# The scalar $B$ meson in the static limit of HQET

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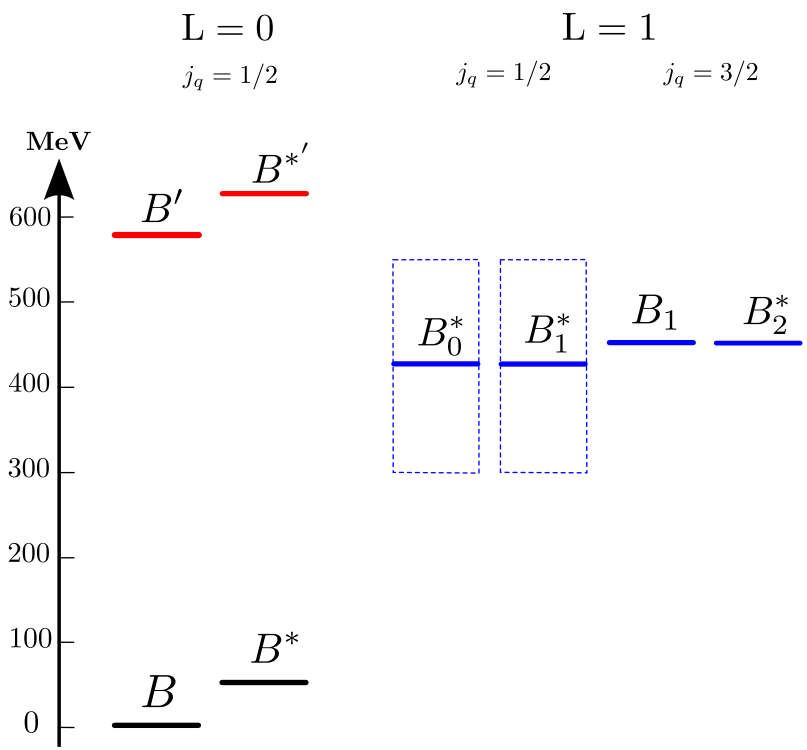
# Introduction: the soft pion couplings $\hat{g}$, $h$ and $\tilde{g}$



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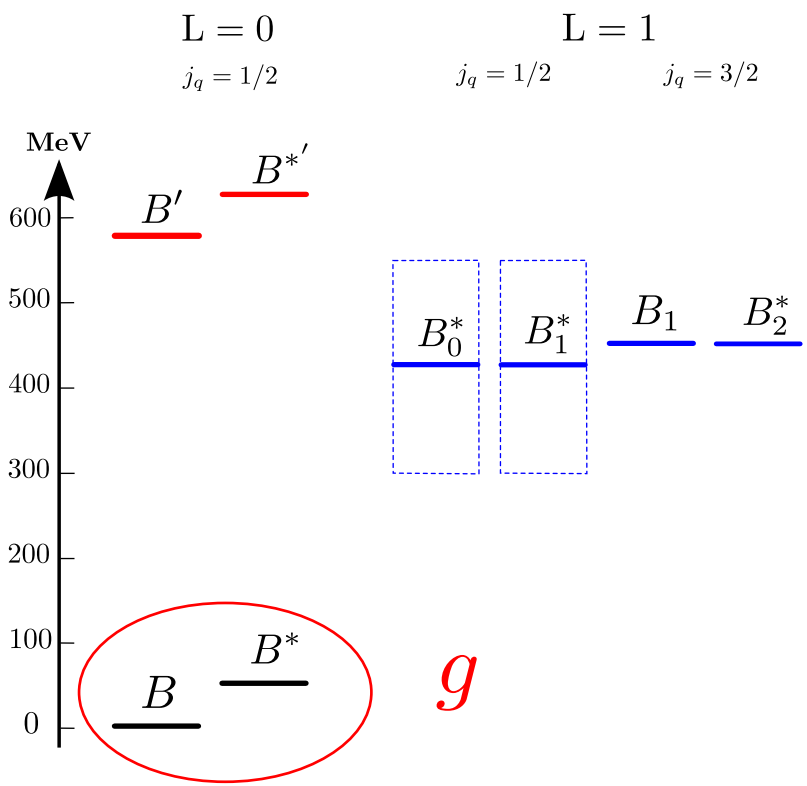
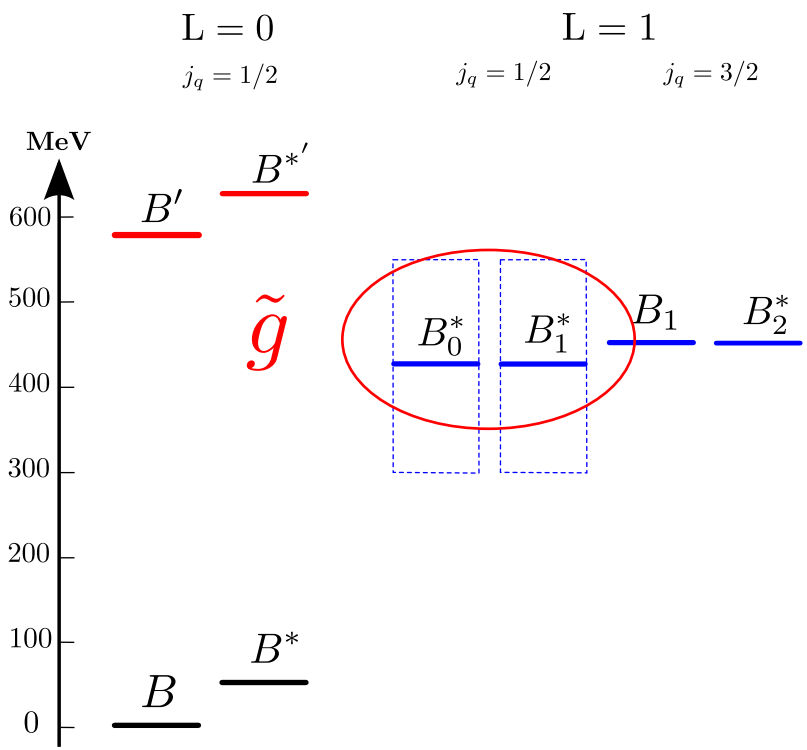
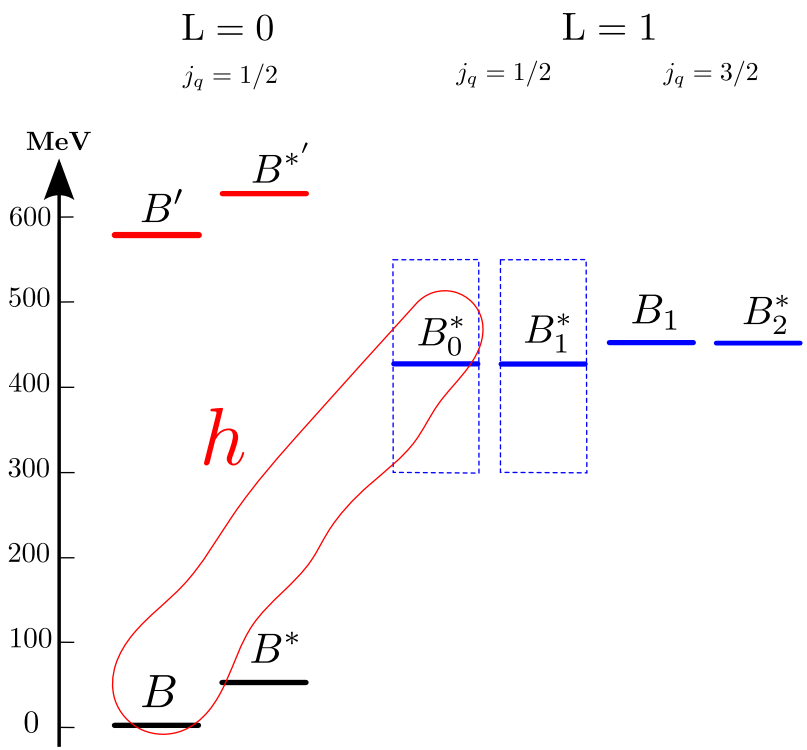


