Transverse Single Spin Asymmetries in Proton-Nucleus collisions

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Based on works done with Arbat, Collins, Kang, Rogers, Vitev, Sterman, Zhang, …

RBRC Workshop on "Forward Physics at RHIC (2012)"
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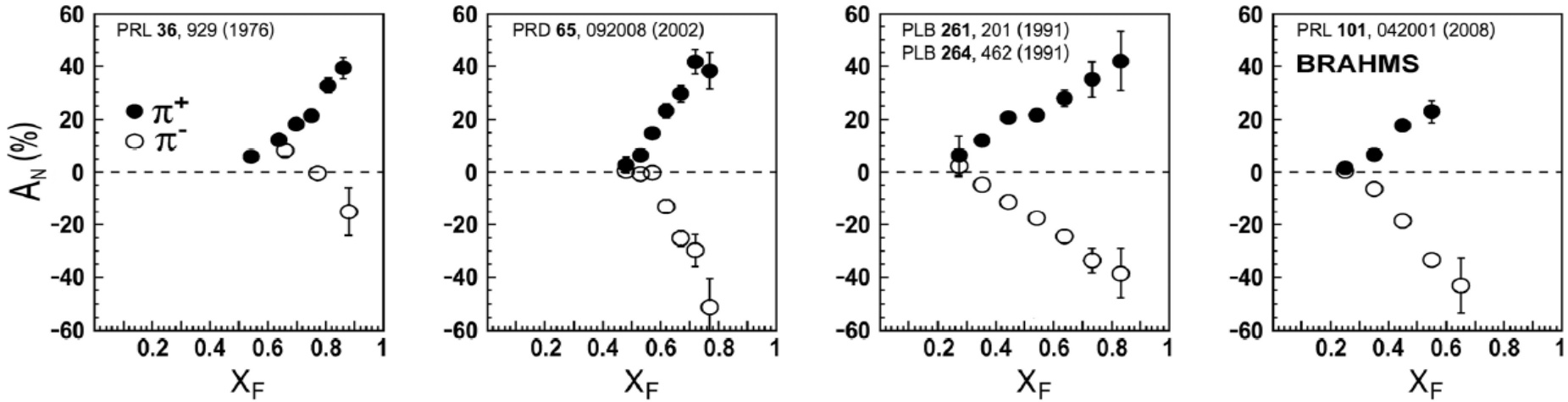
Outline

- ❑ Transverse single spin asymmetry (SSA)
- ❑ Do we understand it?
  - ✧ TMD factorization
  - ✧ Collinear factorization
- ❑ SSA of single hadron production in hadronic collisions
- ❑ SSA in the forward region of pA collisions
- ❑ Summary

Transverse single-spin asymmetry (SSA)

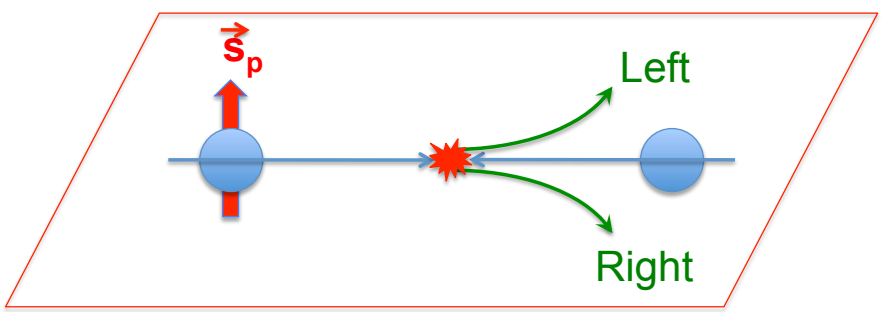
❑ Consistently observed for over 35 years!

ANL – 4.9 GeV | BNL – 6.6 GeV | FNAL – 20 GeV | BNL – 62.4 GeV



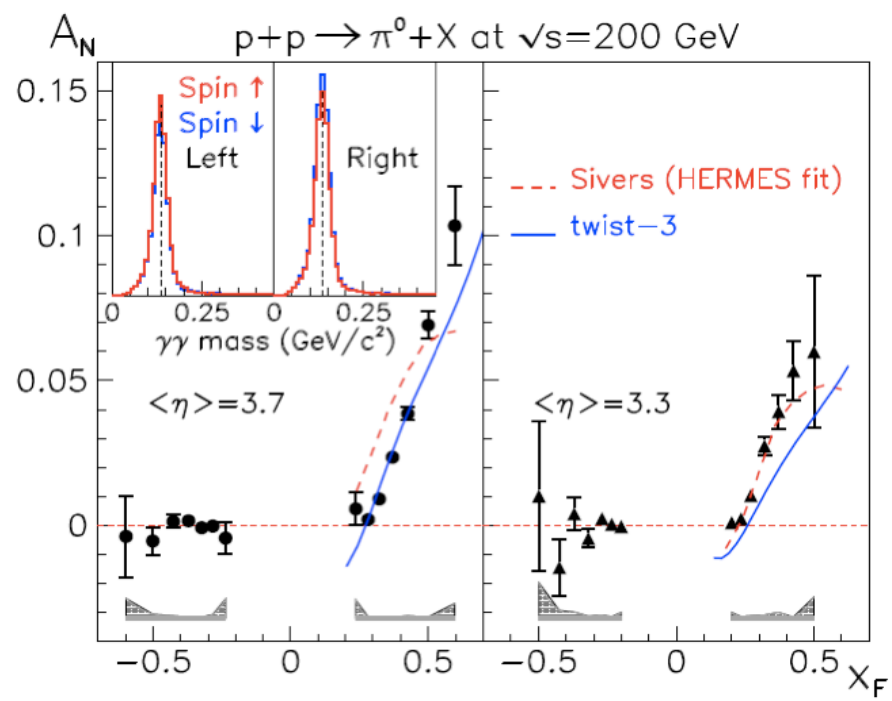
❑ Definition:

$$p(\vec{s}_\perp) + p \to h(\pi^\pm, \pi^0, ...) + X$$



$$A_N \equiv \frac{\Delta\sigma(\ell, \vec{s})}{\sigma(\ell)} = \frac{\sigma(\ell, \vec{s}) - \sigma(\ell, -\vec{s})}{\sigma(\ell, \vec{s}) + \sigma(\ell, -\vec{s})}$$

BNL – 200 GeV

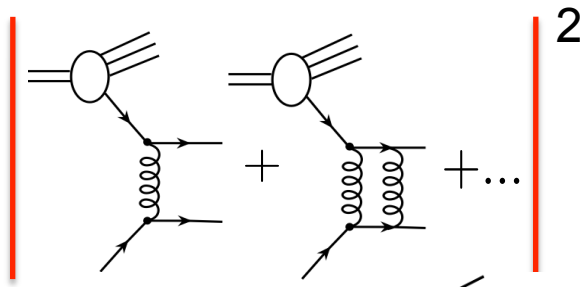


Do we understand it?

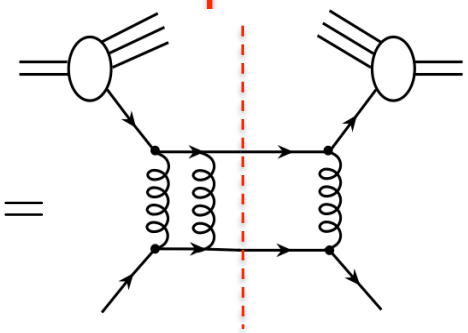
Kane, Pumplin, Repko, PRL, 1978

❑ Early attempt:

Cross section: $\sigma_{AB}(p_T, \vec{s}) \propto$



Asymmetry: $\sigma_{AB}(p_T, \vec{s}) - \sigma_{AB}(p_T, -\vec{s}) =$



$\propto \alpha_s \dfrac{m_q}{p_T}$

Too small to explain available data!

❑ What do we need?

$$A_N \propto i\vec{s}_p \cdot (\vec{p}_h \times \vec{p}_T) \Rightarrow i\epsilon^{\mu\nu\alpha\beta} p_{h\mu} s_\nu p_\alpha p'_{h\beta}$$

Need a phase, a spin flip, enough vectors

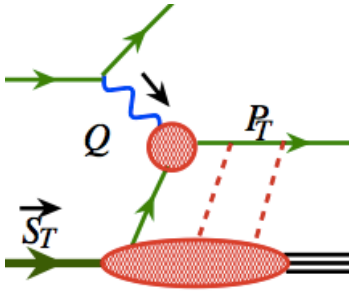
❑ Vanish without parton's transverse motion:

➡ **A direct probe for parton's transverse motion,**

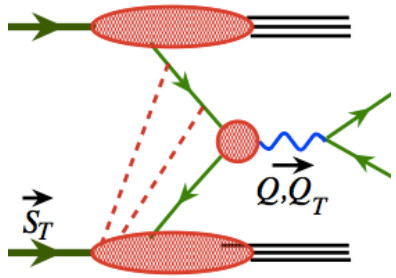
Spin-orbital correlation, QCD quantum interference

Current understanding of SSAs

- Two scales observables – $Q_1 >> Q_2 \sim \Lambda_{QCD}$:



SIDIS: $Q >> P_T$

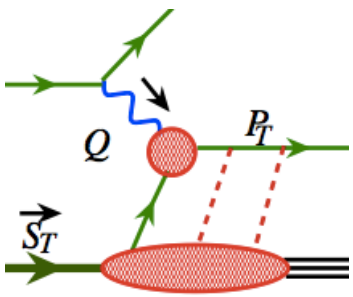


DY: $Q >> Q_T$

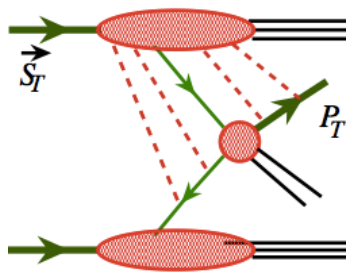
→ TMD factorization
TMD distributions

Talks by Mulders, Yuan, …

- One scale observables – $Q >> \Lambda_{QCD}$:



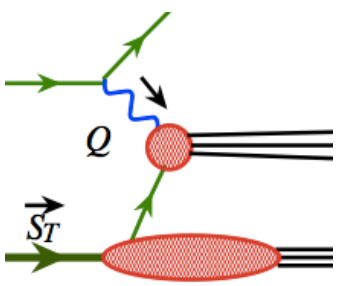
SIDIS: $Q \sim P_T$



Jet, Particle: P_T

→ Collinear factorization
Twist-3 distributions

- Symmetry plays important role:



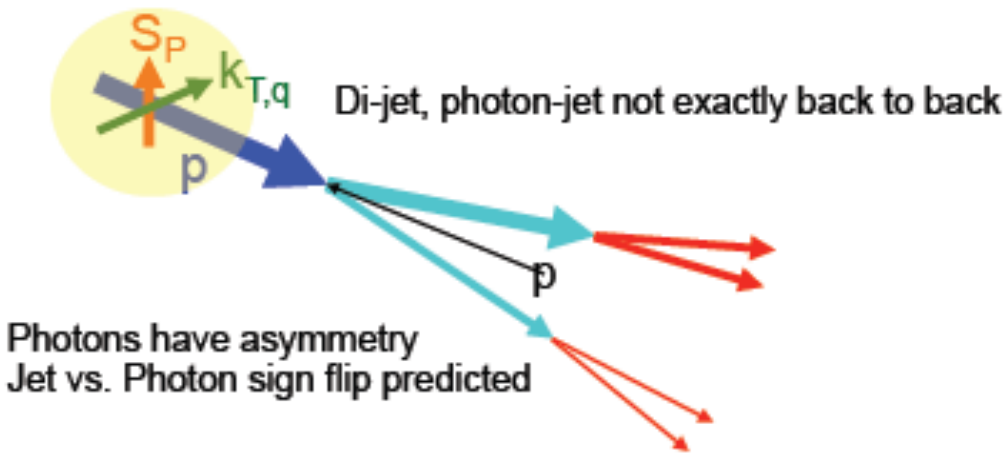
Inclusive DIS
Single scale
Q

Parity
Time-reversal

→ $A_N = 0$

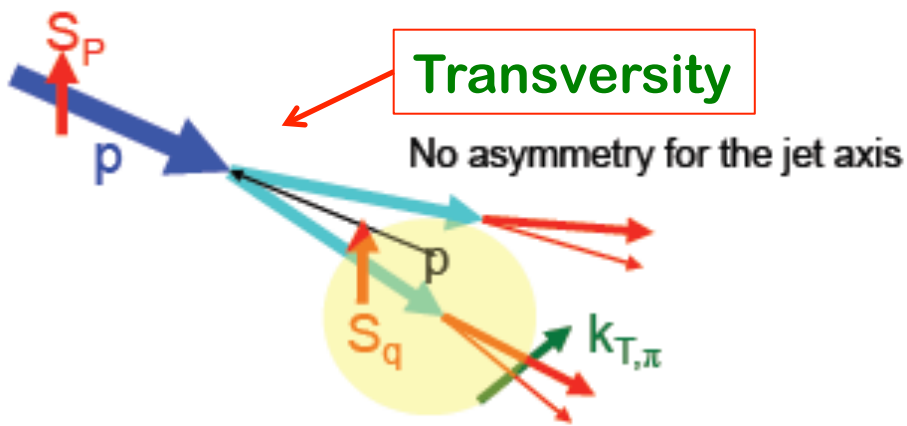
How TMDs generate spin asymmetry?

