The hadronic vacuum polarization contribution to the muon anomalous magnetic moment

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RBC and UKQCD Collaborations

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The anomalous magnetic moment

Potential of particle in magnetic field

$$V(x) = -\vec{\mu} \cdot \vec{B}(x) \tag{1}$$

with

$$\vec{\mu} = g \left(\frac{e}{2m} \right) \vec{S}\,, \tag{2}$$

where $\vec{S}$ is the spin of the particle.

Relativistic description with classical photon (Dirac) yields

$$g = 2 \tag{3}$$

but taking into account QFT yields non-zero anomalous magnetic moment

$$a = (g-2)/2\,. \tag{4}$$

The anomalous magnetic moment

These anomalous moments are measured very precisely. For the electron (Hanneke, Fogwell, Gabrielse 2008)

$$a_e = 0.00115965218073(28) \tag{5}$$

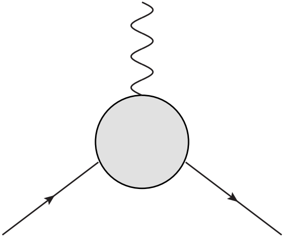
yielding the currently most precise determination of the fine structure constant

$$\alpha = 1/137.035999157(33) \tag{6}$$

via a 5-loop QED computation (Aoyama, Hayakawa, Kinoshita, Nio 2015).

The anomalous magnetic moment

$a \neq 0$ requires QFT: a can be expressed in terms of scattering of particle off a classical photon background



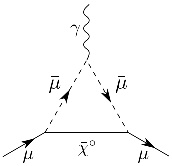
For external photon index μ with momentum q the scattering amplitude can be generally written as

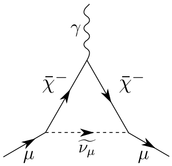
$$(-ie)\left[\gamma_\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q^\nu}{2m}F_2(q^2)\right] \tag{7}$$

with $F_2(0) = a$.

The muon anomalous magnetic moment

The muon anomalous magnetic moment promises to be useful to discover new physics beyond the standard model (SM) of particle physics.
In general, new physics contributions to a_ℓ are given by $a_\ell - a_\ell^{\mathrm{SM}} \propto (m_\ell^2/\Lambda_{\mathrm{NP}}^2)$ for lepton $\ell = e, \mu, \tau$ and new physics scale Λ_{NP}.
With $\ell = \tau$ being experimentally inaccessible, $\ell = \mu$ promises good sensitivity to new physics.





Example contributions: one-loop MSSM neutralino/smuon and chargino/sneutrino contributions to a_μ

The muon anomalous magnetic moment

Currently a tension of more than 3σ exists:

Total SM prediction	11 659 181.5 (4.9)
BNL E821 result	11 659 209.1 (6.3)

$$a_\mu^{\mathrm{EXP}} - a_\mu^{\mathrm{SM}} = (27.6 \pm 8.0) \times 10^{-10} \tag{8}$$

And a new experiment (Fermilab E989) promises a 4$\times$ reduction in experimental uncertainty:

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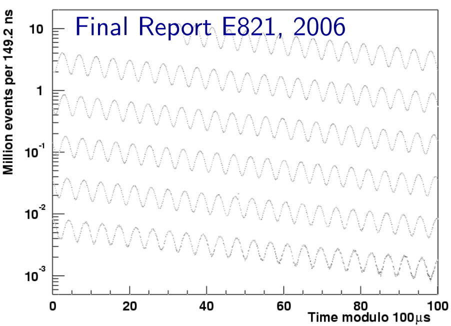


$$\vec{\omega}_a = -\frac{q}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta}\times\vec{E}}{c}\right]$$

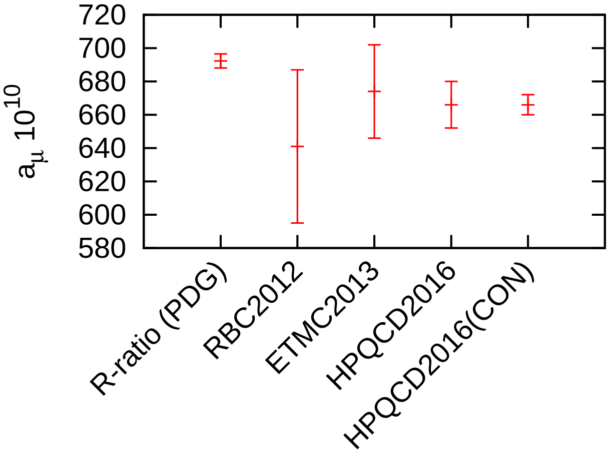
Hadronic contributions to a_μ

Contribution	Value $\times 10^{10}$	Uncertainty $\times 10^{10}$
QED (5 loops)	11 658 471.895	0.008
EW	15.4	0.1
HVP LO	692.3	**4.2**
HVP NLO	-9.84	0.06
HVP NNLO	1.24	0.01
Hadronic light-by-light	10.5	**2.6**
Total SM prediction	11 659 181.5	4.9
BNL E821 result	11 659 209.1	6.3
Fermilab E989 target		$\approx$ **1.6**

