Excited meson radiative transition matrix elements using distillation

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with Jo Dudek and Robert Edwards

arXiv:1501.07457

Framing The Problem

what do we want?

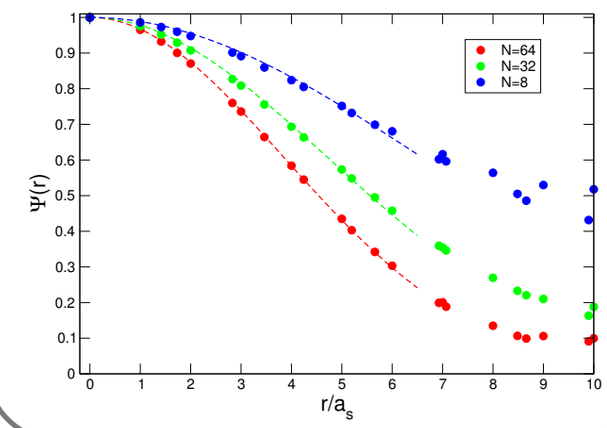
what do we have?

Vector Current Matrix Elements

$$\langle n; p'\lambda' | j^\mu | m; p\lambda \rangle$$

$$j^\mu = \sum_q e_q \bar{\psi}_q \gamma^\mu \psi_q$$

Distillation



HSC, Phys.Rev. D80 (2009) 054506

Optimized Operators

$$\Omega_n^\dagger \sim v_i \mathcal{O}_i^\dagger$$

Variational Method

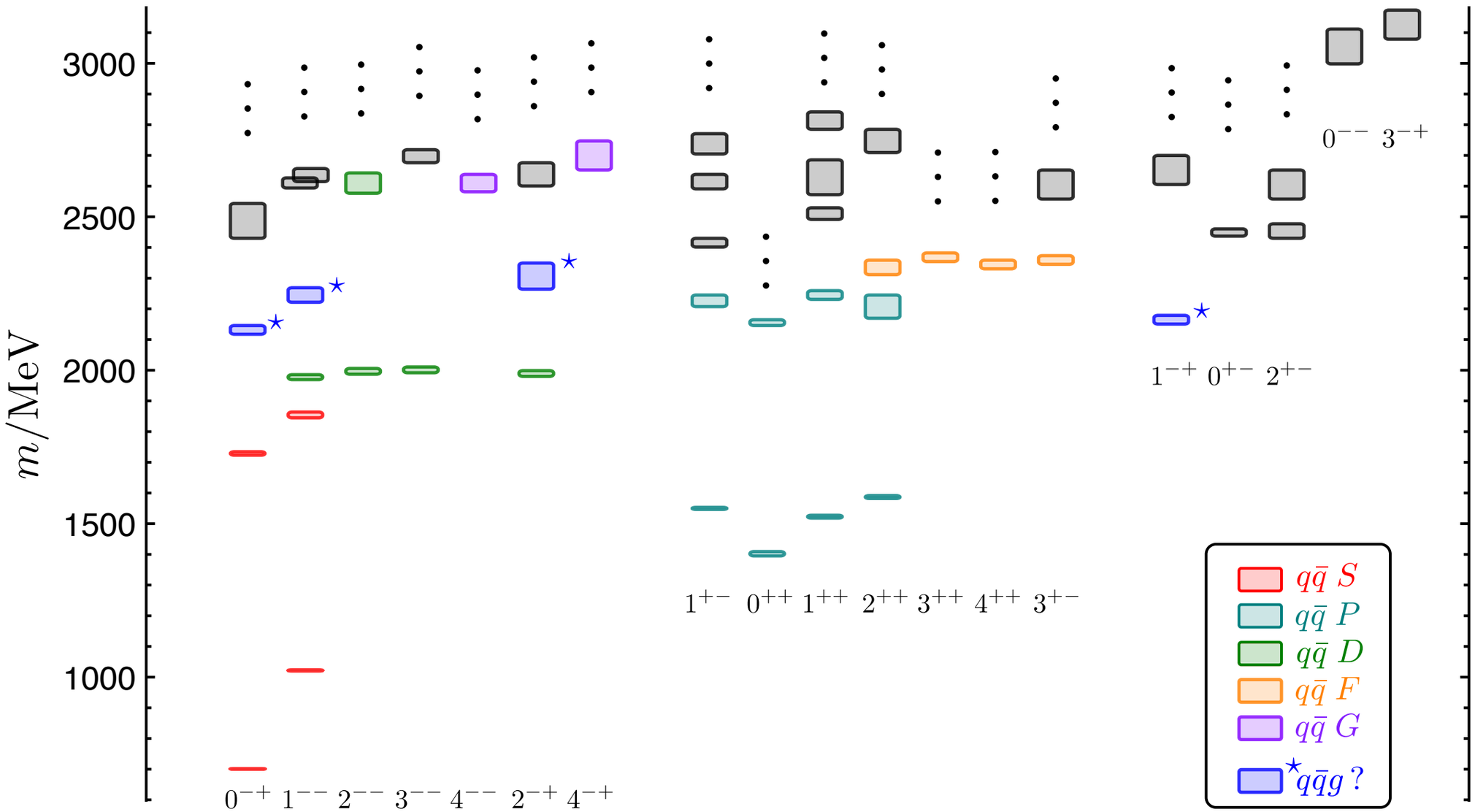
$$C(t)V = C(t_0)V\Lambda$$

Large Basis of Interpolators

$$\mathcal{O}^{J\lambda} \sim \bar{\psi}\Gamma_a \overleftrightarrow{D}_b \overleftrightarrow{D}_c \cdots \psi \times \mathrm{CG}(a, b, c \to J\lambda)$$

spectroscopy

Spectroscopy



$m_\pi \sim 700\mathrm{MeV}$

JJDudek, Phys.Rev. D84 (2011) 074023

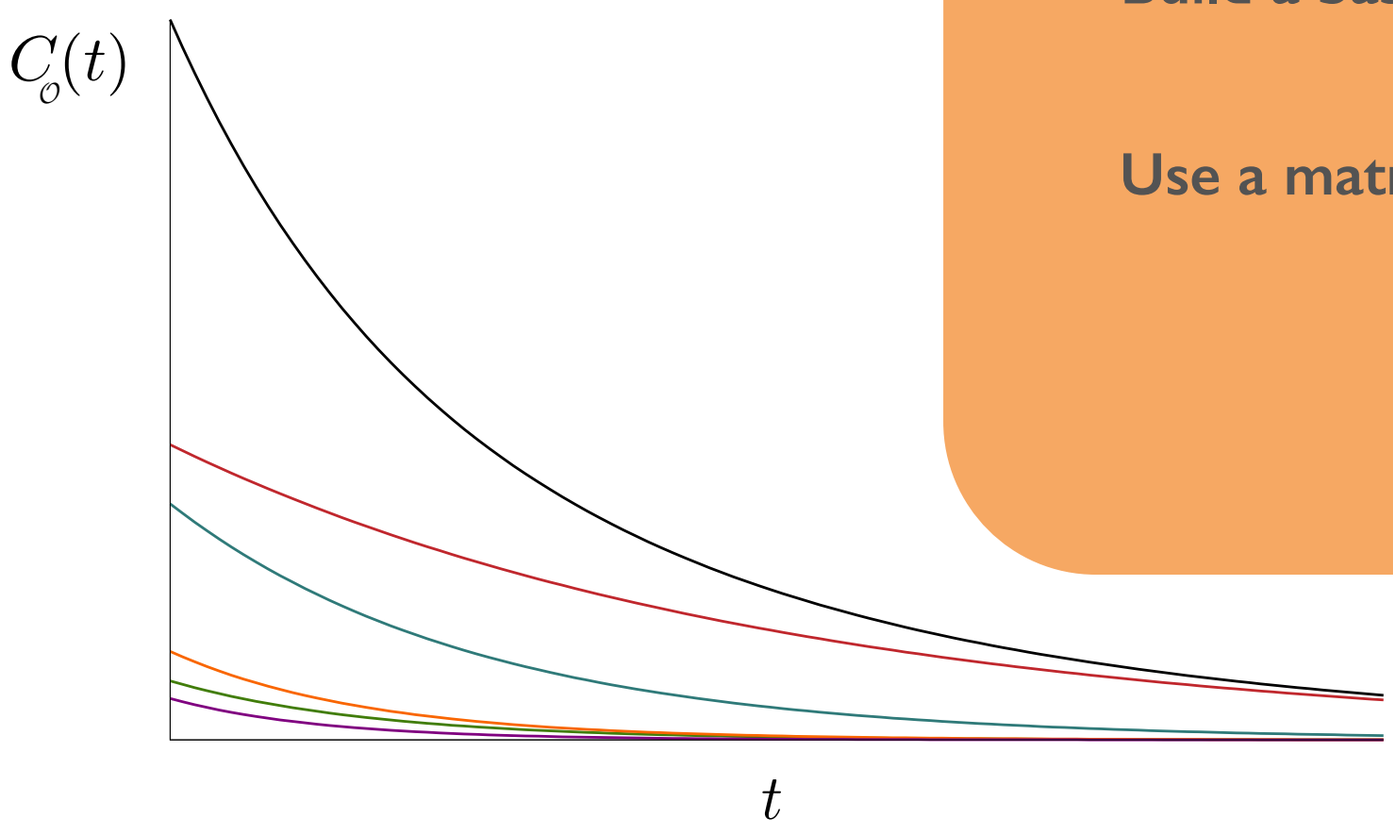
Spectroscopy

Two Point Function

$$C_{\mathcal{O}}(t) = \langle 0|\mathcal{O}(t)\mathcal{O}^\dagger(0)|0\rangle$$
$$= \sum_{\mathfrak{n}} \frac{e^{-E_{\mathfrak{n}} t}}{2E_{\mathfrak{n}}} |\langle 0|\mathcal{O}(0)|\mathfrak{n}\rangle|^2$$

Isolate a single state

What if $|\langle 0|\mathcal{O}(0)|\mathfrak{n}\rangle|^2$ is small (zero) ?



Solution:

Build a basis of operators

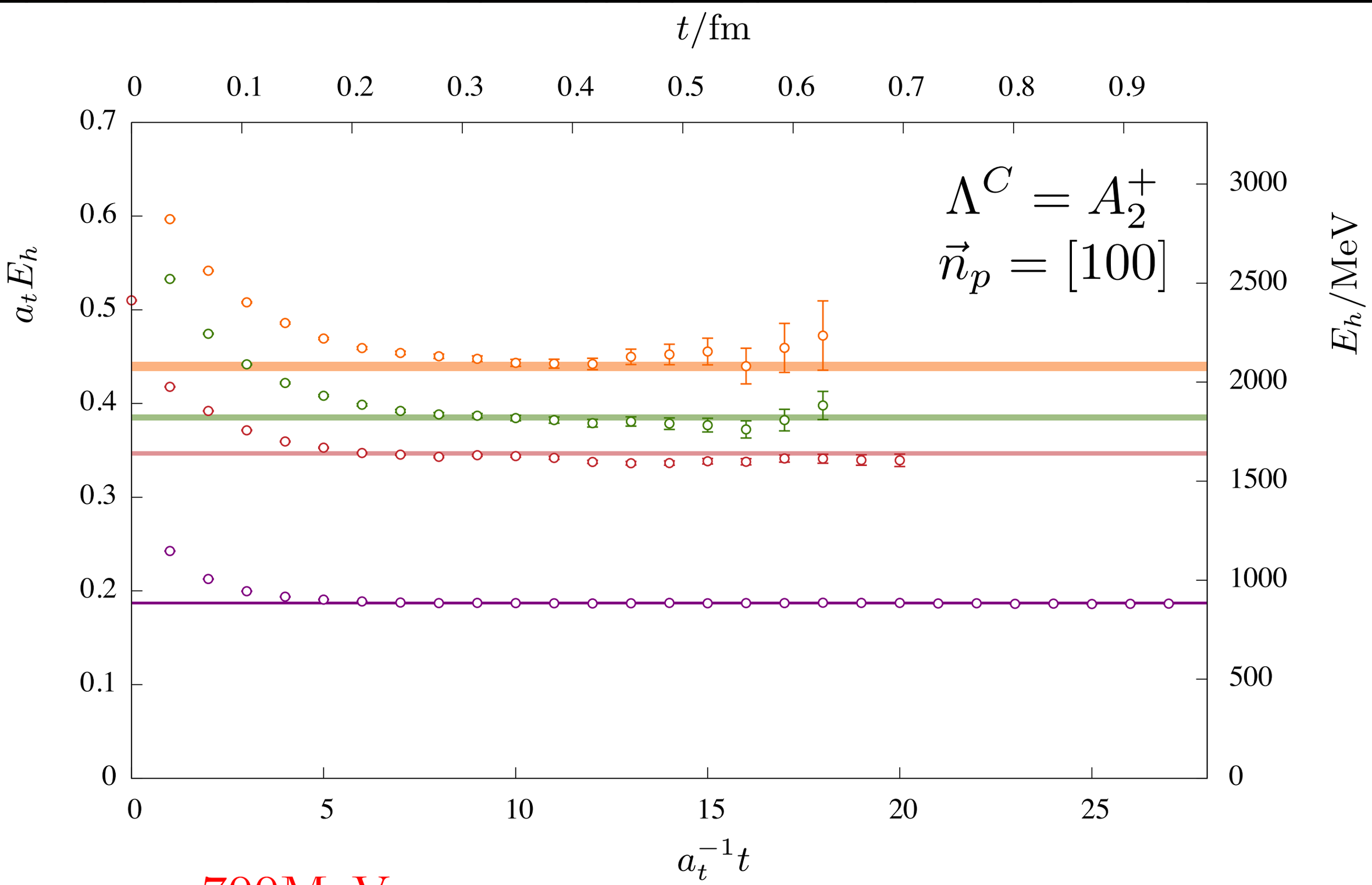
$$\mathcal{O}^{J\lambda} \sim \bar{\psi}\Gamma_a \overleftrightarrow{D}_b \overleftrightarrow{D}_c \cdots \psi \times \mathrm{CG}(a,b,c \to J\lambda)$$

Use a matrix of correlators

$$C(t)v^{(\mathfrak{n})} = \lambda_{\mathfrak{n}}(t) C(t_0) v^{(\mathfrak{n})}$$

Diagonalize the matrix

Projected Ops



$m_\pi \sim 700\mathrm{MeV}$

Matrix Elements

Matrix elements 'hide' in 3-pt functions

$$j^\mu = \sum_q e_q \bar{\psi}_q \gamma^\mu \psi_q$$

$$C^\mu_{\mathfrak{n}\mathfrak{m}} = \langle 0 | \Omega_{\mathfrak{n}}(t) j^\mu(t_\gamma) \Omega^\dagger_{\mathfrak{m}}(0) | 0 \rangle$$

$$= \sum_{\mathfrak{n}'\mathfrak{m}'} \langle \mathfrak{n}' | j^\mu | \mathfrak{m}' \rangle e^{-E_{\mathfrak{n}'}(t-t_\gamma)} e^{-E_{\mathfrak{m}'} t_\gamma}$$

sum over everything

need to isolate

$$\times \frac{\langle 0 | \Omega_{\mathfrak{n}}(0) | \mathfrak{n}' \rangle}{2E_{\mathfrak{n}'}} \frac{\langle \mathfrak{m}' | \Omega^\dagger_{\mathfrak{m}}(0) | 0 \rangle}{2E_{\mathfrak{m}'}}$$

