Medium Induced Transverse Momentum Broadening in Hard Processes

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Joint CTEQ Meeting and POETIC 7

Temple University®

Outline

1. **Motivations**

2. **Vacuum and medium-induced radiation**

3. **QCD correction to quark production in eA collisions**

4. **Summary**

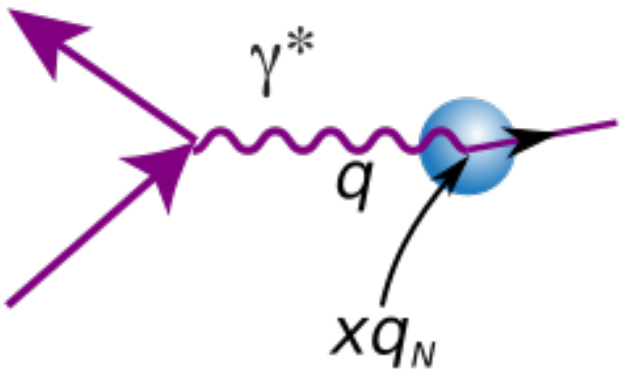
Liou, Mueller and BW, Nucl. Phys. A 916 (2013) 102-125;

BW, JHEP **1110**, 029 (2011); JHEP **1412**, 081 (2014);

Mueller, BW, Xiao and Yuan, arXiv:1608.07339.

1.1. Quark production at LO

When $\frac{1}{q^-} \ll L$, the nucleus size:



an electron-proton collision

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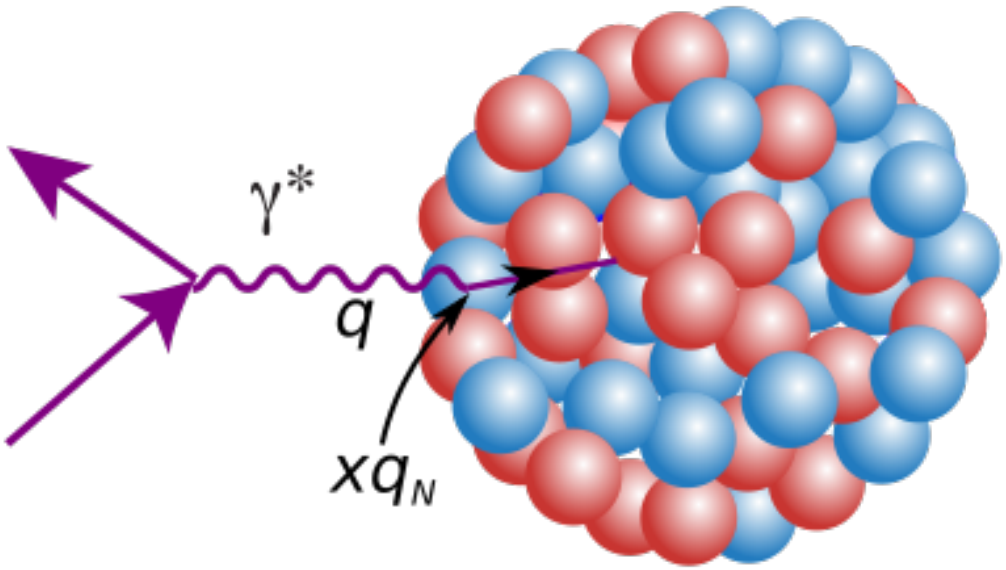

an electron-nucleus collision

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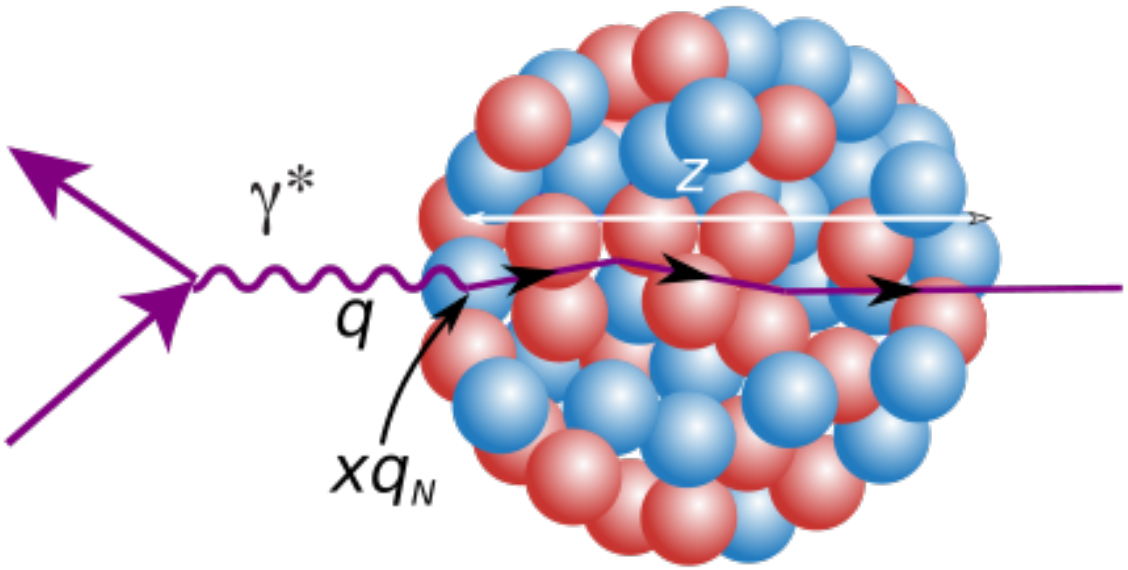

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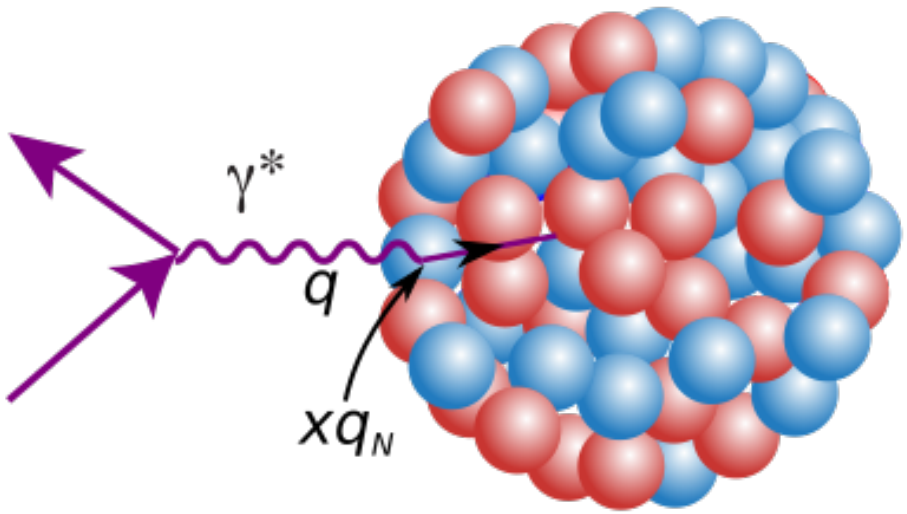

$$\frac{dN}{d^2b d^2k_\perp} = \int \frac{d^2x_\perp}{(2\pi)^2} e^{-ik_\perp \cdot x_\perp} \rho x q_N \left(x, \frac{1}{x_\perp^2 + 1/Q^2}\right)$$

Kovchegov and Mueller, Nucl. Phys. B **529**, 451 (1998).

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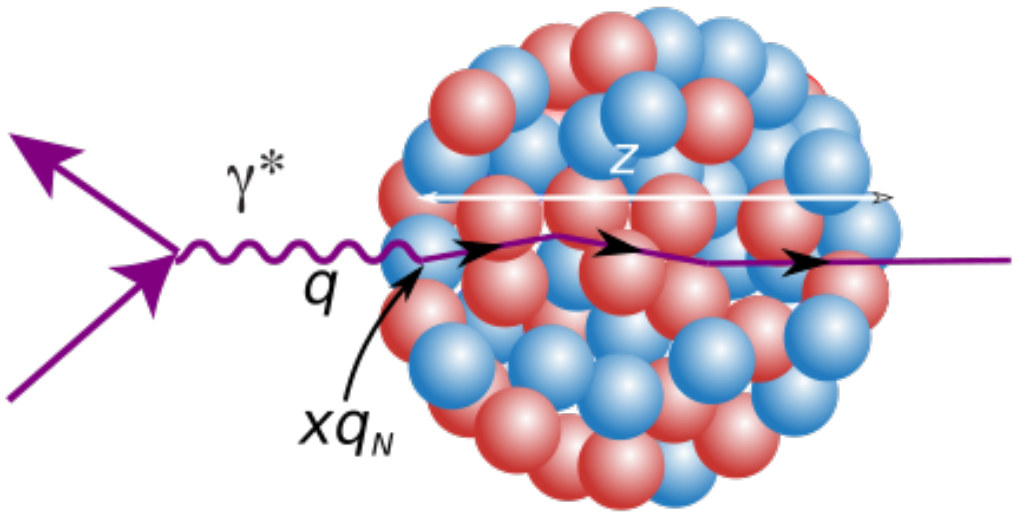

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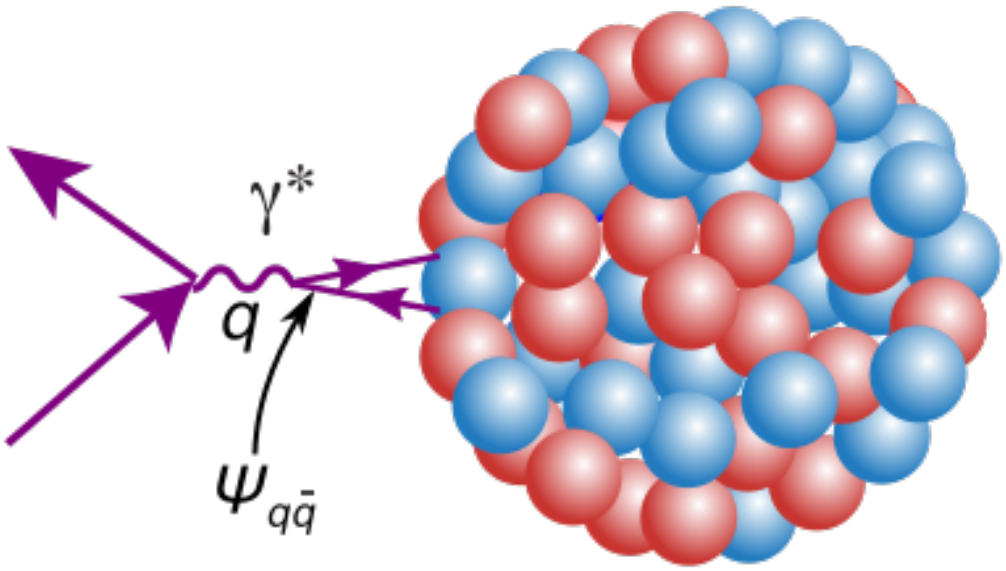
1.1. Quark production at LO

