Lattice study of $\pi\pi$ scattering

using Nf=2+1 Wilson improved quarks with masses down to their physical values

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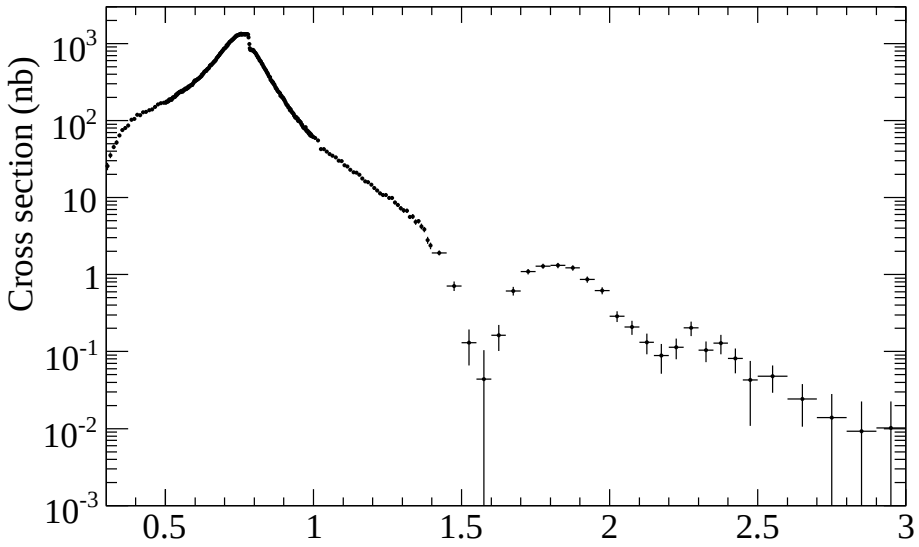
Introduction

ρ

- Resonance in the $\mathrm{L} = 1$, $\mathrm{I} = 1$, $\pi\pi$ scattering channel
- $M_\rho = 775.5$ MeV
- $\Gamma_\rho = 150$ MeV



J.P. Lees et al.
(BABAR Collaboration)
arXiv:1205.2228

Measured cross-section for $e^+e^- \to \pi^+\pi^-$ over the full mass range.

Scattering in finite volume

Lüscher's method

In **finite** volume, $\pi\pi$ states have **discrete** energy levels E_L.