## Introduction: the soft pion couplings $\hat{g}$, $h$ and $\tilde{g}$



# Introduction: the soft pion couplings $\hat{g}$, $h$ and $\tilde{g}$



# Introduction

- The couplings $h$ and $\tilde{g}$ parametrize $\langle \pi(q) B(p') | B_0^*(p) \rangle$ and $\langle B(p') \pi(q) | B_0^*(p) \rangle$ in the Heavy Meson Chiral Lagrangians
- They enter the chiral extrapolations of $f_B$, $f_{B_0^*}$ when positive parity states are taken into account

  $\longrightarrow$ $\Delta = m_{B_0^*} - E_{B\pi}$ is usually not $\gg m_\pi$ on the lattice

  $\longrightarrow$ the coupling $h$ is large
- Computation of the scalar $B$ meson decay constant

| $L$ | $j_q$ | $J^P$ | state | $m$ (MeV) | dom. decay |
|---|---|---|---|---|---|
| 0 | $(1/2)^-$ | $0^-$<br>$1^-$ | $B$<br>$B^*$ | $5279.58 \pm 0.17$<br>$5325.2 \pm 0.4$ | $B\gamma$ |
| 1 | $(1/2)^+$ | $0^+$<br>$1^+$ | $B_0^*$<br>$B_1^*$ | | $B\pi$ (s-wave)<br>$B^*\pi$ (s-wave) |
| 1 | $(3/2)^+$ | $1^+$<br>$2^+$ | $B_1$<br>$B_2^*$ | | $B^*\pi$ (s,d-wave)<br>$B^{(*)}\pi$ (d-wave) |

$$J = j_q \pm \frac{1}{2} \quad \text{with} \quad \vec{j}_q = \vec{S}_q + \vec{L}\,.$$

## Outline


1. Lattice setup
2. Determination of $h$
3. Determination of $\tilde{g}$
4. Determination of $f_{B_0^*}$
5. Conclusion

# Lattice setup

# Correlation functions

- quark-antiquark interpolating operators

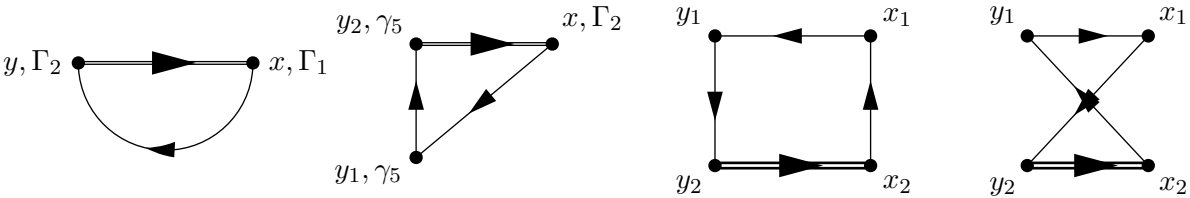
$$\mathcal{O}^B_{\Gamma,n}(t) = \frac{1}{V}\sum_{\vec{x}} \left[\overline{d}^{(n)}(x)\Gamma b(x)\right]$$

- meson-meson interpolating operators

$$\longrightarrow \quad \sqrt{\frac{2}{3}}\pi^+(0)B^-(0) - \sqrt{\frac{1}{3}}\pi^0(0)\overline{B}^0(0)$$

$$\mathcal{O}^{B\pi}_{\Gamma,n} = \frac{1}{V^2}\sum_{\vec{x}_i}\sqrt{\frac{2}{3}}\left[\overline{d}(x_1)\Gamma u(x_1)\right]\left[\overline{u}^{(n)}(x_2)\Gamma b(x_2)\right] - \sqrt{\frac{1}{6}}\left[\overline{u}(x_1)\Gamma u(x_1) - \overline{d}(x_1)\Gamma d(x_1)\right] \times \left[\overline{d}^{(n)}(x_2)\Gamma b(x_2)\right]$$

- local ($\Gamma = \gamma_0,\ \gamma_5$) and derivative ($\Gamma = \gamma_i\gamma_0\gamma_5\nabla_i,\ \gamma_i\nabla_i$) interpolating operators
- 4 levels of gaussian smearing



# Lattice setup

## Lattice discretization

- $N_f = 2$ $O(a)$ improved Wilson-Clover Fermions
- HYP1-2 discretization for the static quark action

## Discretization effects

- 3 lattice spacings $a$ :
  (0.048, 0.065, 0.075) $<$ 0.1 fm

## Light quark mass chiral extrapolations

- different pion masses in the range [280 MeV, 440 MeV]

CLS based

$\Rightarrow$ **total of 4 ensembles**

# Determination of $h$

# Lattice computation

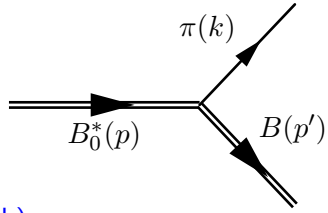
- Continuum

$$\Gamma(B_0^* \to B\pi) = \frac{|\vec{k}|}{8\pi m_{B_0^*}^2} g_{B_0^* B\pi}^2$$

$$|\vec{k}| = \frac{\sqrt{\left(m_{B_0^*}^2 - (m_B + m_\pi)^2\right)\left(m_{B_0^*}^2 - (m_B - m_\pi)^2\right)}}{2m_{B_0^*}}$$

$$g_{B_0^* B\pi} = \langle \pi(k) B(p') | B_0^*(p) \rangle = \sqrt{m_B m_{B_0^*}} \frac{m_{B_0^*}^2 - m_B^2}{m_{B_0^*}} \frac{h}{f_\pi}$$

