Omega-Omega interaction from 2+1 flavor QCD

Masanori Yamada (University of Tsukuba)
for HAL QCD Collaboration

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Hadrons to Atomic nuclei
HAL
from Lattice QCD

Lattice 2014 Jun 26, 2014, Columbia,NY

Outline

- Introduction
- Formulation
  - Construction of the potential [HAL QCD method]
  - Symmetry of Omega-Omega system
- Lattice QCD Simulation results
  - Potential
  - phase shift & binding energy
- Conclusion

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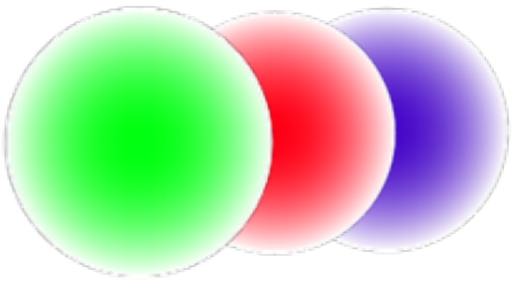
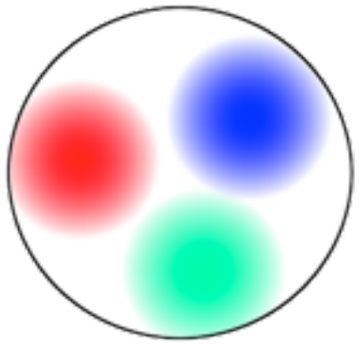
Introduction

Quark $\mathrm{spin}\,\frac{1}{2}$

Baryon





$$\mathrm{spin}\,\frac{1}{2} \otimes \mathrm{spin}\,\frac{1}{2} \otimes \mathrm{spin}\,\frac{1}{2}$$