❑ Sivers' effect – Sivers' function:



Hadron spin influences parton's transverse motion

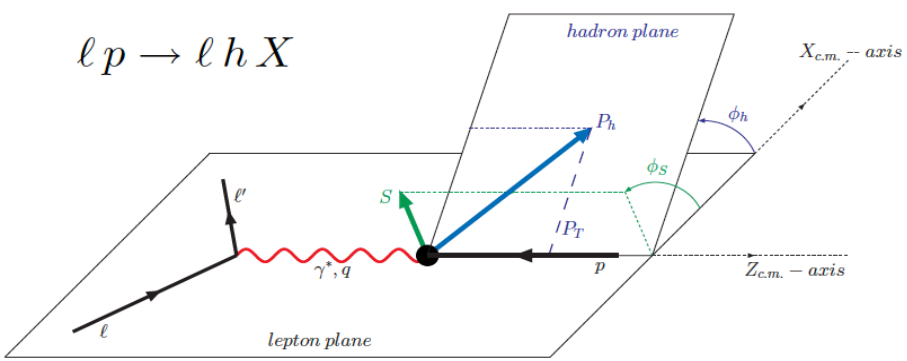
❑ Collin's effect – Collin's function:



Parton's transverse spin affects its hadronization

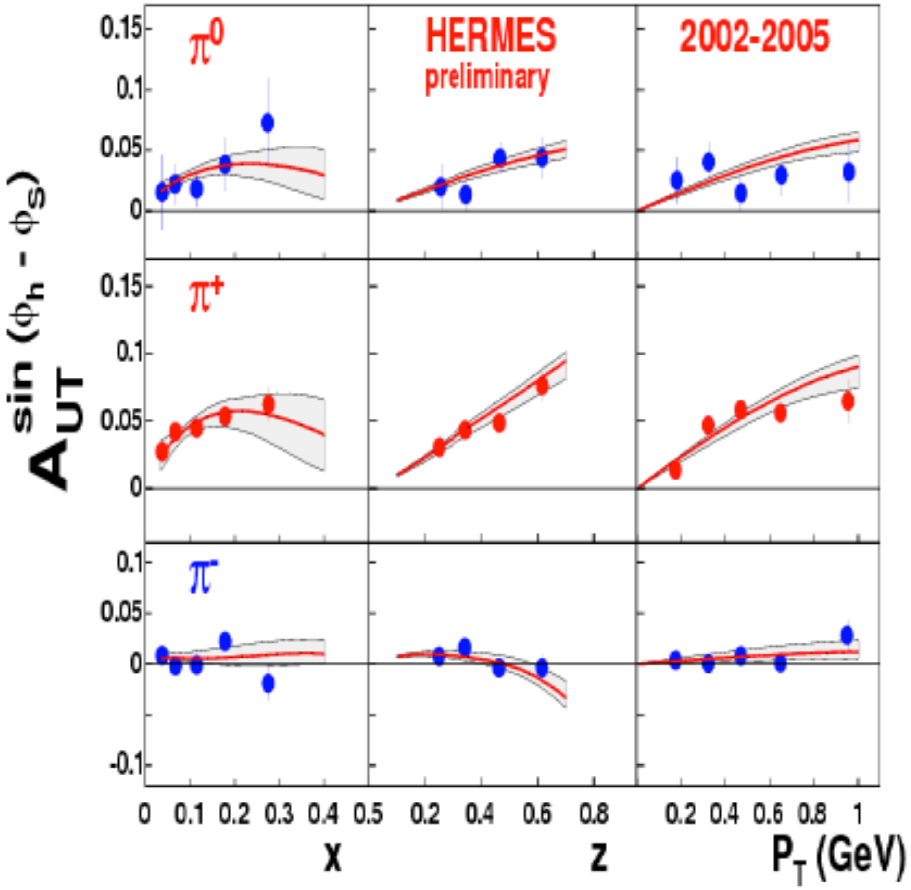
❑ Separation of different effects?

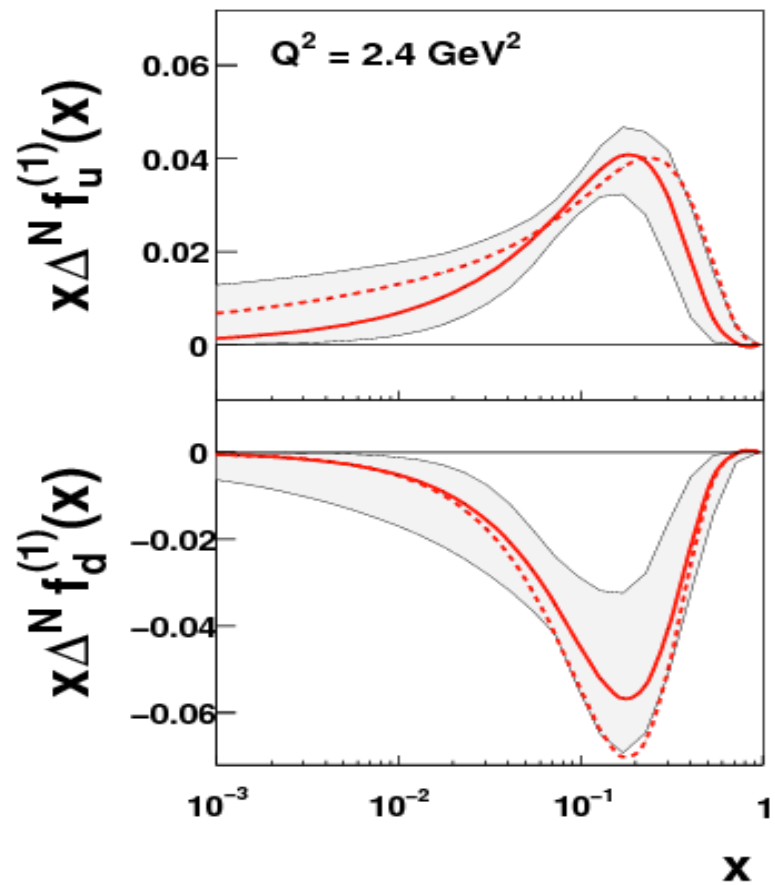
Best in SIDIS, at EIC



Our knowledge of TMDs

- Sivers function from low energy SIDIS:





EIC can do much better job in extracting TMDs

- NO TMD factorization for hadron production in p+p collisions!

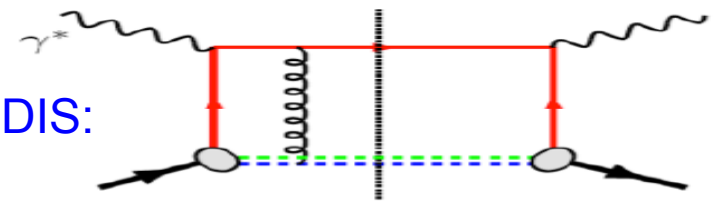
Collins and Qiu, 2007, Vogelsang and Yuan, 2007, Mulders and Rogers, 2010, …

Critical test of TMD factorization

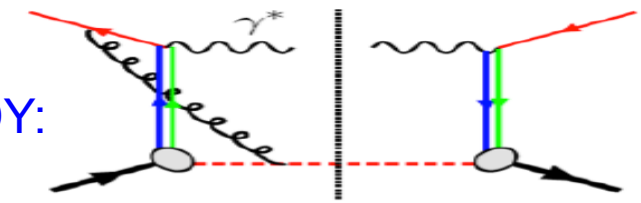
❑ TMD distributions with non-local gauge links:

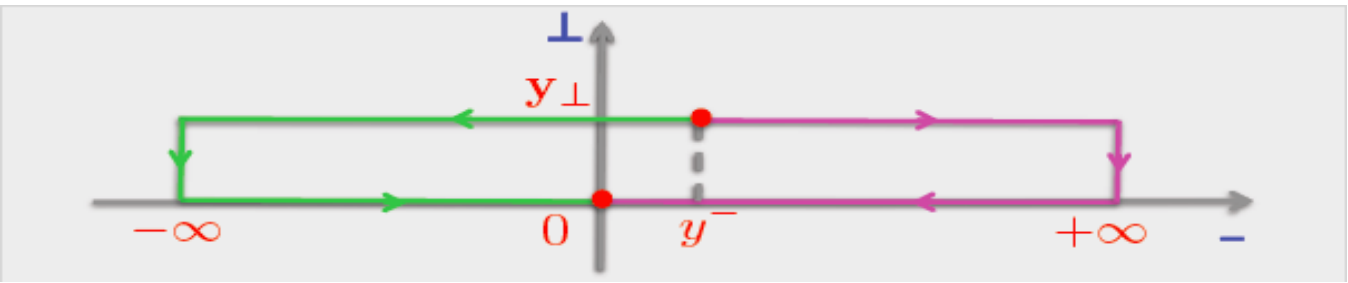
$$f_{q/h^\uparrow}(x,\mathbf{k}_\perp,\vec{S}) = \int \frac{dy^- d^2y_\perp}{(2\pi)^3} e^{ixp^+y^- - i\mathbf{k}_\perp\cdot\mathbf{y}_\perp} \langle p,\vec{S}|\overline{\psi}(0^-,\mathbf{0}_\perp)\,\boxed{\text{Gauge link}}\,\frac{\gamma^+}{2}\psi(y^-,\mathbf{y}_\perp)|p,\vec{S}\rangle$$

SIDIS:



DY:





- For a fixed spin state:

$$f^{\text{SIDIS}}_{q/h^\uparrow}(x,\mathbf{k}_\perp,\vec{S}) \neq f^{\text{DY}}_{q/h^\uparrow}(x,\mathbf{k}_\perp,\vec{S})$$

❑ Parity + Time-reversal invariance:

$$\Rightarrow \quad f^{\text{Sivers}}_{q/h^\uparrow}(x,k_\perp)^{\text{SIDIS}} = -f^{\text{Sivers}}_{q/h^\uparrow}(x,k_\perp)^{\text{DY}}$$

The sign change is a critical test of TMD factorization approach

Another critical test of TMD factorization

❑ Predictive power of QCD factorization:

- ✧ Infrared safety of short-distance hard parts
- ✧ Universality of the long-distance matrix elements
- ✧ QCD evolution or scale dependence of the matrix elements

❑ QCD evolution:

If there is a factorization/invariance, there is an evolution equation

❑ Collinear factorization – DGLAP evolution:

$$\sigma_{\rm phy}(Q,\Lambda_{\rm QCD}) \approx \sum_f \hat{\sigma}_f(Q,\mu) \otimes \phi_f(\mu,\Lambda_{\rm QCD}) \quad \rightarrow \quad \frac{d}{d\mu}\sigma_{\rm phy}(Q,\Lambda_{\rm QCD}) = 0$$

Scaling violation of nonperturbative functions

Evolution kernels are perturbative – a test of QCD

Evolution equations for TMDs

Boer, 2001, 2009, Idilbi, et al, 2004
Aybat, Rogers, 2010
Kang, Xiao, Yuan, 2011
Aybat, Collins, Qiu, Rogers, 2011

❑ Collins-Soper equation:

– b-space quark TMD with γ^+

$$\frac{\partial \tilde{F}_{f/P^\uparrow}(x, \mathbf{b}_\mathrm{T}, S; \mu; \zeta_F)}{\partial \ln \sqrt{\zeta_F}} = \tilde{K}(b_T; \mu) \tilde{F}_{f/P^\uparrow}(x, \mathbf{b}_\mathrm{T}, S; \mu; \zeta_F) \qquad \tilde{K}(b_T; \mu) = \frac{1}{2} \frac{\partial}{\partial y_s} \ln \left(\frac{\tilde{S}(b_T; y_s, -\infty)}{\tilde{S}(b_T; +\infty, y_s)} \right)$$

❑ RG equations:

$$\frac{d\tilde{K}(b_T; \mu)}{d \ln \mu} = -\gamma_K(g(\mu)) \qquad \frac{d\tilde{F}_{f/P^\uparrow}(x, \mathbf{b}_\mathrm{T}, S; \mu; \zeta_F)}{d \ln \mu} = \gamma_F(g(\mu); \zeta_F/\mu^2) \tilde{F}_{f/P^\uparrow}(x, \mathbf{b}_\mathrm{T}, S; \mu; \zeta_F).$$

❑ Evolution equations for Sivers function:

$$F_{f/P^\uparrow}(x, k_T, S; \mu, \zeta_F) = F_{f/P}(x, k_T; \mu, \zeta_F) - F_{1T}^{\perp f}(x, k_T; \mu, \zeta_F) \frac{\epsilon_{ij} k_T^i S^j}{M_p}$$

CS: $$\frac{\partial \ln \tilde{F}_{1T}'^{\perp f}(x, b_T; \mu, \zeta_F)}{\partial \ln \sqrt{\zeta_F}} = \tilde{K}(b_T; \mu) \qquad \tilde{F}_{1T}'^{\perp f}(x, b_T; \mu, \zeta_F) \equiv \frac{\partial \tilde{F}_{1T}^{\perp f}(x, b_T; \mu, \zeta_F)}{\partial b_T}$$