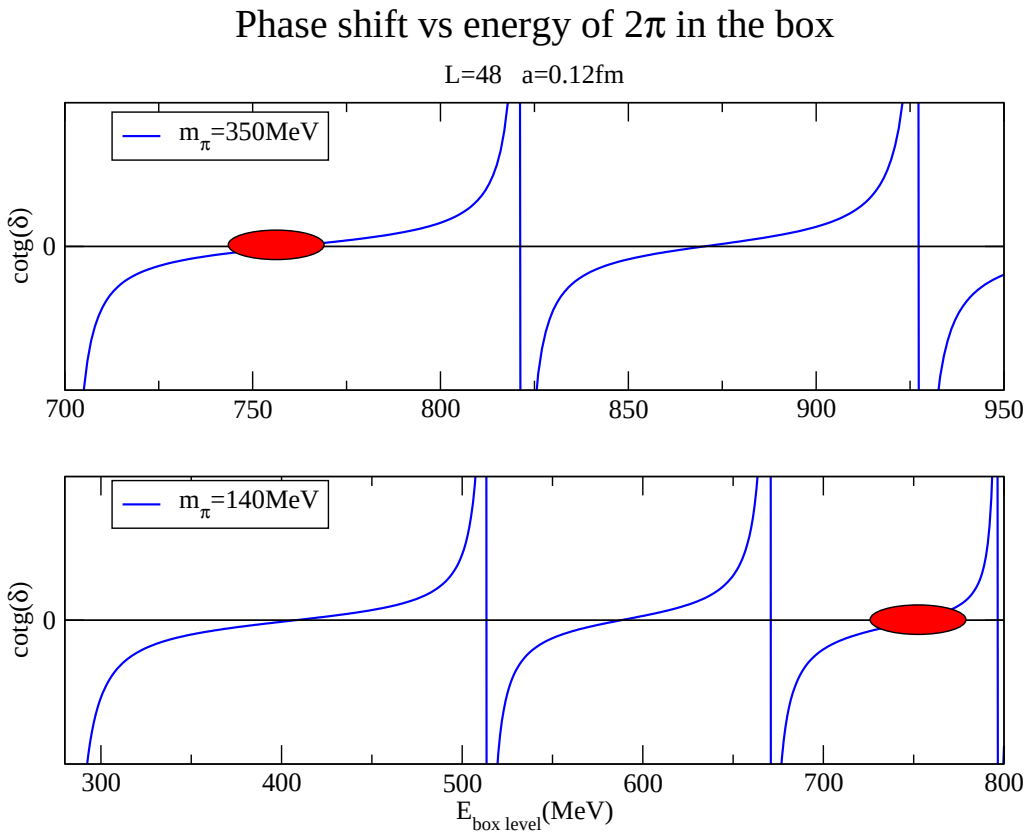
Lüscher's method

Energy levels in finite volume $\longleftrightarrow$ Scattering phase-shift in infinite volume

$$\cot\delta(q) = \frac{\mathcal{Z}_{00}(1;q^2)}{q\,\pi^{3/2}}$$

$$q = \frac{L}{2\pi}\frac{1}{2}\sqrt{E^2 - 2m_\pi^2}$$

$\cot\delta\left(E\right)$



$\Longrightarrow$ strong coupling between ρ and **excited** 2π states at low m_π

Excited states extraction

- Compute on the lattice the matrix of correlators

$$C_{ij}(t) = \langle \mathcal{O}_i(t)\, \bar{\mathcal{O}}_j(0) \rangle$$

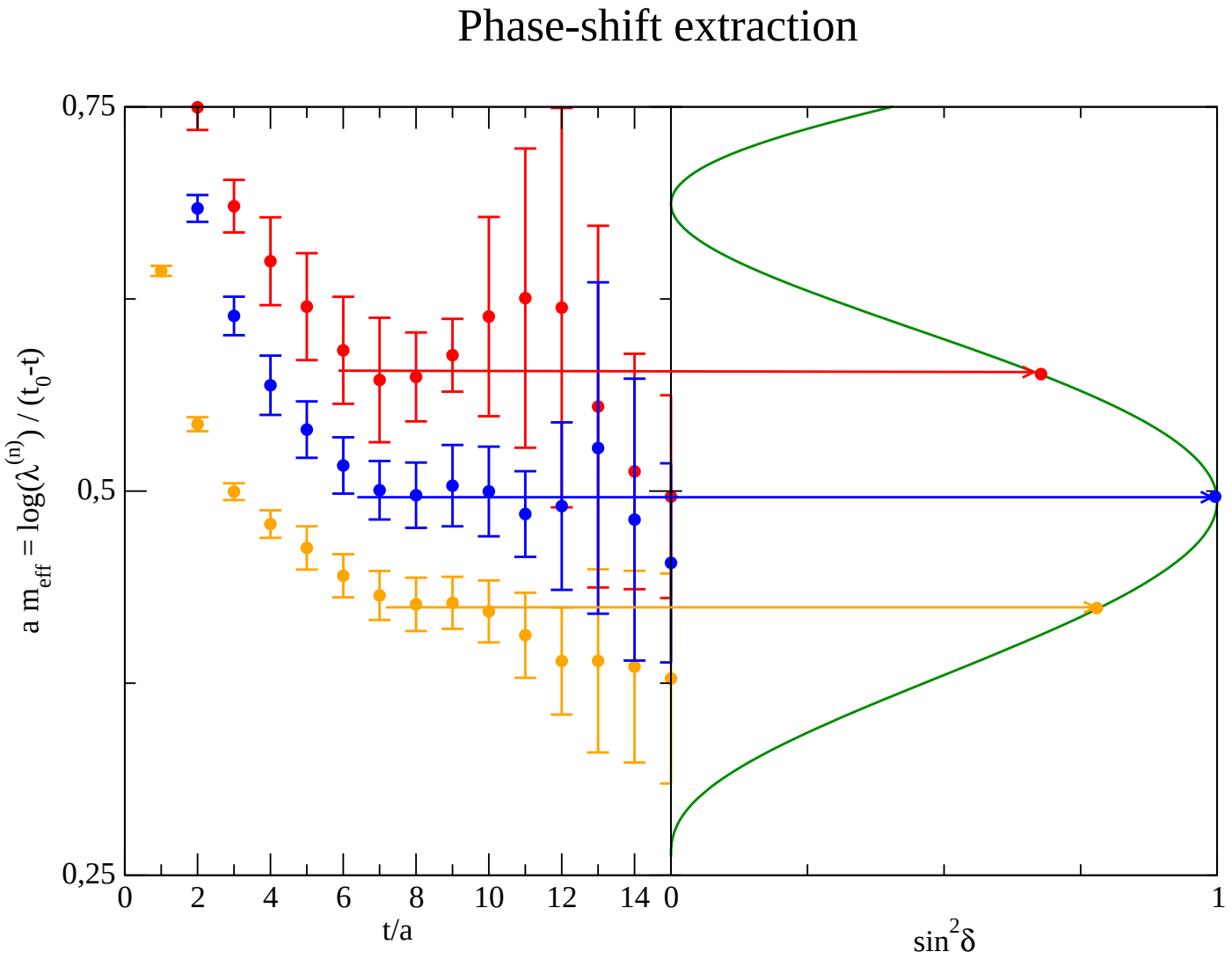
with $\{\mathcal{O}_i\}$ a set of independent appropriate interpolators.

