Factorization and Jet Algorithms in Soft-Collinear Effective Theory

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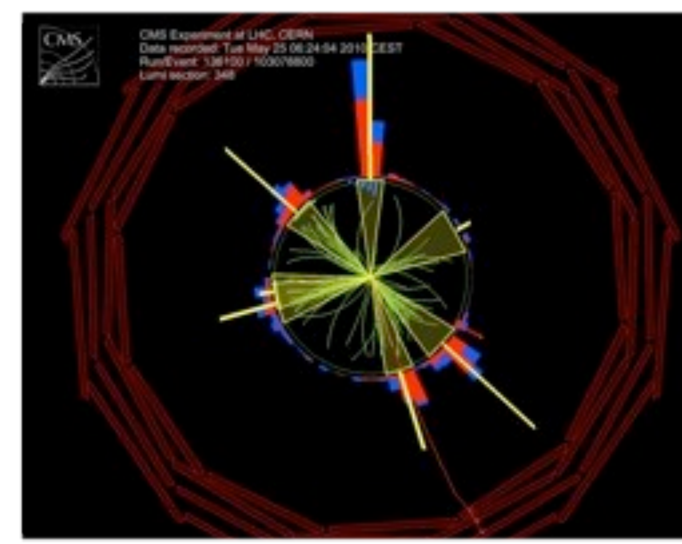
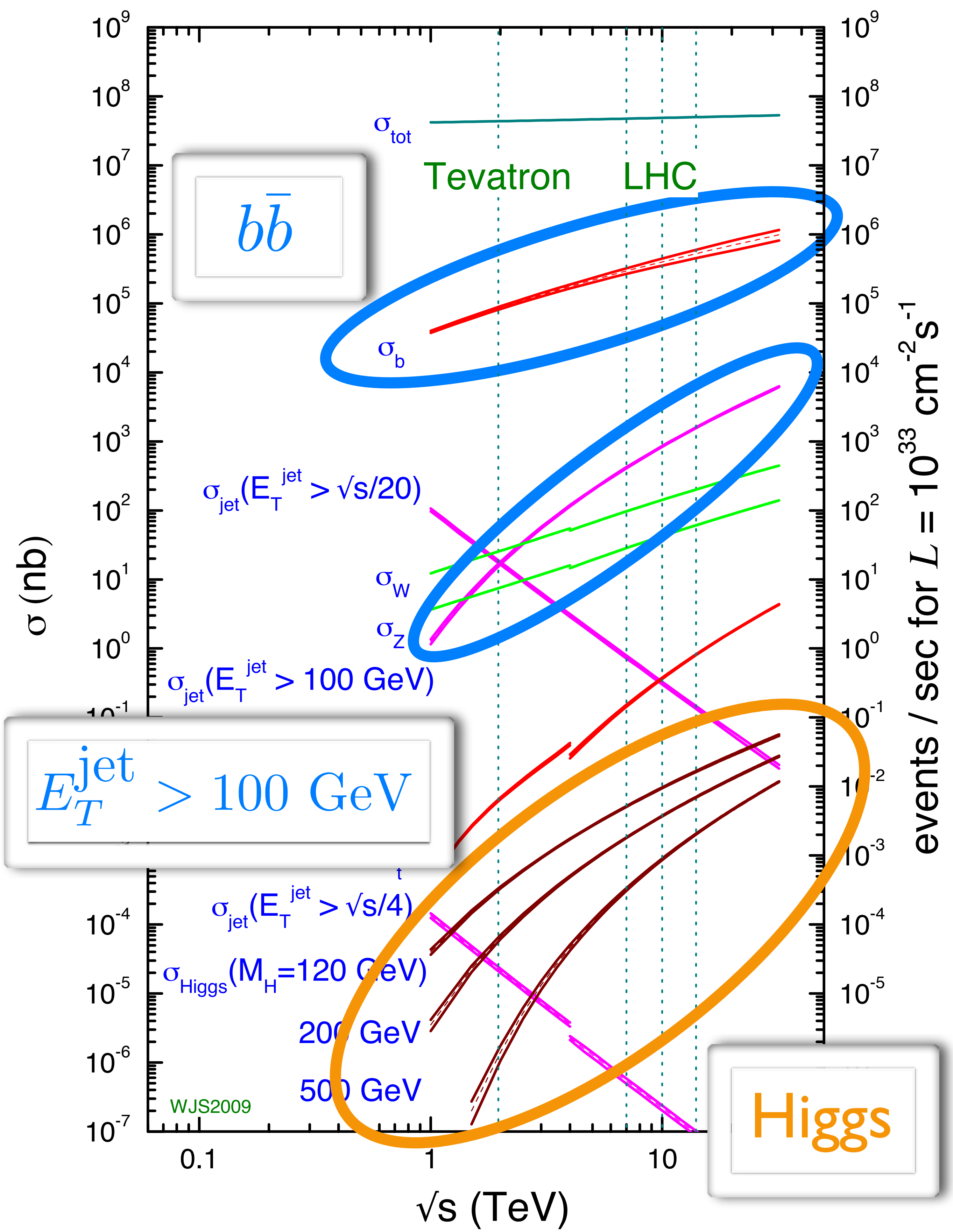
LHC-TI Fellows Meeting BNL, Oct 8, 2010

Outline

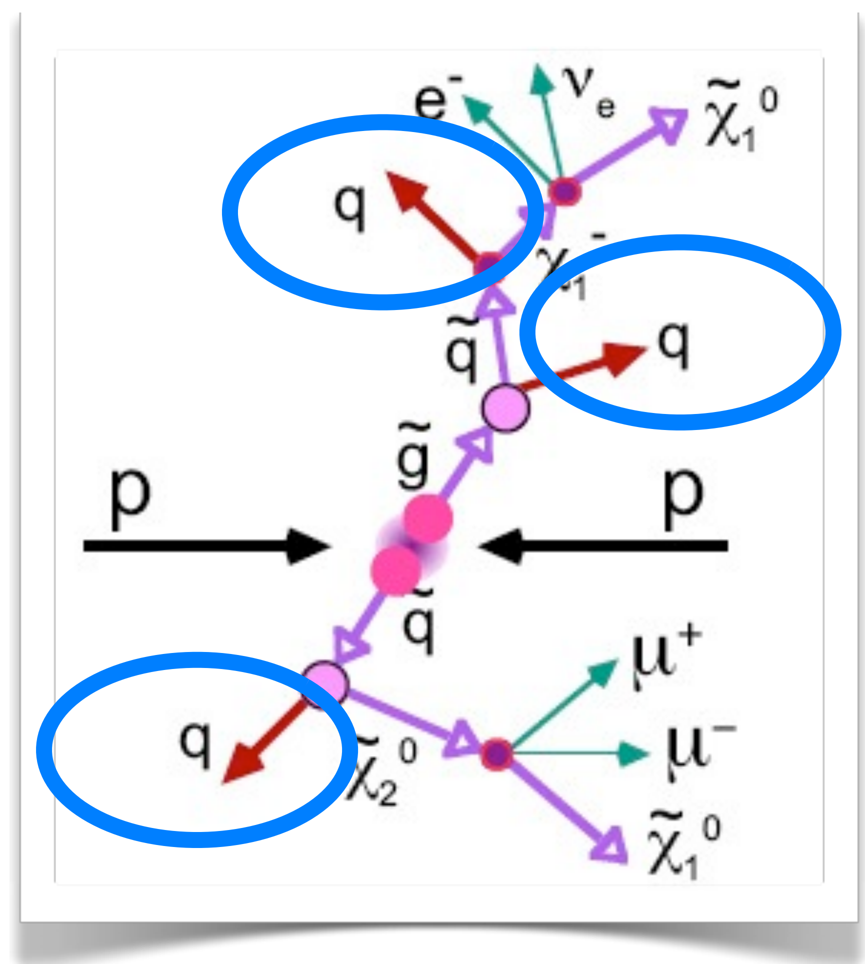
- Factorizing Jet Cross Sections in SCET
- Including the Jet Algorithm
- Results for Sterman-Weinberg Jets
- Conclusions



Why Study Jets?