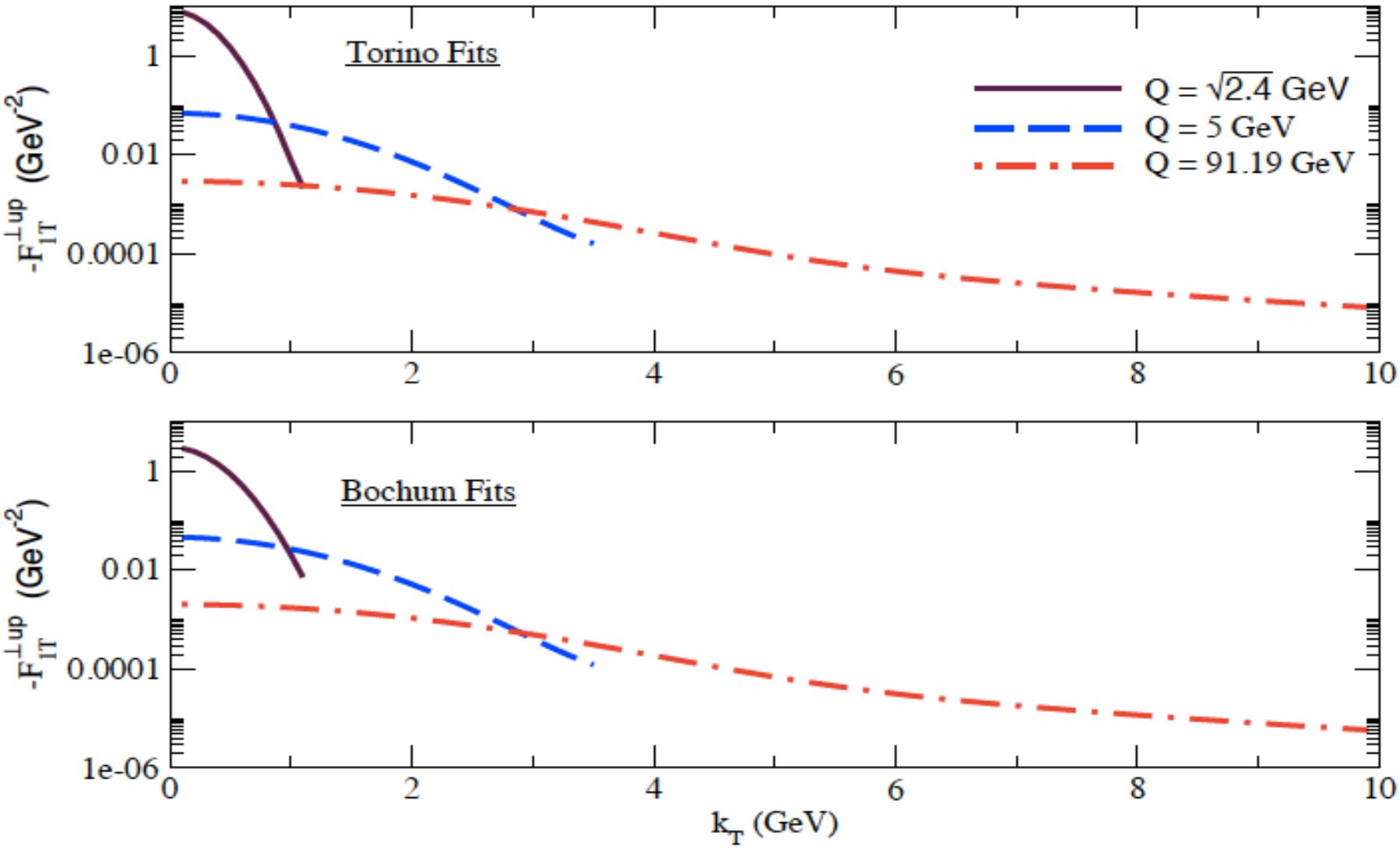
RGs: $$\frac{d\tilde{F}_{1T}'^{\perp f}(x, b_T; \mu, \zeta_F)}{d \ln \mu} = \gamma_F(g(\mu); \zeta_F/\mu^2) \tilde{F}_{1T}'^{\perp f}(x, b_T; \mu, \zeta_F)$$

$$\frac{d\tilde{K}(b_T; \mu)}{d \ln \mu} = -\gamma_K(g(\mu)) \quad \Rightarrow \quad \frac{\partial \gamma_F(g(\mu); \zeta_F/\mu^2)}{\partial \ln \sqrt{\zeta_F}} = -\gamma_K(g(\mu)),$$

Scale dependence of Sivers function

Aybat, Collins, Qiu, Rogers, 2011

- Up quark Sivers function:

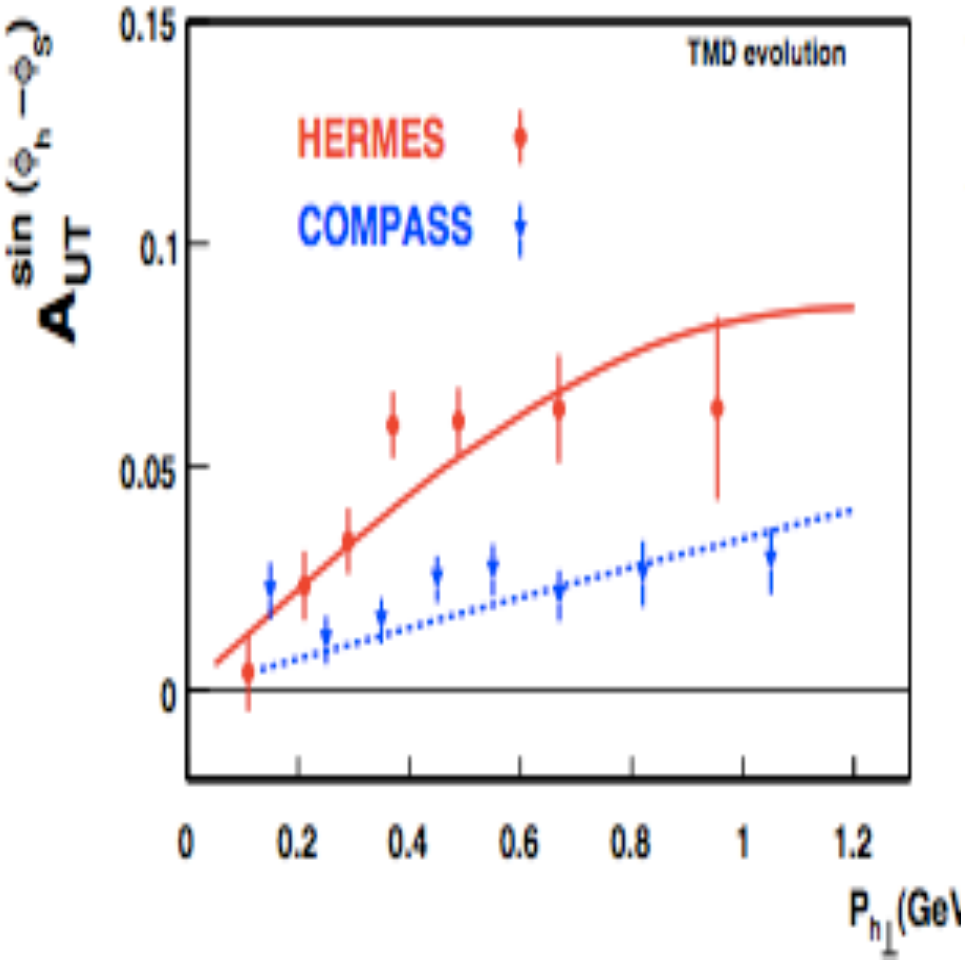


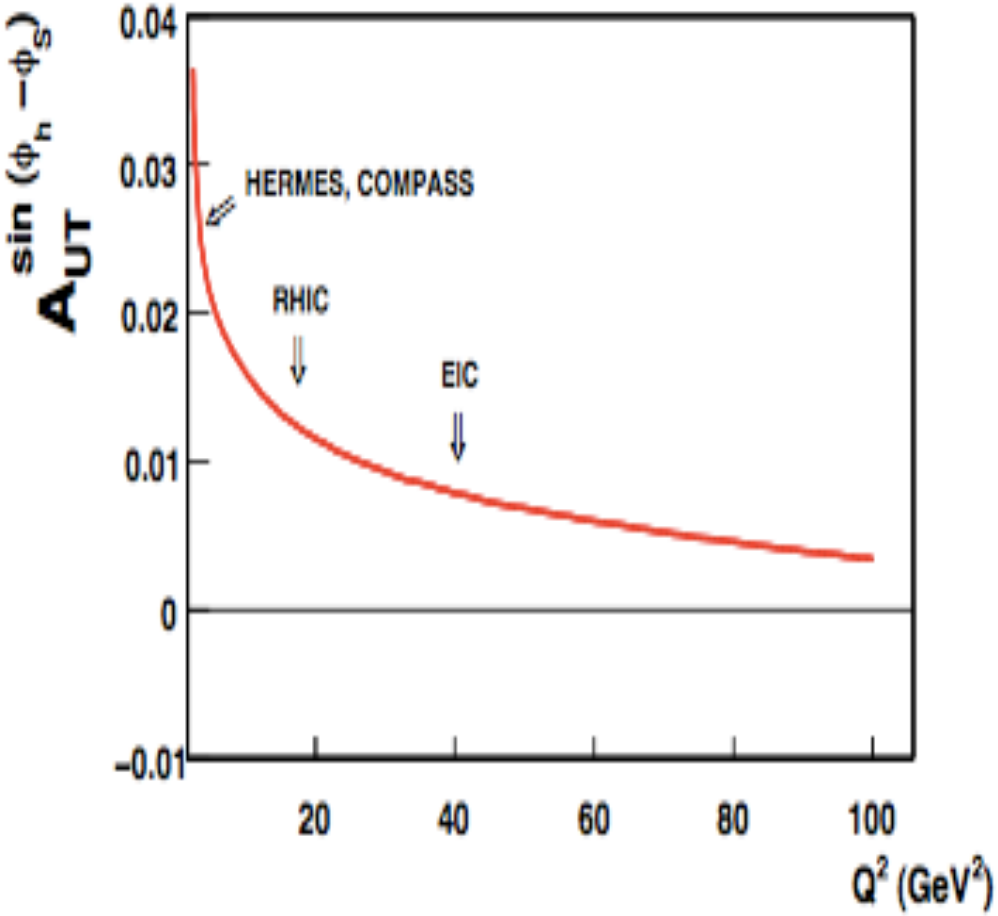
Very significant growth in the width of transverse momentum

Importance of the evolution

Aybat, Rogers, 2012

- SSAs – Sivers function:

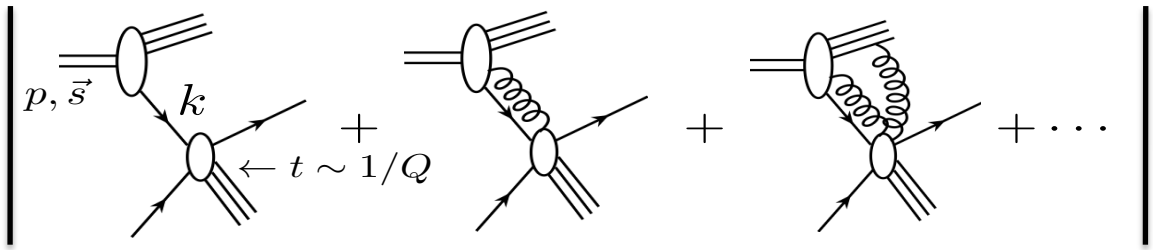




Q^2 dependence – effectiveness of the probe?

How collinear factorization generates SSA?

❑ Collinear factorization beyond leading power:

$\sigma(Q,\vec{s}) \propto$ 

$^2 \left(\frac{\langle k_\perp \rangle}{Q}\right)^n$ – Expansion

$$\sigma(Q, s_T) = H_0 \otimes f_2 \otimes f_2 + (1/Q)\, H_1 \otimes f_2 \otimes f_3 + \mathcal{O}(1/Q^2)$$

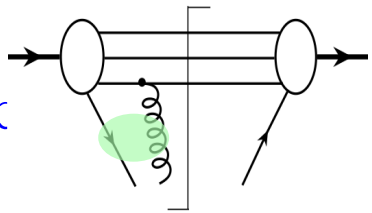
Too large to compete!

Three-parton correlation

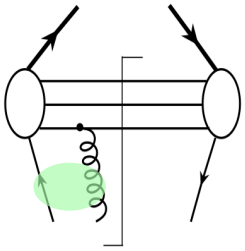
❑ Single transverse spin asymmetry:

Efremov, Teryaev, 82;
Qiu, Sterman, 91, etc.

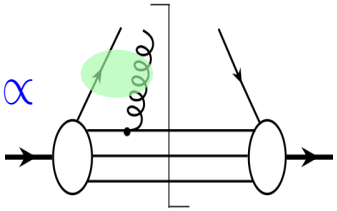
$$\Delta\sigma(s_T) \propto T^{(3)}(x,x) \otimes \hat{\sigma}_T \otimes D(z) + \delta q(x) \otimes \hat{\sigma}_D \otimes D^{(3)}(z,z) + \ldots$$

$T^{(3)}(x,x) \propto$ 

Qiu, Sterman, 1991, …

$D^{(3)}(z,z) \propto$ 

Kang, Yuan, Zhou, 2010

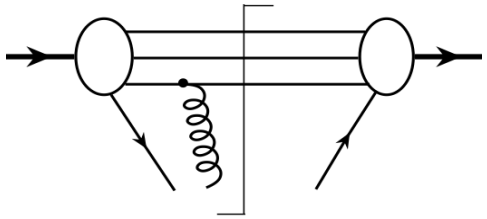
$T^{(3\sigma)}(x,x) \propto$ 

Kanazawa, Koike, 2000

Integrated information on parton's transverse motion!