(static limit)




- Lattice: Fermi Golden rule [Phys.Rev. D63 (2001)] (McNeile et al.)

$$\Gamma\left(B_0^* \to B\pi\right) = (2\pi)\, x^2\, \rho \quad , \quad x = \langle B_0^* | B\pi \rangle$$

$\rho$ is the density of final states:

$$\rho = \frac{L^3 k E_\pi}{2\pi^2}$$

$$\frac{\Gamma\left(B_0^* \to B\pi\right)}{k} = \frac{1}{\pi}\left(\frac{L}{a}\right)^3 (aE_\pi) \times (ax)^2$$

# Lattice computation

$$C_{B_0^*-B\pi}(t) = \langle \mathcal{O}^{B_0^*}(t)\mathcal{O}^{B\pi}(0)^\dagger\rangle = \sum_{t_1} \langle 0|\hat{\mathcal{O}}^{B_0^*}|B_0^*\rangle\, x\, \langle B\pi|\hat{\mathcal{O}}^{B\pi}|0\rangle\, e^{-m_{B_0^*}t_1} e^{-E_{B\pi}(t-t_1)}$$

$$\approx \langle 0|\hat{\mathcal{O}}^{B_0^*}|B_0^*\rangle\, x\, \langle B\pi|\hat{\mathcal{O}}^{B\pi}|0\rangle\, t\, e^{-Et} + \text{excited states}$$

- $x = \langle B_0^*|B\pi\rangle$
- We assume $m_{B_0^*} \approx E_{B\pi} \equiv E$

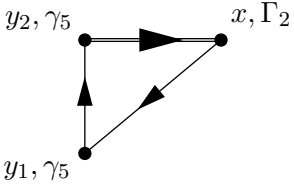
$$\left(\frac{3t^2\Delta^2}{24} \ll 1 \quad \text{for} \quad t \in [0-20]\right)$$

$$\Delta = m_{B_0^*} - E_{B\pi}$$

| CLS | B6 | E5 | F6 | N6 |
|---|---|---|---|---|
| $a\Delta$ | 0.036(4) | −0.012(6) | 0.026(8) | 0.010(3) |

- Excited states contributions vanish only linearly with time

$$R(t) = \frac{C^{(2)}_{B_0^*-B\pi}(t)}{\left(C^{(2)}_{B_0^*-B_0^*}(t)C^{(2)}_{B\pi-B\pi}(t)\right)^{1/2}} \approx A + xt$$

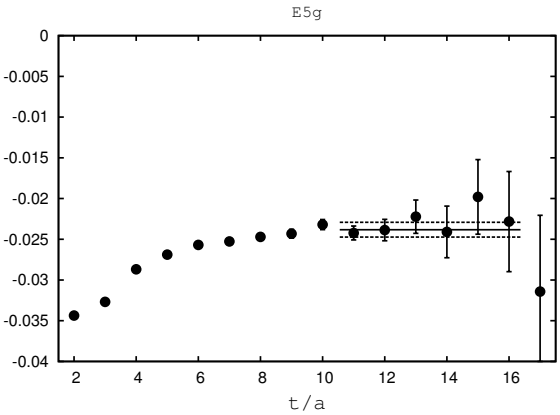



$$R^{\text{GEVP}}(t,t_0) = \frac{\left(v_{B_0^*}(t,t_0), C_{B_0^*-B\pi}(t)\, v_{B\pi}(t,t_0)\right)}{\sqrt{\left(v_{B_0^*}(t,t_0), C_{B_0^*-B_0^*}(t)\, v_{B_0^*}(t,t_0)\right) \times \left(v_{B\pi}(t,t_0), C_{B\pi-B\pi}(t)\, v_{B\pi}(t,t_0)\right)}} = \tilde{A} + xt$$

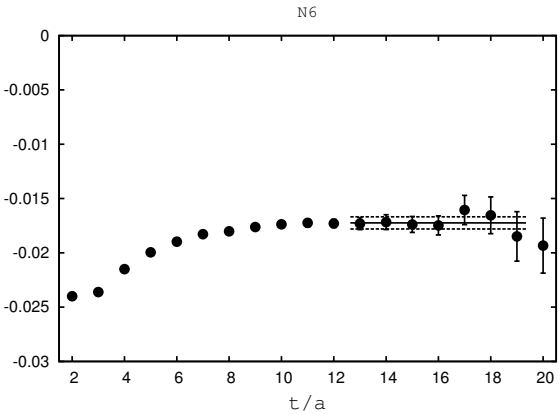
$$x^{\text{eff}}(t) = \partial_t R^{\text{GEVP}}(t) = \frac{R^{\text{GEVP}}(t+a) - R^{\text{GEVP}}(t)}{a} \quad \longrightarrow \quad x$$

$$t_0 = t - a$$

# Results: $x^{\text{eff}}(t)$




E5g : $a = 0.065$ fm, $m_\pi = 440$ MeV

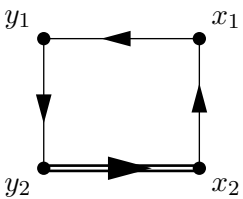



N6 : $a = 0.048$ fm, $m_\pi = 340$ MeV

| CLS | B6 | E5g | F6 | N6 |
|---|---|---|---|---|
| $ax$ | $-0.0156(4)$ | $-0.0241(10)$ | $-0.0159(3)$ | $-0.0174(6)$ |
| $h$ | $0.86(4)$ | $0.84(5)$ | $0.86(3)$ | $0.85(4)$ |

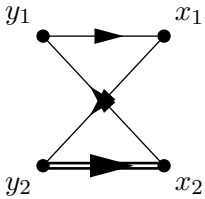
# Cross-check : box and cross diagrams

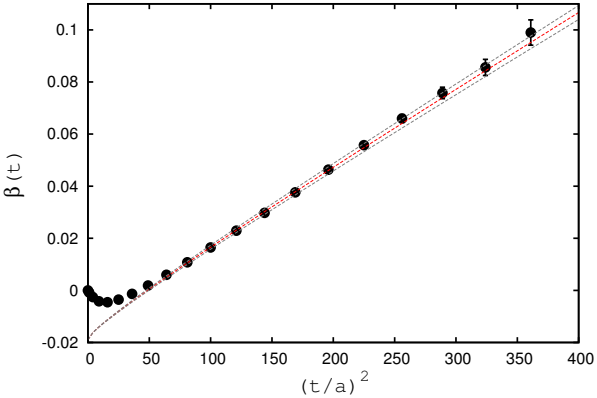
[Phys Lett B556 (2004)] (McNeile et al.)