- Large jet cross sections at the LHC and Tevatron
- Many BSM signatures have multiple jets

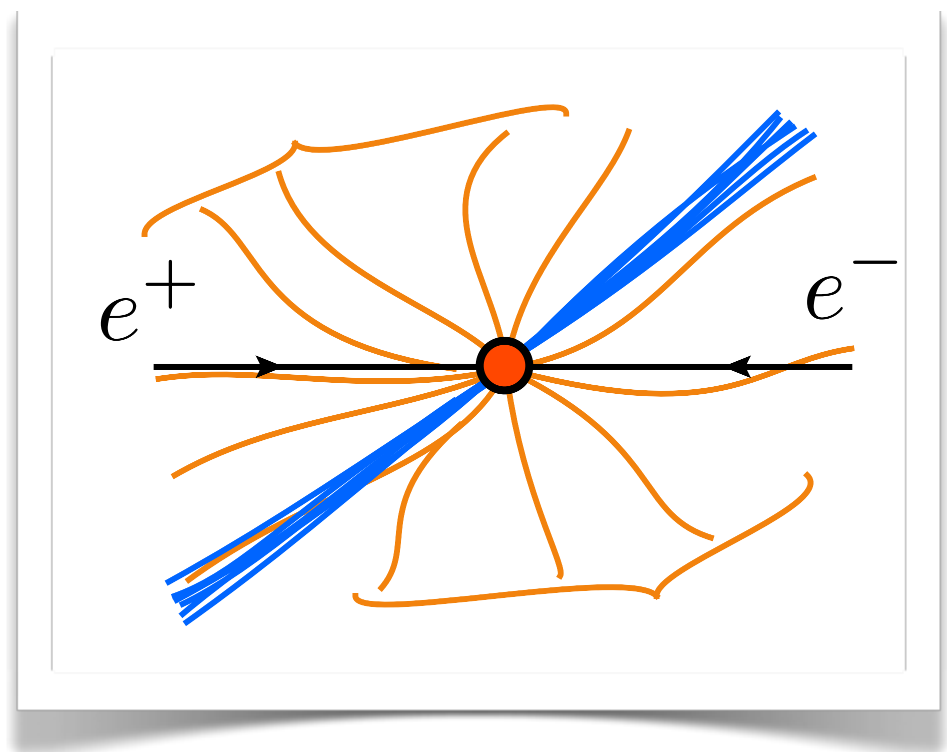


SUSY decay chain

Factorization Separates Physics at Different Scales

- Predicting any QCD observable relies on factorizing perturbative and nonperturbative physics
  Collins, Soper, Sterman

- Some observables depend on several perturbative scales. Consider jet mass m_J in e^+e^- into dijets
  - Hard partonic interaction: Q
  - Collinear radiation in jets: m_J
  - Soft radiation outside jets: m_J^2/Q



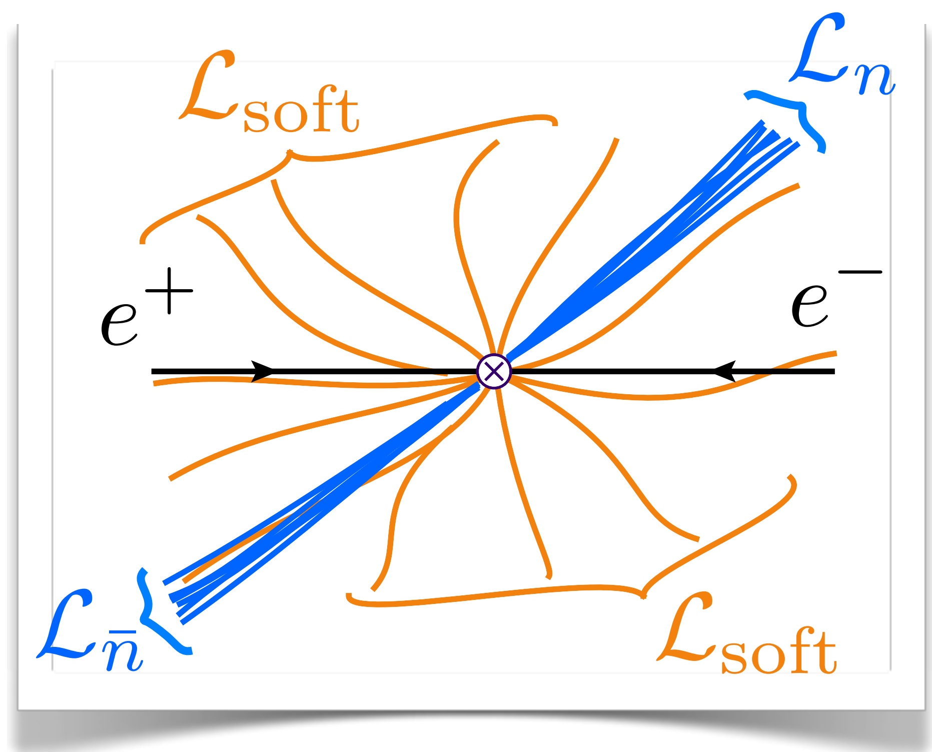
- Want to factorize the cross section into pieces that depend on a single scale
  - For small m_J we must resum large logarithms between scales to obtain reliable predictions

Effective Theories Simplify Factorization

- Effective theory integrates out less relevant degrees of freedom
  - Often use a momentum cutoff Λ
  - Soft-Collinear Effective Theory (SCET) describes soft and collinear modes. Collinearity is with respect to some fixed light-cone directions.

Bauer, Fleming, Pirjol, Stewart

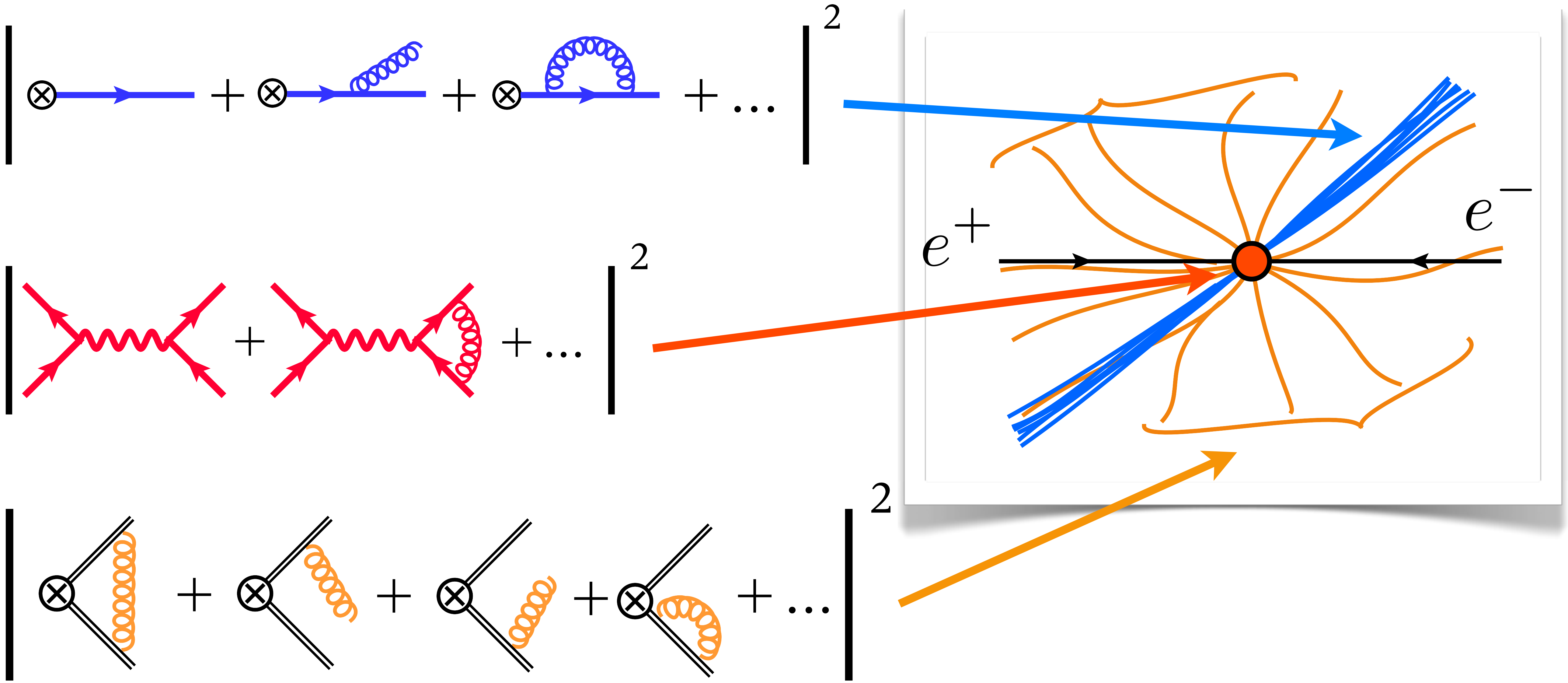
- SCET Lagrangian splits into soft and collinear sectors containing separate fields. This facilitates deriving factorization theorems



