Nuclear matrix elements

William Detmold, MIT

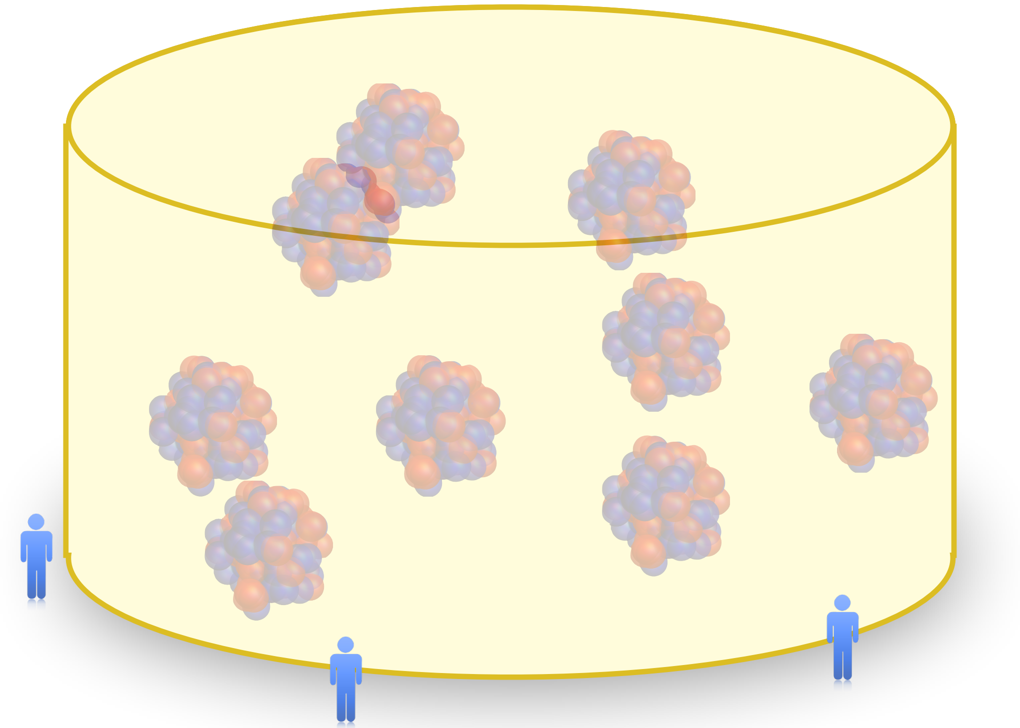
Massachusetts Institute of Technology

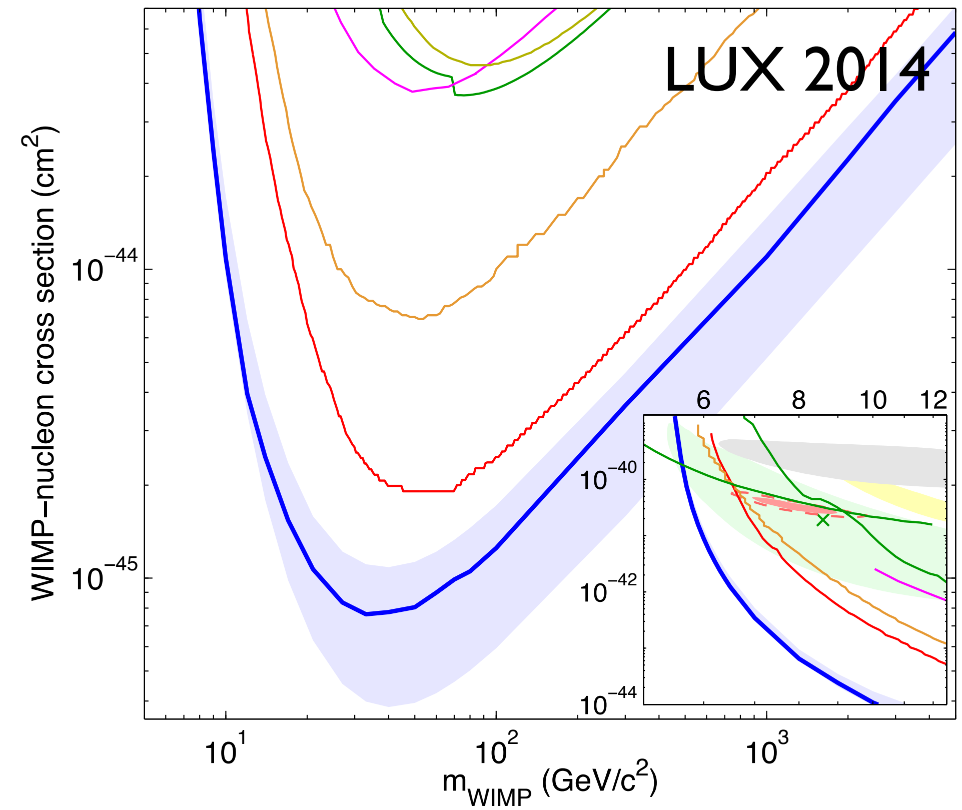
The intensity frontier

- Seek new physics through quantum effects
- Precise experiments
  - Sensitivity to probe the rarest interactions of the SM
  - Look for effects where there is no SM contribution
- Major focus of HEP experimental program over next decade
  - Dark matter direct detection
  - Neutrino physics
  - Charged lepton flavour violation
  - Proton decay, neutron-antineutron oscillations…

Dark matter direct detection

- Search for DM in multiple ways
  - Look for astronomical signals of annihilation
  - Try to produce it at particle colliders
  - Direct detection: wait for DM passing by to hit a nucleus
- Detection rate/bounds depends on
  - Dark matter properties/dynamics
  - Probability for interaction with nucleus: nuclear matrix elements





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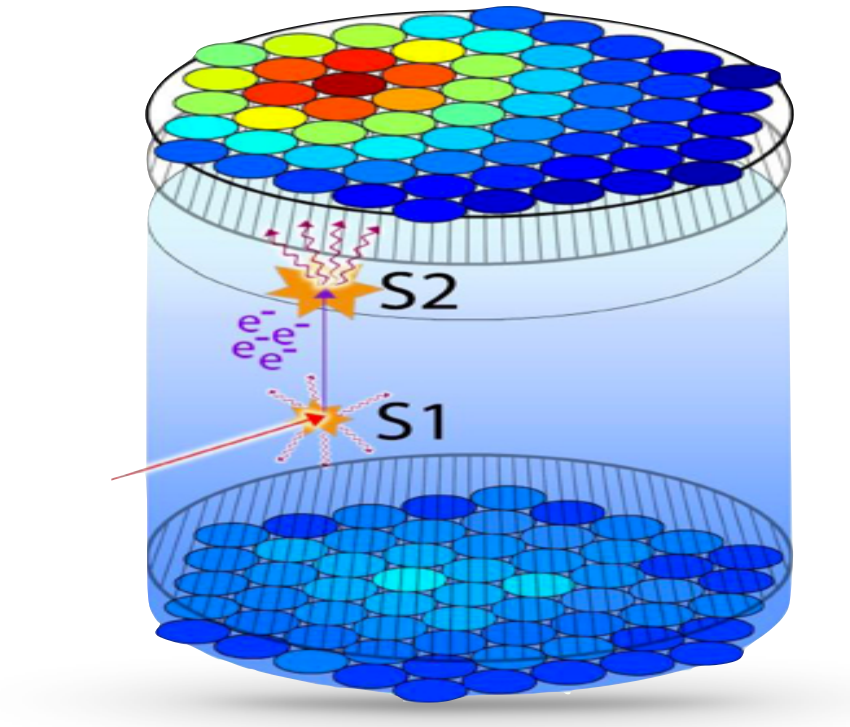
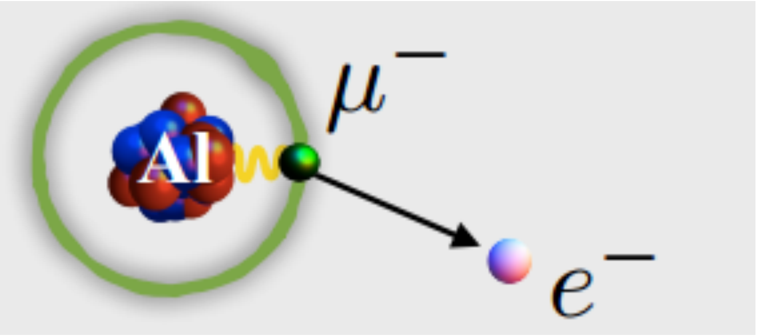
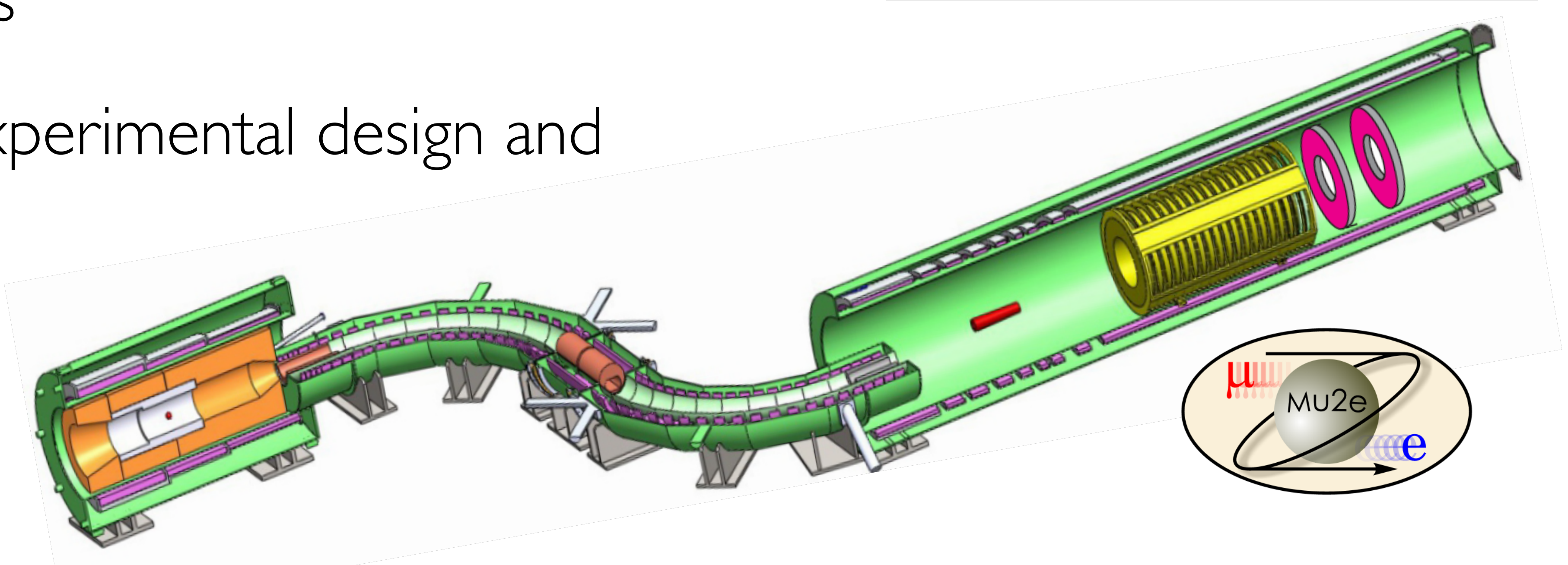


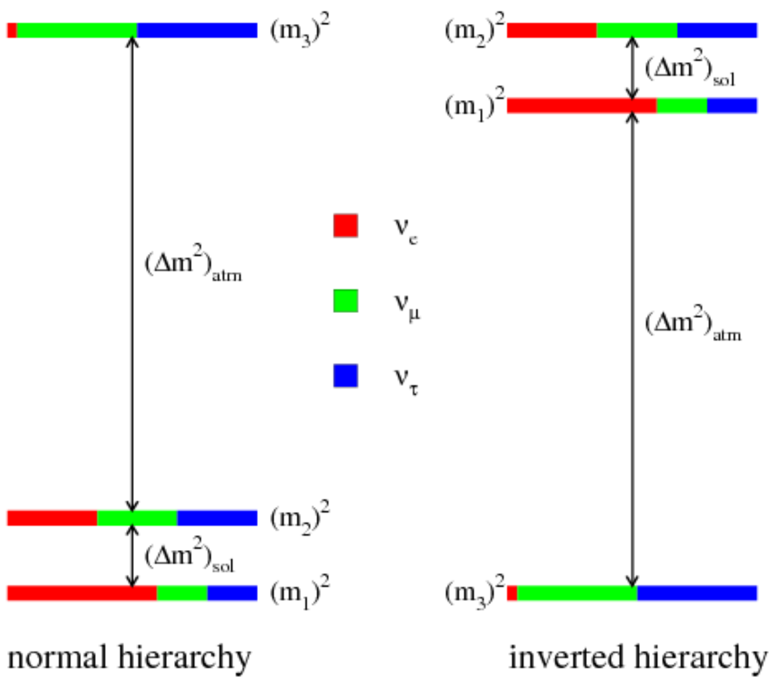
- Dark matter detection: nuclear recoils as signal
  Nuclear matrix elements of exchange current
- μ2e expt: search for charged lepton flavour violation via μ→e conversion in field of Al nucleus
- 😍 Positive signals would be unambiguous



- However <u>need</u> precise nuclear matrix elements with fully quantified uncertainties to discern underlying dynamics
- Also relevant for experimental design and backgrounds

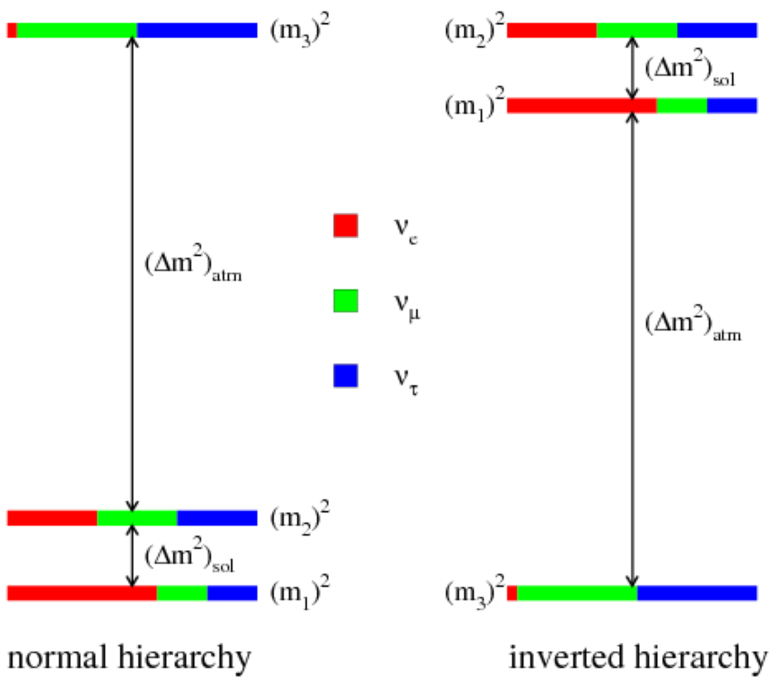


Neutrino oscillations and mass hierarchy



Neutrino oscillations and mass hierarchy

- Long-baseline neutrino beams: further constrain neutrino oscillation parameters and determine mass hierarchy