$$\tilde{R}(t) = \frac{(v_{B\pi}(t,t_0), C_{\text{connected}}(t)\, v_{B\pi}(t,t_0))}{(v_{B\pi}(t,t_0), C_{B\pi-B\pi}(t)\, v_{B\pi}(t,t_0))} = B + \frac{1}{2}x^2t^2 + \mathcal{O}(t)$$

$$C_{\text{connected}}(t) = -\frac{3}{2}C_{\text{box}}(t) + \frac{1}{2}C_{\text{cross}}(t)\,.$$



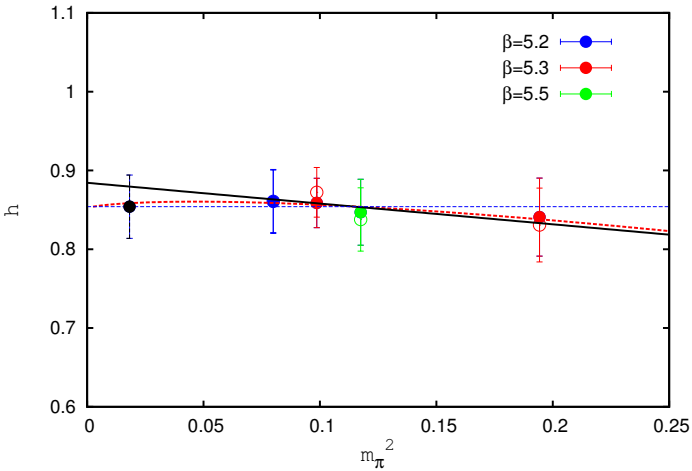




E5g : $a = 0.065$ fm, $m_\pi = 440$ MeV

- $\beta(t) = \partial_t \tilde{R}$
- Previous analysis : $ax = -0.0241(10)$
- Box+Cross diagrams : $ax = -0.0237(8)$

$\hookrightarrow$ Perfect agreement!

# $h$: Chiral and continuum extrapolations




→ small dependence on the lattice spacing

→ small dependence on the quark mass

constant : $h_0 = 0.85(3)$

linear : $h_0 = 0.88(4)$

HM$\chi$PT : $h_0 = 0.86(4)$

$$h_0 = 0.86(4)(2)$$

(preliminary)

$$h = h_0$$

$$h = h_0 + C\left(m_\pi^2 - (m_\pi^{\rm exp})^2\right)$$

$$h = h_0\left[1 - \frac{3}{4}\frac{3\hat{g}^2 + 3\tilde{g}^2 + 2\hat{g}\tilde{g}}{(4\pi f_\pi)^2}\left(m_\pi^2\log(m_\pi^2) - (m_\pi^{\rm exp})^2\log((m_\pi^{\rm exp})^2)\right)\right] + C\left(m_\pi^2 - (m_\pi^{\rm exp})^2\right)$$

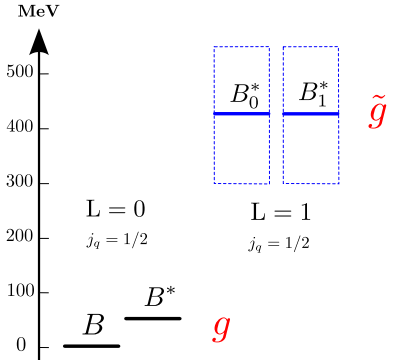
- [Becirevic et al. (2012)]: $h_0 = 0.66(10)(6)$
- PDG: $\Gamma_{D_0^*} = 267(40)$ MeV, $m_{D_0^*} = 2318(29)$ MeV $\Rightarrow h_0 = 0.74(16)$

# Determination of $\tilde{g}$

# The soft coupling $\tilde{g}$

$$\tilde{g}\,\epsilon_i = \langle B_0^*(\vec{0})\,|\,\overline{\psi}_l\gamma_i\gamma_5\psi_l\,|\,B_1^*(\epsilon_i,\vec{0})\rangle$$

- Similar to $g$ but for positive parity states
- Static limit of HQET: $B_0^*$ and $B_1^*$ are degenerate
- $\mathcal{S}(t) = \overline{\psi}_h(x)\Gamma^S\psi_l(x)$
- $\mathcal{A}_k(t) = \overline{\psi}_h(x)\Gamma_k^A\psi_l(x)$




→ three-point correlation function: $A_\mu = \overline{\psi}_l(x)\gamma_\mu\gamma_5\psi_l(x)$

$$C^{(3)}(t,t_1) = Z_A\,\frac{1}{V^3}\sum_{\vec{x},\vec{y},\vec{z}}\sum_{t_x}\frac{1}{3}\langle\mathcal{A}_k(\vec{z},t+t_x)A_k(\vec{y},t_1+t_x)\mathcal{S}^\dagger(\vec{x},t_x)\rangle$$

$Z_A$ was determined non-perturbatively by the ALPHA Collaboration [Nucl.Phys. B865 (2012) 397-429]

→ two-point correlation functions

$$C_{\mathcal{S}}^{(2)}(t) = \left\langle \sum_{\vec{y},\vec{x}}\mathcal{S}(y)\mathcal{S}^\dagger(x)\right\rangle\Big|_{y_0=x_0+t}\quad,\quad C_{\mathcal{A}}^{(2)}(t) = \left\langle \sum_{\vec{y},\vec{x}}\mathcal{A}_k(y)\mathcal{A}_k^\dagger(x)\right\rangle\Big|_{y_0=x_0+t}$$

- Local + Derivative interpolating operators (+ gaussian smearing)

## Extraction on the lattice : "summed" GEVP

- We solve the Generalized Eigenvalue Problem (GEVP):

$$C_{\mathcal{S}}^{(2)}(t)v_n(t,t_0) = \lambda_n(t,t_0)C_{\mathcal{S}}^{(2)}(t_0)v_n(t,t_0)$$

- Eigenvectors and eigenvalues can be used to construct the summed ratio $R_{nn}^{\mathrm{sGEVP}}(t,t_0)$:

$$R_{nn}^{\mathrm{sGEVP}}(t,t_0) = -\partial_t \left( \frac{\left(v_n(t,t_0), \left[K(t,t_0)/\lambda_n(t,t_0) - K(t_0,t_0)\right] v_n(t,t_0)\right)}{\left(v_n(t,t_0), C_{\mathcal{S}}^{(2)}(t_0)v_n(t,t_0)\right)^{1/2} \left(v_n(t,t_0), C_{\mathcal{A}}^{(2)}(t_0)v_n(t,t_0)\right)^{1/2}} \right)$$

$$\text{with} \quad : \quad K_{ij}(t,t_0) = \sum_{t_1} C_{ij}^{(3)}(t,t_1)$$

$$R_{11}^{\mathrm{sGEVP}} \xrightarrow[t_0=t-1]{t\gg 1} \tilde{g} \; + \; \mathcal{O}\left(te^{-\Delta_{N+1,n}t}\right)$$