- Solve the generalized eigenvalue problem

$$C(t)\,\psi = \lambda(t,\, t_0)\; C(t_0)\,\psi$$

- Then $\lambda^{(n)}(t,\, t_0) \sim e^{-E_n(t-t_0)}$ at large t determines the energies

Phase shift


Phase-shift extraction
0,75
0,5
0,25
0
2
4
6
8
10
12
14
t/a
a m_eff = log(λ^(n)) / (t_0-t)
0
1
sin²δ

In practice

Lattice QCD setup

Lattice actions

- Gauge action: tree-level $\mathrm{O}\left(a^2\right)$-improved Symanzik action
- Fermion action: tree-level $\mathrm{O}\left(a\right)$-improved Wilson fermions, $N_f = 2 + 1$, 2 steps of HEX gauge links smearing

Gauge configurations

5 independent gauge ensembles from the **BMW-c**:

- m_π from 135 MeV to 300 MeV
- 3 volumes from $(3.7\ \mathrm{fm})^3$ to $(5.6\ \mathrm{fm})^3$
- 2 lattice spacings (0.12 fm and 0.08 fm)
- $m_\pi L \gtrsim 4$

Interpolating operators

Work in the **center-of-mass** frame.
2 to 5 independent operators with ρ quantum numbers:

- $\mathcal{O}_\rho = \bar{u}\,\gamma_i\,u - \bar{d}\,\gamma_i\,d$
- $\mathcal{O}_{\pi\pi}(\vec{p})$: back-to-back π with up to 4 lattice momenta

