Hadronic transition processes in a finite volume

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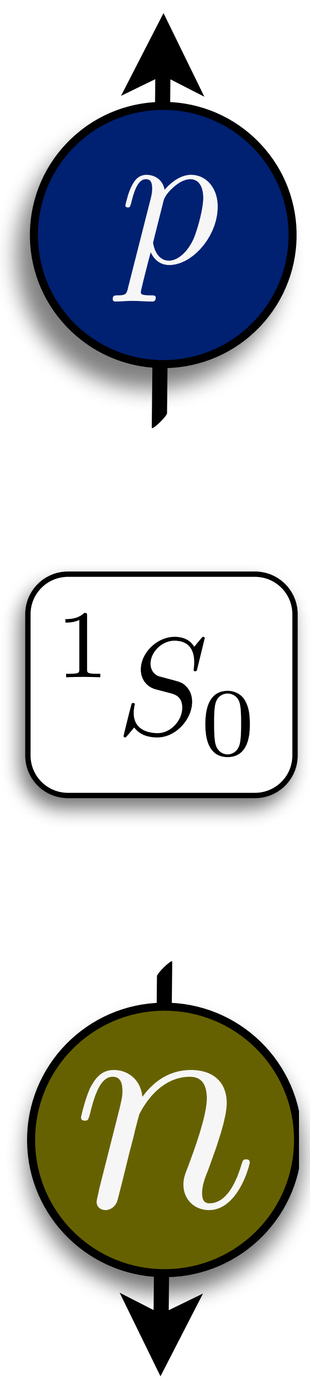
Jefferson Lab

RB, Hansen & Walker-Loud (2014)
RB & Hansen (to appear, 2015)

BNL, Feb. 2015

Transition amplitudes

(e.g., radiative neutron capture [np-to-dγ])



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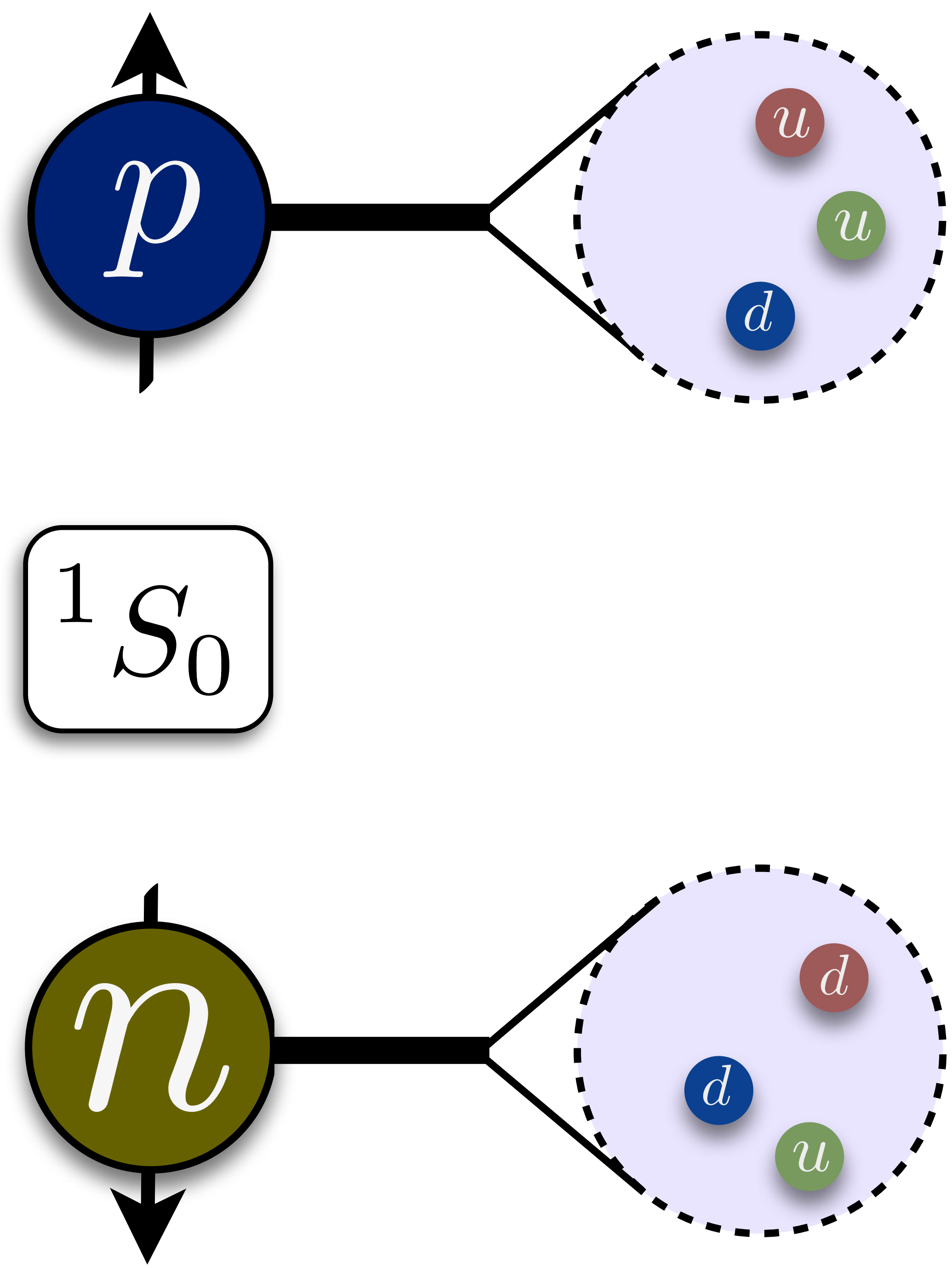

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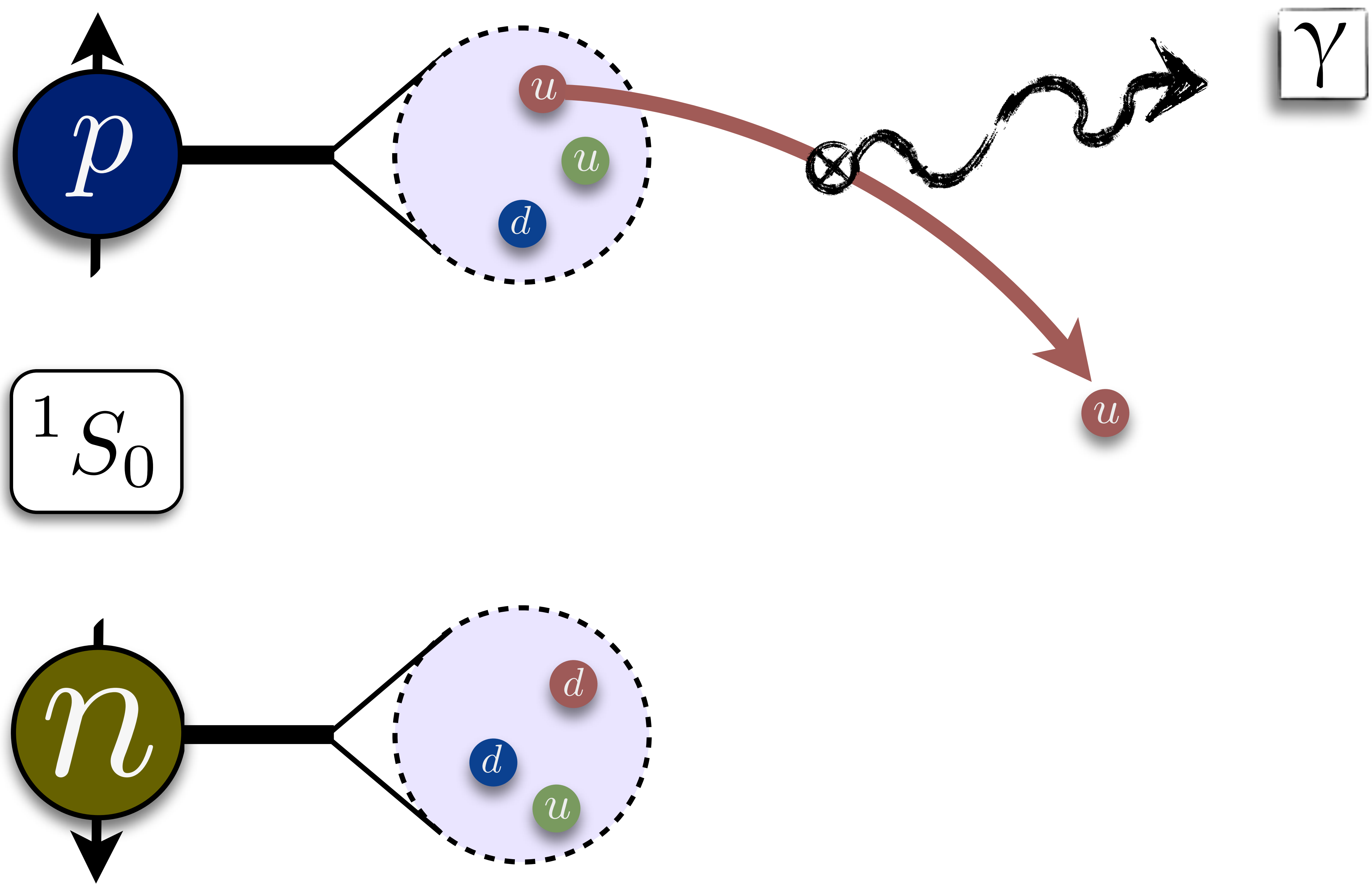

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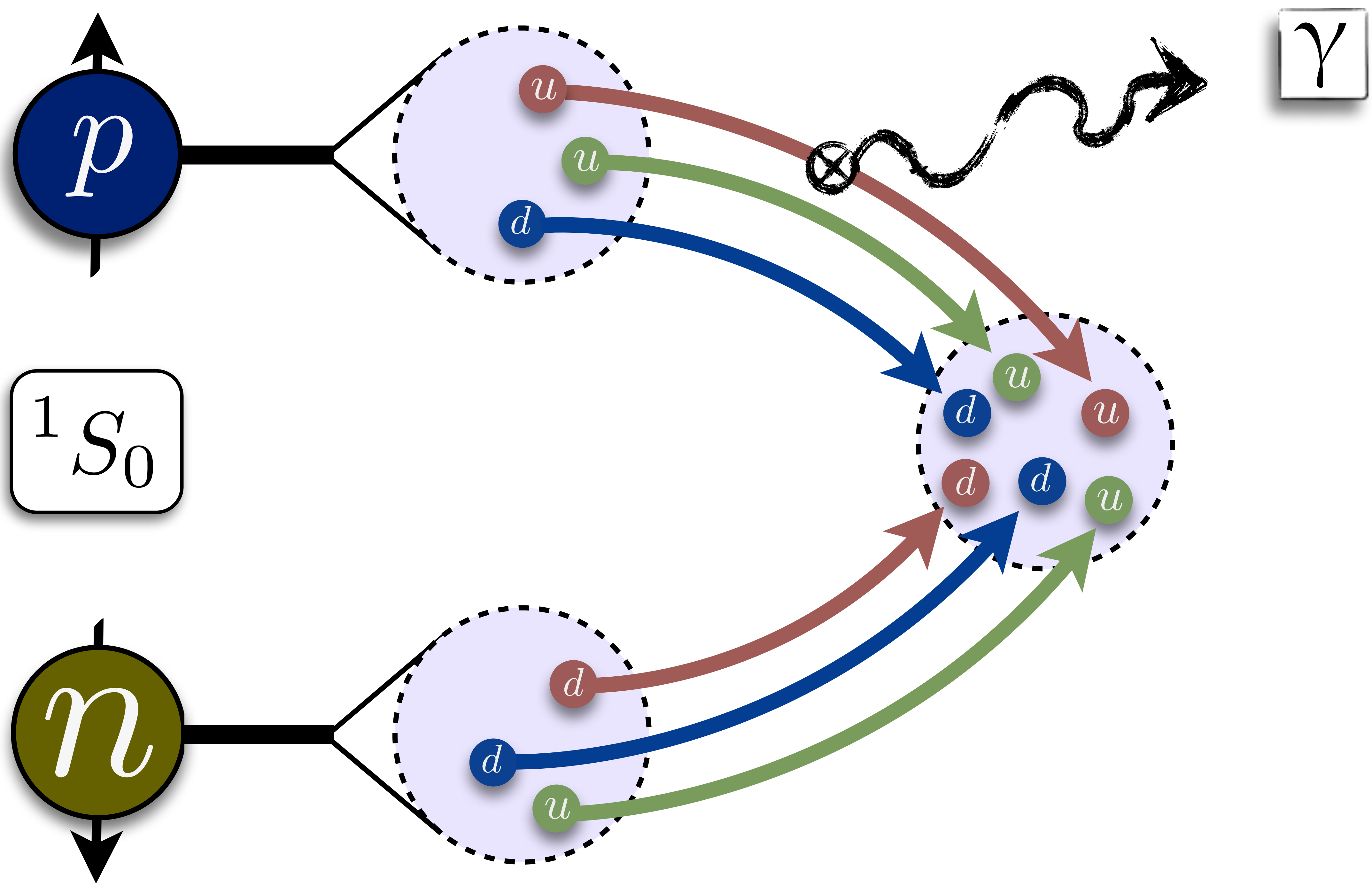

Transition amplitudes

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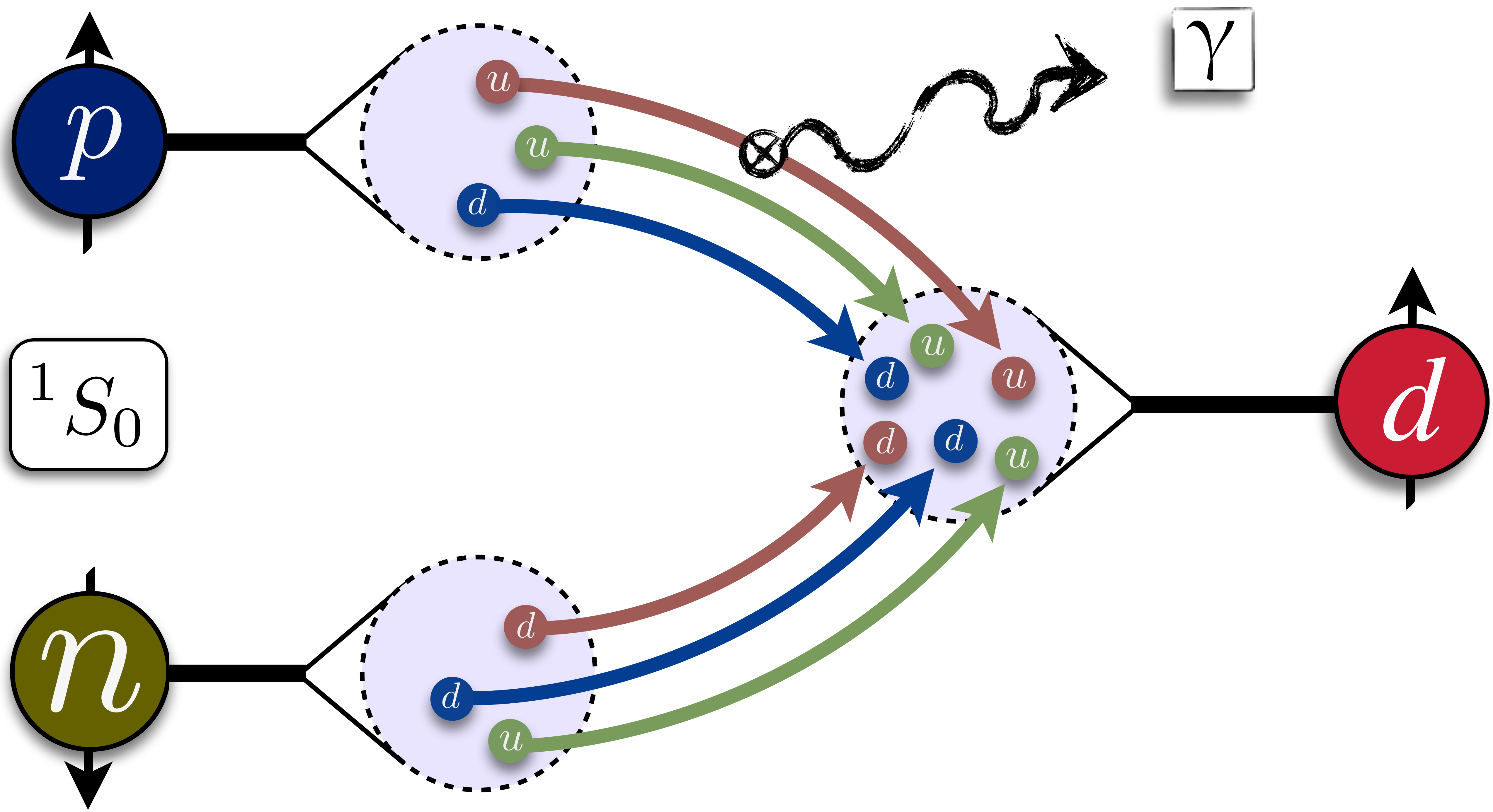
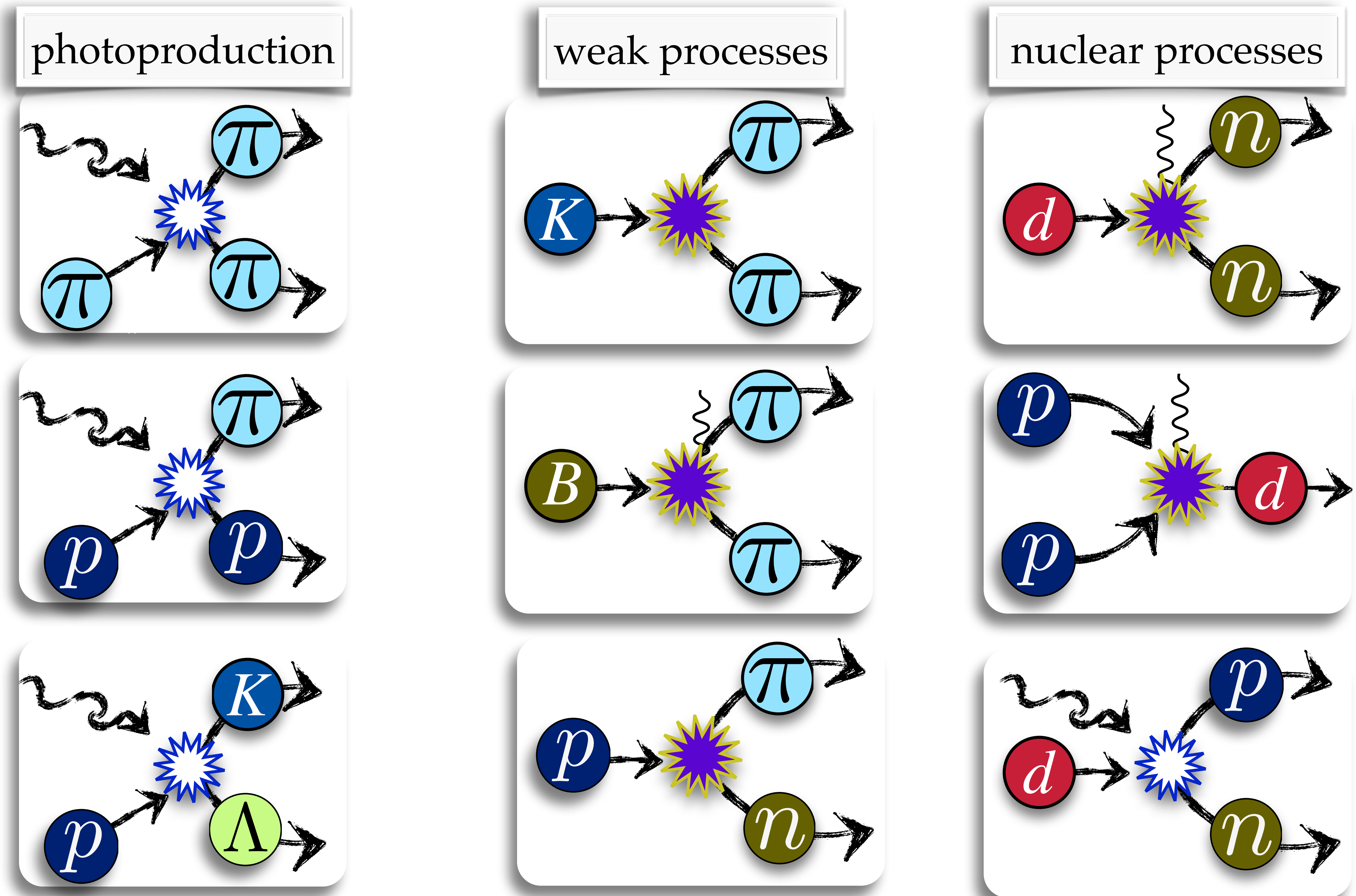