Twist-3 distributions relevant to A_N

❑ Two-sets Twist-3 correlation functions:



No probability interpretation!

$$\widetilde{\mathcal{T}}_{q,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2} e^{ixP^+y_1^-} e^{ix_2P^+y_2^-} \langle P, s_T|\overline{\psi}_q(0)\frac{\gamma^+}{2}\left[\epsilon^{s_T\sigma n\bar{n}}F_\sigma{}^+(y_2^-)\right]\psi_q(y_1^-)|P, s_T\rangle$$

Kang, Qiu, 2009

$$\widetilde{\mathcal{T}}^{(f,d)}_{G,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2} e^{ixP^+y_1^-} e^{ix_2P^+y_2^-} \frac{1}{P^+}\langle P, s_T|F^{+\rho}(0)\left[\epsilon^{s_T\sigma n\bar{n}}F_\sigma{}^+(y_2^-)\right]F^{+\lambda}(y_1^-)|P, s_T\rangle(-g_{\rho\lambda})$$

$$\widetilde{\mathcal{T}}_{\Delta q,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2} e^{ixP^+y_1^-} e^{ix_2P^+y_2^-} \langle P, s_T|\overline{\psi}_q(0)\frac{\gamma^+\gamma^5}{2}\left[i\, s_T^\sigma\, F_\sigma{}^+(y_2^-)\right]\psi_q(y_1^-)|P, s_T\rangle$$

$$\widetilde{\mathcal{T}}^{(f,d)}_{\Delta G,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2} e^{ixP^+y_1^-} e^{ix_2P^+y_2^-} \frac{1}{P^+}\langle P, s_T|F^{+\rho}(0)\left[i\, s_T^\sigma\, F_\sigma{}^+(y_2^-)\right]F^{+\lambda}(y_1^-)|P, s_T\rangle\,(i\epsilon_{\perp\rho\lambda})$$

Role of color magnetic force!

❑ Twist-2 distributions:

- Unpolarized PDFs:

$$q(x) \propto \langle P|\overline{\psi}_q(0)\frac{\gamma^+}{2}\psi_q(y)|P\rangle$$
$$G(x) \propto \langle P|F^{+\mu}(0)F^{+\nu}(y)|P\rangle(-g_{\mu\nu})$$

- Polarized PDFs:

$$\Delta q(x) \propto \langle P, S_\parallel|\overline{\psi}_q(0)\frac{\gamma^+\gamma^5}{2}\psi_q(y)|P, S_\parallel\rangle$$
$$\Delta G(x) \propto \langle P, S_\parallel|F^{+\mu}(0)F^{+\nu}(y)|P, S_\parallel\rangle(i\epsilon_{\perp\mu\nu})$$

❑ Twist-3 fragmentation functions:

See Kang, Yuan, Zhou, 2010, Kang 2010

Test QCD at twist-3 level

❑ Scaling violation – "DGLAP" evolution:

Kang, Qiu, 2009; Yuan, Zhou, 2009
Vogelsang, Yuan, 2009, Braun et al, 2009

$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \begin{pmatrix} \widetilde{\mathcal{T}}_{q,F} \\ \widetilde{\mathcal{T}}_{\Delta q,F} \\ \widetilde{\mathcal{T}}^{(f)}_{G,F} \\ \widetilde{\mathcal{T}}^{(d)}_{G,F} \\ \widetilde{\mathcal{T}}^{(f)}_{\Delta G,F} \\ \widetilde{\mathcal{T}}^{(d)}_{\Delta G,F} \end{pmatrix} = \begin{pmatrix} K_{qq} & K_{q\Delta q} & K^{(f)}_{qG} & K^{(d)}_{qG} & K^{(f)}_{q\Delta G} & K^{(d)}_{q\Delta G} \\ K_{\Delta qq} & K_{\Delta q\Delta q} & K^{(f)}_{\Delta qG} & K^{(d)}_{\Delta qG} & K^{(f)}_{\Delta q\Delta G} & K^{(d)}_{\Delta q\Delta G} \\ K^{(f)}_{Gq} & K^{(f)}_{G\Delta q} & K^{(ff)}_{GG} & K^{(fd)}_{GG} & K^{(ff)}_{G\Delta G} & K^{(fd)}_{G\Delta G} \\ K^{(d)}_{Gq} & K^{(d)}_{G\Delta q} & K^{(df)}_{GG} & K^{(dd)}_{GG} & K^{(df)}_{G\Delta G} & K^{(dd)}_{G\Delta G} \\ K^{(f)}_{\Delta Gq} & K^{(f)}_{\Delta G\Delta q} & K^{(ff)}_{\Delta GG} & K^{(fd)}_{\Delta GG} & K^{(ff)}_{\Delta G\Delta G} & K^{(fd)}_{\Delta G\Delta G} \\ K^{(d)}_{\Delta Gq} & K^{(d)}_{\Delta G\Delta q} & K^{(df)}_{\Delta GG} & K^{(dd)}_{\Delta GG} & K^{(df)}_{\Delta G\Delta G} & K^{(dd)}_{\Delta G\Delta G} \end{pmatrix} \otimes \begin{pmatrix} \widetilde{\mathcal{T}}_{q,F} \\ \widetilde{\mathcal{T}}_{\Delta q,F} \\ \widetilde{\mathcal{T}}^{(f)}_{G,F} \\ \widetilde{\mathcal{T}}^{(d)}_{G,F} \\ \widetilde{\mathcal{T}}^{(f)}_{\Delta G,F} \\ \widetilde{\mathcal{T}}^{(d)}_{\Delta G,F} \end{pmatrix}$$

$$(x, x+x_2, \mu, s_T) \qquad (\xi, \xi+\xi_2; x, x+x_2, \alpha_s) \qquad \int d\xi \int d\xi_2$$

❑ Evolution equation – consequence of factorization:

Factorization: $\Delta\sigma(Q, s_T) = (1/Q) H_1(Q/\mu_F, \alpha_s) \otimes f_2(\mu_F) \otimes f_3(\mu_F)$

DGLAP for f_2: $\dfrac{\partial}{\partial \ln(\mu_F)} f_2(\mu_F) = P_2 \otimes f_2(\mu_F)$

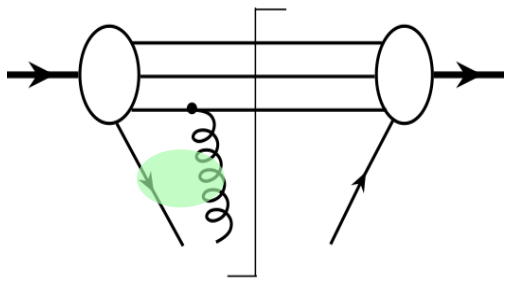
Evolution for f_3: $\dfrac{\partial}{\partial \ln(\mu_F)} f_3 = \left(\dfrac{\partial}{\partial \ln(\mu_F)} H_1^{(1)} - P_2^{(1)} \right) \otimes f_3$

"Interpretation" of twist-3 correlation functions

❑ Measurement of direct QCD quantum interference:

Qiu, Sterman, 1991, …

$T^{(3)}(x, x, S_\perp) \;\propto$



Interference between a single active parton state and an active two-parton composite state

❑ "Expectation value" of QCD operators:

$$\langle P, s|\overline{\psi}(0)\gamma^+\psi(y^-)|P, s\rangle \longrightarrow \langle P, s|\overline{\psi}(0)\gamma^+\left[\epsilon_\perp^{\alpha\beta} s_{T\alpha}\int dy_2^- F_\beta{}^+(y_2^-)\right]\psi(y^-)|P, s\rangle$$

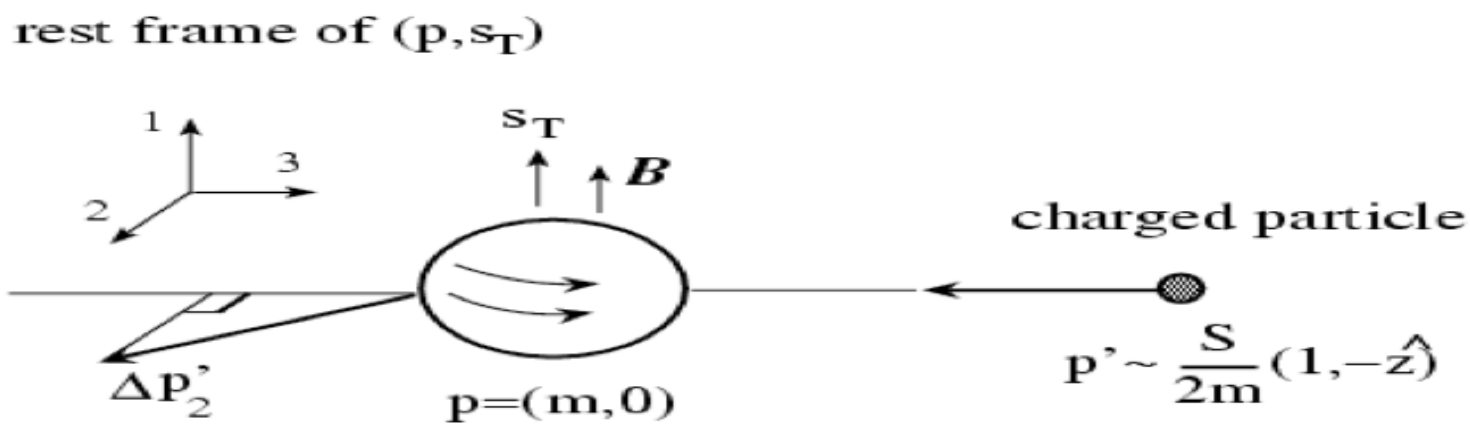
$$\langle P, s|\overline{\psi}(0)\gamma^+\gamma_5\psi(y^-)|P, s\rangle \longrightarrow \langle P, s|\overline{\psi}(0)\gamma^+\left[i\, g_\perp^{\alpha\beta} s_{T\alpha}\int dy_2^- F_\beta{}^+(y_2^-)\right]\psi(y^-)|P, s\rangle$$

How to interpret the "expectation value" of the operators in RED?

A simple example

- The operator in Red – a classical Abelian case:

Qiu, Sterman, 1998



- Change of transverse momentum:

$$\frac{d}{dt}p_2' = e(\vec{v}' \times \vec{B})_2 = -ev_3B_1 = ev_3F_{23}$$

- In the c.m. frame:

$$(m,\vec{0}) \to \bar{n} = (1,0,0_T), \quad (1,-\hat{z}) \to n = (0,1,0_T)$$

$$\Longrightarrow \quad \frac{d}{dt}p_2' = e\,\epsilon^{s_T\sigma n\bar{n}}\,F_\sigma{}^+$$

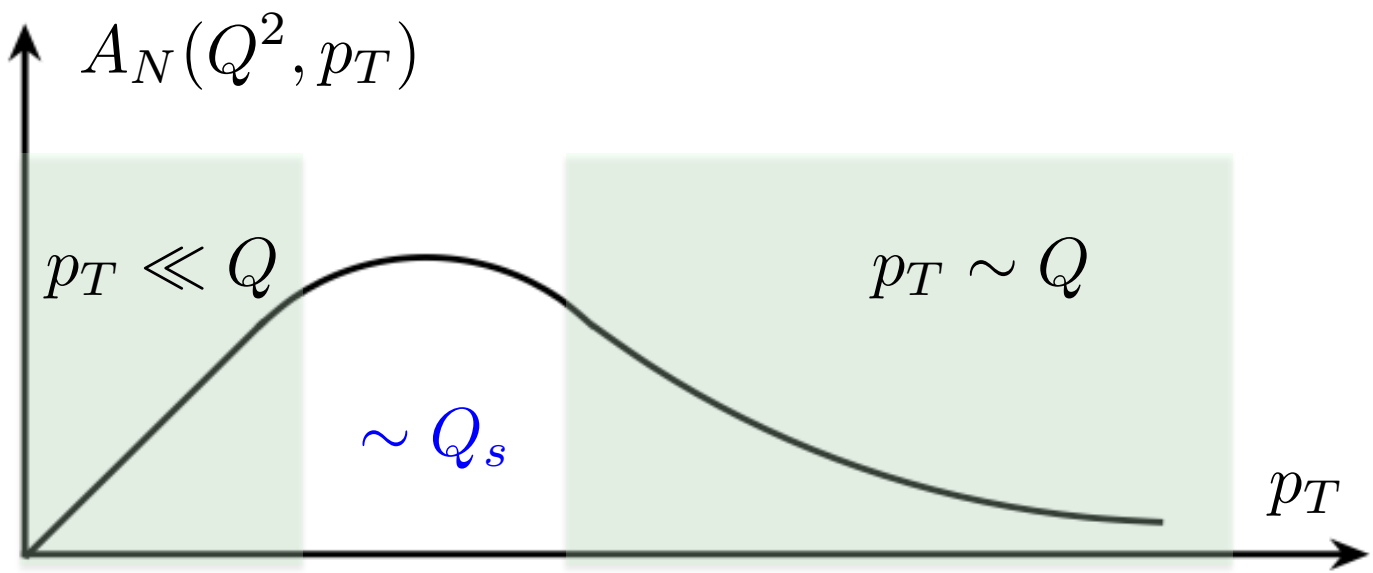
- The total change:

$$\Delta p_2' = e\int dy^-\,\epsilon^{s_T\sigma n\bar{n}}\,F_\sigma{}^+(y^-)$$

Net quark transverse momentum imbalance caused by color Lorentz force inside a transversely polarized proton

Transition from low p_T to high p_T

- Two-scale becomes one-scale:



TMD Collinear Factorization

- TMD factorization to collinear factorization:

Ji,Qiu,Vogelsang,Yuan,
Koike, Vogelsang, Yuan

Two factorization are consistent in the overlap region: $\Lambda_{\rm QCD} \ll p_T \ll Q$

A_N finite – requires correlation of multiple collinear partons

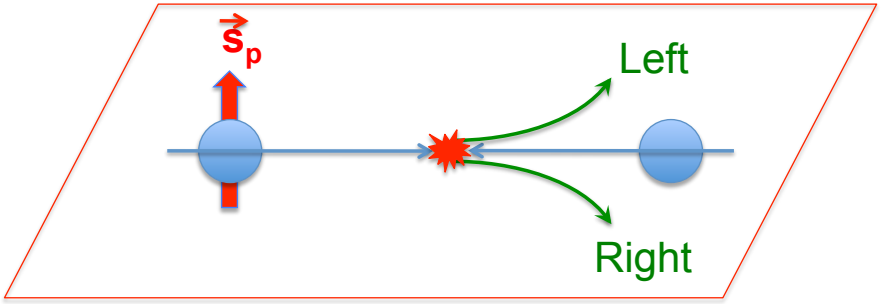
No probability interpretation! New opportunities!