A reduction of uncertainty for HVP and HLbL is needed. For HLbL only model estimations exist. $\Rightarrow$ First-principles non-perturbative determination desired.

The hadronic vacuum polarization from the lattice

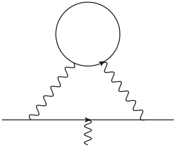
Overview of first-principles lattice QCD results



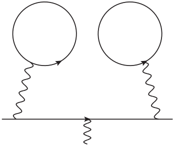
On-going efforts by ETMC, HPQCD+MILC, RBC+UKQCD, ...

HPQCD2016(CON) neglects the systematic error estimates for the HVP disconnected and QED/isospin-breaking corrections.

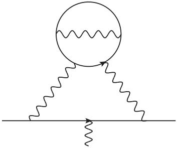
The Hadronic Vacuum Polarization



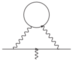
Quark-connected piece with $> 90\%$ of the contribution with by far dominant part from up and down quark loops



Quark-disconnected piece with $\approx 1.5\%$ of the contribution (1/5 suppression already through charge factors); arXiv:1512.09054



QED and isospin-breaking corrections, estimated at the few-per-cent level

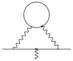


HVP quark-connected contribution

Biggest challenge is to control statistics and potentially large finite-volume errors (Estimated at O(10%) Aubin et al. 2015)

Finite-volume errors are exponentially suppressed in the simulation volume but seem to be sizeable in QCD boxes with $m_\pi L = 4$

Statistics: for strange and charm solved issue, for up and down quarks existing methodology (such as HPQCD moments approach) less effective



HVP quark-connected contribution

Starting from

$$\sum_x e^{iqx} \langle J_\mu(x) J_\nu(0) \rangle = (\delta_{\mu\nu} q^2 - q_\mu q_\nu) \Pi(q^2) \tag{9}$$

with vector current $J_\mu(x) = i \sum_f Q_f \overline{\Psi}_f(x) \gamma_\mu \Psi_f(x)$ and using the subtraction prescription of Bernecker-Meyer 2011

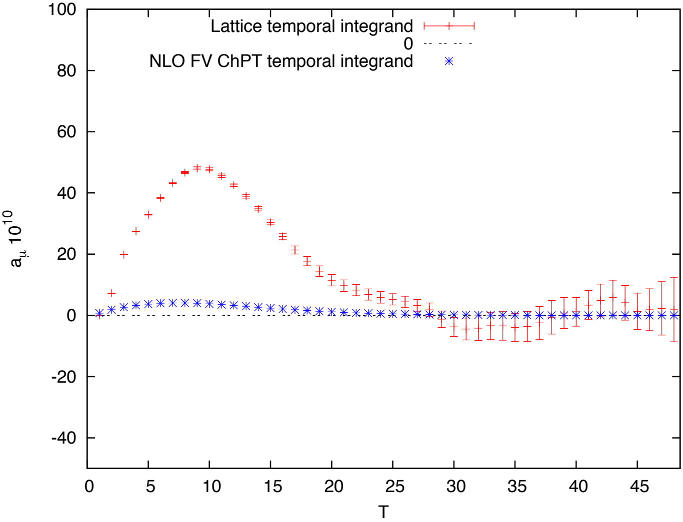
$$\Pi(q^2) - \Pi(q^2 = 0) = \sum_t \left(\frac{\cos(qt) - 1}{q^2} + \frac{1}{2} t^2 \right) C(t) \tag{10}$$

with $C(t) = \frac{1}{3} \sum_{\vec{x}} \sum_{j=0,1,2} \langle J_j(\vec{x}, t) J_j(0) \rangle$ we may write

$$a_\mu^{\mathrm{HVP}} = \sum_{t=0}^{\infty} w_t C(t) \,, \tag{11}$$

where w_t captures the QED part of the diagram.