Matrix Elements

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$$C^\mu_{\mathbf{nm}} = \langle 0 | \Omega_{\mathbf{n}}(t) j^\mu(t_\gamma) \Omega^\dagger_{\mathbf{m}}(0) | 0 \rangle$$

$$= \sum_{\mathbf{n}'\mathbf{m}'} \langle \mathbf{n}' | j^\mu | \mathbf{m}' \rangle e^{-E_{\mathbf{n}'}(t-t_\gamma)} e^{-E_{\mathbf{m}'} t_\gamma}$$

$$\times \frac{\langle 0 | \Omega_{\mathbf{n}}(0) | \mathbf{n}' \rangle}{2E_{\mathbf{n}'}} \frac{\langle \mathbf{m}' | \Omega^\dagger_{\mathbf{m}}(0) | 0 \rangle}{2E_{\mathbf{m}'}}$$

sum over everything

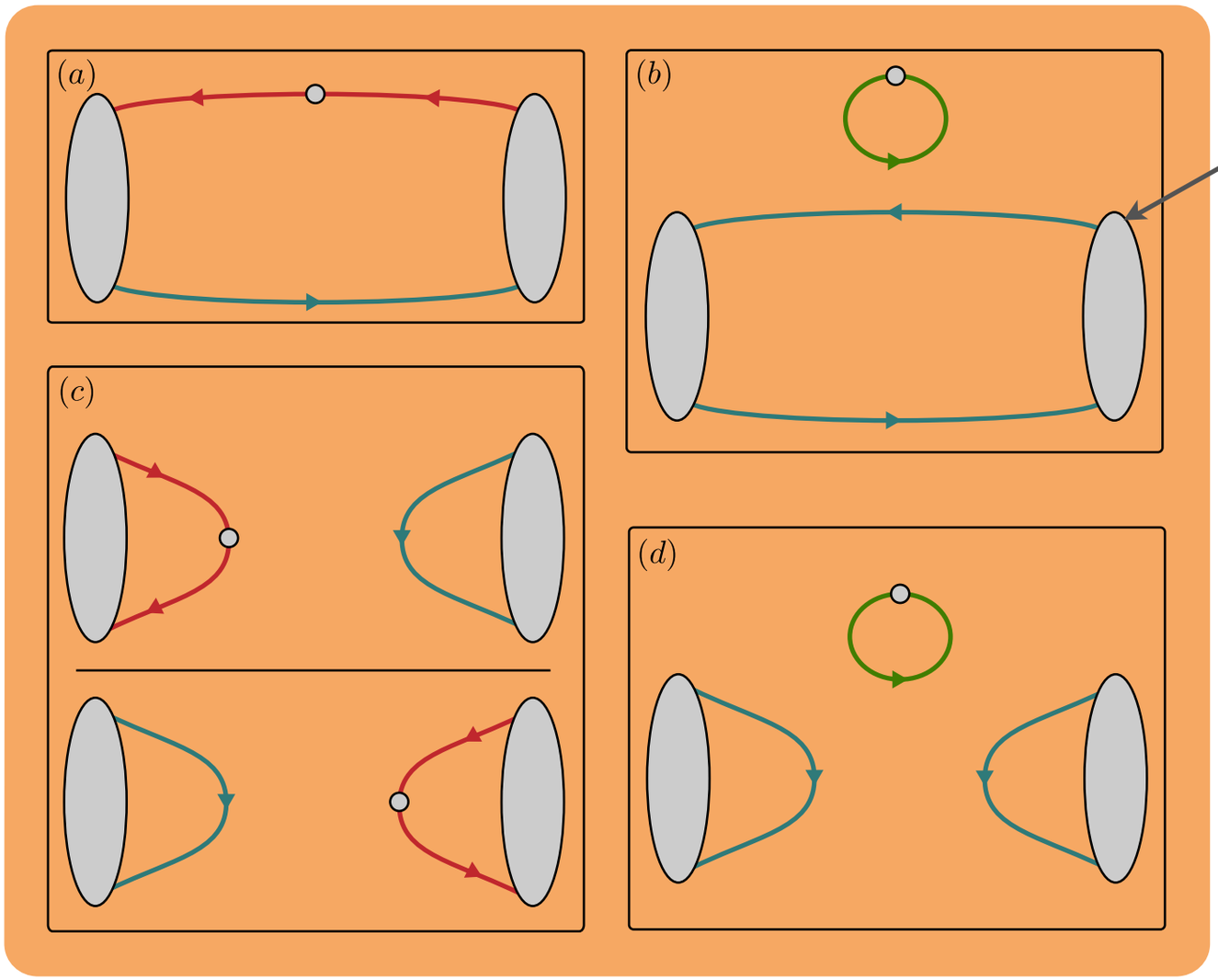
need to isolate

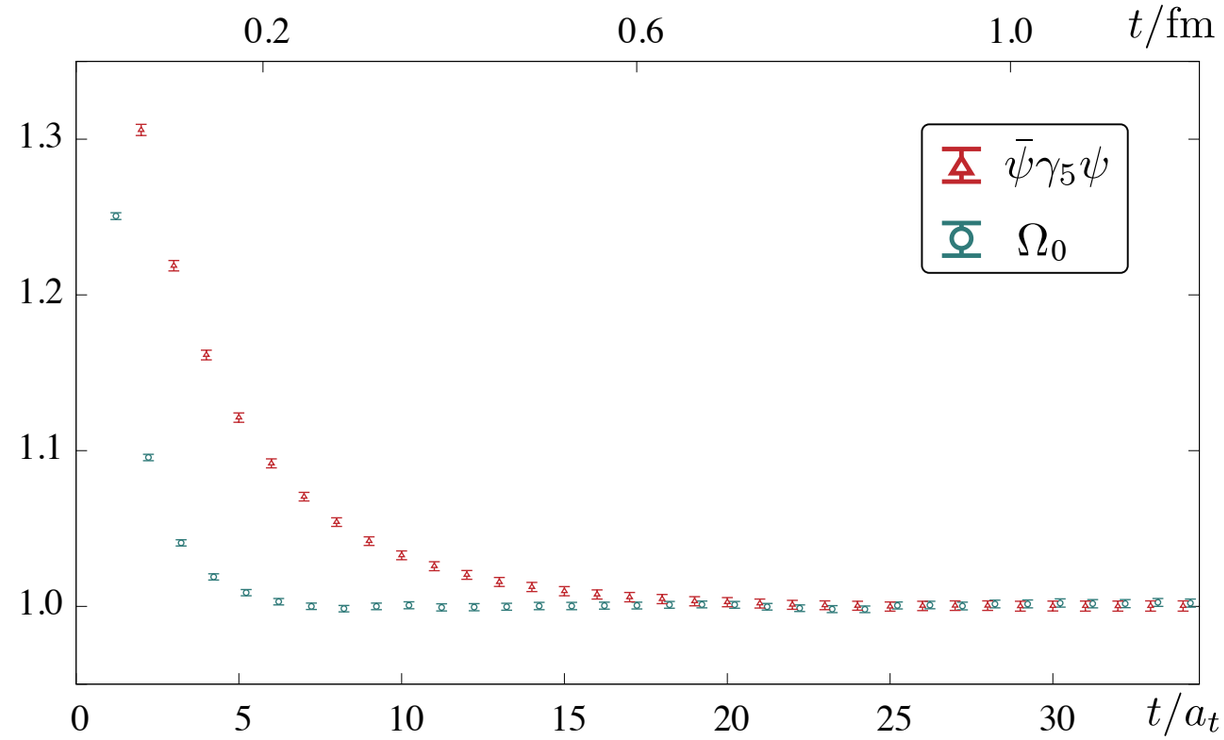
"diagonalize" the sum?

3-pt Functions

$$C^{\mu}_{n,m}(\delta t, t_\gamma) = \langle 0|\Omega_n(\delta t) j^{\mu}(t_\gamma)\Omega^{\dagger}_m(0)|0\rangle$$

Optimized Operators





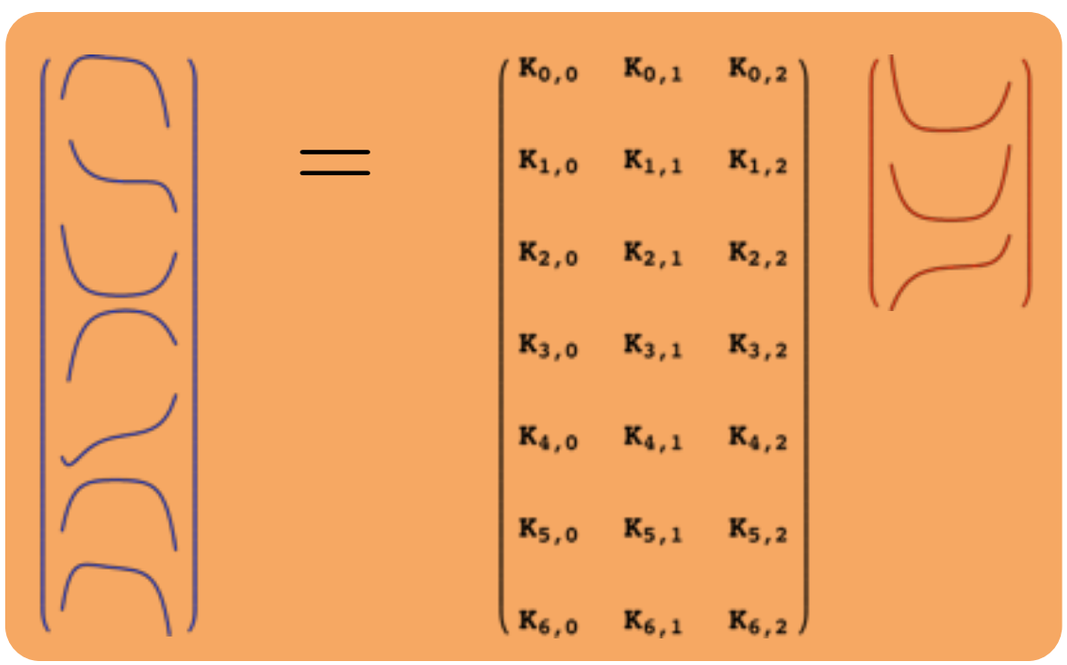
The Linear System

Optimized Operators
'relax' quickly

pollution term

$$C^{\mu}_{n,m}(\delta t, t_\gamma) \sim \frac{e^{-E_n(\delta t - t_\gamma)}}{2E_n}\frac{e^{-E_m t_\gamma}}{2E_m}\langle n|j^\mu|m\rangle + f(\delta t, t_\gamma)$$

Matrix Element Decompositions



Kinematic Factor

Form Factor

$$\langle n|j^\mu|m\rangle = \sum_k K^\mu_k(n,m) F_k(Q^2)$$

<u>Some Examples</u>

$$\langle \pi, p'|j^\mu|\pi, p\rangle = (p' + p)^\mu F_\pi(Q^2)$$

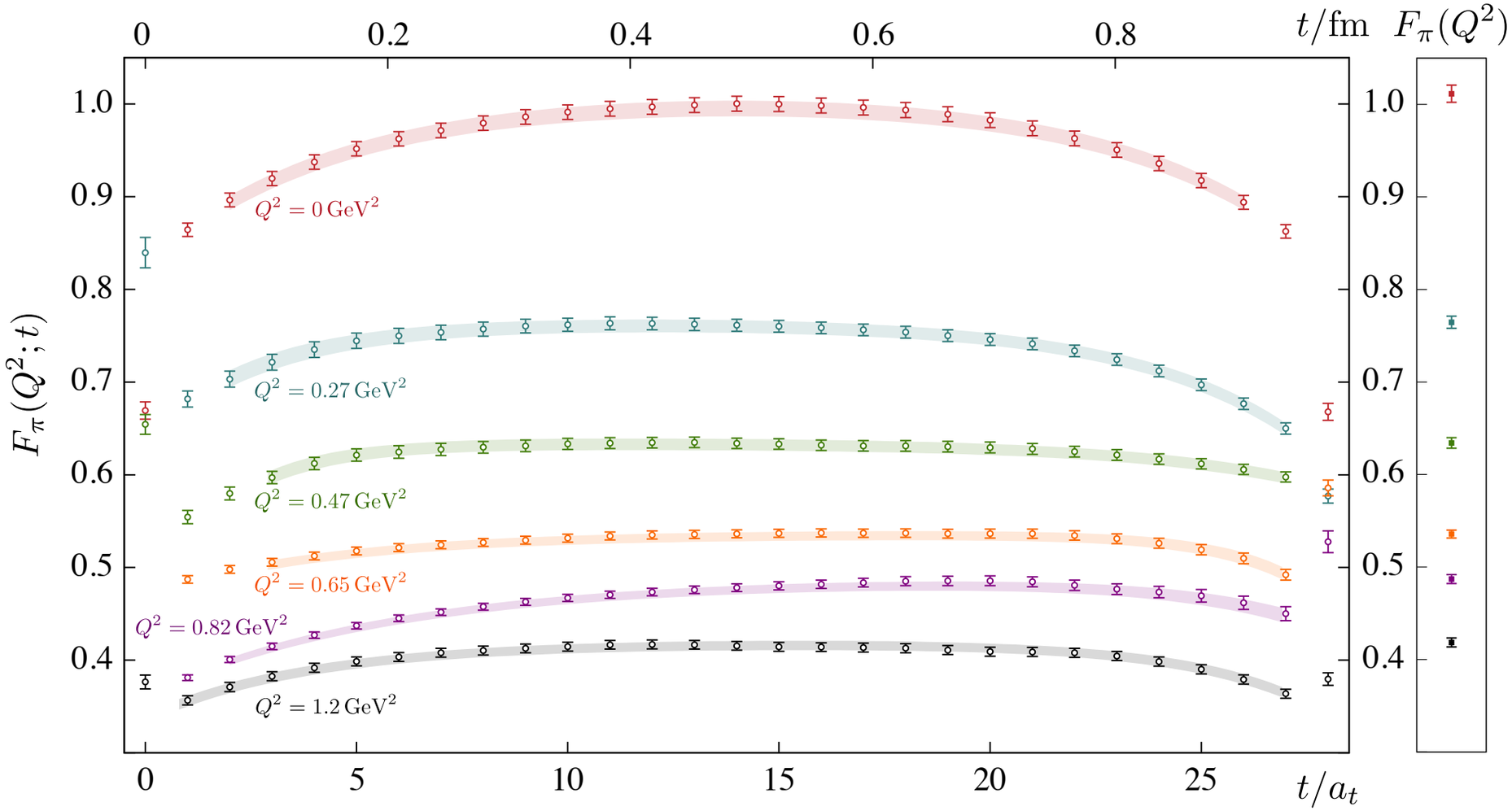
$$\langle \pi, p'|j^\mu|\rho, p\lambda\rangle = \frac{2}{m_\rho + m_\pi}\epsilon^{\mu\nu\sigma\eta} p'_\nu p_\sigma \varepsilon_\eta(p,\lambda) F_{\rho\pi}(Q^2)$$

Signal Isolation

$$C^{\mu}_{\mathfrak{n}\mathfrak{m}} = \langle 0|\Omega_{\mathfrak{n}}(t) j^{\mu}(t_\gamma)\Omega^{\dagger}_{\mathfrak{m}}(0)|0\rangle$$
$$= \sum_{\mathfrak{n}'\mathfrak{m}'} \langle \mathfrak{n}'|j^{\mu}|\mathfrak{m}'\rangle e^{-E_{\mathfrak{n}'}(t-t_\gamma)} e^{-E_{\mathfrak{m}'}t_\gamma}$$