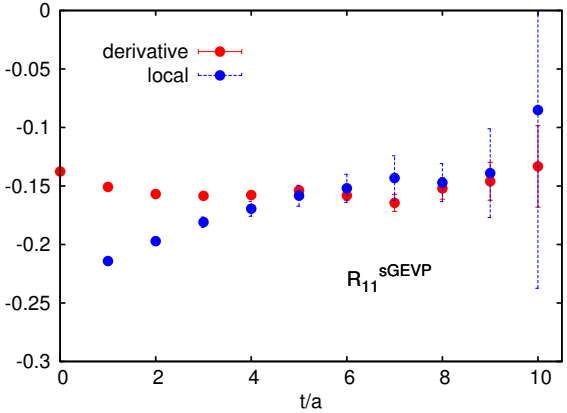
*"summed GEVP"*
[JHEP 1201 (2012) 140]

# Local vs Derivative interpolating operator

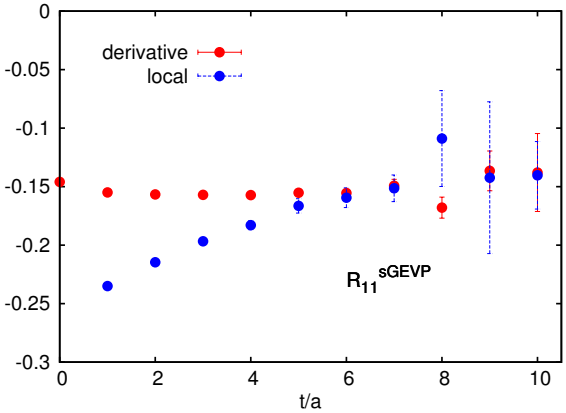
| $J^P$ | Local | Derivative |
|---|---|---|
| $0^+$ | $\Gamma = \gamma_0$ | $\Gamma = \gamma_i \overleftrightarrow{\nabla}_i$ |
| $1^+$ | $\Gamma = \gamma_5\gamma_i$ | $\Gamma = \gamma_5 \overleftrightarrow{\nabla}_i$ |

Table: Interpolating operators

$\rightarrow$ Interpolating operators built from covariant derivatives are beneficial to reduce the contamination from higher excited states



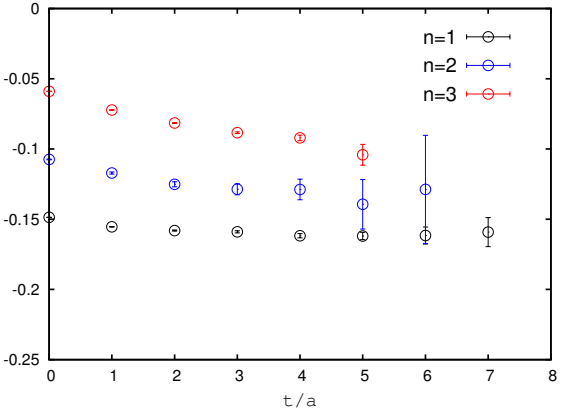

E5g : $a = 0.065$ fm, $m_\pi = 440$ MeV



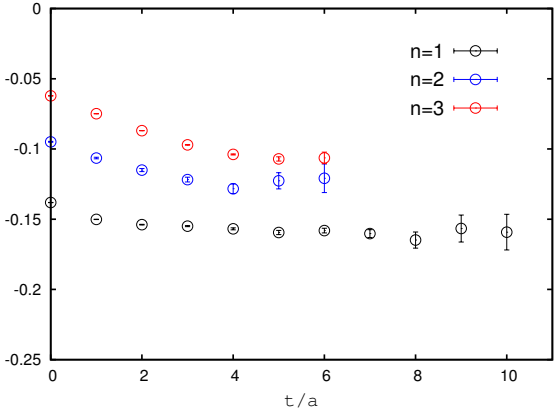

F6 : $a = 0.065$ fm, $m_\pi = 310$ MeV

# GEVP : excited states

$$R_{nn}^{\mathrm{sGEVP}}(t,t_0) = -\partial_t \left( \frac{\left(v_n(t,t_0), \left[K(t,t_0)/\lambda_n(t,t_0) - K(t_0,t_0)\right] v_n(t,t_0)\right)}{\left(v_n(t,t_0), C_{\mathcal{S}}^{(2)}(t_0) v_n(t,t_0)\right)^{1/2} \left(v_n(t,t_0), C_{\mathcal{A}}^{(2)}(t_0) v_n(t,t_0)\right)^{1/2}} \right)$$




B6 : $a = 0.075$ fm, $m_\pi = 280$ MeV

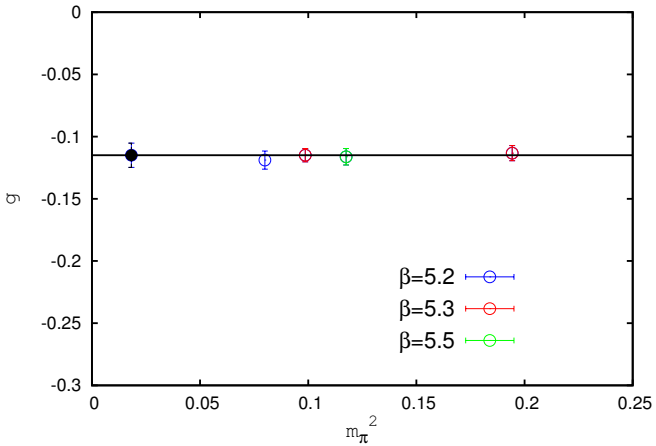



N6 : $a = 0.048$ fm, $m_\pi = 340$ MeV

# Extrapolation to the physical point

$$\tilde{g} = g_0$$

(NLO HM$\chi$PT)