Transition amplitudes

(other applications)

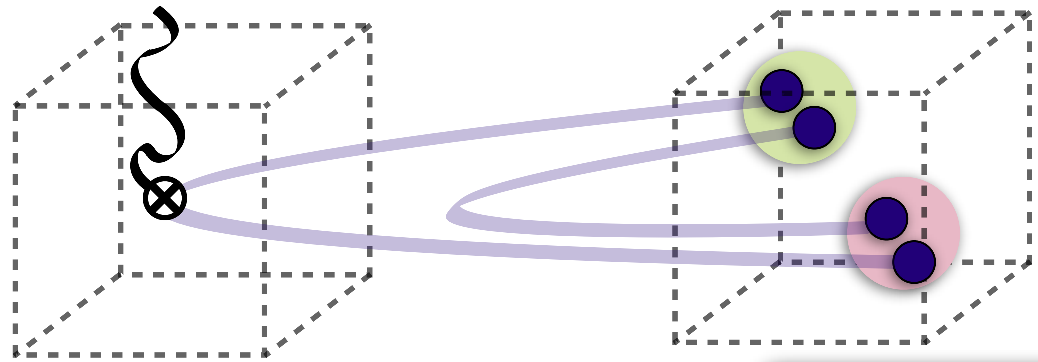


Main results

RB, Hansen & Walker-Loud (2014)
RB & Hansen (2015)

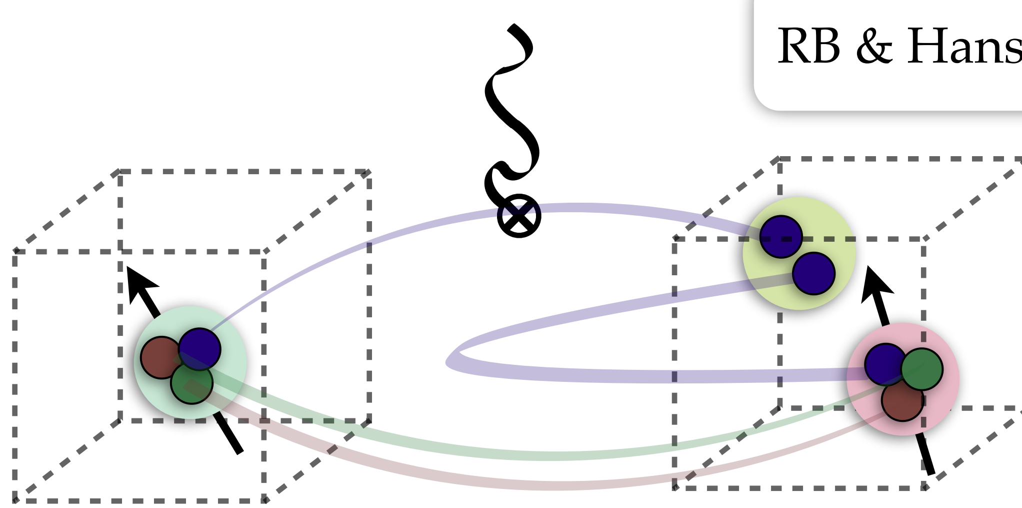
Transition amplitudes

- 0-to-2 with intrinsic spin:



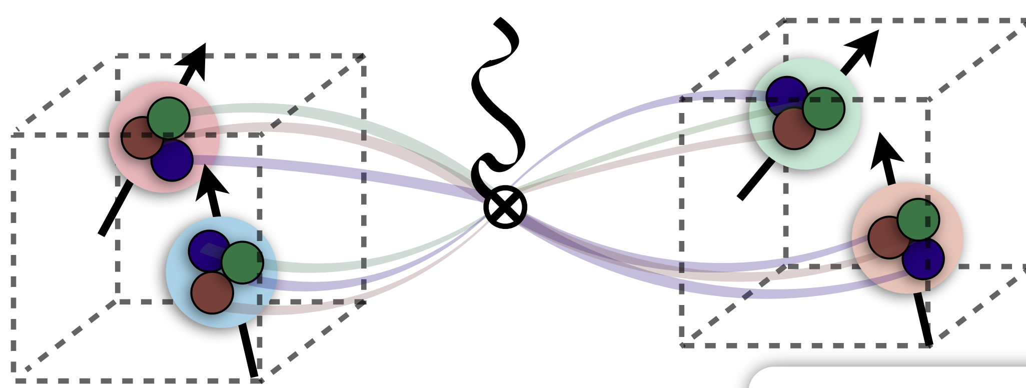
RB & Hansen (2015)

- 1-to-2 with intrinsic spin:



RB, Hansen & Walker-Loud (2014) / RB & Hansen (2015)

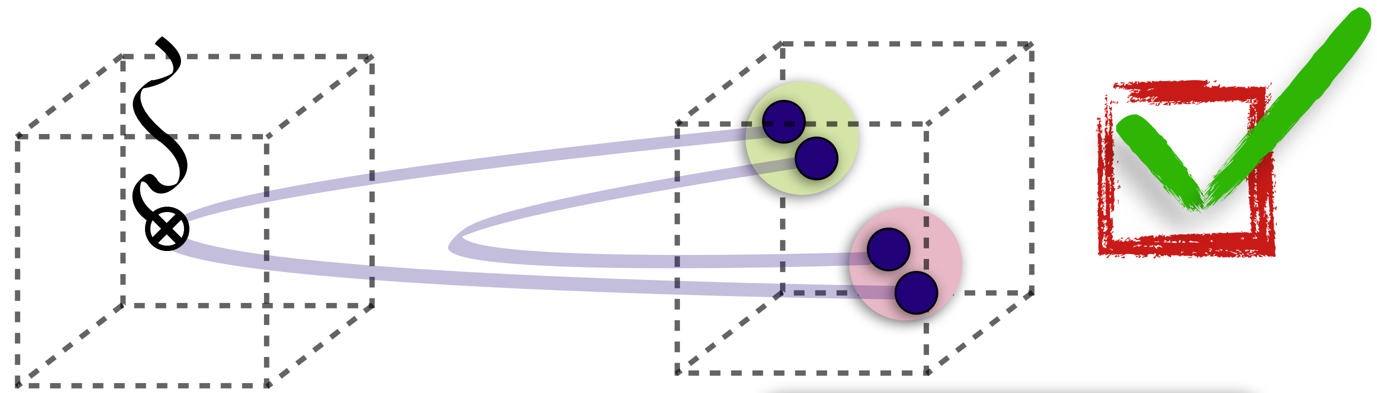
- 2-to-2 with intrinsic spin:



RB & Hansen [in preparation]

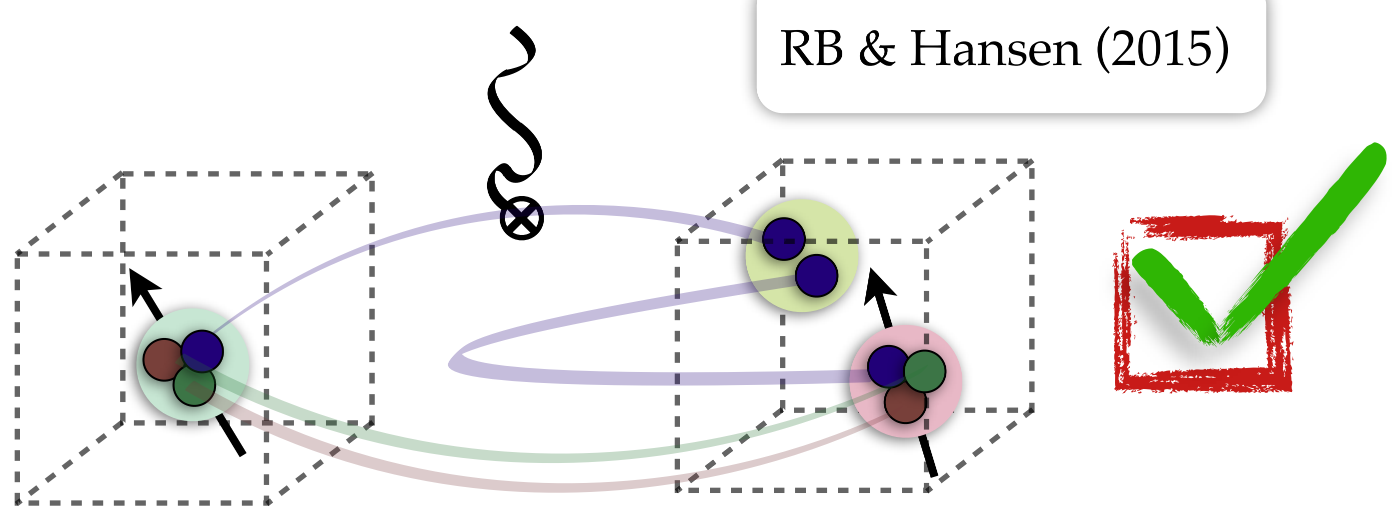
Transition amplitudes

- 0-to-2 with intrinsic spin:



RB & Hansen (2015)

- 1-to-2 with intrinsic spin:



RB, Hansen & Walker-Loud (2014) / RB & Hansen (2015)

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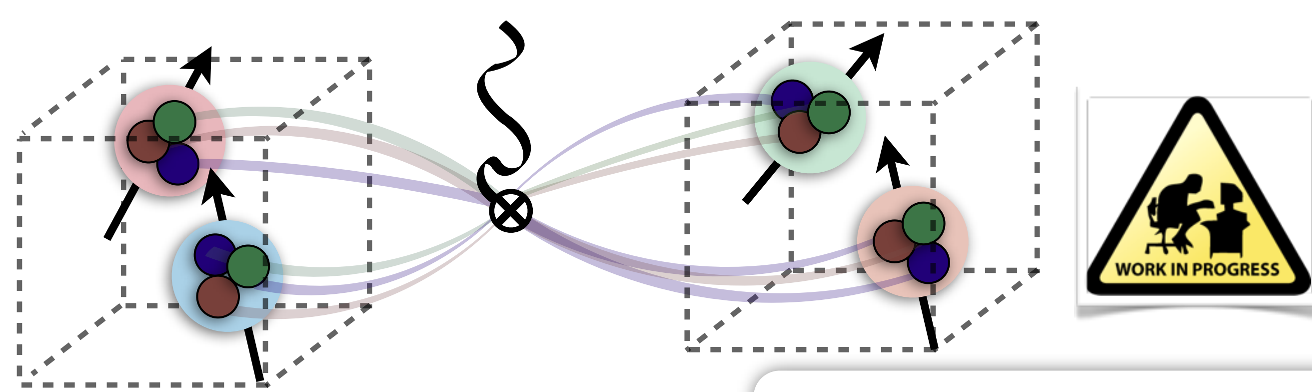


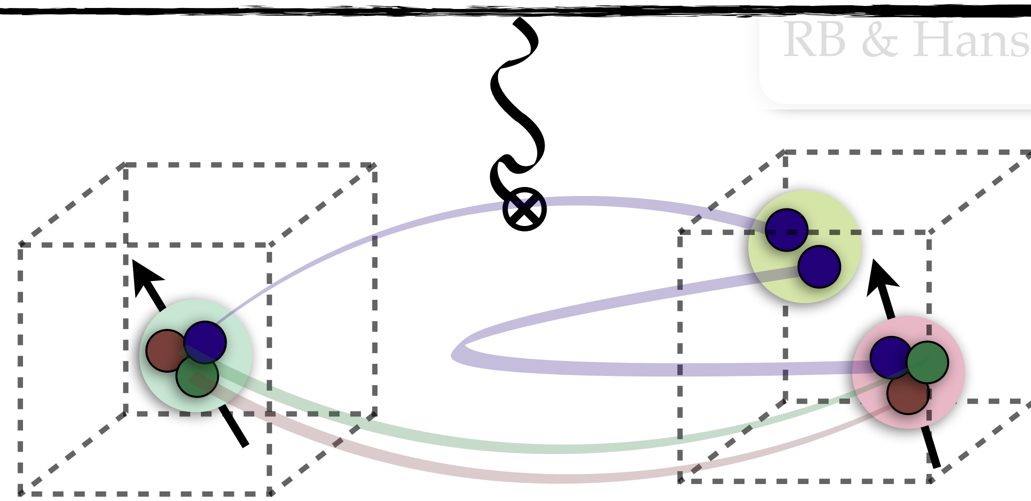
RB & Hansen [in preparation]

Transition amplitudes

0-to-2 with intrinsic spin:

RB & Hansen (2015)

1-to-2 with intrinsic spin:

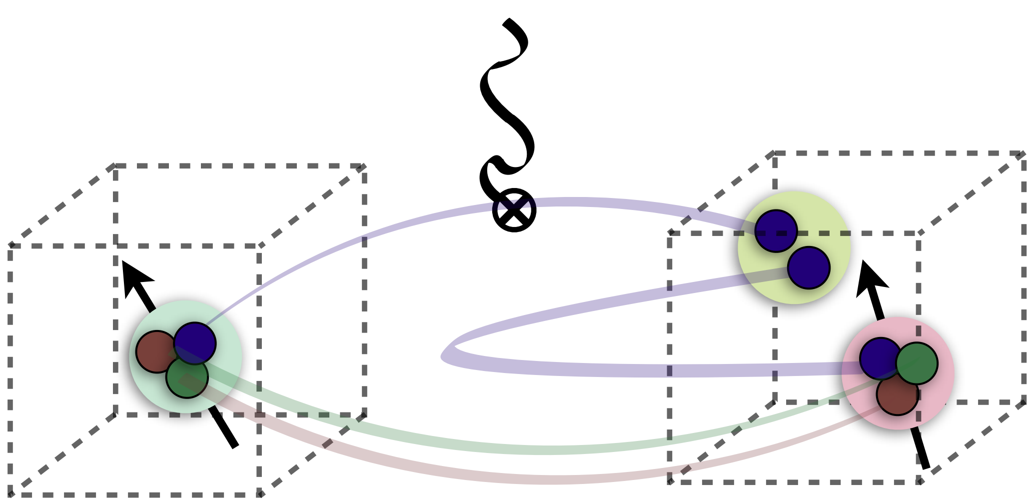


RB, Hansen & Walker-Loud (2014) / RB & Hansen (2015)

2-to-2 with intrinsic spin:

Transition Amplitudes

$$|\langle E_n', \mathbf{P}', L|\widetilde{\mathcal{J}}_A(0, \mathbf{P}-\mathbf{P}')|E_0, \mathbf{P}, L, 1\rangle| = \frac{1}{\sqrt{2E_0}}\sqrt{\mathcal{H}_A^{\text{in}}\ \mathcal{R}(E_n', \mathbf{P}')\ \mathcal{H}_A^{\text{out}}}$$

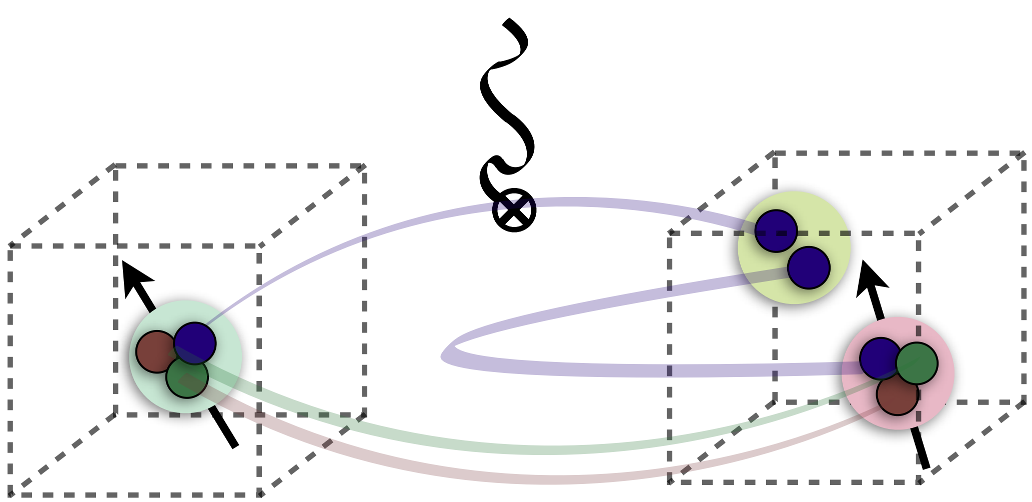


RB, Hansen & Walker-Loud (2014)

RB & Hansen (2015)

Transition Amplitudes

$$|\langle \underbrace{E_n', \mathbf{P}', L}_{\text{final state}}| \underbrace{\widetilde{\mathcal{J}}_A(0, \mathbf{P}-\mathbf{P}')}_{\text{external current}} | \underbrace{E_0, \mathbf{P}, L, 1}_{\text{initial state}} \rangle| = \frac{1}{\sqrt{2E_0}} \sqrt{\mathcal{H}_A^{\text{in}}\ \mathcal{R}(E_n', \mathbf{P}')\ \mathcal{H}_A^{\text{out}}}$$



RB, Hansen & Walker-Loud (2014)

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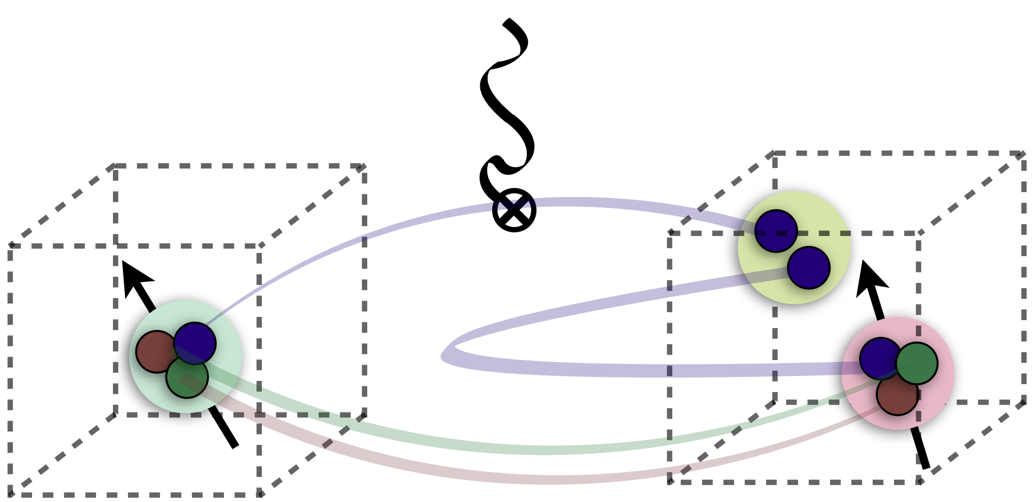
Transition Amplitudes

$$|\langle E_n', \mathbf{P}', L|\widetilde{\mathcal{J}}_A(0, \mathbf{P}-\mathbf{P}')|E_0, \mathbf{P}, L, 1\rangle| = \frac{1}{\sqrt{2E_0}}\sqrt{\mathcal{H}_A^{\text{in}}\ \mathcal{R}(E_n', \mathbf{P}')\ \mathcal{H}_A^{\text{out}}}$$

energy of initial particle



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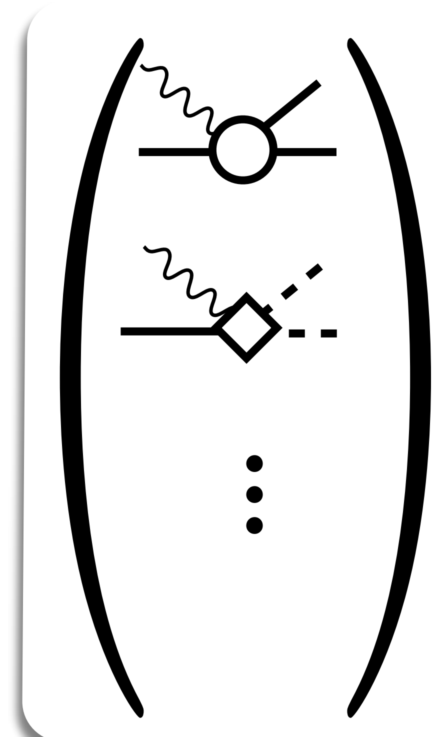
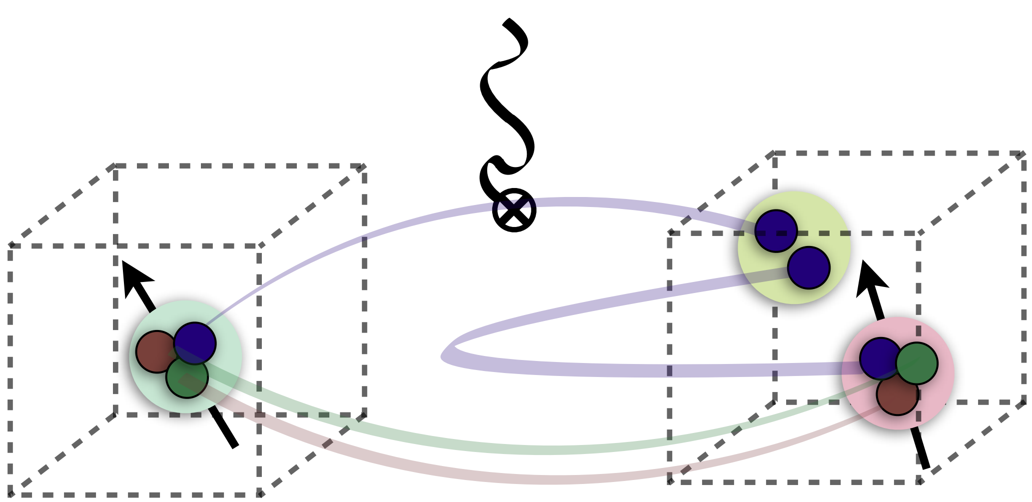
fully dressed, on-shell infinite volume transition amplitude!