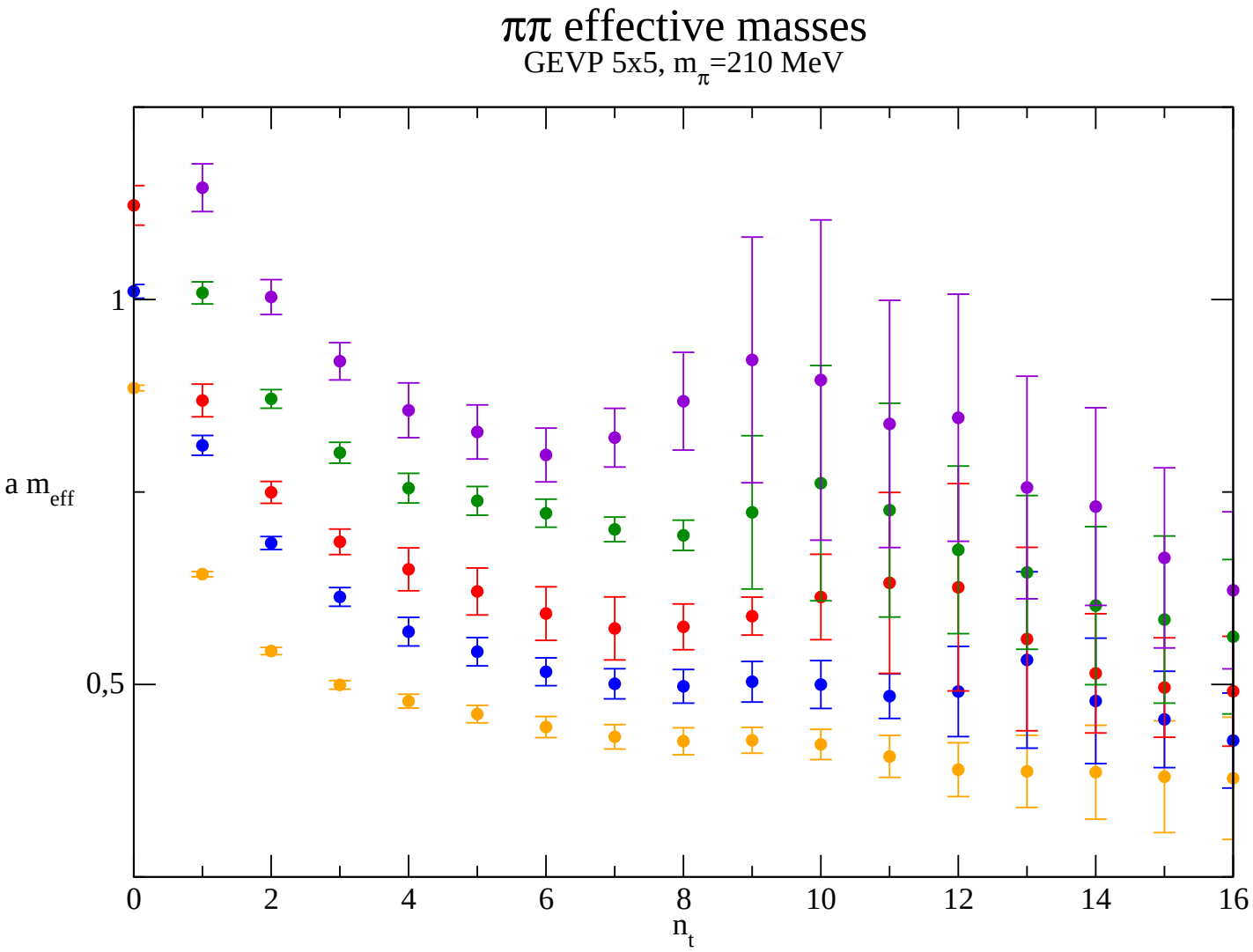
Inversions and contractions

- stochastic sources
- generalized propagators

$\rightarrow$ high cpu cost (between 2M and 10M core-h per ensemble on BG/Q)

Results

Plateaus


ππ effective masses
GEVP 5x5, m_π=210 MeV
a m_eff
1
0,5
0
2
4
6
8
10
12
14
16
n_t