Integrand $w_T C(T)$ for the light-quark connected contribution:

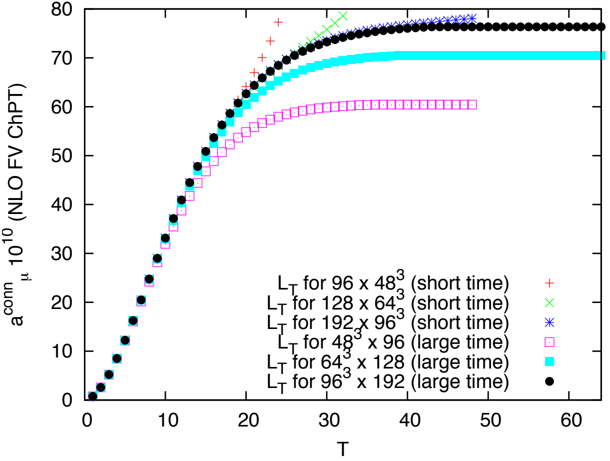


$m_\pi = 140$ MeV, $a = 0.11$ fm (RBC/UKQCD 48^3 ensemble)

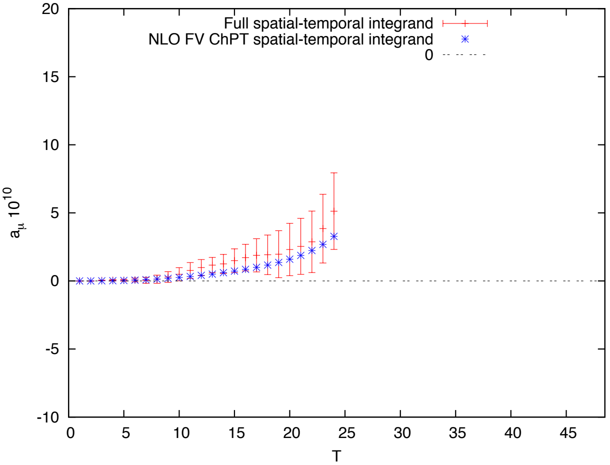
Statistical noise from long-distance region (HPQCD2016 only used lattice data up to 0.5fm–1.5fm)

A closer look at the NLO FV ChPT prediction (1-loop sQED):

We show the partial sum $\sum_{t=0}^{T} w_t C(t)$ for different geometries and volumes:

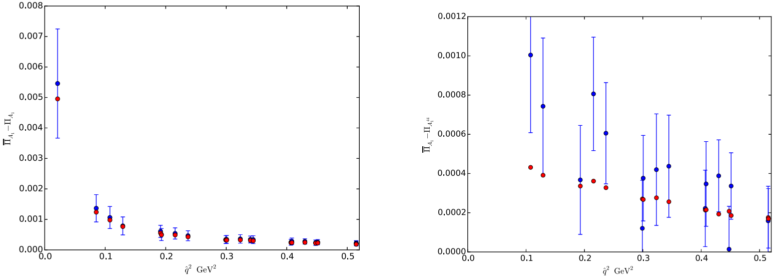


Compare difference of integrand of $48 \times 48 \times 96 \times 48$ (spatial) and $48 \times 48 \times 48 \times 96$ (temporal) geometries with NLO FV ChPT:



$m_\pi = 140$ MeV, $a = 0.11$ fm (RBC/UKQCD 48^3 ensemble)

Similar agreement from Aubin et al. 2015 (arXiv:1512.07555v2)

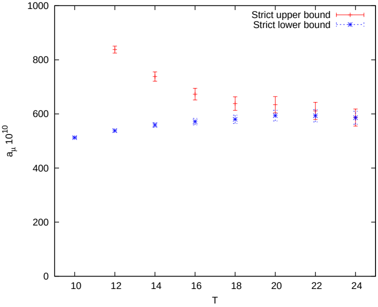


MILC lattice data with $m_\pi L = 4.2$, $m_\pi \approx 220$ MeV; Plot difference of $\Pi(q^2)$ from different irreps of 90-degree rotation symmetry of spatial components versus NLO FV ChPT prediction (red dots)

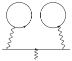
While the absolute value of a_μ is poorly described by the two-pion contribution, the volume dependence may be described sufficiently well to use ChPT to control FV errors at the 1% level; this needs further scrutiny

Regarding statistics:

It is potentially helpful to define stochastic estimator for strict upper and lower bounds of a_μ which has reduced statistical fluctuations C.L. et al. 2016



up and down loop shown here: data shown here is from early stages of computation with 5% statistical error, currently at around 2% statistical error. Within the next year our current setup can produce a continuum limit with 1% statistical error.