a vector in the space of open channels



RB, Hansen & Walker-Loud (2014)

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Transition Amplitudes

$$\left|\langle E'_n, \mathbf{P}', L | \widetilde{\mathcal{J}}_A(0,\mathbf{P}-\mathbf{P}')|E_0,\mathbf{P},L,1\rangle\right| = \frac{1}{\sqrt{2E_0}}\sqrt{\underline{\mathcal{H}^{\rm in}_A}\;\mathcal{R}(E'_n,\mathbf{P}')\;\underline{\mathcal{H}^{\rm out}_A}}$$

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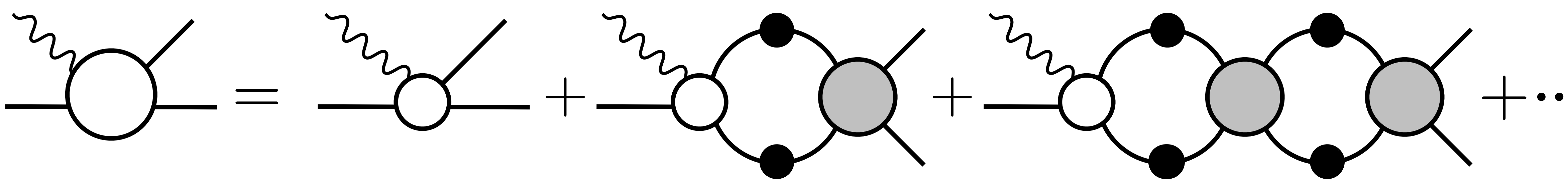

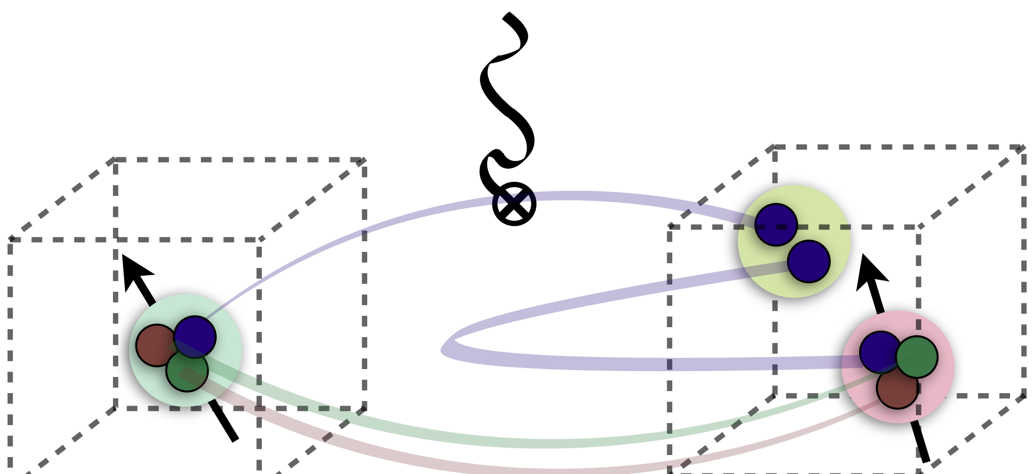

RB, Hansen & Walker-Loud (2014)

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Transition Amplitudes

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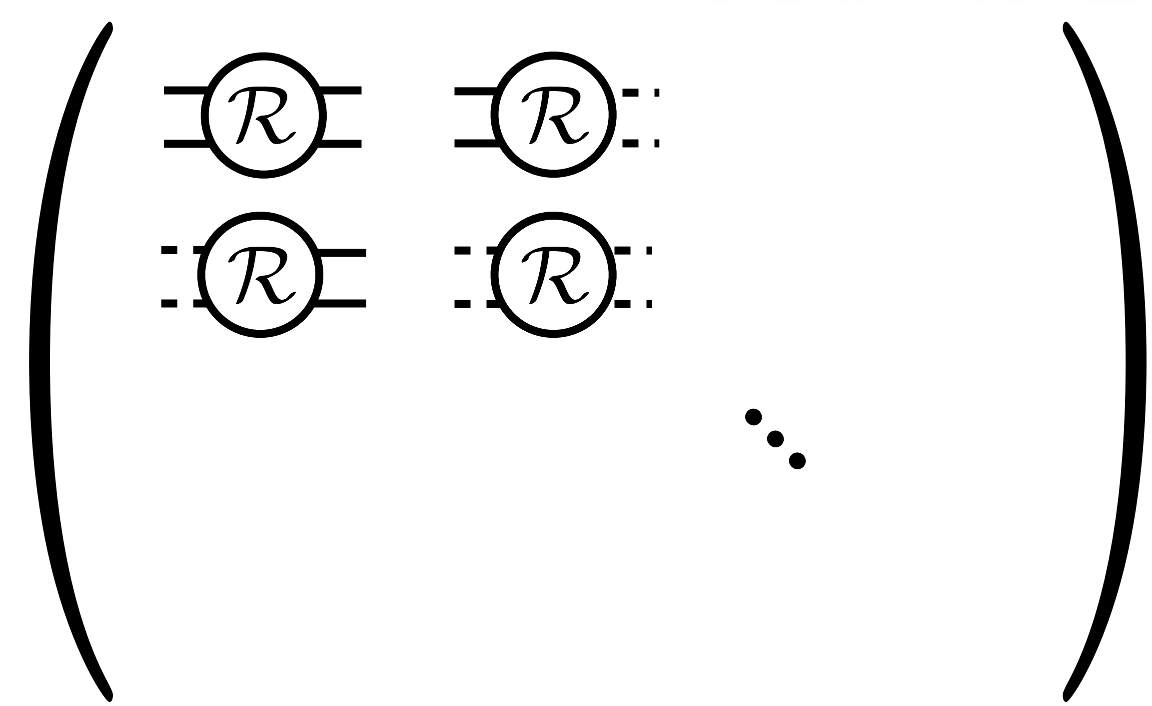
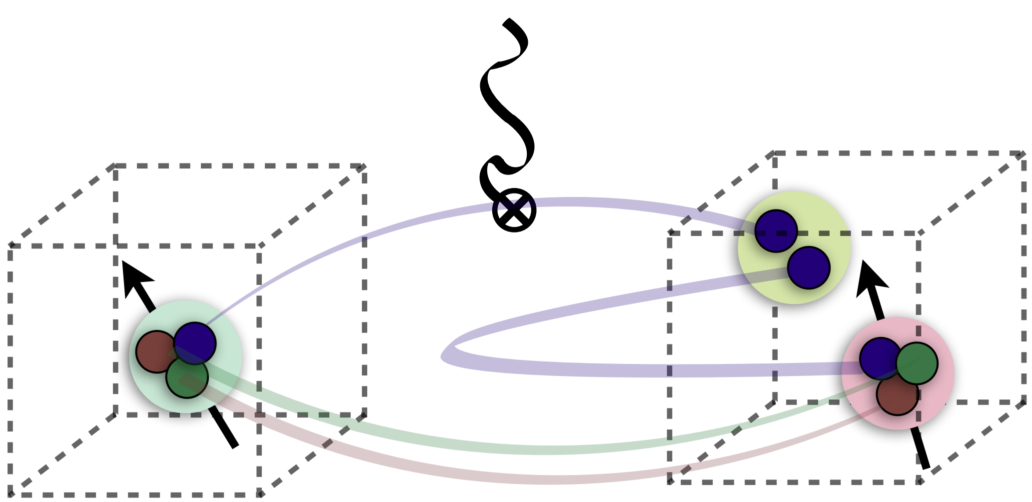
|\langle E'_n, \mathbf{P}', L|\widetilde{\mathcal{J}}_A(0, \mathbf{P}-\mathbf{P}')|E_0, \mathbf{P}, L, 1\rangle| = \frac{1}{\sqrt{2E_0}}\sqrt{\mathcal{H}_A^{\text{in}}\ \underline{\mathcal{R}(E'_n, \mathbf{P}')}\ \mathcal{H}_A^{\text{out}}}
$$

two-particle propagator residue, depends on energy, phase shifts and derivative of phase shifts



a matrix in the space of open channels



RB, Hansen & Walker-Loud (2014)

RB & Hansen (2015)

Transition Amplitudes

$$|\langle E_n', \mathbf{P}', L|\widetilde{\mathcal{J}}_A(0, \mathbf{P}-\mathbf{P}')|E_0, \mathbf{P}, L, 1\rangle| = \frac{1}{\sqrt{2E_0}}\sqrt{\mathcal{H}_A^{\text{in}}\ \mathcal{R}(E_n', \mathbf{P}')\ \mathcal{H}_A^{\text{out}}}$$

- Model independent & non-perturbative
- Universal: lattice QCD, lattice EFT, cold atoms, etc.
- Arbitrary quantum numbers for two particles
- General volumes and boundary conditions: periodic, anti-periodic , or any linear combination on any rectangular prism
- "Just a mapping": between finite/infinite volume physics. Not a one-to-one, but rather a one-to-many.

RB, Hansen & Walker-Loud (2014)

RB & Hansen (2015)

Lüscher formalism

- Lüscher (1986), (1991)
- Rummukainen and Gottlieb (1995)
- Bedaque (2004)
- Li and Liu (2004)
- Feng, Li, and Liu (2004)
- Christ, Kim, and Yamazaki (2005)
- Kim, Sachrajda, and Sharpe (2005)
- Bernard, Lage, Meissner, and Rusetsky (2008)
- Ishizuka (2009)
- Bour, Koenig, Lee, Hammer, and Meissner (2011)
- Gockeler, Horsley, Lage, Meissner, Rakow, Rusetsky, Schierholz, Zanotti (2012)
- Hansen and Sharpe (2012)
- **RB** and Davoudi (2012)
- Li and Liu (2013)
- **RB**, Davoudi, and Luu (2013)
- **RB**, Davoudi, Luu and Savage (2013)
- **RB** (2014)
- ...

Lellouch-Lüscher formalism

- Lellouch & Lüscher (2000)
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Transition Amplitudes

$$|\langle E_n', \mathbf{P}', L|\widetilde{\mathcal{J}}_A(0,\mathbf{P}-\mathbf{P}')|E_0,\mathbf{P},L,1\rangle| = \frac{1}{\sqrt{2E_0}}\sqrt{\mathcal{H}_A^{\text{in}}\ \mathcal{R}(E_n',\mathbf{P}')\ \mathcal{H}_A^{\text{out}}}$$

summarizes everything previously done and more!

Lellouch-Lüscher formalism

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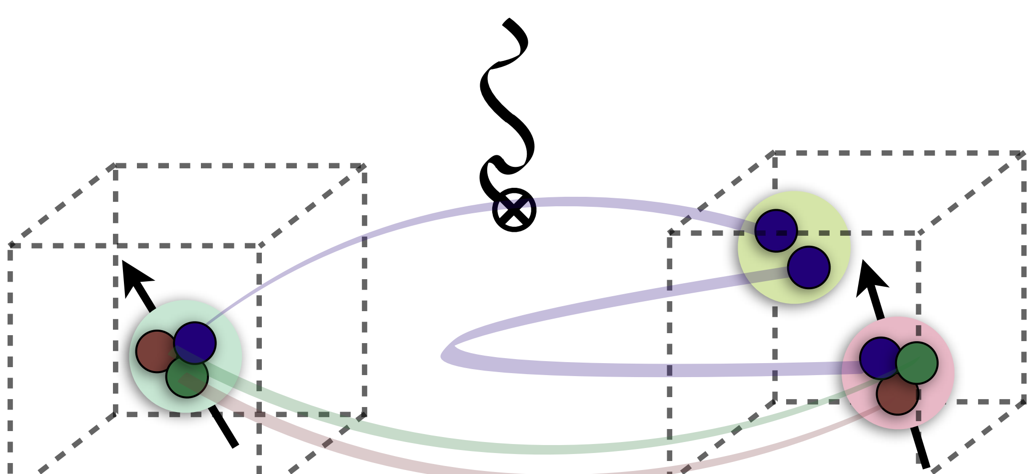


RB, Hansen & Walker-Loud (2014)

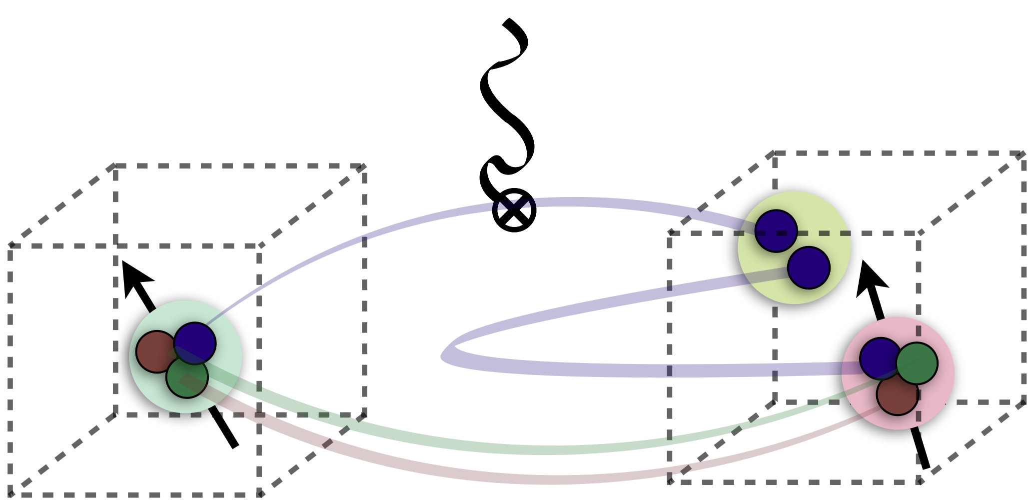
RB & Hansen (2015)

Transition Amplitudes

$$\frac{\langle E_n', \mathbf{P}', L | \widetilde{\mathcal{J}}_{A_1}(0, \mathbf{P} - \mathbf{P}') | E_0, \mathbf{P}, L, 1\rangle}{\langle E_n', \mathbf{P}', L | \widetilde{\mathcal{J}}_{A_2}(0, \mathbf{P} - \mathbf{P}') | E_0, \mathbf{P}, L, 1\rangle} = \frac{\mathcal{X}^\dagger \; \mathcal{R}(E_n', \mathbf{P}') \; \mathcal{H}^{\text{out}}_{A_1}}{\underline{\mathcal{X}^\dagger} \; \mathcal{R}(E_n', \mathbf{P}') \; \mathcal{H}^{\text{out}}_{A_2}}$$

a generic vector in the space of open channels and angular momentum

Absolute sign of matrix elements is unphysical, but relative sign is determinable



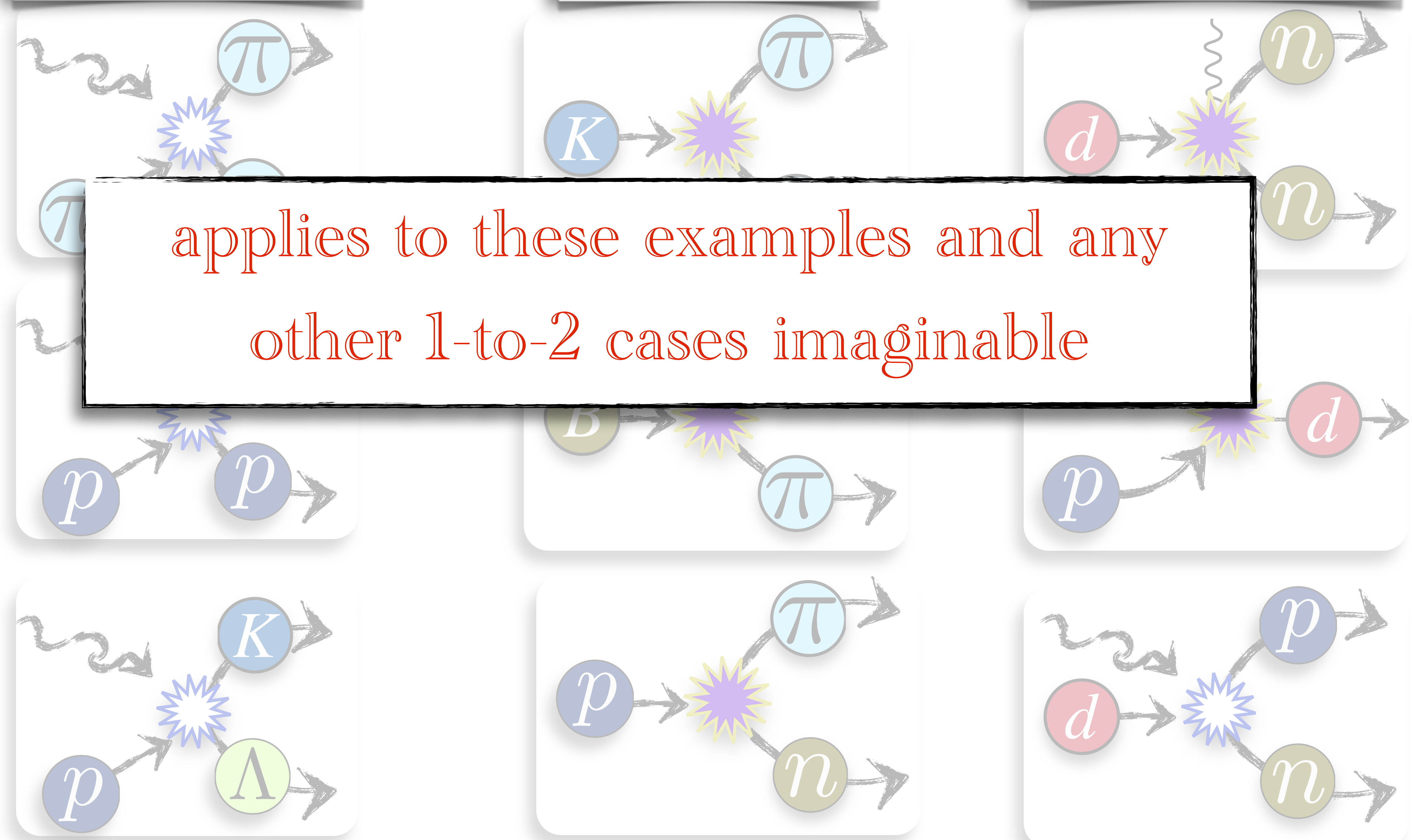
RB, Hansen & Walker-Loud (2014)

RB & Hansen (2015)

Transition amplitudes

(other applications)

photoproduction | weak processes | nuclear processes



applies to these examples and any other 1-to-2 cases imaginable

a sketch of the derivation for 0-to-2 and 1-to-2 processes

à la mode de Kim, Sachrajda, and Sharpe (2005)

Two-point function

$$C_L(x_4 - y_4, \mathbf{P}) \equiv \int_L d\mathbf{x} \int_L d\mathbf{y}\ e^{-i\mathbf{P}\cdot(\mathbf{x}-\mathbf{y})} \left[\langle 0 | T \mathcal{A}(x) \mathcal{B}^\dagger(y) | 0 \rangle\right]_L$$

1. Evaluate two-point correlation function using:

- Complete set of states
- Feynman diagrams

2. Match:

- Spectrum
- Overlap matrix elements

Two-point function

$$C_L(x_4 - y_4, \mathbf{P}) \equiv \int_L d\mathbf{x} \int_L d\mathbf{y}\ e^{-i\mathbf{P}\cdot(\mathbf{x}-\mathbf{y})} \Big[\langle 0|T\mathcal{A}(x)\mathcal{B}^\dagger(y)|0\rangle\Big]_L$$