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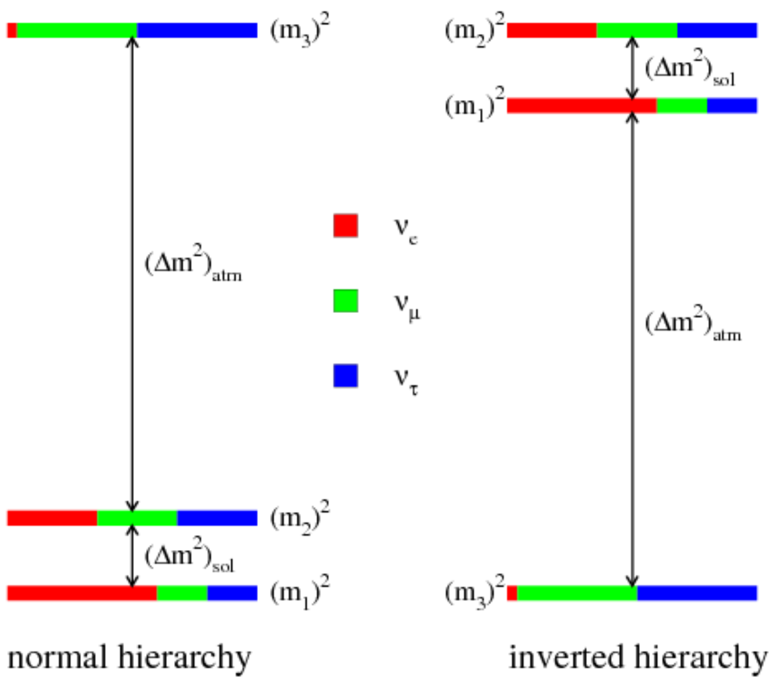

Neutrino oscillations and mass hierarchy

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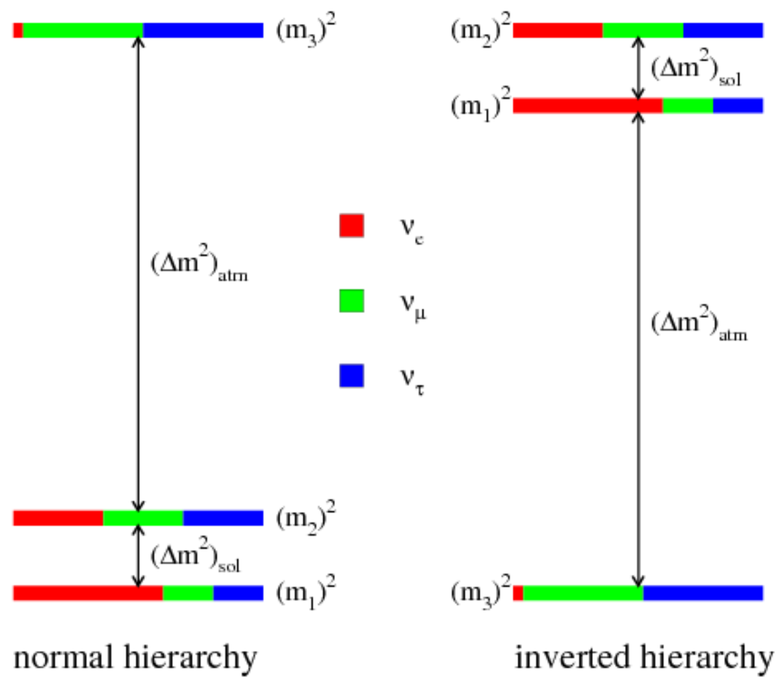
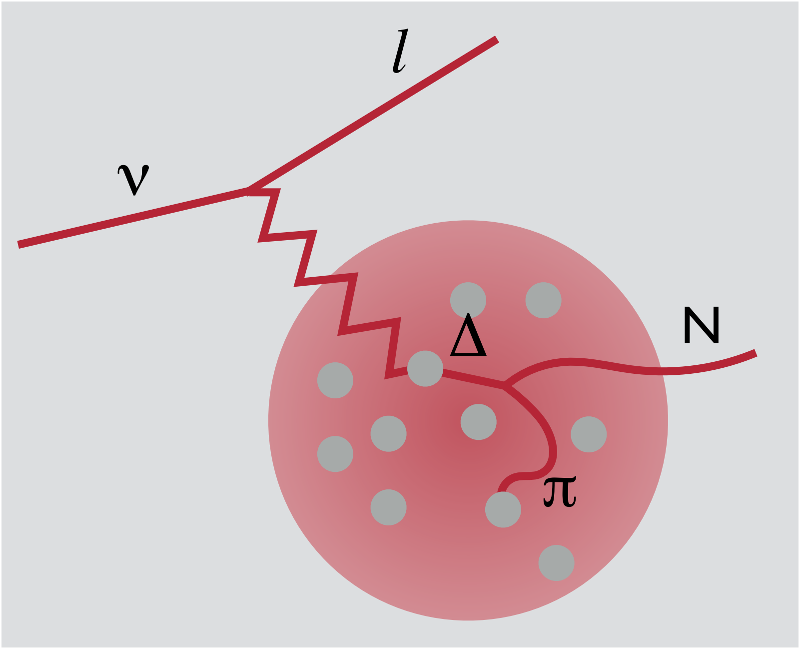
- Future LBNE/O/F requires knowing energies/fluxes to high accuracy
  - Depends on nuclear axial & transition form factors and neutrino-nucleus DIS





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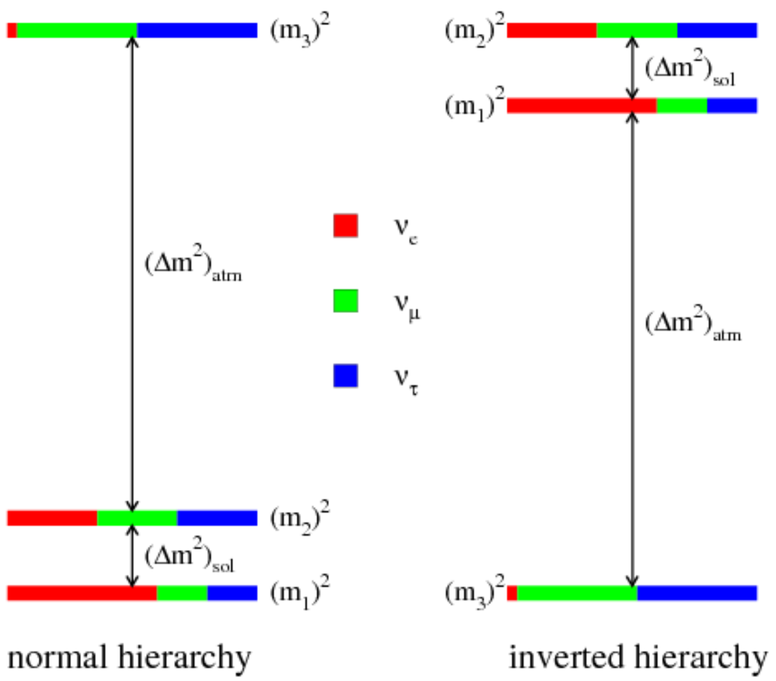
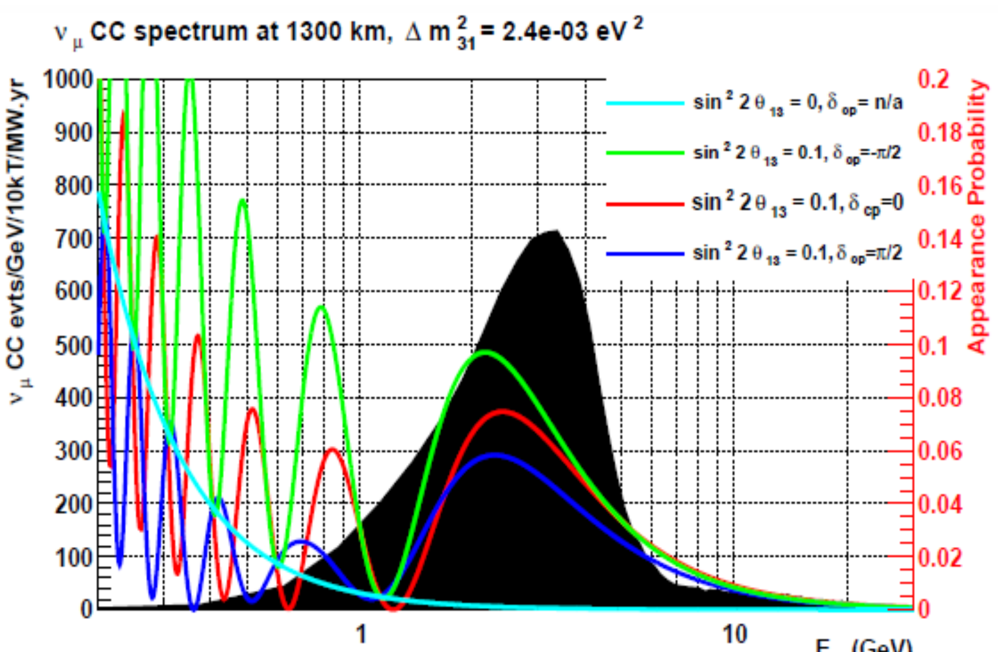
- Future LBNE/O/F requires knowing energies/fluxes to high accuracy
  - Depends on nuclear axial & transition form factors and neutrino-nucleus DIS
  - Current knowledge produces significant uncertainty in determination of oscillation parameters
    [INT workshop 2013]



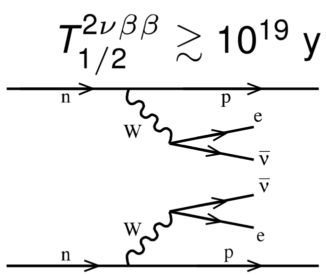


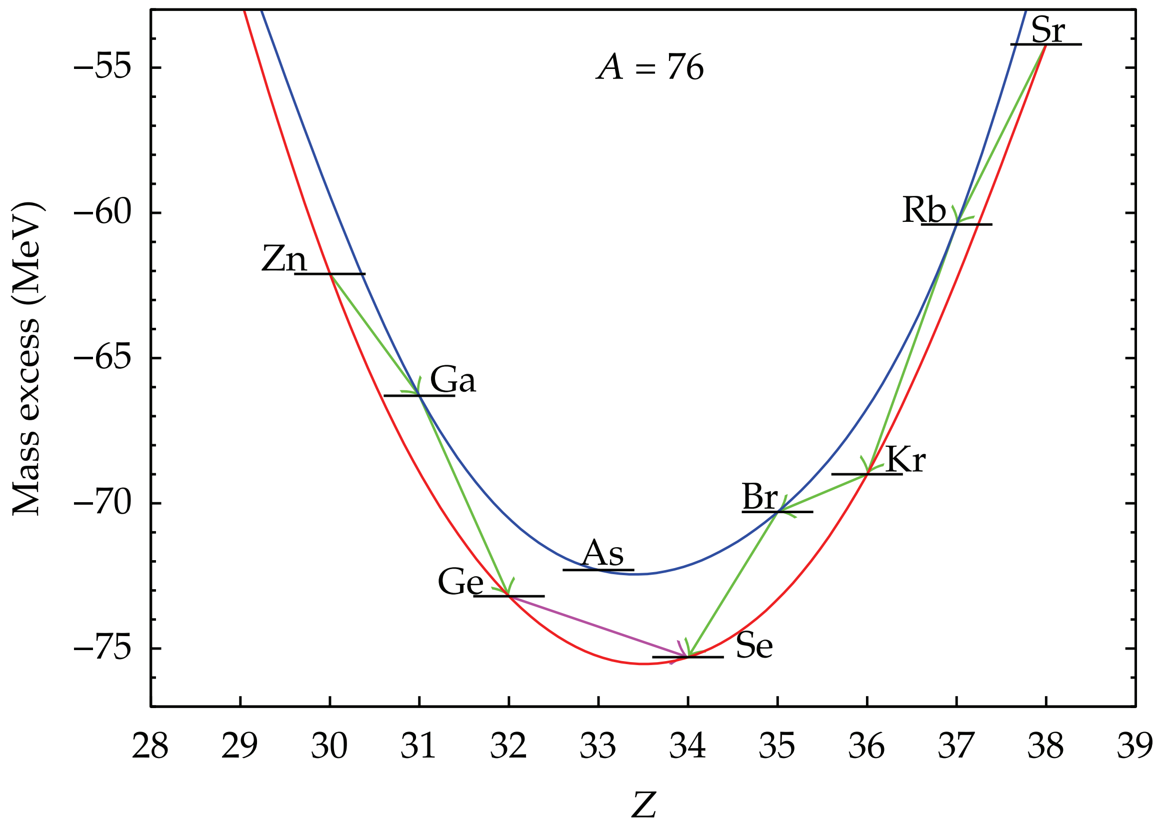
[U Mosel FSNu town

$0\nu\beta\beta$ decay

0νββ decay

- Certain nuclei allow observable **ββ** decay





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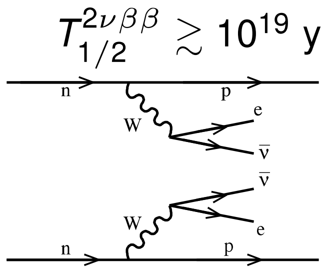