HVP quark-disconnected contribution

First results at physical pion mass with a statistical signal
RBC/UKQCD arXiv:1512.09054

Statistics is clearly the bottleneck

New stochastic estimator allowed us to get result

$$a_\mu^{\mathrm{HVP\ (LO)\ DISC}} = -9.6(3.3)_{\mathrm{stat}}(2.3)_{\mathrm{sys}} \times 10^{-10} \tag{12}$$

from 20 configurations at physical pion mass and 45 propagators/configuration.

Our setup (arXiv:1512.09054):

$$C(t) = \frac{1}{3V} \sum_{j=0,1,2} \sum_{t'} \langle \mathcal{V}_j(t+t') \mathcal{V}_j(t') \rangle_{\mathrm{SU(3)}} \tag{13}$$

where V stands for the four-dimensional lattice volume,
$\mathcal{V}_\mu = (1/3)(\mathcal{V}_\mu^{u/d} - \mathcal{V}_\mu^s)$, and

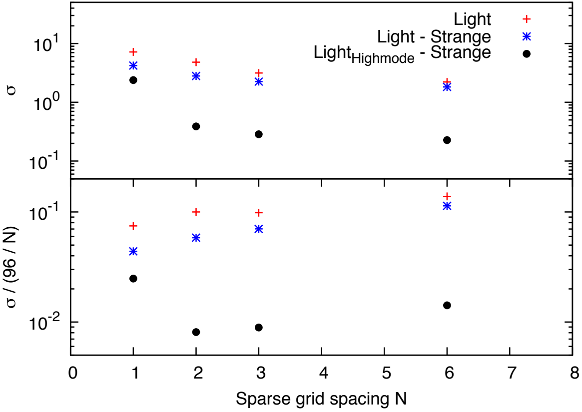
$$\mathcal{V}_\mu^f(t) = \sum_{\vec{x}} \mathrm{Im}\,\mathrm{Tr}[D^{-1}_{\vec{x},t;\vec{x},t}(m_f)\gamma_\mu]\,. \tag{14}$$

We separate 2000 low modes (up to around m_s) from light quark propagator as $D^{-1} = \sum_n v^n (w^n)^\dagger + D^{-1}_{\mathrm{high}}$ and estimate the high mode stochastically and the low modes as a full volume average Foley 2005.

We use a sparse grid for the high modes similar to Li 2010 which has support only for points x_μ with $(x_\mu - x_\mu^{(0)}) \bmod N = 0$; here we additionally use a random grid offset $x_\mu^{(0)}$ per sample allowing us to stochastically project to momenta.

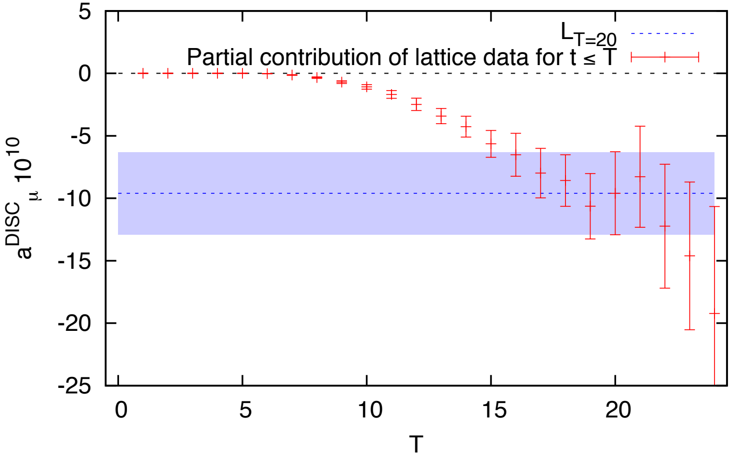
Combination of both ideas is crucial for noise reduction at physical pion mass!