- To focus on jets, here study e+e- to dijets
- Results can be used for factorization theorems at hadron colliders

$$\mathcal{L}_{\rm SCET} = \mathcal{L}_n + \mathcal{L}_{\bar{n}} + \mathcal{L}_{\rm soft}$$

Factorizing a Cross Section in SCET

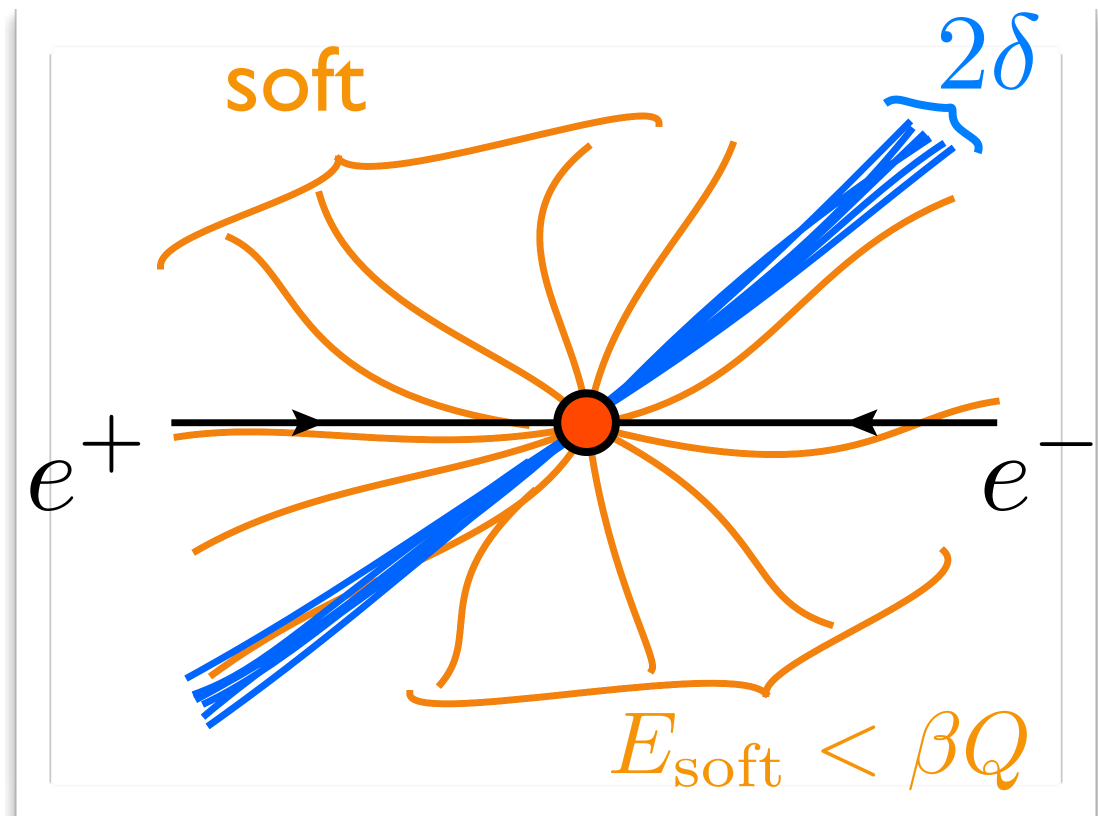


$$d\sigma \sim H \, \left(J \otimes J \otimes S \right)$$

- The present work focuses on the jet function J

Factorizing a Cross Section in SCET

- Sterman-Weinberg (SW) algorithm is defined in terms of the cone half-angle δ and soft radiation fraction β
- Previous SCET work on jet algorithms
  - Factorization with generic jet algorithm
    Bauer, Hornig, Tackmann
  - Resum phase space logs for SW jets
    Trott
  - Phase space for JADE, SW, kT jets
    Cheung, Luke, Zuberi



- Factorization theorem for jets with cone and recombination algorithms has recently been derived Ellis, Hornig, Lee, Vermilion, Walsh 0912.0262, 1001.0014

$$d\sigma^{\mathrm{SW}(\delta,\beta)} \sim H \times \left(J^{\mathrm{SW}(\delta,\beta)} \otimes J^{\mathrm{SW}(\delta,\beta)} \otimes S^{\mathrm{SW}(\delta,\beta)} \right)$$

  - NLL resummation for logs of m_J
  - Calculation for jet function agrees with the one presented here (TJ 0912.5509)
  - Some restrictions for parameters specifying the algorithm

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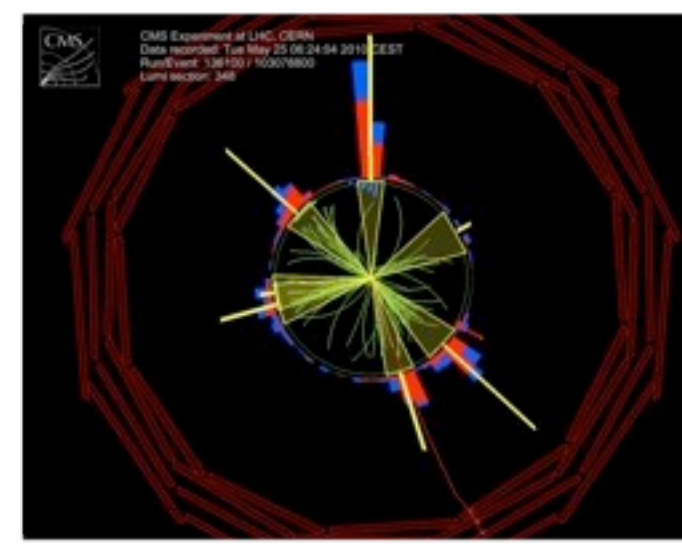
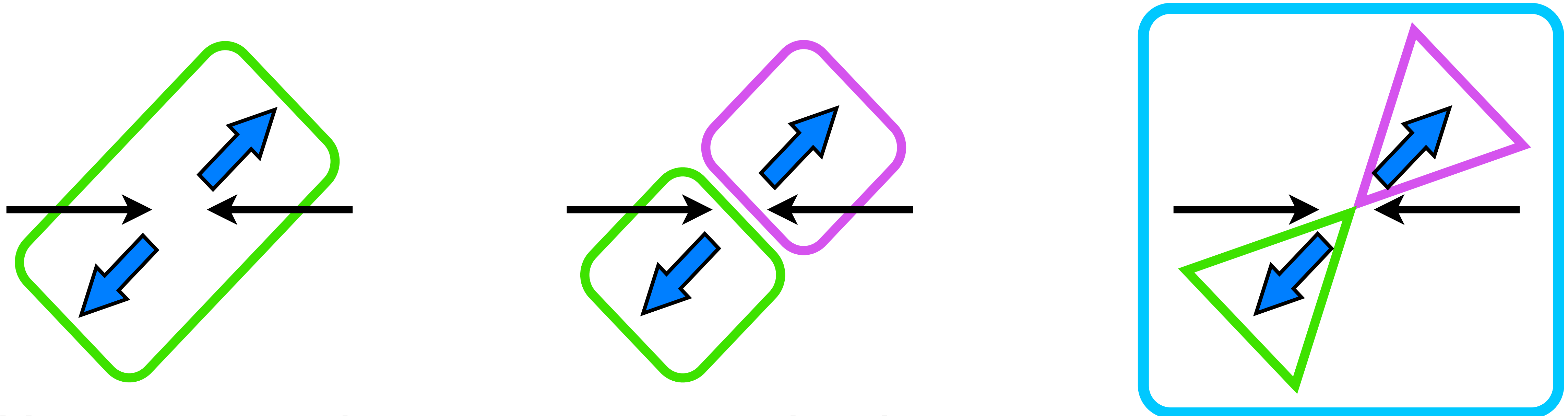


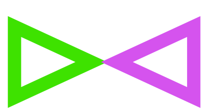
What is Included in a Jet Function?