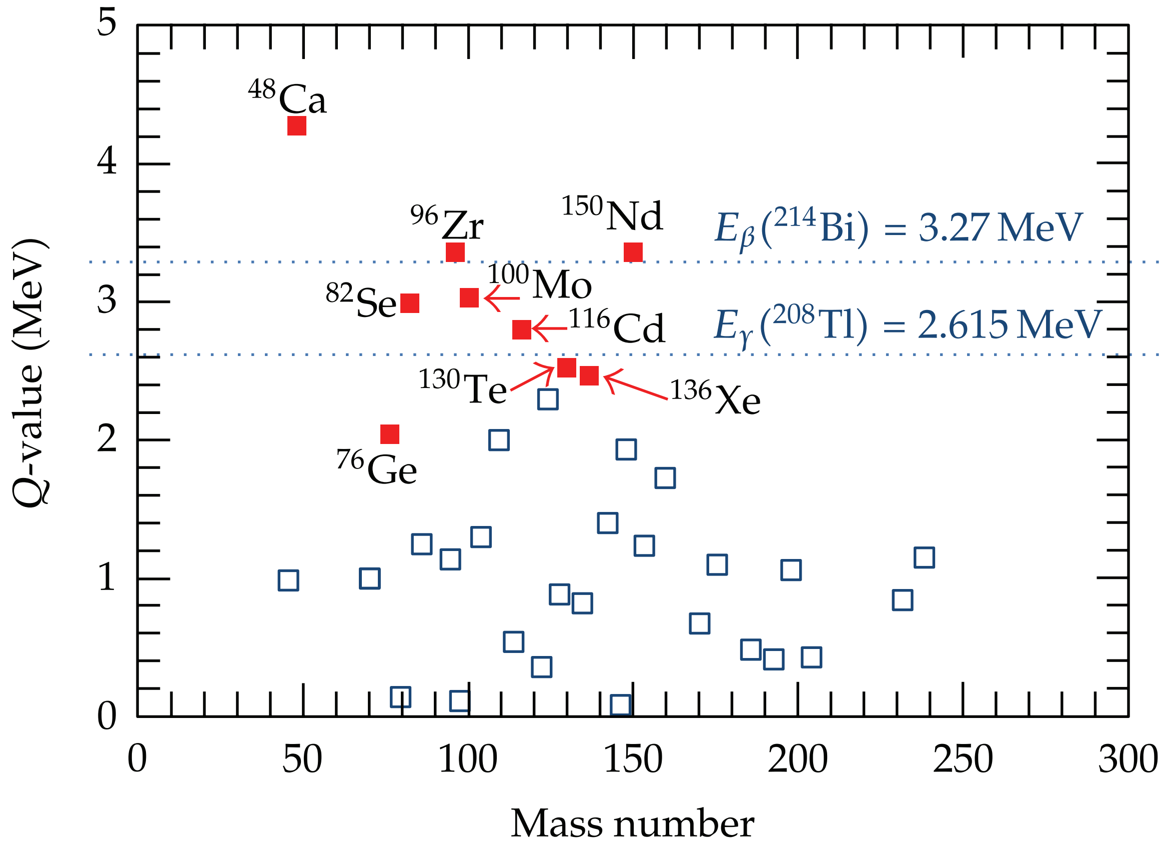

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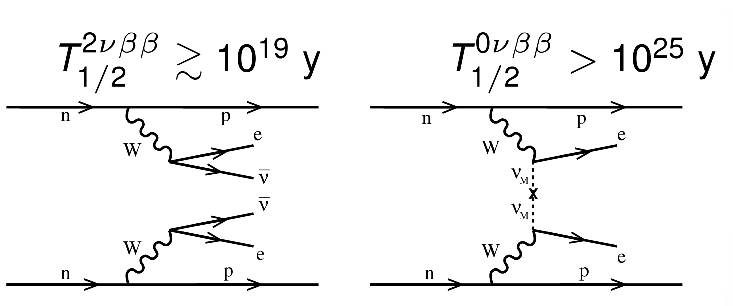
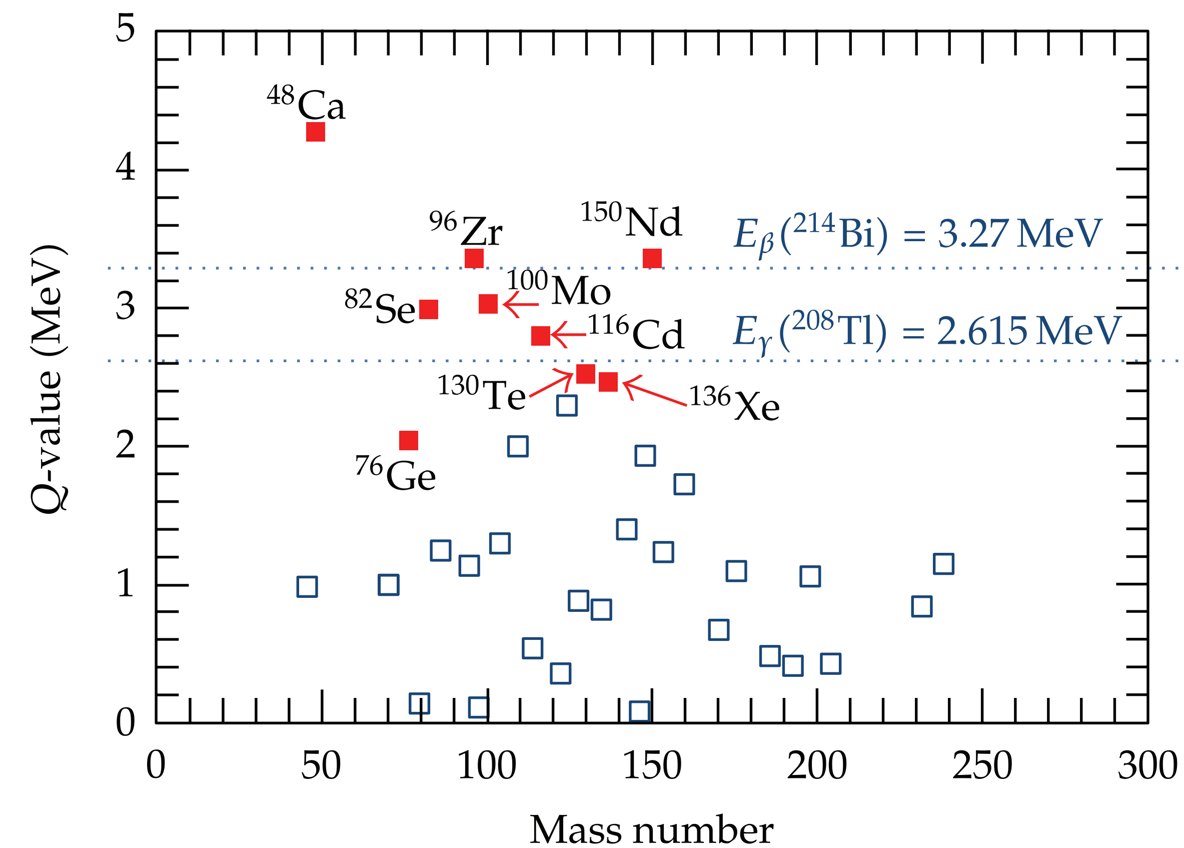
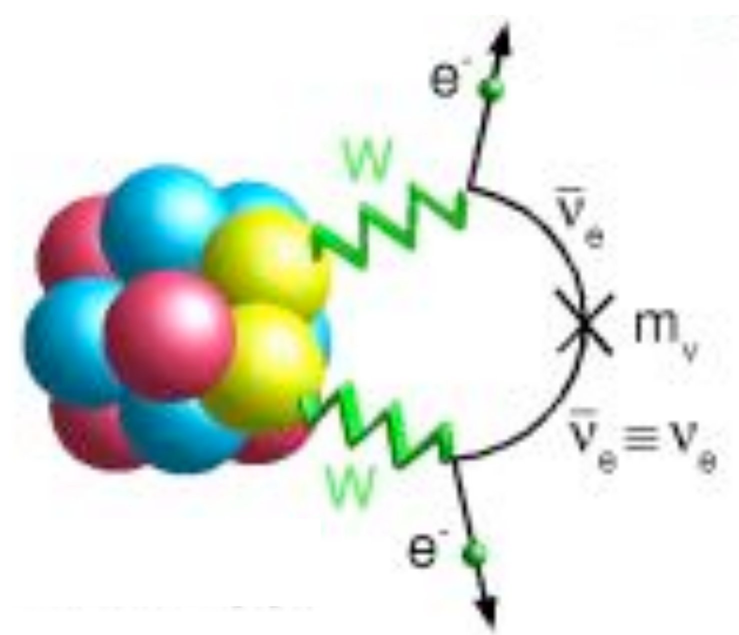


- If neutrinos are massive Majorana fermions 0**νββ** decay is possible





0νββ decay

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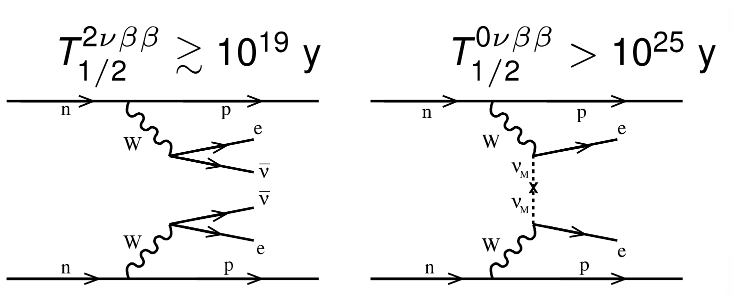
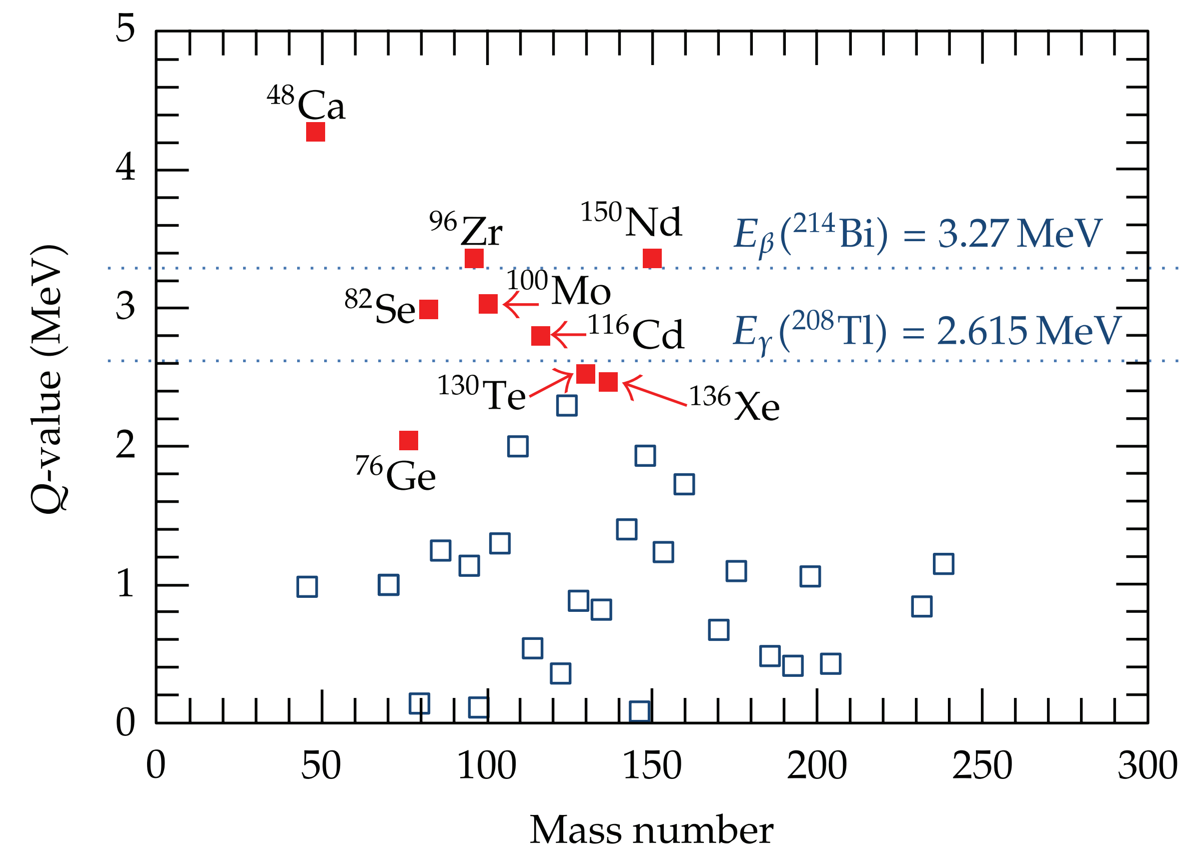
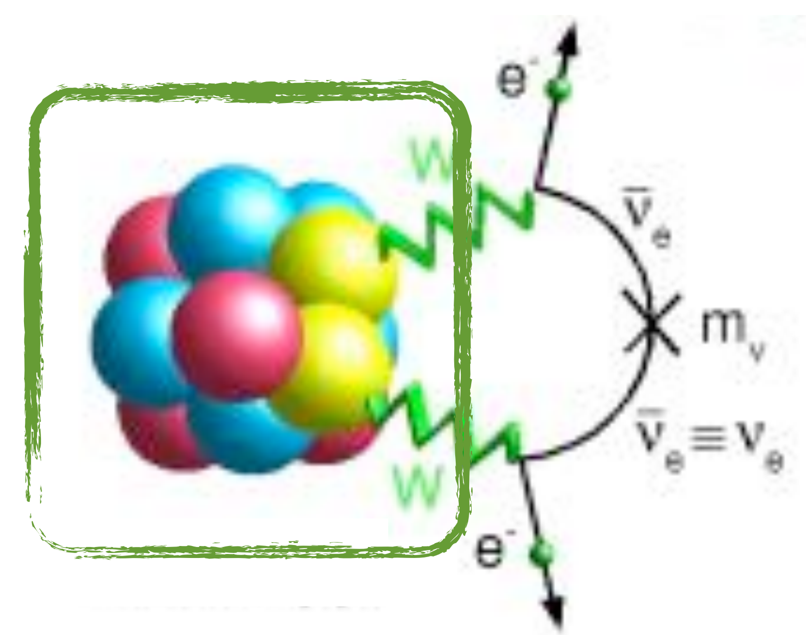


- If neutrinos are massive Majorana fermions 0**ν**ββ decay is possible

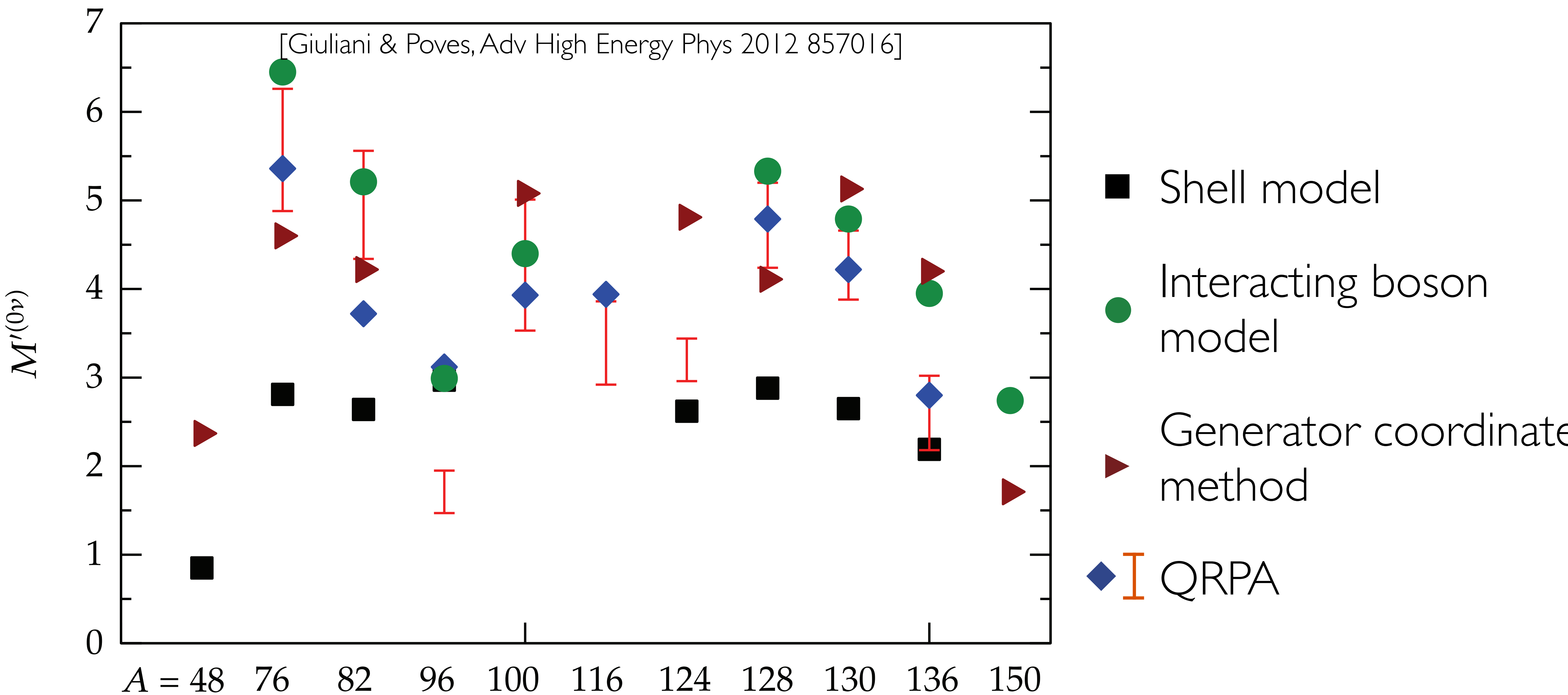
- Half-life depends critically on the nuclear matrix elements of two weak currents!

$$\left(T_{1/2}^{0\nu\beta\beta}(0^+ \rightarrow 0^+)\right)^{-1} = G_{01}\left|M^{0\nu\beta\beta}\right|^2\left(\frac{\langle m_\nu\rangle}{m_e}\right)^2$$