Parametrization of the ρ resonance

Breit-Wigner resonance

Assume **narrow** resonance dominating $\pi\pi$ scattering. Then

$$\sin^2\delta \simeq \frac{\Gamma_\rho^2}{4(E - M_\rho^2)^2 + \Gamma_\rho^2}$$

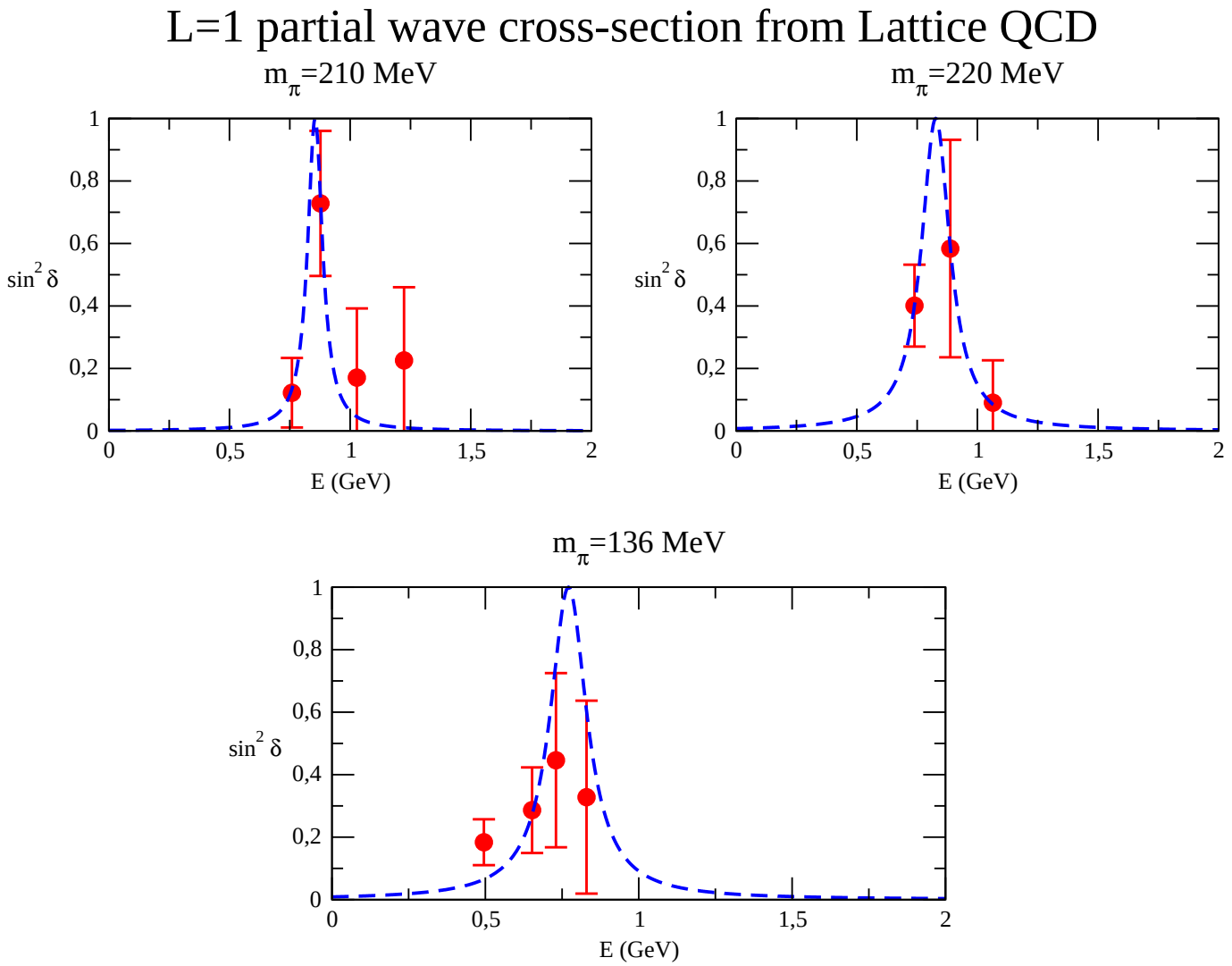
Effective interaction Lagrangian