- We want to study jets in increasing detail

 Sum over everything - no jet function

 Hemisphere jets - inclusive jet function
  - Almost always used to describe jets in SCET
  - No explicit restrictions on collinear particles but require jet mass to be small

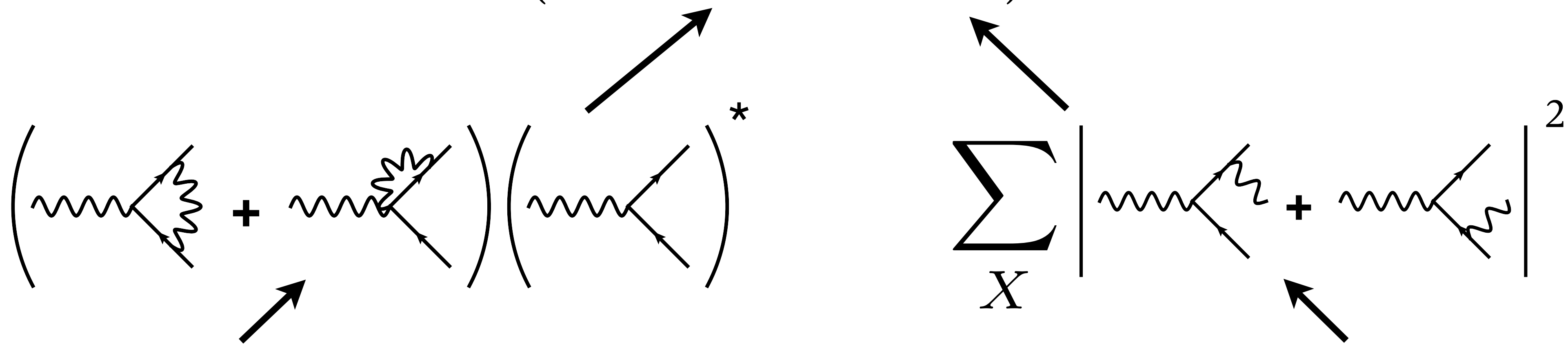
 Realistic jet algorithm - algorithm dependent jet function
  - Explicitly cut final state phase space
  - Threshold factorization for pp has been studied in the presence of a jet algorithm

Kidonakis, Odera, Sterman

Aside: Optical Theorem and IR Divergences

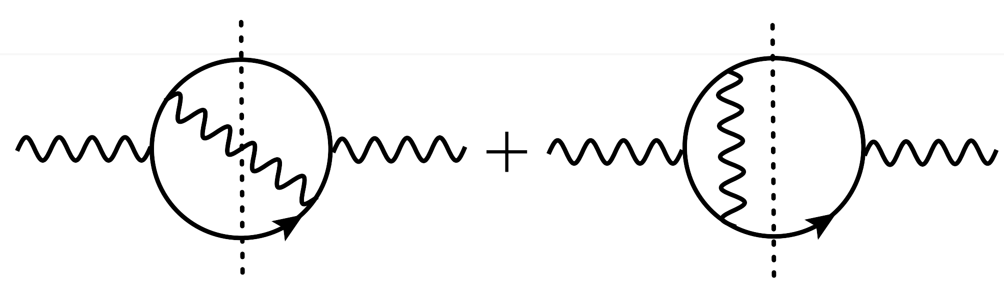
- Infrared divergences cancel between virtual and real graphs

$$\sigma \sim \sigma_B(1 + A_V + A_R)$$



$$-\ln\left(\frac{q^2}{\mu^2}\right)\ln\left(\frac{q^2}{m^2}\right) \qquad\qquad +\ln\left(\frac{\Delta}{\mu^2}\right)\ln\left(\frac{q^2}{m^2}\right)$$

$A_V + A_R \sim$

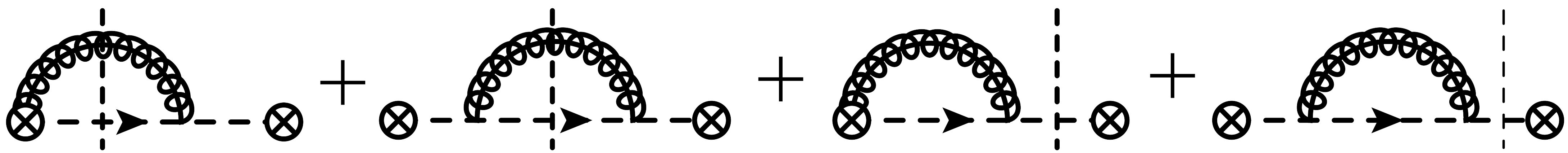


$+ \ldots$

$$\frac{1}{p^2 + i0} \to -2\pi i\delta\left(p^2\right)\theta(p^0)$$

(Here get $\Delta = q^2$)

Define the Jet Function

$J_n^{(\mathrm{inc})} \sim$ 

$$\sim \mathrm{Disc} \int d^4x\, e^{ir\cdot x} \langle 0|\mathrm{T}\{\bar{\chi}(0)\chi(x)\}|0\rangle \propto \sum_X \int d^4x\, e^{ir\cdot x} \langle 0|\chi(x)|X\rangle\langle X|\bar{\chi}(0)|0\rangle$$

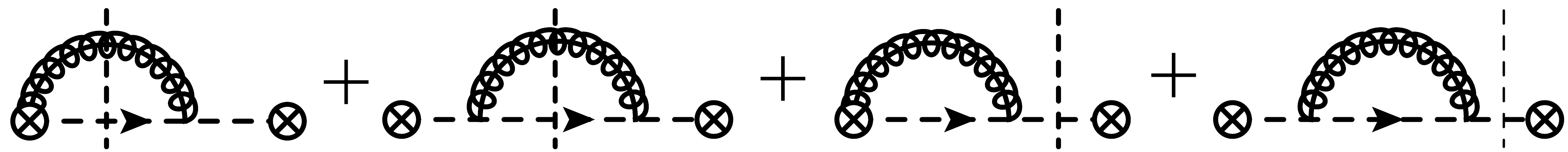
$$\chi \equiv W^\dagger \xi$$

Optical theorem

$$\frac{1}{p^2 + i0} \to -2\pi i\delta\left(p^2\right)\theta(p^0)$$

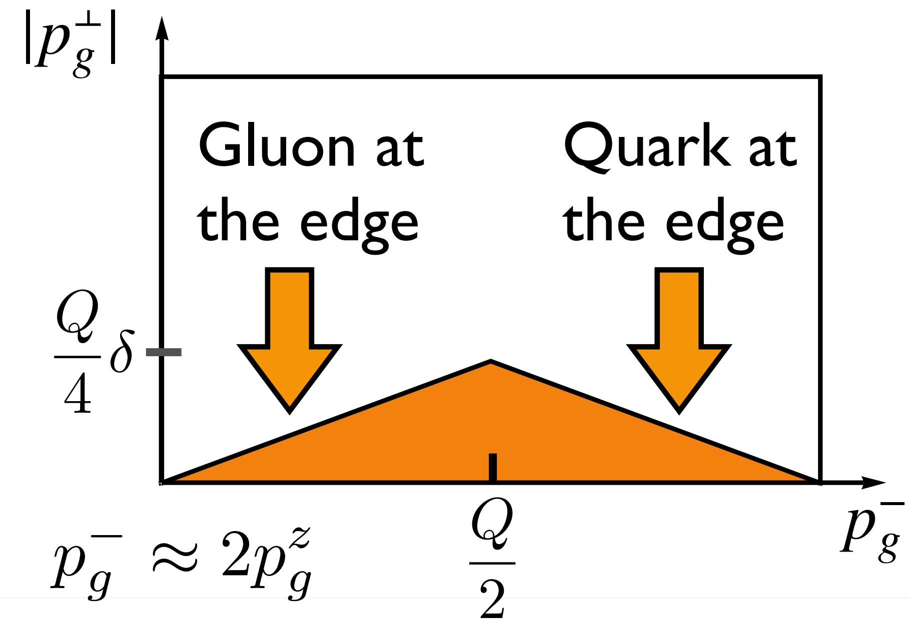
Cutkosky cutting rules

Define the Jet Function

$J_n^F \sim$ 

$$\sim \sum_X \int d^4x\, e^{ir\cdot x} \langle 0|\chi(x)|X\rangle \boxed{F(p_j)} \langle X|\bar{\chi}(0)|0\rangle$$