0νββ decay nuclear matrix elements



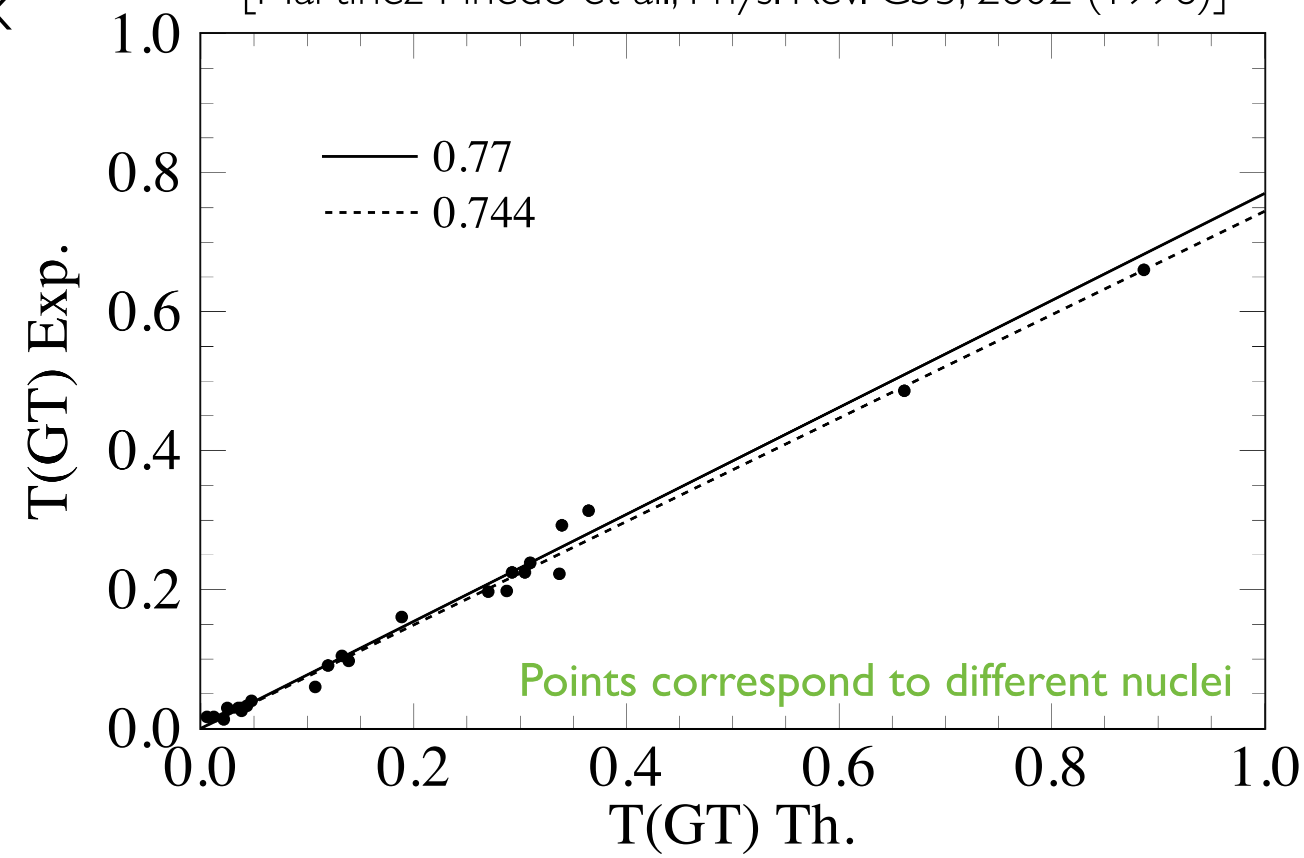
- Large spread of calculations
- Is the spread representative of the true uncertainty?

- How well do we know nuclear matrix elements?

😢 Stark example of problems: Gamow-Teller transitions in nuclei

- Well measured for large range of nuclei (30<A<60)
- Many nuclear structure calcs (QRPA, shell-model,…) – spectrum well described
- Matrix elements systematically off by 20–30%
- “Correct” by “quenching” axial charge in nuclei ...

[Martinez-Pinedo et al., Phys. Rev. C53, 2602 (1996)]



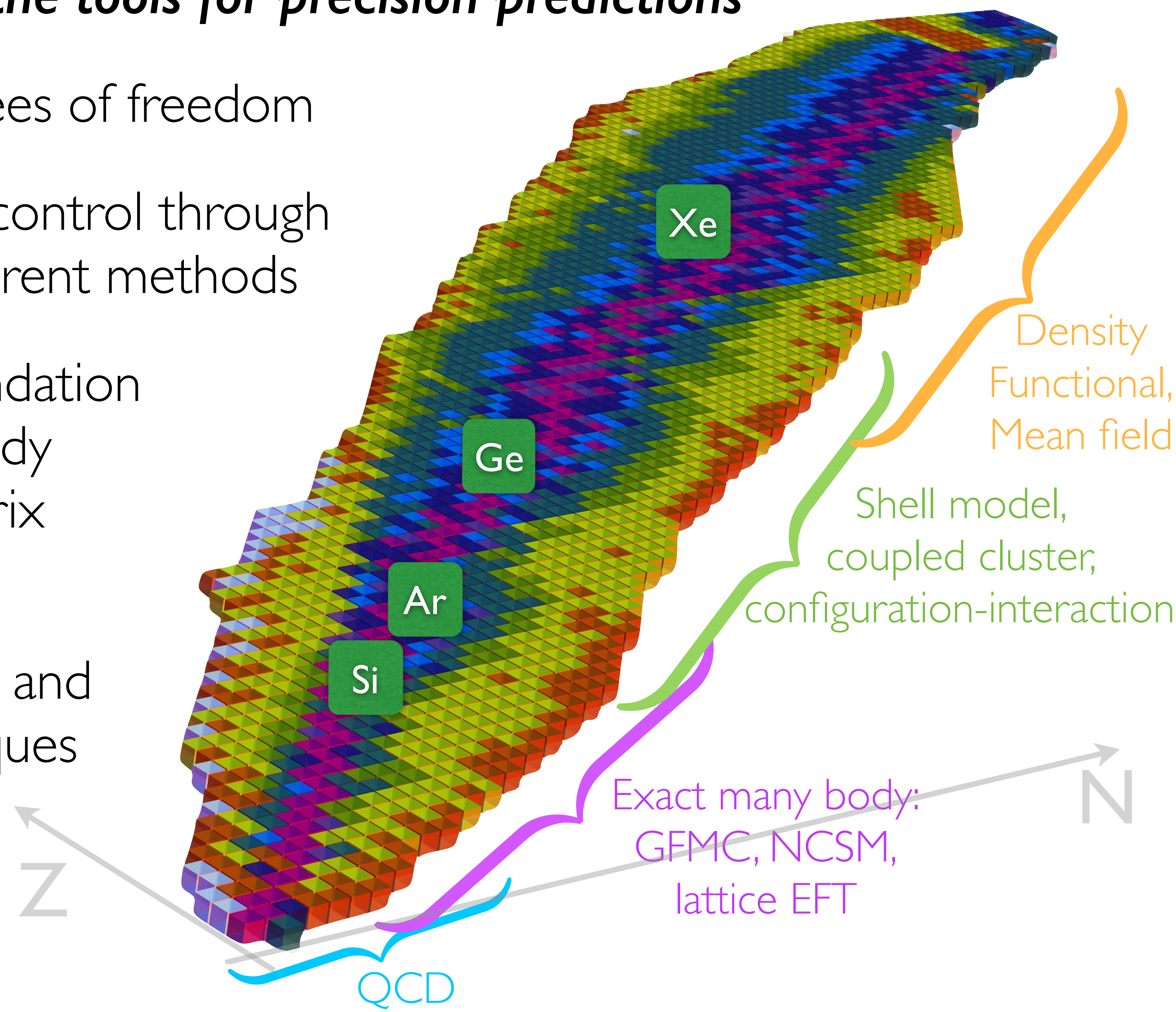
$$T(GT) \sim \sqrt{\sum_f \langle \boldsymbol{\sigma} \cdot \boldsymbol{\tau} \rangle_{i \to f}}$$

$$\langle \boldsymbol{\sigma\tau} \rangle = \frac{\langle f || \sum_k \boldsymbol{\sigma}^k \boldsymbol{t}^k_{\pm} || i \rangle}{\sqrt{2J_i + 1}}$$

Nuclear theory at the intensity frontier

- Definitive need for precision determinations of nuclear matrix elements
  - Must be based on the Standard Model
  - Must have fully quantified uncertainties
  - Timeframe and precision goals set by experiment
- Current state is far from this
- *Nuclear physics is the new flavour physics!*
  - Develop appropriate tools

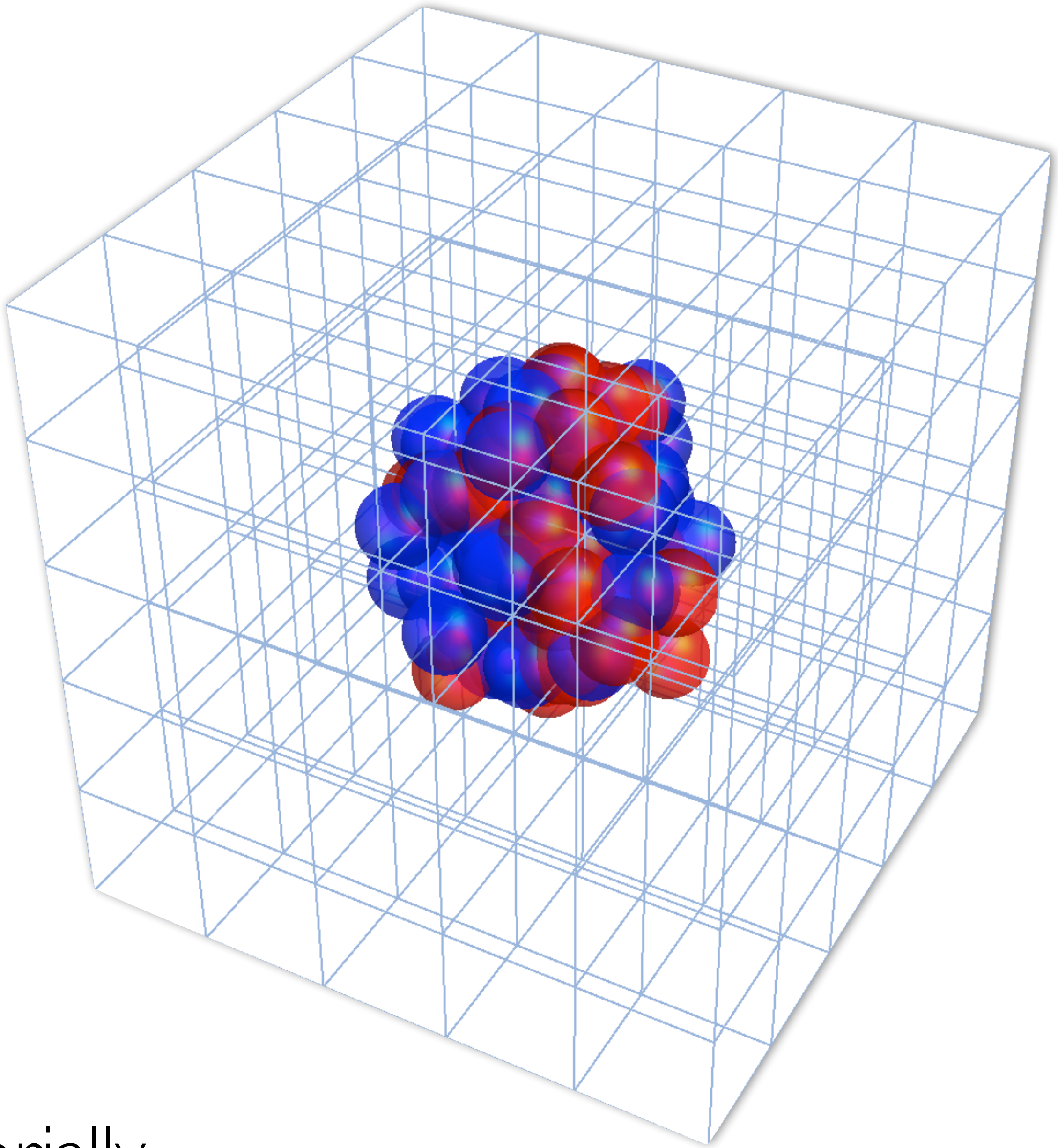
- ***We need to develop the tools for precision predictions***
- Exploit effective degrees of freedom
- Establish quantitative control through linkages between different methods
  - QCD forms a foundation determines few body interactions & matrix elements
  - Match existing EFT and many body techniques onto QCD



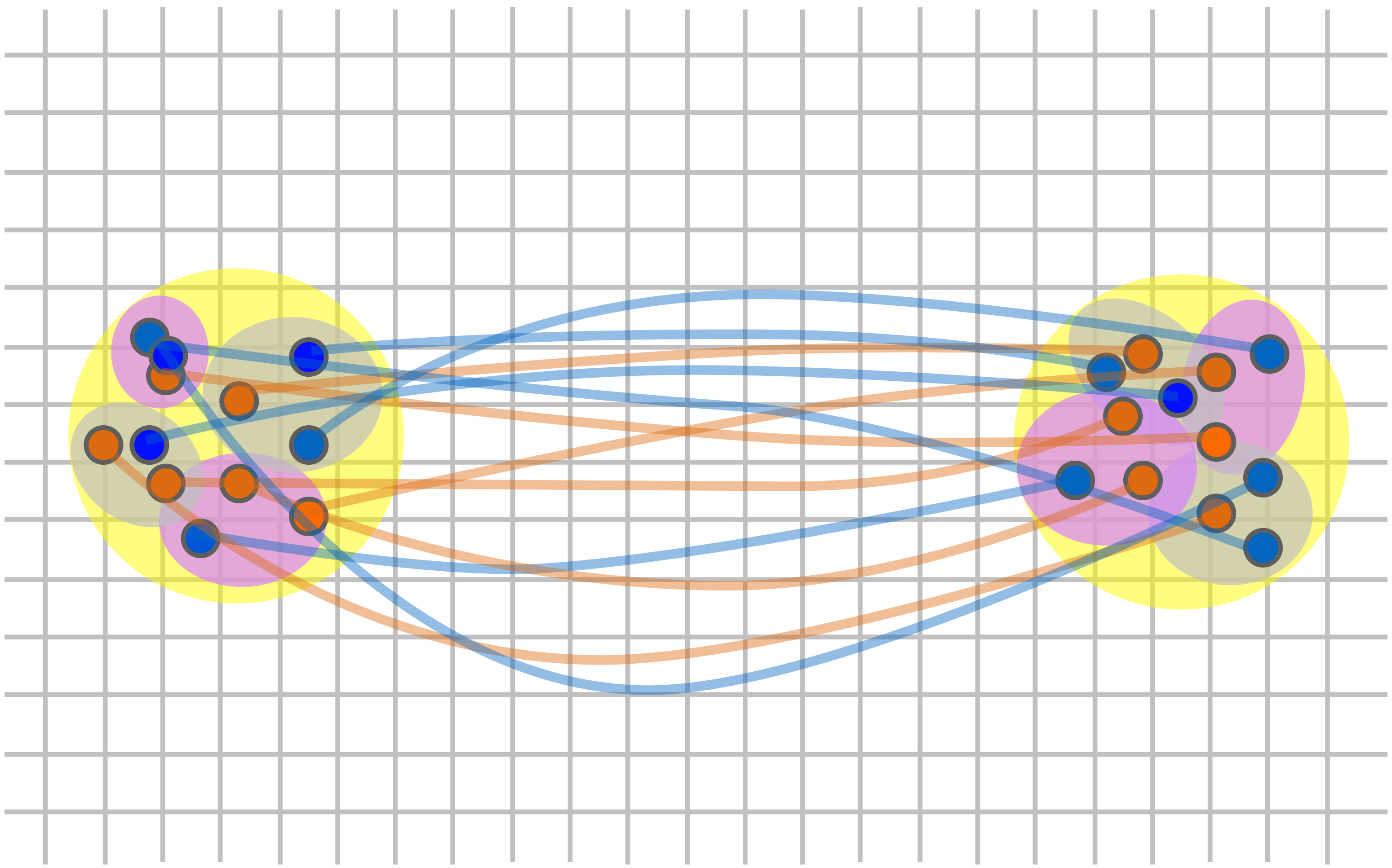
Nuclear Spectra

QCD for Nuclei

- QCD (+EW) describes nuclei
  - Can compute the mass of any nucleus ... in principle
- In practice: a hard problem
- Multiple exponentially difficult challenges
  - Physics at multiple scales
  - Noise: probabilistic method so statistical uncertainty grows exponentially with A
  - Contraction complexity grows factorially
- Large nuclei are challenging, A=2,3,4 are feasible



