Lattice study of the phase structure of the strong Yukawa model

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The collaboration

- Germany
  - NIC, DESY Zeuthen and Humboldt University Berlin
    John Bulava (CERN), Philipp Gerhold ($\rightarrow$ d-Fine), Karl Jansen, Attila Nagy.
- Japan
  - Kobayashi-Maskawa Institute, Nagoya University
    Kei-Ichi Nagai.
- Taiwan
  - National Chiao-Tung University, Hsinchu
    C.-J.David Lin, Kenji Ogawa.
  - National Taiwan University, Taipei
    George W.-S. Hou, Bastian Knippschild, Brian Smigielski ($\rightarrow$ U.S.).

Motivation

Heavy fermions beyond SM3?

- Not much is known for strong (*non-perturbative*) Yukawa theory.

- Heavy extra generation of fermions may
  - enhance CP violation.

    G.W.S. Hou, 2008

  - offer an alternative way to break EW symmetry dynamically and induces bound states to unitarise WW scattering.

    B. Holdom, 2007

  - UV stablise the SM.

    P.Q. Hung, C. Xiong, 2009

Outline

- Goals, general issues and recent developments.
- Simultation setup.
- The phase structure.
- Exploratory numerical studies.
  - VEV.
  - Susceptibility and critical exponents.
  - Binder's cumulant.
- Future plan.

Targets for the bare strong-Yukawa regime

- The nature of the phase transitions.

  $\Rightarrow$ Connection to the continuum world (next slide).

- Possible bound states.

  $\Rightarrow$ Computation of the spectrum.

- Possible new mechanism for dynamical symmetry breaking.

  $\Rightarrow$ Heavy scalar with fermion condensate?

General issues and strategy

- The triviality (Landau-pole) problem.

  $\Rightarrow$ Non-trivial to take the lattice spacing to zero.

- Look for 2nd-order phase transitions via "scanning simulations".

  $\Rightarrow$ $\xi \to \infty$.

- Problem: Finite-volume effects.

  $\Rightarrow$ Phase transitions are washed out.

  $\Rightarrow$ Severe near the critical points since $L = \hat{L}a$.

- Chiral fermions required.

New ingredients in current work

- Previous studies (*circa* 1990):

  Lee, Shigemitsu, Shrock; Bock *et al.*,...

  - Use fermions without exact chiral symmetry.

    $\Rightarrow$ Ambiguity in defining chiral fermions.

  - Small ($\sim 8^3 \times 16$) volumes and no $L \to \infty$ limit taken.

- Current new-generation simulations:
  - Use the overlap fermion (exact chiral symmetry).
  - Several large volumes and $L \to \infty$ limit taken.

    $\Rightarrow$ Test finite-size scaling behaviour.

    $\Rightarrow$ Determine the order of the phase transition.

Reminder: Notaion for scalar field theory

- The discretised Euclidean scalar action ($a = 1$)

$$S_\varphi \;=\; -\sum_{x,\mu} \varphi_x^\alpha \varphi_{x+\hat{\mu}}^\alpha + \sum_x \left[\frac{1}{2}(2d + m_0^2)\varphi_x^\alpha \varphi_x^\alpha + \frac{1}{4}\lambda_0 (\varphi_x^\alpha \varphi_x^\alpha)^2 \right].$$

- $\varphi = \sqrt{2\kappa}\phi, \quad m_0^2 = \frac{1-2\hat{\lambda}}{\kappa} - 2d, \quad \lambda_0 = \frac{\hat{\lambda}}{\kappa^2}$

$$S_\phi = -2\kappa \sum_{x,\mu} \phi_x^\alpha \phi_{x+\hat{\mu}}^\alpha + \sum_x \left[\phi_x^\alpha \phi_x^\alpha + \hat{\lambda}(\phi_x^\alpha \phi_x^\alpha - 1)^2 \right],$$

$$Z_\phi = \int \prod_{x,\alpha} d\phi_x^\alpha \ \mathrm{exp}(-S_\phi) = \int \prod_{x,\alpha} d\mu(\phi_x^\alpha) \ \mathrm{exp}\left(2\kappa \sum_{x,\mu} \phi_x^\alpha \phi_{x+\hat{\mu}}^\alpha \right),$$