$$F_{\rm cone} = \theta\left(\tan\delta - \frac{|p_g^\perp|}{|p_g^z|}\right)\theta\left(\tan\delta - \frac{|p_q^\perp|}{|p_q^z|}\right)$$

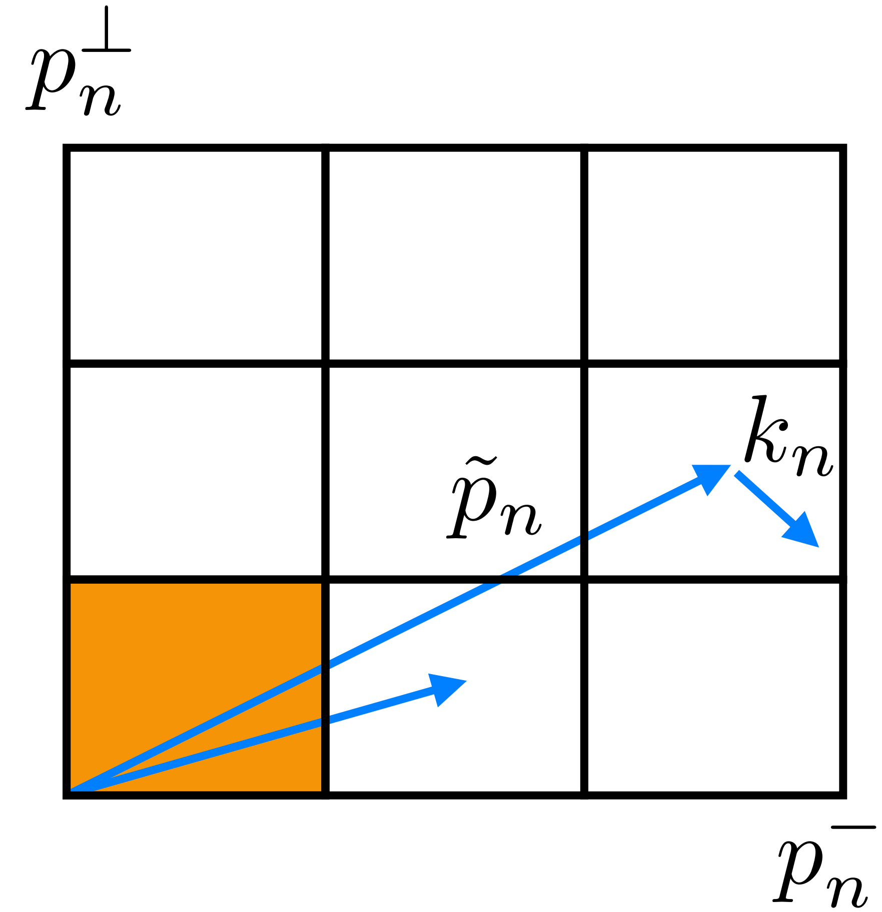


- The jet algorithm imposes cuts on final state phase space
  - Cutkosky rules get modified, need to add a factor F

SCET and Zero-bin

Manohar, Stewart

- To keep track of soft and collinear degrees of freedom, split collinear momenta $p_n = \tilde{p}_n + k_n$
  - Large "label" component $\tilde{p}_n$ is discrete, chooses a square in momentum grid
  - Small "residual" component k_n is continuous, moves inside a square



- Label component of collinear momentum cannot be zero
  - Physics of zero-bin region encoded in the soft function
- For perturbative results at leading power the zero-bin subtraction is equivalent to collinear subtraction in CSS factorization

Lee, Sterman

Momentum Integrals in SCET

- Sums are inconvenient, want integrals over collinear momenta

$$\sum_{\tilde{p}_n \neq 0} \int \mathrm{d}k_n\, \mathcal{M}(\tilde{p}_n + k_n) = \underbrace{\int \mathrm{d}\tilde{p}_n\, \mathcal{M}(\tilde{p}_n)}_{\text{naive collinear}} - \underbrace{\int \mathrm{d}\tilde{p}_n\, \mathcal{M}^{\tilde{p}_n \to 0}(\tilde{p}_n)}_{\text{zero-bin}}$$

- Within dimensional regularization the zero-bin term is most often zero (scaleless). Some people felt the zero-bin was just a technicality.

- In the past couple of years the importance of the zero-bin subtraction for obtaining correct results for calculations was demonstrated
  - Jet function with a jet algorithm has a nontrivial zero-bin

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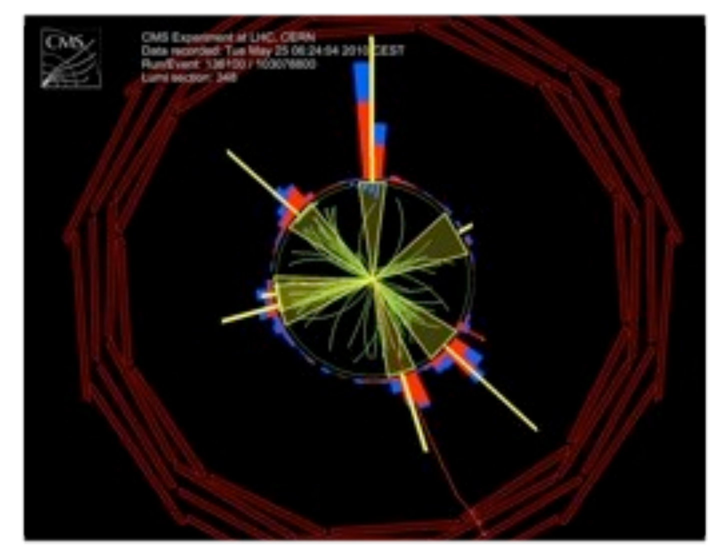
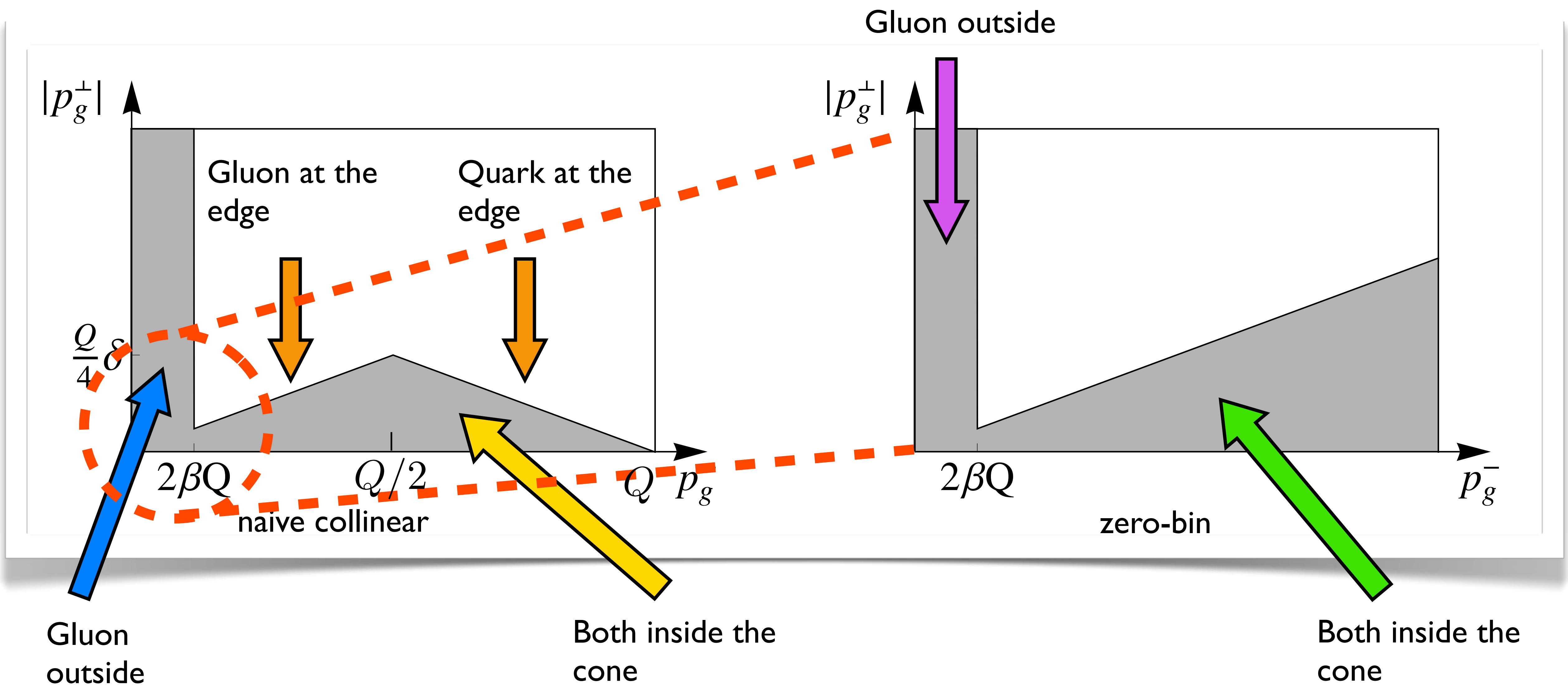