Using complete set of states:

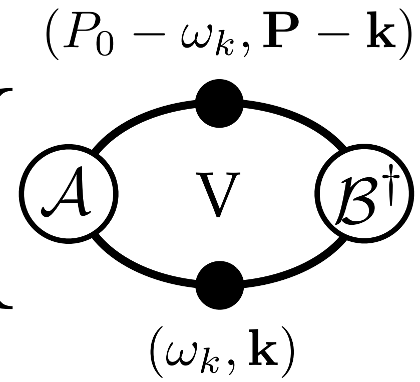
$$\begin{aligned}
&C_L(x_4 - y_4, \mathbf{P}) \\
&\quad = \int_L d\mathbf{x} \int_L d\mathbf{y}\ e^{-i\mathbf{P}\cdot(\mathbf{x}-\mathbf{y})} \sum_n \Big[\langle 0|\mathcal{A}(x_4, \mathbf{x})|E_n, \mathbf{P}, L\rangle\Big]_L \Big[\langle E_n, \mathbf{P}, L|\mathcal{B}^\dagger(y_4, \mathbf{y})|0\rangle\Big]_L \\
&\quad = L^6 \sum_n e^{-E_n(x_4 - y_4)} \Big[\langle 0|\mathcal{A}(0)|E_n, \mathbf{P}, L\rangle\Big]_L \Big[\langle E_n, \mathbf{P}, L|\mathcal{B}^\dagger(0)|0\rangle\Big]_L .
\end{aligned}$$

assuming finite volume states
are normalized to 1

Two-point function

Using Feynman diagrams:

$$\int \frac{dP_0}{2\pi} e^{iP_0(x_0-y_0)} \Bigg\{$$

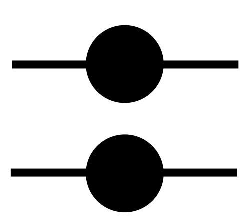


$$+ \cdots \Bigg\}$$

- λ = helicity
- This holds for any particle with spin
- **The particle must be stable**
- Consequence of:
  - relativity
  - helicity conservation

Single body propagator with arbitrary spin:

smooth and finite

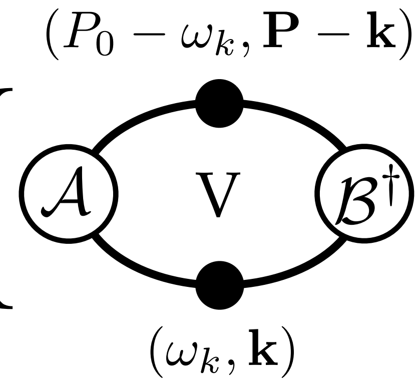


$$[\ \ ]_{\lambda\alpha,\lambda'\alpha'} = \left[\frac{\delta_{\lambda,\lambda'}\delta_{\alpha,\alpha'}}{(4\omega_k\omega_{Pk})(E - \omega_k - \omega_{Pk} + i\epsilon)} + \cdots \right]$$

Two-point function

Using Feynman diagrams:

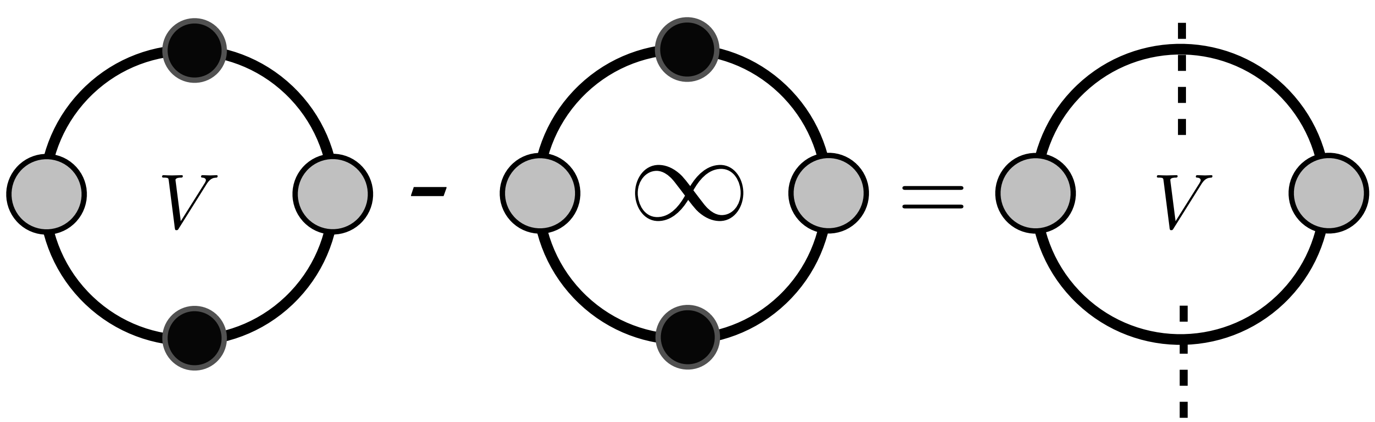
$$\int \frac{dP_0}{2\pi} e^{iP_0(x_0-y_0)} \Big\{$$



$$+ \cdots \Big\}$$

On-shell states can sample boundaries of your volume and lead to power law volume dependence

Matrices in *angular momentum* / *open channel* space

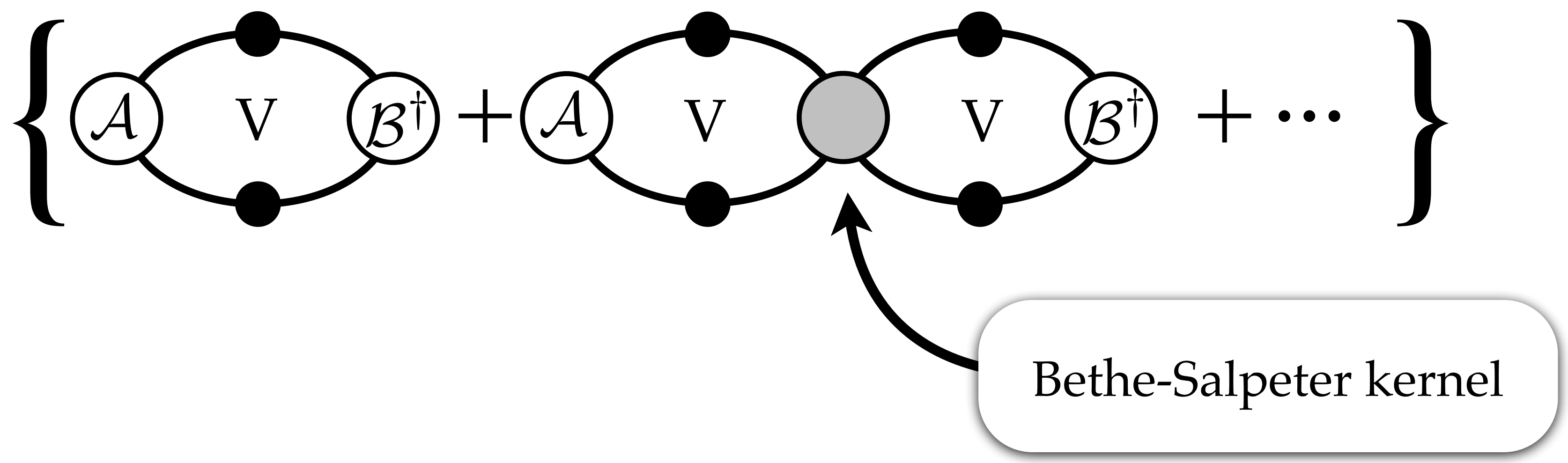


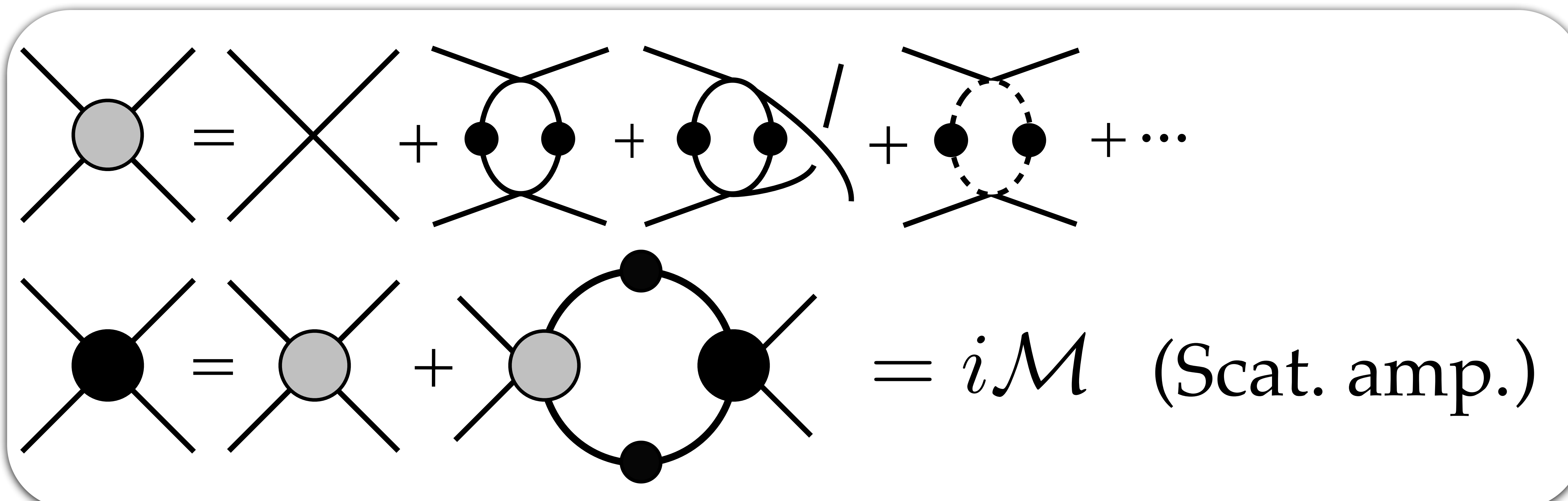
$$= -L\ F(P, L)\ R$$

Two-point function

Using Feynman diagrams:

$$\int \frac{dP_0}{2\pi} e^{iP_0(x_0-y_0)}$$

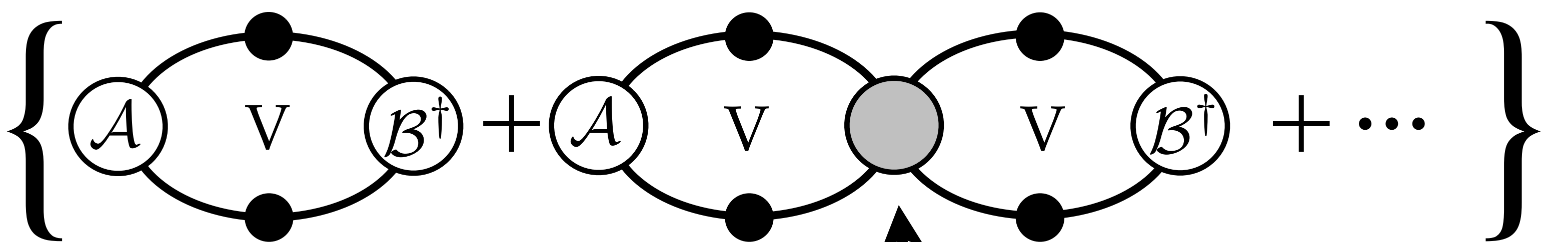




Two-point function

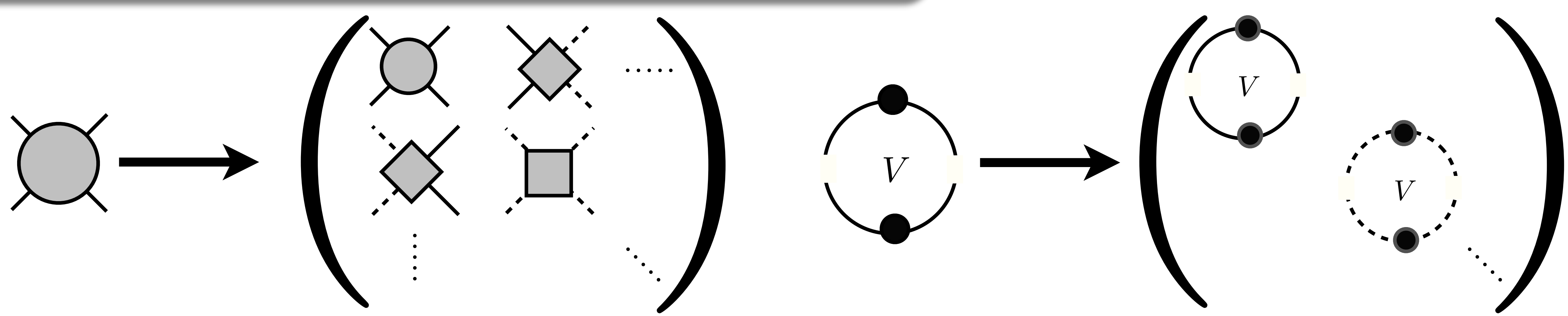
Using Feynman diagrams:

$$\int \frac{dP_0}{2\pi} e^{iP_0(x_0-y_0)}$$



Bethe-Salpeter kernel

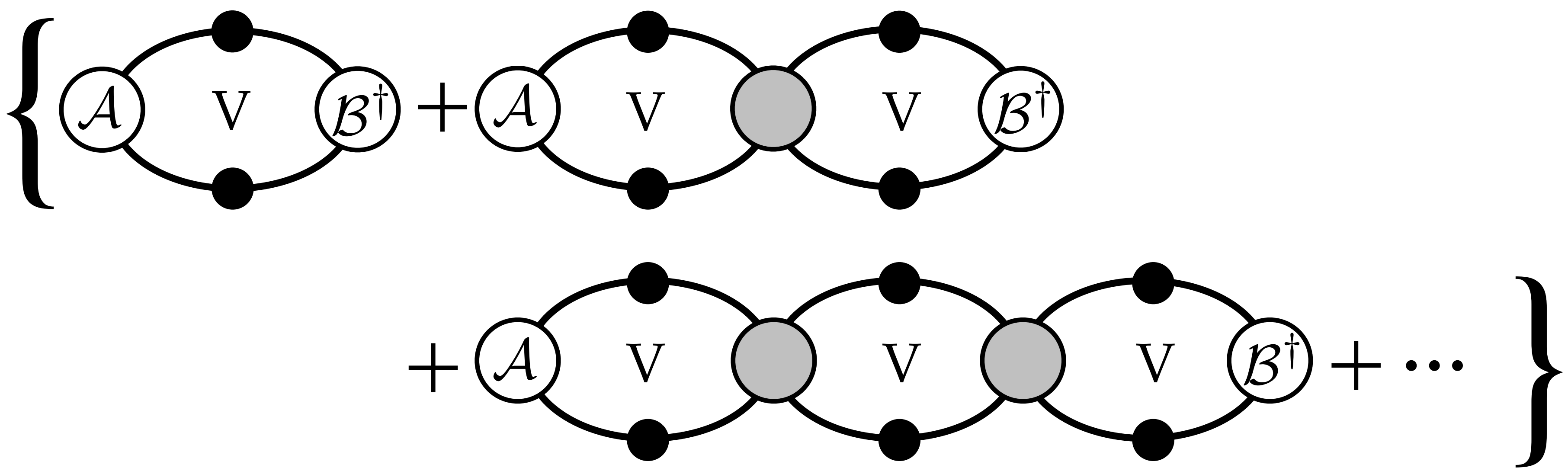
Generalization to two-particle multichannels:



Two-point function

Using Feynman diagrams:

$$\int \frac{dP_0}{2\pi} e^{iP_0(x_0-y_0)}$$



$$= L^3 \int \frac{dP_0}{2\pi} e^{iP_0(x_0-y_0)} \left\{ C_\infty(P) - A(P) \frac{1}{F^{-1}(P,L) + \mathcal{M}(P)} B^\dagger(P) \right\}$$

$\langle 0|\mathcal{A}(0)|\underline{E, \mathbf{P}, \{J\}, \mathrm{in}}\rangle$

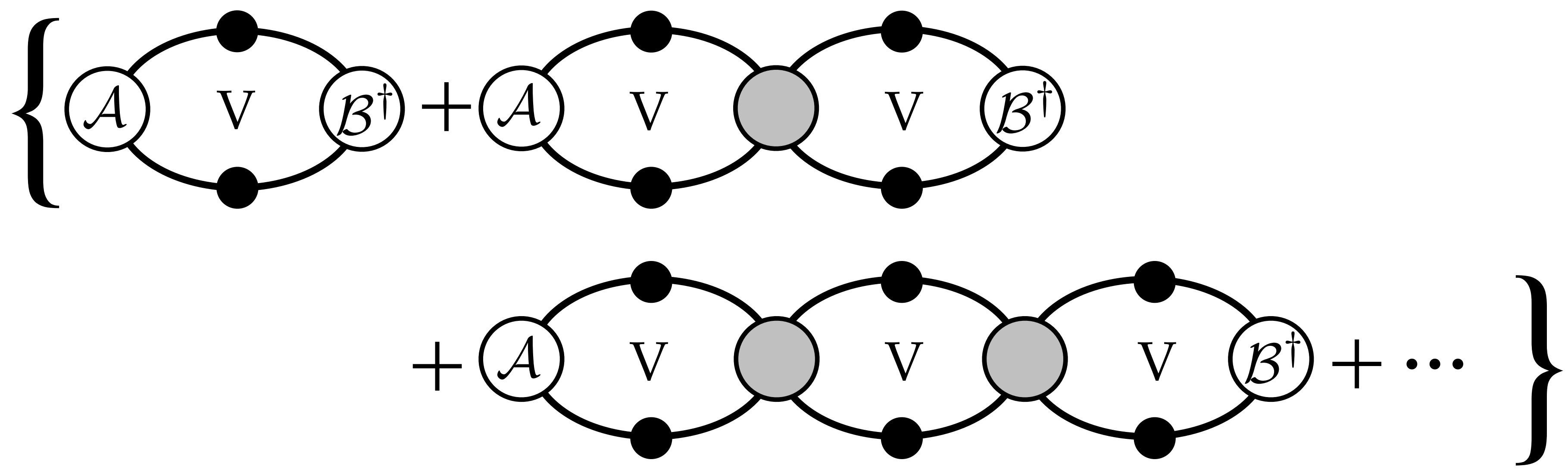
$\langle \underline{E, \mathbf{P}, \{J'\}, \mathrm{out}}|\mathcal{B}^\dagger(0)|0\rangle$

asymptotic infinite volume, two-particle states

Two-point function

Using Feynman diagrams:

$$\int \frac{dP_0}{2\pi} e^{iP_0(x_0-y_0)}$$



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poles satisfy: $\det[F^{-1}(P,L) + \mathcal{M}(P)] = 0$

generalization of Lüscher formalism for arbitrary spin, multichannel, two-particle systems **[RB (2014)]**

Lüscher formalism

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summarizes everything previously done and more!