When $\frac{1}{q^-} \gg L$: a dipole picture



an electron-nucleus collision

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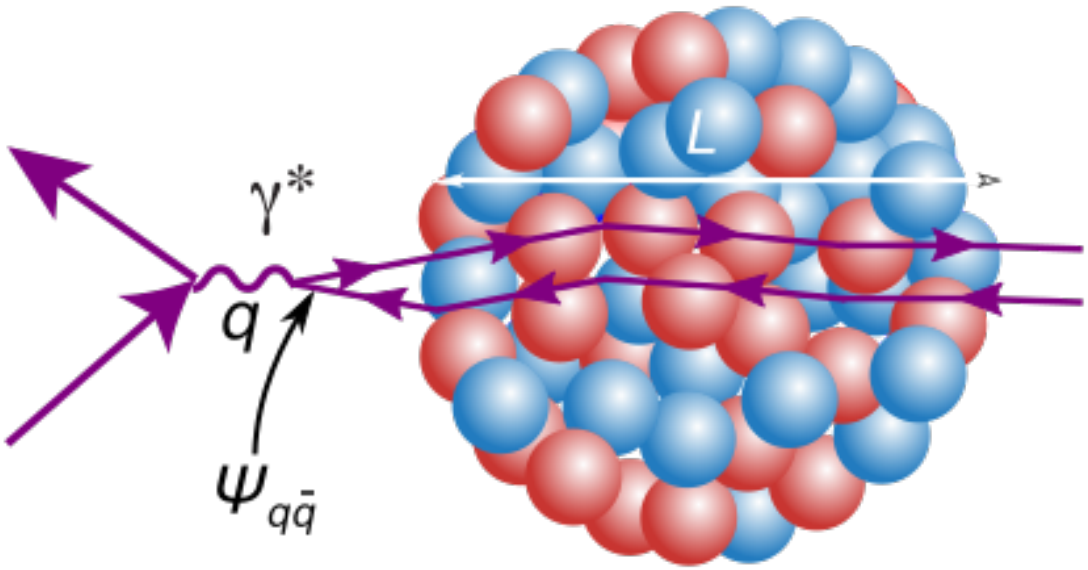
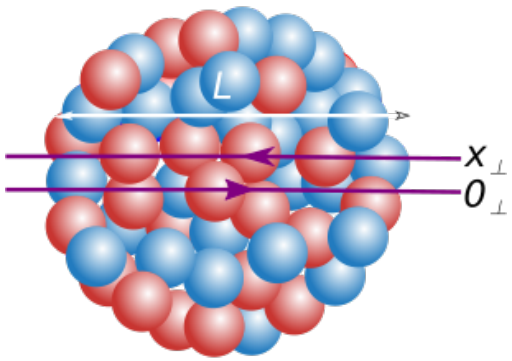


an electron-nucleus collision

1.1. Quark production at LO

$S(x_\perp)$: the S-matrix for a dipole



$$= \exp\left[-\frac{1}{4}\hat{q}(1/x_\perp^2)Lx_\perp^2\right]$$ in the leading log approx.

with transport coefficient (jet quenching parameter)

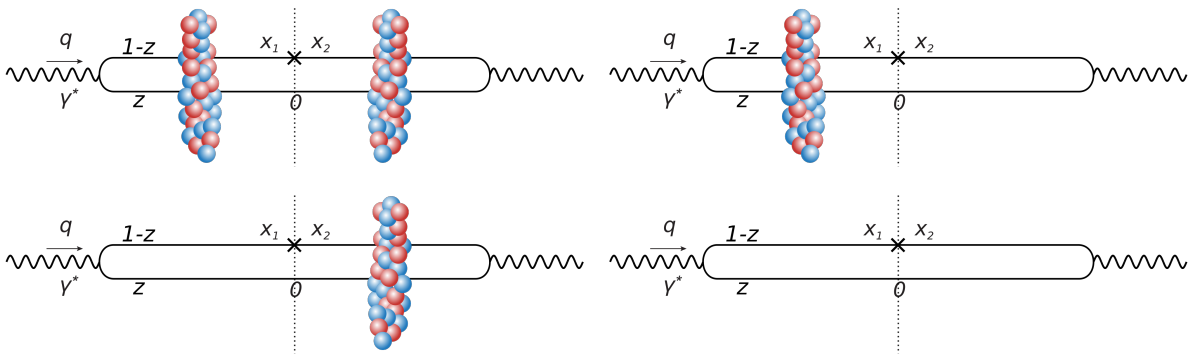
$$\boxed{\hat{q}(1/x_\perp^2) = \frac{4\pi^2\alpha_s C_R}{N_c^2-1}\rho x G(x, 1/x_\perp^2)}$$

Baier, Dokshitzer, Mueller, Peigne and Schiff, Nucl. Phys. B **484**, *265 (1997).*

In McLerran-Venugopalan (MV) model: $Q_s^2 = \hat{q}L$

1.1. Quark production at LO

When $\frac{1}{q^-} \gg L$:



$$\frac{dN}{d^2b_\perp d^2k_\perp} = \frac{Q^2 N_c}{32\pi^6}\int d^2x_1 d^2x_2 \int_0^1 [z^2+(1-z)^2] e^{-ik_\perp\cdot(x_1-x_2)}$$
$$\times \nabla_{x_1} K_0(\sqrt{Q^2x_1^2 z(1-z)})\nabla_{x_2} K_0(\sqrt{Q^2x_2^2 z(1-z)})$$
$$\times [S(x_1-x_2)-S(x_1)-S(x_2)+1]$$

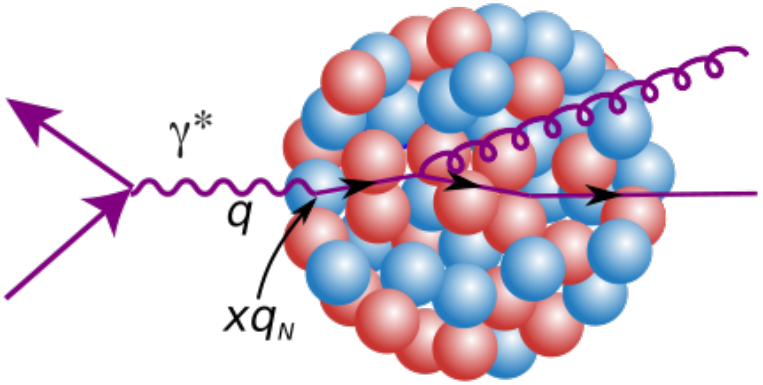
in a frame in which the nucleus is moving along the negative z-direction.

A. H. Mueller, Nucl. Phys. B **558**, *285 (1999).*

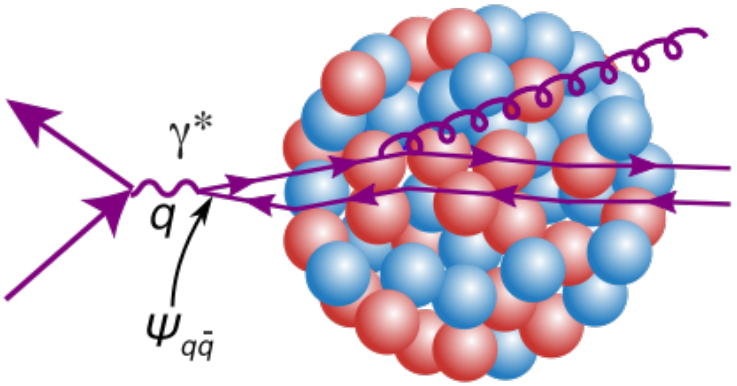
1.2 QCD corrections to quark production

To calculate vacuum and medium-induced radiation:

(a) for $\frac{1}{q^-} \ll L$

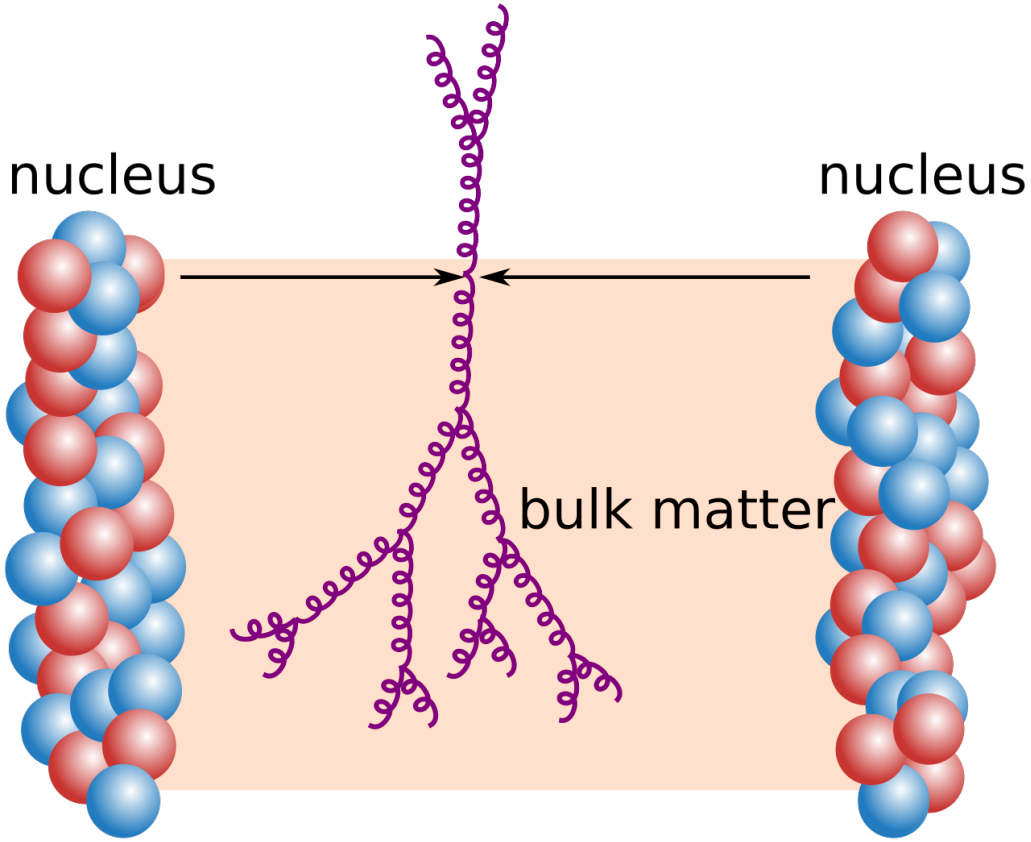


(b) for $\frac{1}{q^-} \gg L$



1.2 QCD corrections to quark production

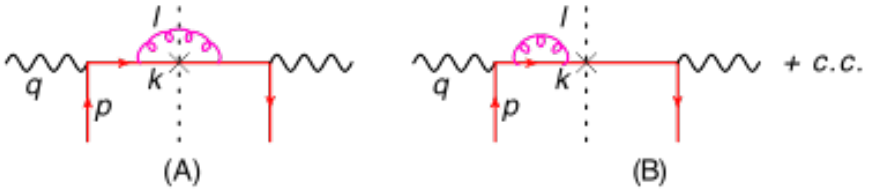
Help to solve one open question in heavy-ion collisions:



interplay between vacuum and medium-induced radiation

2.1 The vacuum radiation

The Sudakov double log ($A^+ = 0$ gauge):



$$A + B \approx -\frac{\alpha_s C_F}{\pi} \rho x q_N \int_{k_\perp^2}^{Q^2} \frac{dl_\perp^2}{l_\perp^2} \int_0^{q^+} \frac{dl^+}{l^+},$$

.

Mueller, BW, Xiao and Yuan, arXiv:1608.07339.

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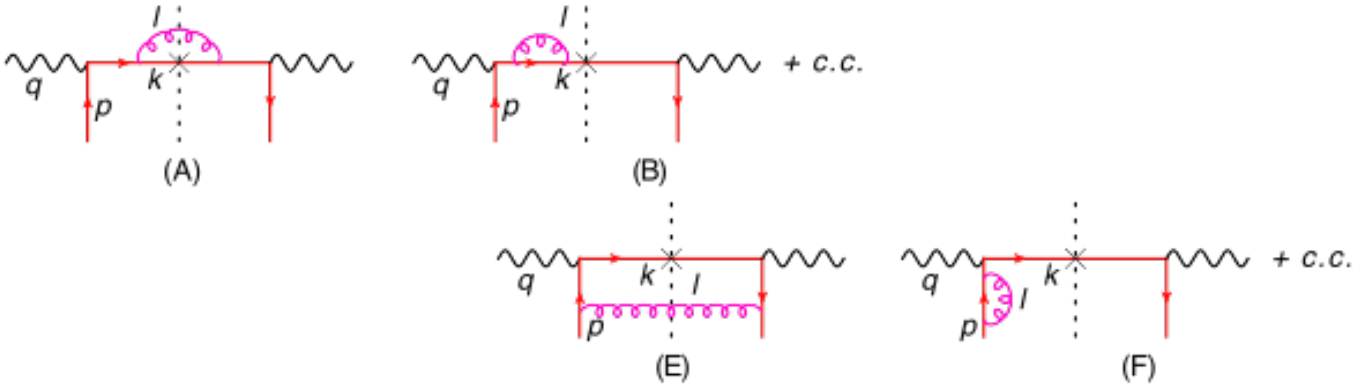

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Mueller, BW, Xiao and Yuan, arXiv:1608.07339.

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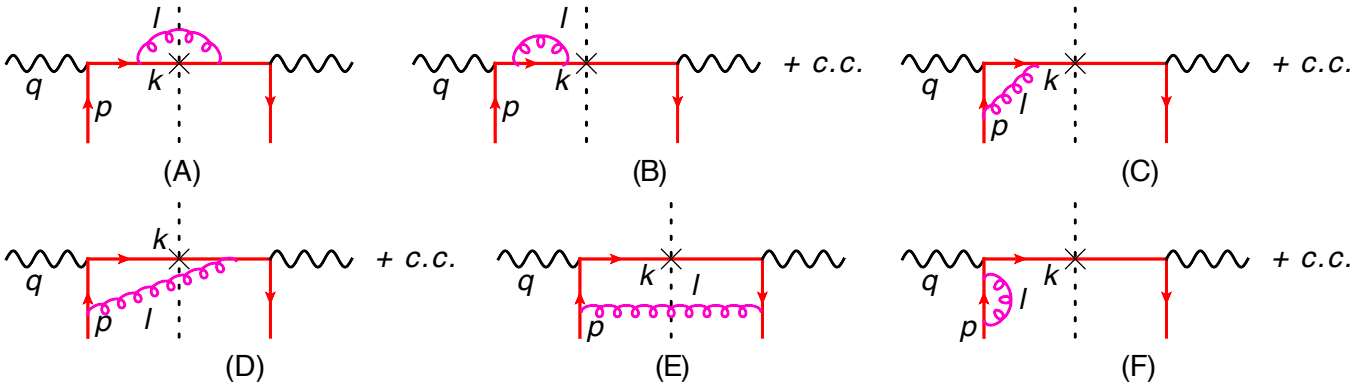

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$$C + D \approx \frac{\alpha_s C_F}{\pi} \rho x q_N \int_{k_\perp^2}^{Q^2} \frac{dl_\perp^2}{l_\perp^2} \int_{l^- > q^-} \frac{dl^+}{l^+} = \frac{\alpha_s C_F}{\pi} \rho x q_N \int_{k_\perp^2}^{Q^2} \frac{dl_\perp^2}{l_\perp^2} \int_0^{q^+ \frac{l_\perp^2}{Q^2}} \frac{dl^+}{l^+},$$

$$A + B + C + D \approx -\frac{\alpha_s C_F}{2\pi} \rho x q_N \ln^2 \frac{Q^2}{k_\perp^2}.$$

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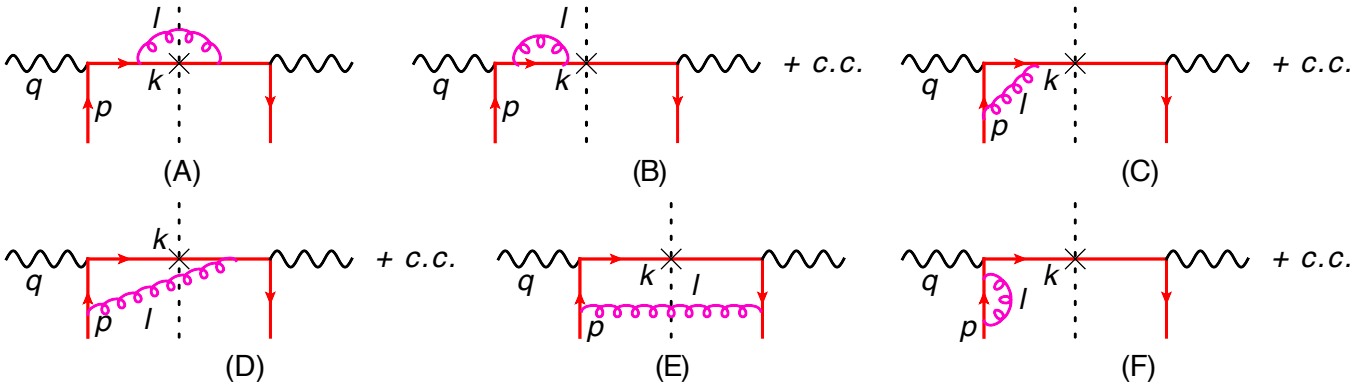

$$A + B \approx -\frac{\alpha_s C_F}{\pi} \rho x q_N \int_{k_\perp^2}^{Q^2} \frac{dl_\perp^2}{l_\perp^2} \int_0^{q^+} \frac{dl^+}{l^+},$$