Sterman-Weinberg Algorithm



$$F_{\rm SW} = \boxed{\theta\left(\frac{|p_g^\perp|}{|p_g^z|} - \tan\delta\right)\theta\left(\beta Q - p_g^0\right)} + \boxed{\theta\left(\tan\delta - \frac{|p_g^\perp|}{|p_g^z|}\right)\theta\left(\tan\delta - \frac{|p_q^\perp|}{|p_q^z|}\right)}$$

$$F_{0,\rm SW} = \boxed{\theta\left(\frac{|p_g^\perp|}{|p_g^z|} - \tan\delta\right)\theta\left(\beta Q - p_g^0\right)} + \boxed{\theta\left(\tan\delta - \frac{|p_g^\perp|}{|p_g^z|}\right)}$$

Impact of the Zero-bin Contribution

$$\boxed{J_n^{\mathrm{SW}}(s) \equiv J_n^{(\mathrm{inc})}(s) + \Delta J_n^{\mathrm{SW}}(s)} \qquad s = q^2$$

$$\begin{aligned}\Delta\tilde{J}_n^{\mathrm{SW}}(s) =& \frac{\alpha_s C_F}{4\pi}\left(\delta^2 - \frac{4s}{Q^2}\right)\left\{\delta(s)\left[2\ln^2 2\beta - \ln^2\left(\frac{Q^2\delta^2}{4\mu^2}\right)\right]\right.\\ &\left. + \frac{4}{\mu^2}\left[\frac{\mu^2\theta(s)}{s}\right]_+ \left[\frac{6s}{4s+Q^2\delta^2 w} + \ln 2\beta + \ln\left(\frac{Q^2\delta^2}{4\mu^2}\right)\right] - \frac{4}{\mu^2}\left[\frac{\mu^2\theta(s)\ln\left(s/\mu^2\right)}{s}\right]_+\right\}\\ & + \frac{\alpha_s C_F}{4\pi}\left(\frac{4s}{Q^2} - \delta^2\right)\frac{4}{\mu^2}\left[\frac{\mu^2\theta(s)}{s}\right]_+ \left\{\frac{3}{4} + \ln 2\beta\right\}\end{aligned}$$

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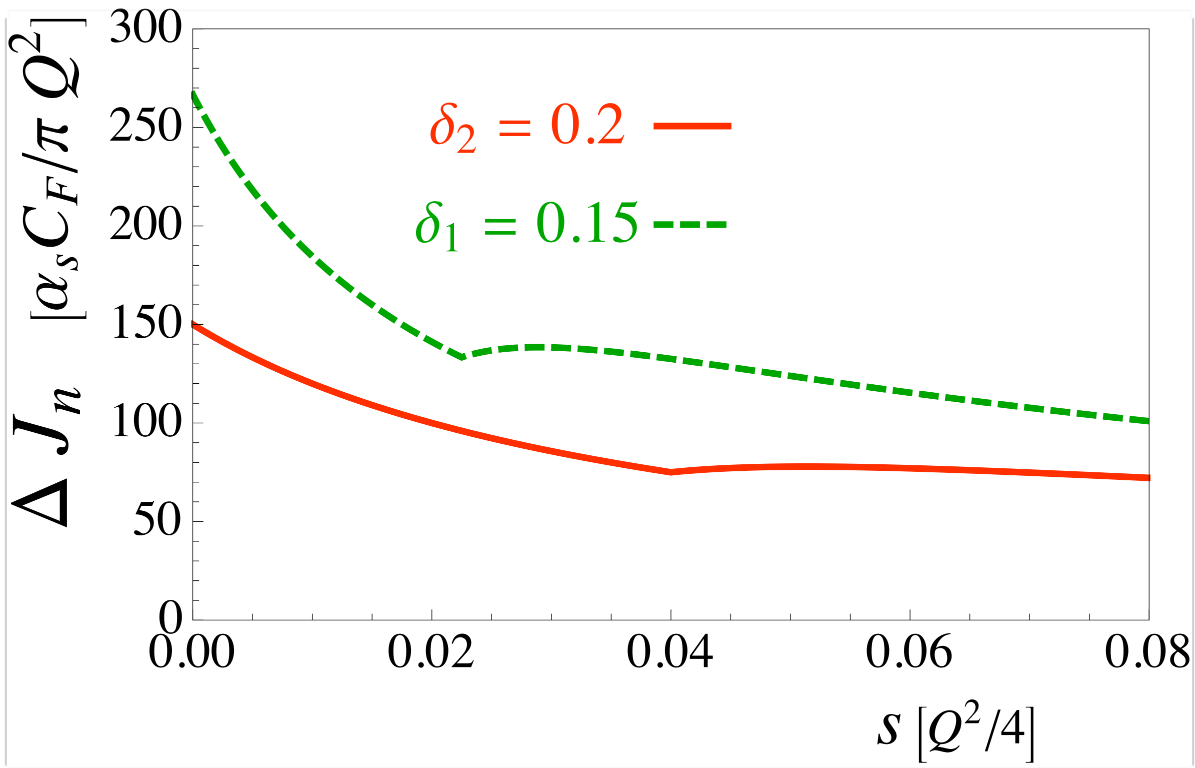
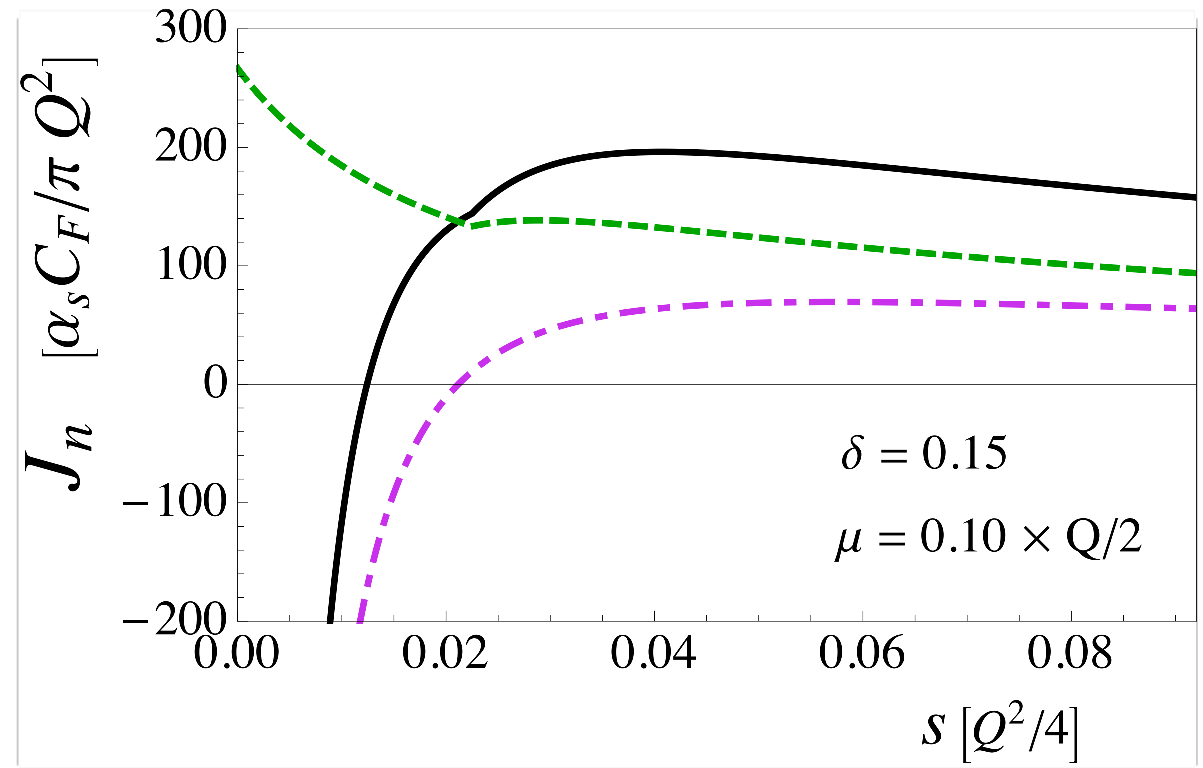
$$\begin{aligned}
\Delta J_n^{\mathrm{SW}}(s) = & \frac{\alpha_s C_F}{4\pi}\left(\delta^2 - \frac{4s}{Q^2}\right)\Bigg\{ \\
& + \frac{4}{\mu^2}\left[\frac{\mu^2\theta(s)}{s}\right]_+ \left[\frac{6s}{4s + Q^2\delta^2 w}\right]\Bigg\} \\
& + \frac{\alpha_s C_F}{4\pi}\left(\frac{4s}{Q^2} - \delta^2\right)\frac{4}{\mu^2}\left[\frac{\mu^2\theta(s)}{s}\right]_+ \left\{\frac{3}{4} + \ln\left(\frac{4s}{Q^2\mu^2}\right)\right\}
\end{aligned}$$