Single hadron production in hadronic collisions

❑ Process:

$$p(\vec{s}_\perp) + p \to h(\pi^\pm, \pi^0, ...) + X$$

$$A_N \equiv \frac{\Delta\sigma(\ell, \vec{s})}{\sigma(\ell)} = \frac{\sigma(\ell, \vec{s}) - \sigma(\ell, -\vec{s})}{\sigma(\ell, \vec{s}) + \sigma(\ell, -\vec{s})}$$



❑ Single hard scale – collinear factorization:

Qiu, Sterman, 1991, 1998

$$A_N \propto \sigma(p_T, S_\perp) - \sigma(p_T, -S_\perp)$$

$$= T^{(3)}_{a/A}(x, x, S_\perp) \otimes \phi_{b/B}(x') \otimes \hat{\sigma}^T_{ab\to c} \otimes D_{h/c}(z)$$

Efremov, Teryaev, Qiu, Sterman, ...

$$+ \delta q_{a/A}(x, S_\perp) \otimes T^{(3\sigma)}_{b/B}(x', x') \otimes \hat{\sigma}^\phi_{ab\to c} \otimes D_{h/c}(z)$$

Kanazawa, Koike, ...

$$+ \delta q_{a/A}(x, S_\perp) \otimes \phi_{b/B}(x', x') \otimes \hat{\sigma}^D_{ab\to c} \otimes D^{(3)}_{h/c}(z, z)$$

Kang, Yuan, Zhou, ...

$$+ \text{ power corrections}$$

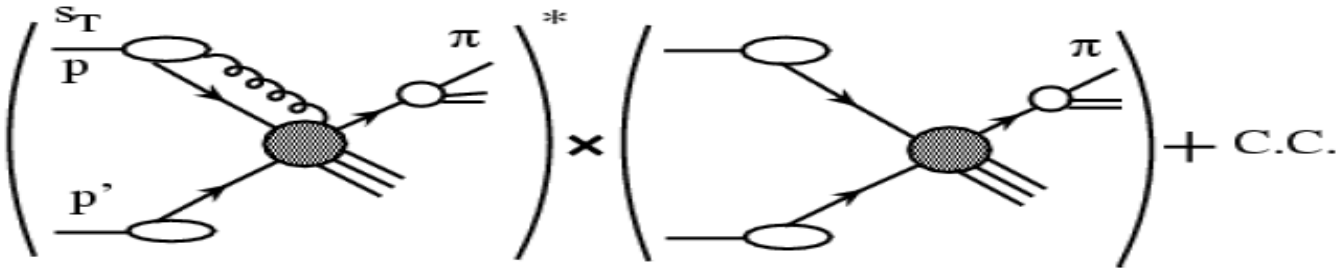
Leading power contribution to cross section cancels!

Only one twist-3 distribution at each term!

Valid when $p_T^2 \gg Q_s^2$, **Corrections:** $\propto \left(\frac{Q_2^2}{p_T^2}\right)^n$

SSAs generated by twist-3 PDFs

- First non-vanish contribution – interference:



- Dominated by the derivative term – forward region:

$$E\frac{d\Delta\sigma}{d^3\ell} \propto \epsilon^{\ell_T s_T n \bar{n}}\, D_{c\to\pi}(z) \otimes \left[-x\frac{\partial}{\partial x}T_F(x,x)\right]$$

$$\otimes \frac{1}{-\hat{u}}\left[G(x')\otimes\Delta\hat{\sigma}_{qg\to c} + \sum_{q'} q'(x')\otimes\Delta\hat{\sigma}_{qq'\to c}\right]$$

Qiu, Sterman, 1998, …

$$A_N \propto \left(\frac{\ell_\perp}{-\hat{u}}\right)\frac{n}{1-x} \quad \text{if } T_F(x,x)\propto q(x)\propto(1-x)^n$$

Kouvaris, Qiu, Vogelsang, Yuan, 2006

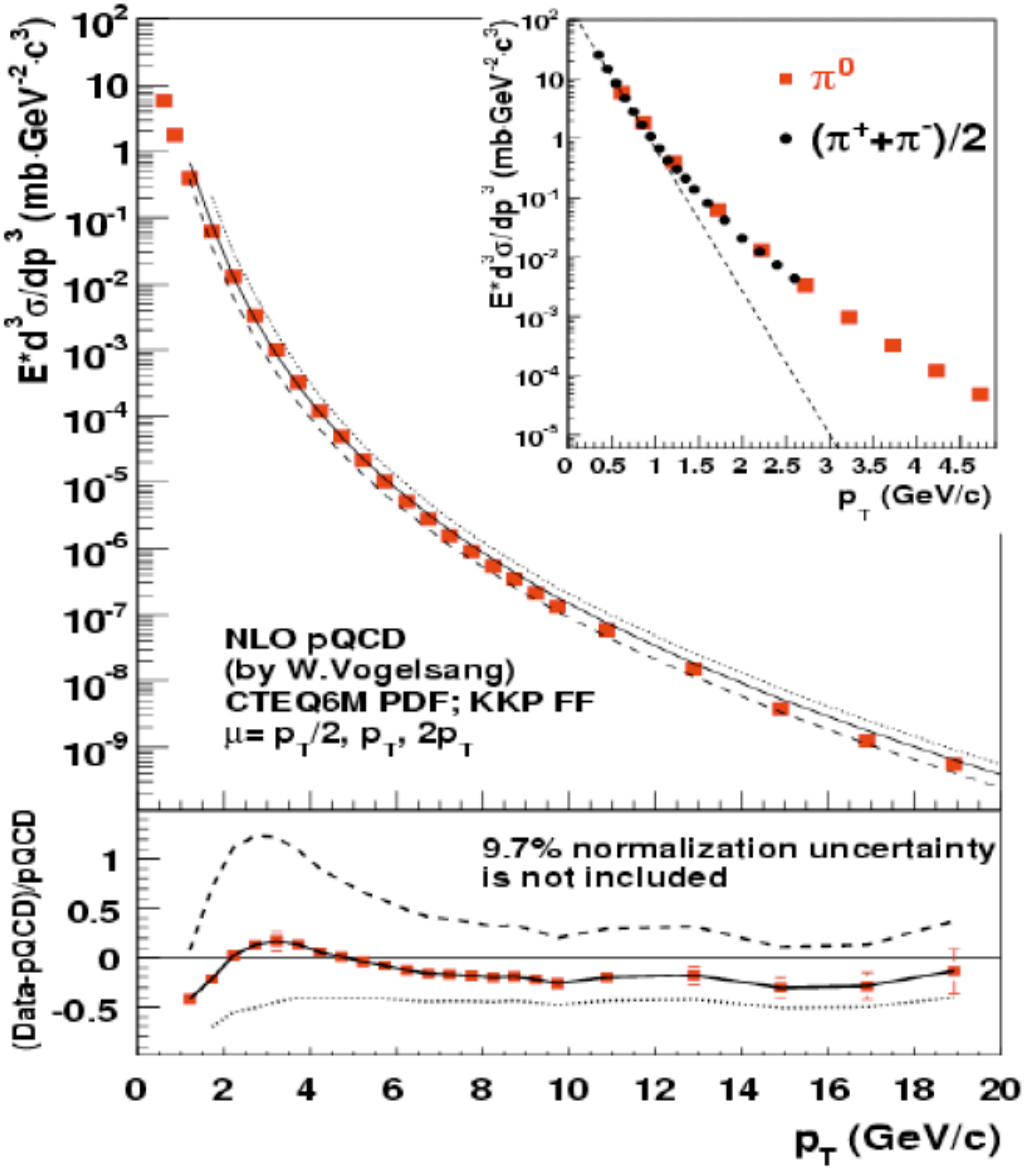
- Complete leading order contribution:

$$E_\ell\frac{d^3\Delta\sigma(\vec{s}_T)}{d^3\ell} = \frac{\alpha_s^2}{S}\sum_{a,b,c}\int_{z_{\min}}^1\frac{dz}{z^2}D_{c\to h}(z)\int_{x'_{\min}}^1\frac{dx'}{x'}\frac{1}{x'S+T/z}\phi_{b/B}(x')\sqrt{4\pi\alpha_s}\left(\frac{\epsilon^{\ell s_T n\bar{n}}}{z\hat{u}}\right)$$

$$\times\frac{1}{x}\left[T_{a,F}(x,x) - x\left(\frac{d}{dx}T_{a,F}(x,x)\right)\right]H_{ab\to c}(\hat{s},\hat{t},\hat{u})$$

Unpolarized single hadron production

- **PHENIX:**



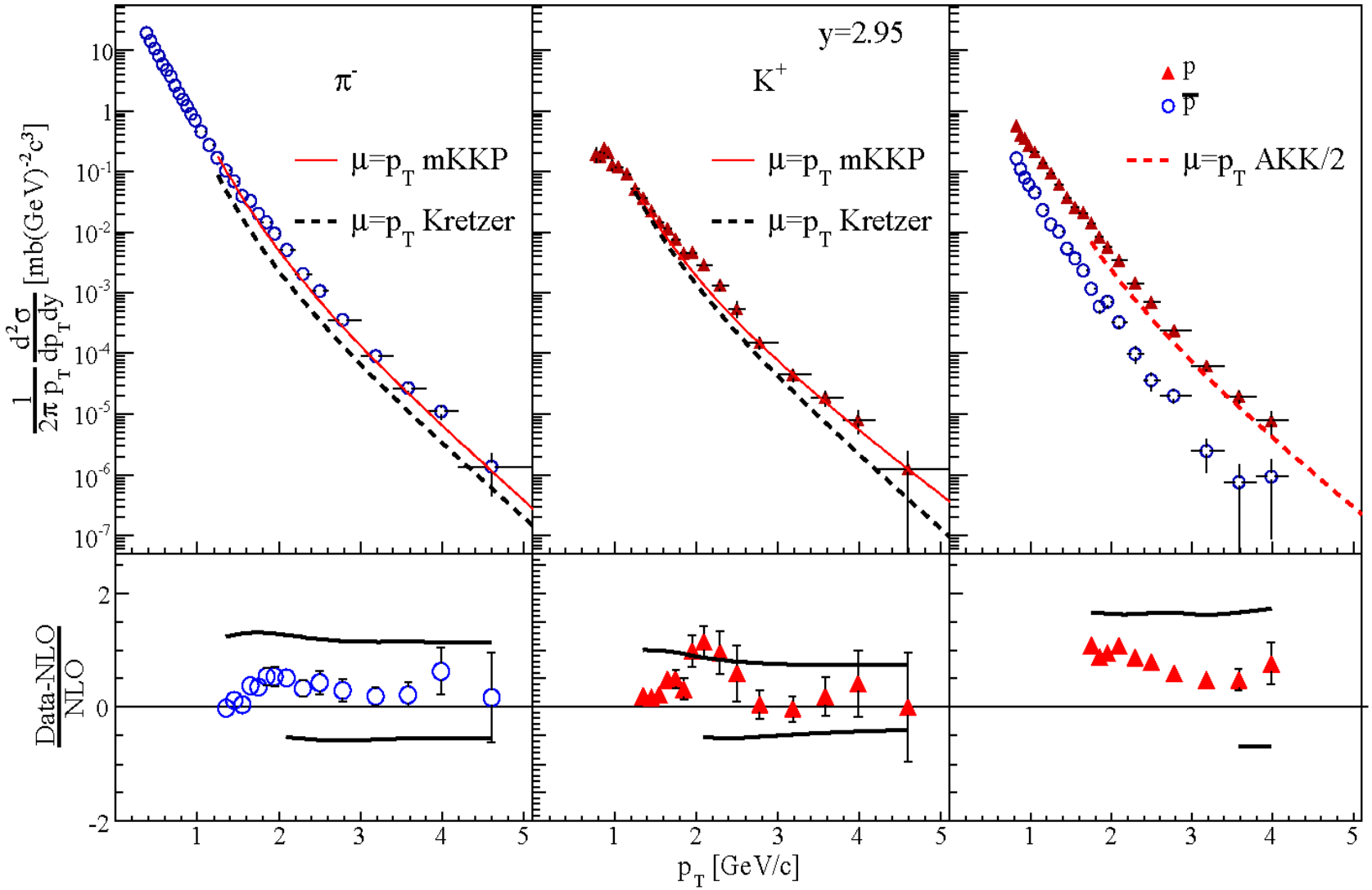
PRD76, 051106 (2007)

QCD factorization/calculation works at RHIC energies!