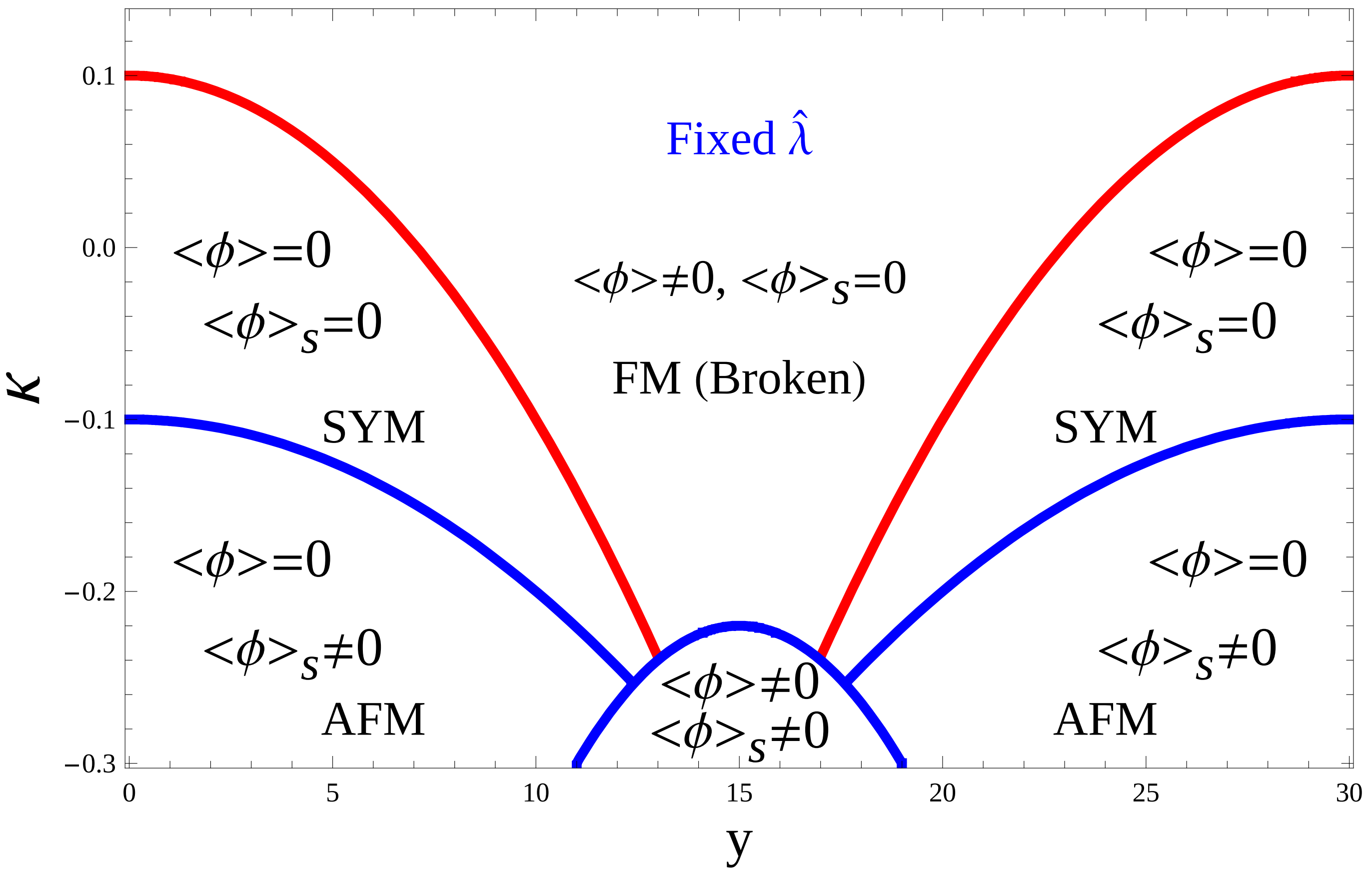
$$d\mu(\phi_x^\alpha) = d\phi_x^\alpha \ \mathrm{exp}\left[-\phi_x^\alpha \phi_x^\alpha - \hat{\lambda}(\phi_x^\alpha \phi_x^\alpha - 1)^2 \right].$$