Provides a convenient parametrization of the resonance

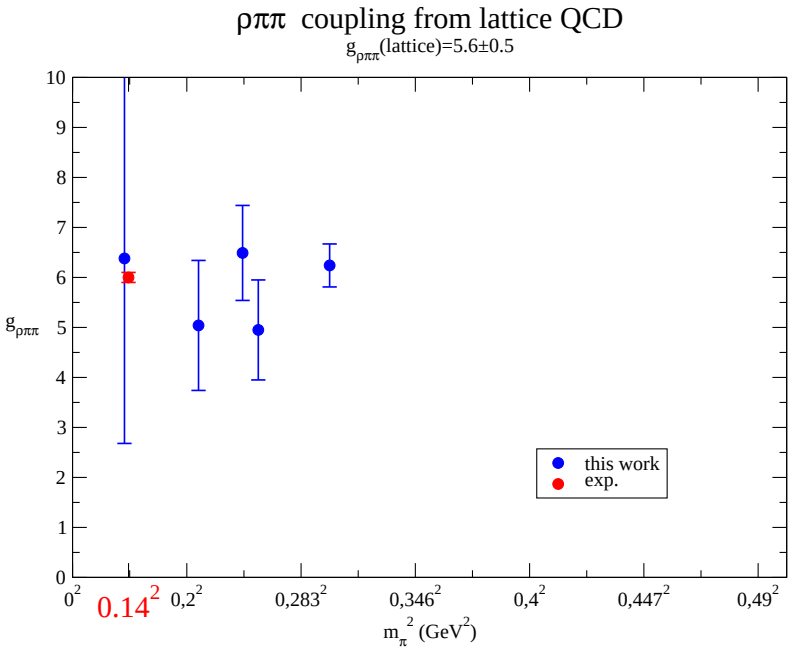
$$\mathcal{L}_{int} = g_{\rho\pi\pi}\ \epsilon_{abc}\ \rho_\mu^a\ \pi^b \partial^\mu \pi^c$$

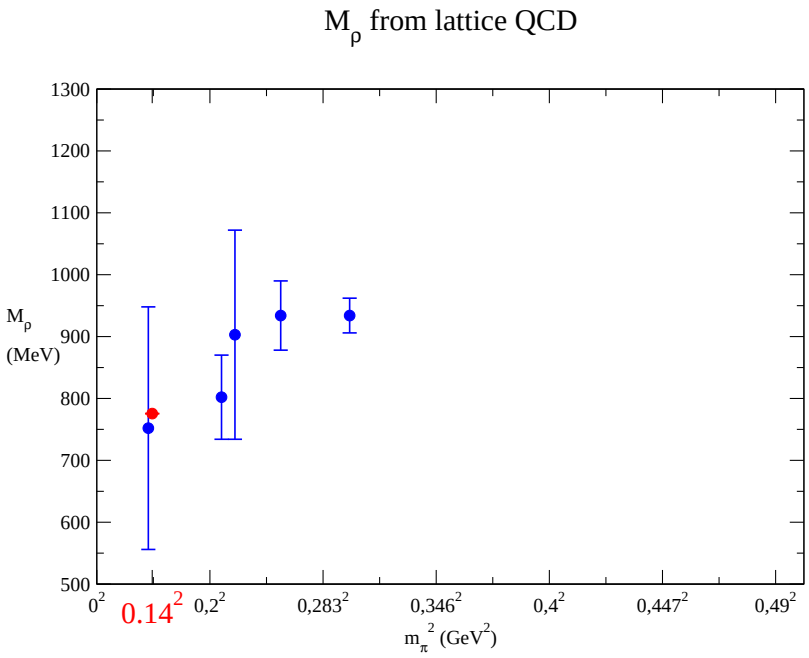
$$\Longrightarrow\ \sin^2\delta\ \left(g_{\rho\pi\pi},\ M_\rho\right)$$