Extending x coverage and particle type

- **BRAHMS:** Large rapidity p,K,p cross sections for p+p, √s=200 GeV

PRL98, 252001 (2007)

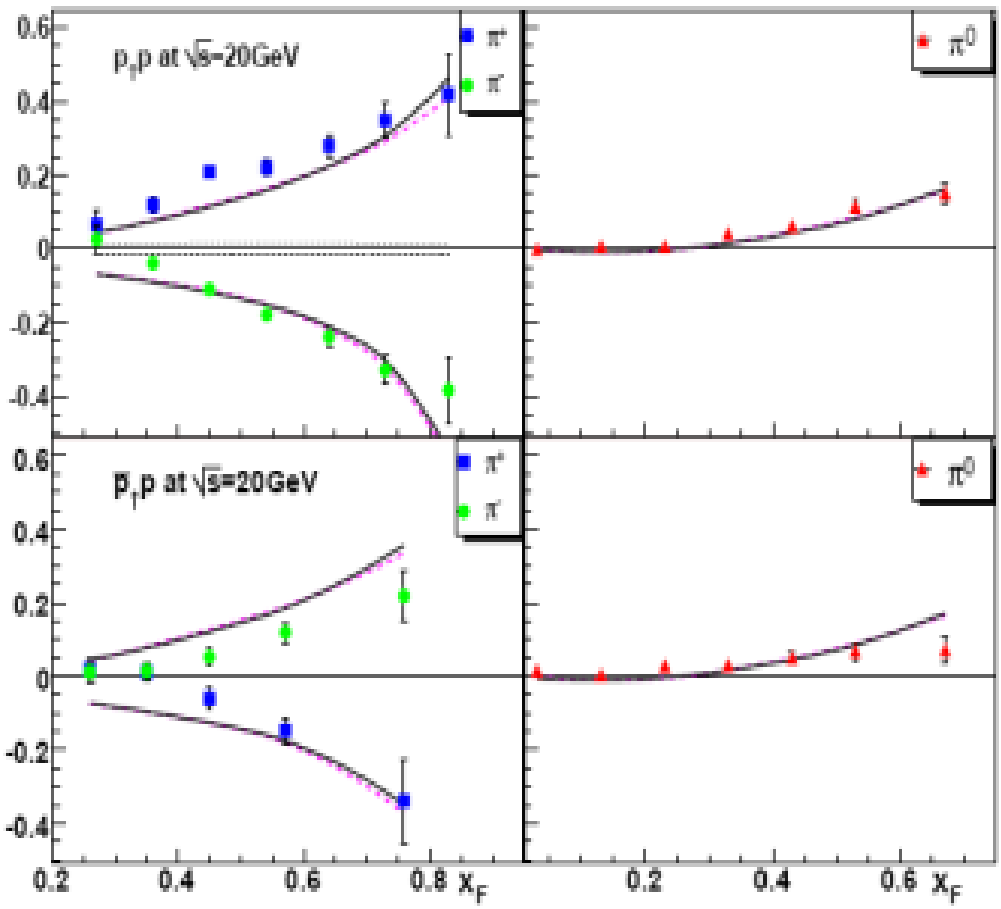


QCD factorization/calculation works at RHIC energies!

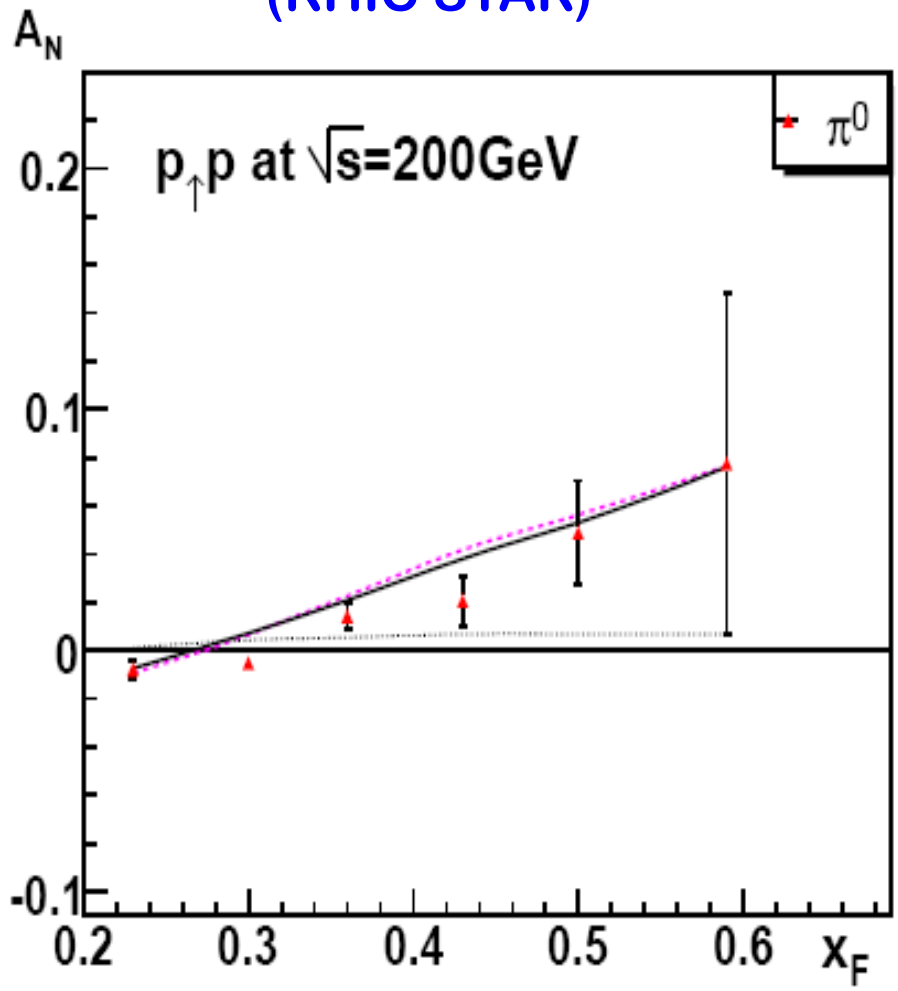
Asymmetries from the $T_F(x,x)$

- Lowest order tree-level contribution: Kouvaris, Qiu, Vogelsang, Yuan, 2006

(FermiLab E704)



(RHIC STAR)



- A_N for jet, D, … production at RHIC and EIC, …

Nonvanish twist-3 function → Nonvanish transverse motion

SSA in the forward region of pA collisions

Excellent probe for distinguishing various contributions to SSA

Excellent probe for studying small-x Physics

SSA increases as x_F (or y) increases

Ideal kinematics for SSA and small-x

Large xF (or y) ⟷ Large SSA ⟷ Small-x

❑ Leading power predictions – single scale:

$$p(\vec{s}_\perp) + p \to h(\pi^\pm, \pi^0, ...) + X$$

$$A_N \equiv \frac{\Delta\sigma(\ell, \vec{s})}{\sigma(\ell)} = \frac{\sigma(\ell, \vec{s}) - \sigma(\ell, -\vec{s})}{\sigma(\ell, \vec{s}) + \sigma(\ell, -\vec{s})}$$

$$\begin{aligned} A_N &\propto \sigma(p_T, S_\perp) - \sigma(p_T, -S_\perp) \\ &= T^{(3)}_{a/A}(x, x, S_\perp) \otimes \phi_{b/B}(x') \otimes \hat{\sigma}^T_{ab\to c} \otimes D_{h/c}(z) \\ &+ \delta q_{a/A}(x, S_\perp) \otimes T^{(3\sigma)}_{b/B}(x', x') \otimes \hat{\sigma}^\phi_{ab\to c} \otimes D_{h/c}(z) \\ &+ \delta q_{a/A}(x, S_\perp) \otimes \phi_{b/B}(x', x') \otimes \hat{\sigma}^D_{ab\to c} \otimes D^{(3)}_{h/c}(z, z) \end{aligned}$$

"Sivers effect" + "Collins effect"

➡ nuclear gluon distribution at $G^A(x)$

Contribution from the 2nd term seems to be small!

Kanazawa, Koike, …

Power corrections

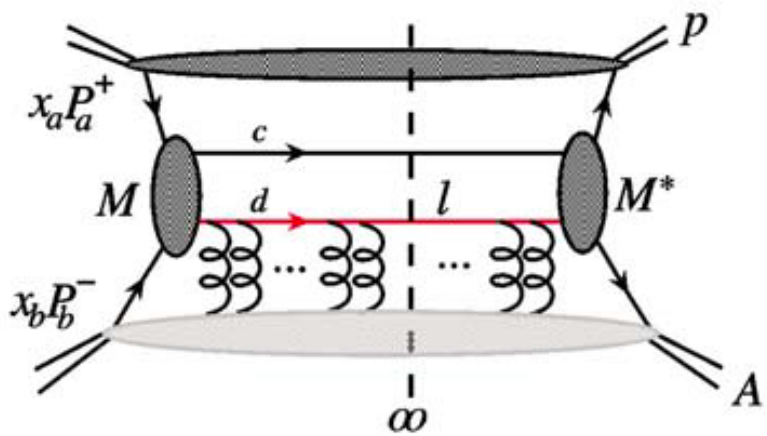
❑ Power corrections:

To both numerator and denominator: $A_N \equiv \frac{\Delta\sigma(\ell,\vec{s})}{\sigma(\ell)} = \frac{\sigma(\ell,\vec{s})-\sigma(\ell,-\vec{s})}{\sigma(\ell,\vec{s})+\sigma(\ell,-\vec{s})}$

❑ Inclusive cross section in the forward region:

Coherent multiple scattering – dominated by t-channel

Qiu, Vitev, 2006

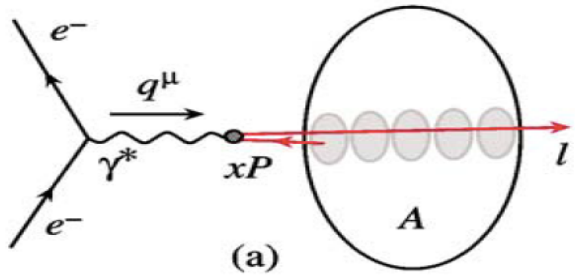


$$\sigma(\ell) = \sigma^{(2)}(\ell) + \left(\frac{Q_s^2}{\ell_T^2}\right)\sigma^{(4)}(\ell) + \left(\frac{Q_s^2}{\ell_T^2}\right)^2 \sigma^{(6)}(\ell) + \ldots$$

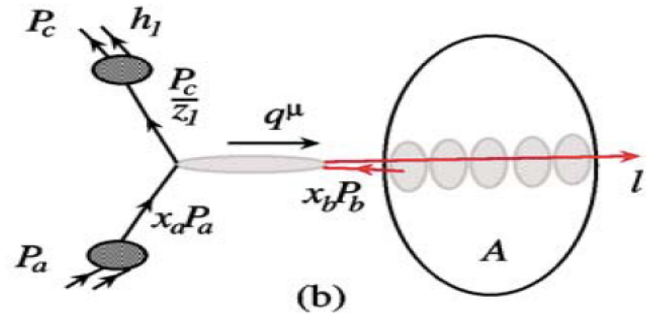
Size of power correction: $\alpha_s \frac{1/\ell_T^2}{R_N^2} \langle F^{+\perp} F^{+\perp} \rangle A^{1/3}$

Coherence at small-x

- Similarity to DIS:

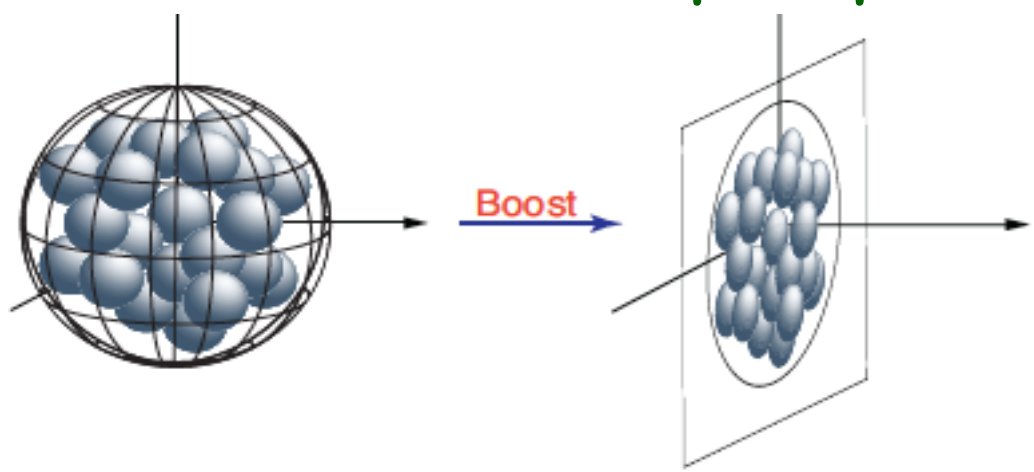


DIS



T-channel of pA

- "Snapshot" does not have a "sharp" depth at small x_B



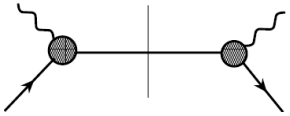
Probe size: transverse - $\frac{1}{Q} \ll 1\ \mathrm{fm}$, longitudinal size - $\frac{1}{xp} \sim \frac{1}{Q} \ll 1\ \mathrm{fm}$