- "staggered symmetry": $\kappa \to -\kappa$ and $\phi_x^\alpha \to (-1)^{x_1+x_2+\ldots+x_d}\phi_x^\alpha$.

Fermions and the Yukawa couplings

- Use the overlap Dirac operator with exact lattice chiral symmetry.
- The Yukawa terms $S_{HY} = \sum_x y(\bar{t}_x, \bar{b}_x)_L \Phi_x b_{x,R} + y(\bar{t}_x, \bar{b}_x)_L \tilde{\Phi}_x t_{x,R} +$ h.c..
  - Φ is a complex scalar doublet and $\tilde{\Phi} = i\tau_2 \Phi^*$.
- Results presented in this talk are from $8^3 \times 16$, $12^3 \times 24$ and $16^3 \times 32$.

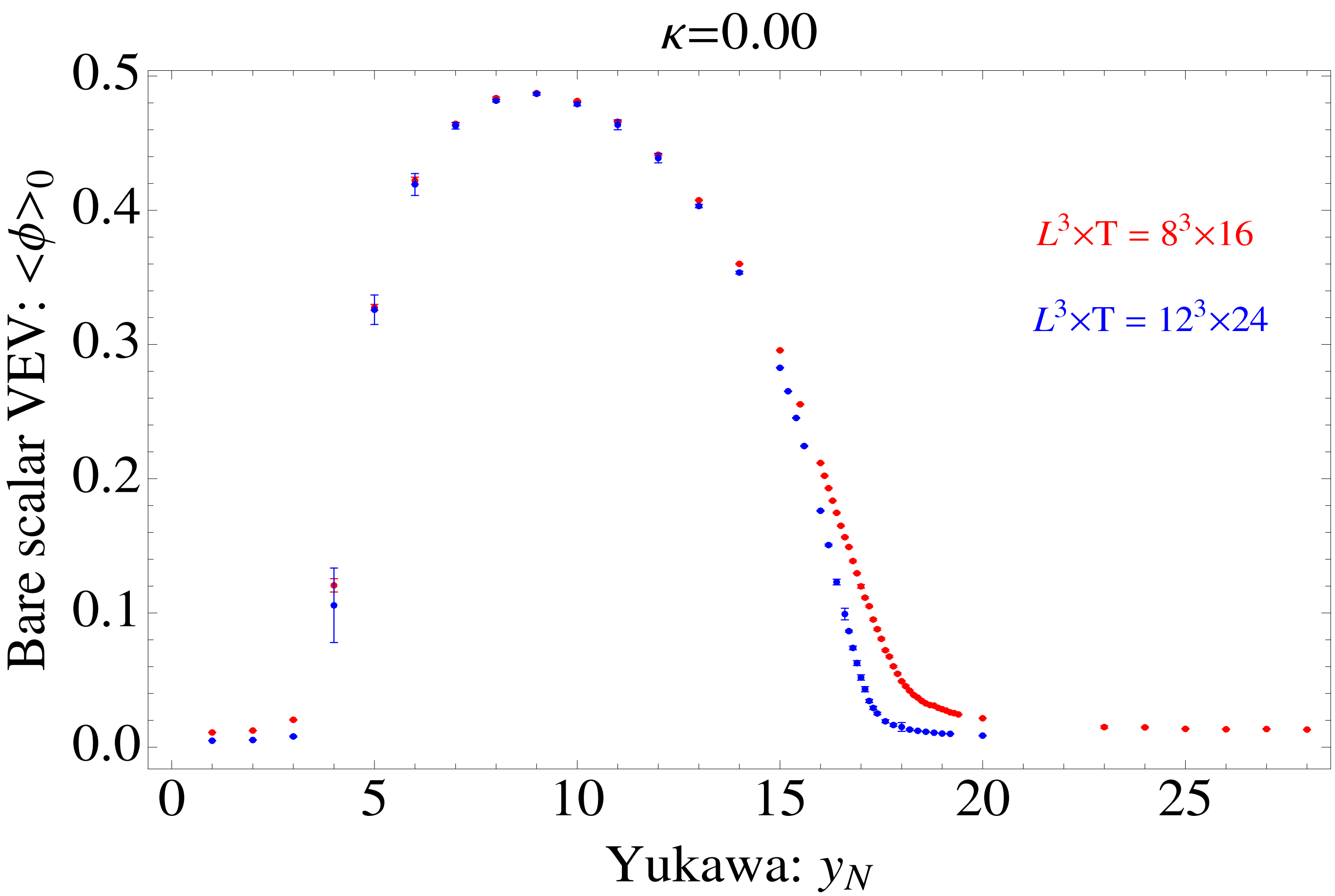
Phase diagram of the H-Y model (qualitative)