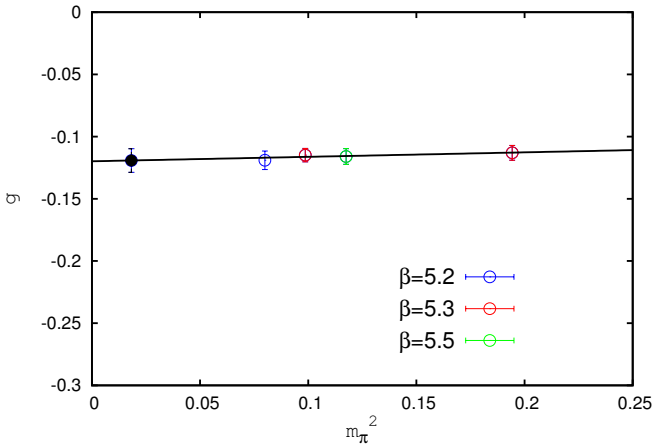

- small dependence on the lattice spacing
- small dependence of the pion mass

$$\tilde{g} = -0.115(5)$$

# Extrapolation to the physical point

$$\tilde{g} = g_0$$
$$\tilde{g} = g_0 + C\left(m_\pi^2 - (m_\pi^{\rm exp})^2\right)$$

(NLO HM$\chi$PT)



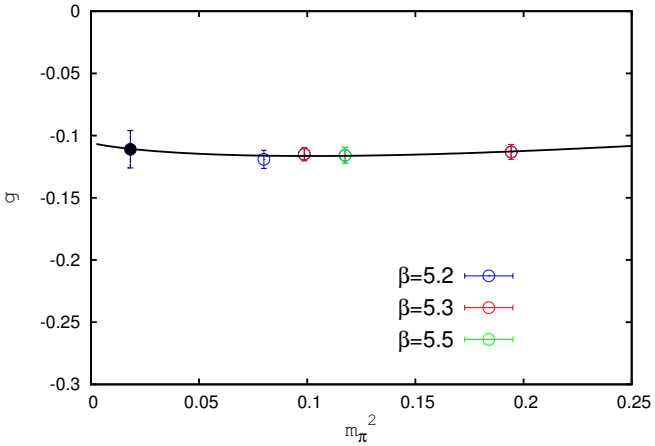

- small dependence on the lattice spacing
- small dependence of the pion mass

$$\tilde{g} = -0.115(5)$$
$$\tilde{g} = -0.119(11)$$

# Extrapolation to the physical point

$$\tilde{g} = g_0$$
$$\tilde{g} = g_0 + C\left(m_\pi^2 - (m_\pi^{\rm exp})^2\right)$$
$$\tilde{g} = \alpha\left[1 - \frac{2+4\tilde{g}^2}{(4\pi f_\pi)^2} m_\pi^2 \log(m_\pi^2) + \frac{h^2}{(4\pi f_\pi)^2}\frac{m_\pi^2}{8\Delta^2}\left(3+\frac{g}{\tilde{g}}\right) m_\pi^2 \log(m_\pi^2)\right] + C m_\pi^2$$

(NLO HM$\chi$PT)




- small dependence on the lattice spacing
- small dependence of the pion mass

$$\tilde{g} = -0.115(5)$$
$$\tilde{g} = -0.119(11)$$
$$\tilde{g} = -0.111(15)$$

$$\tilde{g} = -0.115(15)(5)$$

(preliminary)

# Determination of $f_{B_0^*}$

## Method

- Solve the Generalized Eigenvalue Problem

$$C^{(2)}(t)v_n(t,t_0) = \lambda_n(t,t_0)C^{(2)}(t_0)v_n(t,t_0)$$

- Compute $f_{B_0^*}^{\rm stat}(t,t_0)$ and $f_{B_0^*}^{\rm dV}(t,t_0)$

$$f_{B_0^*}^{\rm stat}(t,t_0) = R_1^{\rm stat}(t,t_0) \times \left(v_1^{\rm stat}(t,t_0), C_{V,\mathsf{loc}}^{\rm stat}(t)\right) \quad \underset{t\gg 1}{\longrightarrow} \quad \langle 0|\hat{V}_0|B_0^*\rangle$$

$$f_{B_0^*}^{\rm dV}(t,t_0) = R_1^{\rm stat}(t,t_0) \times \left(v_1^{\rm stat}(t,t_0), C_{\delta V,\mathsf{loc}}(t)\right) \quad (\mathcal{O}(a)\text{-improvement})$$

$$R_n^{\rm stat} = \left(v_n^{\rm stat}(t,t_0), C_V(t)v_n^{\rm stat}(t,t_0)\right)^{-1/2} \left(\frac{\lambda_n(t,t_0)}{\lambda_n(t+1,t_0)}\right)^{t/(2a)}$$

$$(C_{V,\mathsf{loc}})_i = \langle V_i(0)V_0(t)\rangle \quad , \quad (C_{\delta V,\mathsf{loc}})_i = \langle V_i(0)\delta V(t)\rangle$$

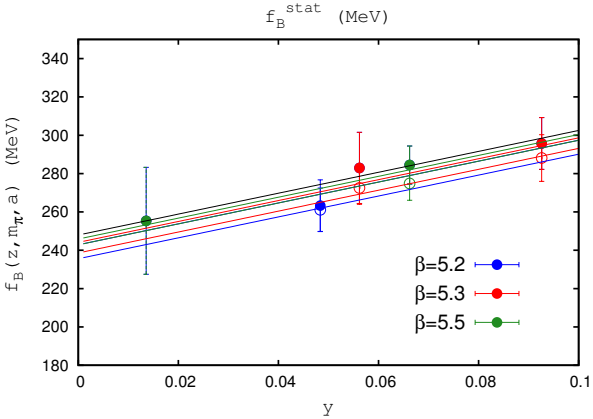
- In the static limit the decay constant is given by:

$$f_{B_0^*}\sqrt{\frac{m_{B_0^*}}{2}} = Z_V^{\rm HQET}\left(1 + b_V^{\rm stat} m_q\right) \times \left(f_{B_0^*}^{\rm stat} + c_V f_{B_0^*}^{\rm dV}\right)$$

[ALPHA, 13] [Blossier, 11]

# $f_{B_0^*}$ : Chiral and continuum extrapolations

$$f_{B_0^*}\sqrt{m_{B_0^*}/2} = \alpha + \beta\left(y - y^{\text{exp}}\right) + \gamma_{\text{HYP}_i}\left(\frac{a}{a_{\beta=5.3}}\right)^2 \quad , \quad y = \frac{m_{\text{PS}}^2}{8\pi^2 f_{\text{PS}}^2}$$



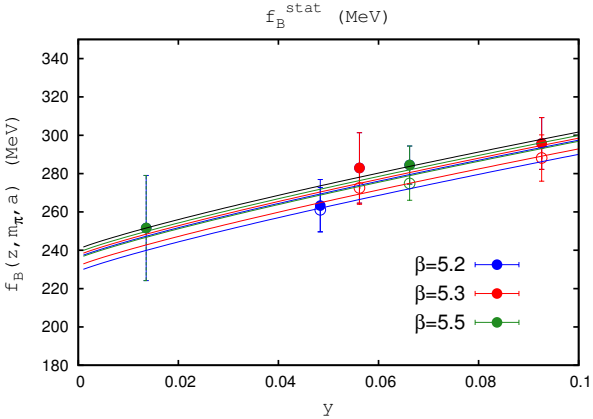

$f_{B_0^*} = 255(28)$ MeV

(preliminary)