Longitudinal size > Lorentz contracted nucleon: $\frac{1}{xp} > 2R\frac{m}{p}$

$$x < x_c = \frac{1}{2mR} \sim 0.1$$

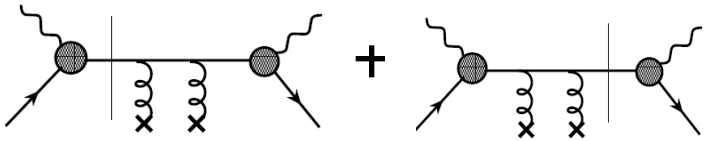
Coherent multiple scattering

- **LO contribution to DIS cross section:**



$\delta(x - x_B)$

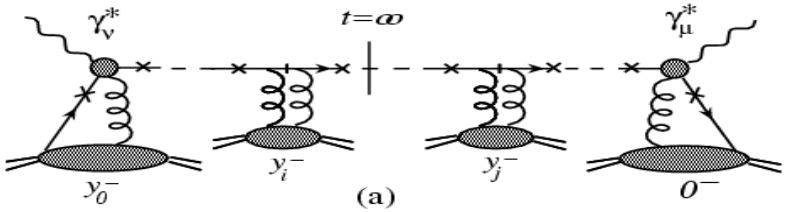
- **NLO contribution:**



$$\frac{g^2}{Q^2}\left(\frac{1}{2N_c}\right)\left[2\pi^2\tilde{F}^2(0)\right] x_B \lim_{x_1\to x}\left[\frac{1}{x-x_1}\delta(x_1-x_B)+\frac{1}{x_1-x}\delta(x-x_B)\right]$$

$$\int \frac{dy_2^- dy_1^-}{(2\pi)^2}\left[F^{+\alpha}(y_2^-)F_\alpha{}^+(y_1^-)\right]\theta(y_2^-) \qquad x_B\left[-\frac{d}{dx}\delta(x-x_B)\right]$$

- **Nth order contribution:**



$$\left[\frac{g^2}{Q^2}\left(\frac{1}{2N_c}\right)\left[2\pi^2\tilde{F}^2(0)\right]\right]^N x_B^N \lim_{x_i\to x}\sum_{m=0}^{N}\delta(x_m-x_B)\left[\prod_{i=1}^{m}\left(\frac{1}{x_{i-1}-x_m}\right)\right]\left[\prod_{j=1}^{N-m}\left(\frac{1}{x_{m+j}-x_m}\right)\right]$$

Infrared safe!

$$x_B^N\left[(-1)^N\frac{1}{N!}\frac{d^N}{dx^N}\delta(x-x_B)\right]$$

Resummation of power corrections

Qiu and Vitev, PRL (2004)

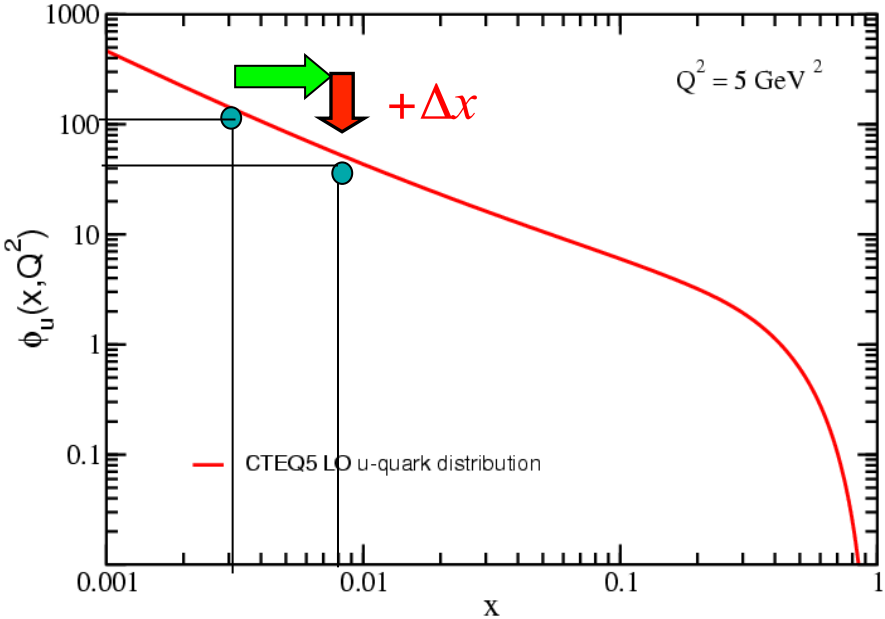
❑ Transverse structure function:

$$F_T(x_B, Q^2) = \sum_{n=0}^{N} \frac{1}{n!} \left[\frac{\xi^2}{Q^2} \left(A^{1/3} - 1 \right) \right]^n x_B^n \frac{d^n}{dx_B^n} F_T^{(0)}(x_B, Q^2)$$

$$\approx F_T^{(0)}(x_B(1+\Delta), Q^2)$$

$$\Delta \equiv \frac{\xi^2}{Q^2} \left(A^{1/3} - 1 \right)$$

$$\xi^2 = \frac{3\pi\alpha_s}{8R^2} \langle F^{+\alpha} F_\alpha{}^{+} \rangle$$



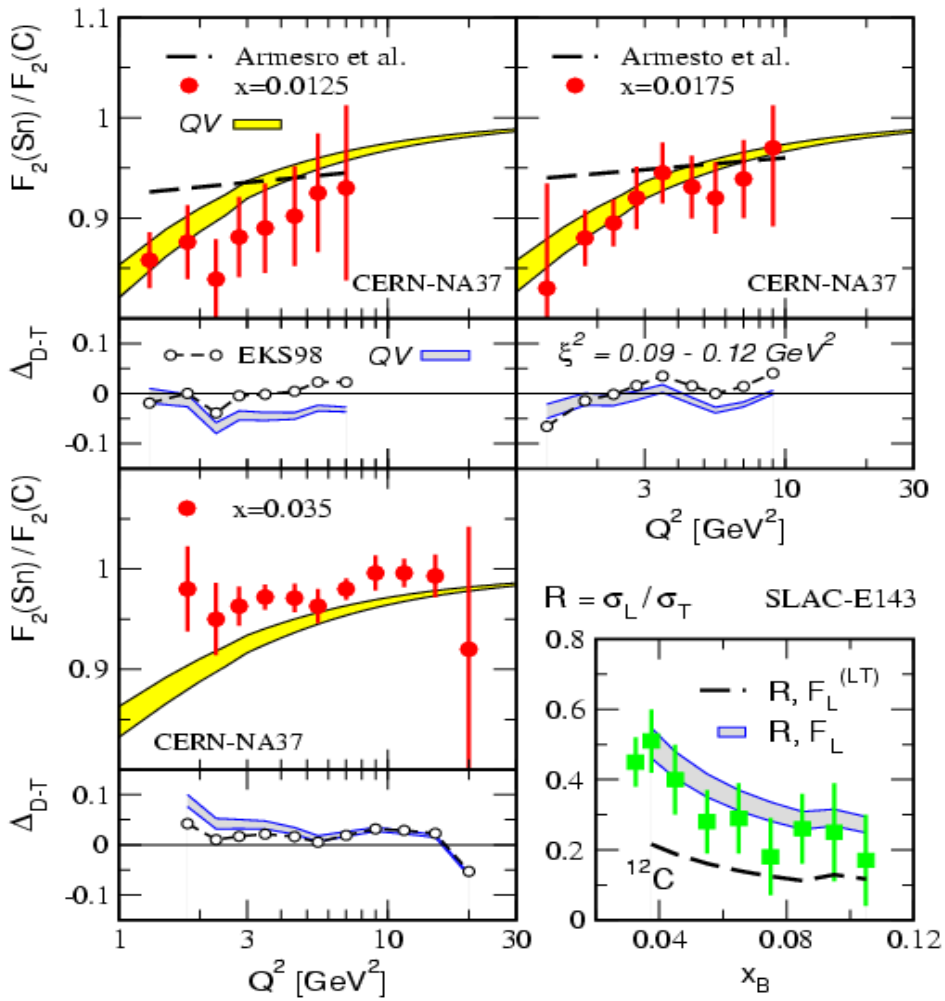
Single parameter for the power correction, and is proportional to the same characteristic scale

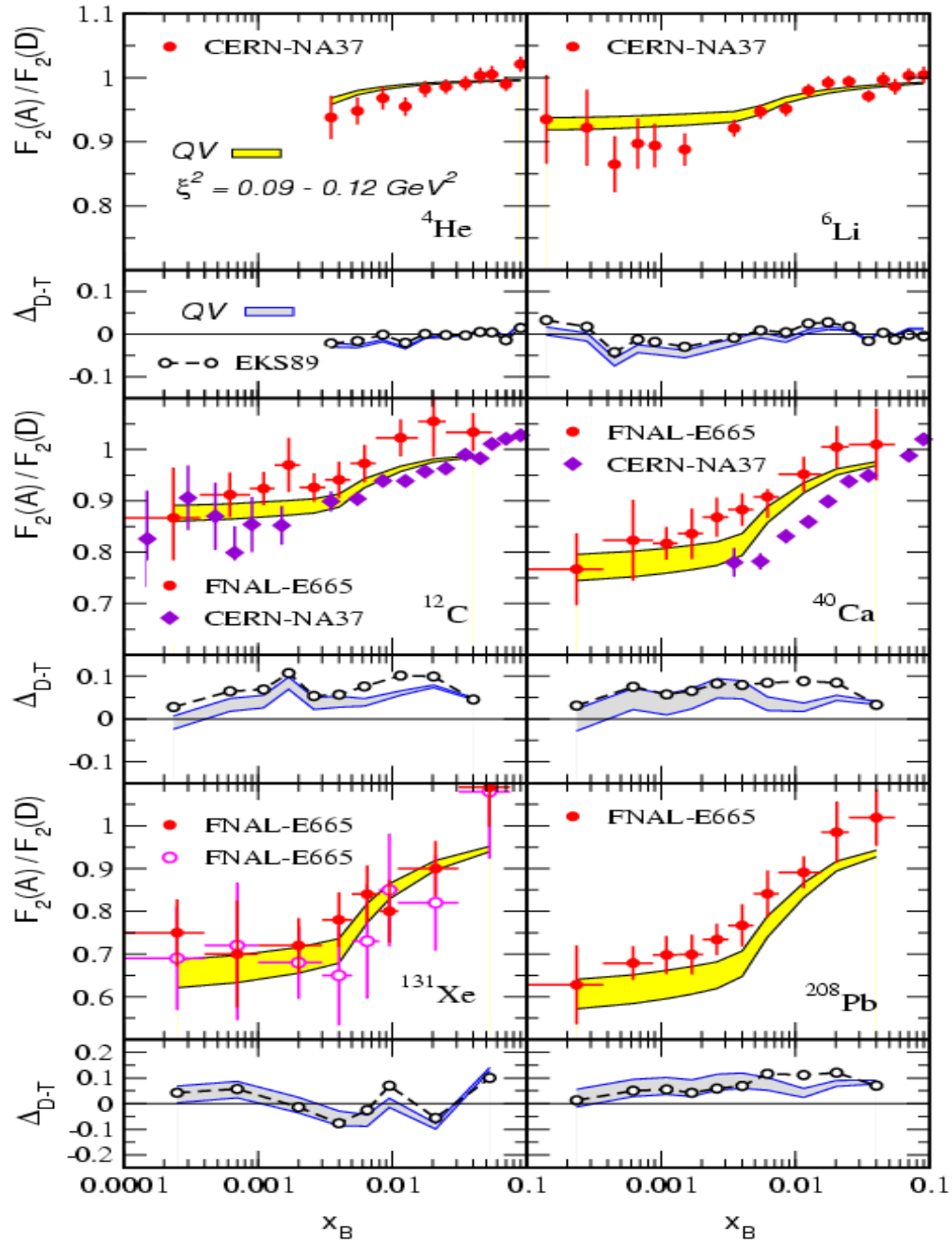
❑ Similar result for longitudinal structure function:

Power corrections to the shadowing

- **Largest power corrections:**

ξ^2 : 0.09 - 0.12 GeV^2





Qiu, Vitev, 2004

Leading power corrections in forward pA

- **Factorized formula:**

$$\frac{d\sigma_{NN}^{h_1}}{dy_1\, d^2 p_{T_1}} = \sum_{abcd} \int \frac{dz_1}{z_1^2} D_{h_1/c}(z_1) \int dx_a \frac{\phi_{a/N}(x_a)}{x_a} \left[\frac{1}{x_a S + U/z_1}\right] \frac{\alpha_s^2}{S} \int dx_b\, \delta(x_b - \bar{x}_b) F_{ab\to cd}(x_b)$$

- **LO proton-proton scattering:**

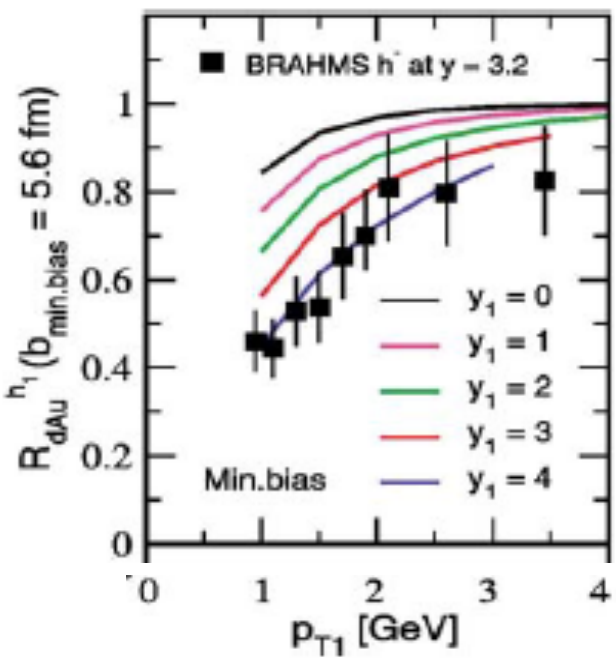
$$F_{ab\to cd}(x_b) \equiv \frac{\phi_{b/N}(x_b)}{x_b} |\bar{M}_{ab\to cd}|^2$$