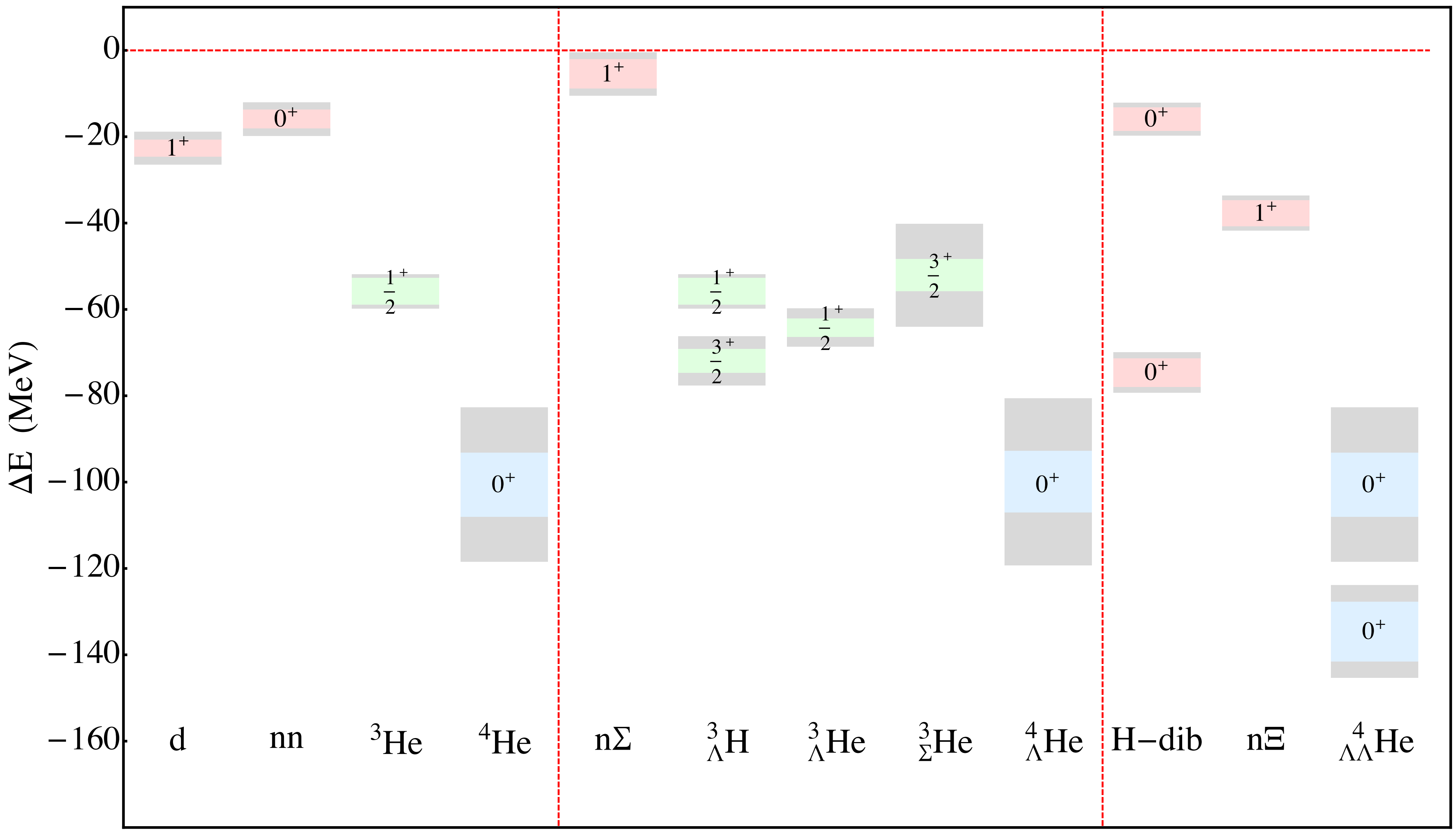
- Quarks need to be tied together in all possible ways
  - $N_{\text{contractions}} = N_u! N_d! N_s!$



- Managed using algorithmic trickery [WD & Savage Doi & Endres, WD & Orginos, Günther&Varnhorst]
  - Study up to N=72 pion systems, A=5/28 nuclei

Light nuclei

- Light hypernuclear spectrum @ 800 MeV



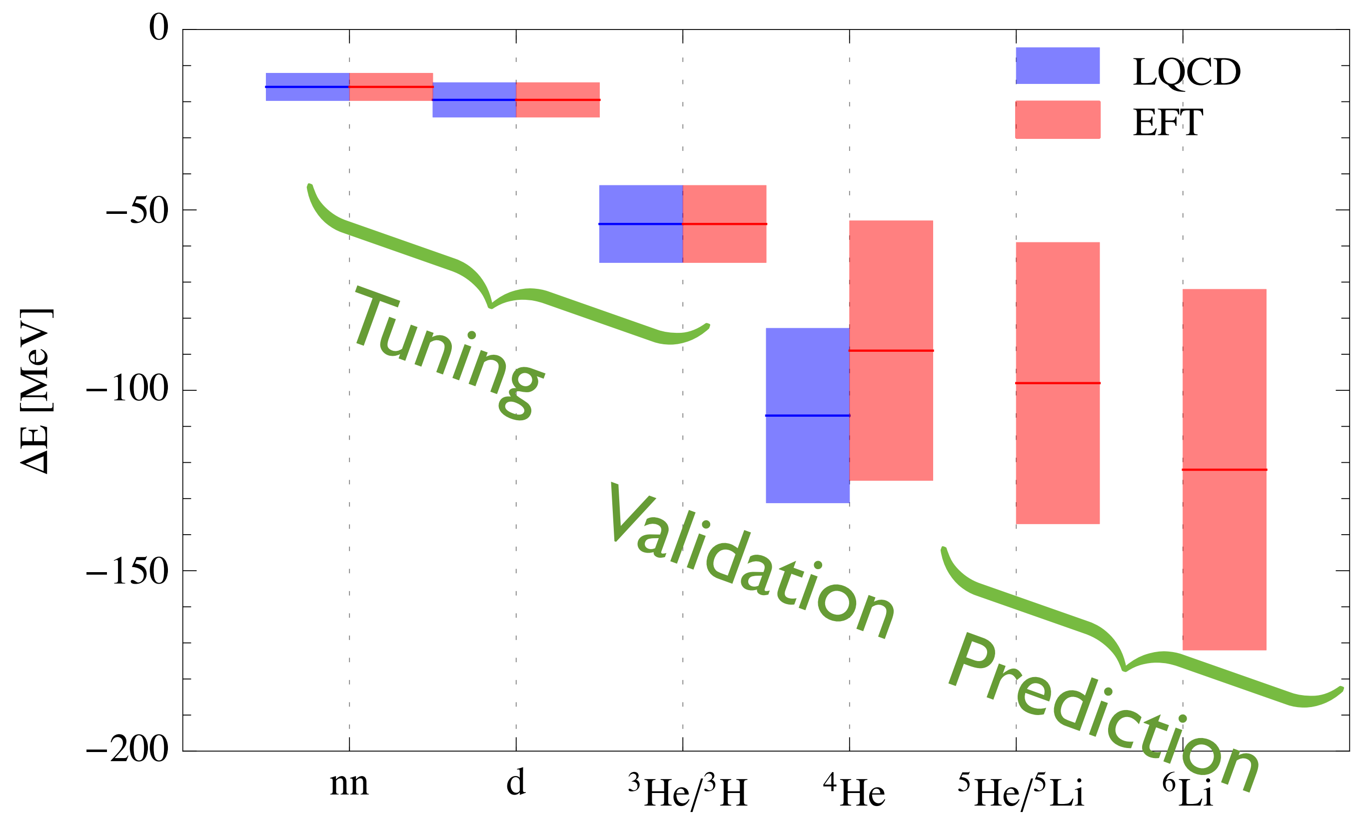
See also talks in previous session

[NPLQCD Phys.Rev. D87 (2013), 034506]

Heavy quark universe

[Barnea et al. 1311.4966 to appear in PRL]

- Combining LQCD and nuclear EFT (pionless EFT)
- For heavy quarks, even spectroscopy requires QCD matching:



In a world @ m_π = 800 MeV

- Equally important for matrix elements

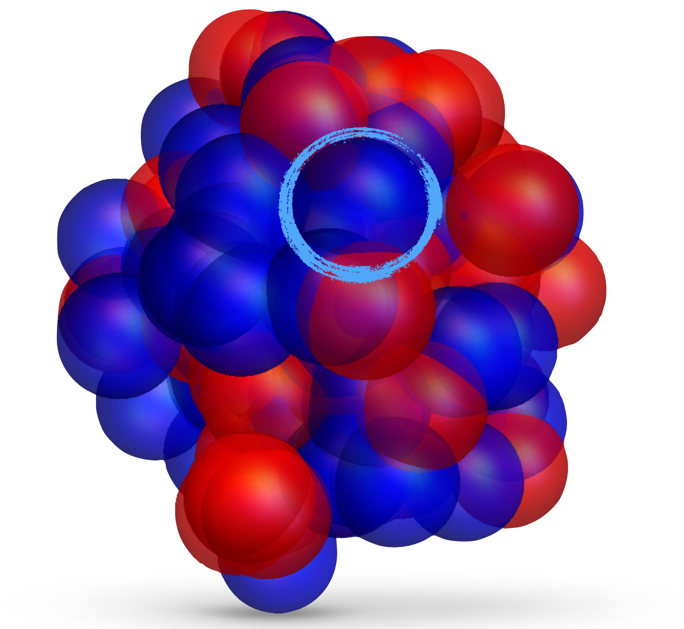
Nuclear Structure

External currents and nuclei

External currents and nuclei

- Current-nucleus interaction
  - Born approximation – interacts with a single nucleon

$$\sigma \sim |A \langle N|J|N\rangle|^2$$



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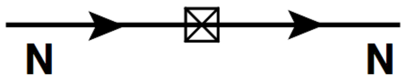

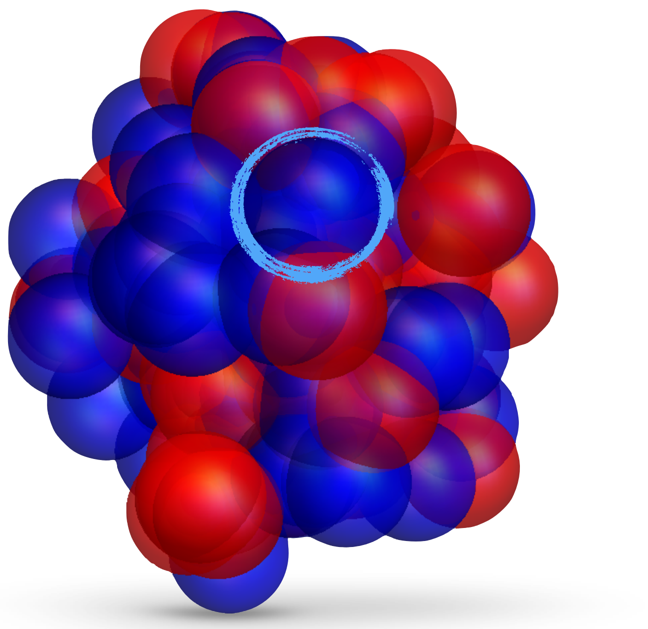

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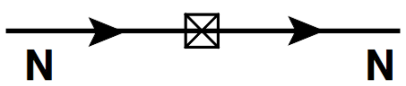
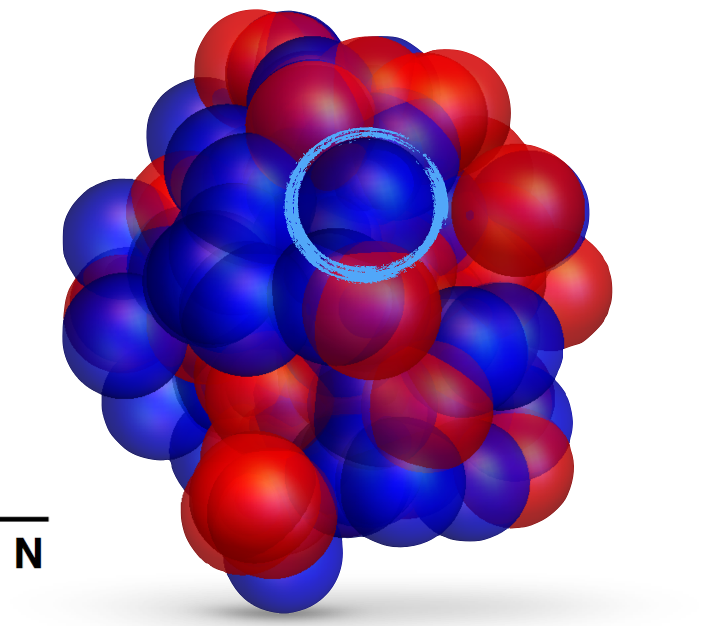
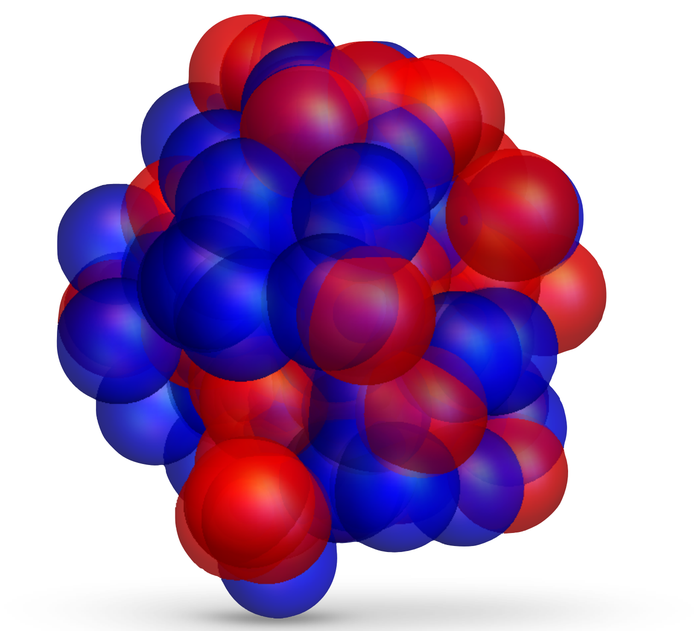




  - Interact non-trivially with multiple nucleons

$$\sigma \sim |A \langle N|J|N\rangle + \alpha \langle NN|J|NN\rangle + \ldots|^2$$



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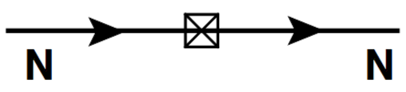