# $f_{B_0^*}$ : Chiral and continuum extrapolations

$$f_{B_0^*}\sqrt{m_{B_0^*}/2} = \alpha + \beta\left(y - y^{\text{exp}}\right) + \gamma_{\text{HYP}_i}\left(\frac{a}{a_{\beta=5.3}}\right)^2 \quad , \quad y = \frac{m_{\text{PS}}^2}{8\pi^2 f_{\text{PS}}^2}$$

$$f_{B_0^*}\sqrt{m_{B_0^*}/2} = \alpha\left[1 - \frac{3}{4}\frac{1+3\tilde{g}^2}{2} y\log y - y^{\text{exp}}\log y^{\text{exp}}\right] + \beta\left(y - y^{\text{exp}}\right) + \gamma_{\text{HYP}_i}\left(\frac{a}{a_{\beta=5.3}}\right)^2$$




$$f_{B_0^*} = 255(28)\ \text{MeV}$$

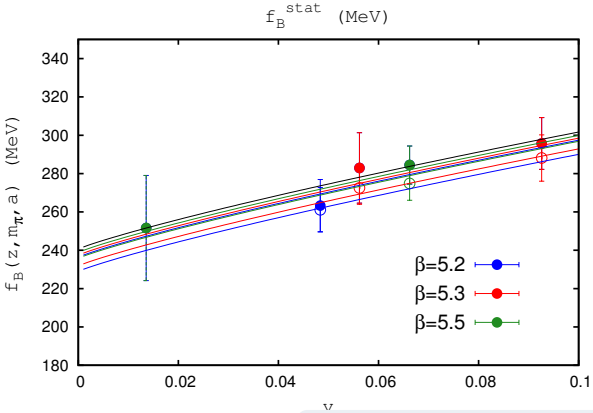
$$f_{B_0^*} = 252(27)\ \text{MeV}$$

(preliminary)

# $f_{B_0^*}$ : Chiral and continuum extrapolations

$$f_{B_0^*}\sqrt{m_{B_0^*}/2} = \alpha + \beta\left(y - y^{\text{exp}}\right) + \gamma_{\text{HYP}_i}\left(\frac{a}{a_{\beta=5.3}}\right)^2 \quad , \quad y = \frac{m_{\text{PS}}^2}{8\pi^2 f_{\text{PS}}^2}$$

$$f_{B_0^*}\sqrt{m_{B_0^*}/2} = \alpha\left[1 - \frac{3}{4}\frac{1+3\tilde{g}^2}{2} y\log y - y^{\text{exp}}\log y^{\text{exp}}\right] + \beta\left(y - y^{\text{exp}}\right) + \gamma_{\text{HYP}_i}\left(\frac{a}{a_{\beta=5.3}}\right)^2$$




$f_{B_0^*} = 255(28)$ MeV

$f_{B_0^*} = 252(27)$ MeV

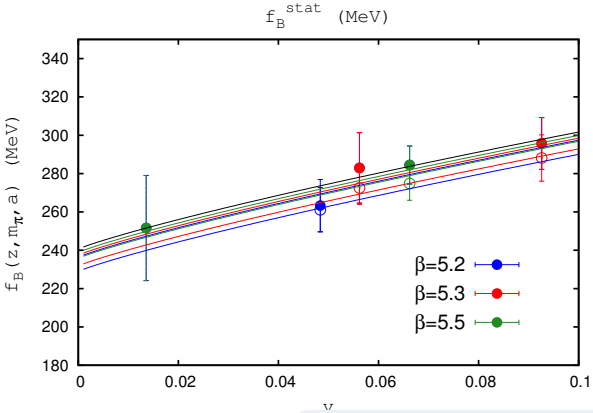
$f_{B_0^*} = 252(27)(3)$ MeV

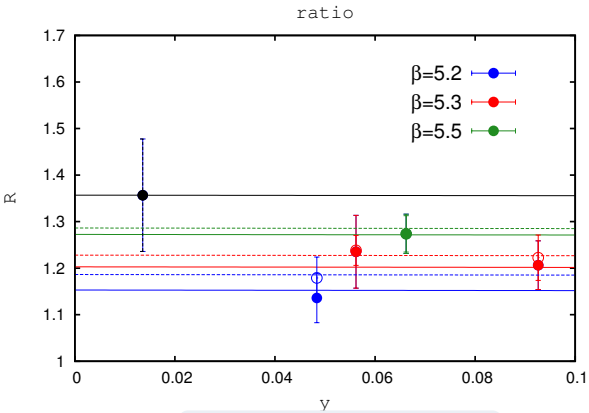
(preliminary)

# $f_{B_0^*}$ : Chiral and continuum extrapolations

$$f_{B_0^*}\sqrt{m_{B_0^*}/2} = \alpha + \beta\left(y - y^{\text{exp}}\right) + \gamma_{\text{HYP}_i}\left(\frac{a}{a_{\beta=5.3}}\right)^2 \quad , \quad y = \frac{m_{\text{PS}}^2}{8\pi^2 f_{\text{PS}}^2}$$

$$f_{B_0^*}\sqrt{m_{B_0^*}/2} = \alpha\left[1 - \frac{3}{4}\frac{1+3\tilde{g}^2}{2} y \log y - y^{\text{exp}} \log y^{\text{exp}}\right] + \beta\left(y - y^{\text{exp}}\right) + \gamma_{\text{HYP}_i}\left(\frac{a}{a_{\beta=5.3}}\right)^2$$




$f_{B_0^*} = 255(28)$ MeV

$f_{B_0^*} = 252(27)$ MeV

$$f_{B_0^*} = 252(27)(3) \text{ MeV}$$

(preliminary)

$$\frac{f_{B_0^*}}{f_B} = 1.36(12)$$

(preliminary)

# Conclusion

- We have computed the soft pion couplings $h$ and $\tilde{g}$ with $N_f = 2$ dynamical quarks

  $\rightarrow$ Static limit of HQET

  $\rightarrow$ Small dependence on the lattice spacing $\quad a \in [0.05 - 0.075]\ \mathrm{fm}$

  $\rightarrow$ Interpolating operators with covariant derivative are beneficial for three-point correlation functions

  $\rightarrow$ Our results are (preliminary):

$$h = 0.86(4)(2)$$
$$\tilde{g} = -0.115(15)(5)$$

- Scalar $B$ meson decay constant

  $\rightarrow$ Static limit of HQET

$$f_{B_0^*} = 252(27)(3)\ \mathrm{MeV} \quad , \quad \frac{f_{B_0^*}}{f_B} = 1.36(12)$$

# Systematic errors: $h$

- Excited states contribution

$$C_{B_0^*-B\pi}(t) = \sum_{nm}\sum_{t_1}\langle 0|\mathcal{O}^{B_0^*}|X_n\rangle x_{nm}\langle X_m|\mathcal{O}^{B\pi}|0\rangle e^{-E_n t_1} e^{-E_m(t-t_1)}$$

where $x_{nm} = \langle X_n|X_m\rangle$. Here, $X_1 = B_0^*$, $X_2 = B\pi$.