$$\mathrm{spin}\,\frac{3}{2} \oplus \mathrm{spin}\,\frac{1}{2}$$

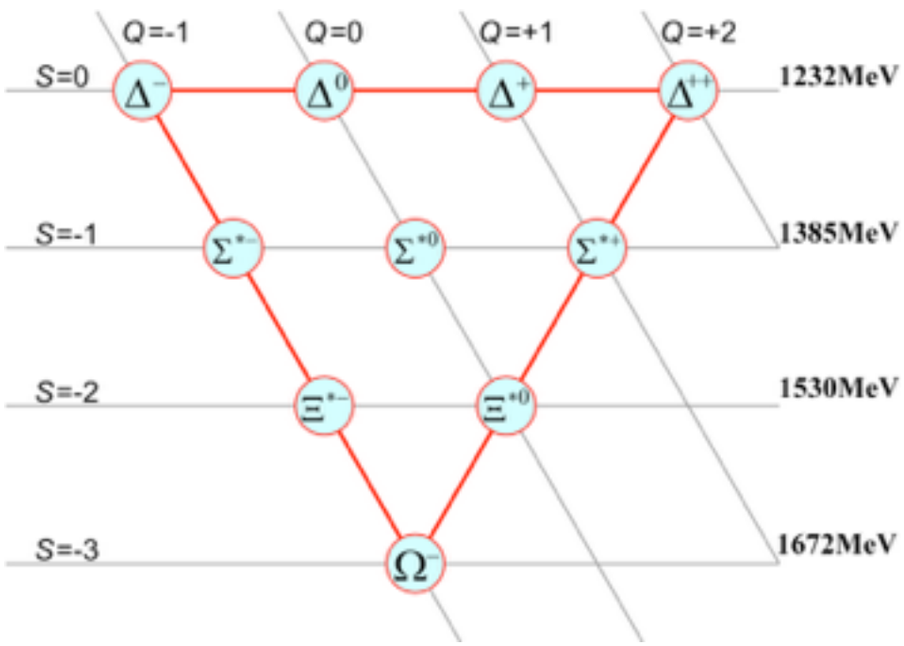
Decuplet Baryon

Octet Baryon

- Proton
- Neutron
- etc…

Introduction

Decuplet Baryon

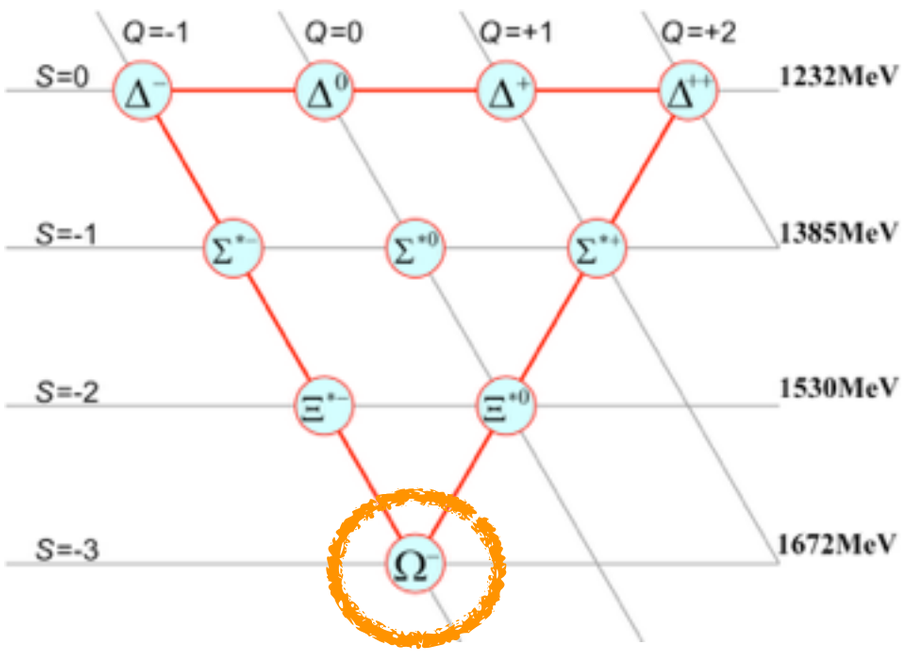


Motivation

- Omega baryon is stable in QCD
- It's a first target of Decuplet-Decuplet interactions at HAL QCD method.
- There have been different model calculations in the J=0 channel

Introduction

Decuplet Baryon



My target **Ω-Ω interaction**

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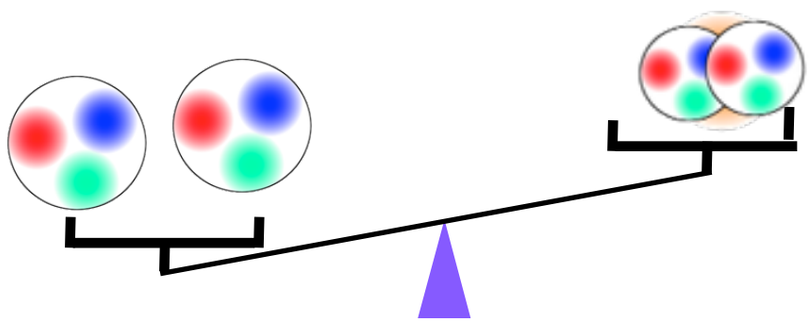

interaction energy

$$\Delta M_{\Omega\Omega} = E_{\Omega\Omega} - 2M_{\Omega} = -166\mathrm{MeV}$$

(SU(3) Chiral Quark Model)

$$E_{\Omega\Omega} \equiv 2\sqrt{k^2 + M_{\Omega}^2}$$

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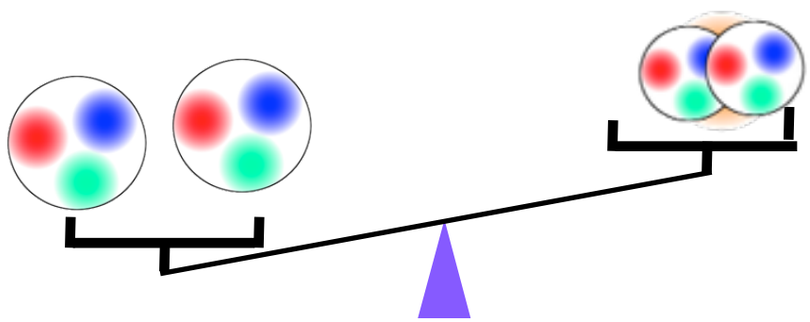

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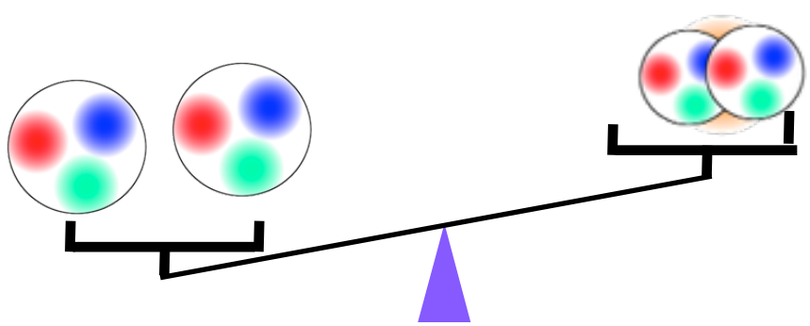