- Anomalous dimension unchanged, algorithm modifies parts not involving large logs. The latter are needed for NNLL resummation.

Plot Renormalized Jet Function





- Compare algorithm contributions
  - $\delta_1 = 0.15$
  - $\delta_2 = 0.20$

- Compare jet functions
  - Algorithm contribution
  - NLO inclusive jet function
  - NLO SW jet function

$$\Delta J_n^{\mathrm{SW}}(s) = \frac{\alpha_s C_F}{4\pi}\theta\left(\delta^2 - \frac{4s}{Q^2}\right)\boxed{\frac{24}{4s + Q^2\delta^2}} + \frac{\alpha_s C_F}{4\pi}\theta\left(\frac{4s}{Q^2} - \delta^2\right)\boxed{\left[\frac{3}{s} + \frac{4}{s}\ln\left(\frac{4s}{Q^2\delta^2}\right)\right]}$$

Compare to Full Cross Section

- Three jets in Mercedes-Benz configuration
  - Measure $\tau = m_J/Q$ for one jet
  - Resummed logs of τ to NLL
  - Three different cone sizes

$$\frac{1}{\sigma^{(0)}}\frac{d\sigma}{d\tau} \sim H \times (J \otimes J \otimes J \otimes S)$$

Ellis, Hornig, Lee, Vermilion, Walsh

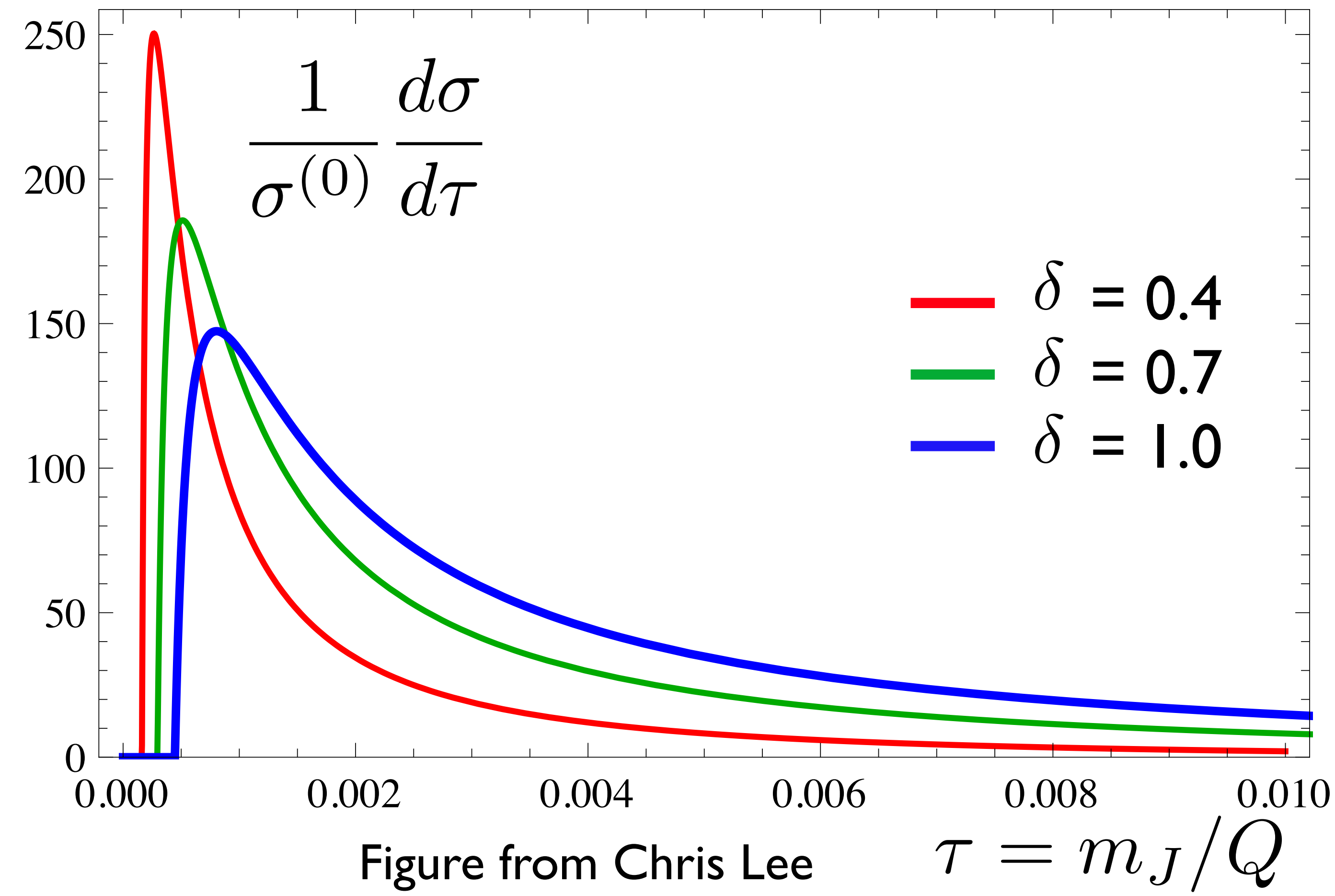


Figure from Chris Lee

- Relative shapes with different cone sizes are similar to jet function
  - For small cone size the cross section has a higher peak for small jet mass, i.e. it favors narrow jets
  - For larger cone size the distribution in jet mass is more even