$*$ From earlier work using Wilson fermions.

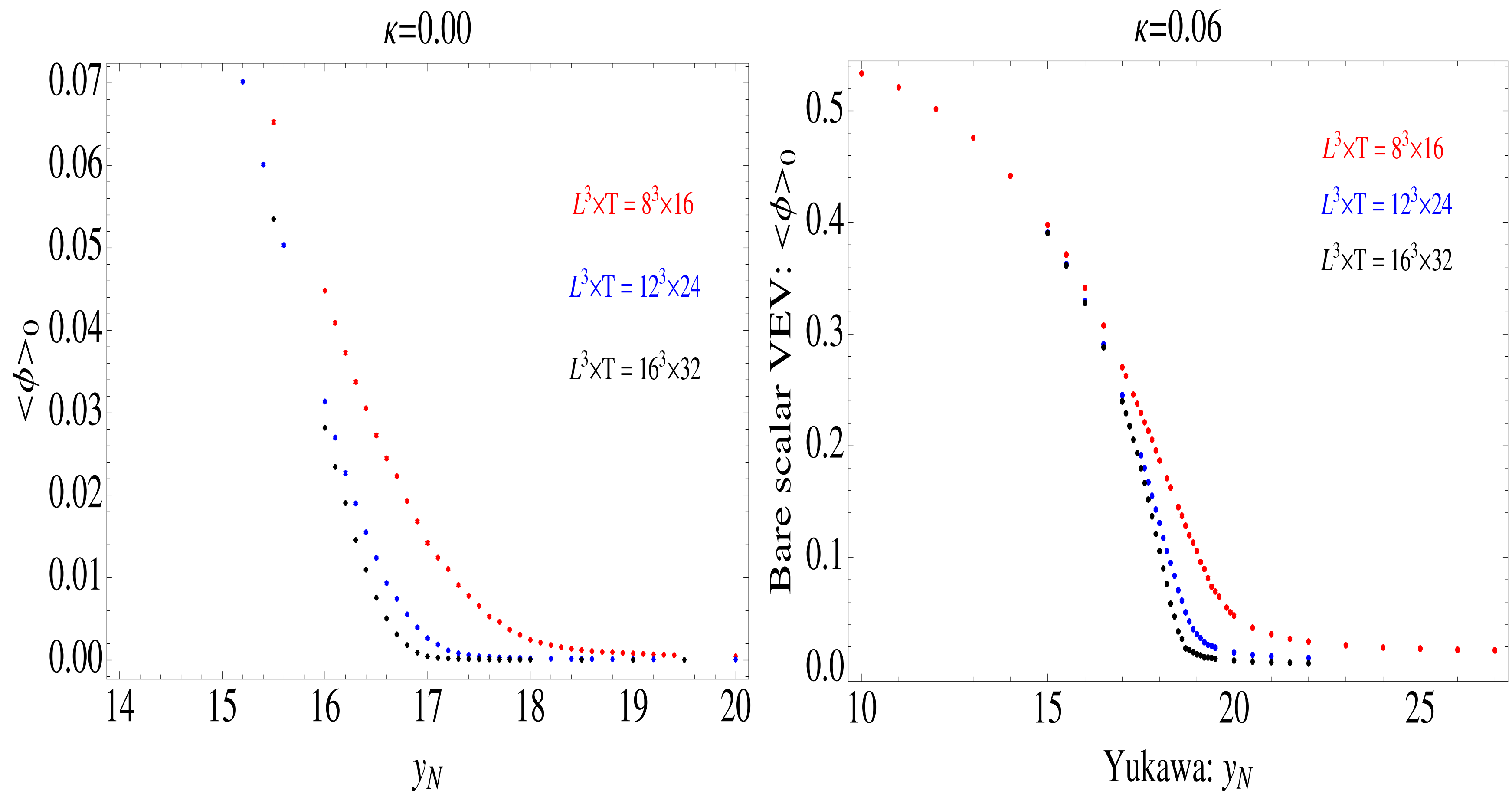
$\Rightarrow$ Controversy from staggered-fermion calculations.