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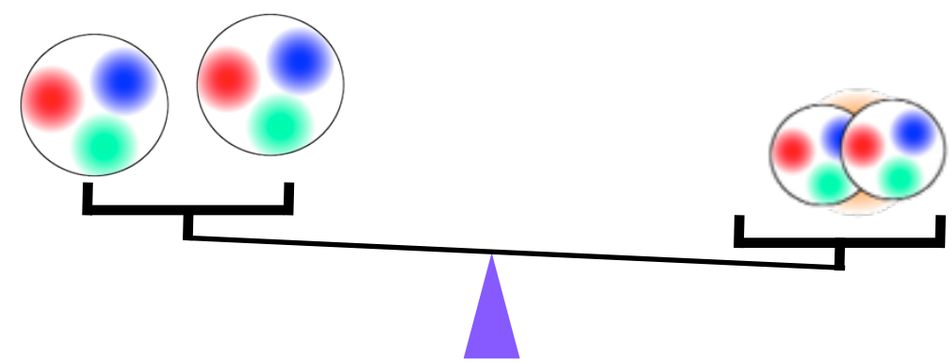

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$$\Delta M_{\Omega\Omega} = E_{\Omega\Omega} - 2M_{\Omega} = 43 \pm 18\mathrm{MeV}$$

(Quark Disloc./Color-screen Model)

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[Z.Y.Zhang et al. Phys.Rev.C .61, 065204]
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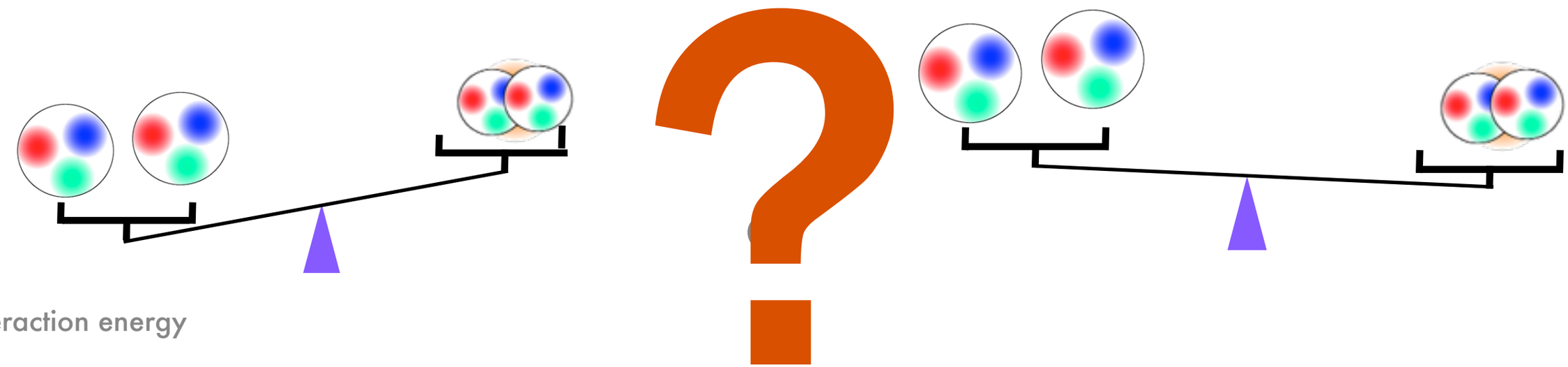

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Introduction

Report from another group (Lattice QCD simulation)

Lüscher's method [Lüscher CMP105(86)153, NPB354(91)531]

Buchoff et al. : L=3fm Ω=1628[MeV]

J=0 : weak repulsion a= - 0.16 ± 0.22 fm [arXiv:1201.3596]

J=2 : strong repulsion

J.Wasem @Lattice2012

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determine a nature of J=0 Omega-Omega interaction, attractive or repulsive

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Construction of the potential

[S. Aoki,etal. Prog. Theor. Phys., 123:89]

[N.Ishizuka,arXiv:0910.2772.]

Basic Idea

Commonly used method

Schrödinger eq

in Quantum mechanics

Potential(Given) → wave function(result)

via Schrödinger eq

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We can get

Given by Lattice QCD calculation

Construction of the potential

Q. What is the wave function in QCD ?

Construction of the potential

Q. What is the wave function in QCD ?

A. Nambu-Bethe-Salpeter(NBS) wave function

Ω interpolating filed

$$\psi_k(r) \equiv \langle 0| \Omega(r)\Omega(0) \left|\overline{\Omega}(k)\overline{\Omega}(-k); in\right\rangle$$

same quntum number Ω-Ω

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Because

NBS wave has the same asymptotic form of the scattering wave in quantum mechanics.

Wave function↔phase shift↔S-matrix

$$\psi_k(r) \simeq e^{i\delta(k)} \frac{\sin(kr - \frac{l\pi}{2} + \delta(k))}{kr}$$

[C.-J.D. Lin et al., NPB619(2001)467.]