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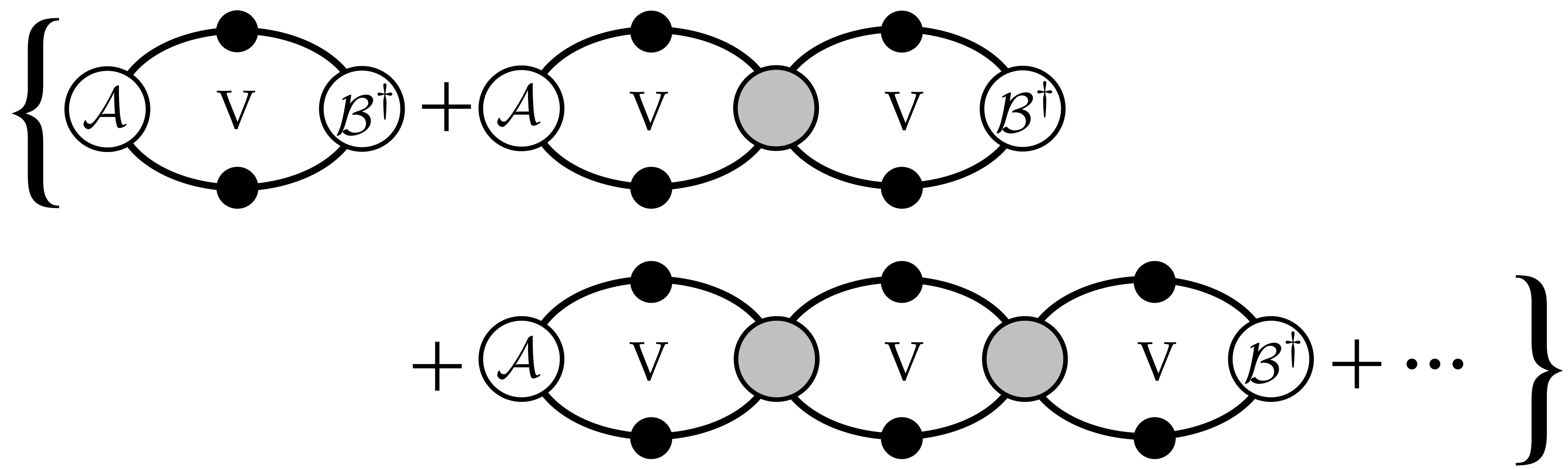
Using Feynman diagrams:

$$\int \frac{dP_0}{2\pi} e^{iP_0(x_0-y_0)}$$



$$= L^3 \int \frac{dP_0}{2\pi} e^{iP_0(x_0-y_0)} \left\{ C_\infty(P) - A(P) \frac{1}{F^{-1}(P,L) + \mathcal{M}(P)} B^\dagger(P) \right\}$$

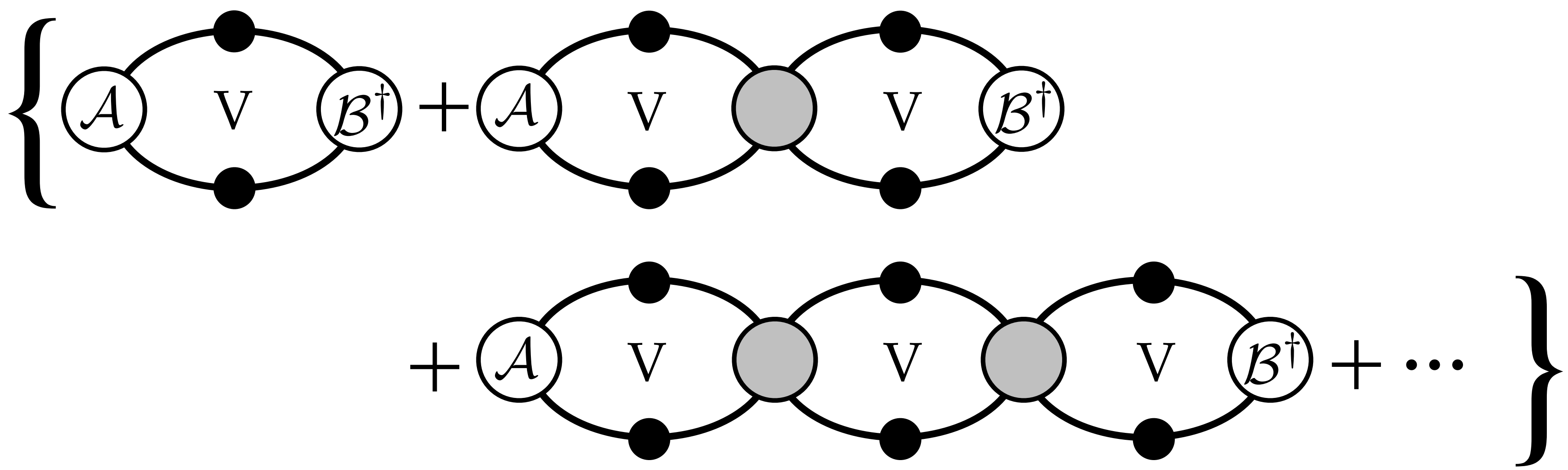
Residue matrix: $\mathcal{R}_{\{J\},\{J'\}}(E_n, \mathbf{P}) \equiv \lim_{P_4 \to iE_n} \left[-(iP_4 + E_n) \frac{1}{F^{-1}(P,L) + \mathcal{M}(P)} \right]_{\{J\},\{J'\}}$

$$|E, \mathbf{P}, \{J\}, \text{in}\rangle \equiv |E, \mathbf{P}, a, J, M, l, S, s_1^a, s_2^a, \text{in}\rangle$$

Two-point function

Using Feynman diagrams:

$$\int \frac{dP_0}{2\pi} e^{iP_0(x_0-y_0)}$$



$$= L^3 \int \frac{dP_0}{2\pi} e^{iP_0(x_0-y_0)} \left\{ C_\infty(P) - A(P) \frac{1}{F^{-1}(P,L) + \mathcal{M}(P)} B^\dagger(P) \right\}$$

$$= \sum_n e^{-E_n(x_4-y_4)} L^3 \langle 0|\mathcal{A}(0)|E_n, \mathbf{P}, \{J\}, \mathrm{in}\rangle \Big[\mathcal{R}_{\{J\},\{J'\}}(E_n, \mathbf{P}) \Big] \langle E_n, \mathbf{P}, \{J'\}, \mathrm{out}|\mathcal{B}^\dagger(0)|0\rangle$$

mimics the outer product of finite volume states

Master equation

Equating both representation of the correlation functions:

$$\Big[\langle 0|\mathcal{A}(0)|E_n,\mathbf{P},L\rangle\Big]_L\Big[\langle E_n,\mathbf{P},L|\mathcal{B}^\dagger(0)|0\rangle\Big]_L =$$
$$\frac{1}{L^3}\langle 0|\mathcal{A}(0)|E_n,\mathbf{P},\{J\},\mathrm{in}\rangle\Big[\mathcal{R}_{\{J\},\{J'\}}(E_n,\mathbf{P})\Big]\langle E_n,\mathbf{P},\{J'\},\mathrm{out}|\mathcal{B}^\dagger(0)|0\rangle$$

"Relating finite volume and infinite volume states"

Master equation

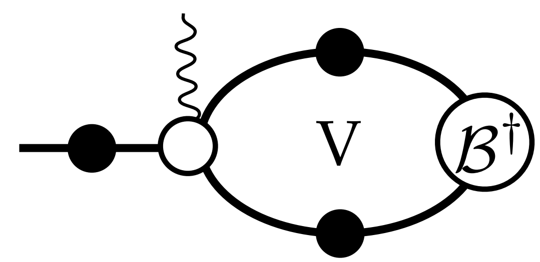
Equating both representation of the correlation functions:

$$\Big[\langle 0|\mathcal{A}(0)|E_n,\mathbf{P},L\rangle\Big]_L\Big[\langle E_n,\mathbf{P},L|\mathcal{B}^\dagger(0)|0\rangle\Big]_L = \frac{1}{L^3}\langle 0|\mathcal{A}(0)|E_n,\mathbf{P},\{J\},\mathrm{in}\rangle\Big[\mathcal{R}_{\{J\},\{J'\}}(E_n,\mathbf{P})\Big]\langle E_n,\mathbf{P},\{J'\},\mathrm{out}|\mathcal{B}^\dagger(0)|0\rangle$$

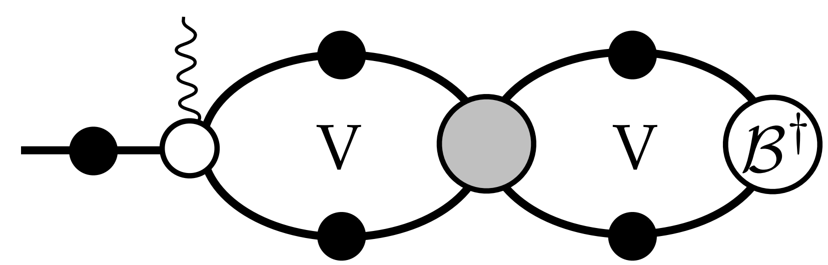
"Relating finite volume and infinite volume states"

Similarly, can study three-point function:

$$\int \frac{dP_{i,0}}{2\pi}\frac{dP_{f,0}}{2\pi}e^{iP_{i,0}(x_{f,0}-y_0)}e^{iP_{f,0}(y_0-x_{i,0})}\Bigg\{$$



$+$



$+\cdots\Bigg\}$

Alternatively, one can use clever choices for $\mathcal{A}(0),\ \mathcal{B}^\dagger(0)$

1-to-2 transitions

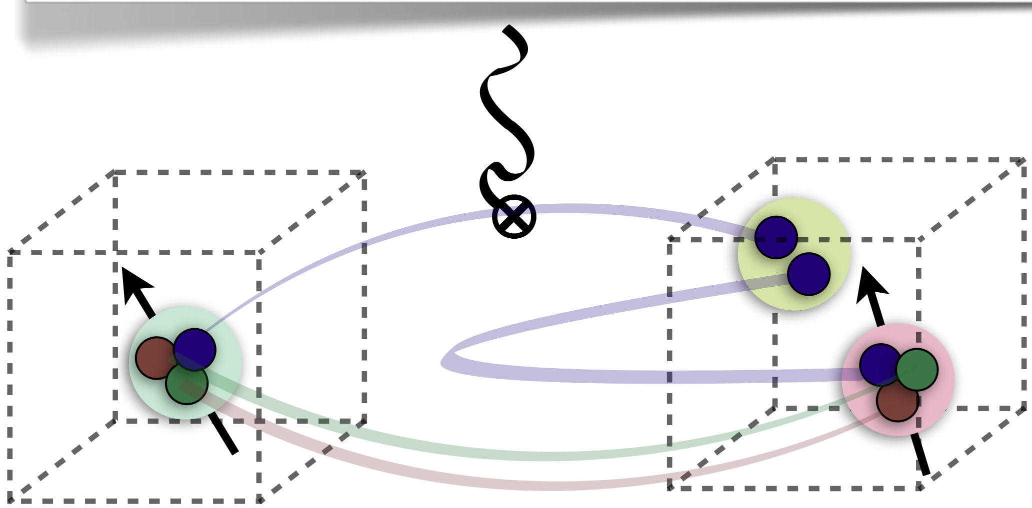
Using clever choices for $\mathcal{A}(0),\ \mathcal{B}^\dagger(0)$

$$\Big[\langle 0|\mathcal{A}(0)|E_n,\mathbf{P},L\rangle\Big]_L\Big[\langle E_n,\mathbf{P},L|\mathcal{B}^\dagger(0)|0\rangle\Big]_L = \frac{1}{L^3}\langle 0|\mathcal{A}(0)|E_n,\mathbf{P},\{J\},\mathrm{in}\rangle\Big[\mathcal{R}_{\{J\},\{J'\}}(E_n,\mathbf{P})\Big]\langle E_n,\mathbf{P},\{J'\},\mathrm{out}|\mathcal{B}^\dagger(0)|0\rangle$$

$$\mathcal{B}^\dagger(0) = \frac{1}{\sqrt{2E_0'L^3}}\mathcal{J}_A(0)a^\dagger_{E_0',\mathbf{P}'}\,,\quad \mathcal{A}(0) = \frac{1}{\sqrt{2E_0'L^3}}a_{E_0',\mathbf{P}'}\mathcal{J}_A^\dagger(0)$$

and some massaging...

$$|\langle E_n,\mathbf{P},L|\widetilde{\mathcal{J}}_A(0,\mathbf{P}'-\mathbf{P})|E_0',\mathbf{P}',L,1\rangle| = \frac{1}{\sqrt{2E_0'}}\sqrt{\mathcal{H}_A^{\mathrm{in}}\ \mathcal{R}(E_n,\mathbf{P})\ \mathcal{H}_A^{\mathrm{out}}}$$



Subducing this equation onto cubic irreps is straightforward, once one understands how to subduce the two-body quantization condition

0-to-2 transitions

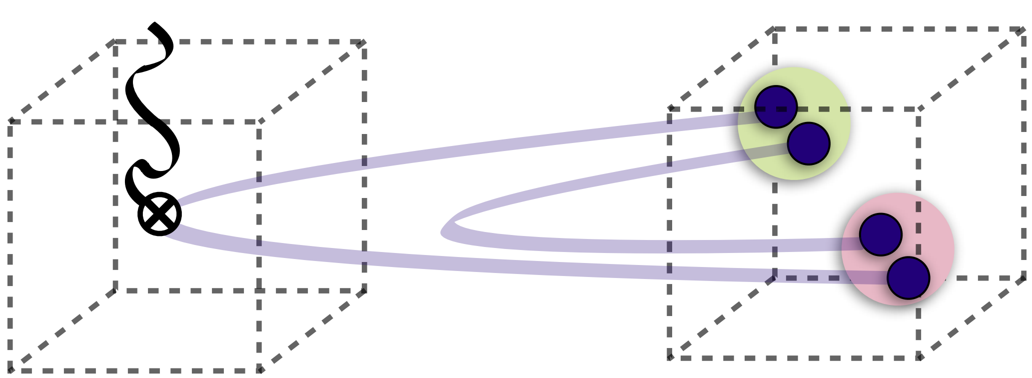
Using clever choices for $\mathcal{A}(0),\ \mathcal{B}^\dagger(0)$

$$\Big[\langle 0|\mathcal{A}(0)|E_n,\mathbf{P},L\rangle\Big]_L \Big[\langle E_n,\mathbf{P},L|\mathcal{B}^\dagger(0)|0\rangle\Big]_L = \frac{1}{L^3}\langle 0|\mathcal{A}(0)|E_n,\mathbf{P},\{J\},\mathrm{in}\rangle\Big[\mathcal{R}_{\{J\},\{J'\}}(E_n,\mathbf{P})\Big]\langle E_n,\mathbf{P},\{J'\},\mathrm{out}|\mathcal{B}^\dagger(0)|0\rangle$$

$$\mathcal{B}^\dagger(0) = \mathcal{J}_A(0)\,, \quad \mathcal{A}(0) = \mathcal{J}_A^\dagger(0)$$

and some massaging...

$$|\langle E_n,\mathbf{P},L|\widetilde{\mathcal{J}}_A(0,-\mathbf{P})|0\rangle| = \sqrt{\mathcal{V}^{\mathrm{in}}_{A,\{J\}}\Big[L^3\mathcal{R}_{\{J\},\{J'\}}(E_n,\mathbf{P})\Big]\mathcal{V}^{\mathrm{out}}_{A,\{J'\}}}$$



Subducing this equation onto cubic irreps is straightforward, once one understands how to subduce the two-body quantization condition

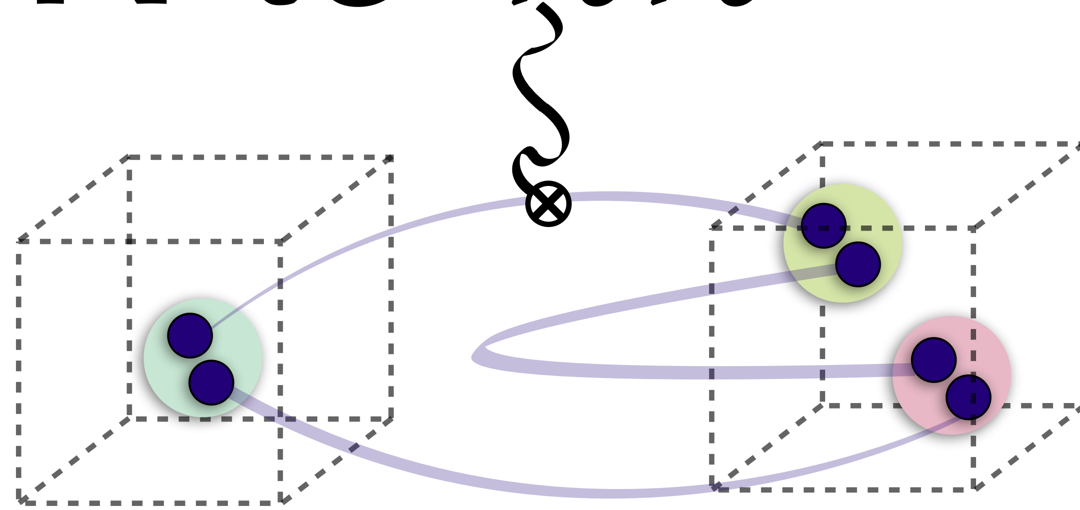
Examples: X-to-$\pi\pi$

X -to-$\pi\pi$ could stand for:

- $\pi\gamma^*$-to-$\pi\pi$, B-to-$\pi\pi$ll, D-to-$\pi\pi$ll, ...

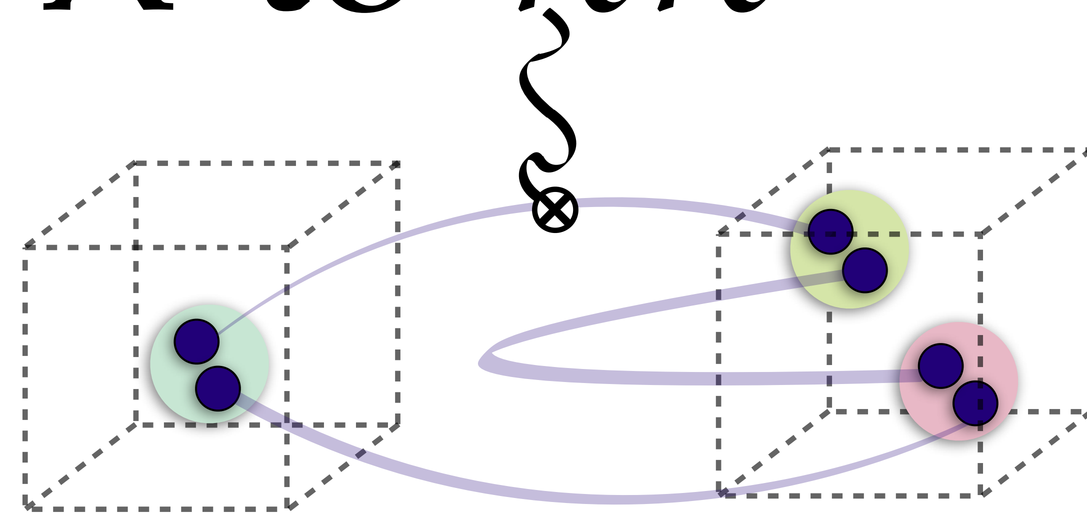
in the ϱ-channel

let's ignore partial wave mixing for now...



$$\frac{|\mathcal{H}_{X\to\pi\pi}|^2}{|\langle\pi\pi,n|\mathcal{J}|X\rangle_L|^2} = \frac{32\pi E_X\ E^*_{\pi\pi,n}}{q^*_{\pi\pi,n}}\ \left.\frac{\partial(\delta_P+\phi^{\mathbf{d}})}{\partial P_0}\right|_{P_0=E_{\pi\pi,n}}$$

Examples: X-to-$\pi\pi$



X -to-ππ could stand for:

- πγ*-to-ππ, B-to-ππll, D-to-ππll, ...

in the ϱ-channel

let's ignore partial wave mixing for now...

$$\frac{|\mathcal{H}_{X\to\pi\pi}|^2}{|\langle\pi\pi,n|\mathcal{J}|X\rangle_L|^2} = \frac{32\pi E_X\ E^*_{\pi\pi,n}}{q^*_{\pi\pi,n}}\ \frac{\partial(\delta_P+\phi^{\mathbf{d}})}{\partial P_0}\bigg|_{P_0=E_{\pi\pi,n}}$$

$$\frac{|\mathcal{H}_{X\to\pi\pi}|^2}{|\langle\pi\pi,n|\mathcal{J}|X\rangle_L|^2} = 4E_X\frac{E^2_{\pi\pi,n}}{\nu_n}L^3$$

free limit

Examples: X-to-$\pi\pi$

X -to-$\pi\pi$ could stand for:

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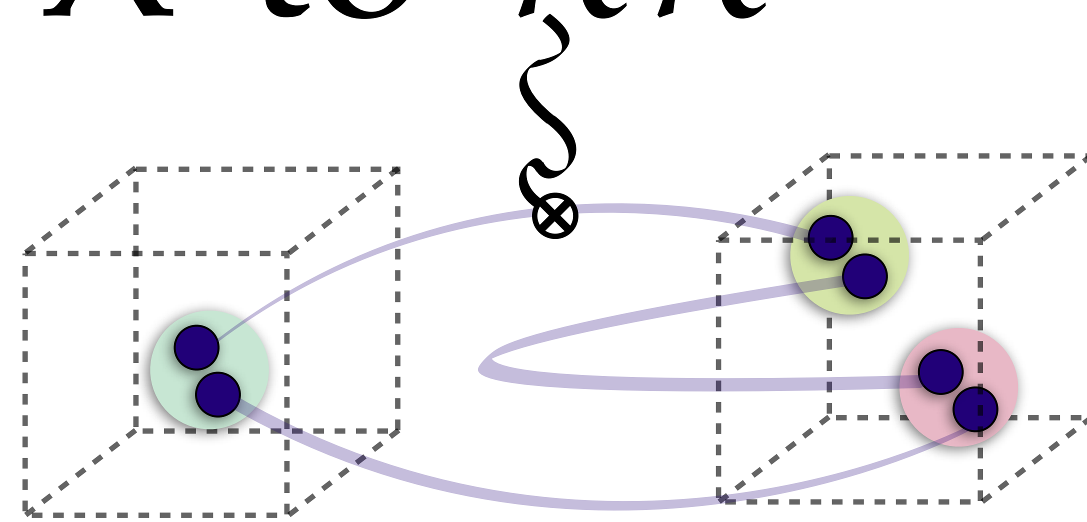
in the ϱ-channel

let's ignore partial wave mixing for now...