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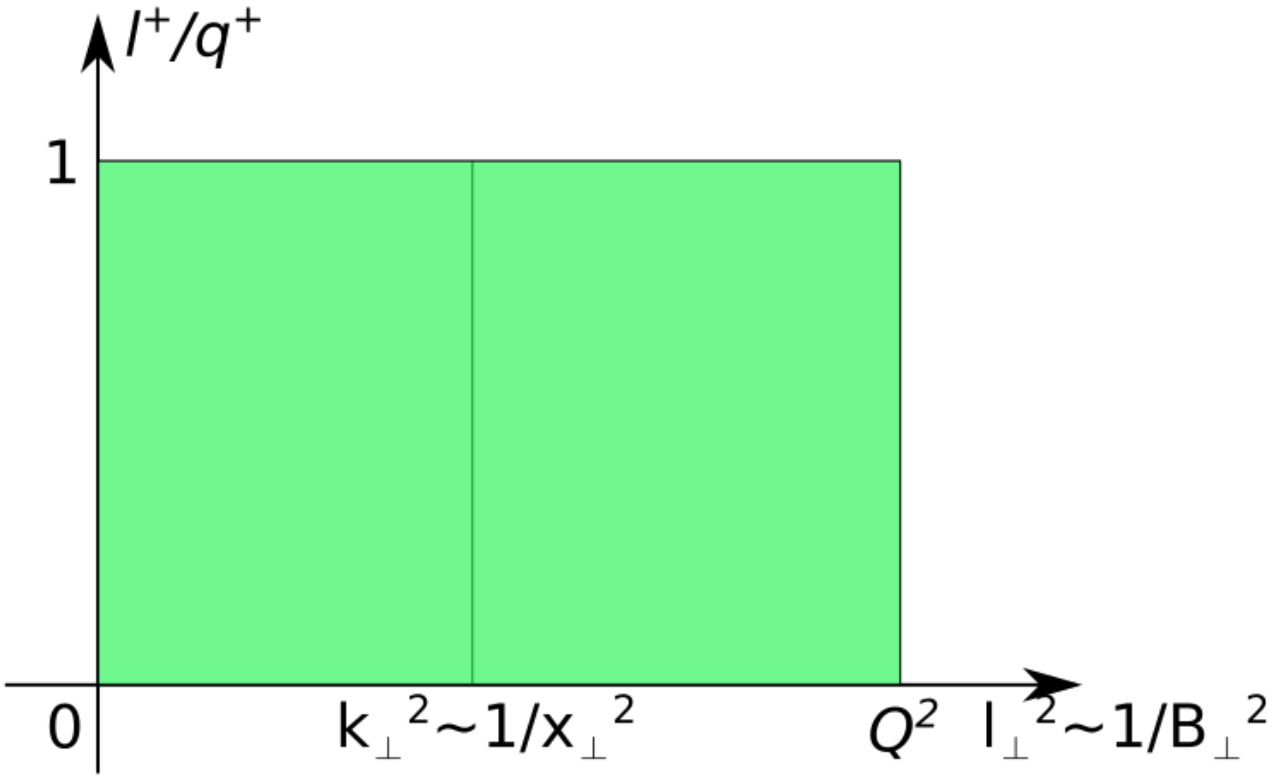
2.2 Sudakov double log and multiple scattering

Conclusion:

Multiple scattering does not modify the Sudakov factor.

Reason:



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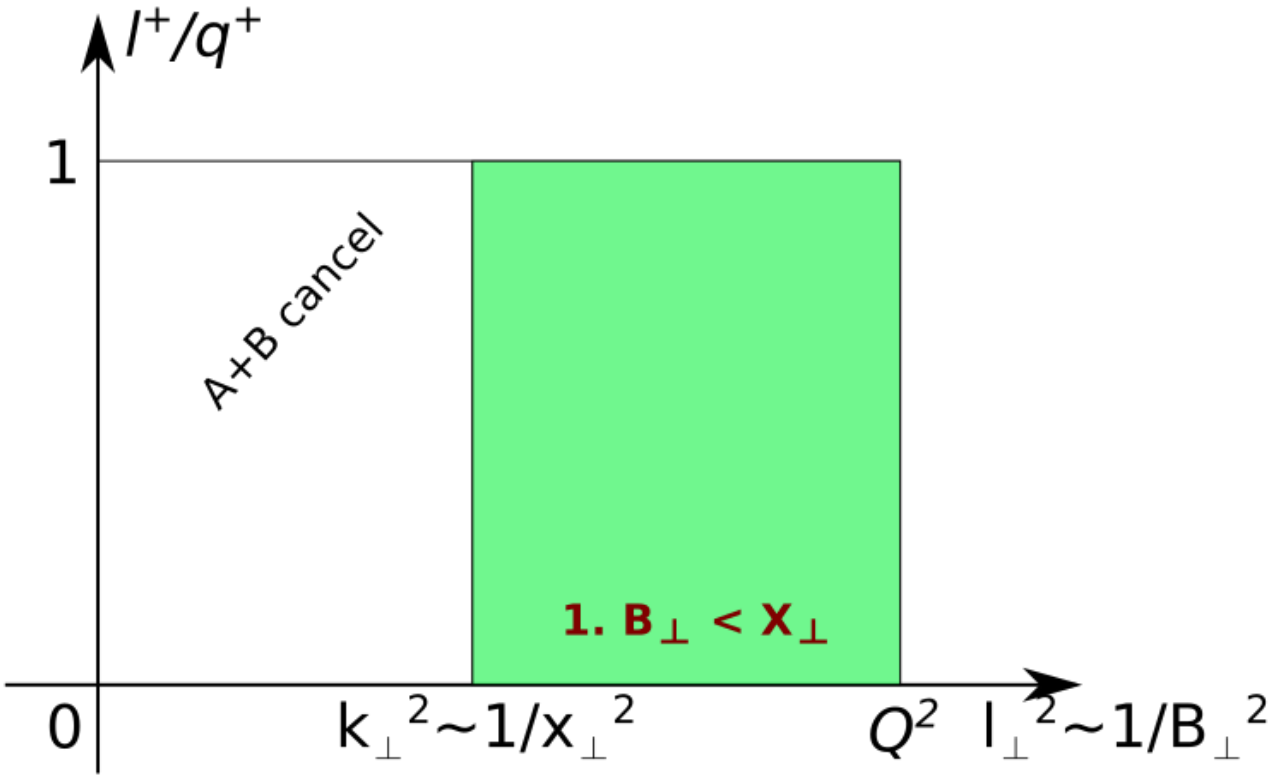

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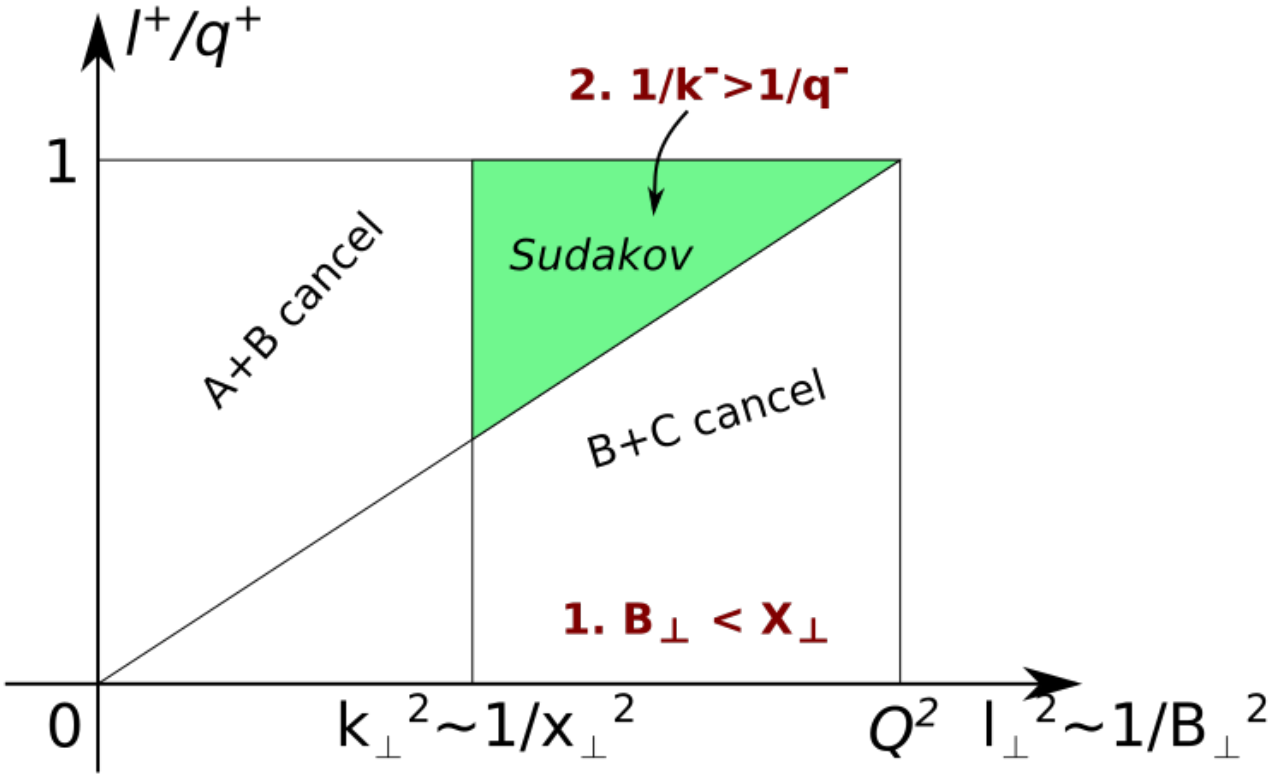
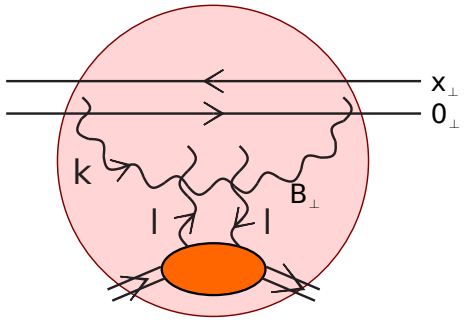


The medium can not "see" the radiated gluon.

2.3 Medium-induced radiation

Medium induced double log with $B_\perp > x_\perp$:

$$\langle p_\perp^2 \rangle_{rad} =$$



$$= \frac{\alpha_s N_c}{\pi} \hat{q} L \int_{l_0}^{L} \frac{dz}{z} \int_{\hat{q} z^2}^{\hat{q} L z} \frac{d\omega}{\omega}$$

$$= \frac{\alpha_s N_c}{8\pi} \hat{q} L \log^2 \left(\frac{L}{r_0} \right)^2$$

with r_0 the nucleon size.

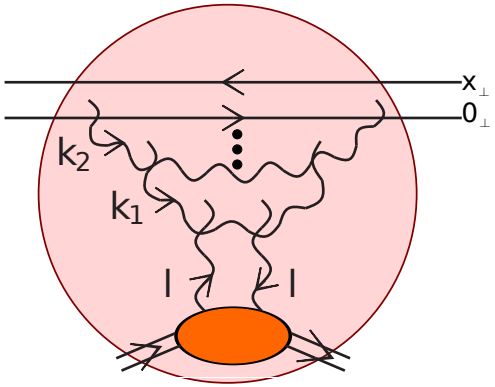
Liou, Mueller and BW (2013).

Interpretation: radiative correction to $\hat{q}$

Blaizot & Mehtar-Tani (2014); Iancu (2014).