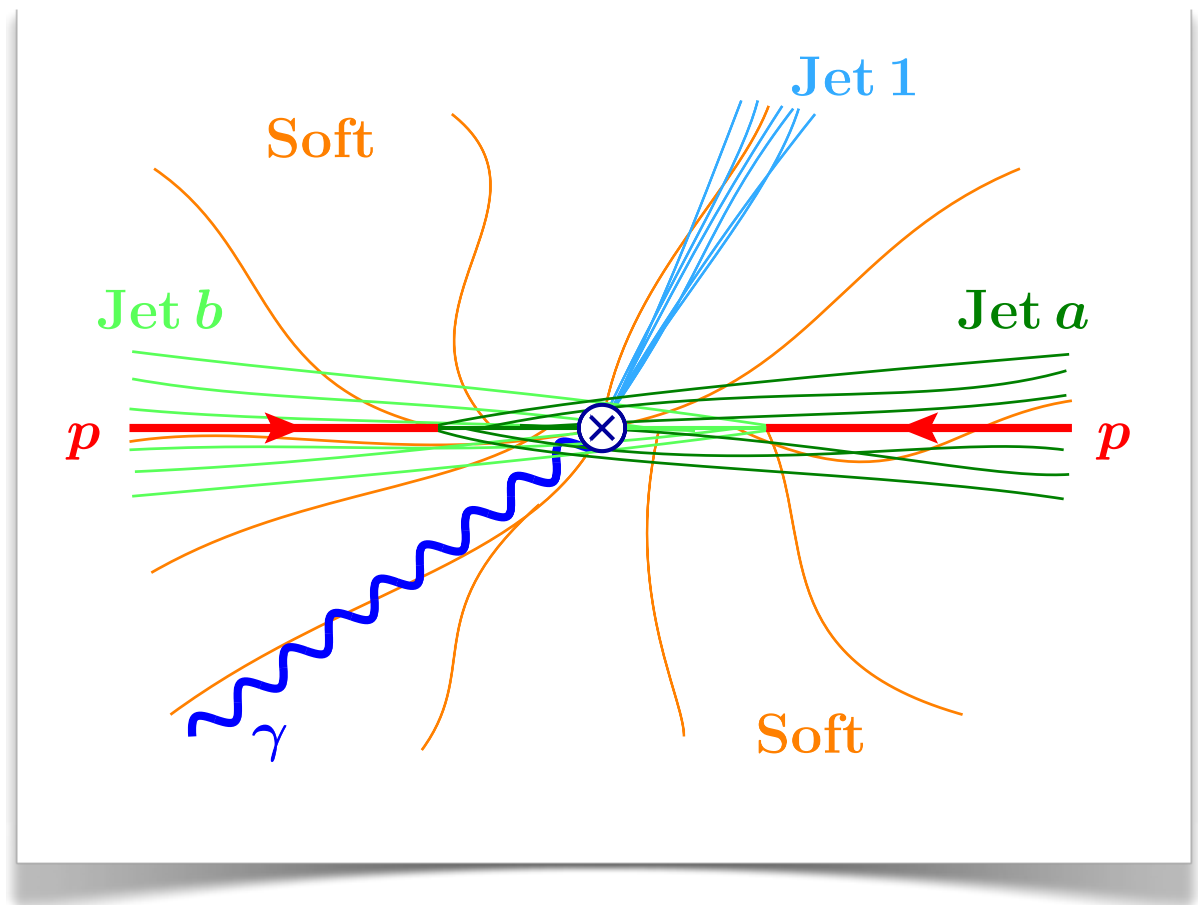
Conclusions

- Have taken steps toward precision calculation of e^+e^- jet observables with jets defined by realistic jet algorithms
  - Same methods applicable to LHC, critical for precise comparisons with experiment
- Developed a general procedure to calculate jet function with a jet algorithm.
  - Explicit results for the Sterman-Weinberg algorithm
  - At NLL, RG evolution same with and without algorithm, parts without large logs are different from the inclusive jet function
- Next steps:
  - Apply jet algorithms for hadron colliders using SCET
  - Resum the remaining logs of δ, β
  - NNLL calculations where jet algorithm dependent corrections become crucial will soon be within reach

Current Work

- N-jettiness - a new observable for pp colliders Stewart, Tackmann, Waalewijn
  - Global event shape for events with N signal jets
  - Can be used as an inclusive jet-veto

- 1-jettiness TJ et al.
  - Study events in a pp collider with one jet recoiling against a photon
  - Beam functions describe the initial state radiation of incoming hadrons
  - Jet function describes the outgoing final state radiation in the jet
  - Currently calculating the soft function which is the remaining ingredient to study the cross section as a function of 1-jettiness



Backup slides

Full Algorithm Contributions

$$\Delta \tilde{J}_n^{\rm SW}(s) = \frac{\alpha_s C_F}{4\pi} A(\epsilon)\theta\left(\delta^2 - \frac{4s}{Q^2}\right)\left[\delta(s)\left\{\frac{2}{\epsilon^2} - \frac{4}{\epsilon}\left(\ln 2\beta + \frac{1}{2}\ln\left(\frac{Q^2\delta^2}{4\mu^2}\right)\right) + 2\ln^2 2\beta - \ln^2\left(\frac{Q^2\delta^2}{4\mu^2}\right)\right\}\right.$$
$$+ \frac{4}{\mu^2}\left[\frac{\mu^2\theta(s)}{s}\right]_+\left\{\ln 2\beta + \ln\left(\frac{Q^2\delta^2}{4\mu^2}\right) + \frac{6s}{4s+Q^2\delta^2}\right\} - \left.\frac{4}{\mu^2}\left[\frac{\mu^2\theta(s)\ln\left(s/\mu^2\right)}{s}\right]_+\right]$$
$$+ \frac{\alpha_s C_F}{4\pi}\theta\left(\frac{4s}{Q^2} - \delta^2\right)\frac{4}{\mu^2}\left[\frac{\mu^2\theta(s)}{s}\right]_+\left(\frac{3}{4} + \ln 2\beta\right).$$

$$\Delta J_{n0}^{\rm SW}(s) = \frac{\alpha_s C_F}{4\pi} A(\epsilon)\theta\left(\delta^2 - \frac{4s}{Q^2}\right)\left[\delta(s)\left\{\frac{2}{\epsilon^2} - \frac{4}{\epsilon}\left(\ln 2\beta + \frac{1}{2}\ln\left(\frac{Q^2\delta^2}{4\mu^2}\right)\right) + 2\ln^2 2\beta - \ln^2\left(\frac{Q^2\delta^2}{4\mu^2}\right)\right\}\right.$$
$$+ \frac{4}{\mu^2}\left[\frac{\mu^2\theta(s)}{s}\right]_+\left\{\ln 2\beta + \ln\left(\frac{Q^2\delta^2}{4\mu^2}\right)\right\} - \left.\frac{4}{\mu^2}\left[\frac{\mu^2\theta(s)\ln\left(s/\mu^2\right)}{s}\right]_+\right]$$
$$+ \frac{\alpha_s C_F}{4\pi}\theta\left(\frac{4s}{Q^2} - \delta^2\right)\left[\frac{4}{\mu^2}\left[\frac{\mu^2\theta(s)}{s}\right]_+\left\{\ln 2\beta + \ln\left(\frac{Q^2\delta^2}{4\mu^2}\right)\right\} - \frac{4}{\mu^2}\left[\frac{\mu^2\theta(s)\ln\left(s/\mu^2\right)}{s}\right]_+\right].$$

$$\Delta J_n^{\rm SW}(s) = \frac{\alpha_s C_F}{4\pi}\theta\left(\delta^2 - \frac{4s}{Q^2}\right)\frac{24}{4s+Q^2\delta^2} + \frac{\alpha_s C_F}{4\pi}\theta\left(\frac{4s}{Q^2} - \delta^2\right)\left[\frac{3}{s} + \frac{4}{s}\ln\left(\frac{4s}{Q^2\delta^2}\right)\right]$$