If $m_{B_0^*} \approx E_{B\pi} \equiv E$, the contribution of an exited state with a non-negligible overlap with $\mathcal{O}^{B_0^*}$ is:

$$\begin{aligned}
&\sum_{t_1}\langle 0|\mathcal{O}^{B_0^*}|X_3\rangle x_{32}\langle B\pi|\mathcal{O}^{B\pi}|0\rangle e^{-E_3 t_1} e^{-E(t-t_1)} \\
&\qquad = \langle 0|\mathcal{O}^{B_0^*}|X_3\rangle x_{32}\langle B\pi|\mathcal{O}^{B\pi}|0\rangle\, e^{-Et}\sum_{t_1} e^{(E_3-E)t_1} \\
&\qquad = \underbrace{t\langle 0|\mathcal{O}^{B_0^*}|B_0^*\rangle x\langle B\pi|\mathcal{O}^{B\pi}|0\rangle\, e^{-Et}}_{\text{ground state contribution}} \times \underbrace{\frac{1}{t}\frac{\langle 0|\mathcal{O}^{B_0^*}|X_3\rangle}{\langle 0|\mathcal{O}^{B_0^*}|B_0^*\rangle}\frac{x_{32}}{x}\sum_{t_1} e^{(E_3-E)t_1}}_{},
\end{aligned}$$

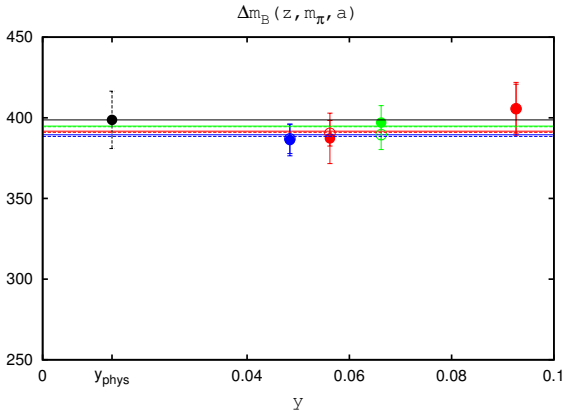
- Corrections from $\Delta = m_{B_0^*} - E_{B\pi} \neq 0$

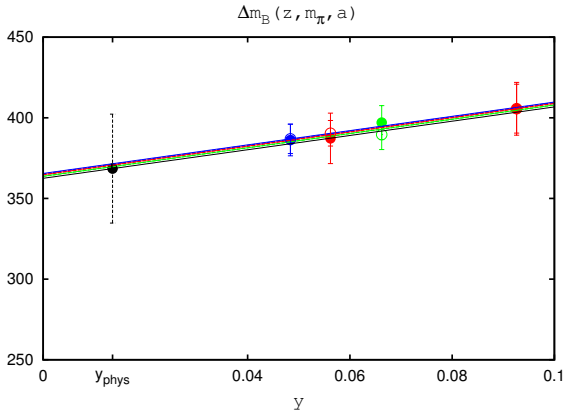
$$t \longrightarrow \frac{2}{\Delta}\sinh\left(\frac{\Delta}{2}t\right) = t + \frac{\Delta^2 t^3}{24} + \mathcal{O}(\Delta^4)$$

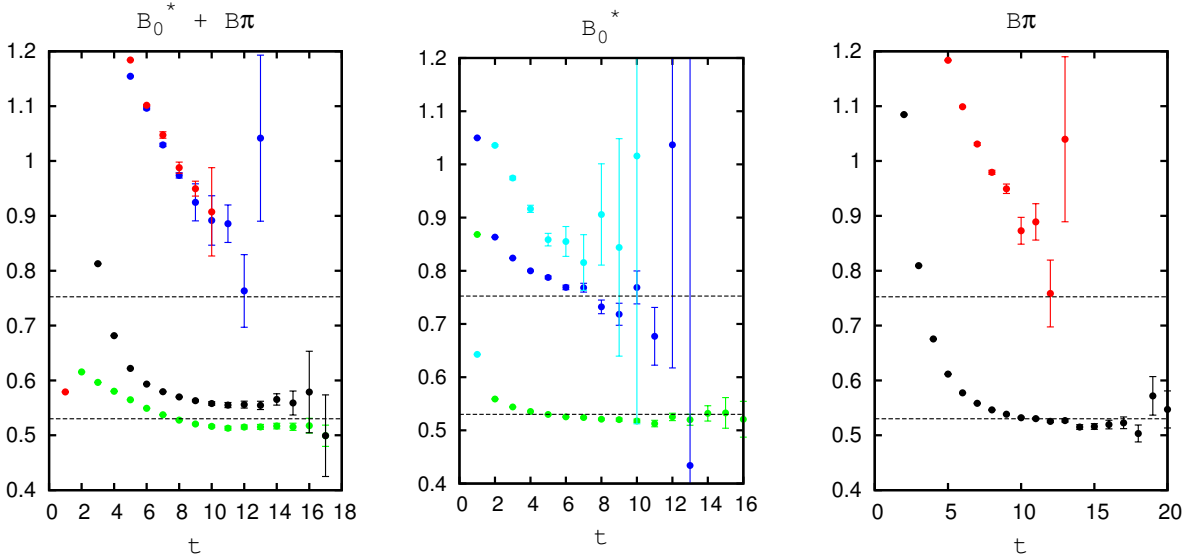
# Mass of the scalar $B_0^*$ meson

$$a\Delta m(a, m_\pi) = E^s_{\text{stat}}(a, m_\pi) - E^{ps}_{\text{stat}}(a, m_\pi)$$

$$E_n^{\text{eff}}(t, t_0) = a^{-1} \log \frac{\lambda_n^{\text{stat}}(t, t_0)}{\lambda_n^{\text{stat}}(t + a, t_0)}$$




$$\Delta m_{B_0^*} = 385(21)_{\text{stat}}(30)_{\text{syst}}$$

# Scalar $B_0^*$ meson