Evidence of a symmetric phase at large y



Consistent with recent results in P. Gerhold and K. Jansen, 2007.

The bare scalar vev at large Y



Finite-size scaling of susceptibility

- Susceptibility: $\chi = V_4 \left(\langle \phi^2 \rangle - \langle \phi \rangle \langle \phi \rangle \right)$.
- The scaling behaviour from solving the RGE,
  - Universal function $\chi L_s^{-\gamma/\nu} \sim g(\tilde{t} L_s^{1/\nu})$, where $\tilde{t} = (y/y_{\rm crit} - 1)$.
  - critical exponents γ and ν.
  - Modelling the scaling violation from

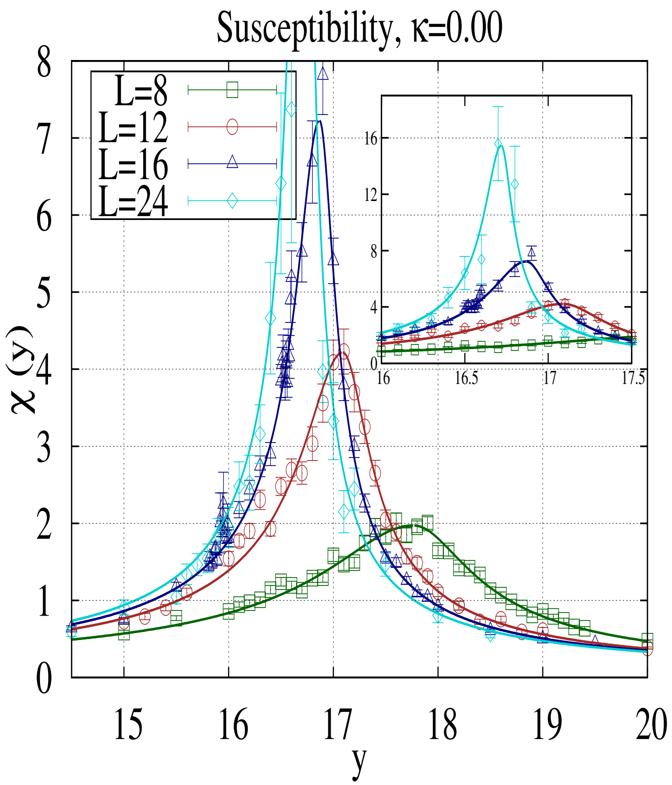
    M. Fisher and M. Barber, 1972

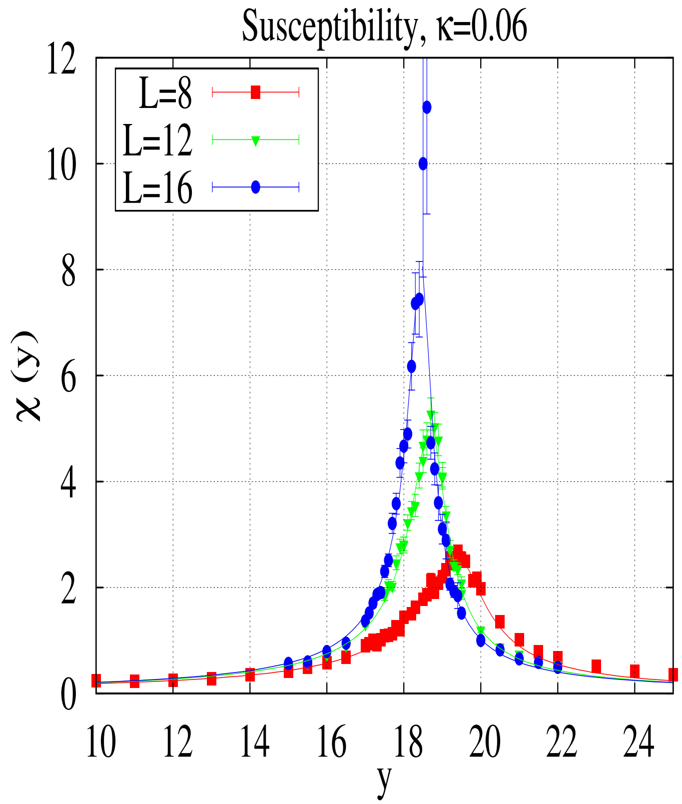
    $\Rightarrow \chi L_s^{-\gamma/\nu} \sim g(t L_s^{1/\nu})$, where $t = (y/(y_{\rm crit} - A_4/L_s^b) - 1)$.
  - Fit all the data to the (partly empirical) function at fixed κ

    K. Jansen and P. Seuferling, 1990

$$\chi \;\; = \;\; A_1 \left\{ L_s^{-2/\nu} + A_{2,3} \left(y - y_{\rm crit} - A_4/L_s^b \right)^2 \right\}^{-\gamma/2}.$$