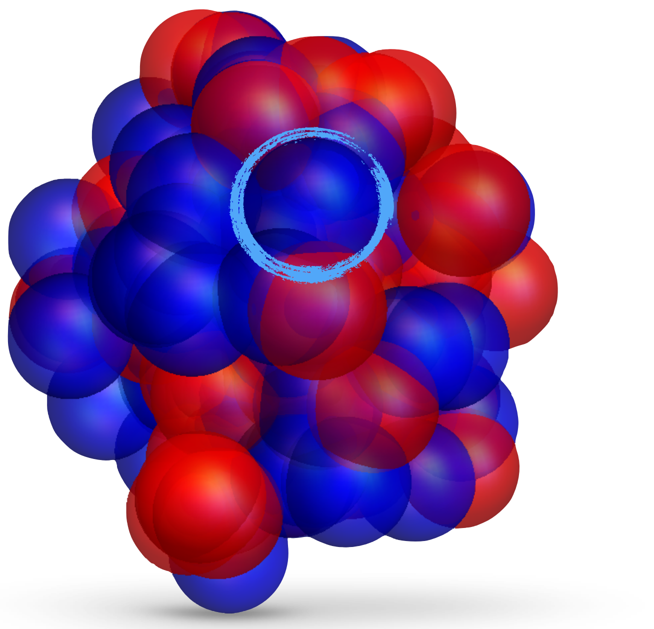

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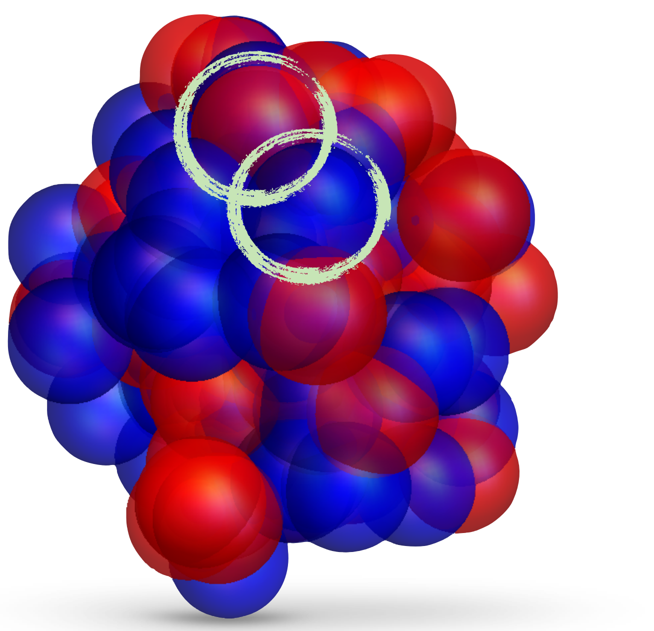

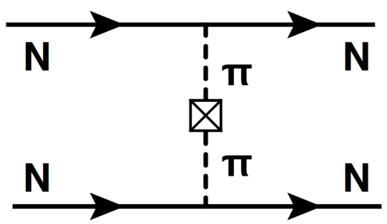

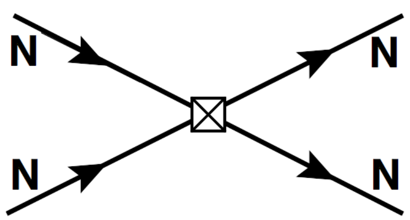

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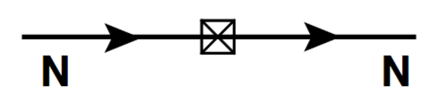

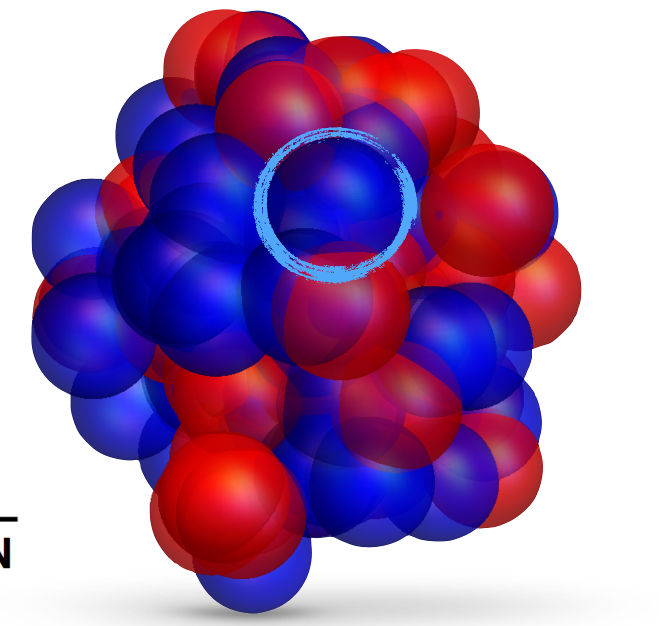

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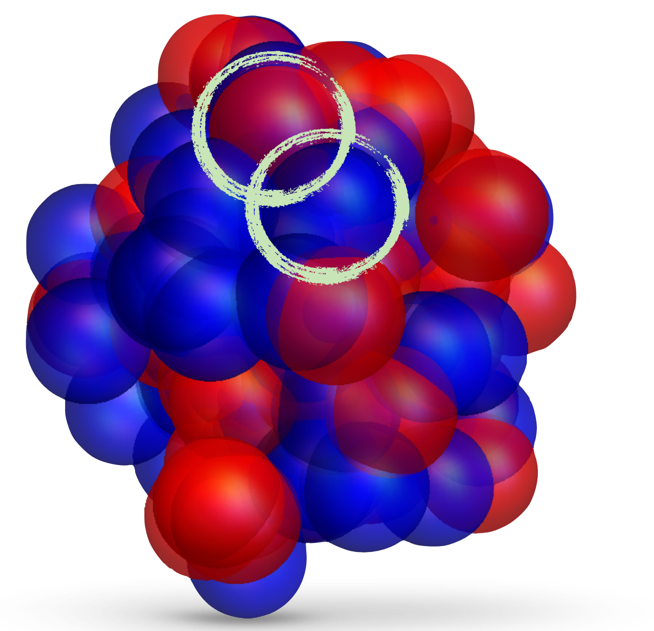
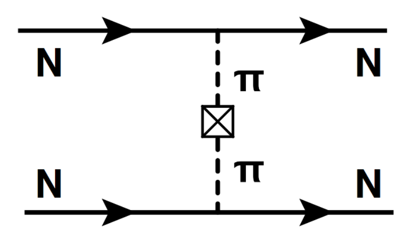
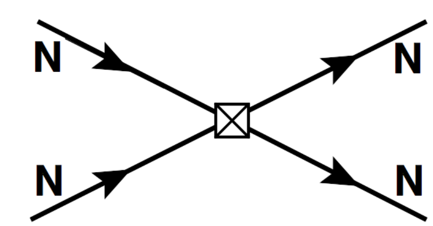


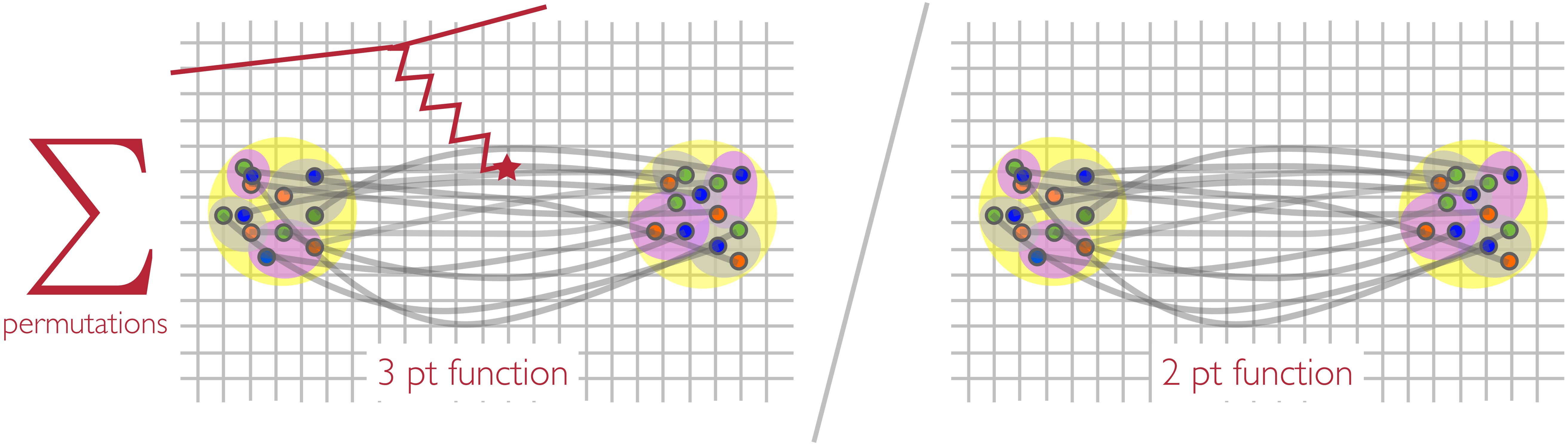
- Second term may be significant
  - May shift cross sections
  - May scale differently with Z and A
  - Leads to significant uncertainty





Nuclear matrix elements

- For deeply bound nuclei, use the techniques as for single hadron matrix elements



  - At large separations gives ground-state matrix element of current

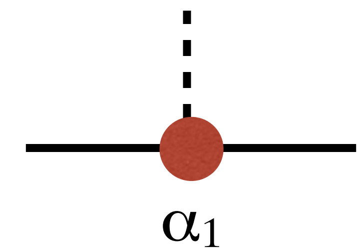
- For near threshold states and scattering states need to be careful with volume effects

- Calculations of matrix elements of currents in light nuclei just beginning for A<5

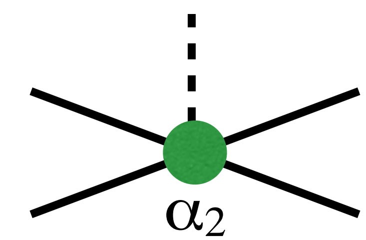
Multi-pion matrix elements

- Perturbatively interacting multi-hadron systems
  [WD & M Flynn arXiv:1412.3895]

- External current (no momentum injection)

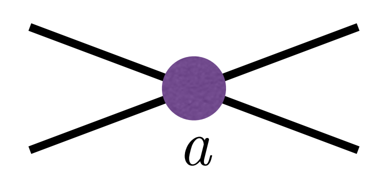


$$J = \sum_{\mathbf{k}} \alpha_1 h^\dagger_{\mathbf{k}} h_{\mathbf{k}} + \sum_{\mathbf{k},\mathbf{Q},\mathbf{p}} \alpha_2 h^\dagger_{\frac{\mathbf{Q}}{2}+\mathbf{k}} h^\dagger_{\frac{\mathbf{Q}}{2}-\mathbf{k}} \, h_{\frac{\mathbf{Q}}{2}+\mathbf{p}} \, h_{\frac{\mathbf{Q}}{2}-\mathbf{p}}$$



- Matrix element of current J in n boson state in finite volume

$$\begin{aligned}\langle n|J|n\rangle \;=\; & n\alpha_1 + \frac{n\alpha_1 a^2}{\pi^2 L^2}\binom{n}{2}\mathcal{J} + \frac{\alpha_2}{L^3}\binom{n}{2} \\ & + \frac{2n\alpha_1 a^3}{\pi^3 L^3}\binom{n}{2}\left\{\mathcal{K}\binom{n}{2} - \left[\mathcal{I}\,\mathcal{J} + 4\mathcal{K}\binom{n-2}{1} + \mathcal{K}\binom{n-2}{2}\right]\right\} - \frac{2\alpha_2 a}{\pi L^4}\binom{n}{2}\mathcal{I} \\ & + \frac{n\alpha_1 a^4}{\pi^4 L^4}\Bigg[3\mathcal{I}^2\mathcal{J} + \mathcal{L}\left(186 - \frac{241n}{2} + \frac{29}{2}n^2\right) + \mathcal{J}^2\left(\frac{n^2}{4} + \frac{3n}{4} - \frac{7}{2}\right) \\ & \qquad + \mathcal{I}\,\mathcal{K}(4n-14) + \mathcal{U}(32n-64) + \mathcal{V}(16n-32)\Bigg] + \mathcal{O}(1/L^5)\,.\end{aligned}$$



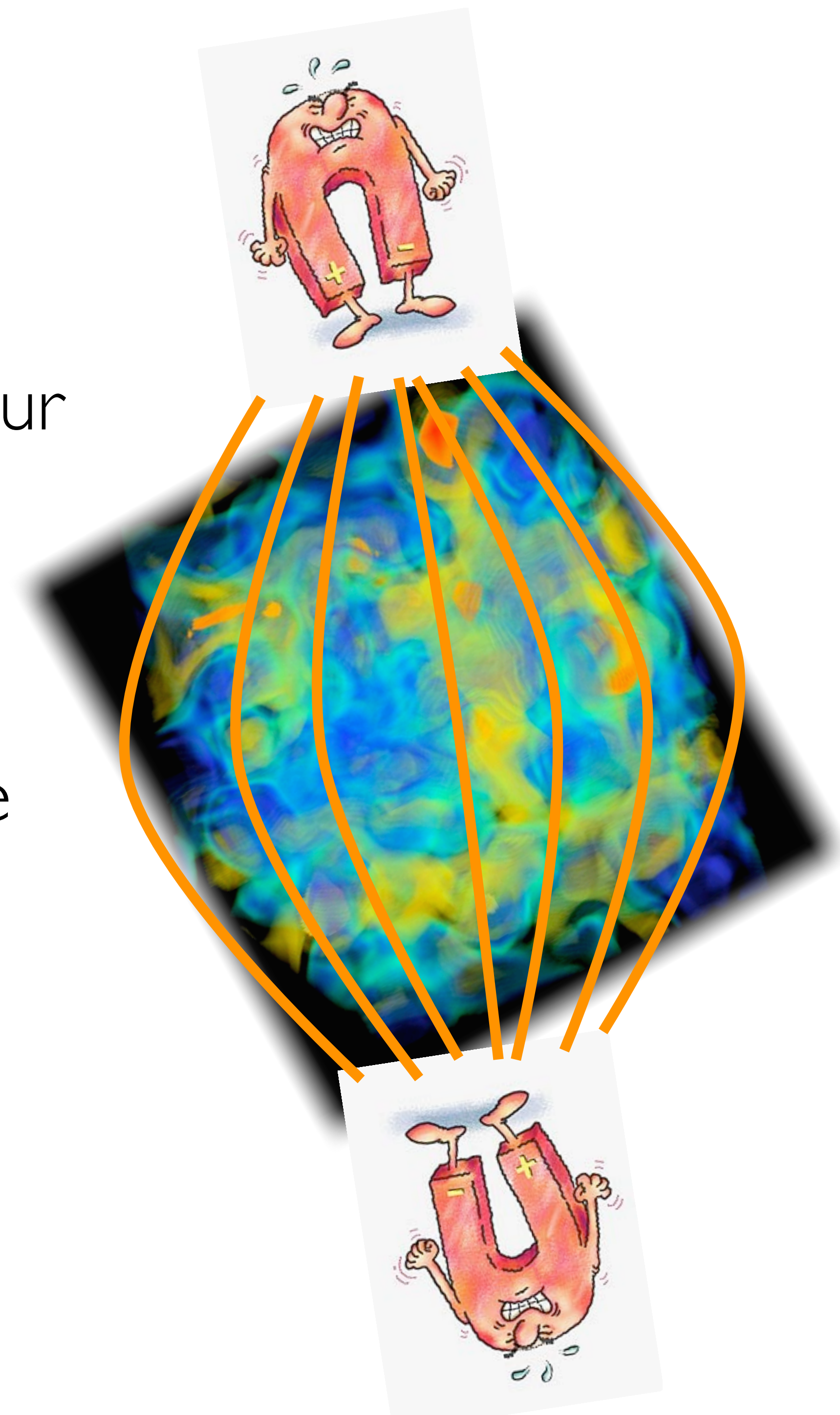
- Compare with FV lattice calculations to extract α_1 and α_2
  (previous calculation with HW Lin for <x> operator)

Background field method

- Hadron/nuclear two-point functions are modified in presence of fixed eternal fields
- Eg: fixed B field: modified exponential behaviour

$$E(\mathbf{B}) = M + \frac{|Q\,e\,\mathbf{B}|}{2M} - \boldsymbol{\mu}\cdot\mathbf{B} - 2\pi\beta_{M0}\,|\mathbf{B}|^2 - 2\pi\beta_{M2}T_{ij}B_iB_j + \ldots$$

- QCD spectroscopy with multiple fields enable extraction of coefficients of response
  - Eg: magnetic moments, polarisabilities, …
  - Not restricted to simple EM fields (axial, twist-2,…)