Resonance on the lattice



$g_{\rho\pi\pi}$ and M_ρ





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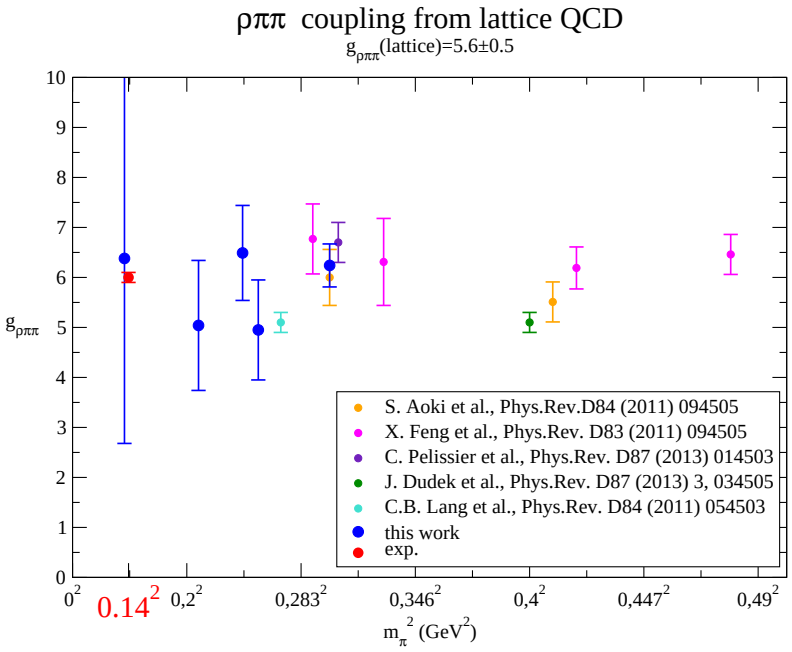
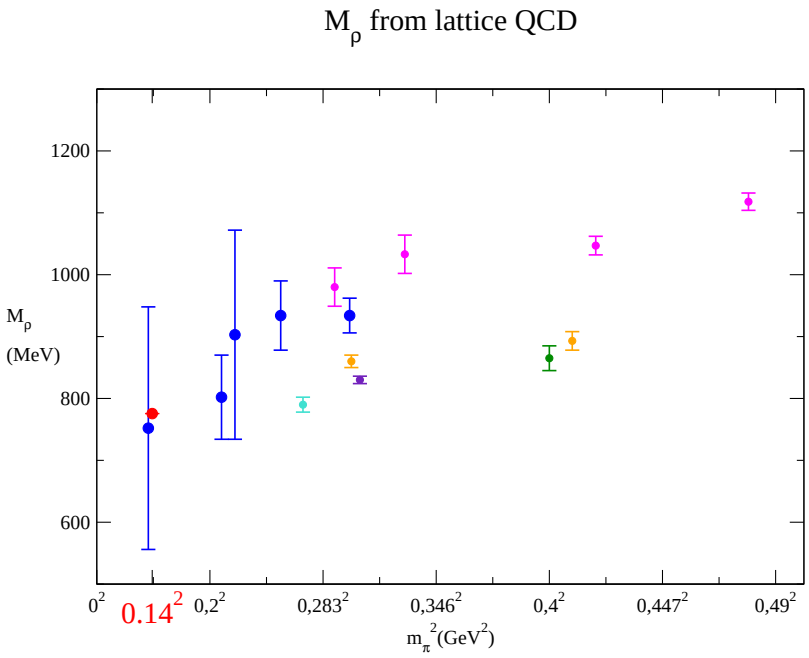




Conclusion

Conclusion

Results

- first lattice computation of the ρ resonance parameters at the physical π mass
- good agreement with experiment
- confirmation of the **very weak** dependence of $g_{\rho\pi\pi}$ on the pion mass

Current investigations

- continuum extrapolation
- analysis of the pion mass dependence of M_ρ
- systematic error analysis
- improve statistics ? (computationally expensive)