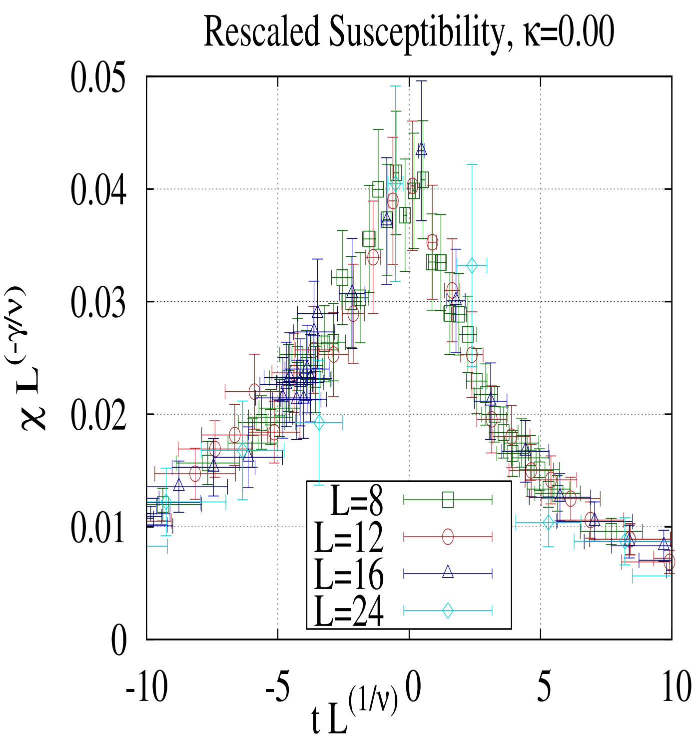
Finite-size fit of susceptibility

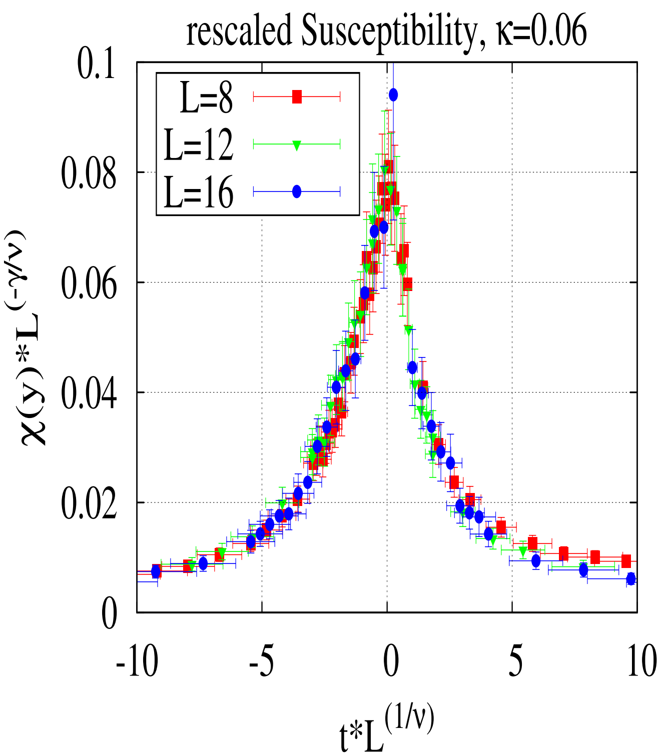