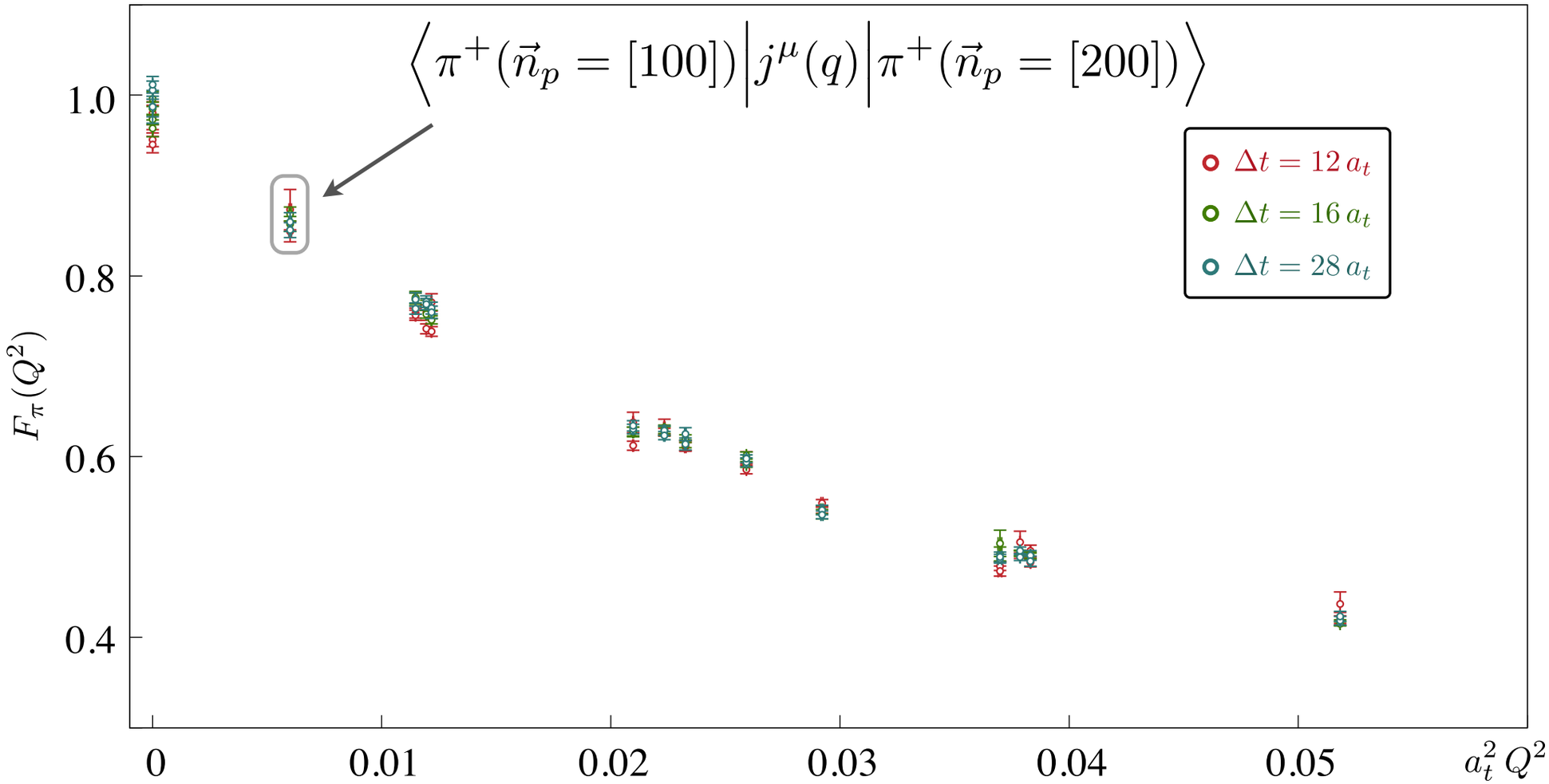
$$\times \frac{\langle 0|\Omega_{\mathfrak{n}}(0)|\mathfrak{n}'\rangle}{2E_{\mathfrak{n}'}} \frac{\langle \mathfrak{m}'|\Omega^{\dagger}_{\mathfrak{m}}(0)|0\rangle}{2E_{\mathfrak{m}'}}$$

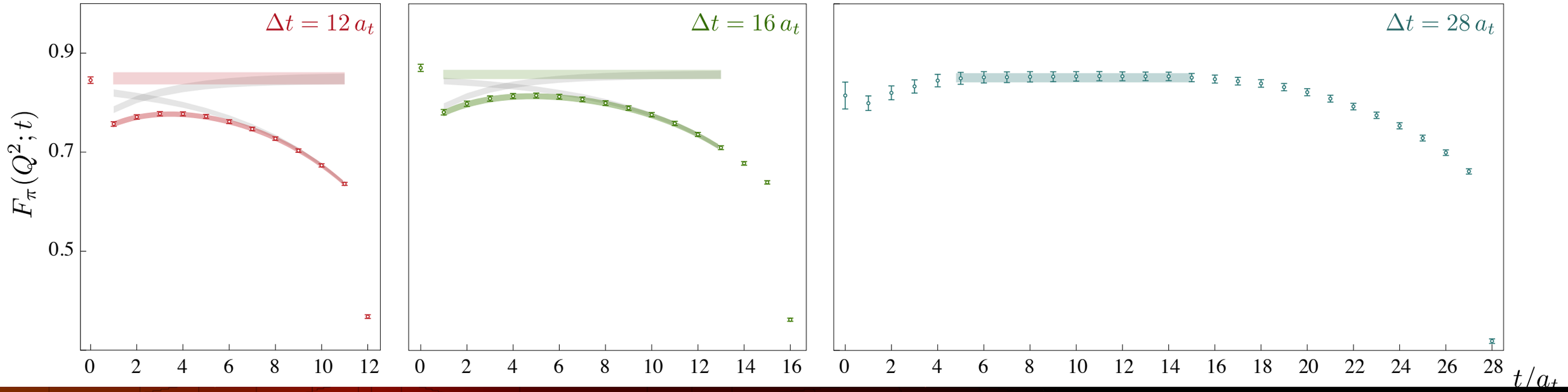
How well does it work?