2.3 Medium-induced radiation

Resummation of medium-induced double log:



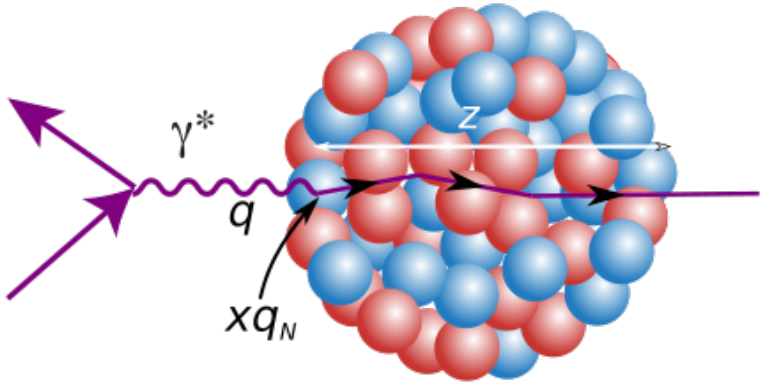
$$\hat{q}_t = \begin{cases} \hat{q} & \text{if } x_\perp^2 > 1/\hat{q}r_0, \\ \hat{q}\frac{1}{\sqrt{\bar{\alpha}_s}K} I_1(2\sqrt{\bar{\alpha}_s}K) & \text{if } 1/\hat{q}r_0 \geq x_\perp^2 \geq 1/\hat{q}L, \\ \hat{q}\left[\frac{1}{\sqrt{\bar{\alpha}_s K\tau}} I_1(2\sqrt{\bar{\alpha}_s K\tau}) + \left(1-\frac{\tau}{K}\right) I_2(2\sqrt{\bar{\alpha}_s K\tau})\right] & \text{if } x_\perp^2 \leq 1/\hat{q}L. \end{cases}$$

with $K = \ln \frac{1}{\hat{q}r_0 x_\perp^2}$ and $\tau = \ln \frac{L}{r_0}$.

Liou, Mueller and BW (2013); Iancu & Triantafyllopoulos (2014).

3.1 QCD correction to quark production: $\frac{1}{q^-} \ll L$

For $\frac{1}{q^-} \ll L$:



$$\frac{dN}{d^2 b d^2 k_\perp} = \int \frac{d^2 x_\perp}{(2\pi)^2} e^{-ik_\perp \cdot x_\perp} \rho \, x q_N \left(x, \frac{1}{x_\perp^2 + 1/Q^2} \right) \times \int_0^L dz \exp \left[-\frac{1}{4} \hat{q} \, x_\perp^2 z \right].$$

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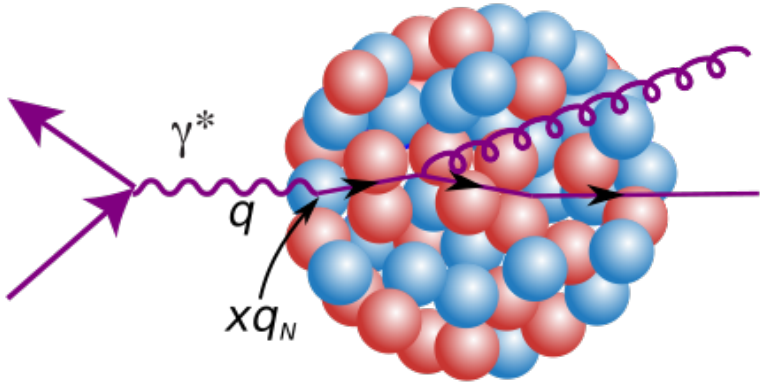

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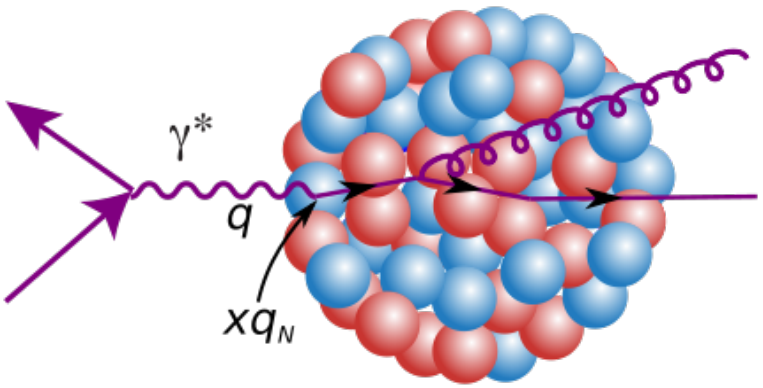

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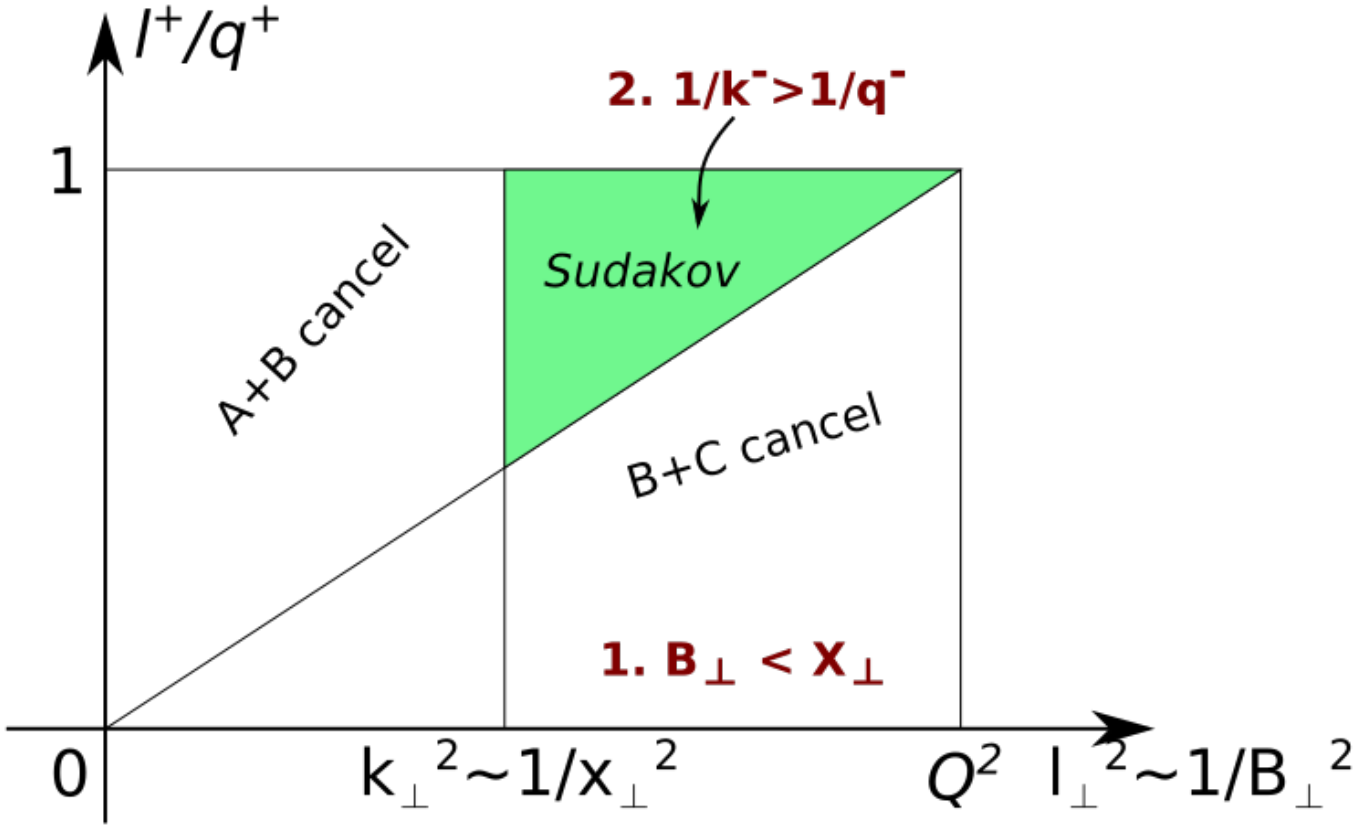
See, for resummation, Collins, Soper & Sterman (1985); Mueller, Xiao & Yuan (2013).

3.1 QCD correction to quark production: $\frac{1}{q^-} \ll L$

Two double logs are factorized:



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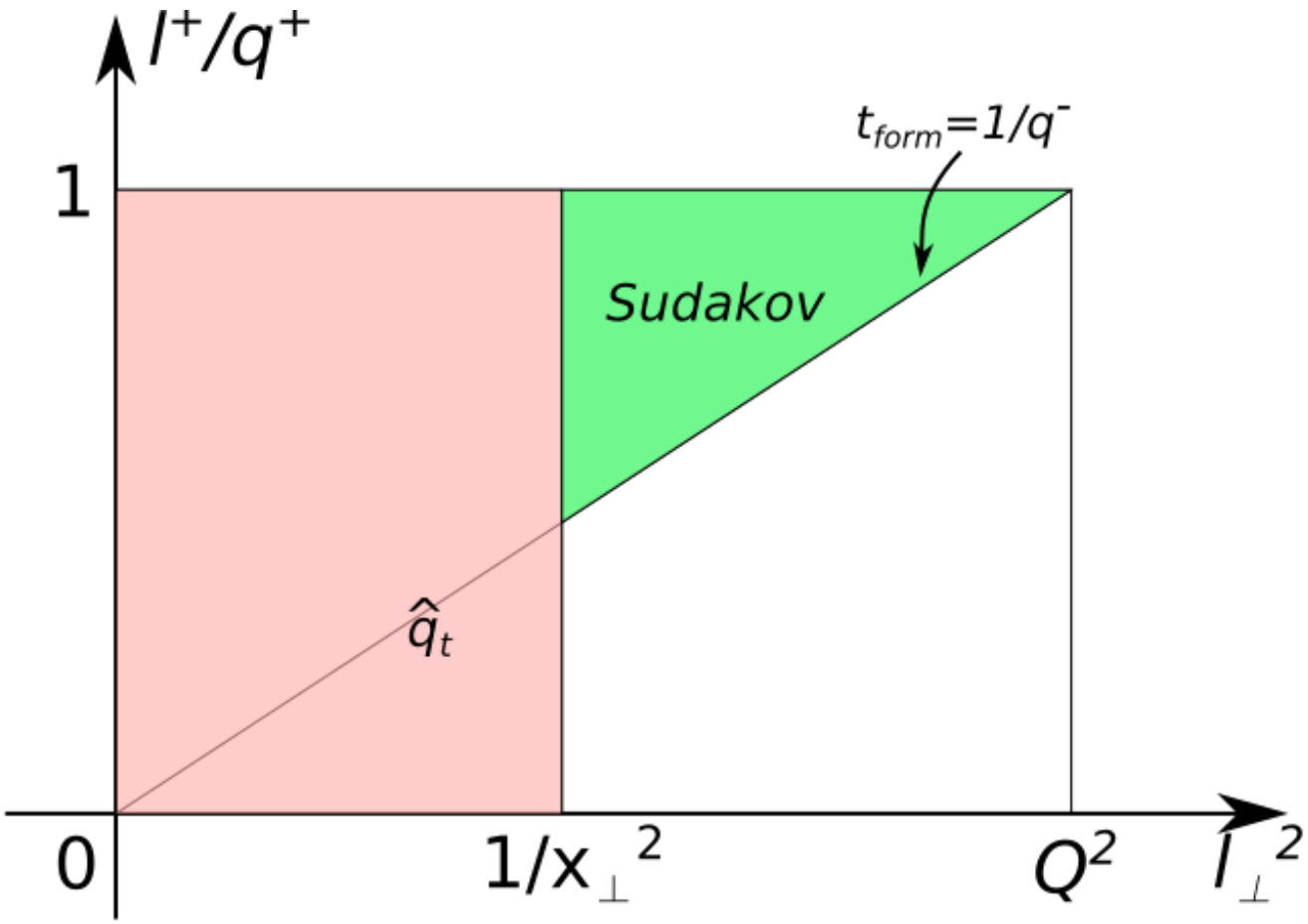