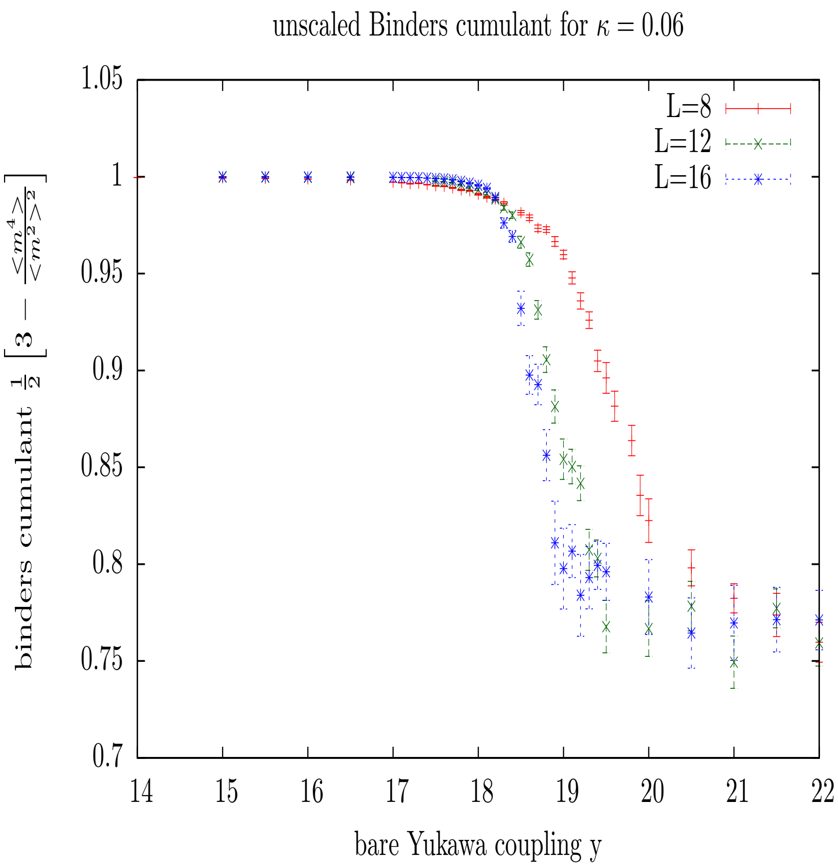
Fit range: $y = 15.0 \sim 20.0$ $y = 14.0 \sim 22.0$