$$\Omega^{\dagger}_{\pi} \sim \sum_i v_i^{(\pi)} \mathcal{O}^{\dagger}_i$$

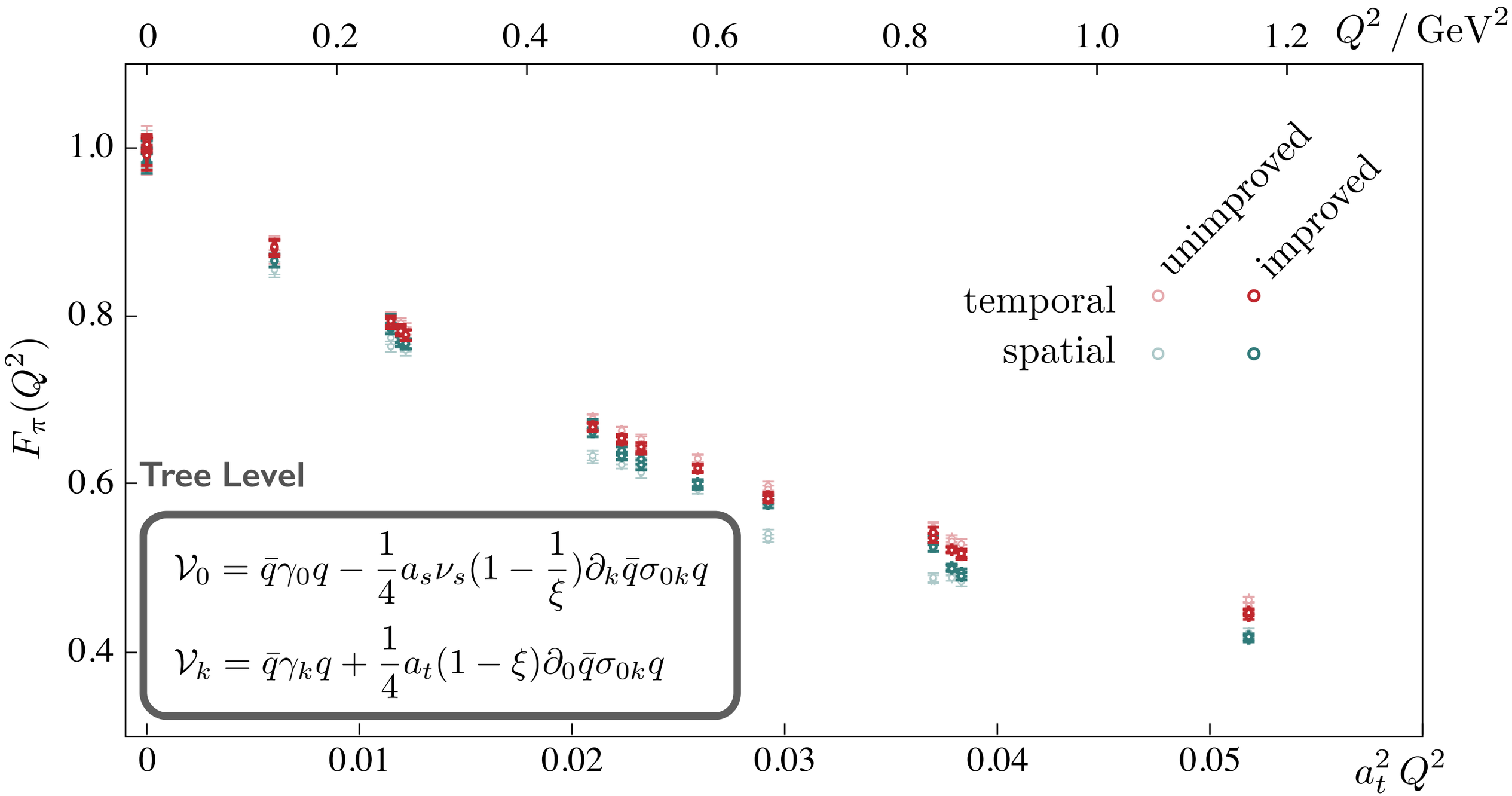
Time Dependence



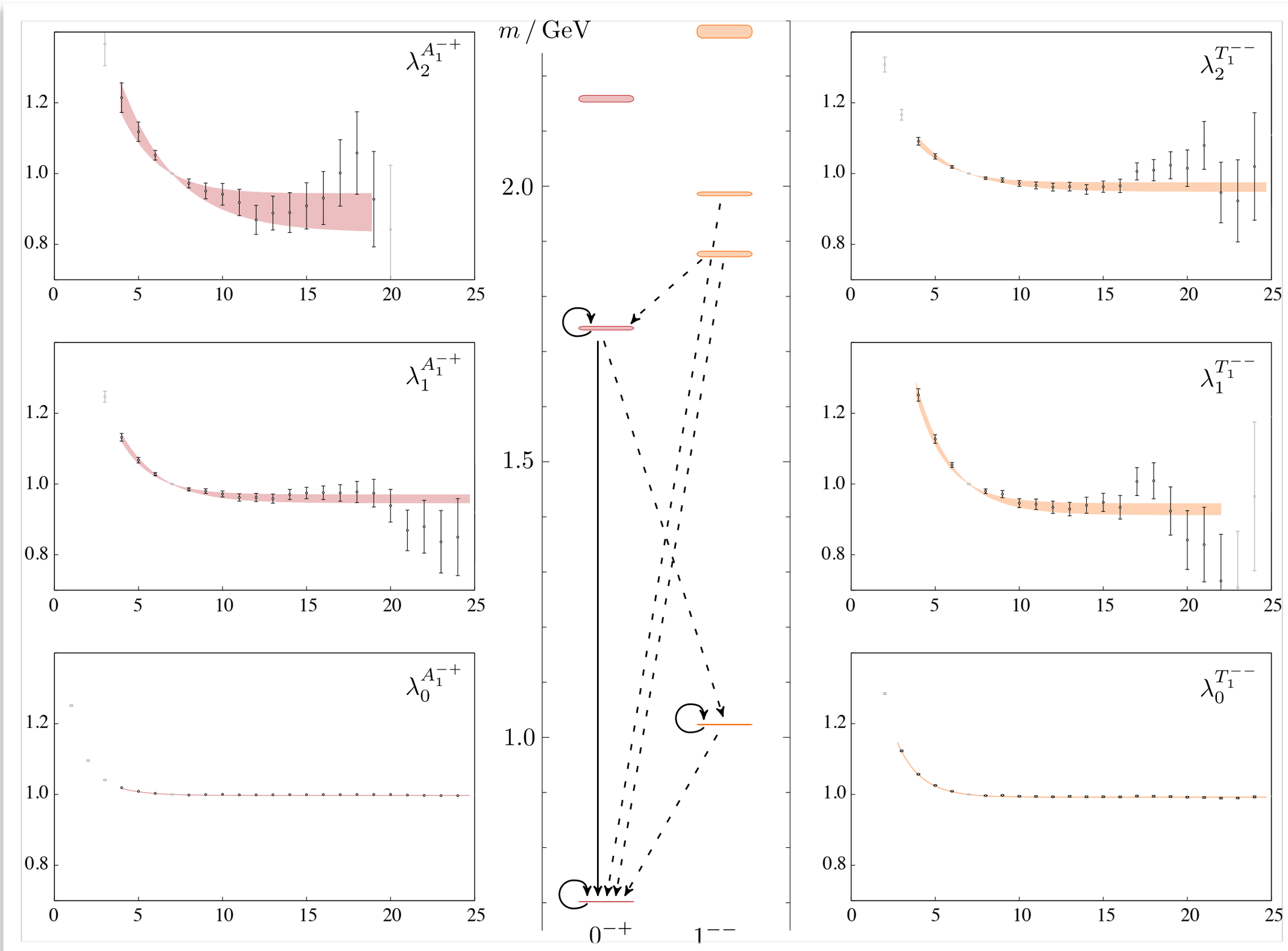


A Lattice Artifact

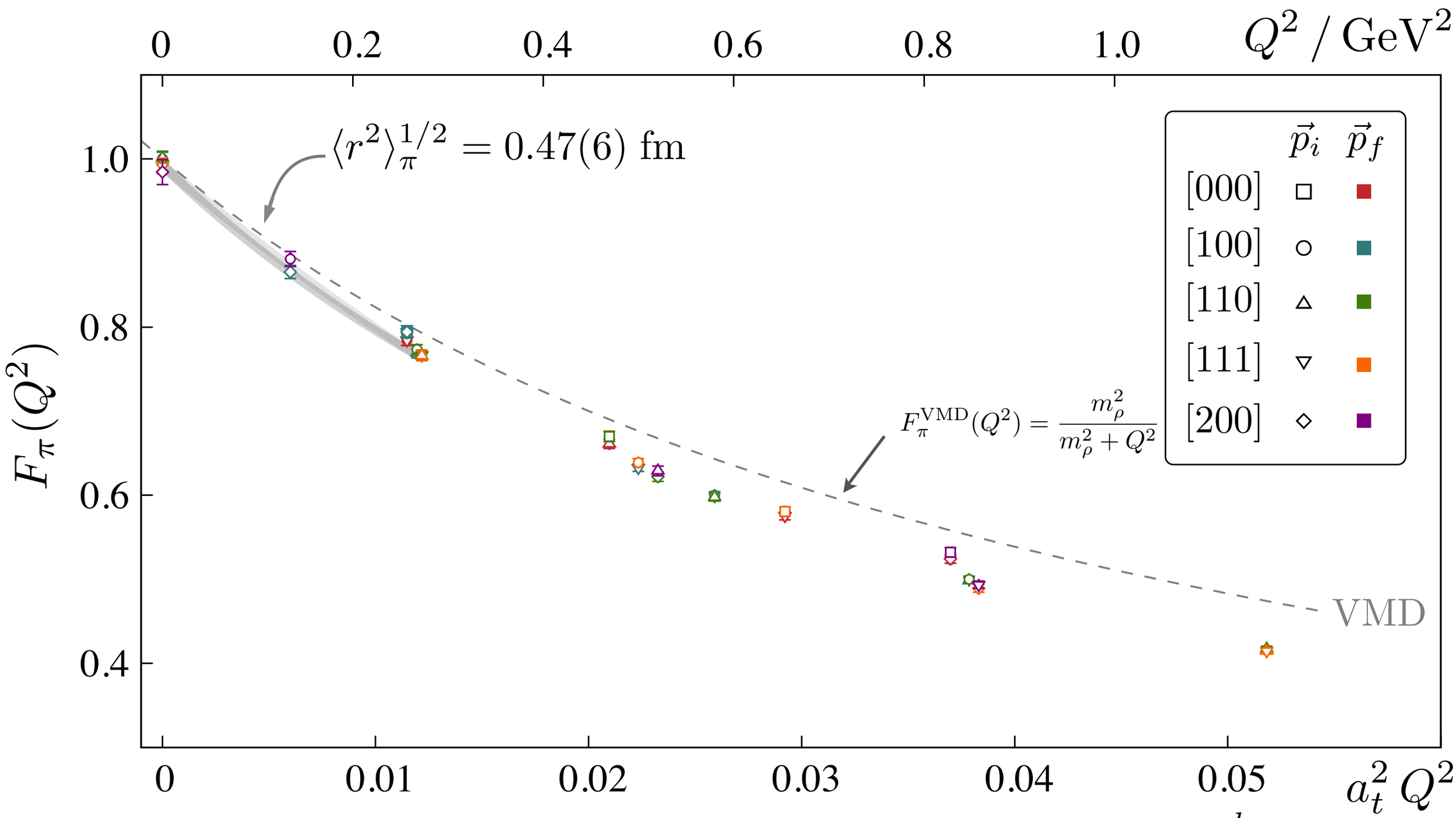
$\pi \to \pi\gamma$



The Calculation



Pion Form-factor



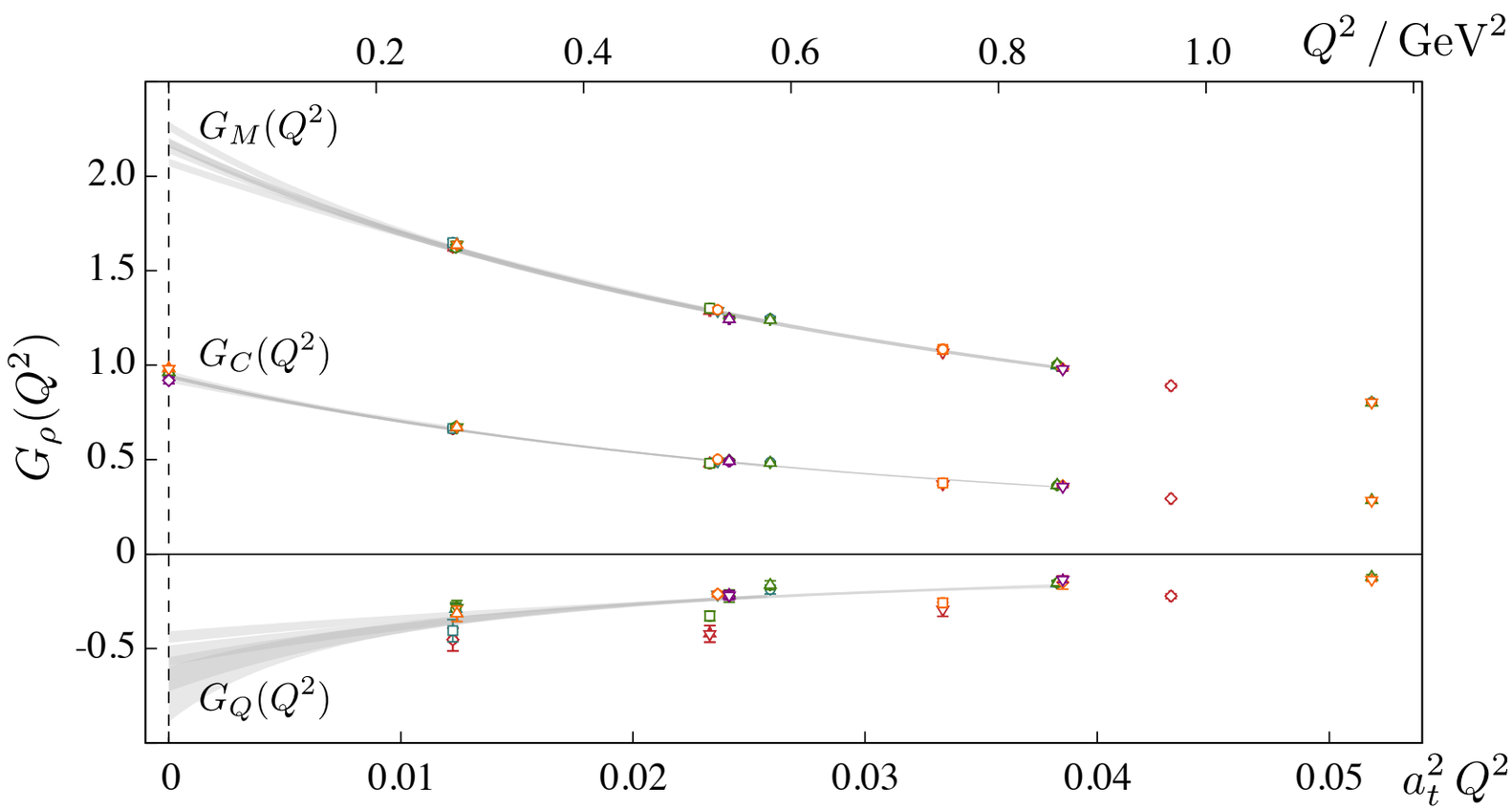
$$\langle r^2 \rangle = -6 \frac{d}{dQ^2} F(Q^2)|_{Q^2=0}$$

Rho Form-factors

$\rho \to \rho\gamma$

linear combinations of multipoles

$$\langle \rho, p'\lambda' | j^\mu | \rho, p\lambda \rangle = -(p'+p)^\mu \epsilon^*_\alpha(p'\lambda')\epsilon^\alpha(p\lambda) G_1(Q^2)$$
$$+ \left[\epsilon^\mu(p\lambda)\epsilon^*_\alpha(p'\lambda')p^\alpha + \epsilon^{\mu *}(p'\lambda')\epsilon^\alpha(p\lambda)p'_\alpha\right] G_2(Q^2)$$
$$- (p'+p)^\mu \epsilon^*_\alpha(p'\lambda')p^\alpha \epsilon^\beta(p\lambda)p'_\beta \frac{G_3(Q^2)}{2m_\rho^2}$$



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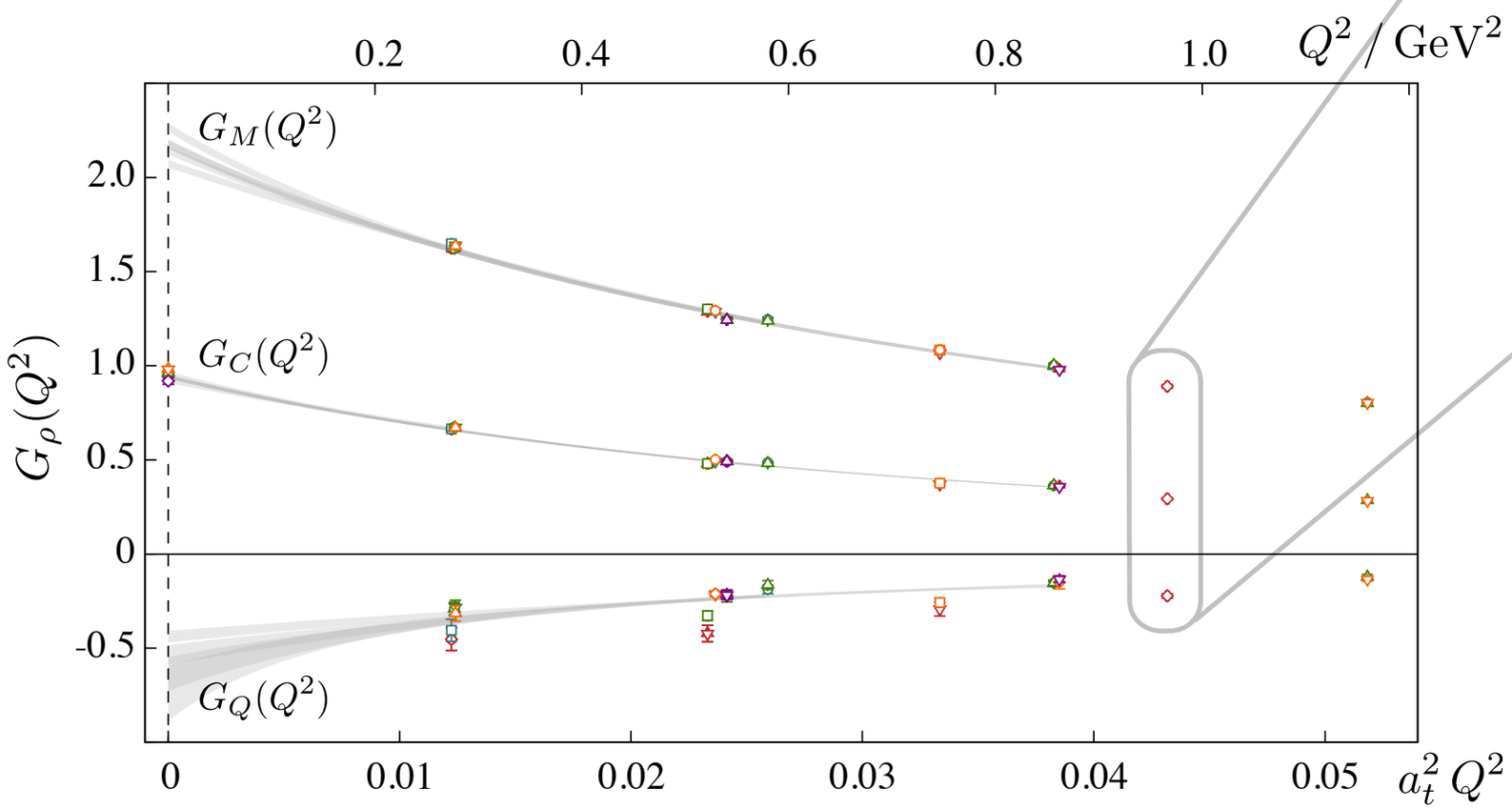
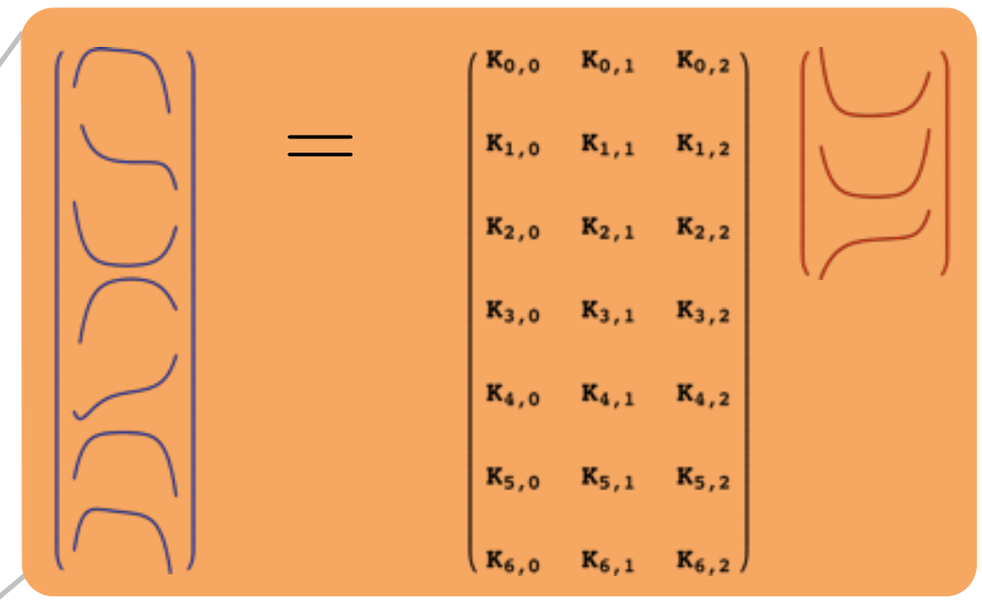


solved simultaneously



$$\langle n | j^\mu | m \rangle = \sum_k K^\mu_k(n,m) F_k(Q^2)$$

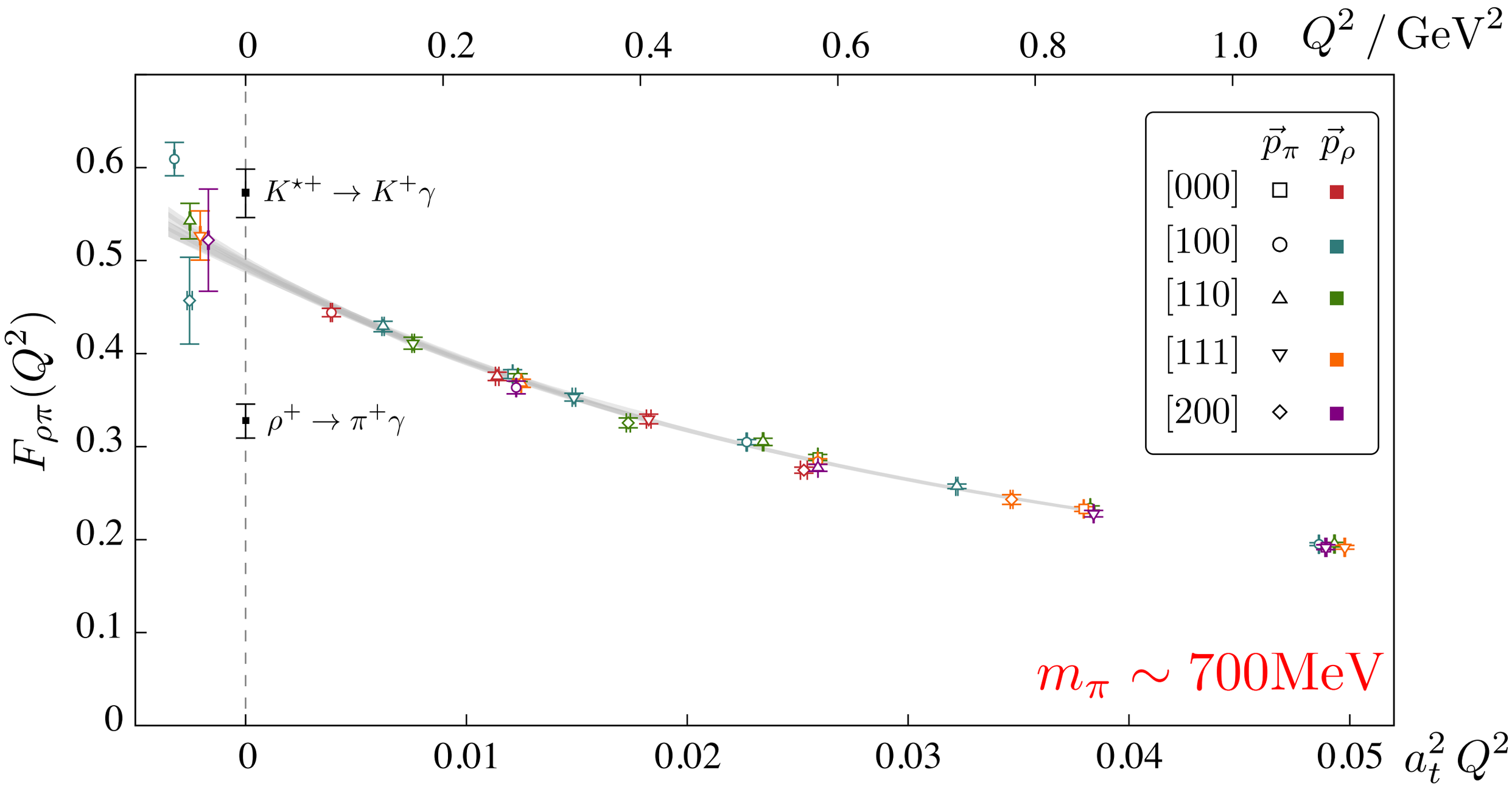
Kinematic Factor

Form Factor

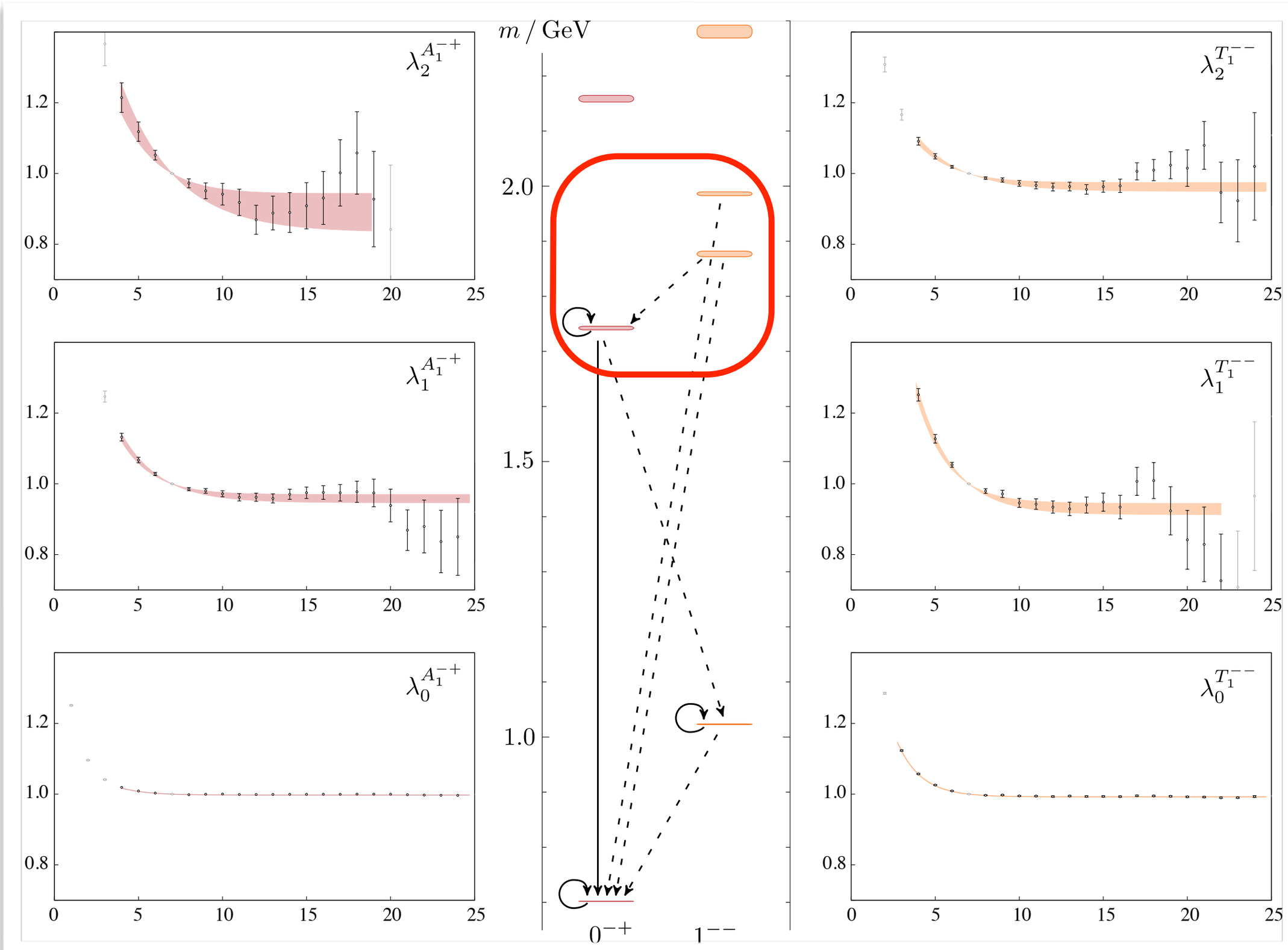
Transitions

$\rho \to \pi\gamma$

$$\langle \rho; p'\lambda | j^\mu | \pi; p \rangle = \frac{2F(Q^2)}{m_\rho + m_\pi} \epsilon^{\mu\nu\eta\sigma} p'_\nu p_\eta \varepsilon^*_\sigma(p', \lambda)$$

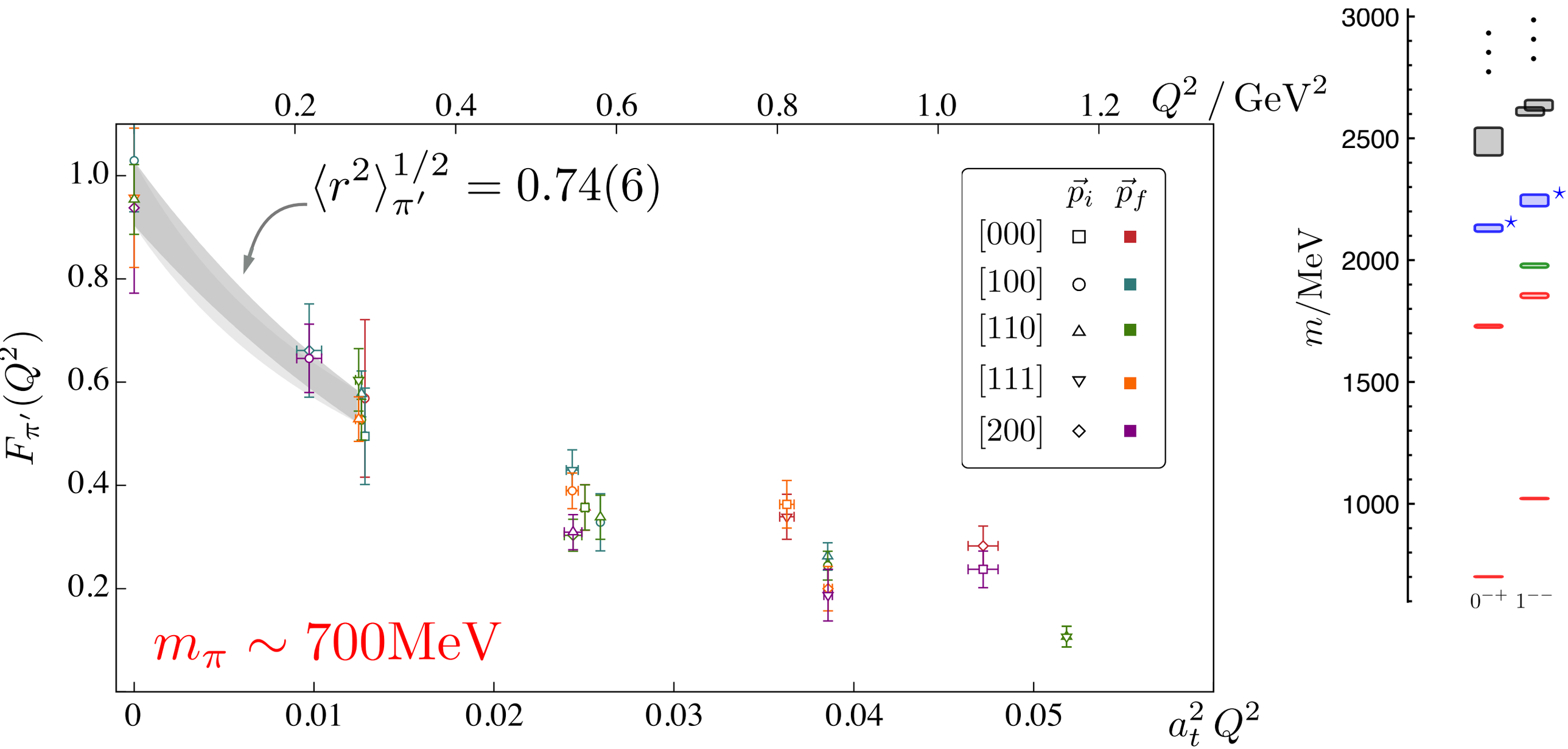


The 'Illegal Stuff'



Something New

$\pi' \to \pi'\gamma$

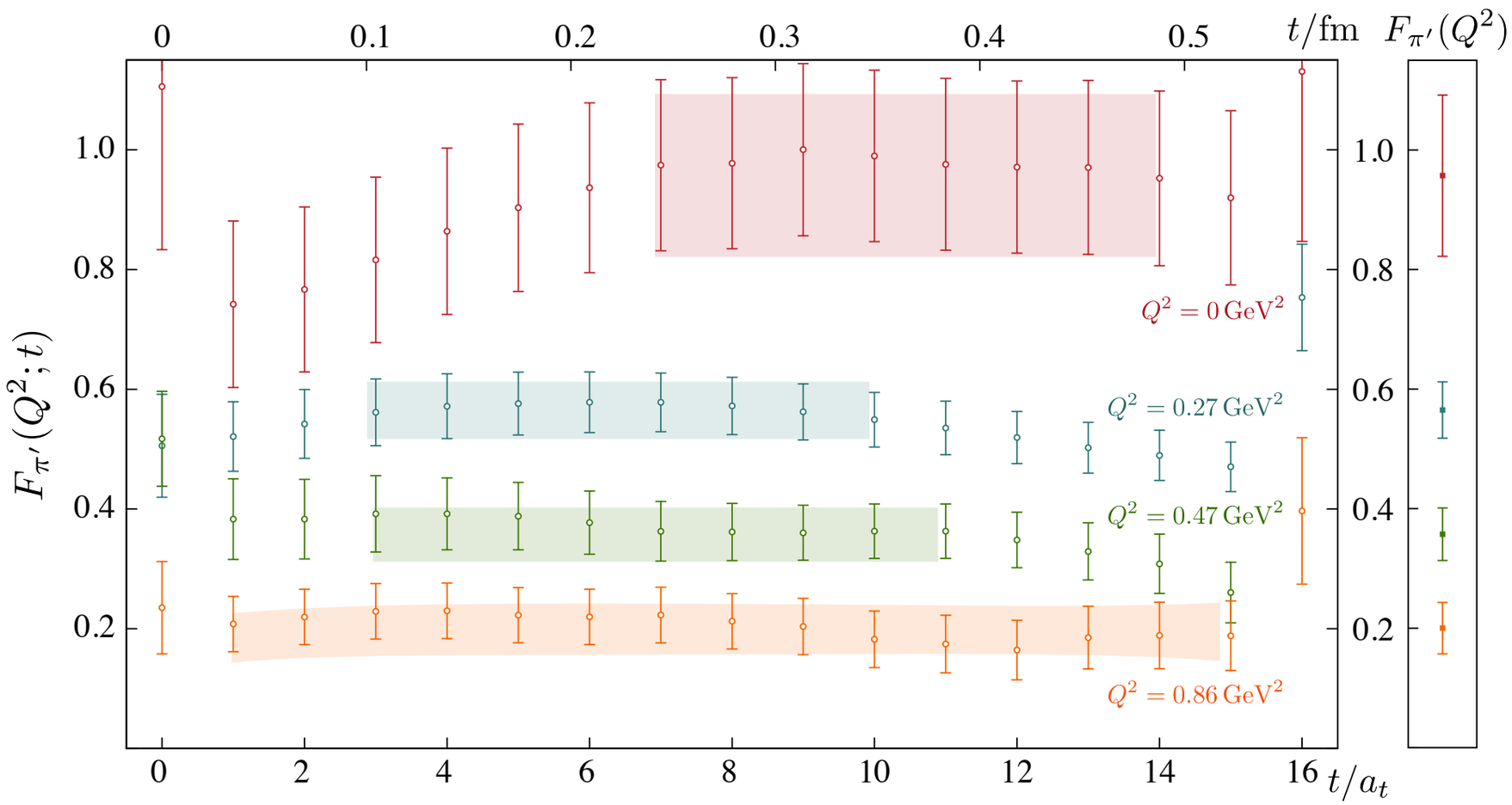


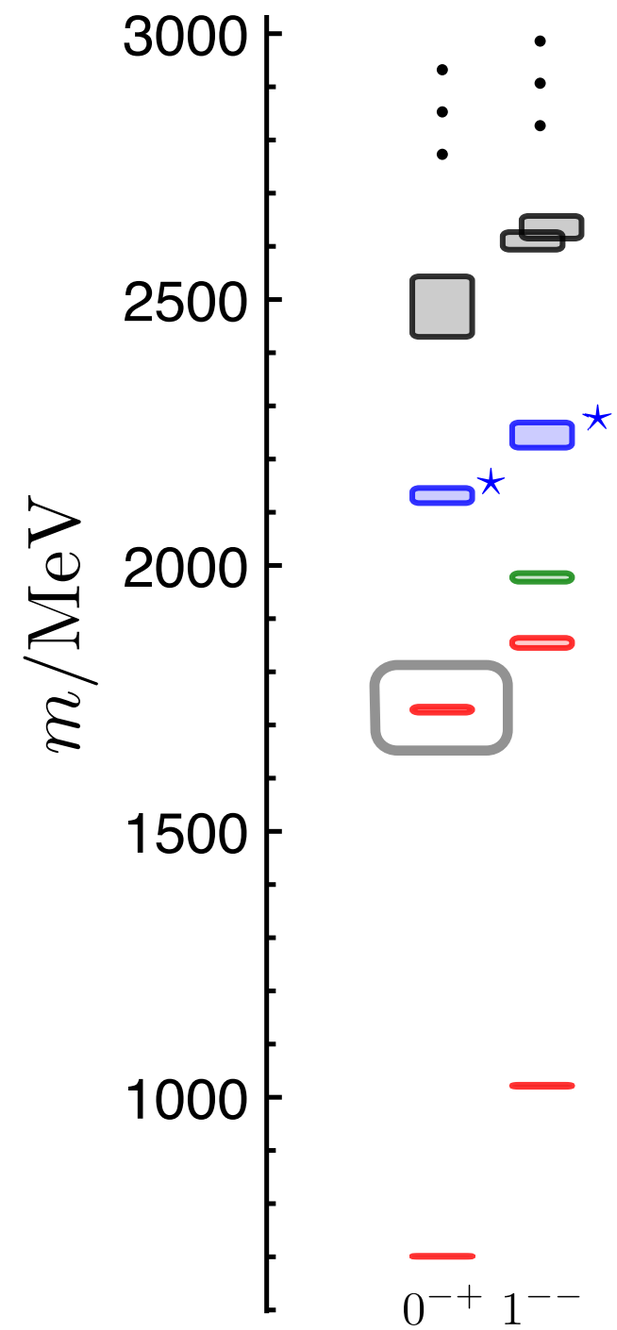
A Look 'Under the Hood'

$$C^{\mu}_{\mathfrak{n}\mathfrak{m}} = \langle 0|\Omega_{\mathfrak{n}}(t) j^{\mu}(t_\gamma)\Omega^{\dagger}_{\mathfrak{m}}(0)|0\rangle$$

$$= \sum_{\mathfrak{n}'\mathfrak{m}'} \langle \mathfrak{n}'|j^{\mu}|\mathfrak{m}'\rangle e^{-E_{\mathfrak{n}'}(t-t_\gamma)} e^{-E_{\mathfrak{m}'}t_\gamma}$$

$$\times \frac{\langle 0|\Omega_{\mathfrak{n}}(0)|\mathfrak{n}'\rangle}{2E_{\mathfrak{n}'}} \frac{\langle \mathfrak{m}'|\Omega^{\dagger}_{\mathfrak{m}}(0)|0\rangle}{2E_{\mathfrak{m}'}}$$





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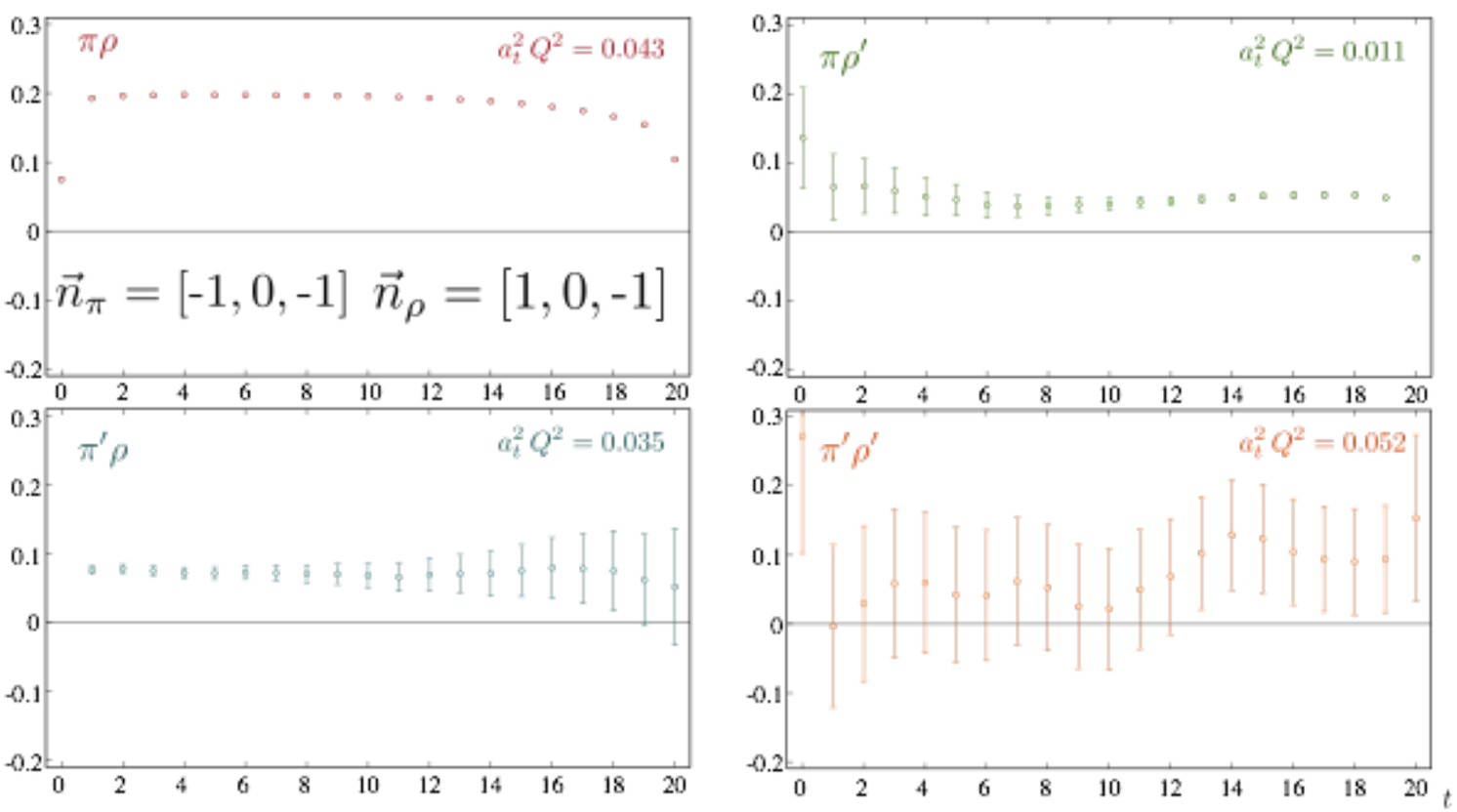
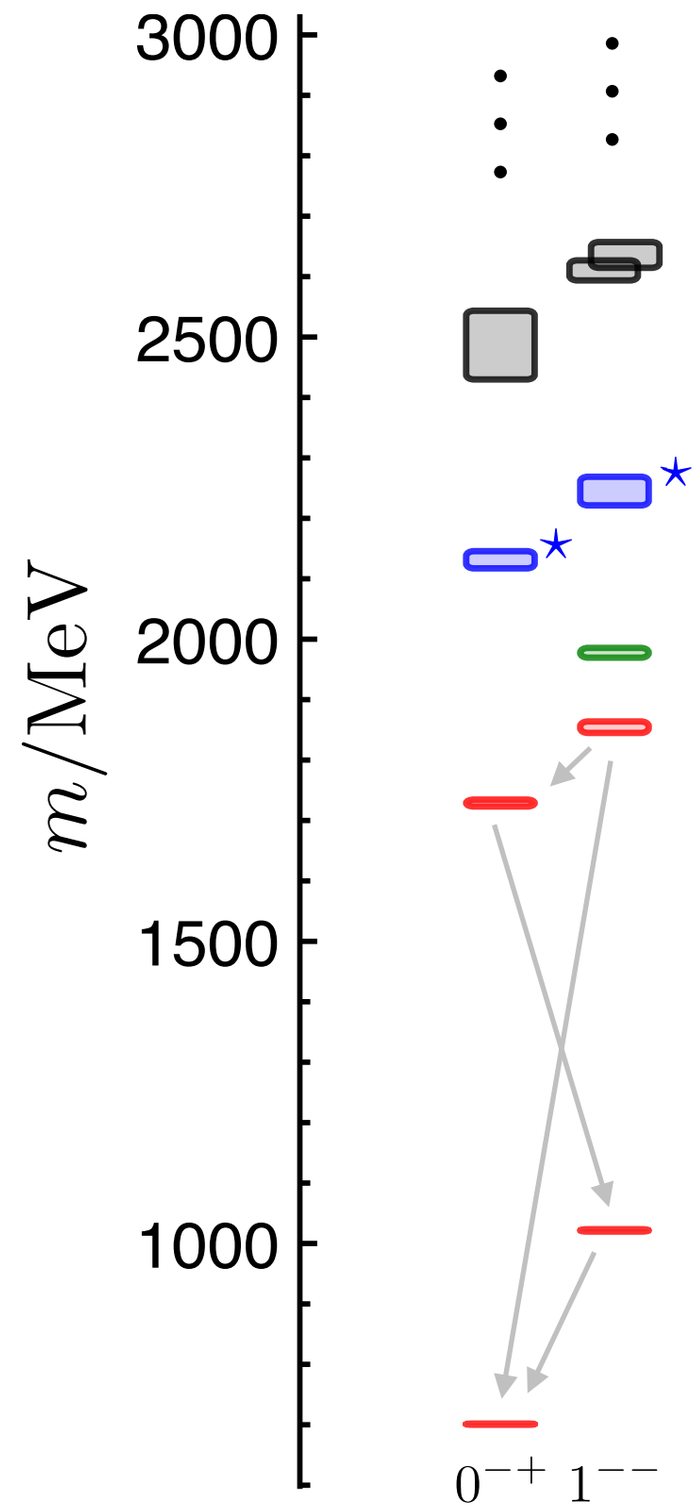
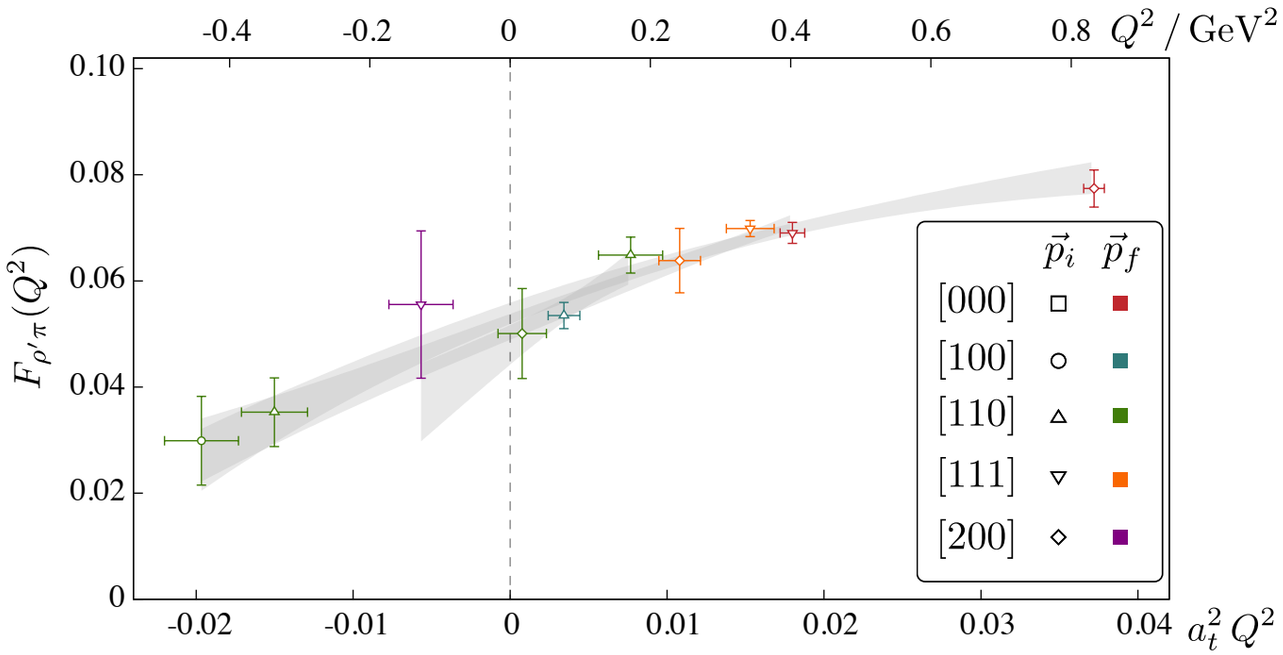
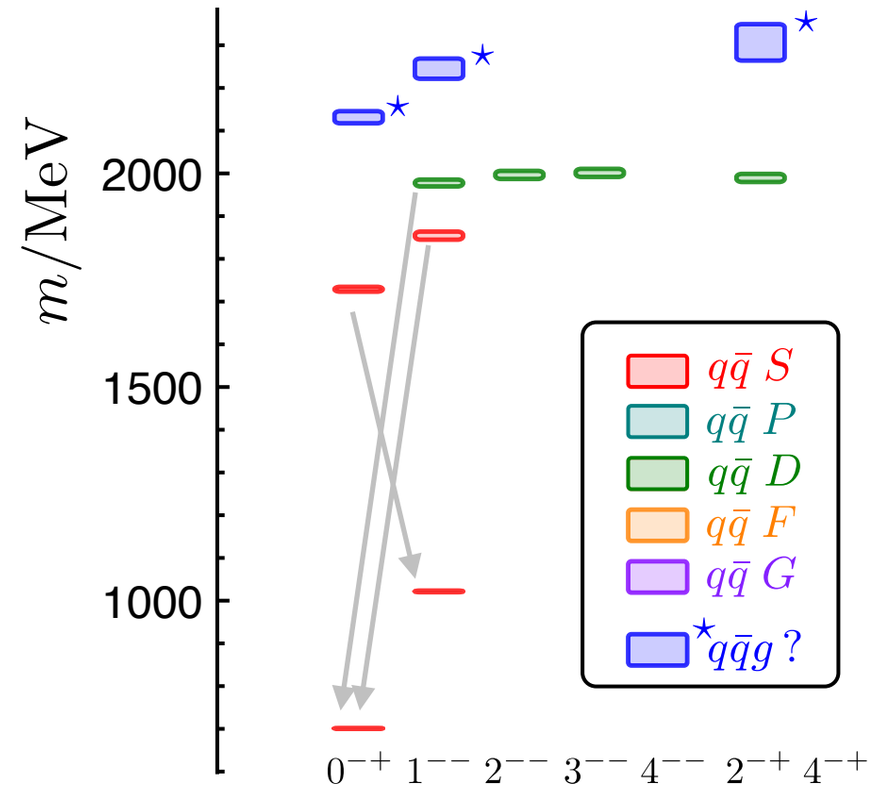
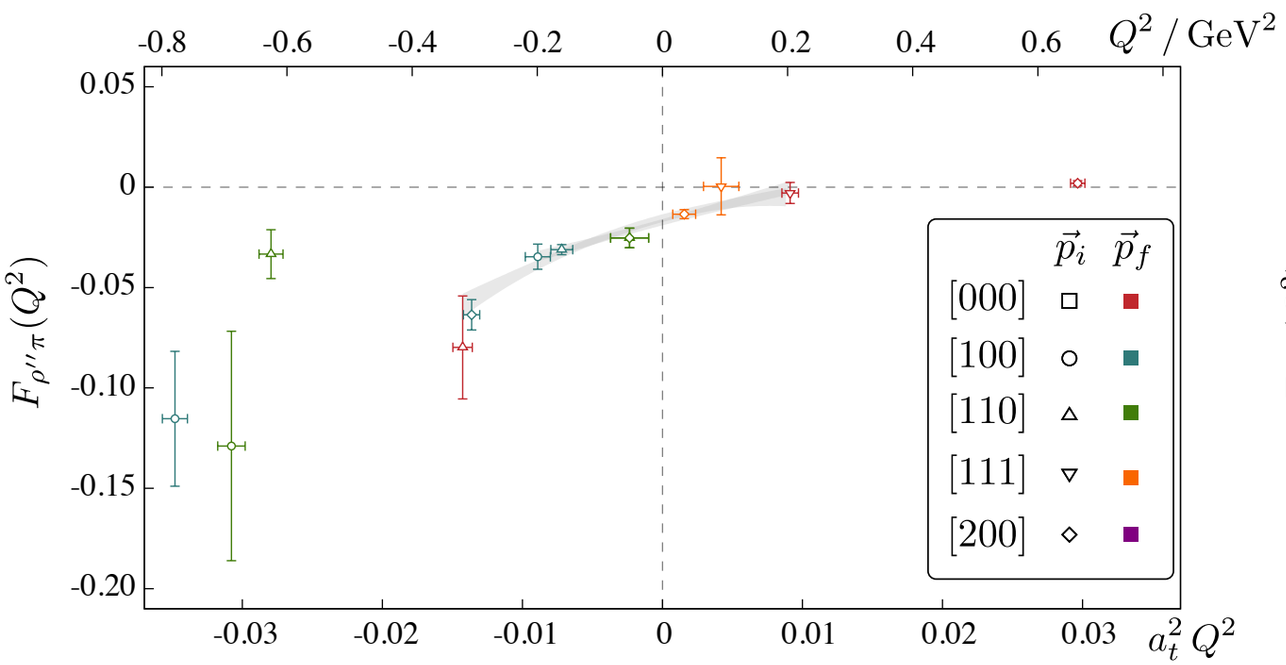


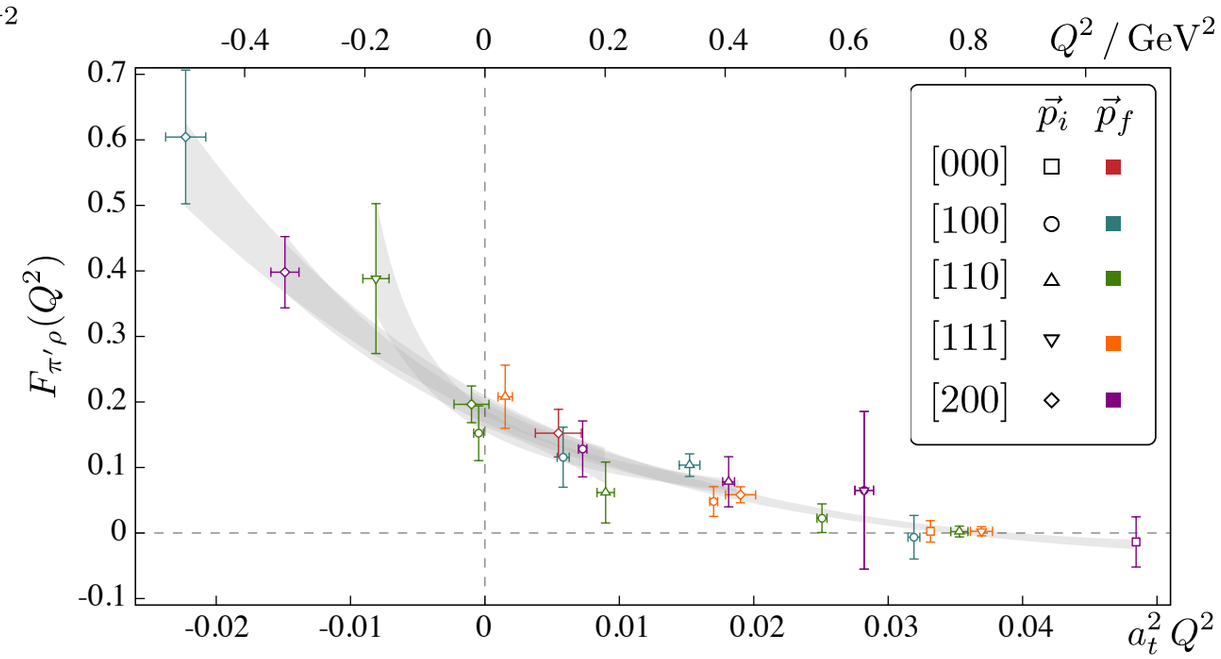


‘Hindered’ Transitions





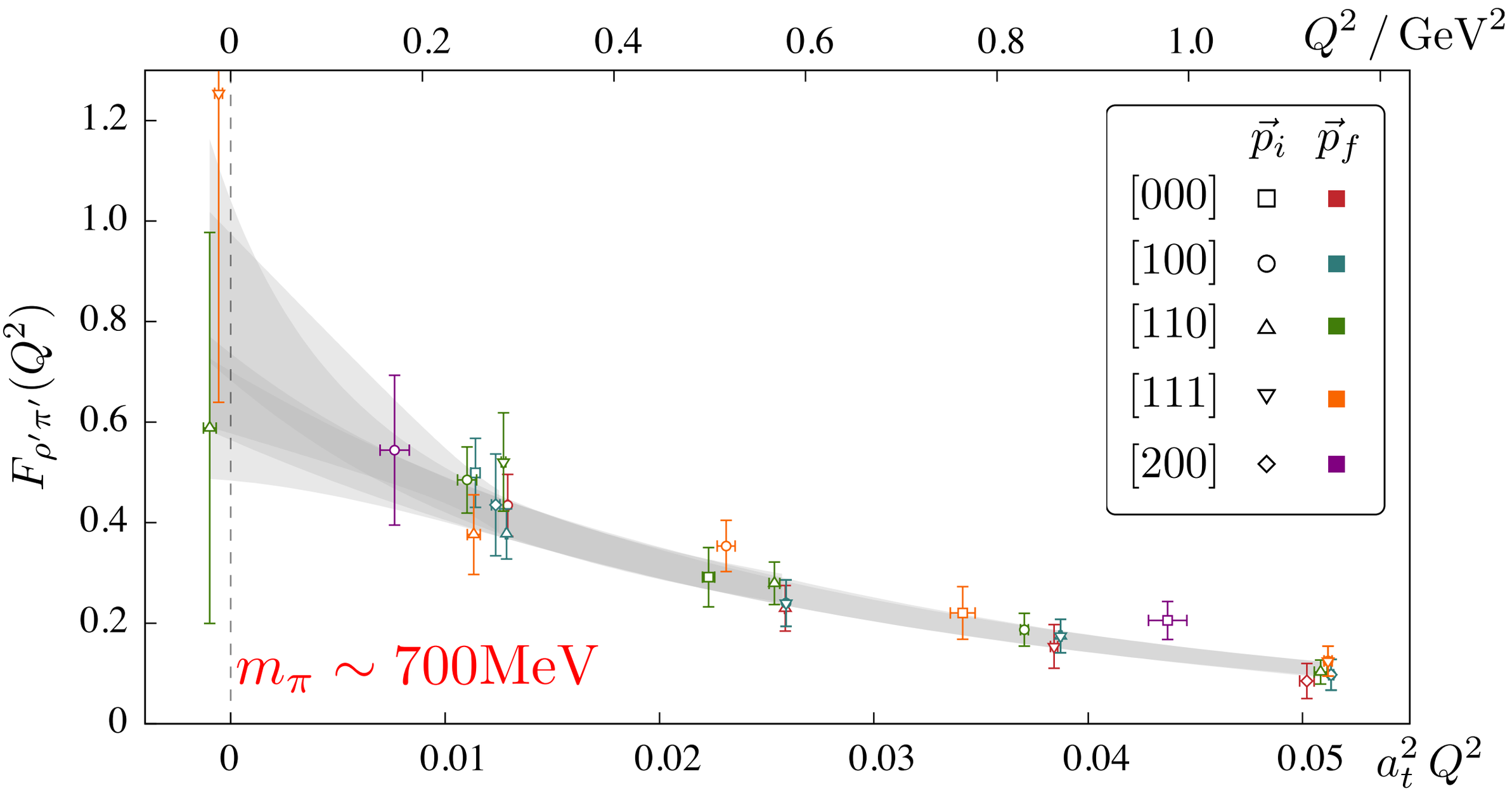




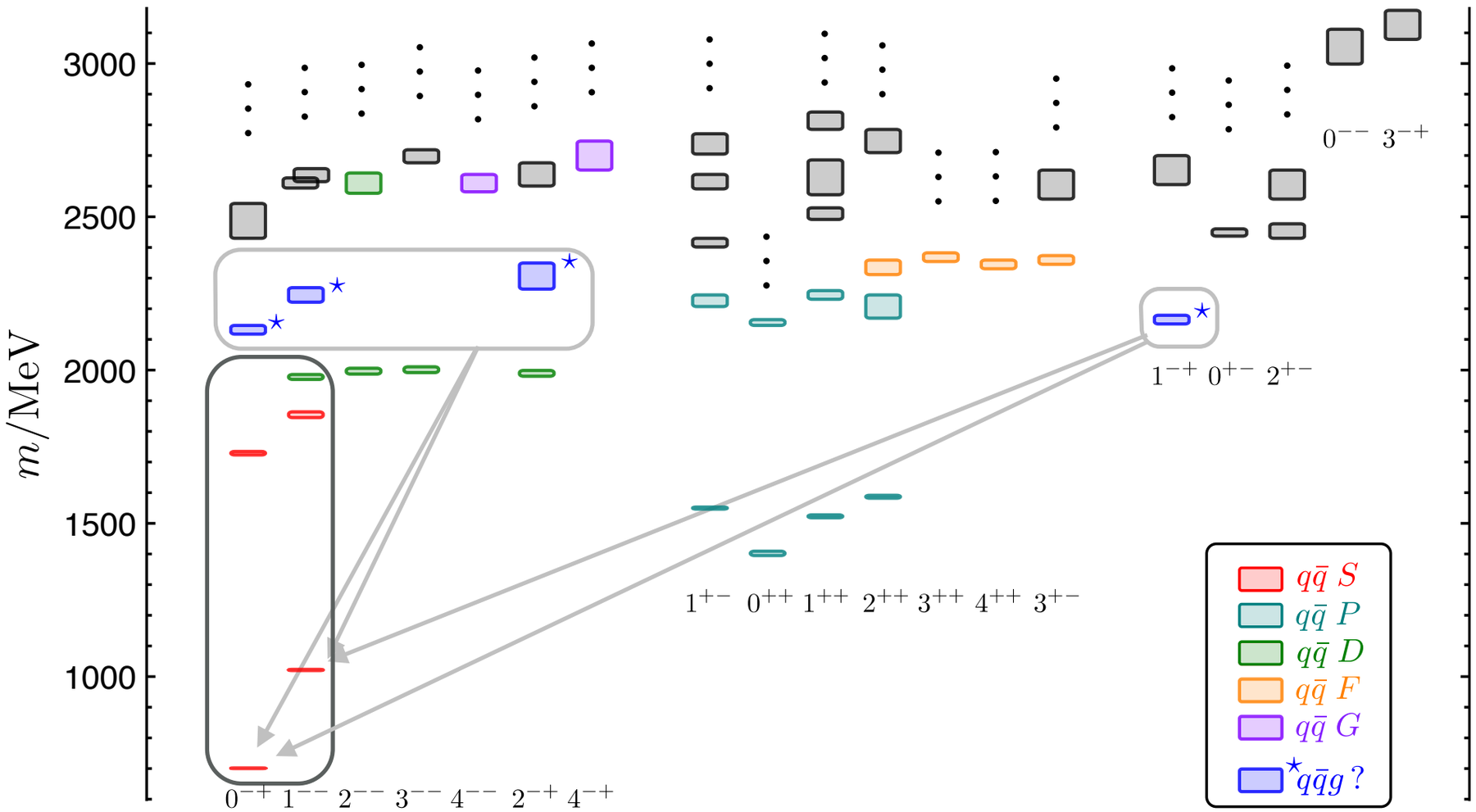
Excited to Excited Transition

$\rho' \to \pi' \gamma$

$$\langle \rho; p' \lambda | j^\mu | \pi; p \rangle = \frac{2F(Q^2)}{m_\rho + m_\pi} \epsilon^{\mu\nu\eta\sigma} p'_\nu p_\eta \varepsilon^*_\sigma(p', \lambda)$$



The Future



what have we done?

- implemented the formalism
- moved beyond ground states
- explored systematics
- seen real physics

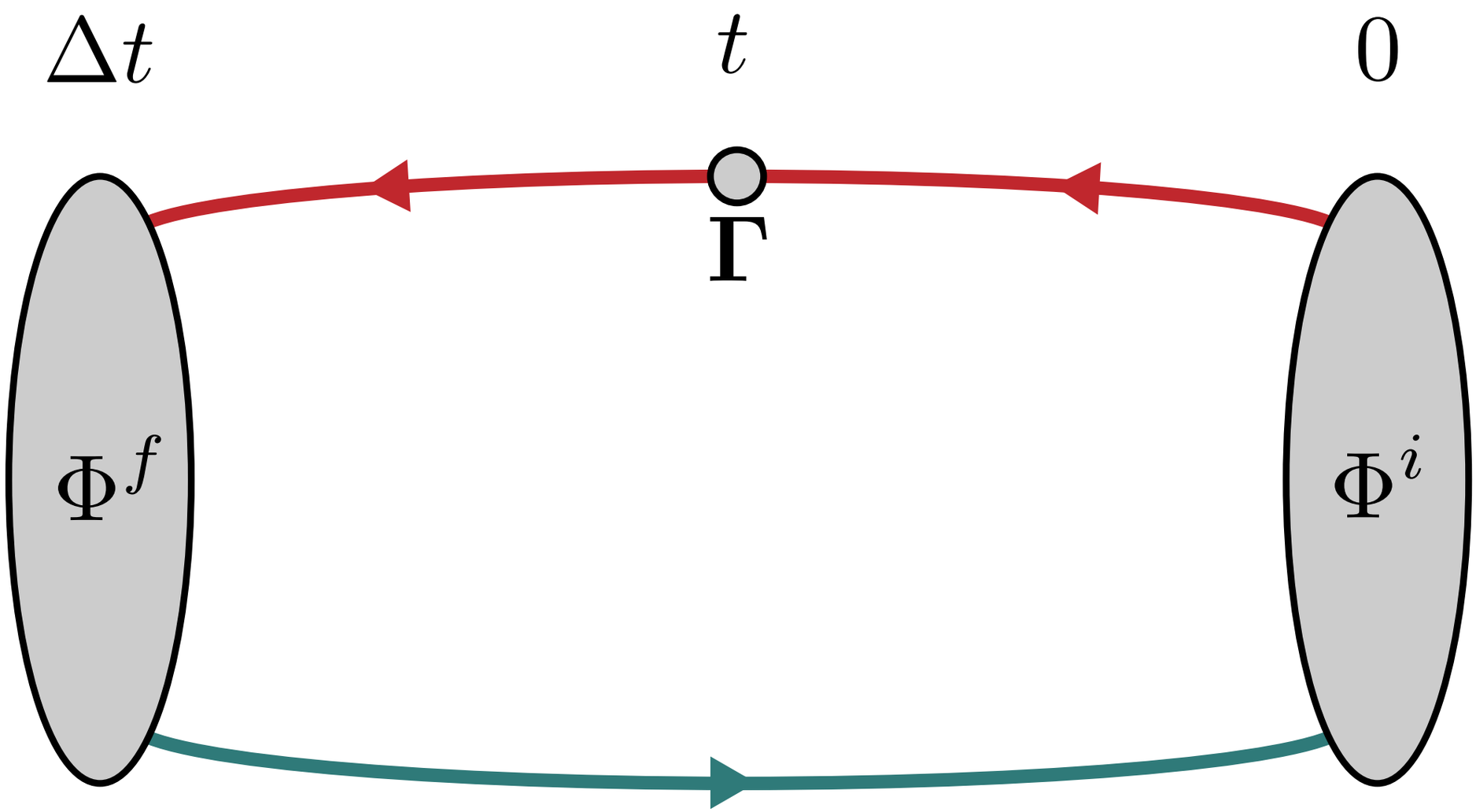
In the works

- explore more of the spectrum
- unstable to stable transition

other directions

- charm, tetraquarks, baryons?

Correlator Construction



Clover Action

Add irrelevant operators

$$S_C \equiv a_t a_s^3 \sum_x \bar{\psi}(x) \left[m_0 + \nu_t \not{D}_t^W + \nu_s \sum_i \not{D}_s^W - \frac{a_s}{2} \left(c_t \sum_i \sigma_{4i} F_{4i} - c_s \sum_{i<i'} \sigma_{ii'} F_{ii'} \right) \right] \psi(x)$$

$$S_C = \bar{\psi} \overleftarrow{\Omega} \left[m + \not{\nabla} \right] \overrightarrow{\Omega} \psi$$

$$\overrightarrow{\Omega} = 1 + a_t \Omega_m m + a_t \Omega_t \overrightarrow{\not{\nabla}}_t + a_s \Omega_s \sum_i \overrightarrow{\not{\nabla}}_i$$

Redefine QED fields

$$\tilde{V}^\mu = \bar{\psi} \overleftarrow{\Omega} \gamma^\mu \overrightarrow{\Omega} \psi$$

Tree Level

$$\mathcal{V}_0 = \bar{\psi}\gamma_0\psi - \frac{1}{4} a_s \nu_s (1 - \frac{1}{\xi}) \partial_k \bar{\psi} \sigma_{0k} \psi$$

$$\mathcal{V}_k = \bar{\psi}\gamma_k\psi - \frac{1}{4} a_t \left(1 - \xi\right) \partial_0 \bar{\psi} \sigma_{0k} \psi$$