Nuclear magnetic moments

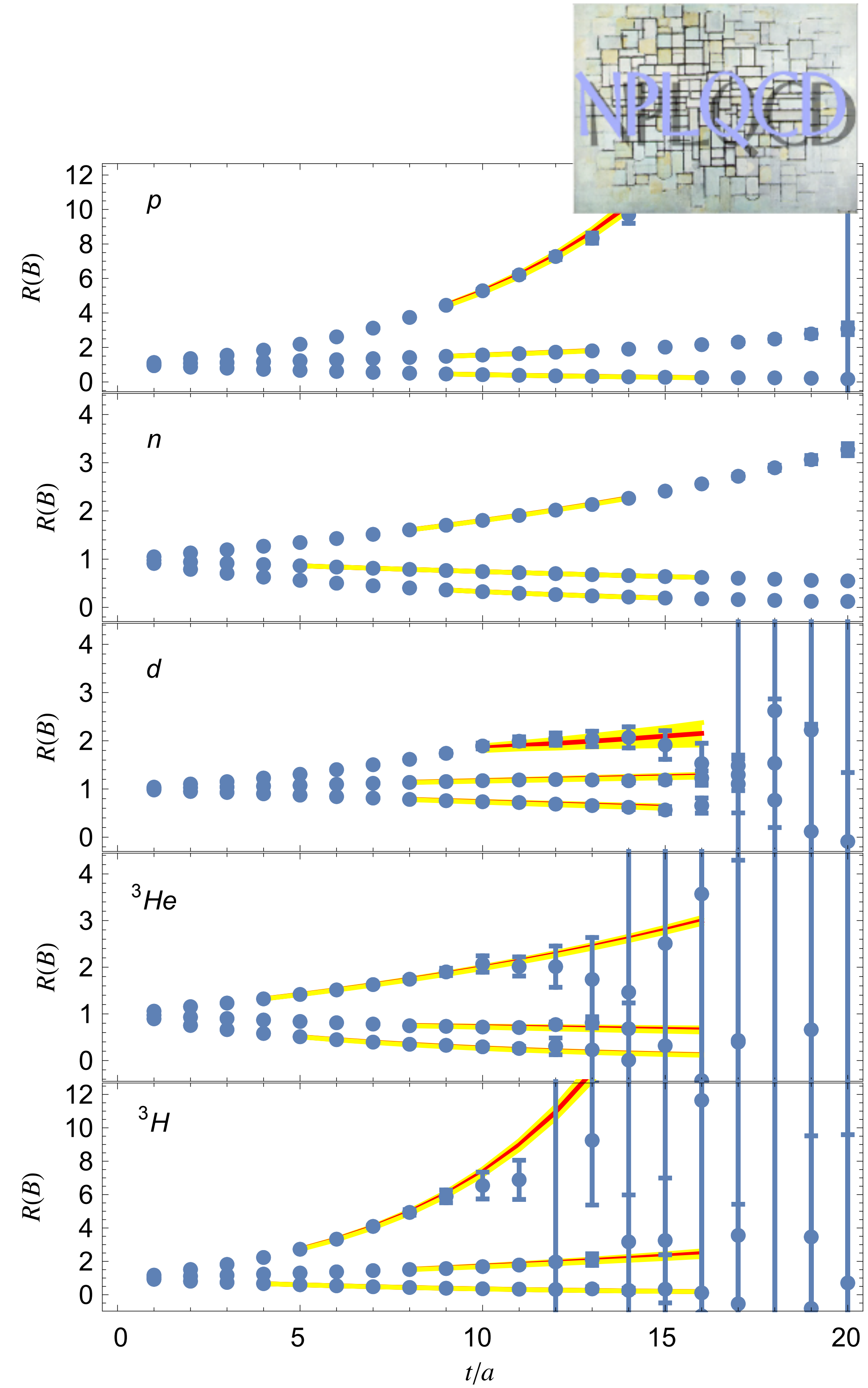
- Magnetic moments from spin splittings

$$\delta E^{(B)} \equiv E^{(B)}_{+j} - E^{(B)}_{-j} = -2\mu|\mathbf{B}| + \gamma|\mathbf{B}|^3 + \ldots$$

- Extract splittings from ratios of correlation functions

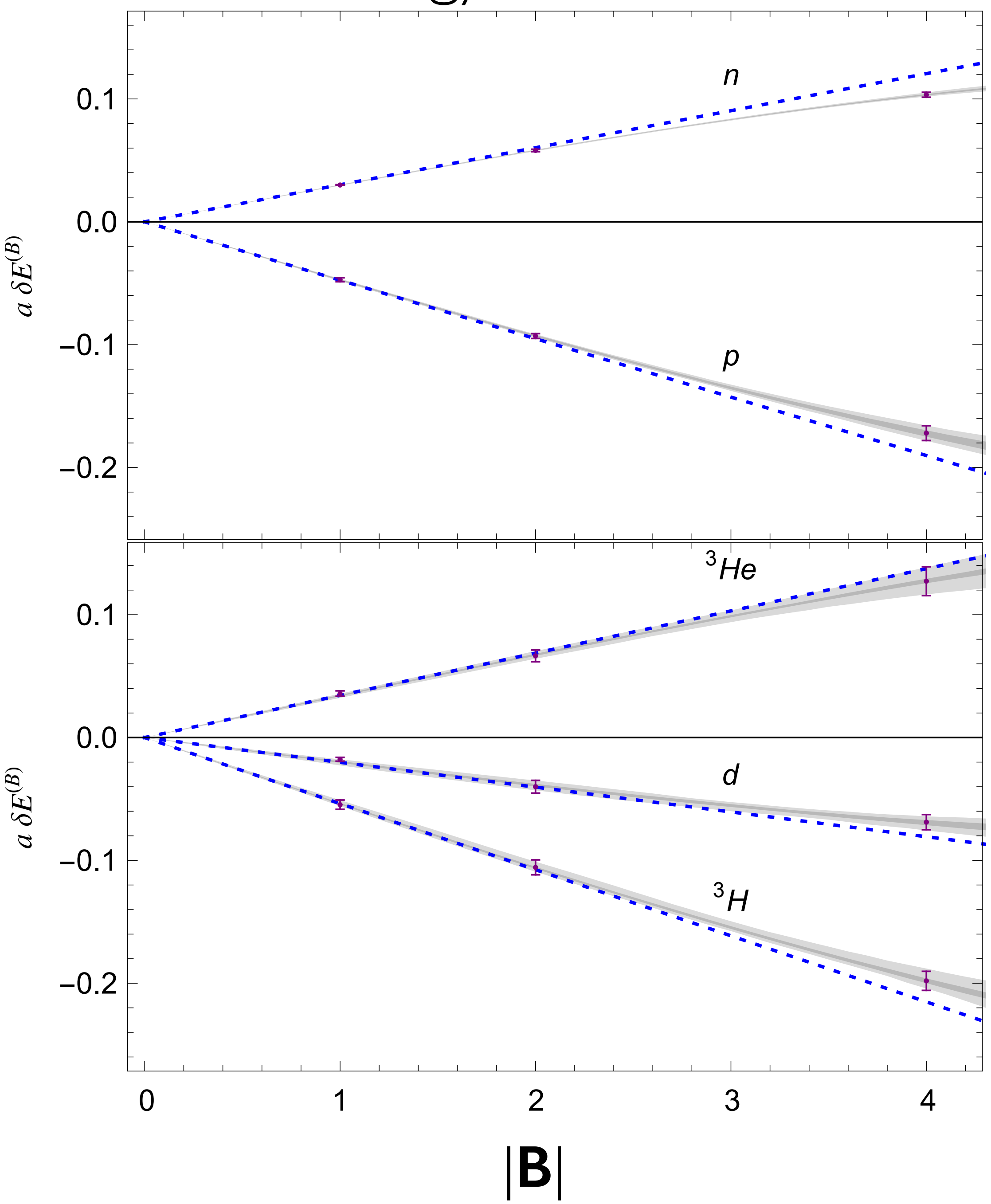
$$R(B) = \frac{C^{(B)}_{j}(t)\; C^{(0)}_{-j}(t)}{C^{(B)}_{-j}(t)\; C^{(0)}_{j}(t)} \;\overset{t\to\infty}{\longrightarrow}\; Z e^{-\delta E^{(B)} t}$$

- Careful to be in single exponential region of each correlator



[NPLQCD 1409.3556, PRL to appear]

Energy shift vs B



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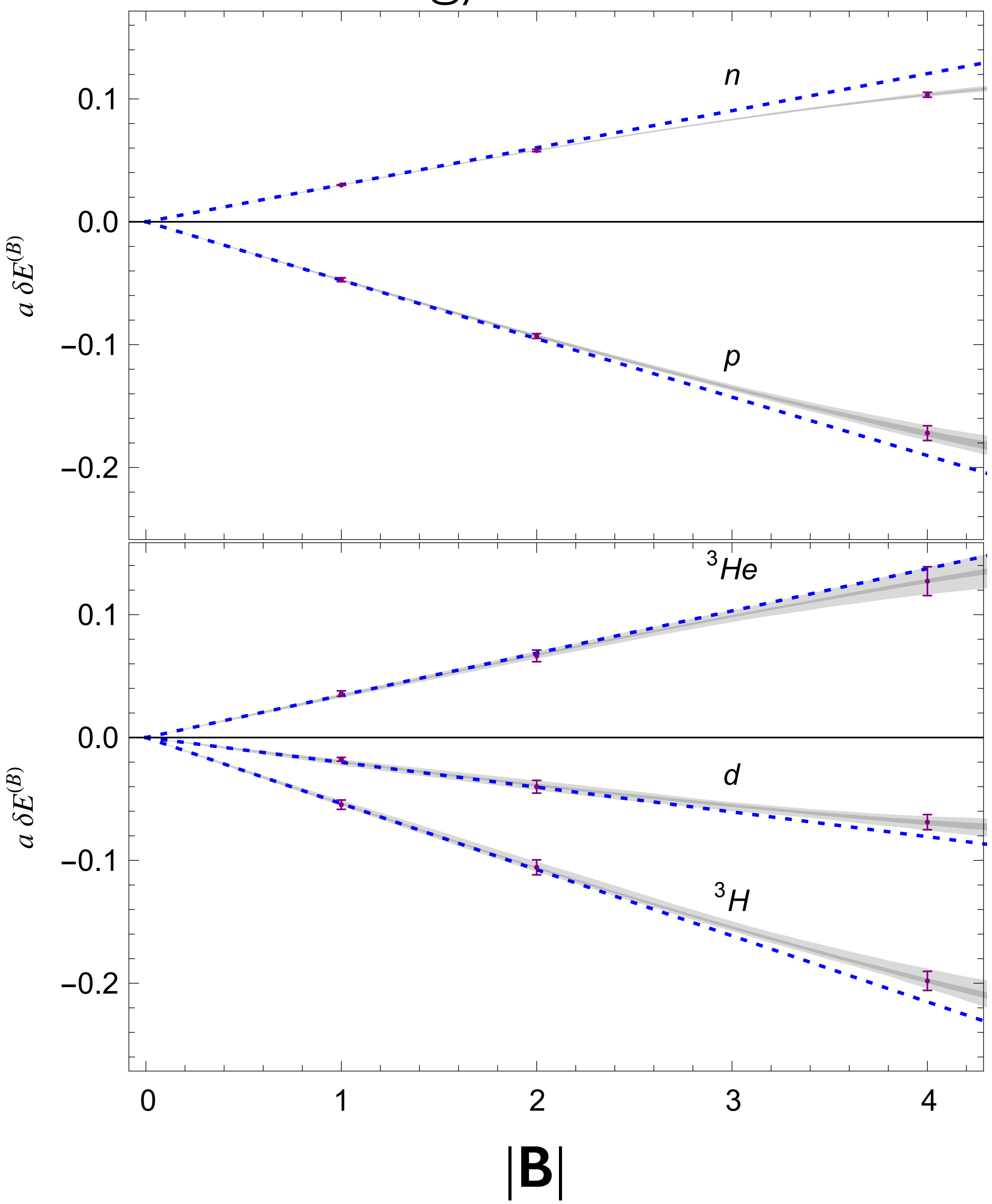
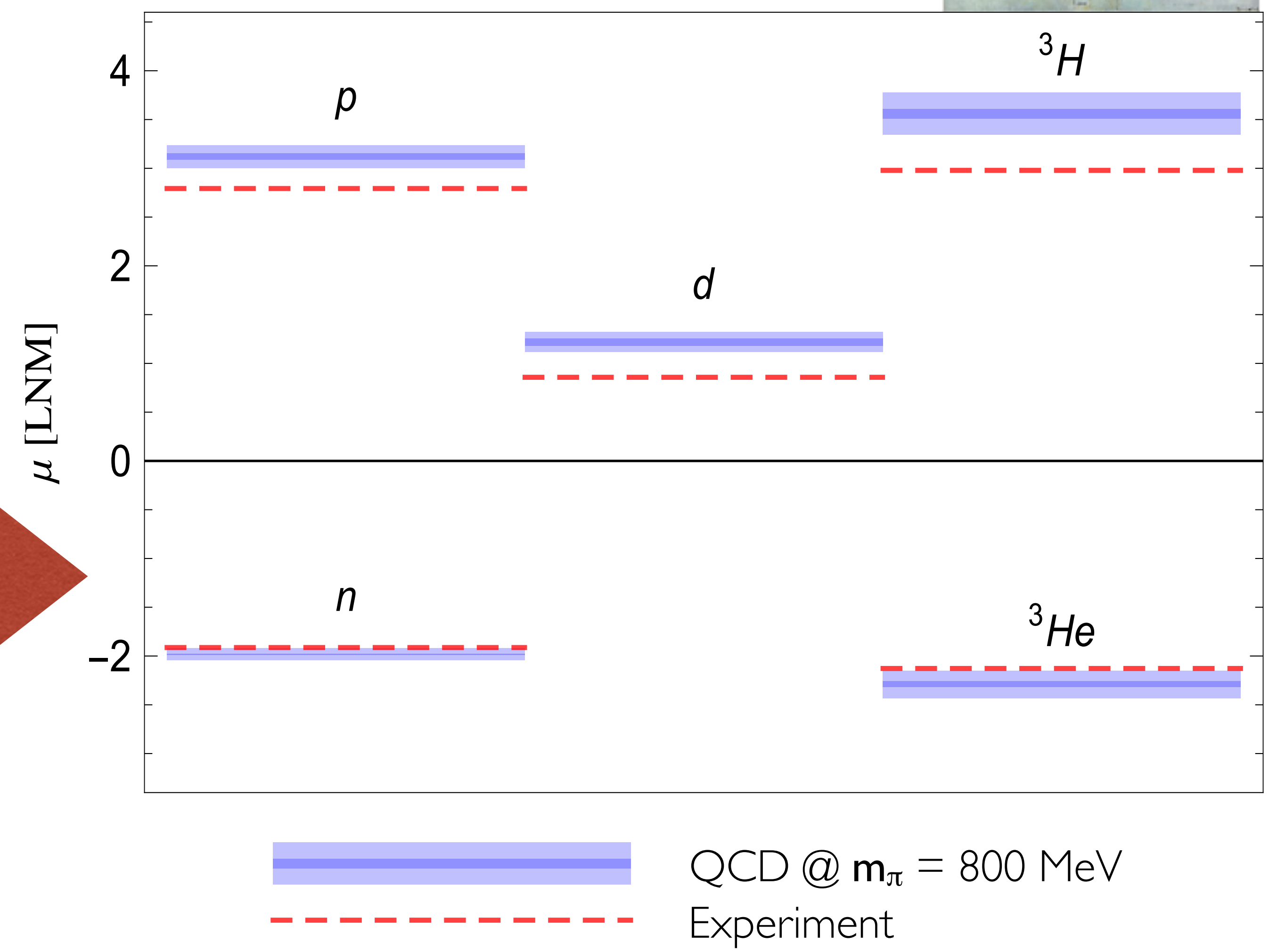




	n	p	d	3	3
μ	-1.98(1)(2)	3.21(3)(6)	1.22(4)(9)	-2.29(3)(12)	3.56(5)(18)