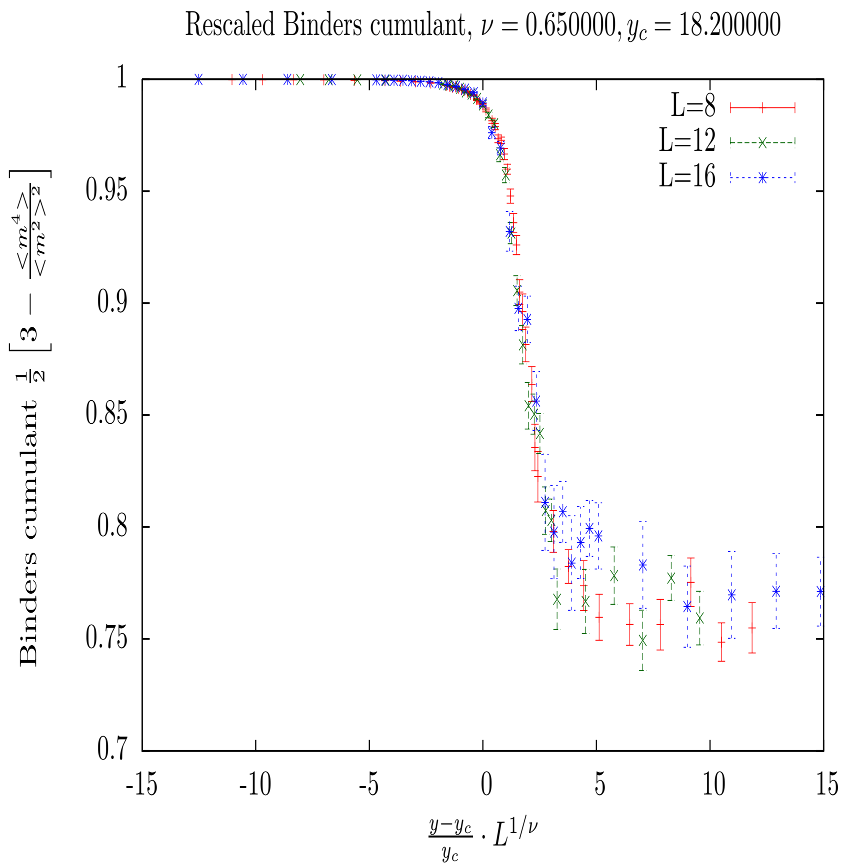
Finite-size scaling of susceptibility





Finite-size scaling of Binder's cumulant





Probing the phase structure using susceptibility

	$\kappa = 0.00$	$\kappa = 0.06$	O(4) scalar model
y_{crit}	16.57 ± 0.06	18.11 ± 0.06	N/A
γ	0.97 ± 0.02	1.08 ± 0.01	1
ν	0.52 ± 0.01	0.66 ± 0.02	0.5
b	2.18 ± 0.09	2.04 ± 0.20	?

- Quoted errors are statistical.
- Estimate systematics by changing the fit range in y.
- Systematic effects
  - y_{crit} is very stable.
  - γ can change by $\sim 2\%$.
  - ν can vary by $\sim 8\%$. $\Rightarrow$ Different from O(4) scalar model?

Outlook

- Improving results by
  - running more at large lattices, $24^3 \times 48$. (finishing soon.)
  - using more sophisticated procedure to investigate the susceptibility.
  - studying the details of the scaling behaviour of Binder's cummulant.
- More information:
  - Compute three renormalised couplings to "trade" with κ, $\widehat{\lambda}$ and y.
  - Study the spectrum in the strong Yukawa regime.

A lot more to do and to understand.