- **Resummation for pA collisions:**

$$\int dx_b \frac{(-1)^N}{N!} \left[C_d \frac{\bar{x}_b \xi^2}{-t}(A^{1/3}-1)\right]^N \frac{d^N \delta(x_b - \bar{x}_b)}{dx_b^N} F_{ab\to cd}(x_b)$$

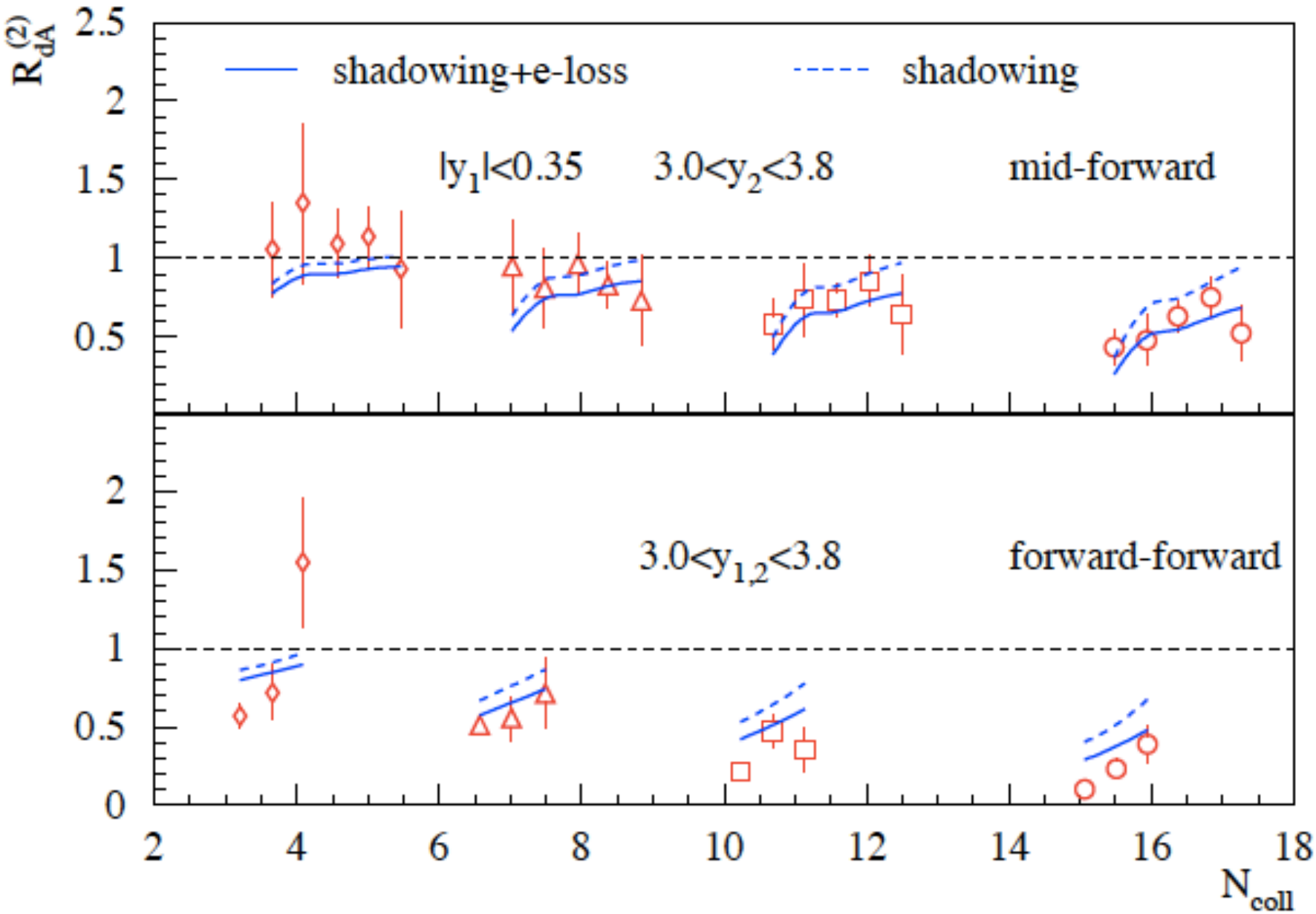
$$= F_{ab\to cd}\left(x_b\left[1 + C_d \frac{\xi^2}{-t}(A^{1/3}-1)\right]\right).$$

In principle, no free parameter!



Di-hadron in d-A collisions

Kang, Vitev, Xing, 2012

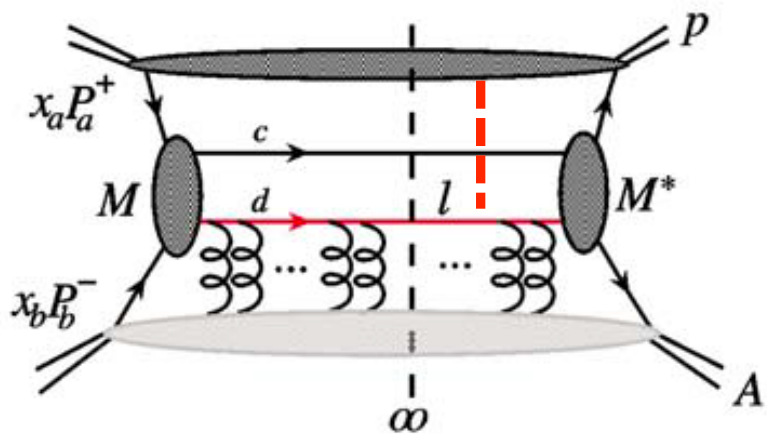


SSA in pA collisions

- To numerator:

$$A_N \equiv \frac{\Delta\sigma(\ell,\vec{s})}{\sigma(\ell)} = \frac{\sigma(\ell,\vec{s}) - \sigma(\ell,-\vec{s})}{\sigma(\ell,\vec{s}) + \sigma(\ell,-\vec{s})}$$

- Leading power approximation:



The momenta of all additional scatterings are fixed by the unpinched poles!

- Same shift from the coherent multiple scattering:

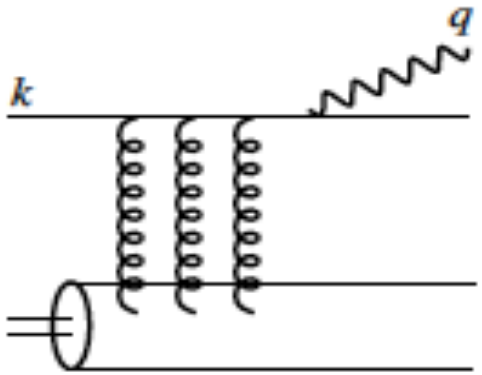
$$A_N(pp) \sim A_N(pA) \qquad \text{When} \qquad \ell_T^2 \gg Q_s^2$$

Only small difference due to the convolution over $T_F(x,x)$ vs. $q(x)$

Note: A_N(photon) =\= 0!

TMD-type approach

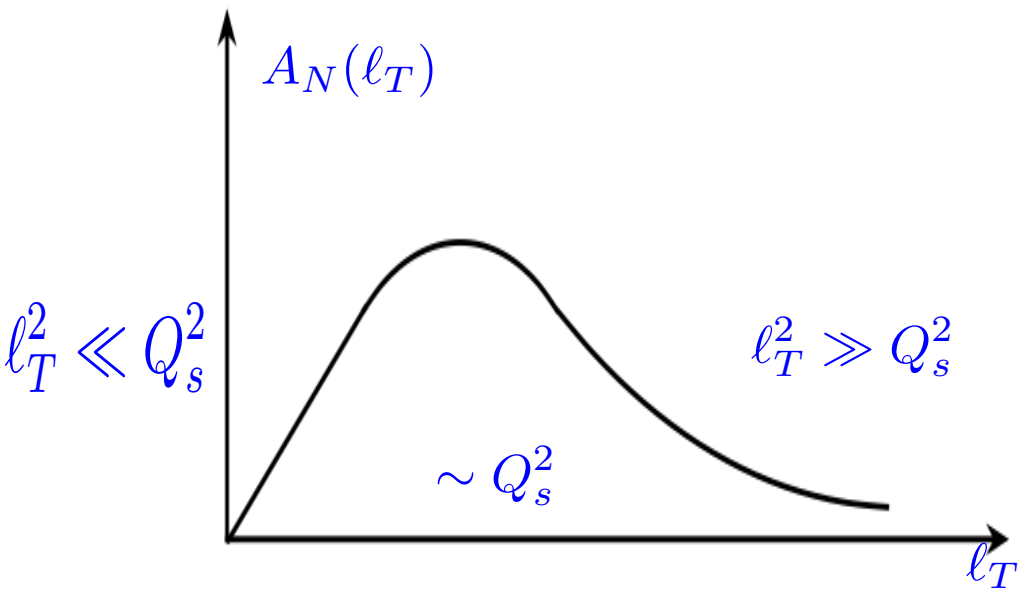
- **TMD or small-x approach:**



No TMD factorization for single hadron production

Relevant when $\ell_T^2 \leq Q_s^2$

- **Matching:**



Recall:

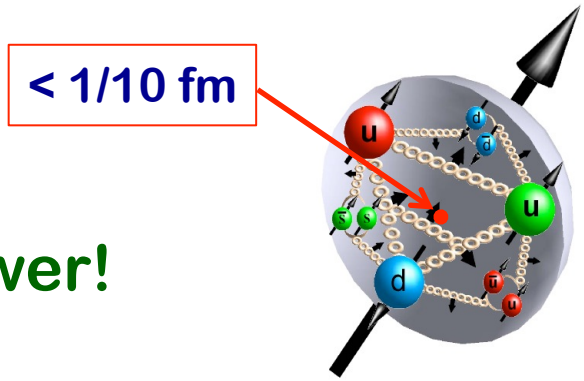
Single hadron spectrum

$$\frac{d\sigma}{d\ell_T^2} \propto \left(\frac{1}{\ell_T^2 + a}\right)^{\alpha}$$

$\Rightarrow$ $a \propto Q_s^2$

Summary

- QCD factorization/calculation have been very successful in interpreting HEP scattering data



- QCD is much richer than the leading power!

- Transverse spin opens a new domain to test QCD dynamics and new observables for parton correlations

- SSA in pA is an excellent observable to study small-x physics in a nucleus

Thank you!

Backup slices

Scale dependence of Sivers function

Aybat, Collins, Qiu, Rogers, 2011

❑ Kernel is not perturbative for all b:

CSS prescription:
(not unique)

$$\mathbf{b}_* = \frac{\mathbf{b}_T}{\sqrt{1+b_T^2/b_{\max}^2}} \qquad \mu_b = \frac{C_1}{b_*}$$

$$\tilde{K}(b_T;\mu) = \tilde{K}(b_*;\mu_b) - \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'}\gamma_K(g(\mu')) - g_K(b_T)$$

❑ Q^2-dependence of Sivers function:

$$\tilde{F}'^{\perp\, f}_{1T}(x,b_T;\mu,\zeta_F) = \tilde{F}'^{\perp\, f}_{1T}(x,b_T;\mu_0,Q_0^2)\exp\left\{\ln\frac{\sqrt{\zeta_F}}{Q_0}\tilde{K}(b_*;\mu_b) + \int_{\mu_0}^{\mu}\frac{d\mu'}{\mu'}\left[\gamma_F(g(\mu');1) - \ln\frac{\sqrt{\zeta_F}}{\mu'}\gamma_K(g(\mu'))\right]\right.$$
$$\left. + \int_{\mu_0}^{\mu_b}\frac{d\mu'}{\mu'}\ln\frac{\sqrt{\zeta_F}}{Q_0}\gamma_K(g(\mu')) - g_K(b_T)\ln\frac{\sqrt{\zeta_F}}{Q_0}\right\}$$

$$F^{\perp\, f}_{1T}(x,k_T;\mu,\zeta_F) = \frac{-1}{2\pi k_T}\int_0^\infty db_T\, b_T J_1(k_T b_T)\tilde{F}'^{\perp\, f}_{1T}(x,b_T;\mu,\zeta_F)$$ – Evolved Sivers function

❑ Small-b perturbative contribution – match to twist-3:

$$\tilde{F}'^{\perp\, f}_{1T}(x,b_T;\mu,\zeta_F) = \sum_j \frac{M_p b_T}{2}\int_x^1 \frac{d\hat{x}_1\, d\hat{x}_2}{\hat{x}_1\,\hat{x}_2}\tilde{C}^{\rm Sivers}_{f/j}(\hat{x}_1,\hat{x}_2,b_*;\mu_b^2,\mu_b,g(\mu_b))\, T_{F\, j/P}(\hat{x}_1,\hat{x}_2,\mu_b)$$
$$\times\exp\left\{\ln\frac{\sqrt{\zeta_F}}{\mu_b}\tilde{K}(b_*;\mu_b) + \int_{\mu_b}^{\mu}\frac{d\mu'}{\mu'}\left[\gamma_F(g(\mu');1) - \ln\frac{\sqrt{\zeta_F}}{\mu'}\gamma_K(g(\mu'))\right]\right\}\times\exp\left\{-g^{\rm Sivers}_{j/P}(x,b_T) - g_K(b_T)\ln\frac{\sqrt{\zeta_F}}{Q_0}\right\}$$

Kang, Xiao, Yuan, 2011