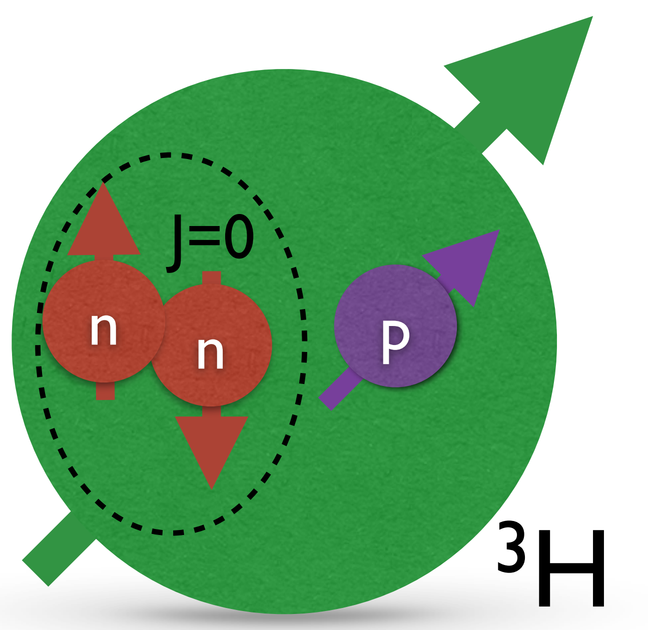
In units of appropriate nuclear magnetons (heavy M_N)

[NPLQCD 1409.3556, PRL to appear]

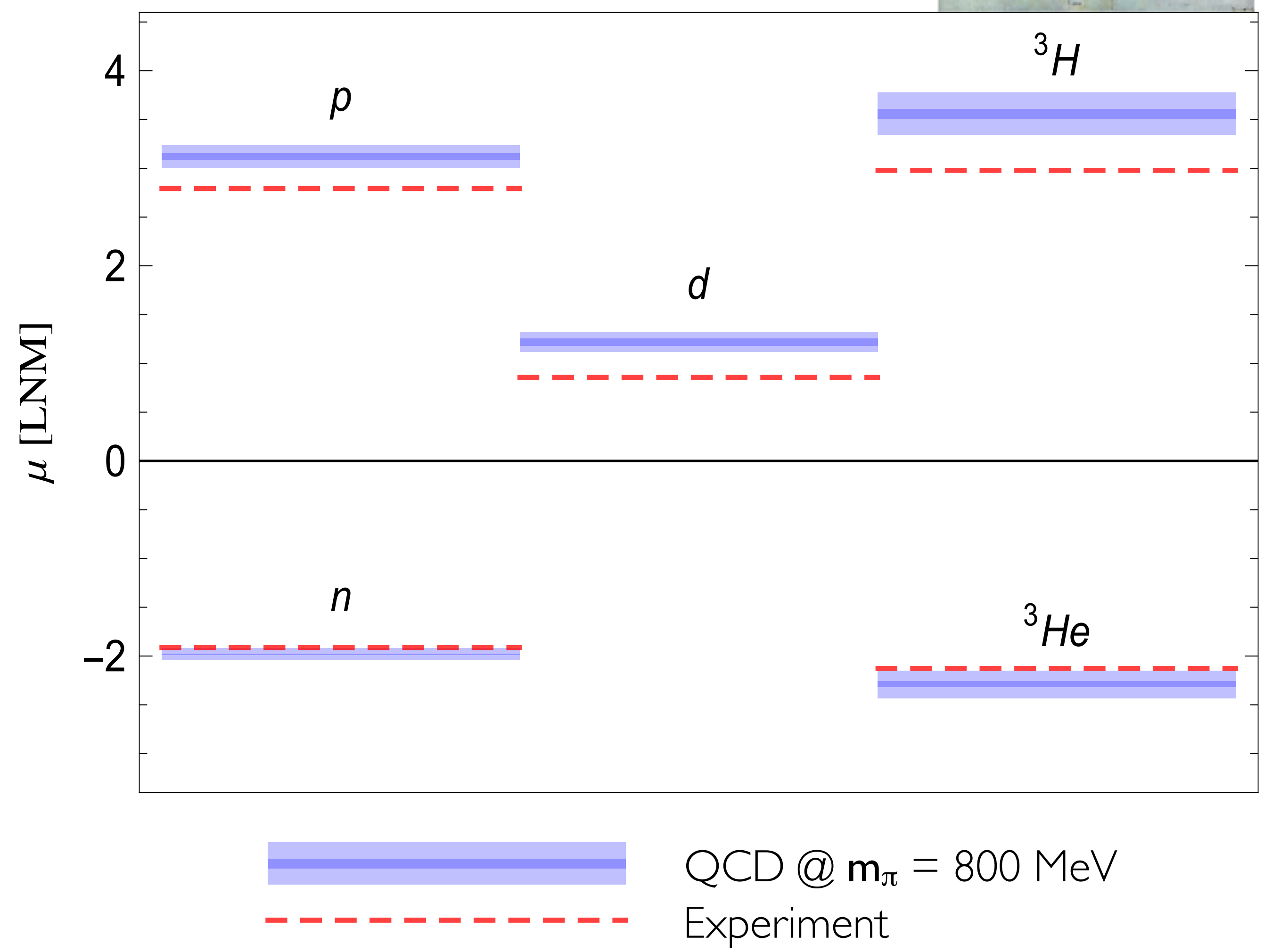
Nuclear magnetic moments

- Numerical values are surprisingly interesting
- Shell model expectations

$$\mu_d = \mu_p + \mu_n$$
$$\mu_{^3\mathrm{H}} = \mu_p$$
$$\mu_{^3\mathrm{He}} = \mu_n$$



- Lattice results appear to suggest heavy quark nuclei are shell-model like!



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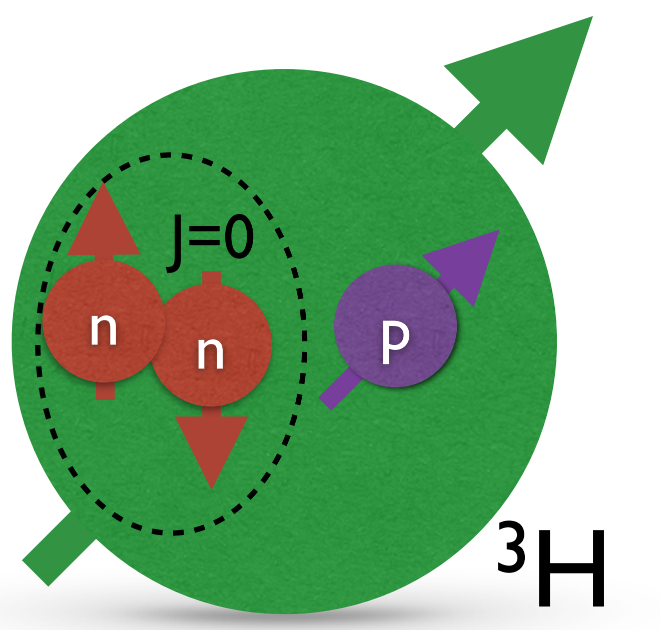

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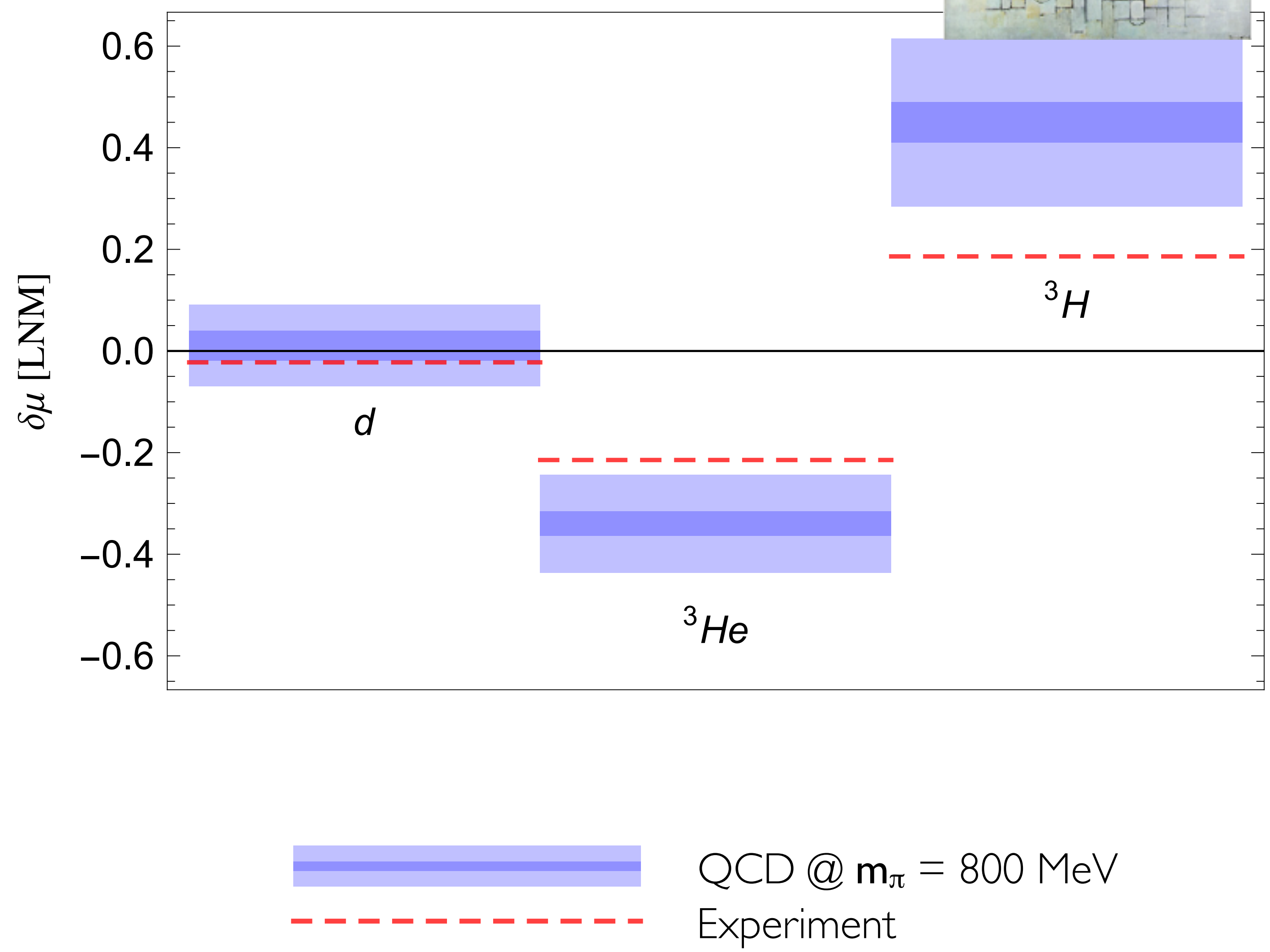


	d	3	3
δμ	0.01(3)(7)	-0.34(2)(9)	0.45(4)(16)

Difference from NSM expectation

[NPLQCD 1409.3556, PRL to appear]

Nuclear sigma terms

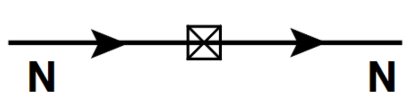
- One possible DM interaction is through scalar exchange

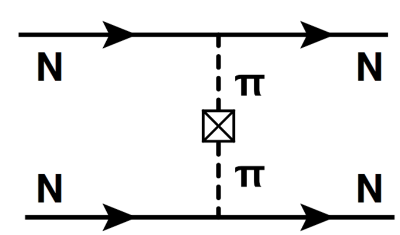
$$\mathcal{L} = \frac{G_F}{2} \sum_q a_S^{(q)} (\overline{\chi}\, \chi)(\overline{q}\, q)$$

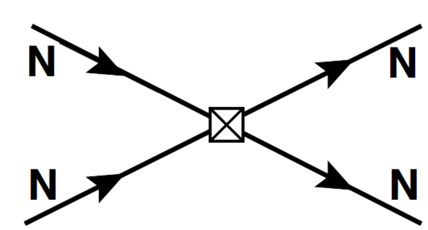
- Accessible via Feynman-Hellman theorem
- At hadronic/nuclear level

$$\mathcal{L} \;\to\; G_F\, \overline{\chi}\chi\, \Big(\frac{1}{4}\langle 0|\overline{q}q|0\rangle\; \mathrm{Tr}\left[a_S\Sigma^\dagger + a_S^\dagger\Sigma\right] \;+\; \frac{1}{4}\langle N|\overline{q}q|N\rangle N^\dagger N \mathrm{Tr}\left[a_S\Sigma^\dagger + a_S^\dagger\Sigma\right]$$
$$-\;\frac{1}{4}\langle N|\overline{q}\tau^3 q|N\rangle \left(N^\dagger N \mathrm{Tr}\left[a_S\Sigma^\dagger + a_S^\dagger\Sigma\right] \;-\; 4N^\dagger a_{S,\xi} N\right) \;+\; ...\Big)$$

  - Contributions:





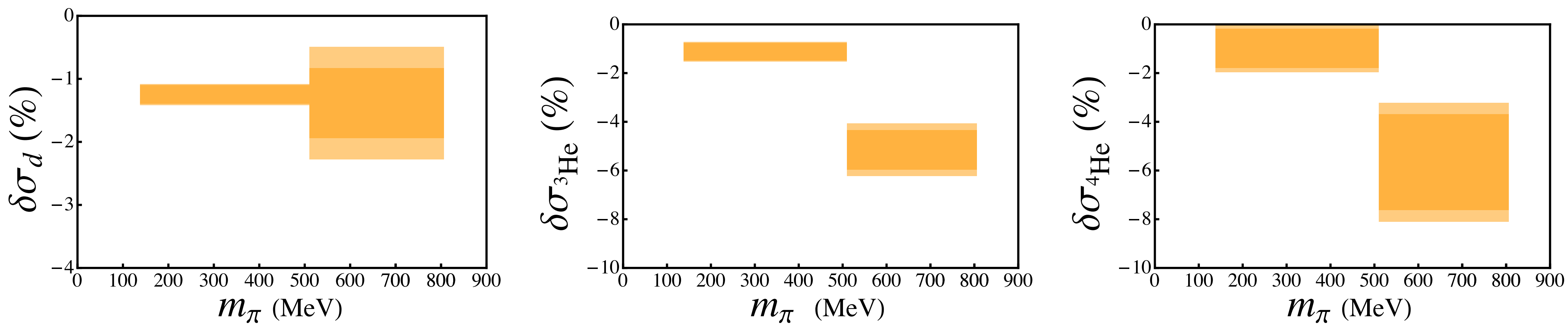


Nuclear sigma terms

- Previous work suggested scalar dark matter couplings to nuclei have O(50%) uncertainty arising from MECs [Prezeau et al 2003]
- Quark mass dependence of nuclear binding energies bounds such contributions

$$\delta\sigma_{Z,N} = \frac{\langle Z,N(\mathrm{gs})|\ \overline{u}u + \overline{d}d|Z,N(\mathrm{gs})\rangle}{A\ \langle N|\ \overline{u}u + \overline{d}d|N\rangle} - 1 = -\frac{1}{A\sigma_N}\frac{m_\pi}{2}\frac{d}{dm_\pi}B_{Z,N}$$

  - Lattice calculations + physical point suggest such contributions are O(10%) or less for light nuclei (A<4)



[NPLQCD PRD to appear]

QCD for nuclear physics

- Nuclei are under serious study directly from QCD
  - Spectroscopy of light nuclei and exotic nuclei (strange, charmed, …)
  - Nuclear properties/matrix elements
- Prospect of a quantitative connection to QCD makes this a very exciting time for nuclear physics
  - Critical role in current and upcoming intensity frontier experimental program
- Learn many interesting things about nuclear physics along the way

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