$$\frac{|\mathcal{H}_{X\to\pi\pi}|^2}{|\langle\pi\pi,n|\mathcal{J}|X\rangle_L|^2} = \frac{32\pi E_X\ E^*_{\pi\pi,n}}{q^*_{\pi\pi,n}}\ \frac{\partial(\delta_P+\phi^{\mathbf{d}})}{\partial P_0}\bigg|_{P_0=E_{\pi\pi,n}}$$

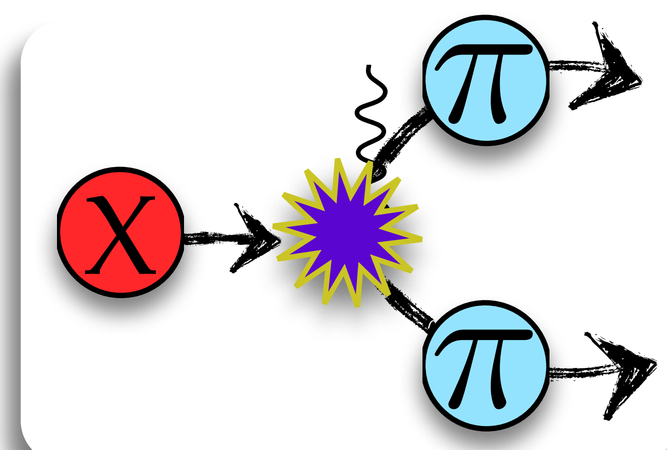
$$\frac{|\mathcal{H}_{X\to\pi\pi}|^2}{|\langle\pi\pi,n|\mathcal{J}|X\rangle_L|^2} = 4E_X\frac{E^2_{\pi\pi,n}}{\nu_n}L^3$$

free limit

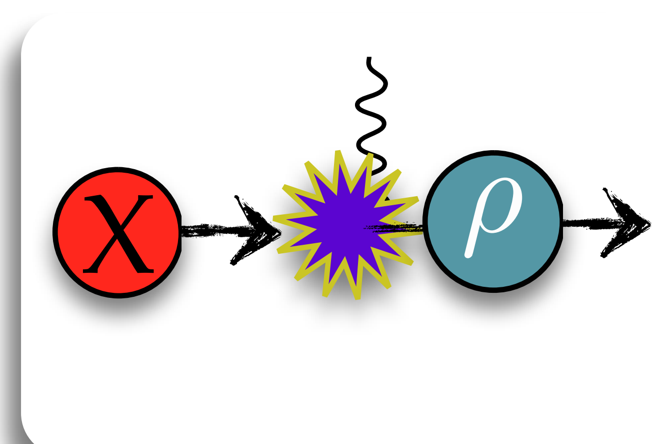
$$\frac{|\mathcal{H}_{X\to\pi\pi}|^2}{|\langle\pi\pi,n|\mathcal{J}|X\rangle_L|^2} = 2E_X\frac{32\pi E_\rho}{q^*_\rho\Gamma_\rho}\left[1+\mathcal{O}(\Gamma_\rho/m_\rho)\right]$$

narrow-width limit

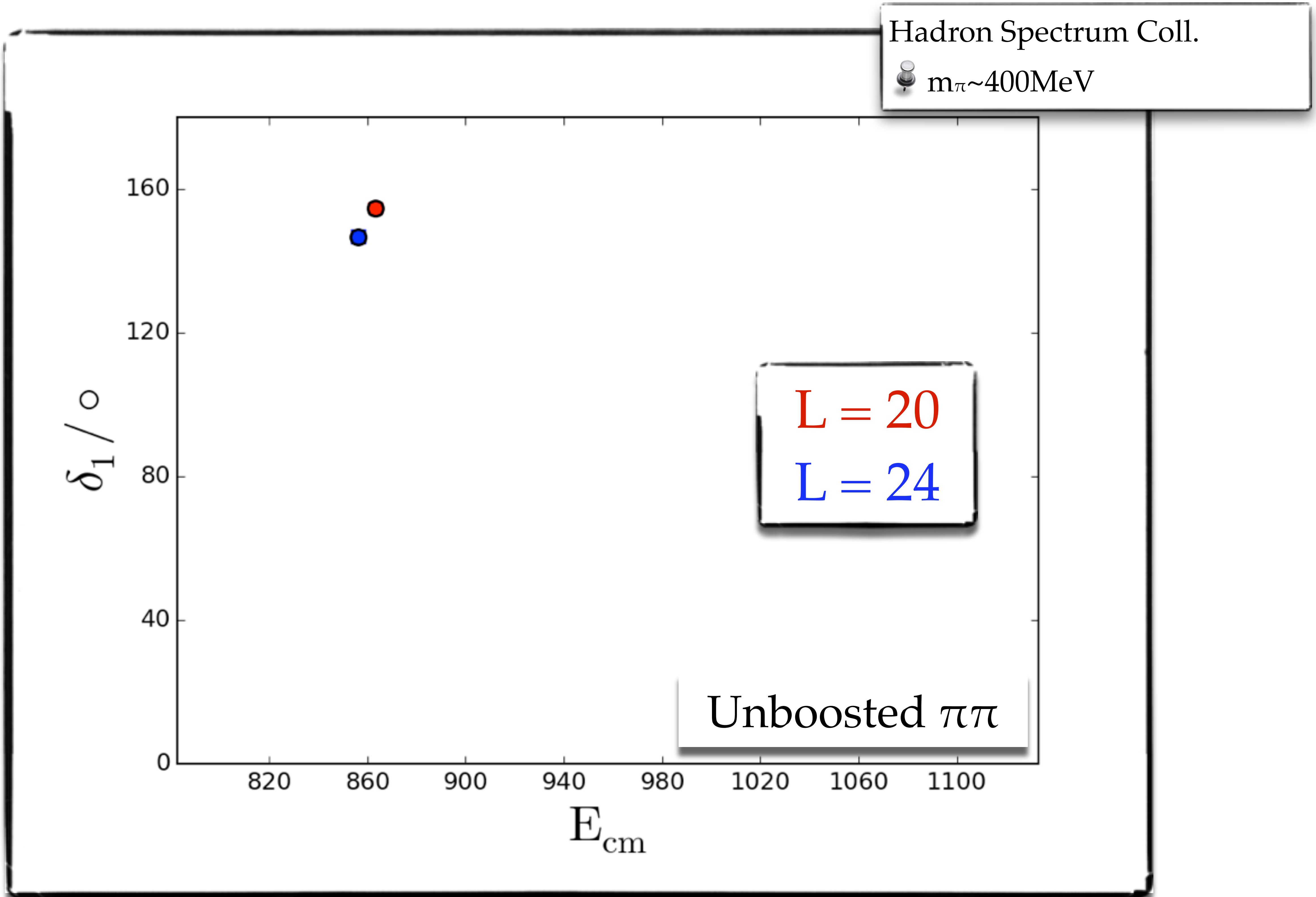
Million dollar question...



v.s.

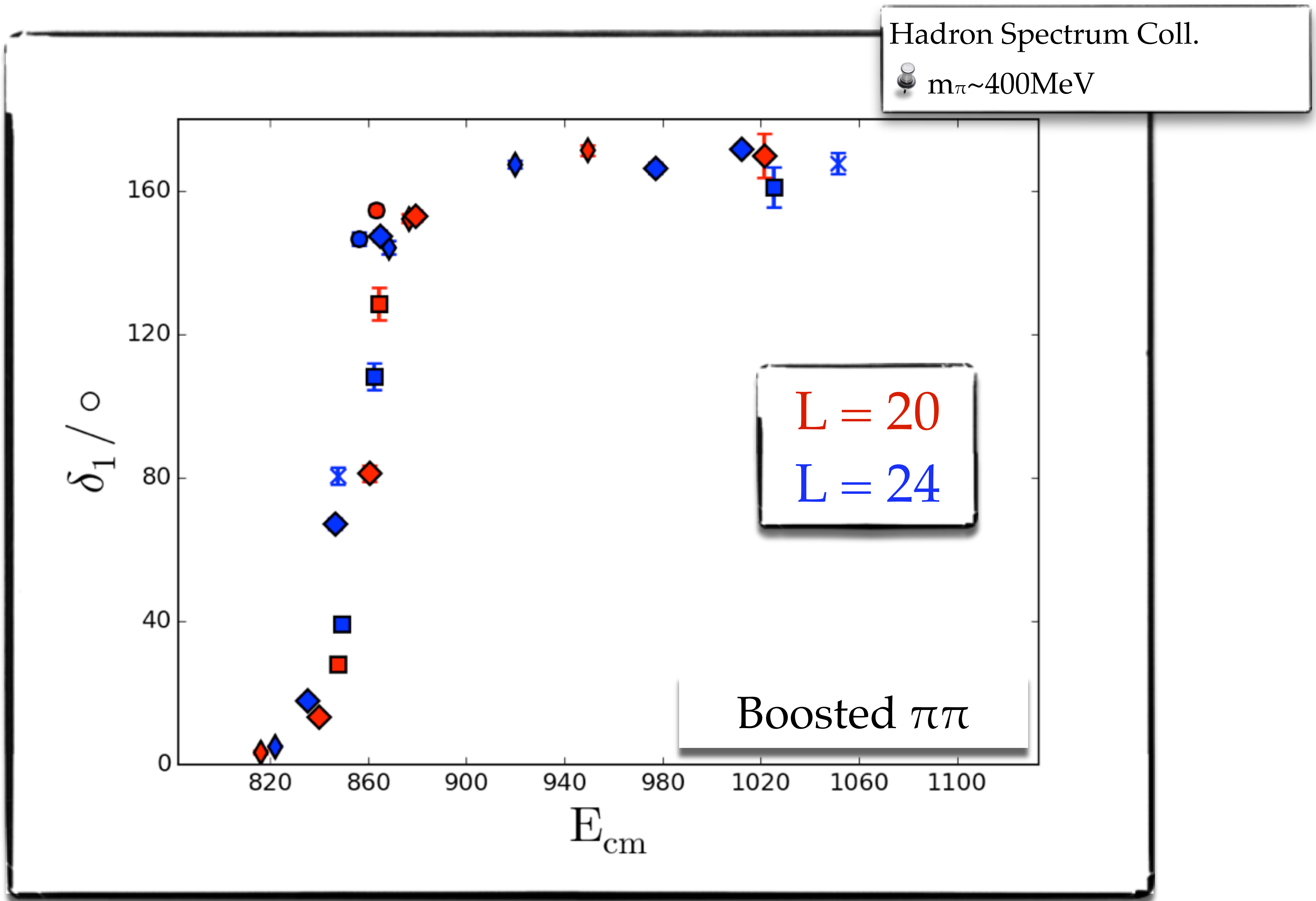


X-to-$\pi\pi$



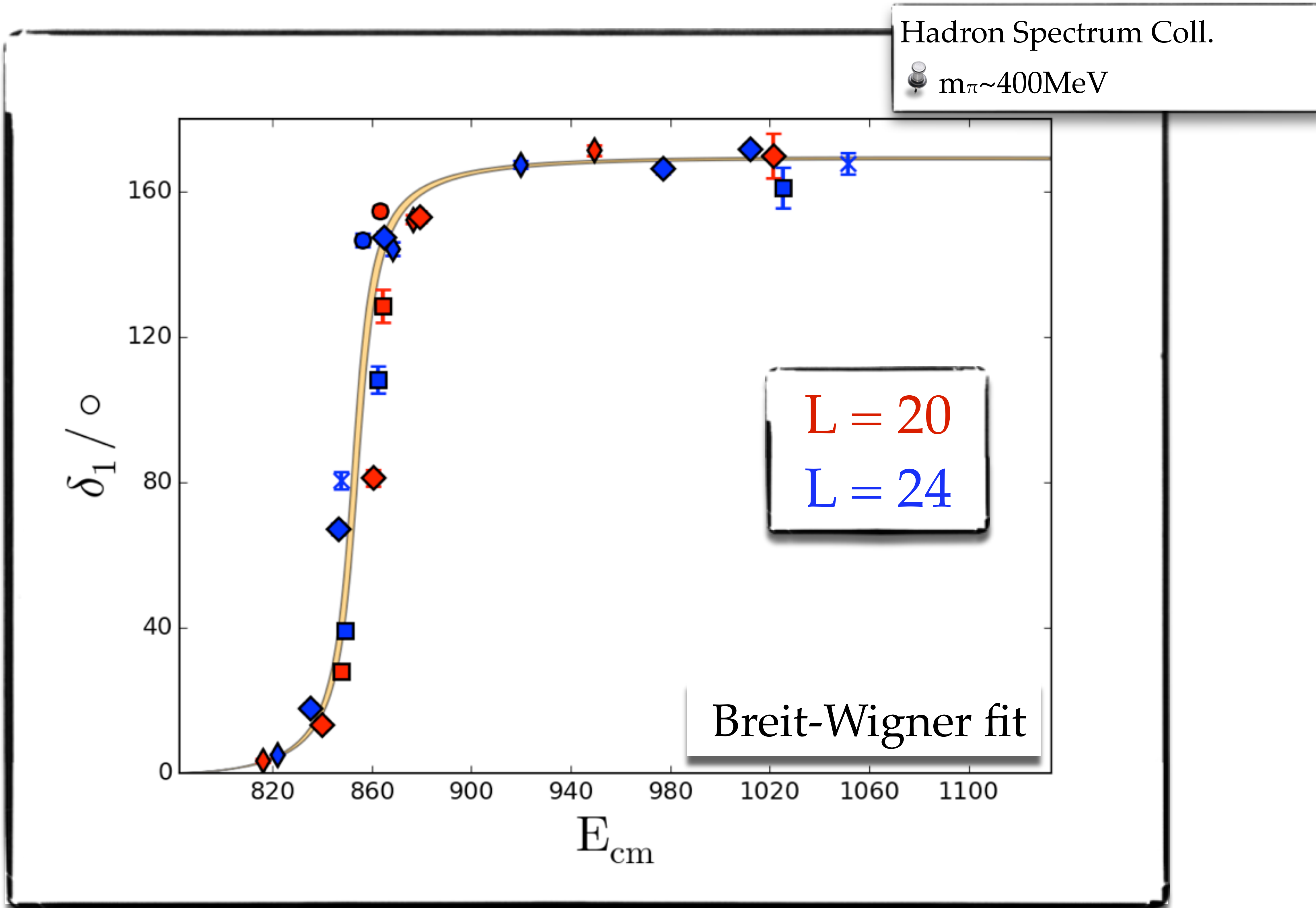
Hadron Spectrum Coll.
m_π~400MeV
160
120
80
40
0
δ_1 / ○
820
860
900
940
980
1020
1060
1100
E_{cm}
L = 20
L = 24
Unboosted $\pi\pi$

X-to-$\pi\pi$

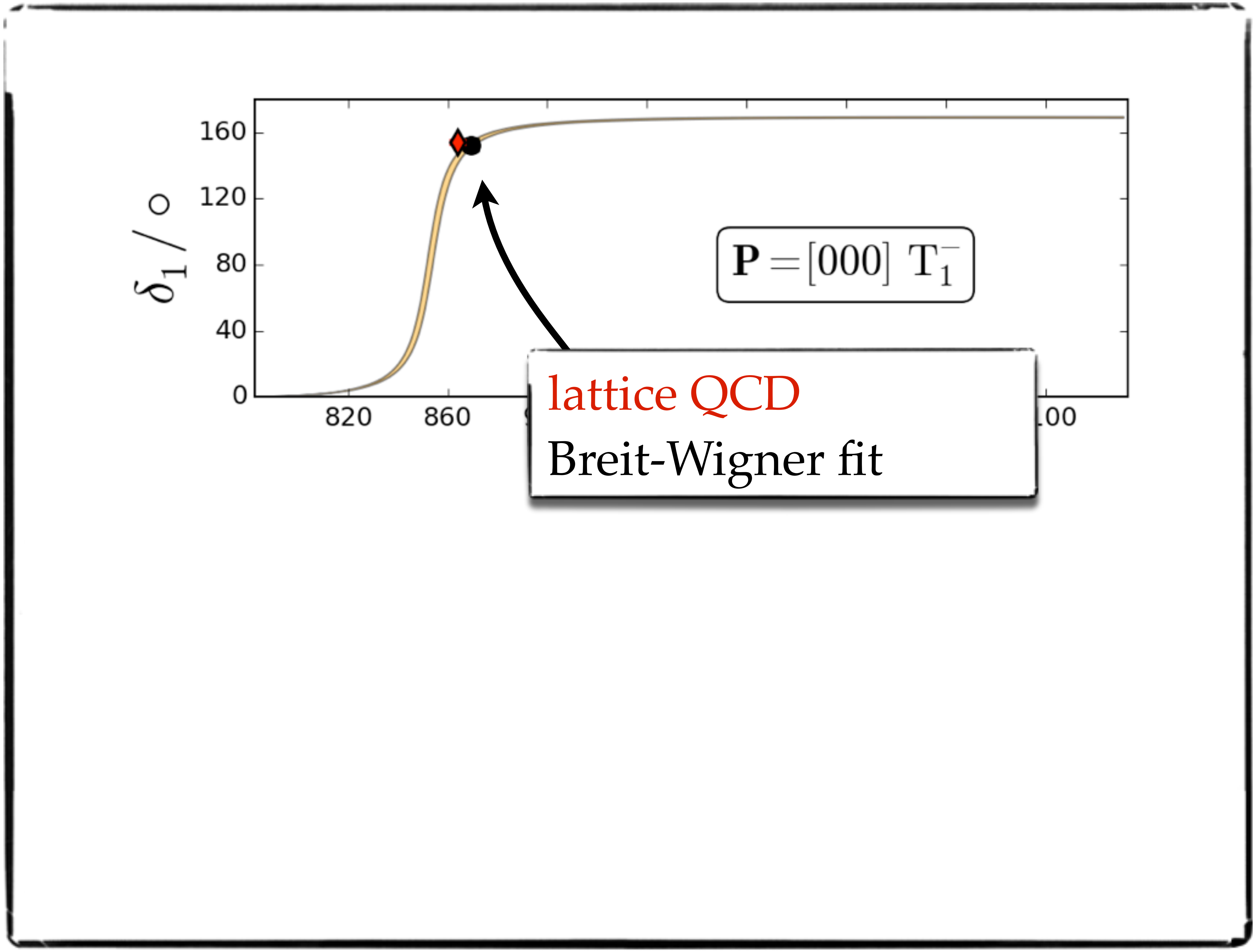


Hadron Spectrum Coll.
mπ~400MeV
160
120
80
40
0
820
860
900
940
980
1020
1060
1100
δ1 / ○
Ecm
L = 20
L = 24
Boosted ππ