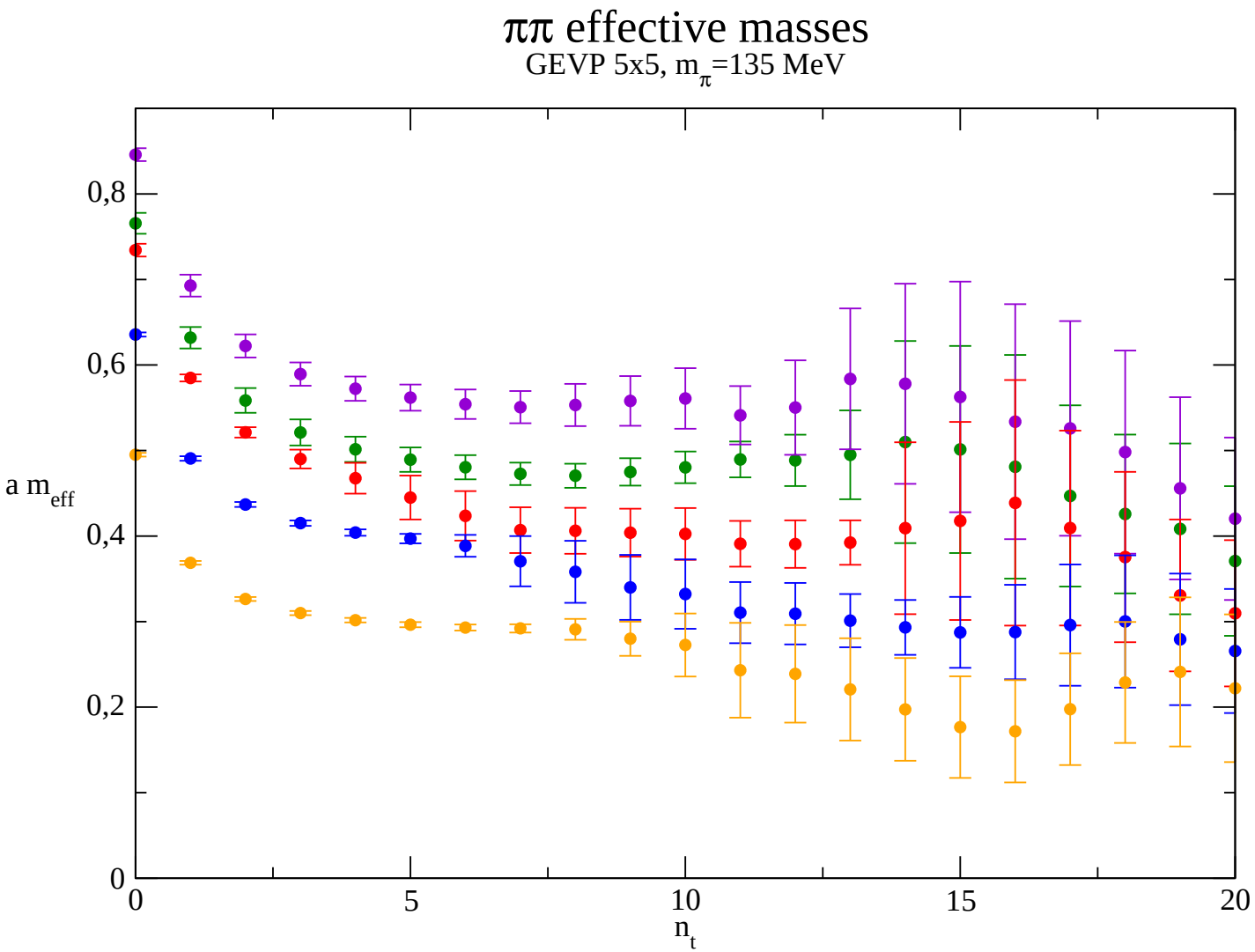
Thank you

We thank GENCI-IDRIS and FZ Juelich for access to their super-computing resources.

Gauge ensembles

β	L^3xT	a[fm]	m_π[MeV]	# confs
3.31	48^3x48	0.12	135	483
3.31	32^3x48	0.12	210	225
3.31	32^3x48	0.12	300	451
3.61	48^3x48	0.08	220	200
3.61	48^3x48	0.08	255	210

Plateaus


ππ effective masses
GEVP 5x5, m_π=135 MeV
a m_eff
0,8
0,6
0,4
0,2
0
0
5
10
15
20
n_t