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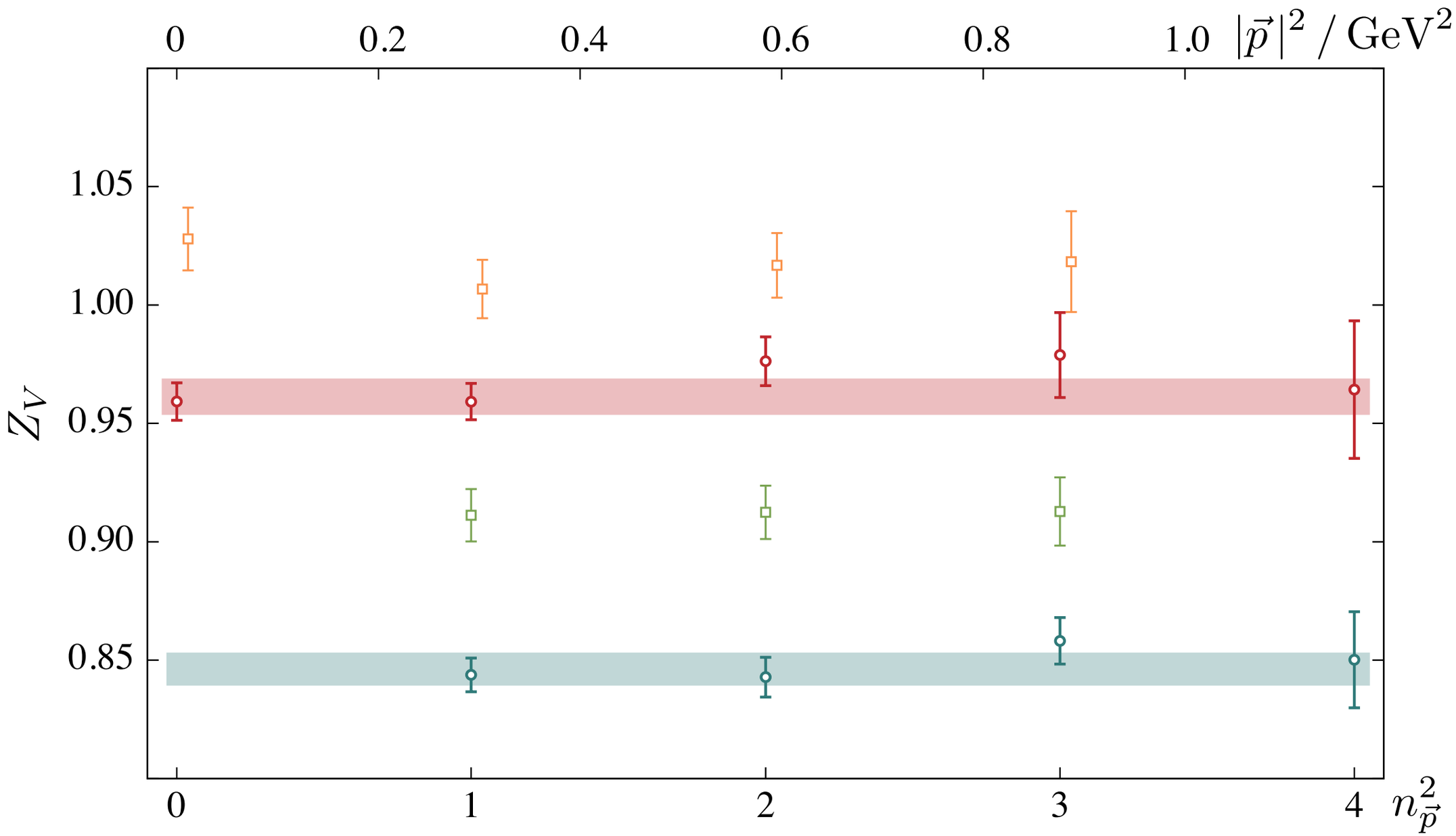
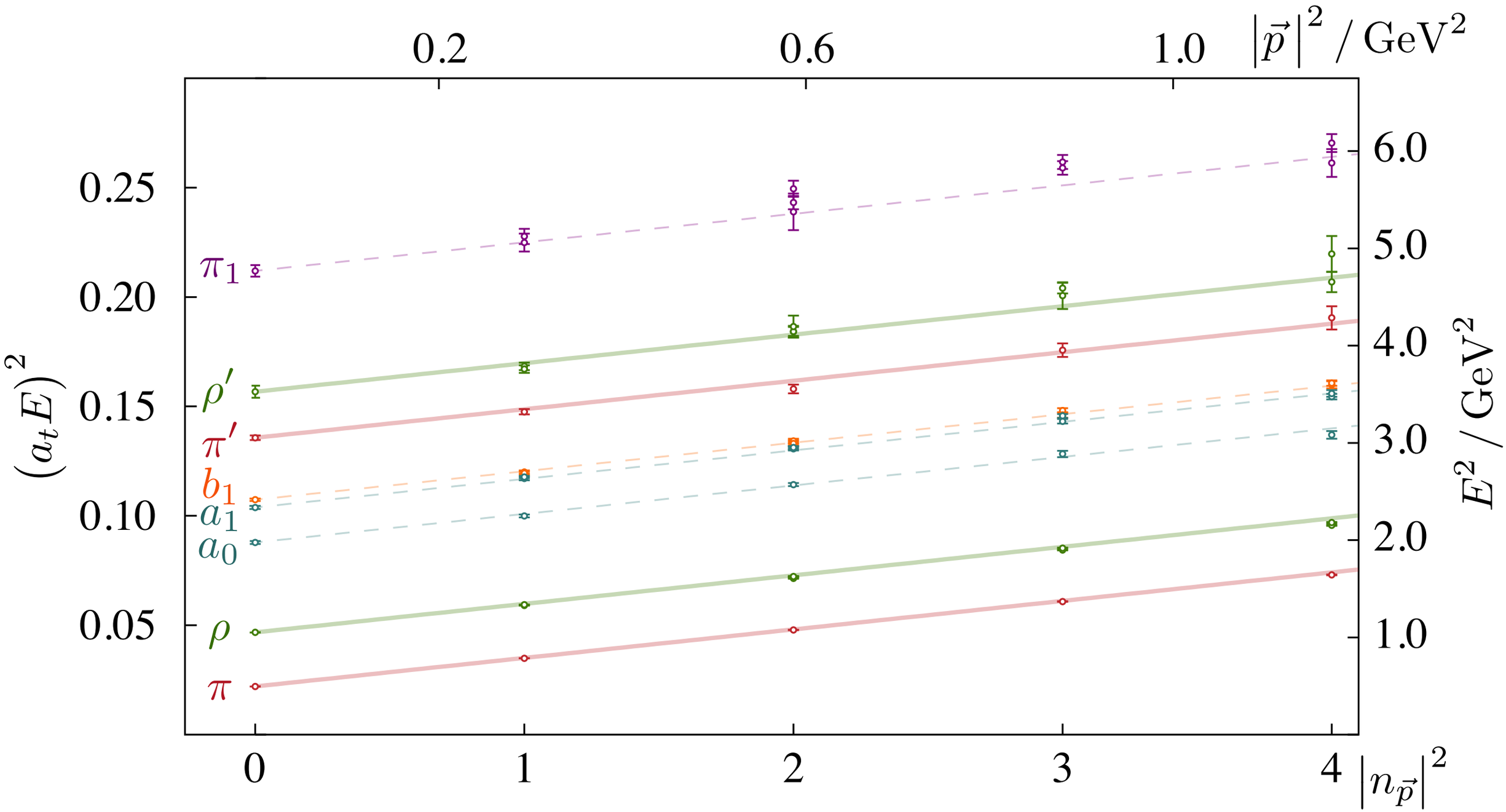
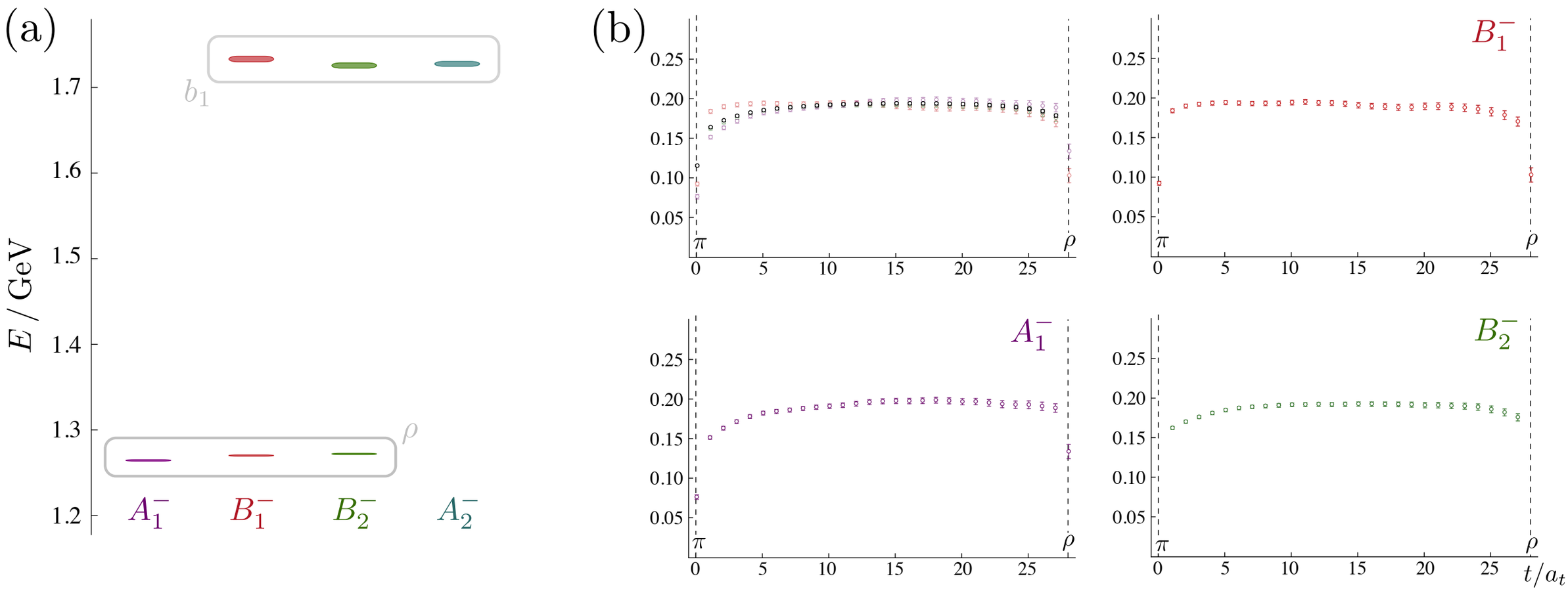
Renormalization



Dispersion


0.2
0.6
1.0
$|\vec{p}|^2$ / GeV^2
0.25
0.20
0.15
0.10
0.05
$(a_t E)^2$
6.0
5.0
4.0
3.0
2.0
1.0
E^2 / GeV^2
π_1
ρ'
π'
b_1
a_1
a_0
ρ
π
0
1
2
3
4
$|n_{\vec{p}}|^2$

Rotational Symmetry



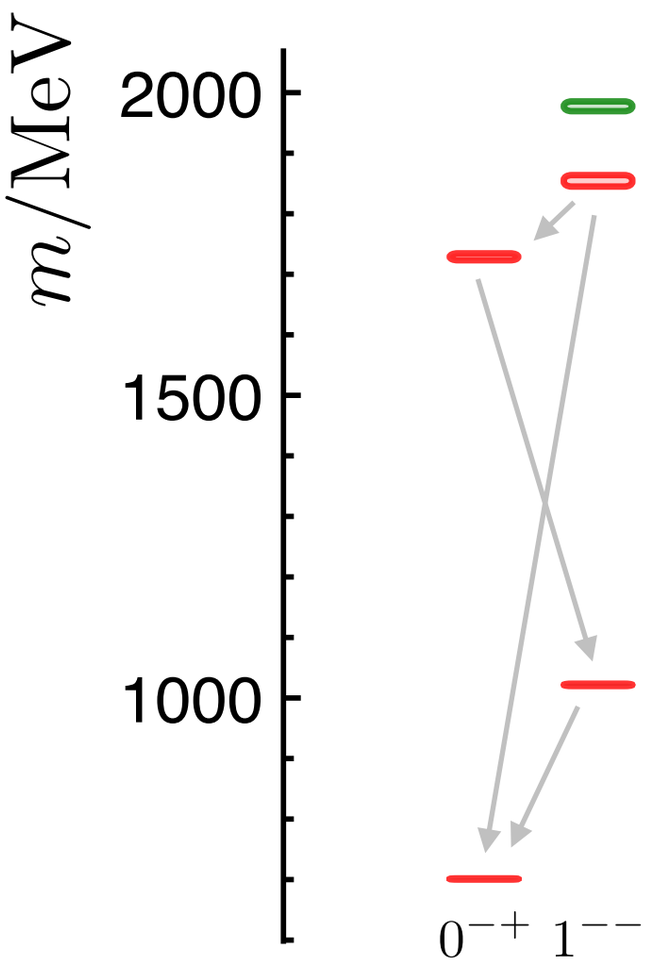
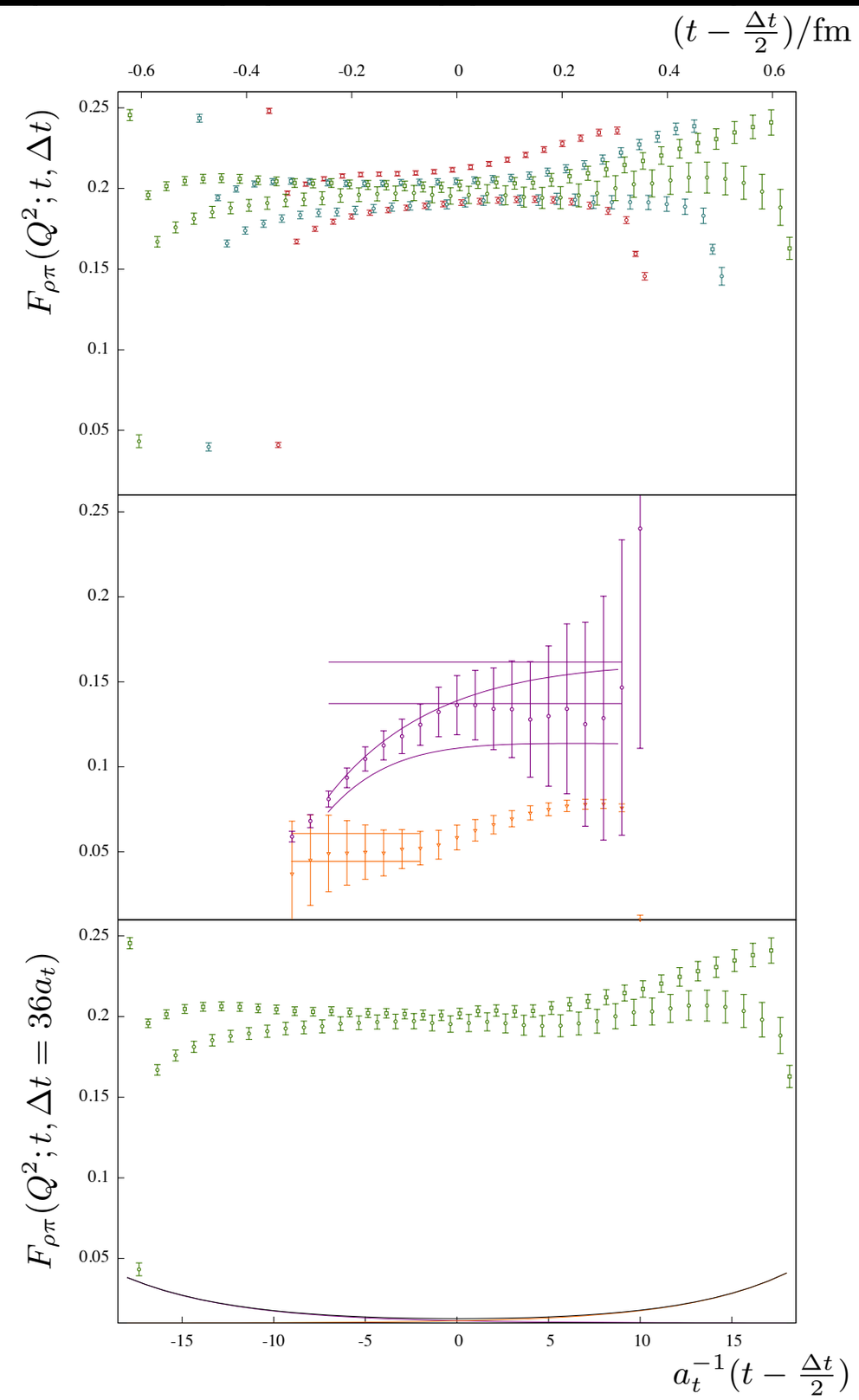
$$\vec{n}_p = [110]$$

$$\langle 0|\Omega_\rho^{(\Lambda)}(28,\vec{p}\,')j^{\mu(E_2)}(t,\vec{q})\Omega_\pi^\dagger(0,\vec{p})|0\rangle$$

$$\vec{n}_{p'} = [0,1,1] \qquad \vec{n}_p = [0,\text{-}1,1]$$

Reconstruction

$$C^{\mu}_{\mathfrak{n}\mathfrak{m}} = \langle 0|\Omega_{\mathfrak{n}}(t) j^{\mu}(t_\gamma)\Omega^{\dagger}_{\mathfrak{m}}(0)|0\rangle$$

$$= \sum_{\mathfrak{n}'\mathfrak{m}'} \langle \mathfrak{n}'|j^{\mu}|\mathfrak{m}'\rangle e^{-E_{\mathfrak{n}'}(t-t_\gamma)} e^{-E_{\mathfrak{m}'}t_\gamma}$$

$$\times \frac{\langle 0|\Omega_{\mathfrak{n}}(0)|\mathfrak{n}'\rangle}{2E_{\mathfrak{n}'}} \frac{\langle \mathfrak{m}'|\Omega^{\dagger}_{\mathfrak{m}}(0)|0\rangle}{2E_{\mathfrak{m}'}}$$