X-to-$\pi\pi$

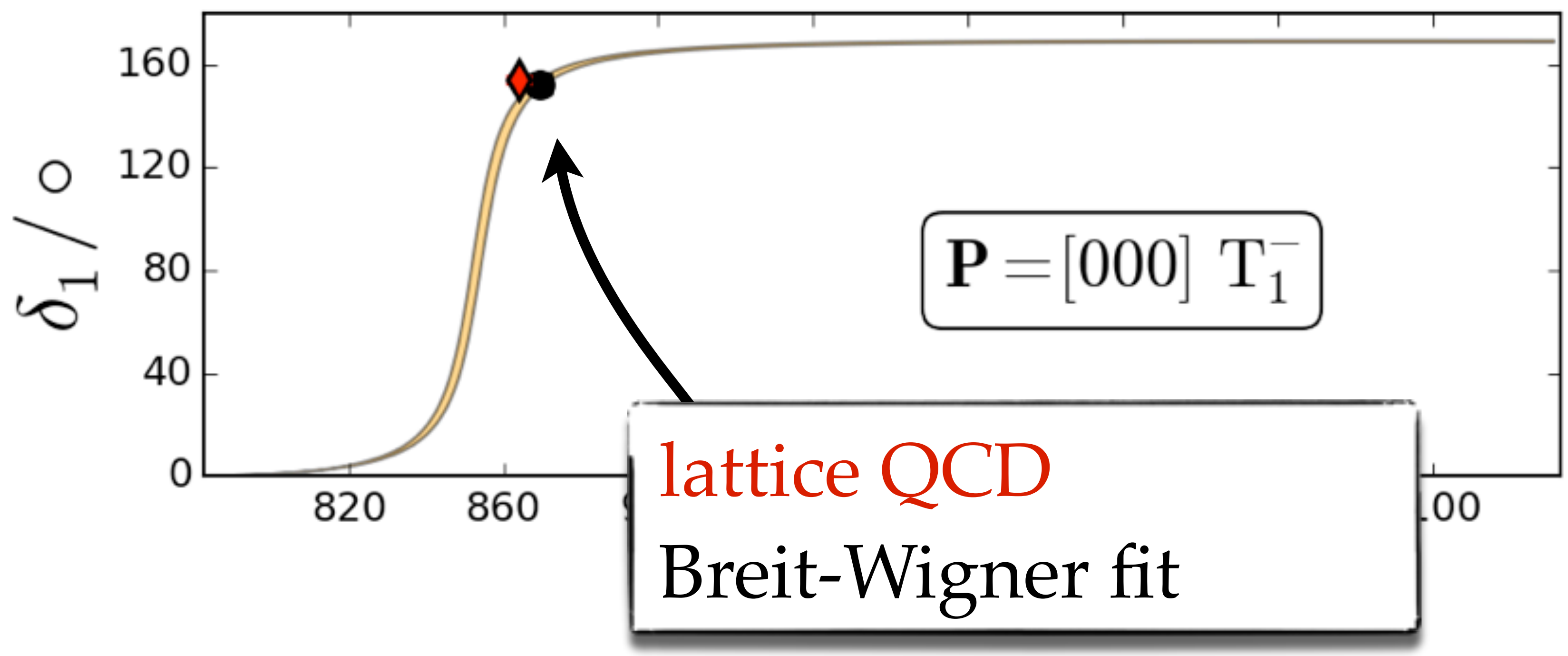

Hadron Spectrum Coll.
m_π~400MeV
δ_1 / ○
0
40
80
120
160
820
860
900
940
980
1020
1060
1100
E_{cm}
L = 20
L = 24
Breit-Wigner fit

X-to-$\pi\pi$



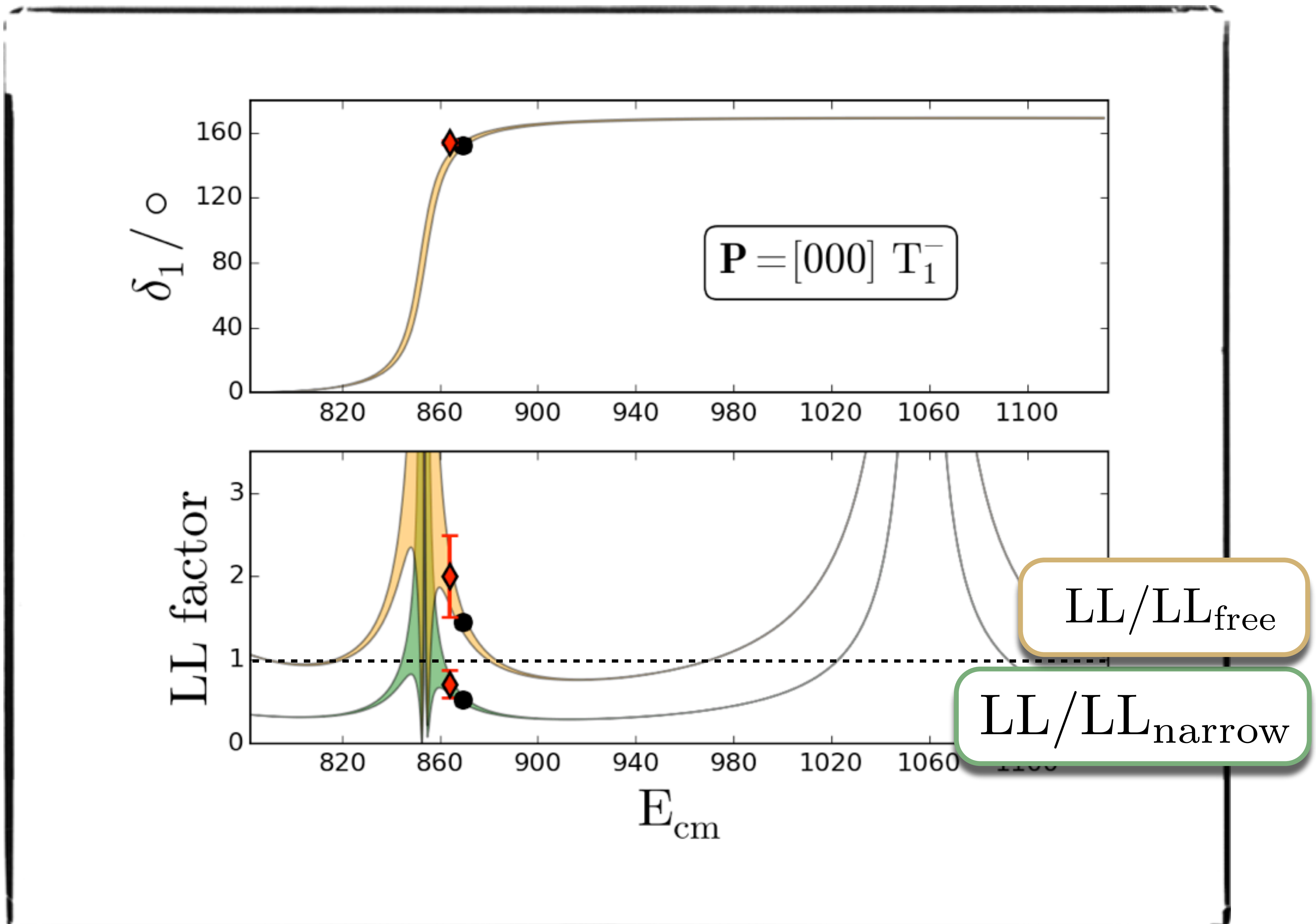
160
120
80
40
0
$\delta_1 / \circ$
820
860
$\mathbf{P} = [000]\ T_1^-$
lattice QCD
Breit-Wigner fit

X-to-$\pi\pi$



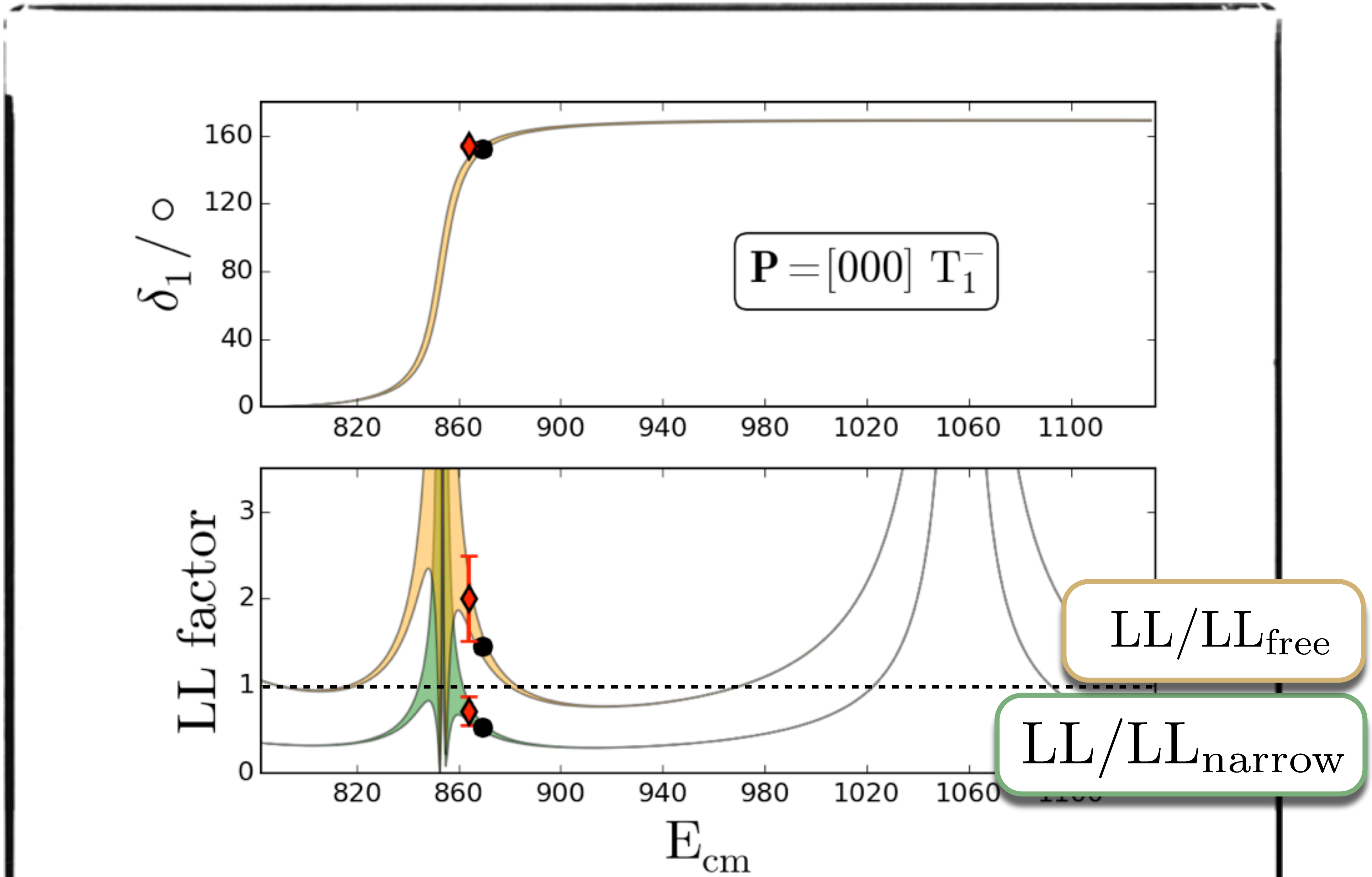
Note: there is an ambiguity as to which energy level is chosen: lattice QCD vs. "model"

X-to-$\pi\pi$



160
120
80
40
0
δ_1 / ○
$\mathbf{P} = [000]\ T_1^-$
820
860
900
940
980
1020
1060
1100
3
2
1
0
LL factor
E_{cm}
LL/LL$_{free}$
LL/LL$_{narrow}$

X-to-$\pi\pi$



Tension for the free and narrow width approximations!

Even though $\boldsymbol{\Gamma_\rho / m_\rho \sim 1\%}$

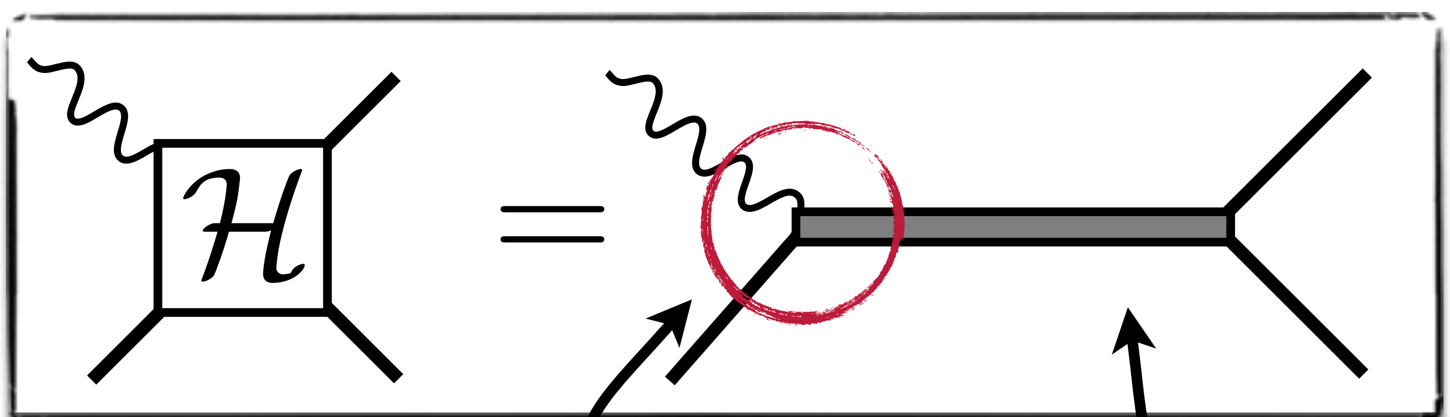
X-to-$\pi\pi$

Consider the following parametrization:

$$\mathcal{H}_{X\to\pi\pi} = \underline{F_{X\to\rho}(E^*, Q^2)} \frac{\sqrt{E^*\Gamma(E^*)}}{m_\rho^2 - E^{*2} - iE^*\Gamma(E^*)} \sqrt{\frac{16\pi E^*}{q^*}}$$

energy-dependent amplitude

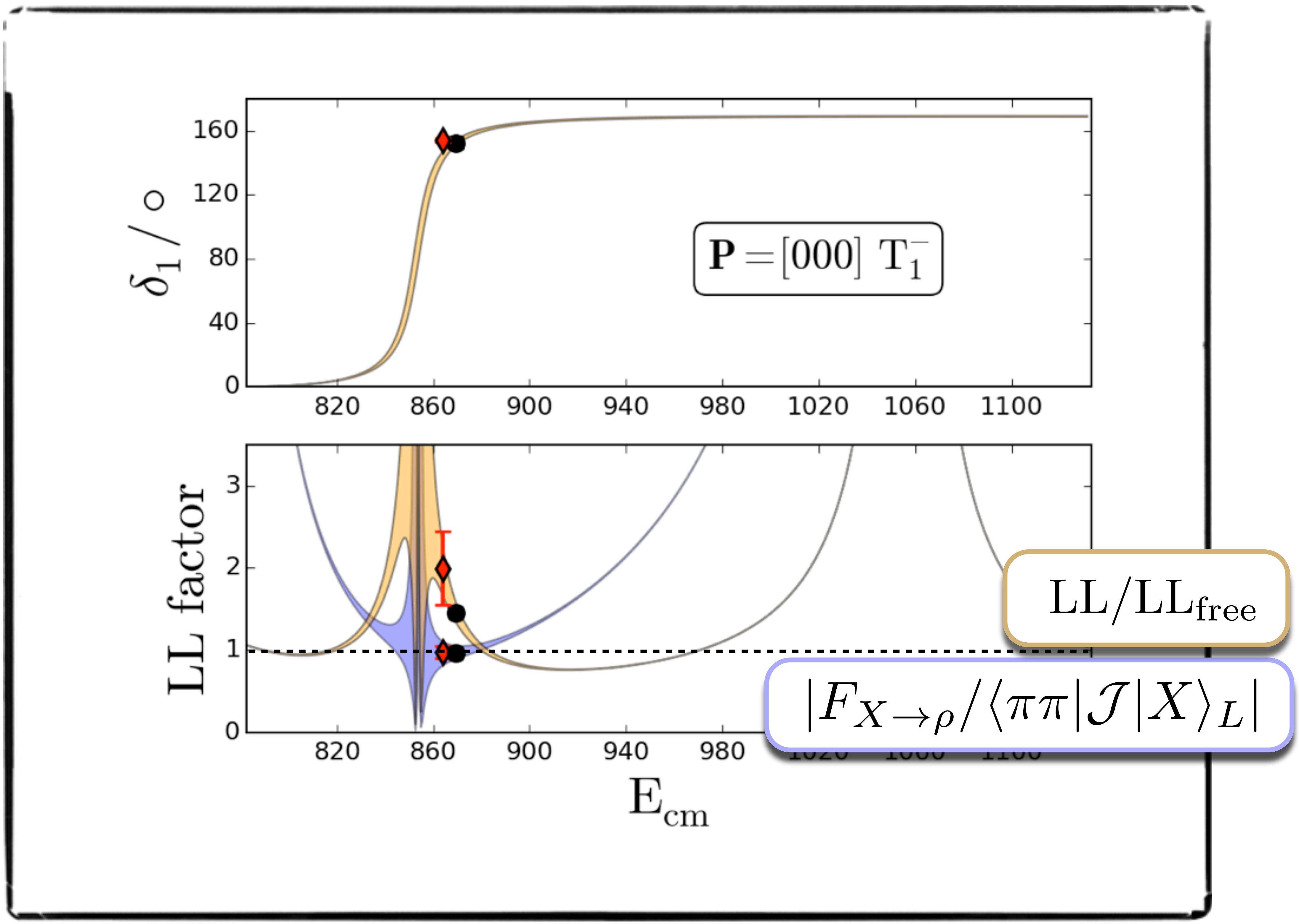
Intuitive picture:



current couples to incoming state to create an "off-shell" ϱ-meson

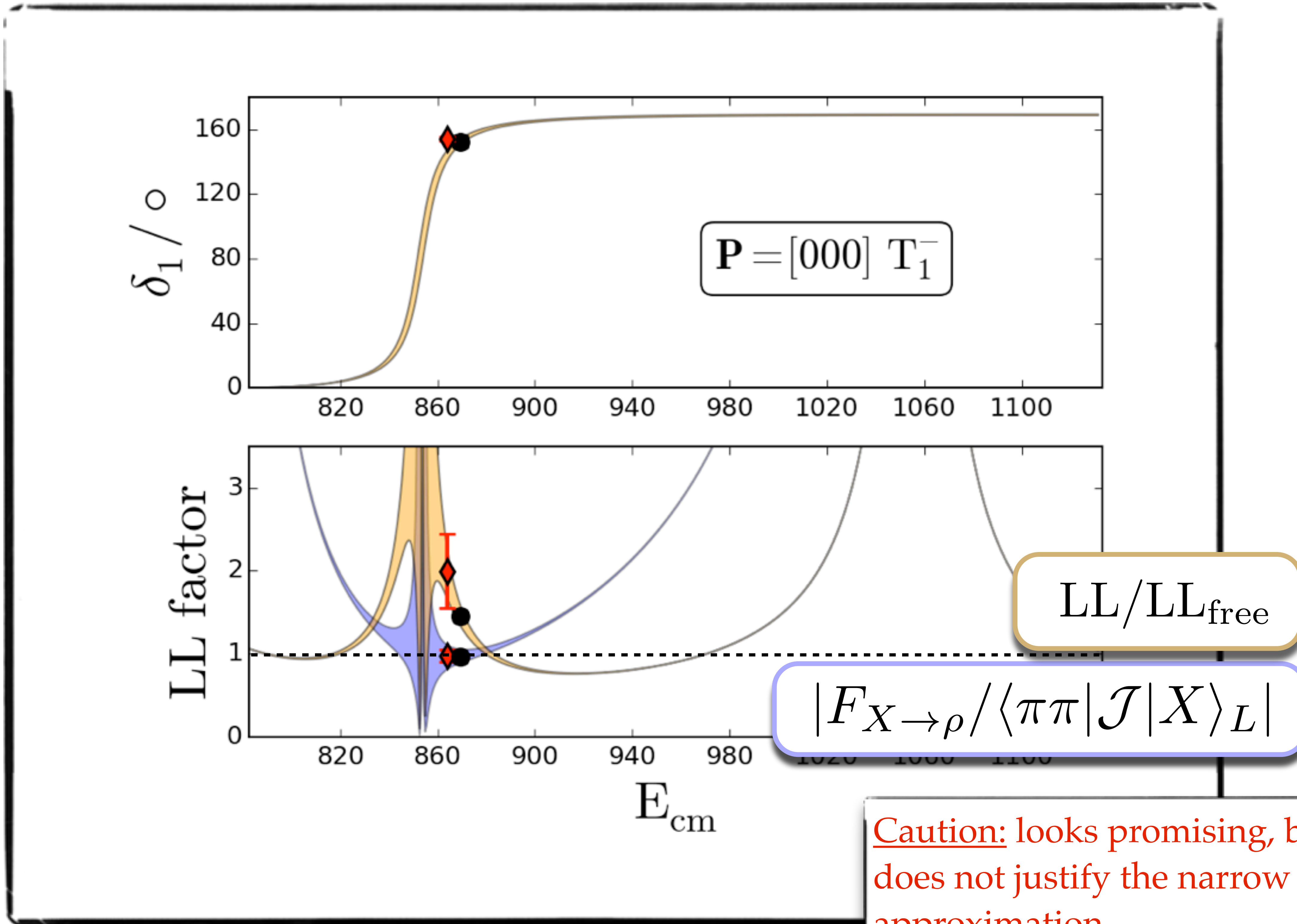
the ϱ-meson propagates and decays to two pions