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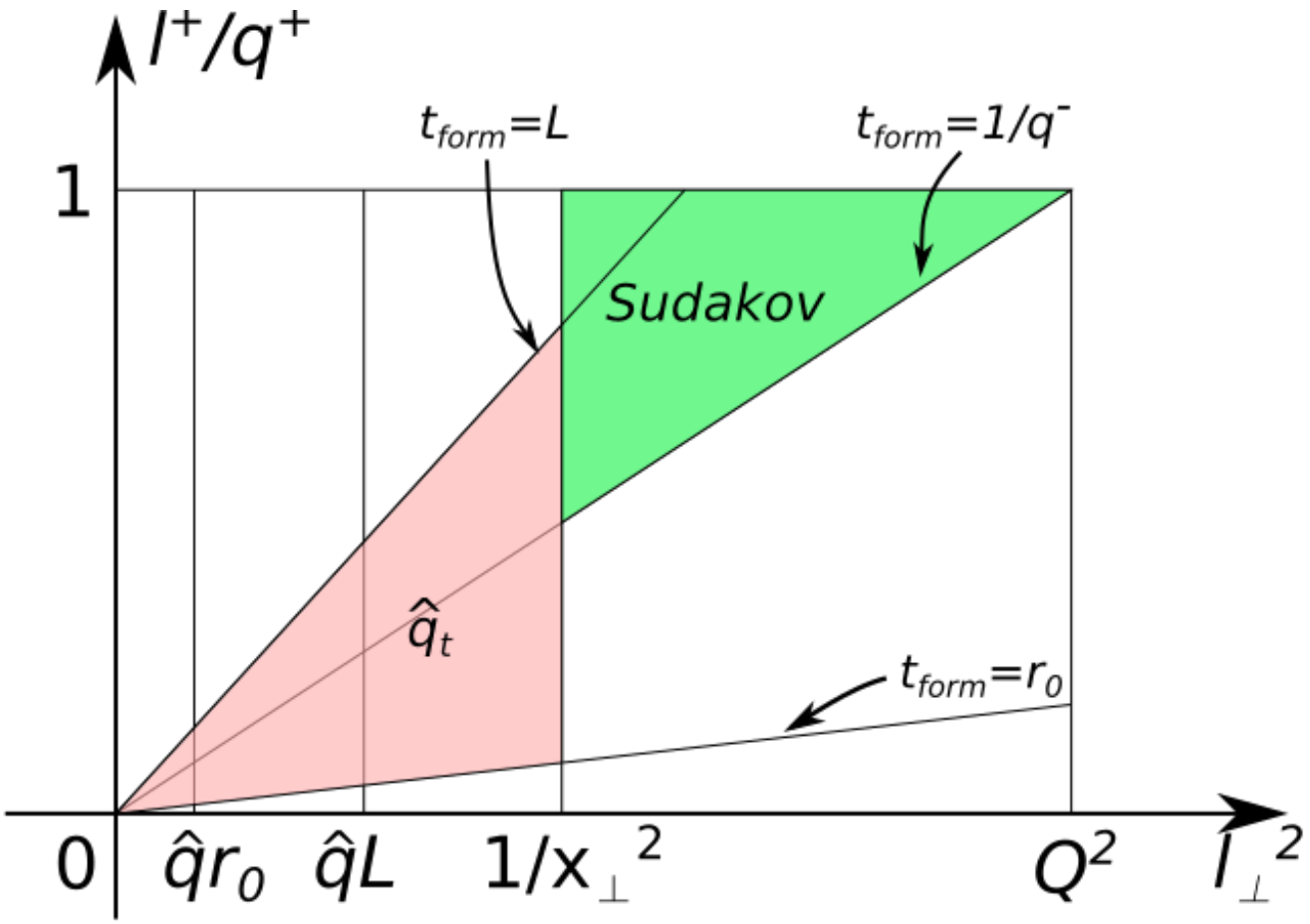

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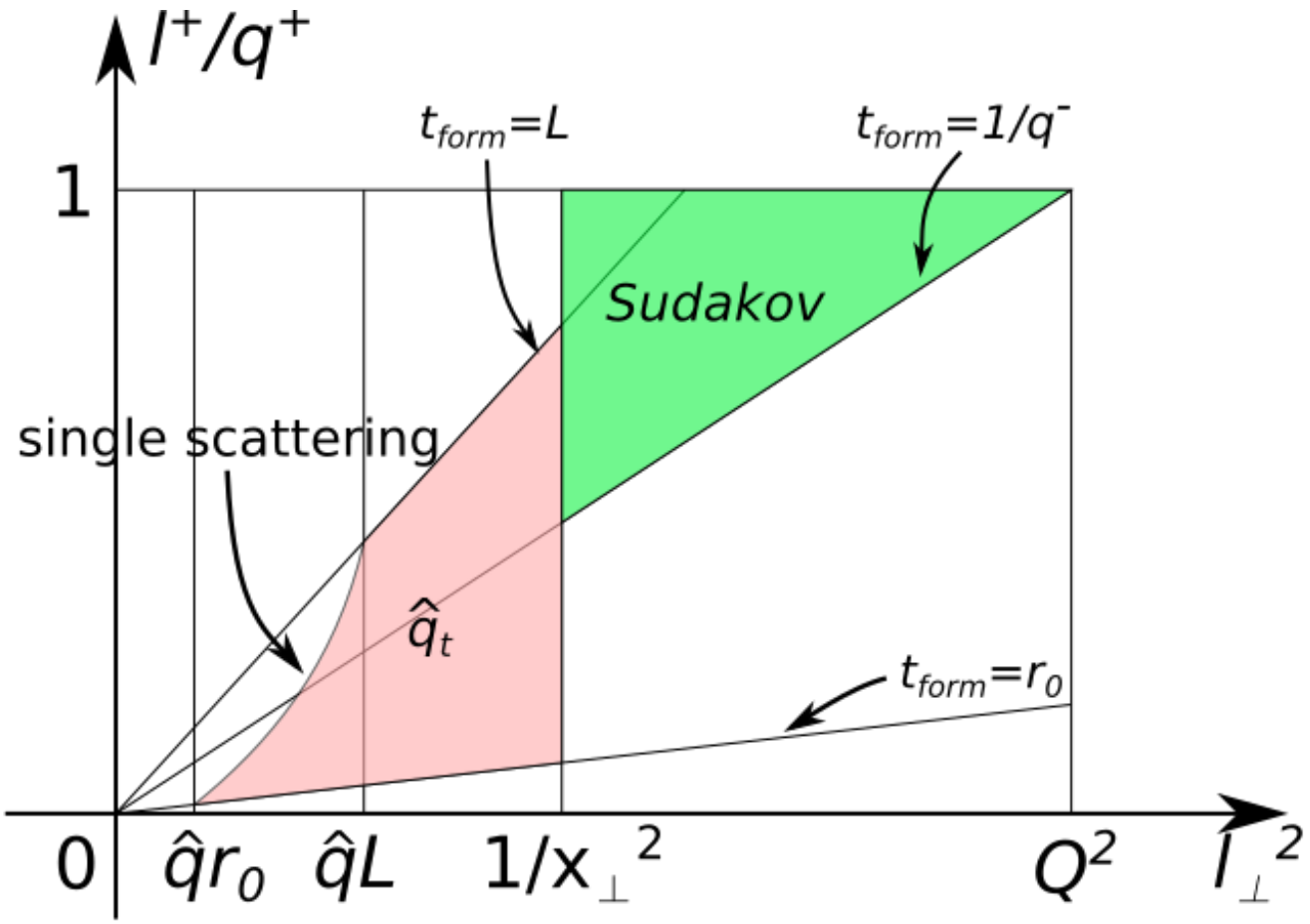
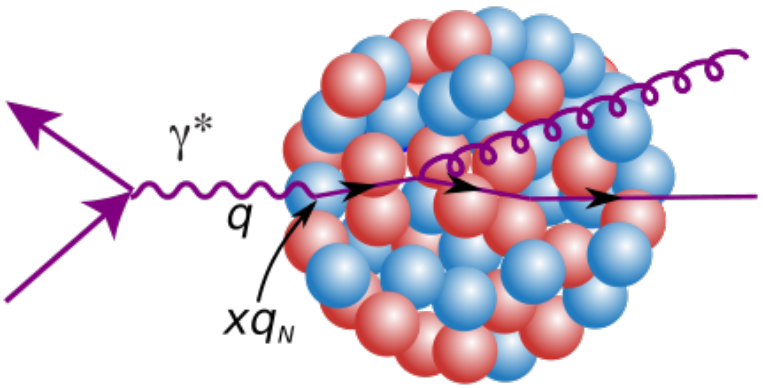


