{b}})\right] - C_F\ln\zeta_2\right\}$$

$$\gamma^{[1]}_{\mathcal{C}_g} = 4C_A\left[-\ln\left(B\mu^2\, e^{2\gamma_E}\right) + \ln R^2 - \ln(4c_{\boldsymbol{b}}^2) - i\pi\text{sign}(c_{\boldsymbol{b}})\right]$$

$$\gamma^{[1]}_{\mathcal{C}_f} = 4C_F\left[-\ln\left(B\mu^2\, e^{2\gamma_E}\right) + \ln R^2 - \ln(4c_{\boldsymbol{b}}^2) + i\pi\text{sign}(c_{\boldsymbol{b}})\right]$$

$$\gamma^{[1]}_{\mathcal{C}_{\bar{f}}} = 4C_F\left[-\ln\left(B\mu^2\, e^{2\gamma_E}\right) + \ln R^2 - \ln(4c_{\boldsymbol{b}}^2) - i\pi\text{sign}(c_{\boldsymbol{b}})\right]$$

Imaginary parts in the anomalous dimension must be treated carefully.
They cancel with angle integration.

Conclusion

- We have shown the factorization of two processes involving gluon TMD. A new soft function appear, whose properties need to be studied.
- Zeta-prescription of this new soft function is currently studied and we plan to implement in artemide
- The factorization theorem for jets works for small radii. We expect R :0.4 at EIC.
- Unpolarized and linearly polarized gluons appear together, but the latter has a suppressed matching coefficient.


Back up

Kinematics in the Breit frame

$$Q^2 = -q^2; \quad \boxed{x = \frac{Q^2}{2P\cdot q};} \quad q^\mu = (0,0,0,Q); \quad P^\mu = \frac{1}{2x}(Q,0,0,-Q); \quad \xi = \frac{k^+}{P^+}$$

parton / target hadron

$$\eta_\pm = \frac{\eta_1 \pm \eta_2}{2}$$

$$Q = 2p_T \cosh(\eta_-)\exp(\eta_+); \quad \xi = 2x\cosh(\eta_+)\exp(-\eta_+)$$

Mandelstam variables

$$\hat{s} = (q+k)^2 = +4p_T^2\cosh^2(\eta_-) \,,$$

$$\hat{t} = (q-p_2)^2 = -4p_T^2\cosh(\eta_-)\cosh(\eta_+)\exp(\eta_1) \,,$$

$$\hat{u} = (q-p_1)^2 = -4p_T^2\cosh(\eta_-)\cosh(\eta_+)\exp(\eta_2) \,,$$

$$\hat{s}+\hat{t}+\hat{u} = -Q^2$$