X-to-$\pi\pi$



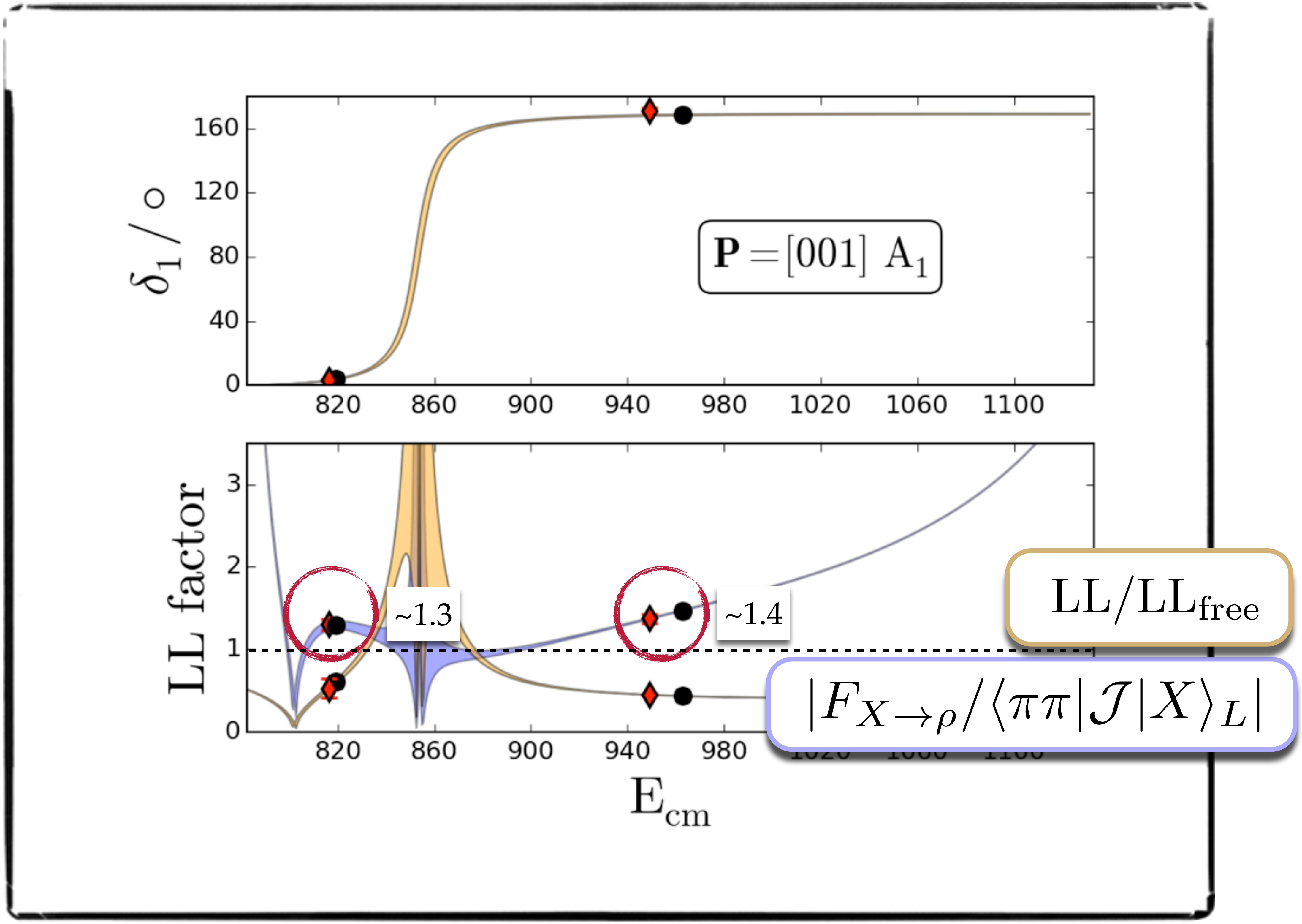
$\delta_1\,/\,\circ$
160
120
80
40
0
$\mathbf{P} = [000]\ \mathrm{T}_1^-$
820
860
900
940
980
1020
1060
1100
LL factor
3
2
1
0
E_{cm}
$\mathrm{LL}/\mathrm{LL}_{\mathrm{free}}$
$|F_{X\to\rho}/\langle\pi\pi|\mathcal{J}|X\rangle_L|$

X-to-$\pi\pi$



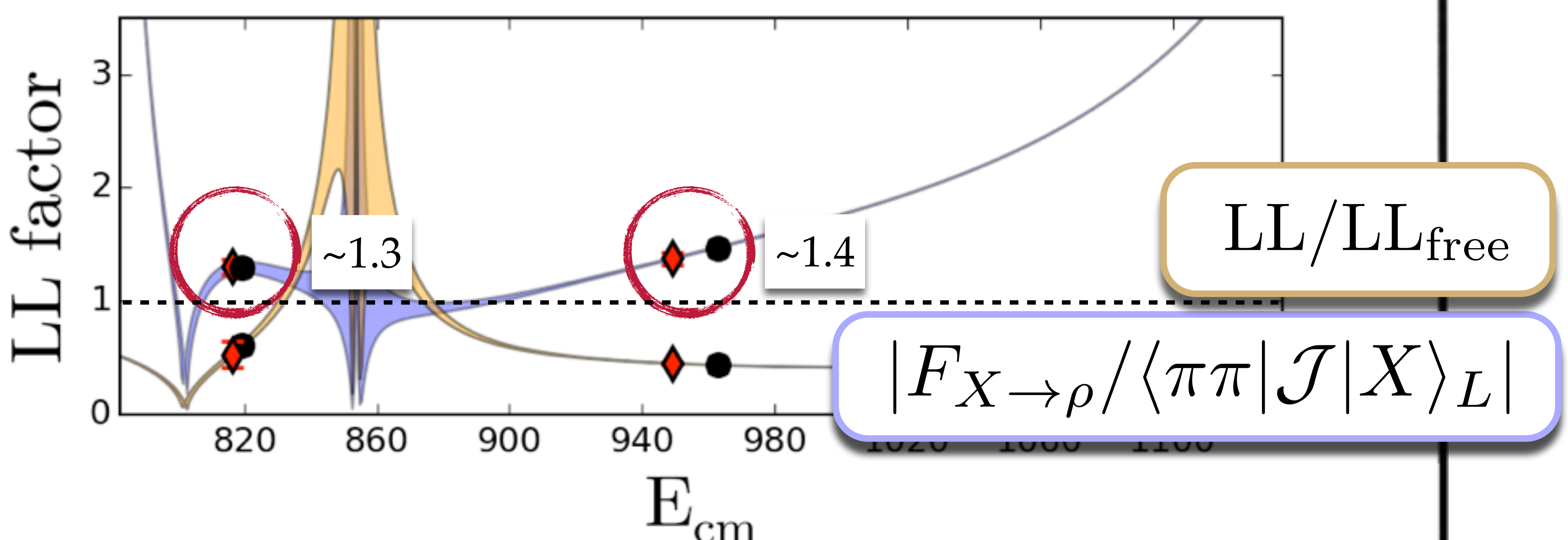
Caution: looks promising, but it does not justify the narrow width approximation

X-to-$\pi\pi$


$\delta_1 / \circ$
160
120
80
40
0
820
860
900
940
980
1020
1060
1100
$\mathbf{P} = [001]\ \mathrm{A}_1$
LL factor
3
2
1
0
~1.3
~1.4
LL/LL$_{\mathrm{free}}$
$|F_{X\to\rho}/\langle\pi\pi|\mathcal{J}|X\rangle_L|$
$\mathrm{E_{cm}}$

X-to-$\pi\pi$

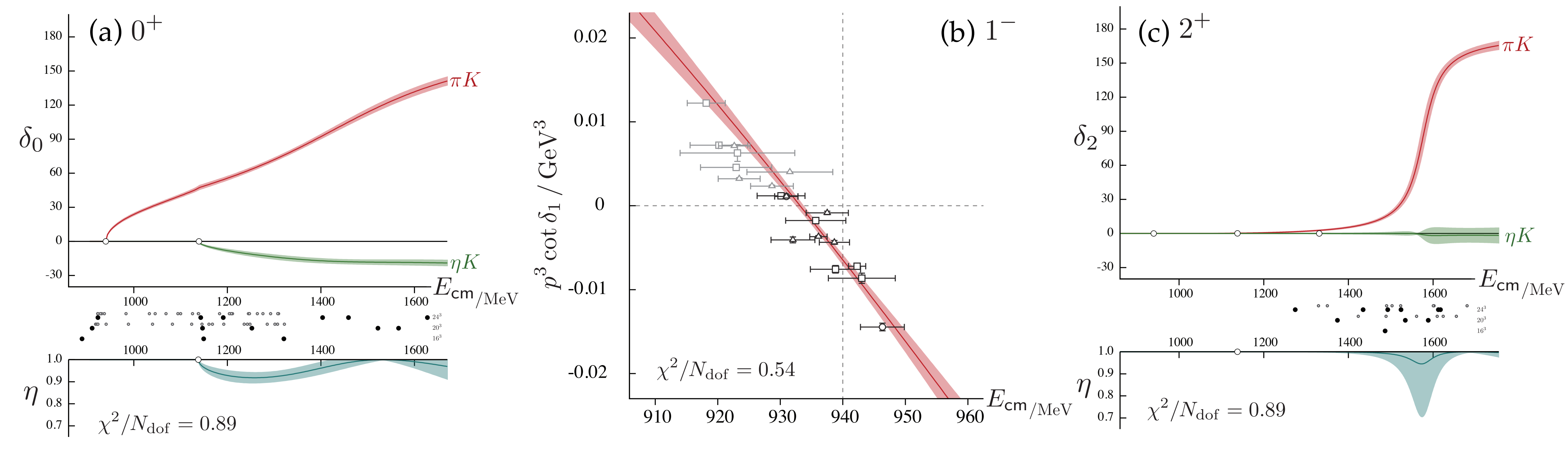
Conclusion: if precision is what you're seeking, the finite volume formalism is your best hope!



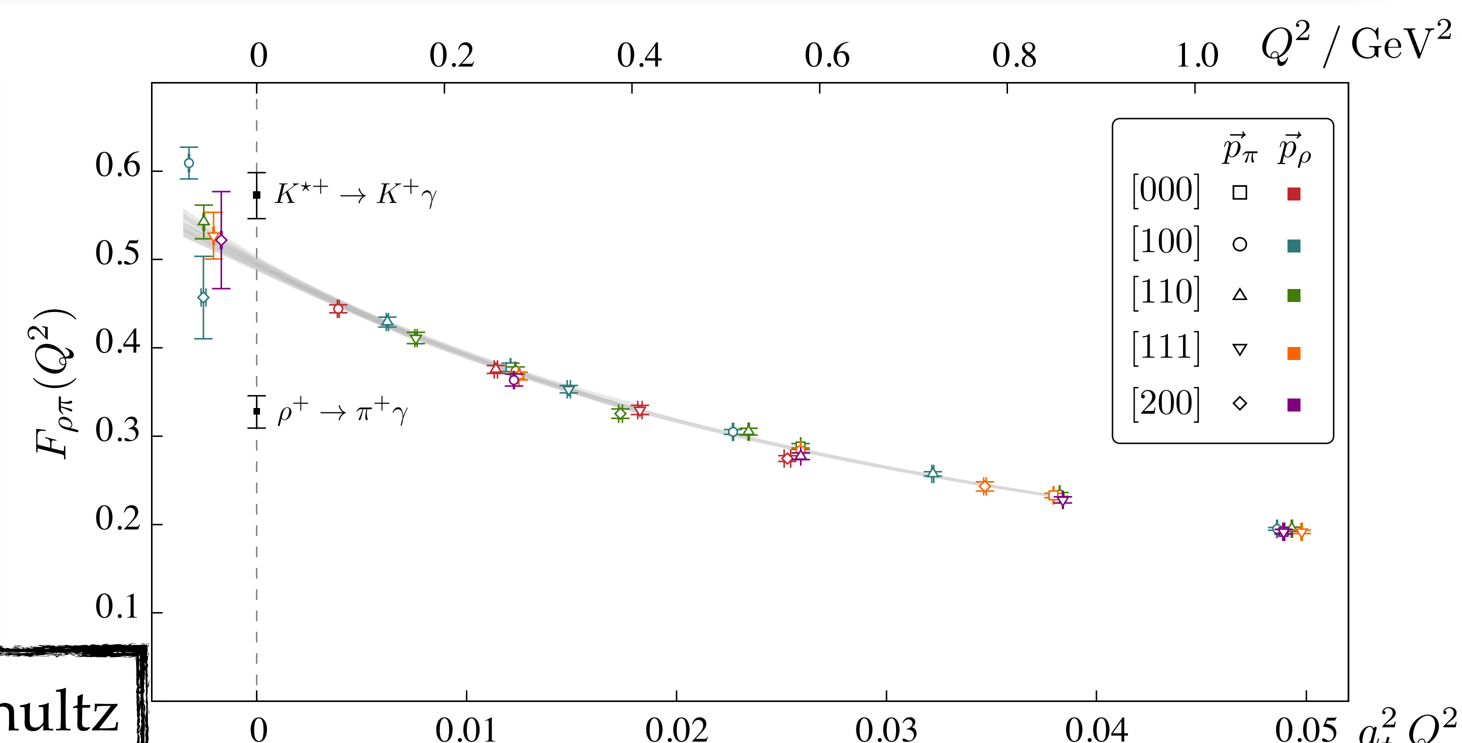
Final comments

Partial wave mixing and coupled channels:

David Wilson



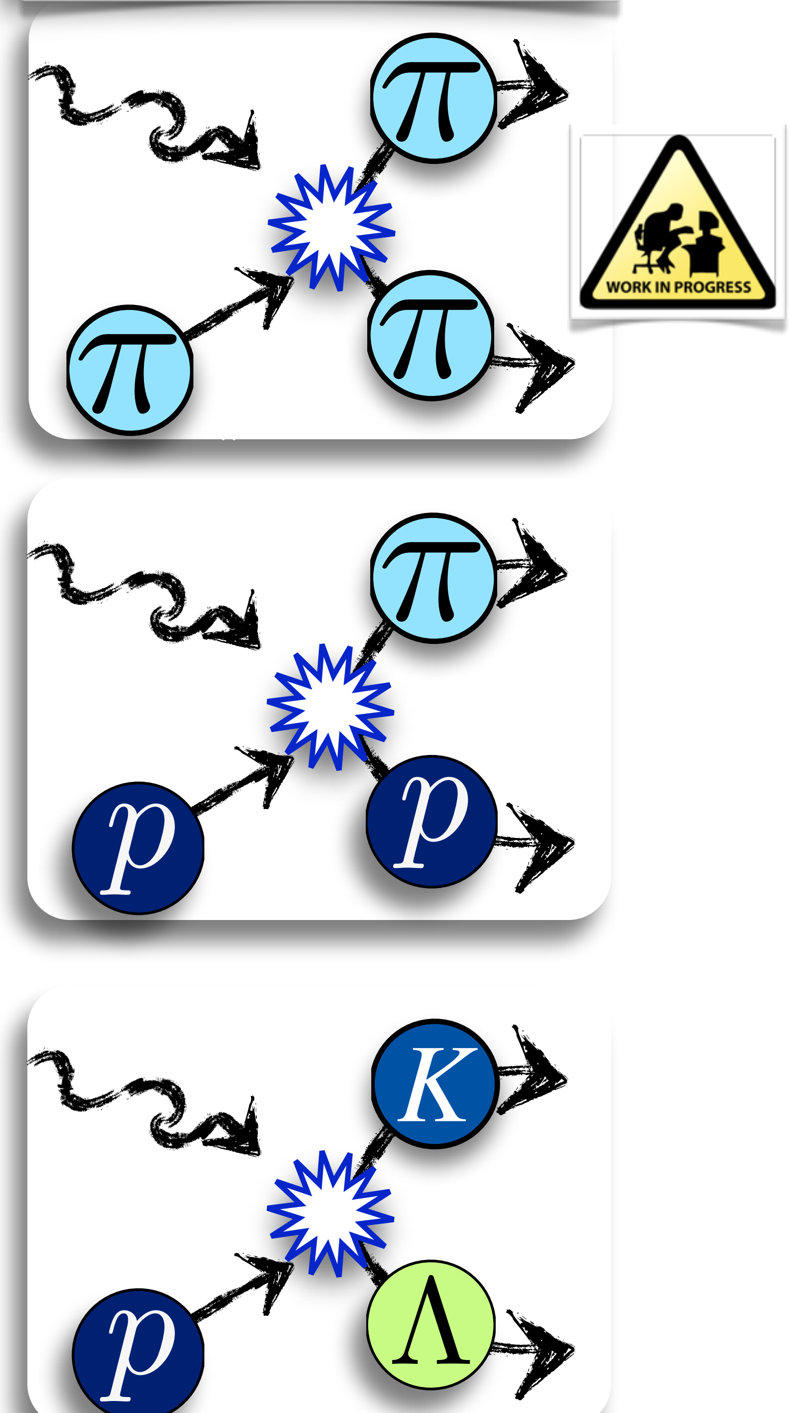
Matrix elements of not just ground states:



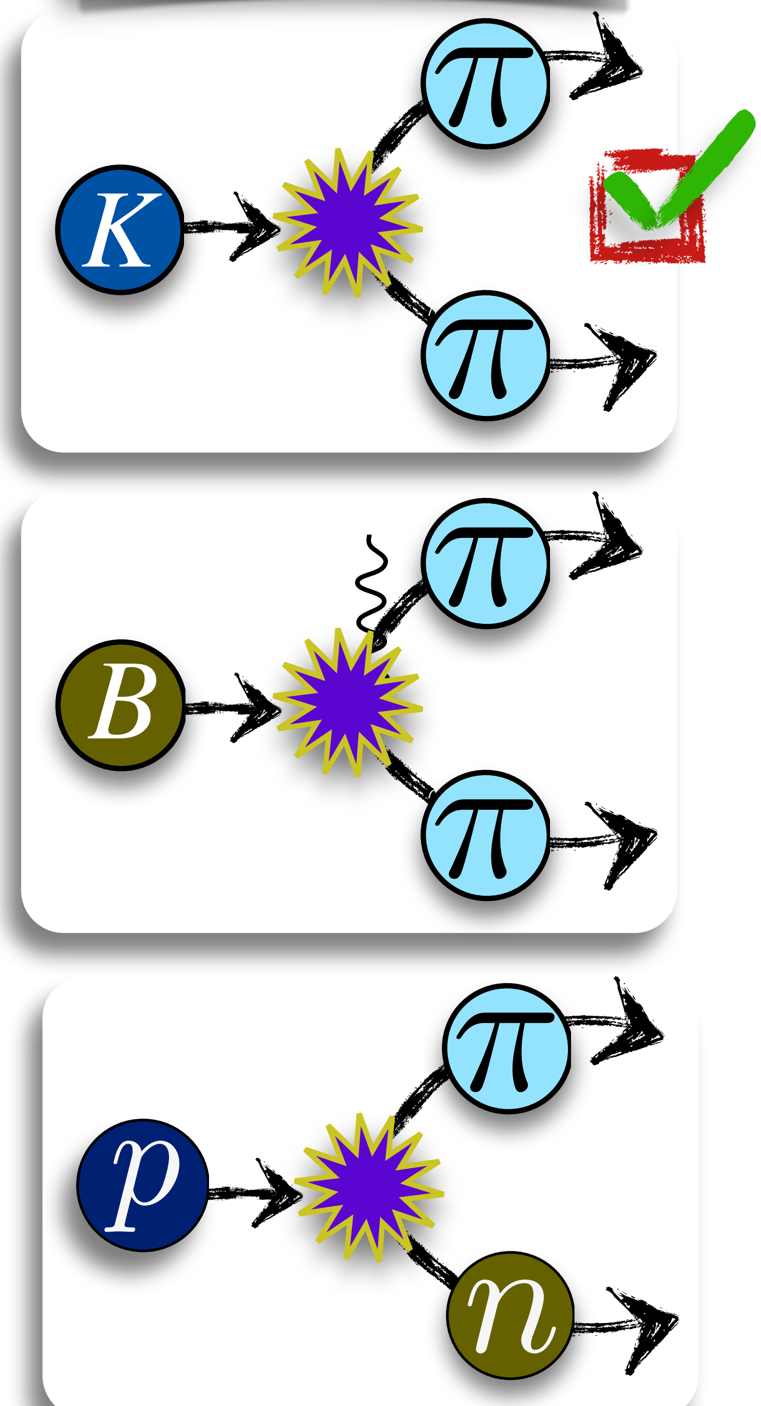
Christian Shultz

What the future holds!

photoproduction



weak processes



nuclear processes