Fluctuation of $\mathcal{V}_\mu$ (σ):



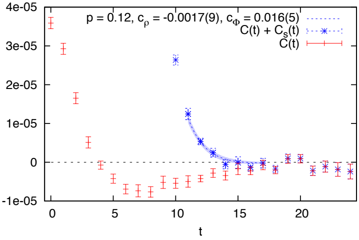
Since $C(t)$ is the autocorrelator of $\mathcal{V}_\mu$, we can create a stochastic estimator whose noise is potentially reduced linearly in the number of random samples, hence the normalization in the lower panel

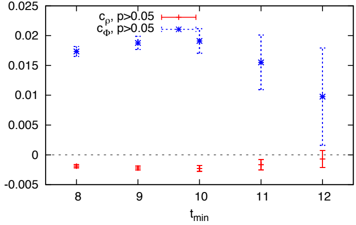
Result for partial sum $L_T = \sum_{t=0}^{T} w_t C(t)$:



For $t \geq 15$ $C(t)$ is consistent with zero but the stochastic noise is t-independent and $w_t \propto t^4$ such that it is difficult to identify a plateau region based only on this plot

Resulting correlators and fit of $C(t) + C_s(t)$ to $c_\rho e^{-E_\rho t} + c_\phi e^{-E_\phi t}$ in the region $t \in [t_{\min}, \ldots, 17]$ with fixed energies $E_\rho = 770$ MeV and $E_\phi = 1020$. $C_s(t)$ is the strange connected correlator.

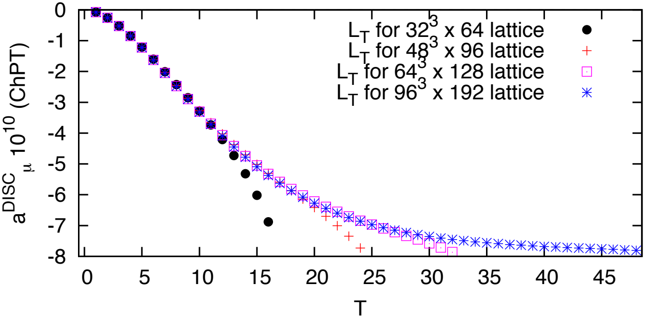




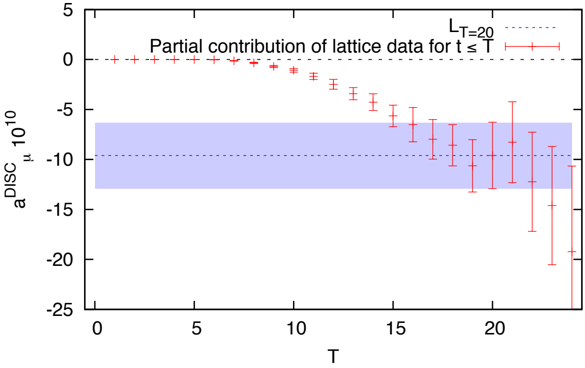
We fit to $C(t) + C_s(t)$ instead of $C(t)$ since the former has a spectral representation.

We could use this model alone for the long-distance tail to help identify a plateau but it would miss the two-pion tail

We therefore additionally calculate the two-pion tail for the disconnected diagram in ChPT:

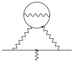


We then pick a point in the potential plateau region such as $T = 20$ and use a combined estimate of the resonance model and the two-pion tail to estimate $\sum_{t=T+1}^{\infty} w_t C(t)$ as a systematic uncertainty.



Combined with an estimate of discretization errors, we find

$$a_\mu^{\mathrm{HVP\ (LO)\ DISC}} = -9.6(3.3)_{\mathrm{stat}}(2.3)_{\mathrm{sys}} \times 10^{-10}\,. \qquad (15)$$



HVP QED corrections

Largely unexplored but finite-volume errors are likely substantial

New methods with potential to control large finite-volume errors in lattice QCD+QED simulations may prove useful (C^* boundary conditions Lucini et al. 2015, massive QED Endres et al. 2015, QED_∞ C.L. et al. Lattice 2015)

We are actively working on this measurement using technology similar to our on-going hadronic light-by-light calculation (next talk by Luchang)

- First-principles determination of the HVP contribution comparable with Fermilab E989 uncertainty (0.3% uncertainty on HVP) is very challenging

- Substantial progress in the last year both for the HVP light connected and disconnected contributions

- Active effort on necessary sub-leading contributions such as QED/isospin-breaking corrections

First-principles predictions for the HVP on time-scale of and with errors comparable to Fermilab E989 appear possible!

Thank you

Experimental setup: muon storage ring with tuned momentum of muons to cancel leading coupling to electric field

$$\vec{\omega}_a = -\frac{q}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta}\times\vec{E}}{c}\right]$$

Because of parity violation in weak decay of muon, a correlation between muon spin and decay electron direction exists, which can be used to measure the anomalous precession frequency ω_a: