Twist-3 spin observables and multi-parton correlations in *ep* collisions at an EIC

Daniel Pitonyak

Penn State University-Berks, Reading, PA

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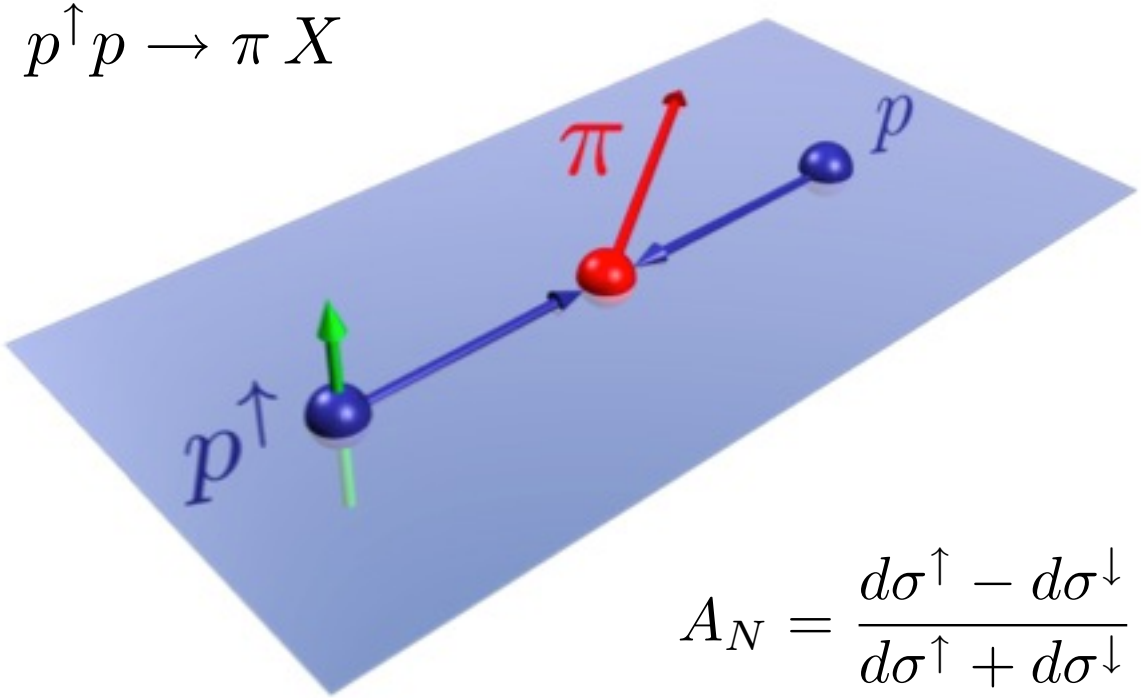
Outline

- Background
  - Transverse single-spin asymmetries (TSSAs): a 40 year-old puzzle
  - Collinear twist-3 factorization
- Transverse spin observables in *ep* collisions
  - Motivation for analyzing them
  - A specific example: A_{UTU} $\ell\, N^{\uparrow} \to \pi\, X$
  - A_{LTU} , A_{UUT} , A_{LUT} spin asymmetries
- Summary and outlook

Background

- TSSAs: a 40 year-old puzzle

$$p^{\uparrow} p \to \pi X$$



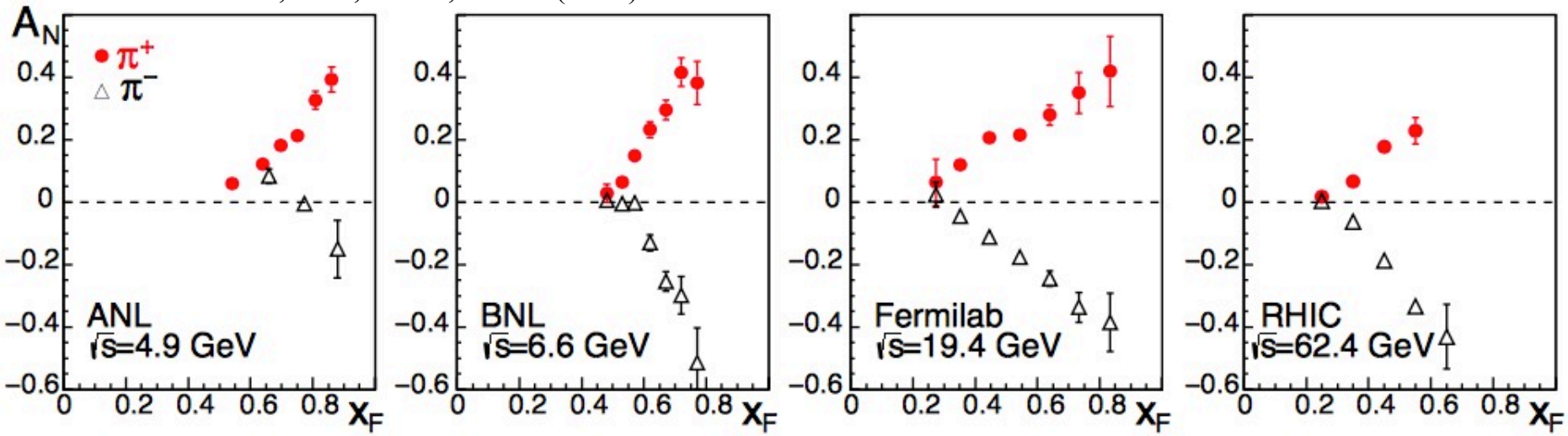
$$A_N = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}} = \frac{d\sigma_L - d\sigma_R}{d\sigma_L + d\sigma_R}$$

Data available from RHIC (BRAHMS, PHENIX, STAR), FNAL (E704, E581), AGS, and ANL

Background

- TSSAs: a 40 year-old puzzle

Plot from Aidala, Bass, Hasch, Mallot (2013)



1976 ⟶

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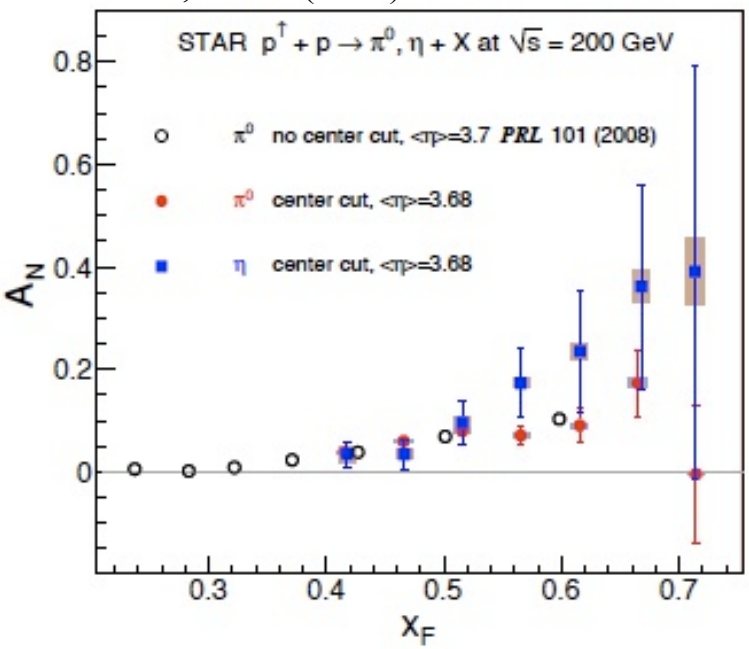
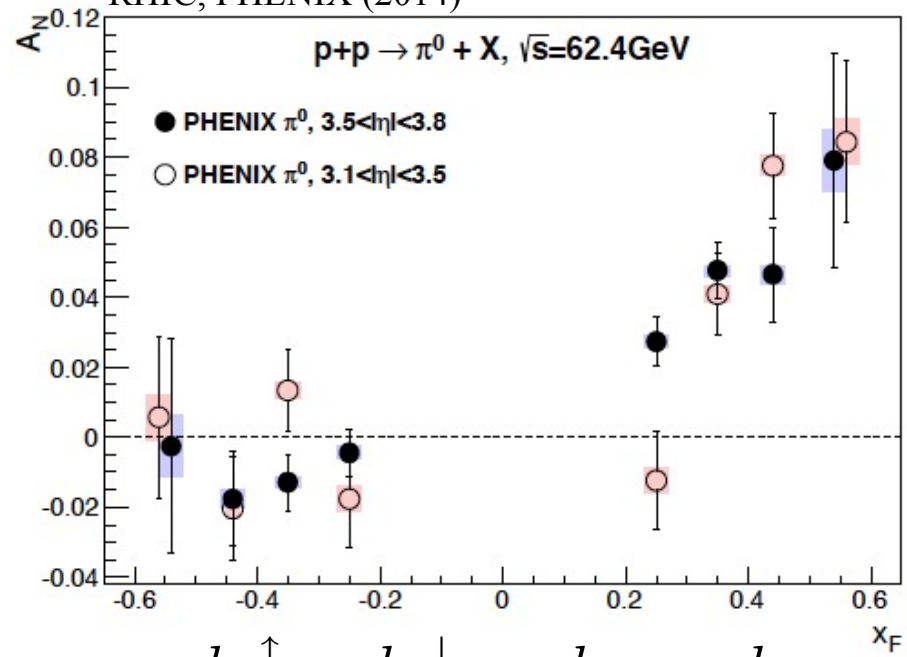
Background

- TSSAs: a 40 year-old puzzle

RHIC, STAR (2012)



RHIC, PHENIX (2014)



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- Collinear twist-3 factorization

$$d\sigma = H \otimes f_{a/A(3)} \otimes f_{b/B(2)} \otimes D_{c/C(2)} + H' \otimes f_{a/A(2)} \otimes f_{b/B(3)} \otimes D_{c/C(2)} + H'' \otimes f_{a/A(2)} \otimes f_{b/B(2)} \otimes D_{c/C(3)}$$

A + B -> C + X

-One hadron T polarized (others U or L)

-One large scale ($P_{C,T}$)

- Collinear twist-3 factorization

$$
\begin{aligned}
d\sigma = \; & H \otimes f_{a/A(3)} \otimes f_{b/B(2)} \otimes D_{c/C(2)} \\
+ \; & H' \otimes f_{a/A(2)} \otimes f_{b/B(3)} \otimes D_{c/C(2)} \\
+ \; & H'' \otimes f_{a/A(2)} \otimes f_{b/B(2)} \otimes D_{c/C(3)}
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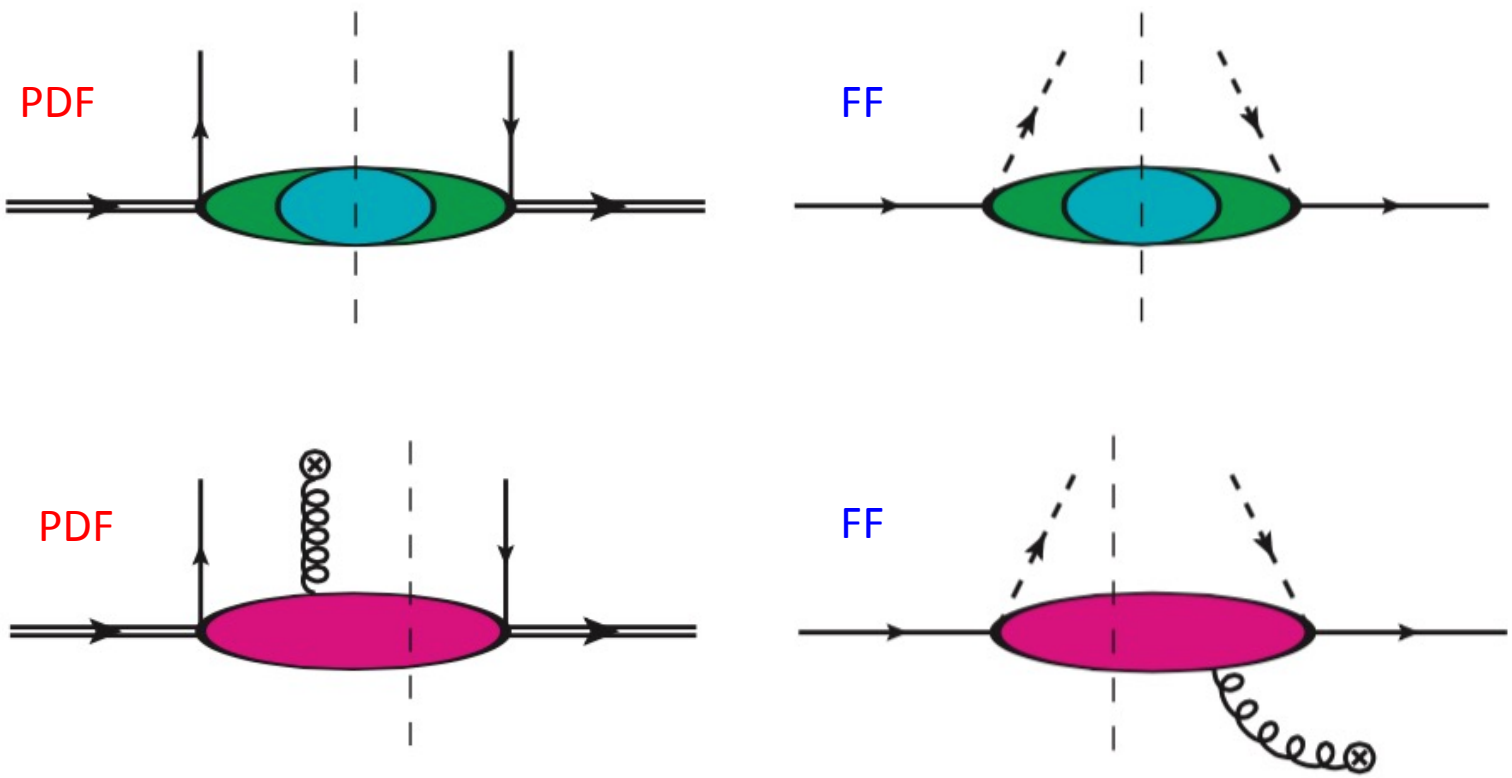

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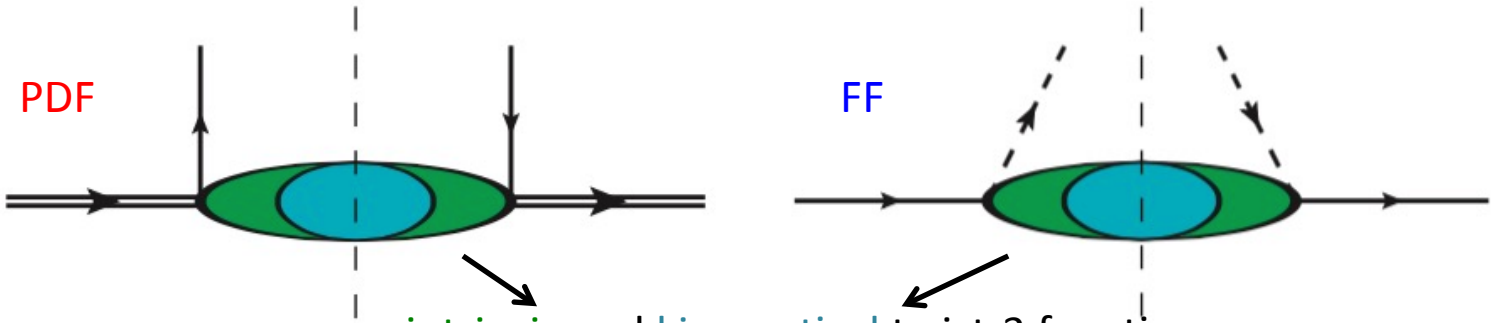
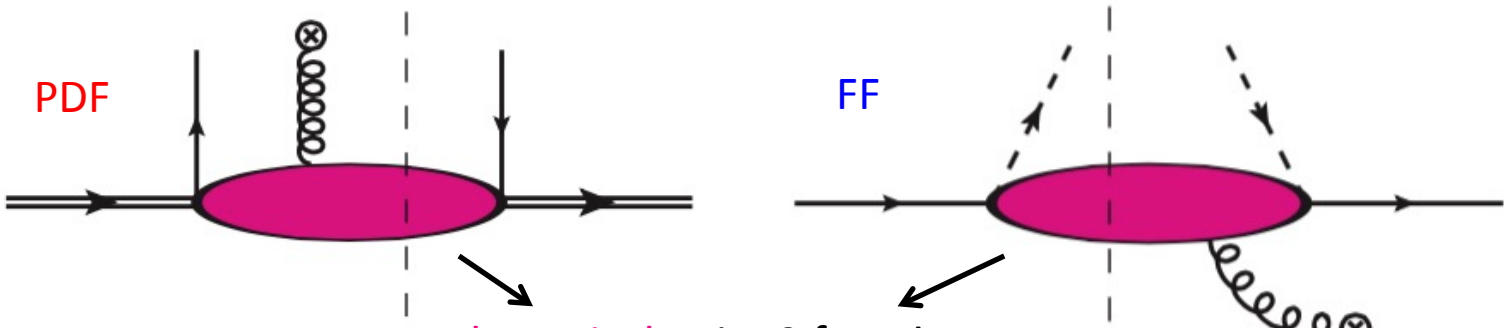
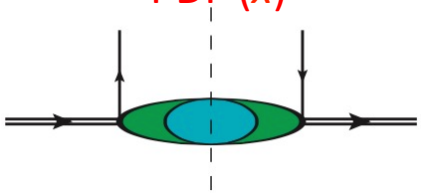
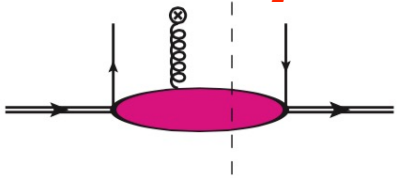
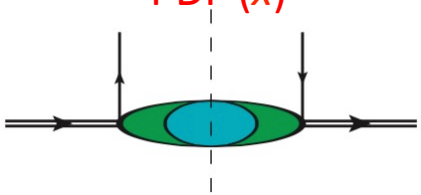
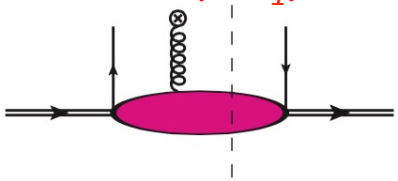


intrinsic and kinematical twist-3 functions



dynamical twist-3 functions

Hadron Pol.	PDF (x) 		PDF (x, x_1) 
	intrinsic	kinematical	dynamical
U	e	$h_1^{\perp(1)}$	H_{FU}
L	h_L	$h_{1L}^{\perp(1)}$	H_{FL}
T	g_T	$f_{1T}^{\perp(1)}$, $g_{1T}^{\perp(1)}$	F_{FT}, G_{FT}

Hadron Pol.	PDF (x)		PDF (x, x_1)
			
	intrinsic	kinematical	dynamical
U	e	$h_1^{\perp(1)}$	H_{FU} $h_1^{\perp(1)}(x) \sim H_{FU}(x,x)$
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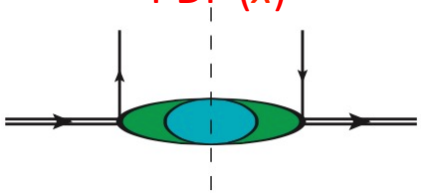
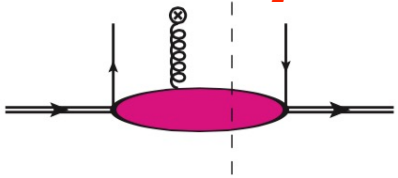
Hadron Pol.	PDF (x) 		PDF (x, x_1) 
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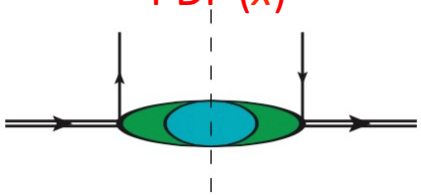
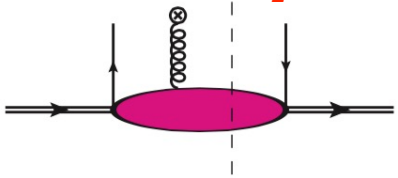
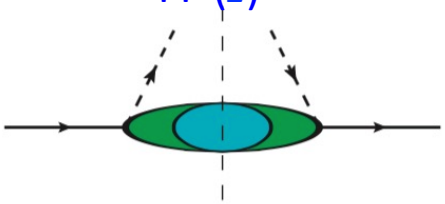
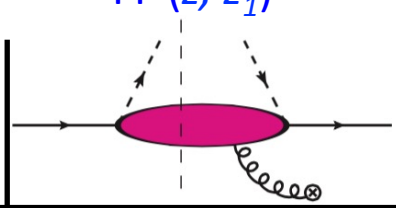
Qiu-Sterman function

Hadron Pol.	PDF (x)		PDF (x, x_1)	FF (z)		FF (z, z_1)
						
	intrinsic	kinematical	dynamical	intrinsic	kinematical	dynamical
U	e	$h_1^{\perp(1)}$	H_{FU}	E, H	$H_1^{\perp(1)}$	$\hat{H}_{FU}^{\Re,\Im}$
L	h_L	$h_{1L}^{\perp(1)}$	H_{FL}	H_L, E_L	$H_{1L}^{\perp(1)}$	$\hat{H}_{FL}^{\Re,\Im}$
T	g_T	$f_{1T}^{\perp(1)}$, $g_{1T}^{\perp(1)}$	F_{FT}, G_{FT}	D_T, G_T	$D_{1T}^{\perp(1)}$, $G_{1T}^{\perp(1)}$	$\hat{D}_{FT}^{\Re,\Im}, \hat{G}_{FT}^{\Re,\Im}$

Transverse Spin Observables in Electron-Proton Collisions

- Motivation
  - *pp* collisions are a challenge to reveal the underlying physics
  - Data already available on most polarization configurations

    A_{UTU} $\ell\{p^{\uparrow}, n^{\uparrow}\} \to \{\pi, K\} X$ (HERMES (2013); JLab Hall A (2013))

    A_{LTU} $\vec{\ell} n^{\uparrow} \to \{\pi, K\} X$ (JLab Hall A (2015))

    A_{UUT} $\ell p \to \Lambda^{\uparrow} X$ (HERMES (2014))
  - Playground to solidify theory (gauge invariance, frame independence, higher-order corrections)
  - Explore potential of an EIC to measure them

- A_{UTU} $\ell N^{\uparrow} \to \pi X$

$$d\sigma = H \otimes f_{a/N^{\uparrow}(3)} \otimes D_{\pi/c(2)} + H' \otimes f_{a/N^{\uparrow}(2)} \otimes D_{\pi/c(3)}$$

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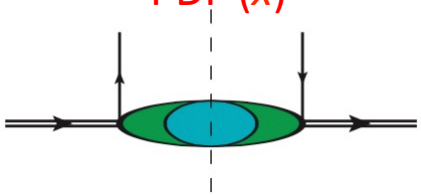
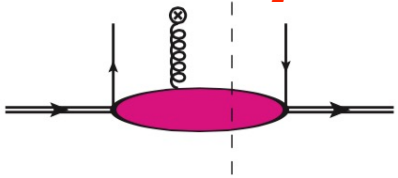
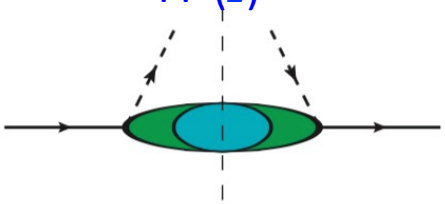
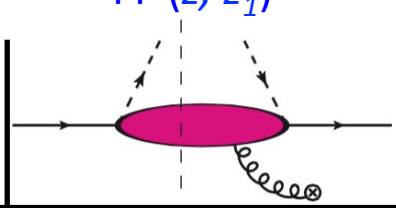
$$d\sigma = \boxed{H \otimes f_{a/N^{\uparrow}(3)} \otimes D_{\pi/c(2)}} + H' \otimes f_{a/N^{\uparrow}(2)} \otimes D_{\pi/c(3)}$$

Hadron Pol.	PDF (x)		PDF (x, x_1)	FF (z)		FF (z, z_1)
						
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	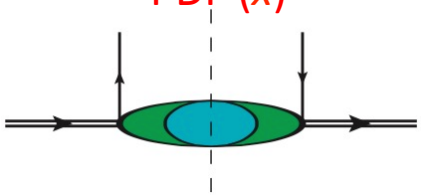		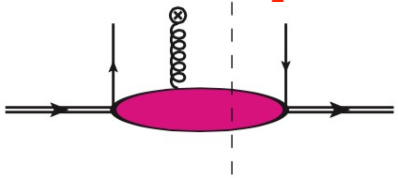	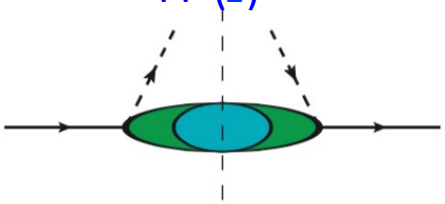		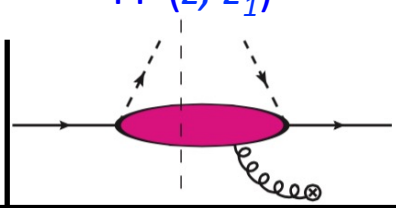
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	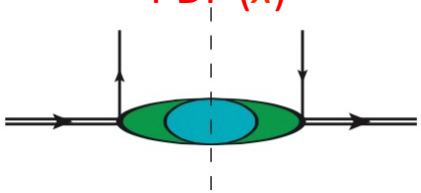		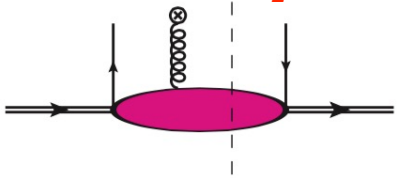	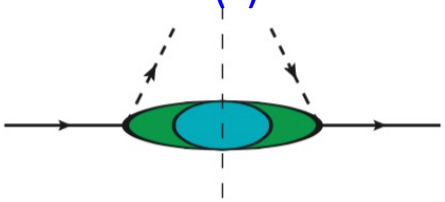		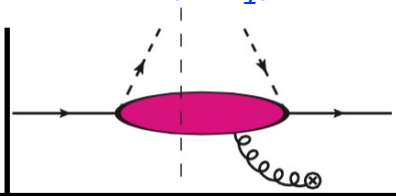
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	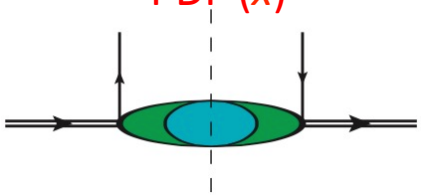		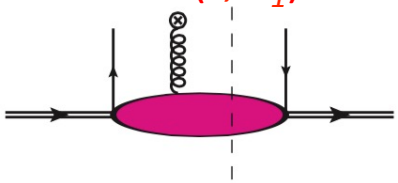	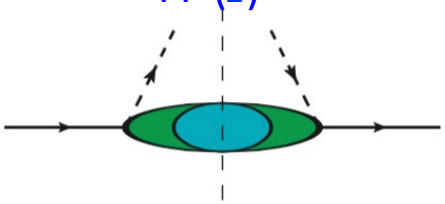		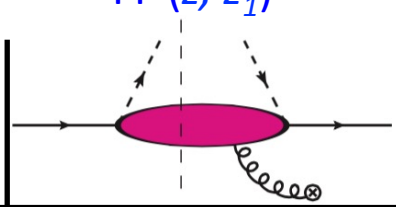
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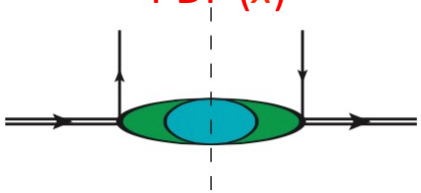
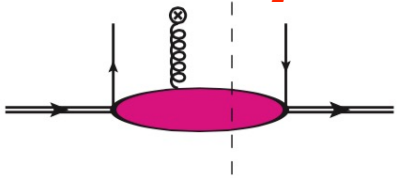
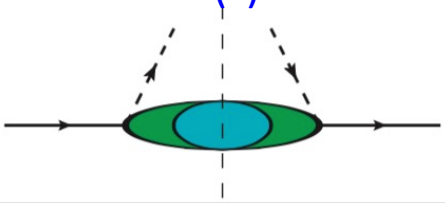
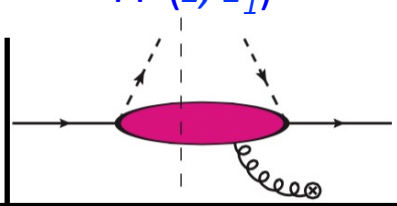
T	g_T	$f_{1T}^{\perp(1)}$, $g_{1T}^{\perp(1)}$	F_{FT}, G_{FT}	D_T, G_T	$D_{1T}^{\perp(1)}$, $G_{1T}^{\perp(1)}$	$\hat{D}_{FT}^{\Re,\Im}, \hat{G}_{FT}^{\Re,\Im}$

Hadron Pol.	PDF (x)		PDF (x, x_1)	FF (z)		FF (z, z_1)
						
	intrinsic	kinematical	dynamical	intrinsic	kinematical	dynamical
U	e	$h_1^{\perp(1)}$	H_{FU}	E, H	$H_1^{\perp(1)}$	$\hat{H}_{FU}^{\Re,\Im}$
L	h_L	$h_{1L}^{\perp(1)}$	H_{FL}	H_L, E_L	$H_{1L}^{\perp(1)}$	$\hat{H}_{FL}^{\Re,\Im}$
T	g_T	$f_{1T}^{\perp(1)}$, $g_{1T}^{\perp(1)}$	F_{FT}, G_{FT}	D_T, G_T	$D_{1T}^{\perp(1)}$, $G_{1T}^{\perp(1)}$	$\hat{D}_{FT}^{\Re,\Im}, \hat{G}_{FT}^{\Re,\Im}$

- A_{UTU} $\ell N^{\uparrow} \to \pi X$

$$d\sigma = \boxed{H \otimes f_{a/N^{\uparrow}(3)} \otimes D_{\pi/c(2)}} + \boxed{H' \otimes f_{a/N^{\uparrow}(2)} \otimes D_{\pi/c(3)}}$$

$$P_h^0 \frac{d\sigma_{UT}}{d^3\vec{P}_h} = -\frac{8\alpha_{\rm em}^2}{S}\,\varepsilon_{\perp\mu\nu}\,S^\mu_{P\perp}\,P^\nu_{h\perp}\sum_q e_q^2 \int_{z_{\min}}^1 \frac{dz}{z^3}\,\frac{1}{S+T/z}\,\frac{1}{x}$$

$$\times\left\{-\frac{\pi M}{\hat{u}}\,D_1^{h/q}(z)\left(F_{FT}^q(x,x) - x\frac{dF_{FT}^q(x,x)}{dx}\right)\left[\frac{\hat{s}(\hat{s}^2+\hat{u}^2)}{2\hat{t}^3}\right]\right.$$

$$+\frac{M_h}{-x\hat{u}-\hat{t}}\,h_1^q(x)\left\{\left(\hat{H}^{h/q}(z) - z\frac{d\hat{H}^{h/q}(z)}{dz}\right)\left[\frac{(1-x)\hat{s}\hat{u}}{\hat{t}^2}\right]\right.$$

$$\left.\left.+\frac{1}{z}\,H^{h/q}(z)\left[\frac{\hat{s}(\hat{s}^2+(x-1)\hat{u}^2)}{\hat{t}^3}\right] + 2z^2\int_z^\infty \frac{dz_1}{z_1^2}\,\frac{1}{\frac{1}{z}-\frac{1}{z_1}}\,\hat{H}_{FU}^{h/q,\Im}(z,z_1)\left[\frac{x\hat{s}^2\hat{u}}{\xi_z\,\hat{t}^3}\right]\right\}\right\}$$

***e-p c.m.* frame**

$$\xi_z = z\left(\frac{1}{z} - \frac{1}{z_1}\right)$$

$$\hat{H}(z) = H_1^{\perp(1)}(z)$$

(Gamberg, Kang, Metz, DP, Prokudin - PRD **90** (2014))

$$P_h^0 \frac{d\sigma_{UT}}{d^3\vec{P}_h} = -\frac{8\alpha_{\rm em}^2}{S}\,\varepsilon_{\perp\mu\nu}\,S^\mu_{P\perp}\,P^\nu_{h\perp}\sum_q e_q^2 \int_{z_{\min}}^1 \frac{dz}{z^3}\,\frac{1}{S+T/z}\,\frac{1}{x}$$

$$\times\left\{-\frac{\pi M}{\hat{u}}\,D_1^{h/q}(z)\left(F_{FT}^q(x,x) - x\frac{dF_{FT}^q(x,x)}{dx}\right)\left[\frac{\hat{s}(\hat{s}^2+\hat{u}^2)}{2\hat{t}^3}\right]\right.$$

$$+\frac{M_h}{-x\hat{u}-\hat{t}}\,h_1^q(x)\left\{\left(\hat{H}^{h/q}(z) - z\frac{d\hat{H}^{h/q}(z)}{dz}\right)\left[\frac{(1-x)\hat{s}\hat{u}}{\hat{t}^2}\right]\right.$$

$$\left.\left.+\frac{1}{z}\,H^{h/q}(z)\left[\frac{\hat{s}(\hat{s}^2+(x-1)\hat{u}^2)}{\hat{t}^3}\right] + 2z^2\int_z^\infty \frac{dz_1}{z_1^2}\,\frac{1}{\frac{1}{z}-\frac{1}{z_1}}\,\hat{H}_{FU}^{h/q,\Im}(z,z_1)\left[\frac{x\hat{s}^2\hat{u}}{\xi_z\,\hat{t}^3}\right]\right\}\right\}$$

***e-p c.m.* frame**

$$\xi_z = z\left(\frac{1}{z} - \frac{1}{z_1}\right)$$

$$\hat{H}(z) = H_1^{\perp(1)}(z)$$

(Gamberg, Kang, Metz, DP, Prokudin - PRD **90** (2014))

$$2z^3\int_z^\infty \frac{dz_1}{z_1^2}\,\frac{1}{\frac{1}{z}-\frac{1}{z_1}}\,\hat{H}_{FU}^{\Im}(z,z_1) = H(z) + 2zH_1^{\perp(1)}(z)$$

QCD e.o.m. relation

$$
\begin{aligned}
P_h^0 \frac{d\sigma_{UT}}{d^3\vec{P}_h} = & -\frac{8\alpha_{\rm em}^2}{S}\,\varepsilon_{\perp\mu\nu}\,S^\mu_{P\perp}\,P^\nu_{h\perp}\sum_q e_q^2 \int_{z_{\min}}^1 \frac{dz}{z^3}\,\frac{1}{S+T/z}\,\frac{1}{x} \\
& \times\Bigg\{-\frac{\pi M}{\hat{u}}\,D_1^{h/q}(z)\left(F^q_{FT}(x,x) - x\frac{dF^q_{FT}(x,x)}{dx}\right)\left[\frac{\hat{s}(\hat{s}^2+\hat{u}^2)}{2\hat{t}^3}\right] \\
& + \frac{M_h}{-x\hat{u}-\hat{t}}\,h_1^q(x)\Bigg\{\left(\hat{H}^{h/q}(z) - z\frac{d\hat{H}^{h/q}(z)}{dz}\right)\left[\frac{(1-x)\hat{s}\hat{u}}{\hat{t}^2}\right] \\
& + \frac{1}{z}\,H^{h/q}(z)\left[\frac{\hat{s}(\hat{s}^2+(x-1)\hat{u}^2)}{\hat{t}^3}\right] + 2z^2\int_z^\infty \frac{dz_1}{z_1^2}\,\frac{1}{\frac{1}{z}-\frac{1}{z_1}}\,\hat{H}^{h/q,\Im}_{FU}(z,z_1)\left[\frac{x\hat{s}^2\hat{u}}{\xi_z\,\hat{t}^3}\right]\Bigg\}\Bigg\}
\end{aligned}
$$

$$P_h^0 \frac{d\sigma_{UT}}{d^3\vec{P}_h} = -\frac{8\alpha_{\rm em}^2}{S}\,\varepsilon_{\perp\mu\nu}\,S^\mu_{P\perp}\,P^\nu_{h\perp}\sum_q e_q^2 \int_{z_{\min}}^1 \frac{dz}{z^3}\,\frac{1}{S+T/z}\,\frac{1}{z}$$

$$\times\left\{-\frac{\pi M}{\hat{u}}\,D_1^{h/q}(z)\left(F_{FT}^q(x,x) - x\frac{dF_{FT}^q(x,x)}{dx}\right)\left[\frac{\hat{s}(\hat{s}^2+\hat{u}^2)}{2\hat{t}^3}\right]\right.$$

$$+\frac{M_h}{-x\hat{u}-\hat{t}}\,h_1^q(x)\left\{\left(\hat{H}^{h/q}(z) - z\frac{d\hat{H}^{h/q}(z)}{dz}\right)\left[\frac{(1-x)\hat{s}\hat{u}}{\hat{t}^2}\right]\right.$$

$$\left.\left.+\frac{1}{z}H^{h/q}(z)\left[\frac{\hat{s}(\hat{s}^2+(x-1)\hat{u}^2)}{\hat{t}^3}\right] + 2z^2\int_z^\infty \frac{dz_1}{z_1^2}\,\frac{1}{\frac{1}{z}-\frac{1}{z_1}}\,\hat{H}_{FU}^{h/q,\Im}(z,z_1)\left[\frac{x\hat{s}^2\hat{u}}{\xi_z\,\hat{t}^3}\right]\right\}\right\}$$

Use Sivers function from SIDIS (Anselmino, et al. (2009))

Use Collins function from SIDIS/e^+e^- (Anselmino, et al. (2013))

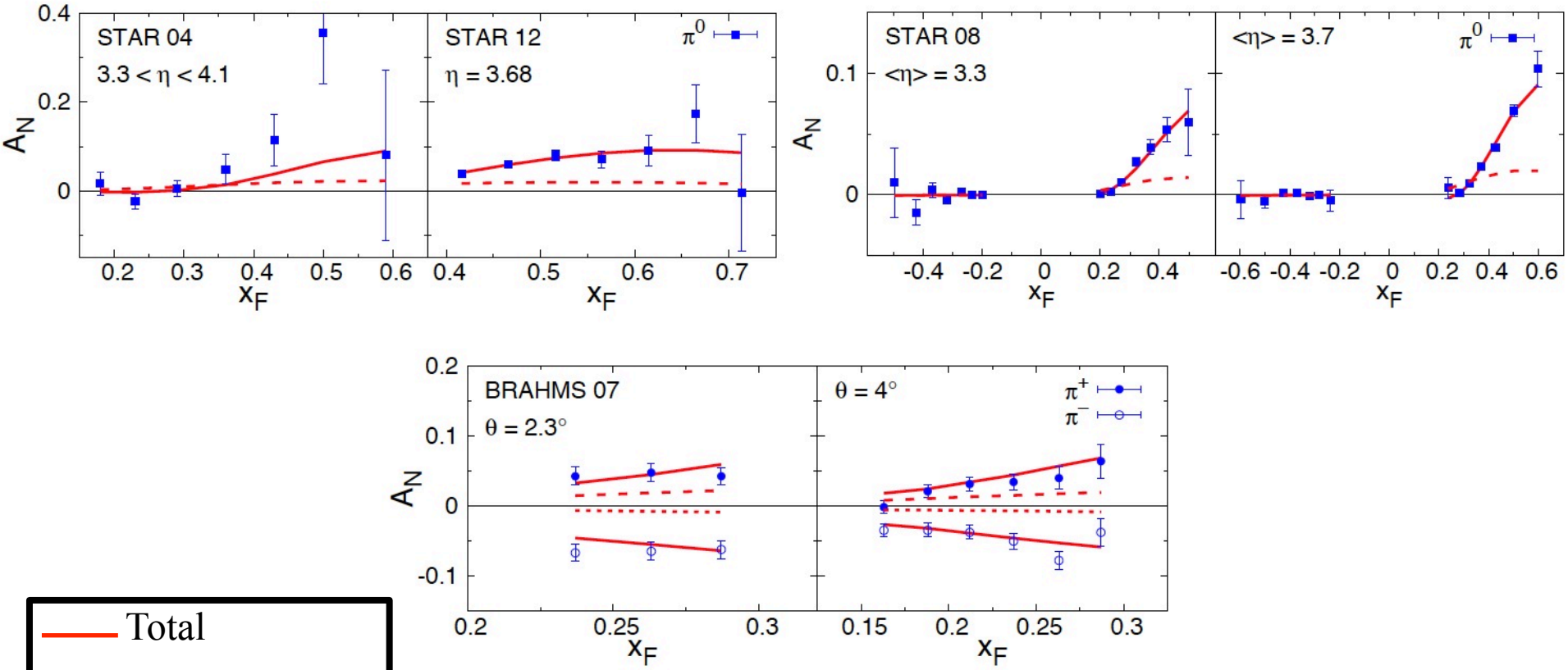
Take from *pp* fit by Kanazawa, Koike, Metz, DP

A_N in $p^{\uparrow}p \to \pi X$

χ^2/d.o.f. = 1.03

Fragmentation (Metz and DP - PLB **723** (2013))

+ Qiu-Sterman (fix through Sivers function)



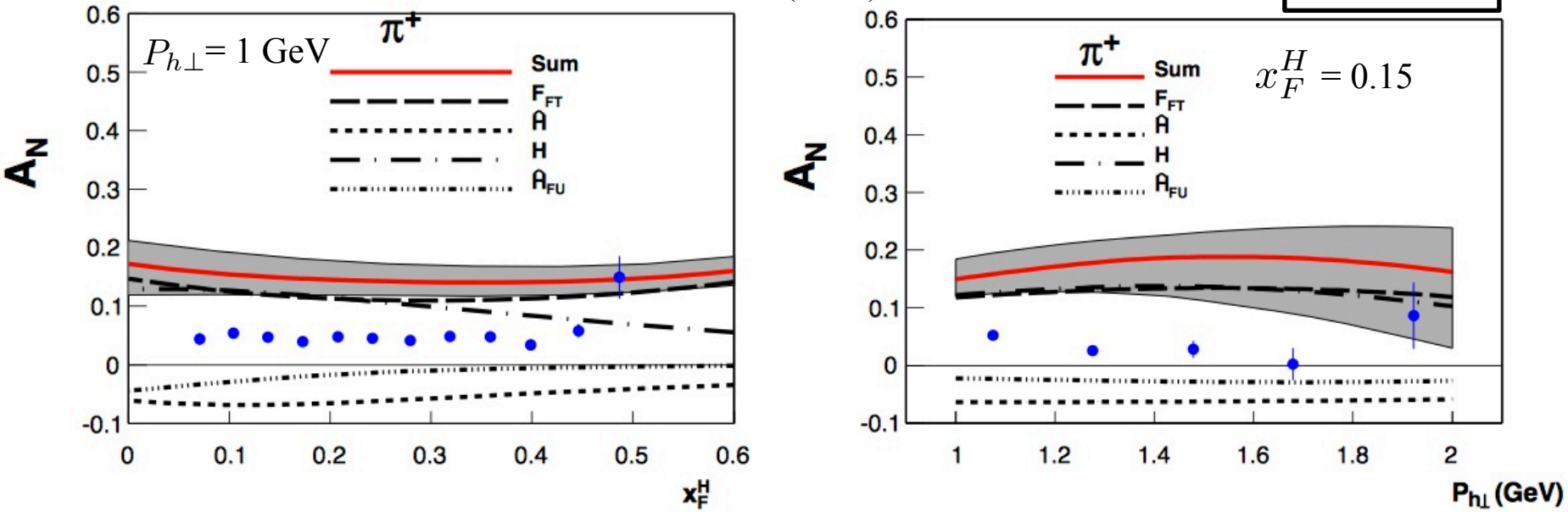
(Kanazawa, Koike, Metz, DP - PRD **89**(RC) (2014))

A_N in $e\,p^{\uparrow} \longrightarrow \pi\,X$

(Gamberg, Kang, Metz, DP, Prokudin - PRD **90** (2014))

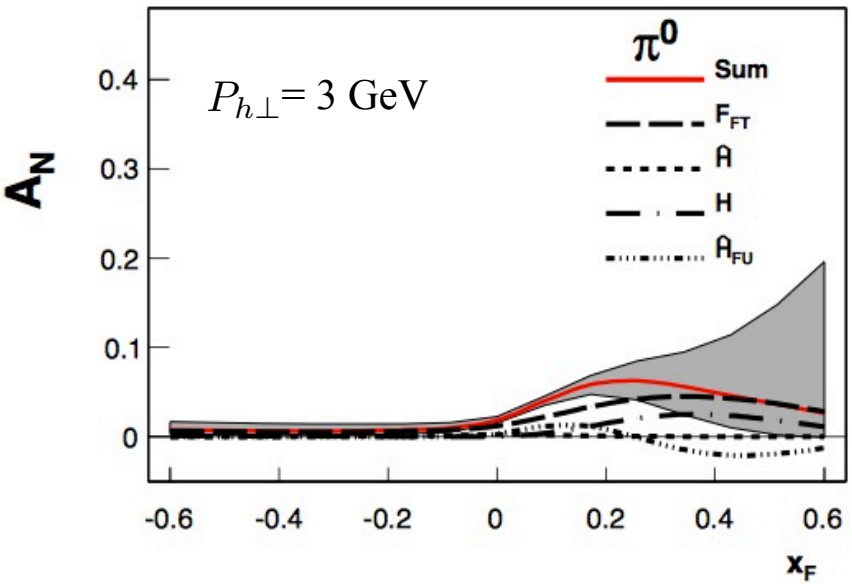
HERMES (2014) $\sqrt{S} = 7.25$ GeV

$x_F^H = -x_F$



- Theoretical results are above the data, but **full NLO calculation most likely needed** (see Hinderer, Schlegel, Vogelsang (2015))
- Jefferson Lab Hall A also has data for a neutron target, but P_T is too low
  - ➔ 12 GeV upgrade will give valuable data at higher P_T
- This process can help better constrain the 3-parton FF that has been fitted in *pp*
  - ➔ crucial to measure at at EIC

EIC $\sqrt{S} = 63$ GeV



- EIC is a unique position to measure A_N in the forward region like in *pp* collisions
- Clearly nonzero signal (~ 10%) predicted for π^0 for $x_F > 0$, like in *pp*
- Can provide further constraints/tests of the mechanism used to describe A_N in *pp*

(Gamberg, Kang, Metz, DP, Prokudin - PRD **90** (2014))

$$P_h^0 \frac{d\sigma_{UT}}{d^3\vec{P}_h} = -\frac{8\alpha_{\rm em}^2}{S}\,\varepsilon_{\perp\mu\nu}\,S^\mu_{P\perp}\,P^\nu_{h\perp}\sum_q e_q^2 \int_{z_{\min}}^1 \frac{dz}{z^3}\,\frac{1}{S+T/z}\,\frac{1}{x}$$
$$\times\Bigg\{-\frac{\pi M}{\hat{u}}\,D_1^{h/q}(z)\left(F_{FT}^q(x,x) - x\frac{dF_{FT}^q(x,x)}{dx}\right)\left[\frac{\hat{s}(\hat{s}^2+\hat{u}^2)}{2\hat{t}^3}\right]$$
$$+\frac{M_h}{-x\hat{u}-\hat{t}}\,h_1^q(x)\Bigg\{\left(\hat{H}^{h/q}(z) - z\frac{d\hat{H}^{h/q}(z)}{dz}\right)\left[\frac{(1-x)\hat{s}\hat{u}}{\hat{t}^2}\right]$$
$$+\frac{1}{z}\,H^{h/q}(z)\left[\frac{\hat{s}(\hat{s}^2+(x-1)\hat{u}^2)}{\hat{t}^3}\right] + 2z^2\int_z^\infty \frac{dz_1}{z_1^2}\,\frac{1}{\frac{1}{z}-\frac{1}{z_1}}\,\hat{H}_{FU}^{h/q,\Im}(z,z_1)\left[\frac{x\hat{s}^2\hat{u}}{\xi_z\,\hat{t}^3}\right]\Bigg\}\Bigg\}$$

***e-p c.m.* frame**

$$\xi_z = z\left(\frac{1}{z} - \frac{1}{z_1}\right)$$

e-p c.m. frame

$$\xi_z = z\left(\frac{1}{z} - \frac{1}{z_1}\right)$$

$$P_h^0 \frac{d\sigma_{UT}}{d^3\vec{P}_h} = -\frac{8\alpha_{\rm em}^2}{S}\,\varepsilon_{\perp\mu\nu}\,S^\mu_{P\perp}\,P^\nu_{h\perp}\sum_q e_q^2 \int_{z_{\min}}^1 \frac{dz}{z^3}\,\frac{1}{S+T/z}\,\frac{1}{x}$$
$$\times\Bigg\{-\frac{\pi M}{\hat{u}}\,D_1^{h/q}(z)\left(F_{FT}^q(x,x) - x\frac{dF_{FT}^q(x,x)}{dx}\right)\left[\frac{\hat{s}(\hat{s}^2+\hat{u}^2)}{2\hat{t}^3}\right]$$
$$+\frac{M_h}{-x\hat{u}-\hat{t}}\,h_1^q(x)\Bigg\{\left(\hat{H}^{h/q}(z) - z\frac{d\hat{H}^{h/q}(z)}{dz}\right)\left[\frac{(1-x)\hat{s}\hat{u}}{\hat{t}^2}\right]$$
$$+\frac{1}{z}\,H^{h/q}(z)\left[\frac{\hat{s}(\hat{s}^2+(x-1)\hat{u}^2)}{\hat{t}^3}\right] + 2z^2\int_z^\infty \frac{dz_1}{z_1^2}\,\frac{1}{\frac{1}{z}-\frac{1}{z_1}}\,\hat{H}_{FU}^{h/q,\Im}(z,z_1)\left[\frac{x\hat{s}^2\hat{u}}{\xi_z\,\hat{t}^3}\right]\Bigg\}\Bigg\}$$

---

p-π c.m. frame

$$P_h^0 \frac{d\sigma_{UT}}{d^3\vec{P}_h} = -\frac{8\alpha_{\rm em}^2}{S}\,\varepsilon_{\perp\mu\nu}\,S^\mu_{P\perp}\,P^\nu_{h\perp}\sum_q e_q^2 \int_{z_{\min}}^1 \frac{dz}{z^3}\,\frac{1}{S+T/z}\,\frac{1}{x}$$
$$\times\Bigg\{-\frac{\pi M}{\hat{u}}\,D_1^{h/q}(z)\left(F_{FT}^q(x,x) - x\frac{dF_{FT}^q(x,x)}{dx}\right)\left[\frac{\hat{s}(\hat{s}^2+\hat{u}^2)}{2\hat{t}^3}\right]$$
$$+\,M_h\,h_1^q(x)\left\{\frac{H^{h/q}(z)}{z}\left[\frac{\hat{s}(\hat{s}-\hat{u})}{\hat{t}^3}\right] + \hat{H}^{h/q}(z)\left[\frac{\hat{s}^2}{\hat{t}^3}\right] + z\frac{d\hat{H}^{h/q}(z)}{dz}\left[\frac{\hat{s}\hat{u}}{\hat{t}^3}\right]\right\}\Bigg\}$$

e-p c.m. frame

$$\xi_z = z\left(\frac{1}{z} - \frac{1}{z_1}\right)$$

$$P_h^0 \frac{d\sigma_{UT}}{d^3\vec{P}_h} = -\frac{8\alpha_{\rm em}^2}{S}\,\varepsilon_{\perp\mu\nu}\,S_{P\perp}^{\mu}\,P_{h\perp}^{\nu}\sum_q e_q^2 \int_{z_{\min}}^{1}\frac{dz}{z^3}\,\frac{1}{S+T/z}\,\frac{1}{x}$$
$$\times\left\{-\frac{\pi M}{\hat{u}}\,D_1^{h/q}(z)\left(F_{FT}^q(x,x) - x\frac{dF_{FT}^q(x,x)}{dx}\right)\left[\frac{\hat{s}(\hat{s}^2+\hat{u}^2)}{2\hat{t}^3}\right]\right.$$
$$+\frac{M_h}{-x\hat{u}-\hat{t}}\,h_1^q(x)\left\{\left(\hat{H}^{h/q}(z) - z\frac{d\hat{H}^{h/q}(z)}{dz}\right)\left[\frac{(1-x)\hat{s}\hat{u}}{\hat{t}^2}\right]\right.$$
$$\left.\left.+\frac{1}{z}\,H^{h/q}(z)\left[\frac{\hat{s}(\hat{s}^2+(x-1)\hat{u}^2)}{\hat{t}^3}\right] + 2z^2\int_z^{\infty}\frac{dz_1}{z_1^2}\,\frac{1}{\frac{1}{z}-\frac{1}{z_1}}\,\hat{H}_{FU}^{h/q,\Im}(z,z_1)\left[\frac{x\hat{s}^2\hat{u}}{\xi_z\,\hat{t}^3}\right]\right\}\right\}$$

---

p-π c.m. frame

$$P_h^0 \frac{d\sigma_{UT}}{d^3\vec{P}_h} = -\frac{8\alpha_{\rm em}^2}{S}\,\varepsilon_{\perp\mu\nu}\,S_{P\perp}^{\mu}\,P_{h\perp}^{\nu}\sum_q e_q^2 \int_{z_{\min}}^{1}\frac{dz}{z^3}\,\frac{1}{S+T/z}\,\frac{1}{x}$$
$$\times\left\{-\frac{\pi M}{\hat{u}}\,D_1^{h/q}(z)\left(F_{FT}^q(x,x) - x\frac{dF_{FT}^q(x,x)}{dx}\right)\left[\frac{\hat{s}(\hat{s}^2+\hat{u}^2)}{2\hat{t}^3}\right]\right.$$
$$\left.+\,M_h\,h_1^q(x)\left\{\frac{H^{h/q}(z)}{z}\left[\frac{\hat{s}(\hat{s}-\hat{u})}{\hat{t}^3}\right] + \hat{H}^{h/q}(z)\left[\frac{\hat{s}^2}{\hat{t}^3}\right] + z\frac{d\hat{H}^{h/q}(z)}{dz}\left[\frac{\hat{s}\hat{u}}{\hat{t}^3}\right]\right\}\right\}$$

e-p c.m. frame

$$\xi_z = z\left(\frac{1}{z} - \frac{1}{z_1}\right)$$

$$P_h^0 \frac{d\sigma_{UT}}{d^3\vec{P}_h} = -\frac{8\alpha_{\rm em}^2}{S}\,\varepsilon_{\perp\mu\nu}\,S^\mu_{P\perp}\,P^\nu_{h\perp}\sum_q e_q^2 \int_{z_{\min}}^1 \frac{dz}{z^3}\,\frac{1}{S+T/z}\,\frac{1}{x}$$
$$\times\left\{-\frac{\pi M}{\hat{u}}\,D_1^{h/q}(z)\left(F_{FT}^q(x,x) - x\frac{dF_{FT}^q(x,x)}{dx}\right)\left[\frac{\hat{s}(\hat{s}^2+\hat{u}^2)}{2\hat{t}^3}\right]\right.$$
$$+\frac{M_h}{-x\hat{u}-\hat{t}}\,h_1^q(x)\left\{\left(\hat{H}^{h/q}(z) - z\frac{d\hat{H}^{h/q}(z)}{dz}\right)\left[\frac{(1-x)\hat{s}\hat{u}}{\hat{t}^2}\right]\right.$$
$$\left.\left.+\frac{1}{z}\,H^{h/q}(z)\left[\frac{\hat{s}(\hat{s}^2+(x-1)\hat{u}^2)}{\hat{t}^3}\right] + 2z^2\int_z^\infty \frac{dz_1}{z_1^2}\,\frac{1}{\frac{1}{z}-\frac{1}{z_1}}\,\hat{H}_{FU}^{h/q,\Im}(z,z_1)\left[\frac{x\hat{s}^2\hat{u}}{\xi_z\,\hat{t}^3}\right]\right\}\right\}$$

---

p-π c.m. frame

$$P_h^0 \frac{d\sigma_{UT}}{d^3\vec{P}_h} = -\frac{8\alpha_{\rm em}^2}{S}\,\varepsilon_{\perp\mu\nu}\,S^\mu_{P\perp}\,P^\nu_{h\perp}\sum_q e_q^2 \int_{z_{\min}}^1 \frac{dz}{z^3}\,\frac{1}{S+T/z}\,\frac{1}{x}$$
$$\times\left\{-\frac{\pi M}{\hat{u}}\,D_1^{h/q}(z)\left(F_{FT}^q(x,x) - x\frac{dF_{FT}^q(x,x)}{dx}\right)\left[\frac{\hat{s}(\hat{s}^2+\hat{u}^2)}{2\hat{t}^3}\right]\right.$$
$$\left.+\,M_h\,h_1^q(x)\left\{\frac{H^{h/q}(z)}{z}\left[\frac{\hat{s}(\hat{s}-\hat{u})}{\hat{t}^3}\right] + \hat{H}^{h/q}(z)\left[\frac{\hat{s}^2}{\hat{t}^3}\right] + z\frac{d\hat{H}^{h/q}(z)}{dz}\left[\frac{\hat{s}\hat{u}}{\hat{t}^3}\right]\right\}\right\}$$

?

e-p c.m. frame

$$\xi_z = z\left(\frac{1}{z} - \frac{1}{z_1}\right)$$

$$P_h^0 \frac{d\sigma_{UT}}{d^3\vec{P}_h} = -\frac{8\alpha_{\rm em}^2}{S}\,\varepsilon_{\perp\mu\nu}\,S^\mu_{P\perp}\,P^\nu_{h\perp}\sum_q e_q^2 \int_{z_{\min}}^1 \frac{dz}{z^3}\,\frac{1}{S+T/z}\,\frac{1}{x}$$
$$\times\left\{-\frac{\pi M}{\hat{u}}\,D_1^{h/q}(z)\left(F_{FT}^q(x,x) - x\frac{dF_{FT}^q(x,x)}{dx}\right)\left[\frac{\hat{s}(\hat{s}^2+\hat{u}^2)}{2\hat{t}^3}\right]\right.$$
$$+\frac{M_h}{-x\hat{u}-\hat{t}}\,h_1^q(x)\left\{\left(\hat{H}^{h/q}(z) - z\frac{d\hat{H}^{h/q}(z)}{dz}\right)\left[\frac{(1-x)\hat{s}\hat{u}}{\hat{t}^2}\right]\right.$$
$$\left.\left.+\frac{1}{z}\,H^{h/q}(z)\left[\frac{\hat{s}(\hat{s}^2+(x-1)\hat{u}^2)}{\hat{t}^3}\right] + 2z^2\int_z^\infty \frac{dz_1}{z_1^2}\,\frac{1}{\frac{1}{z}-\frac{1}{z_1}}\,\hat{H}_{FU}^{h/q,\Im}(z,z_1)\left[\frac{x\hat{s}^2\hat{u}}{\xi_z\,\hat{t}^3}\right]\right\}\right\}$$

---

p-π c.m. frame

$$P_h^0 \frac{d\sigma_{UT}}{d^3\vec{P}_h} = -\frac{8\alpha_{\rm em}^2}{S}\,\varepsilon_{\perp\mu\nu}\,S^\mu_{P\perp}\,P^\nu_{h\perp}\sum_q e_q^2 \int_{z_{\min}}^1 \frac{dz}{z^3}\,\frac{1}{S+T/z}\,\frac{1}{x}$$
$$\times\left\{-\frac{\pi M}{\hat{u}}\,D_1^{h/q}(z)\left(F_{FT}^q(x,x) - x\frac{dF_{FT}^q(x,x)}{dx}\right)\left[\frac{\hat{s}(\hat{s}^2+\hat{u}^2)}{2\hat{t}^3}\right]\right.$$
$$\left.+\,M_h\,h_1^q(x)\left\{\frac{H^{h/q}(z)}{z}\left[\frac{\hat{s}(\hat{s}-\hat{u})}{\hat{t}^3}\right] + \hat{H}^{h/q}(z)\left[\frac{\hat{s}^2}{\hat{t}^3}\right] + z\frac{d\hat{H}^{h/q}(z)}{dz}\left[\frac{\hat{s}\hat{u}}{\hat{t}^3}\right]\right\}\right\}$$

$$2z^3\int_z^\infty \frac{dz_1}{z_1^2}\,\frac{1}{\frac{1}{z}-\frac{1}{z_1}}\,\hat{H}_{FU}^{\Im}(z,z_1) = H(z) + 2zH_1^{\perp(1)}(z)$$ **WILL NOT HELP!** ?

e-p c.m. frame

$$\xi_z = z\left(\frac{1}{z} - \frac{1}{z_1}\right)$$

$$P_h^0 \frac{d\sigma_{UT}}{d^3\vec{P}_h} = -\frac{8\alpha_{\rm em}^2}{S}\,\varepsilon_{\perp\mu\nu}\,S_{P\perp}^{\mu}\,P_{h\perp}^{\nu}\sum_q e_q^2 \int_{z_{\min}}^{1}\frac{dz}{z^3}\,\frac{1}{S+T/z}\,\frac{1}{x}$$

$$\times\left\{-\frac{\pi M}{\hat{u}}\,D_1^{h/q}(z)\left(F_{FT}^q(x,x) - x\frac{dF_{FT}^q(x,x)}{dx}\right)\left[\frac{\hat{s}(\hat{s}^2+\hat{u}^2)}{2\hat{t}^3}\right]\right.$$

$$+\frac{M_h}{-x\hat{u}-\hat{t}}\,h_1^q(x)\left\{\left(\hat{H}^{h/q}(z) - z\frac{d\hat{H}^{h/q}(z)}{dz}\right)\left[\frac{(1-x)\hat{s}\hat{u}}{\hat{t}^2}\right]\right.$$

$$\left.\left.+\frac{1}{z}\,H^{h/q}(z)\left[\frac{\hat{s}^2+(z-1)\hat{u}^2}{\hat{t}^3}\right] + 2z\int_z^{\infty}\frac{dz_1}{z_1^2}\,\frac{1}{\frac{1}{z}-\frac{1}{z_1}}\,\hat{H}_{FU}^{h/q,\Im}(z,z_1)\left[\frac{x\hat{s}^2\hat{u}}{\xi_z\,\hat{t}^3}\right]\right\}\right\}$$

FRAME DEPENDENCE?

p-π c.m. frame

$$P_h^0 \frac{d\sigma_{UT}}{d^3\vec{P}_h} = -\frac{8\alpha_{\rm em}^2}{S}\,\varepsilon_{\perp\mu\nu}\,S_{P\perp}^{\mu}\,P_{h\perp}^{\nu}\sum_q e_q^2 \int_{z_{\min}}^{1}\frac{dz}{z^3}\,\frac{1}{S+T/z}\,\frac{1}{x}$$

$$\times\left\{-\frac{\pi M}{\hat{u}}\,D_1^{h/q}(z)\left(F_{FT}^q(x,x) - x\frac{dF_{FT}^q(x,x)}{dx}\right)\left[\frac{\hat{s}(\hat{s}^2+\hat{u}^2)}{2\hat{t}^3}\right]\right.$$

$$\left.+\,M_h\,h_1^q(x)\left\{\frac{H^{h/q}(z)}{z}\left[\frac{\hat{s}(\hat{s}-\hat{u})}{\hat{t}^3}\right] + \hat{H}^{h/q}(z)\left[\frac{\hat{s}^2}{\hat{t}^3}\right] + z\frac{d\hat{H}^{h/q}(z)}{dz}\left[\frac{\hat{s}\hat{u}}{\hat{t}^3}\right]\right\}\right\}$$

e-p c.m. frame

$$\xi_z = z\left(\frac{1}{z} - \frac{1}{z_1}\right)$$

$$P_h^0 \frac{d\sigma_{UT}}{d^3\vec{P}_h} = -\frac{8\alpha_{\rm em}^2}{S}\,\varepsilon_{\perp\mu\nu}\,S^\mu_{P\perp}\,P^\nu_{h\perp}\sum_q e_q^2 \int_{z_{\min}}^1 \frac{dz}{z^3}\,\frac{1}{S+T/z}\,\frac{1}{x}$$
$$\times\left\{-\frac{\pi M}{\hat{u}}\,D_1^{h/q}(z)\left(F_{FT}^q(x,x) - x\frac{dF_{FT}^q(x,x)}{dx}\right)\left[\frac{\hat{s}(\hat{s}^2+\hat{u}^2)}{2\hat{t}^3}\right]\right.$$
$$+\frac{M_h}{-x\hat{u}-\hat{t}}\,h_1^q(x)\left\{\left(\hat{H}^{h/q}(z) - z\frac{d\hat{H}^{h/q}(z)}{dz}\right)\left[\frac{(1-x)\hat{s}\hat{u}}{\hat{t}^2}\right]\right.$$
$$\left.\left.+\frac{1}{z}\,H^{h/q}(z)\left[\frac{\hat{s}(\hat{s}^2+(x-1)\hat{u}^2)}{\hat{t}^3}\right] + 2z^2\int_z^\infty \frac{dz_1}{z_1^2}\,\frac{1}{\frac{1}{z}-\frac{1}{z_1}}\,\hat{H}_{FU}^{h/q,\Im}(z,z_1)\left[\frac{x\hat{s}^2\hat{u}}{\xi_z\,\hat{t}^3}\right]\right\}\right\}$$

NO!

p-π c.m. frame

$$P_h^0 \frac{d\sigma_{UT}}{d^3\vec{P}_h} = -\frac{8\alpha_{\rm em}^2}{S}\,\varepsilon_{\perp\mu\nu}\,S^\mu_{P\perp}\,P^\nu_{h\perp}\sum_q e_q^2 \int_{z_{\min}}^1 \frac{dz}{z^3}\,\frac{1}{S+T/z}\,\frac{1}{x}$$
$$\times\left\{-\frac{\pi M}{\hat{u}}\,D_1^{h/q}(z)\left(F_{FT}^q(x,x) - x\frac{dF_{FT}^q(x,x)}{dx}\right)\left[\frac{\hat{s}(\hat{s}^2+\hat{u}^2)}{2\hat{t}^3}\right]\right.$$
$$\left.+\,M_h\,h_1^q(x)\left\{\frac{H^{h/q}(z)}{z}\left[\frac{\hat{s}(\hat{s}-\hat{u})}{\hat{t}^3}\right] + \hat{H}^{h/q}(z)\left[\frac{\hat{s}^2}{\hat{t}^3}\right] + z\frac{d\hat{H}^{h/q}(z)}{dz}\left[\frac{\hat{s}\hat{u}}{\hat{t}^3}\right]\right\}\right\}$$

$$2z^3 \int_z^\infty \frac{dz_1}{z_1^2} \frac{1}{\frac{1}{z} - \frac{1}{z_1}} \hat{H}_{FU}^{\Im}(z, z_1) = H(z) + 2zH_1^{\perp(1)}(z)$$

QCD e.o.m. relation

$$2z^3 \int_z^\infty \frac{dz_1}{z_1^2} \frac{1}{\frac{1}{z} - \frac{1}{z_1}} \hat{H}_{FU}^{\Im}(z, z_1) = H(z) + 2z H_1^{\perp(1)}(z)$$

QCD e.o.m. relation

$$\frac{H(z)}{z} = -\left(1 - z\frac{d}{dz}\right) H_1^{\perp(1)}(z) - \frac{2}{z}\int_z^\infty \frac{dz_1}{z_1^2} \frac{\hat{H}_{FU}^{\Im}(z, z_1)}{(1/z - 1/z_1)^2}$$

Lorentz Invariance Relation (LIR)

(Kanazawa, Koike, Metz, DP, Schlegel, PRD **93** (2016))

$$E_h \frac{d\sigma_{\rm LO}(S_N)}{d^3\vec{P}_h} = \frac{8\alpha_{\rm em}^2}{S} \sum_q e_q^2 \int_0^1 dx \int_0^1 \frac{dz}{z^3}\, \delta(\hat{s}+\hat{t}+\hat{u})$$

$$\times\ \epsilon^{lPP_hS_N} \Bigg[\pi M \left(1 - x\frac{\rm d}{{\rm d}x}\right) \boldsymbol{F_{FT}^q(x,x)}\, D_1^q(z) \left(\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^3\hat{u}}\right)$$

$$+ M_h\, h_1^q(x) \left(\frac{\boldsymbol{H^q(z)}}{z} \left(\frac{2(\hat{u}-\hat{s})}{\hat{t}^3}\right) + \left(1 - z\frac{\rm d}{{\rm d}z}\right) \boldsymbol{H_1^{\perp(1),q}(z)} \left(\frac{2\hat{u}}{\hat{t}^3}\right) \right) \Bigg]$$

-Result is the same in <u>any</u> frame

-Calculated in both Feynman gauge and lightcone gauge

-Checked EM gauge invariance

(Kanazawa, Koike, Metz, DP, Schlegel, PRD **93** (2016))

$$E_h \frac{d\sigma_{\rm LO}(S_N)}{d^3\vec{P}_h} = \frac{8\alpha_{\rm em}^2}{S} \sum_q e_q^2 \int_0^1 dx \int_0^1 \frac{dz}{z^3}\, \delta(\hat{s}+\hat{t}+\hat{u})$$

$$\times\ \epsilon^{lPP_hS_N} \left[\pi M \left(1 - x\frac{\rm d}{{\rm d}x}\right) \boldsymbol{F_{FT}^q(x,x)}\, D_1^q(z) \left(\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^3\hat{u}}\right) \right.$$

$$\left. + M_h\, h_1^q(x) \left(\frac{\boldsymbol{H^q(z)}}{z} \left(\frac{2(\hat{u}-\hat{s})}{\hat{t}^3}\right) + \left(1 - z\frac{\rm d}{{\rm d}z}\right) \boldsymbol{H_1^{\perp(1),q}(z)} \left(\frac{2\hat{u}}{\hat{t}^3}\right) \right) \right]$$

-Result is the same in <u>any</u> frame

-Calculated in both Feynman gauge and lightcone gauge

-Checked EM gauge invariance

Recall the LIR: $g_T(x) = g_1(x) + \frac{d}{dx} g_{1T}^{(1)}(x) - 2\,\mathcal{P}\int_{-1}^{1} dx_1 \frac{G_{FT}(x,x_1)}{(x-x_1)^2}$ (e.g., Accardi, et al. (2009))

Now we have derived LIRs for twist-3 fragmentation functions

(Kanazawa, Koike, Metz, DP, Schlegel, PRD **93** (2016))

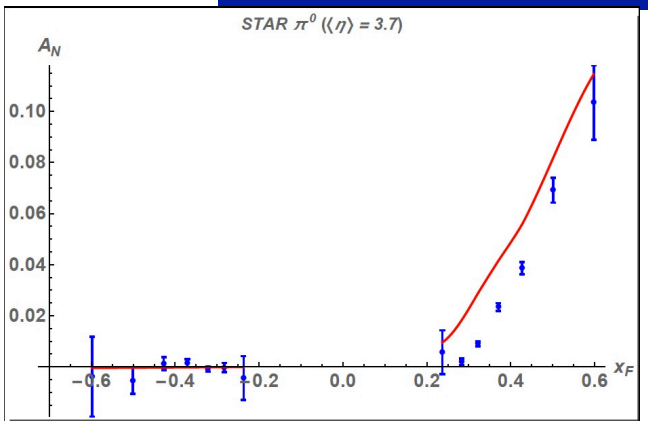
A_N in $p^\uparrow p \to \pi X$

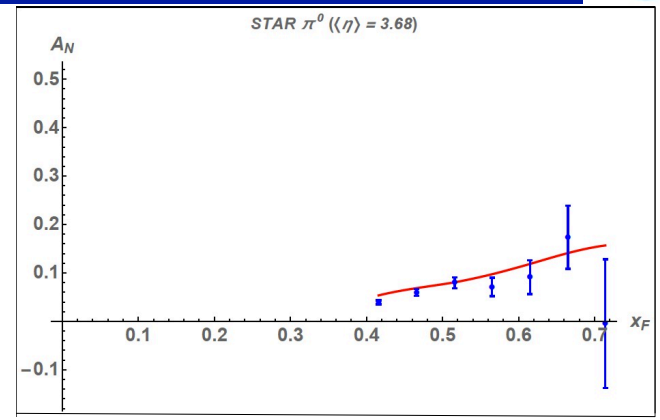
$$\frac{E_h d\sigma^{Frag}(S_P)}{d^3\vec{P}_h} = -\frac{4\alpha_s^2 M_h}{S}\,\epsilon^{P'PP_hS_P}\sum_i\sum_{a,b,c}\int_0^1\frac{dz}{z^3}\int_0^1 dx'\int_0^1 dx\;\delta(\hat{s}+\hat{t}+\hat{u})$$

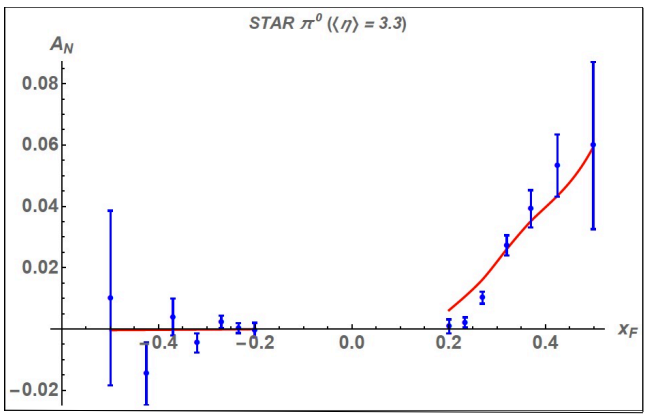
$$\times\,\frac{1}{\hat{s}}\,h_1^a(x)\,f_1^b(x')\left\{\frac{H^c(z)}{z}\tilde{S}_H^i+\left[\left(1-z\frac{d}{dz}\right)H_1^{\perp(1),c}(z)\right]\tilde{S}_{H_1^\perp}^i\right\},$$

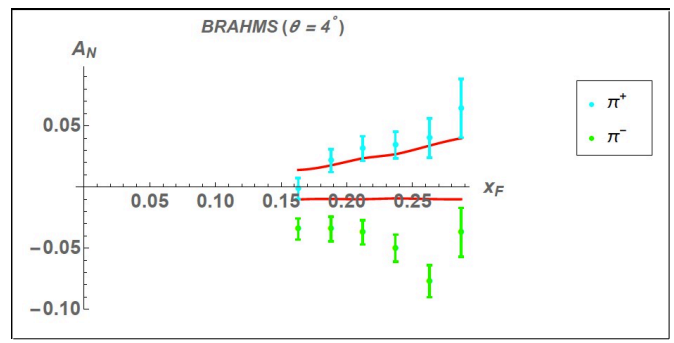
Recall: $H(z) = -2zH_1^{\perp(1)}(z) + 2z^3\int_z^\infty \frac{dz_1}{z_1^2}\frac{1}{\frac{1}{z}-\frac{1}{z_1}}\hat{H}_{FU}^{\Im}(z,z_1)$

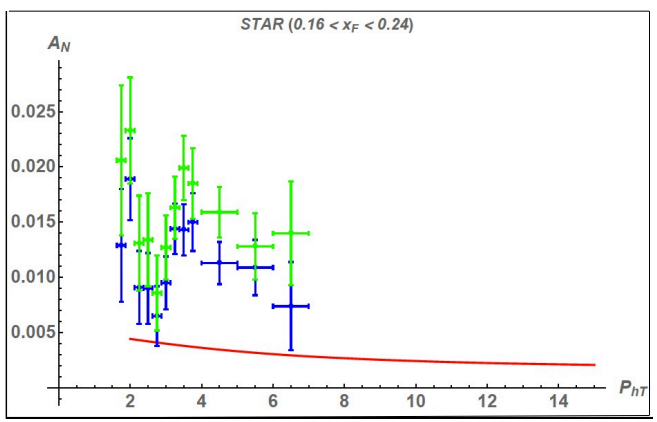
(Gamberg, Kang, DP, Prokudin, in preparation)











-Confirms the original work of Kanazawa, Koike, DP, Metz (2014)

-Encouraged that we will be able to fully describe A_N through the twist-3 fragmentation term even with the additional constraint from the LIR

(Gamberg, Kang, DP, Prokudin, in preparation)

$$2z^3 \int_z^\infty \frac{dz_1}{z_1^2} \frac{1}{\frac{1}{z} - \frac{1}{z_1}} \hat{H}_{FU}^{\Im}(z, z_1) = H(z) + 2z H_1^{\perp(1)}(z)$$

QCD e.o.m. relation

$$\frac{H(z)}{z} = -\left(1 - z\frac{d}{dz}\right) H_1^{\perp(1)}(z) - \frac{2}{z} \int_z^\infty \frac{dz_1}{z_1^2} \frac{\hat{H}_{FU}^{\Im}(z, z_1)}{(1/z - 1/z_1)^2}$$

LIR

$$2z^3 \int_z^\infty \frac{dz_1}{z_1^2} \frac{1}{\frac{1}{z} - \frac{1}{z_1}} \hat{H}_{FU}^{\Im}(z, z_1) = H(z) + 2z H_1^{\perp(1)}(z)$$

QCD e.o.m. relation

$$\frac{H(z)}{z} = -\left(1 - z\frac{d}{dz}\right) H_1^{\perp(1)}(z) - \frac{2}{z}\int_z^\infty \frac{dz_1}{z_1^2} \frac{\hat{H}_{FU}^{\Im}(z, z_1)}{(1/z - 1/z_1)^2}$$

LIR

$$H(z) = \int_z^1 dz_1 \int_{z_1}^\infty \frac{dz_2}{z_2^2}\, 2 \left[\frac{\left(2(\frac{2}{z_1} - \frac{1}{z_2}) + \frac{1}{z_1}\left(\frac{1}{z_1} - \frac{1}{z_2}\right)\delta\left(\frac{1}{z_1} - \frac{1}{z}\right)\right)}{\left(\frac{1}{z_1} - \frac{1}{z_2}\right)^2} \hat{H}_{FU}^{\Im}(z_1, z_2) \right]$$

$$H_1^{\perp(1)}(z) = -\frac{2}{z}\int_z^1 dz_1 \int_{z_1}^\infty \frac{dz_2}{z_2^2} \frac{\left(\frac{2}{z_1} - \frac{1}{z_2}\right)}{\left(\frac{1}{z_1} - \frac{1}{z_2}\right)^2} \hat{H}_{FU}^{\Im}(z_1, z_2)$$

$$2z^3 \int_z^\infty \frac{dz_1}{z_1^2} \frac{1}{\frac{1}{z} - \frac{1}{z_1}} \hat{H}_{FU}^{\Im}(z, z_1) = H(z) + 2z H_1^{\perp(1)}(z)$$

QCD e.o.m. relation

$$\frac{H(z)}{z} = -\left(1 - z\frac{d}{dz}\right) H_1^{\perp(1)}(z) - \frac{2}{z}\int_z^\infty \frac{dz_1}{z_1^2} \frac{\hat{H}_{FU}^{\Im}(z, z_1)}{(1/z - 1/z_1)^2}$$

LIR

$$H(z) = \int_z^1 dz_1 \int_{z_1}^\infty \frac{dz_2}{z_2^2}\, 2 \left[\frac{\left(2(\frac{2}{z_1} - \frac{1}{z_2}) + \frac{1}{z_1}\left(\frac{1}{z_1} - \frac{1}{z_2}\right)\delta\left(\frac{1}{z_1} - \frac{1}{z}\right)\right)}{\left(\frac{1}{z_1} - \frac{1}{z_2}\right)^2} \hat{H}_{FU}^{\Im}(z_1, z_2) \right]$$

$$H_1^{\perp(1)}(z) = -\frac{2}{z}\int_z^1 dz_1 \int_{z_1}^\infty \frac{dz_2}{z_2^2} \frac{\left(\frac{2}{z_1} - \frac{1}{z_2}\right)}{\left(\frac{1}{z_1} - \frac{1}{z_2}\right)^2} \hat{H}_{FU}^{\Im}(z_1, z_2)$$

Entire result for A_{UTU} (& A_N) can be written in terms of **dynamical** twist-3 functions

➢ A_{LTU} , A_{UUT} , A_{LUT} spin asymmetries

-A_{LTU} , A_{UUT} calculated before
(Kanazawa, Metz, DP, Schlegel - PLB **742** (2014), PLB **744** (2015))

Like with A_{UTU}, we have:

-Calculated these observables in a general frame

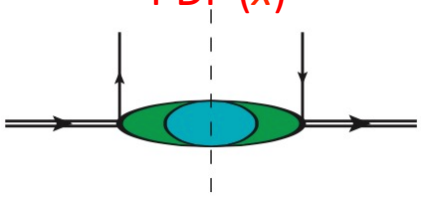
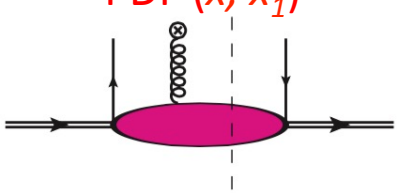
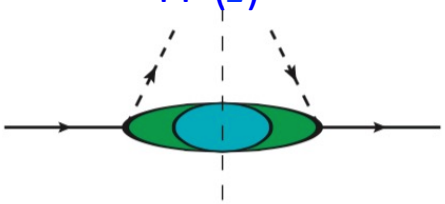
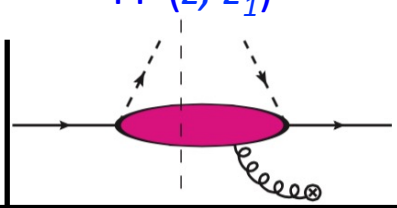
-Derived relevant LIRs (crucial to show frame independence!)

-Shown how all twist-3 observables can be written solely in terms of dynamical functions

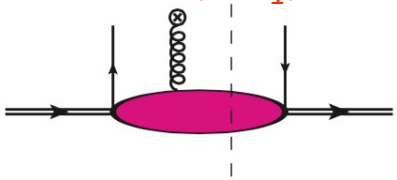
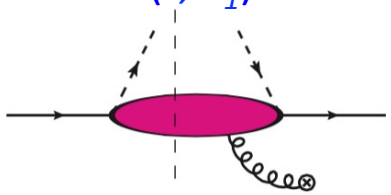
-Calculated in both Feynman gauge and lightcone gauge

-Checked EM gauge invariance

(Kanazawa, Koike, Metz, DP, Schlegel, PRD **93** (2016))

Hadron Pol.	PDF (x)		PDF (x, x_1)	FF (z)		FF (z, z_1)
						
	intrinsic	kinematical	dynamical	intrinsic	kinematical	dynamical
U	f_1	$h_1^{\perp(1)}$	H_{FU}	D_1, E	$H_1^{\perp(1)}$	$\hat{H}_{FU}^{\Re,\Im}$
L	h_L	$h_{1L}^{\perp(1)}$	H_{FL}	D_L, E_L	$H_{1L}^{\perp(1)}$	$\hat{H}_{FL}^{\Re,\Im}$
T	g_T	$f_{1T}^{\perp(1)}$, $g_{1T}^{\perp(1)}$	F_{FT}, G_{FT}	D_T, G_T	$D_{1T}^{\perp(1)}$, $G_{1T}^{\perp(1)}$	$\hat{D}_{FT}^{\Re,\Im}, \hat{G}_{FT}^{\Re,\Im}$

(Kanazawa, Koike, Metz, DP, Schlegel, PRD **93** (2016))

Hadron Pol.	PDF (x, x_1) 	FF (z, z_1) 
U	dynamical H_{FU}	dynamical $\hat{H}_{FU}^{\Re,\Im}$
L	H_{FL}	$\hat{H}_{FL}^{\Re,\Im}$
T	F_{FT}, G_{FT}	$\hat{D}_{FT}^{\Re,\Im}, \hat{G}_{FT}^{\Re,\Im}$

ep observables involving the *transverse spin* of hadrons give direct access to *multi-parton correlations*

(Kanazawa, Koike, Metz, DP, Schlegel, PRD **93** (2016))

Summary and outlook

- TSSAs in proton-proton collisions have been around for close to 40 years, but the underlying mechanism is still unclear
  - ➔ Collinear twist-3 fragmentation could finally give us an explanation
  - ➔ Must continue to find observables to test/understand TSSAs

- Transverse spin observables in lepton-nucleon collisions can test (experimentally *and* theoretically) the collinear twist-3 framework and its explanation of TSSAs
  - ➔ Collinear twist-3 formalism seems to be on sound theoretical ground:
    - -gauge invariance ✔
    - -Lorentz invariance ✔ (LIRs crucial ➔ newly derived for twist-3 FFs)
    - -NLO calculation ★
  - ➔ Already have data on several spin configurations (but LO pQCD applicable???)
  - ➔ EIC would produce hadrons at high enough P_T to safely apply pQCD
    - -Phenomenology still at early stage (A_{UTU} should be ~ 10%)
    - -Can give direct access to dynamical twist-3 functions (Multi-parton correlations)