3.1 QCD correction to quark production: $\frac{1}{q^-} \ll L$

To study nuclear effects, Q should not be large!



$$\frac{dN}{d^2b d^2k_\perp} = \int \frac{d^2x_\perp}{(2\pi)^2} e^{-ik_\perp\cdot x_\perp} \rho\, x q_N\left(x, \frac{1}{x_\perp^2 + 1/Q^2}\right) \times \int_0^L dz \exp\left[\underbrace{-\frac{1}{4}\hat{q}_t x_\perp^2 z}_{\text{medium-induced}} \underbrace{-\frac{\alpha_s C_F}{2\pi}\ln^2\left(Q^2 x_\perp^2\right)}_{\text{vacuum radiation}}\right].$$

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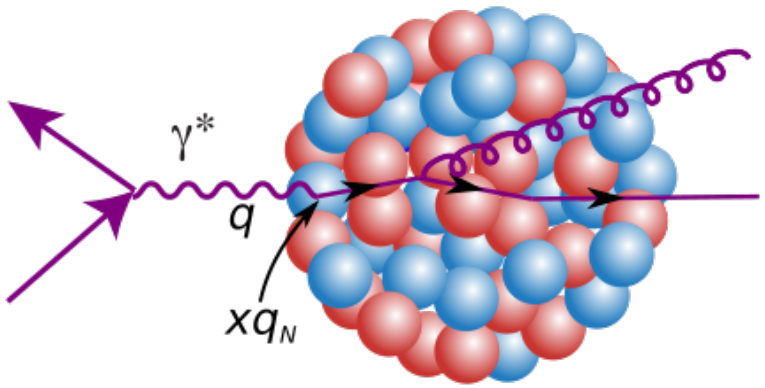


Otherwise,

$$\frac{dN}{d^2b d^2k_\perp} \approx \int \frac{d^2x_\perp}{(2\pi)^2} e^{-ik_\perp \cdot x_\perp} \rho\, x q_N \left(x, \frac{1}{x_\perp^2 + 1/Q^2}\right) \times \int_0^L dz \exp\left[\underbrace{-\frac{\alpha_s C_F}{2\pi} \ln^2\left(Q^2 x_\perp^2\right)}_{\text{vacuum radiation}} \right].$$

3.1 QCD correction to quark production: $\frac{1}{q^-} \ll L$

Dijet azimuthal angular distributions ($Q \to p_\perp$)

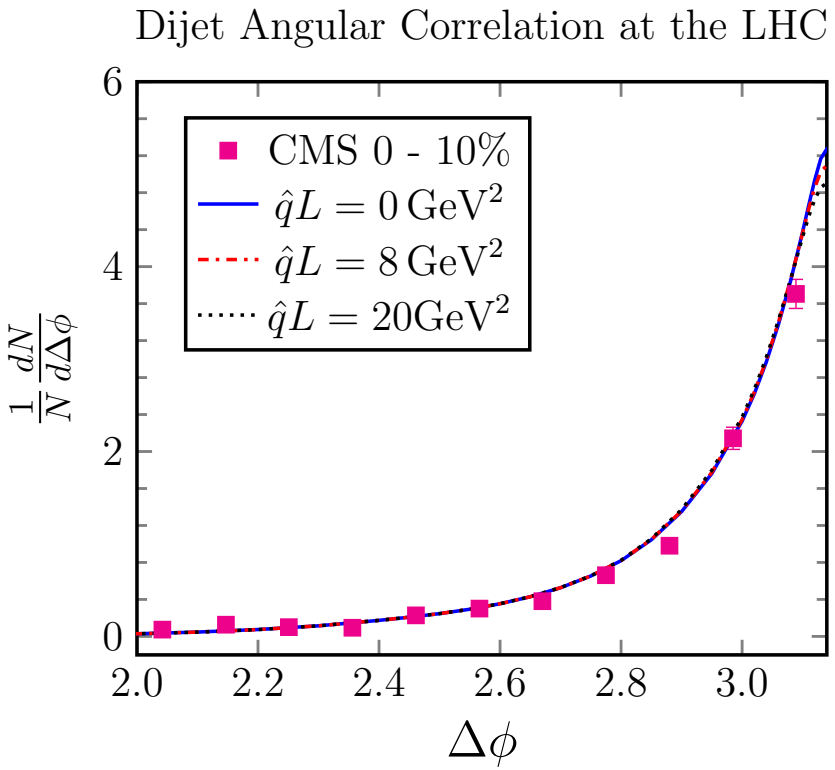
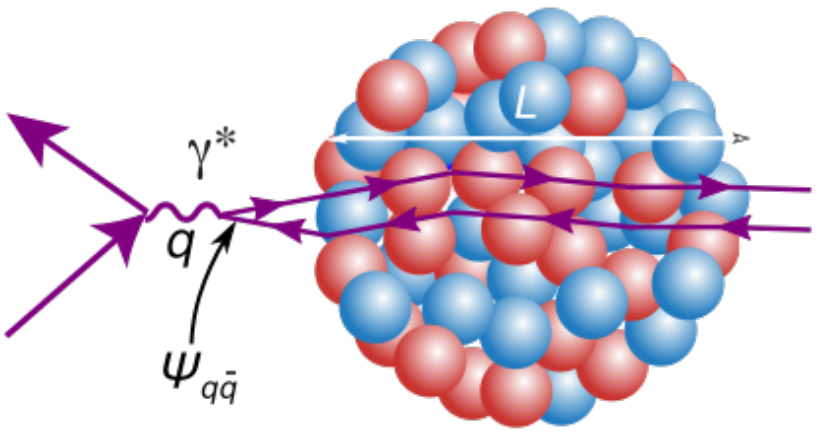


Figure: $p_\perp$=120 GeV and 50 GeV in PbPb collisions at the LHC.

Mueller, BW, Xiao and Yuan, Phys. Lett. B **763**, 208 (2016).

3.2 QCD correction to quark production: $\frac{1}{q^-} \gg L$

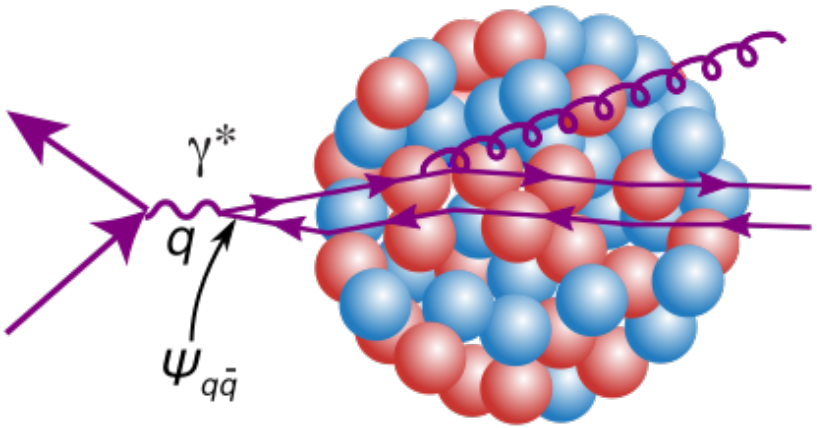
When $\frac{1}{q^-} \gg L$:



$$\frac{dN}{d^2b_\perp d^2k_\perp} = \frac{Q^2 N_c}{32\pi^6} \int d^2x_1 d^2x_2 \int_0^1 [z^2 + (1-z)^2] e^{-ik_\perp \cdot (x_1 - x_2)}$$
$$\times \nabla_{x_1} K_0(\sqrt{Q^2 x_1^2 z(1-z)}) \nabla_{x_2} K_0(\sqrt{Q^2 x_2^2 z(1-z)})$$
$$\times [S(x_1 - x_2) - S(x_1) - S(x_2) + 1]$$

3.2 QCD correction to quark production: $\frac{1}{q^-} \gg L$

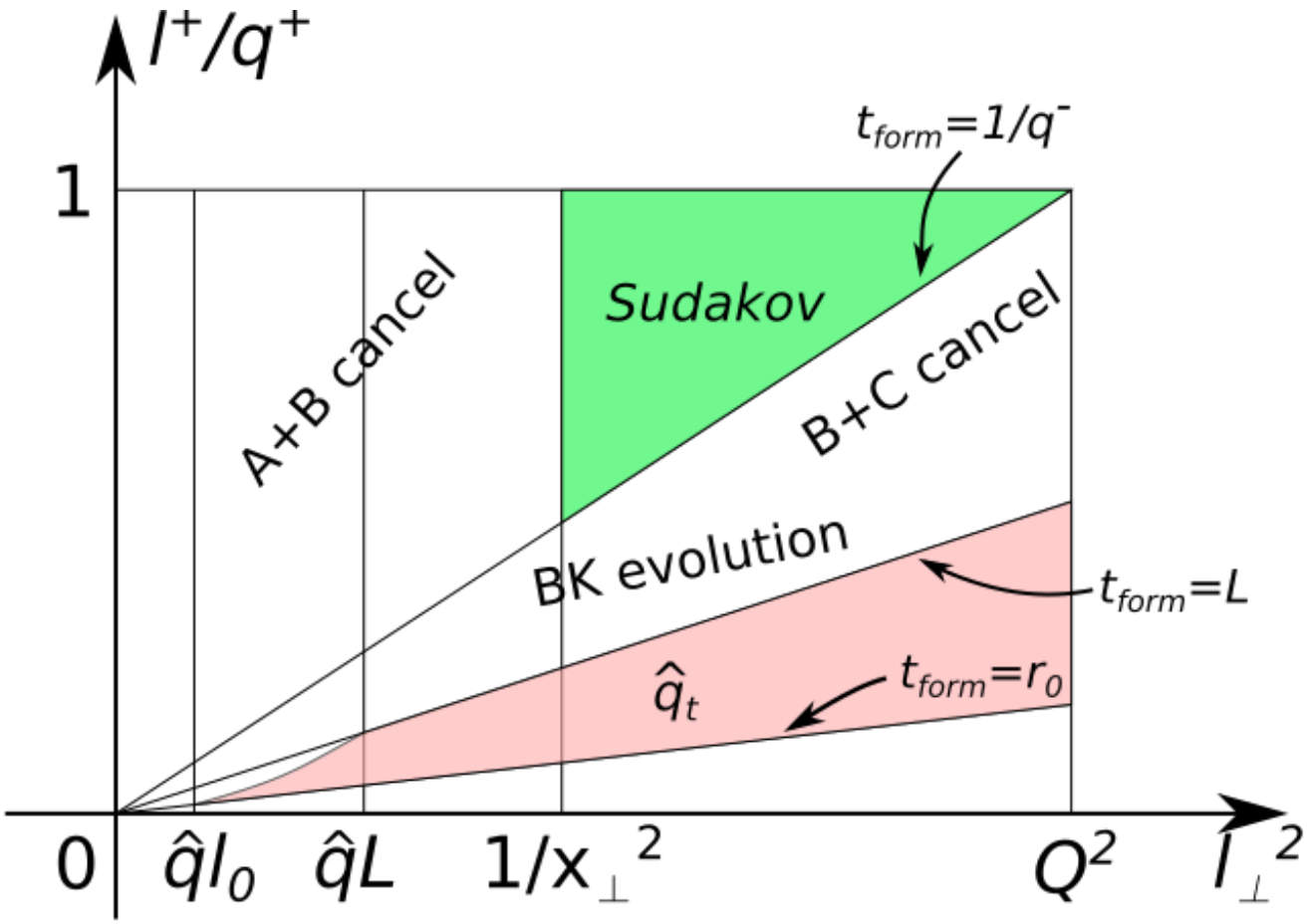
When $\frac{1}{q^-} \gg L$:



$$\frac{dN}{d^2b_\perp d^2k_\perp} = \frac{Q^2 N_c}{32\pi^6}\int d^2x_1 d^2x_2 \int_0^1 [z^2+(1-z)^2] e^{-ik_\perp\cdot(x_1-x_2)}$$
$$\times \nabla_{x_1} K_0(\sqrt{Q^2x_1^2 z(1-z)})\nabla_{x_2} K_0(\sqrt{Q^2x_2^2 z(1-z)})$$
$$\times \underbrace{[S(x_1-x_2)-S(x_1)-S(x_2)+1]}_{\text{medium-induced}} \underbrace{e^{-\frac{\alpha_s C_F}{2\pi}\ln^2(Q^2x_\perp^2)}}_{\text{vacuum radiation}}$$

3.2 QCD correction to quark production: $\frac{1}{q^-} \gg L$

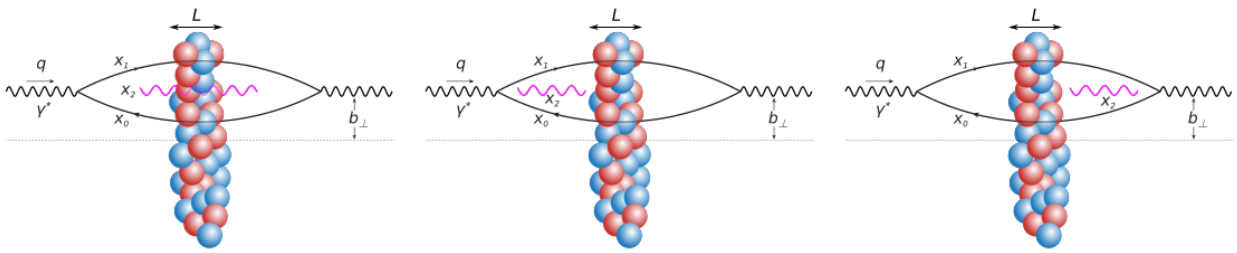
Two double logs are factorized:



3.2 QCD correction to quark production: $\frac{1}{q^-} \gg L$

The small-x evolution

BK:



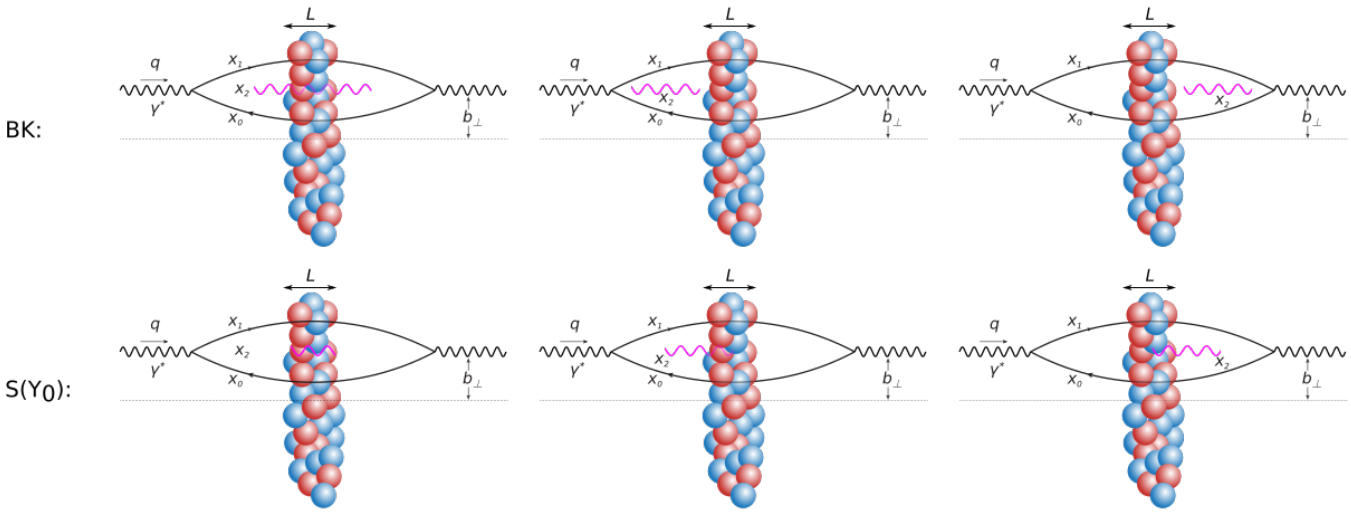
First 3 graphs: The Balitsky-Kovchegov (BK) equation

$$\frac{\partial}{\partial Y} S(x_{10}, b_\perp, Y) = \frac{\alpha_s N_c}{2\pi^2} \int d^2 x_2 \frac{x_{10}^2}{x_{20}^2 x_{21}^2}$$
$$\times [S(x_{12}, b_\perp + \frac{x_{20}}{2}, Y) S(x_{20}, b_\perp + \frac{x_{21}}{2}, Y) - S(x_{10}, b_\perp, Y)]$$

Initial condition: $S(x_\perp, b_\perp, Y_0) = e^{-\frac{1}{4}\hat{q} L x_\perp^2}$ in the MV model

3.2 QCD correction to quark production: $\frac{1}{q^-} \gg L$

The small-x evolution and medium-induced double log



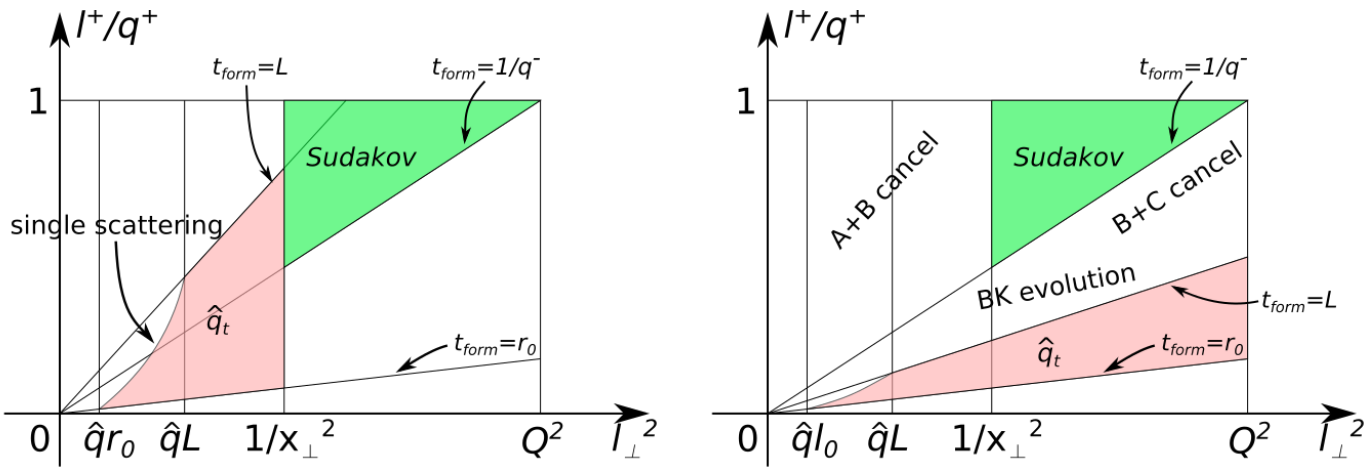
First 3 graphs: The Balitsky-Kovchegov (BK) equation

$$\frac{\partial}{\partial Y} S(x_{10}, b_\perp, Y) = \frac{\alpha_s N_c}{2\pi^2} \int d^2 x_2 \frac{x_{10}^2}{x_{20}^2 x_{21}^2}$$
$$\times [S(x_{12}, b_\perp + \frac{x_{20}}{2}, Y) S(x_{20}, b_\perp + \frac{x_{21}}{2}, Y) - S(x_{10}, b_\perp, Y)]$$

Last 3 graphs: Initial condition $S(x_\perp, b_\perp, Y_0) = e^{-\frac{1}{4}\hat{q}_t L x_\perp^2}$ at $Y = Y_0 = \ln(LM)$

Summary

1. **Sudakov (vacuum) and medium-induced double logs factorize.**



2. **To study parton saturation, Q should not be too large.**

3. **Energy loss from vacuum and medium-induced radiation?**