Energy independent potential U(r,r') is defined from NBS wave function.

because of

$$(\frac{k^2}{m} + \frac{1}{m}\nabla^2)\psi_k(r) = \int d^3r' U(r,r')\psi_k(x')$$

This potential reproduces the phase shift faithfully

we can extract a potential(interaction kernel) which is defined through the NBS wave function which gives the correct scattering phase shift at asymptotic state.

Symmetry of Ω-Ω

Ω operator is defined as

$$\Omega_{\alpha,k} \equiv \varepsilon^{abc} s^a (C\gamma_k) s^b s^c_\alpha$$

blue is $\mathrm{spin1}$ index, red is $\mathrm{spin}\frac{1}{2}$ index

We treat spin 3/2 made from spin 1 and spin 1/2 linear combination by using highest weight construction

- one Ω case($\mathrm{spin}\frac{3}{2}$)

$$\mathrm{spin}\frac{1}{2} \otimes \mathrm{spin1} = \mathrm{spin}\frac{3}{2} \oplus \mathrm{spin}\frac{1}{2}$$

- consider two Ω case (Ω-Ω interaction)

$$\mathrm{spin}\frac{3}{2} \otimes \mathrm{spin}\frac{3}{2} = \mathrm{spin3} \oplus \mathrm{spin2} \oplus \mathrm{spin1} \oplus \mathrm{spin0}$$

Symmetry of Ω-Ω

Conserved quantity J, J_z, P

- parity $P = (-1)^L$
- quantum spin $(-1)^{S+1}$

$$(-1)^L \times (-1)^{S+1} = -1$$

fermionic condition

$$\psi_1 \psi_2 = -\psi_2 \psi_1$$

Which L ,S is allowed at J^P

	P=+	P=-
J=0	S=0 L=0 , S=2 L=2	S=1 L=1 , S=3 L=3
J=1	S=2 L=2	S=1 L=1 , S=3 L=3
J=2	S=2 L=0 , S=0 L=2 , S=2 L=2 , S=2 L=4	S=1 L=1 , S=3 L=1 , S=1 L=3 , S=3 L=3 , S=3 L=5
J=3	S=2 L=2 , S=2 L=4	S=3 L=1 , S=1 L=3 , S=3 L=3 , S=3 L=5
J=4	S=2 L=2 , S=0 L=4 , S=2 L=4 , S=2 L=6	S=3 L=1 , S=1 L=3 , S=3 L=3 , S=1 L=5 , S=3 L=5 , S=3 L=7

	P=+	P=-
J=0	S=0 L=0 , S=2 L=2	S=1 L=1 , S=3 L=3
J=1	S=2 L=2	S=1 L=1 , S=3 L=3
J=2	S=2 L=0 , S=0 L=2 , S=2 L=2 , S=2 L=4	S=1 L=1 , S=3 L=1 , S=1 L=3 , S=3 L=3 , S=3 L=5
J=3	S=2 L=2 , S=2 L=4	S=3 L=1 , S=1 L=3 , S=3 L=3 , S=3 L=5
J=4	S=2 L=2 , S=0 L=4 , S=2 L=4 , S=2 L=6	S=3 L=1 , S=1 L=3 , S=3 L=3 , S=1 L=5 , S=3 L=5 , S=3 L=7

at source

at sink

	P=+	P=-
J=0	S=0 L=0 , S=2 L=2	S=1 L=1 , S=3 L=3
J=1	S=2 L=2	S=1 L=1 , S=3 L=3
J=2	S=2 L=0 , S=0 L=2 , S=2 L=2 , S=2 L=4	S=1 L=1 , S=3 L=1 , S=1 L=3 , S=3 L=3 , S=3 L=5
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at source

L=0 ⇐ We use wall source

at sink

	P=+	P=-
J=0	S=0 L=0 , S=2 L=2	S=1 L=1 , S=3 L=3
J=1	S=2 L=2	S=1 L=1 , S=3 L=3
J=2	S=2 L=0 , S=0 L=2 , S=2 L=2 , S=2 L=4	S=1 L=1 , S=3 L=1 , S=1 L=3 , S=3 L=3 , S=3 L=5
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at sink

	P=+	P=-
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at sink

We can extract S=0 L=0 ,S=2 L=2

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Lattice set up

We used 2 sets (Small , Large)

	Small	Large
Lattice volume	L = 1.950(30) [fm]	L = 2.902(42) [fm]
hopping parameters	$K_s = 0.13710$ $K_{ud} = 0.13760$	$K_s = 0.13640$ $K_{ud} = 0.13700$
	$M_\Omega = 2104(8)$[MeV] $M_\pi = 875(1)$[MeV]	$M_\Omega = 1966(6)$[MeV] $M_\pi = 701(5)$[MeV]
β	β=1.83	β=1.90
lattice spaceing	a = 0.1219(19)[fm]	a = 0.0907(13) [fm]

- RG improved gauge action & O(a) improved Wilson quark action
- flat wall source(P=0)

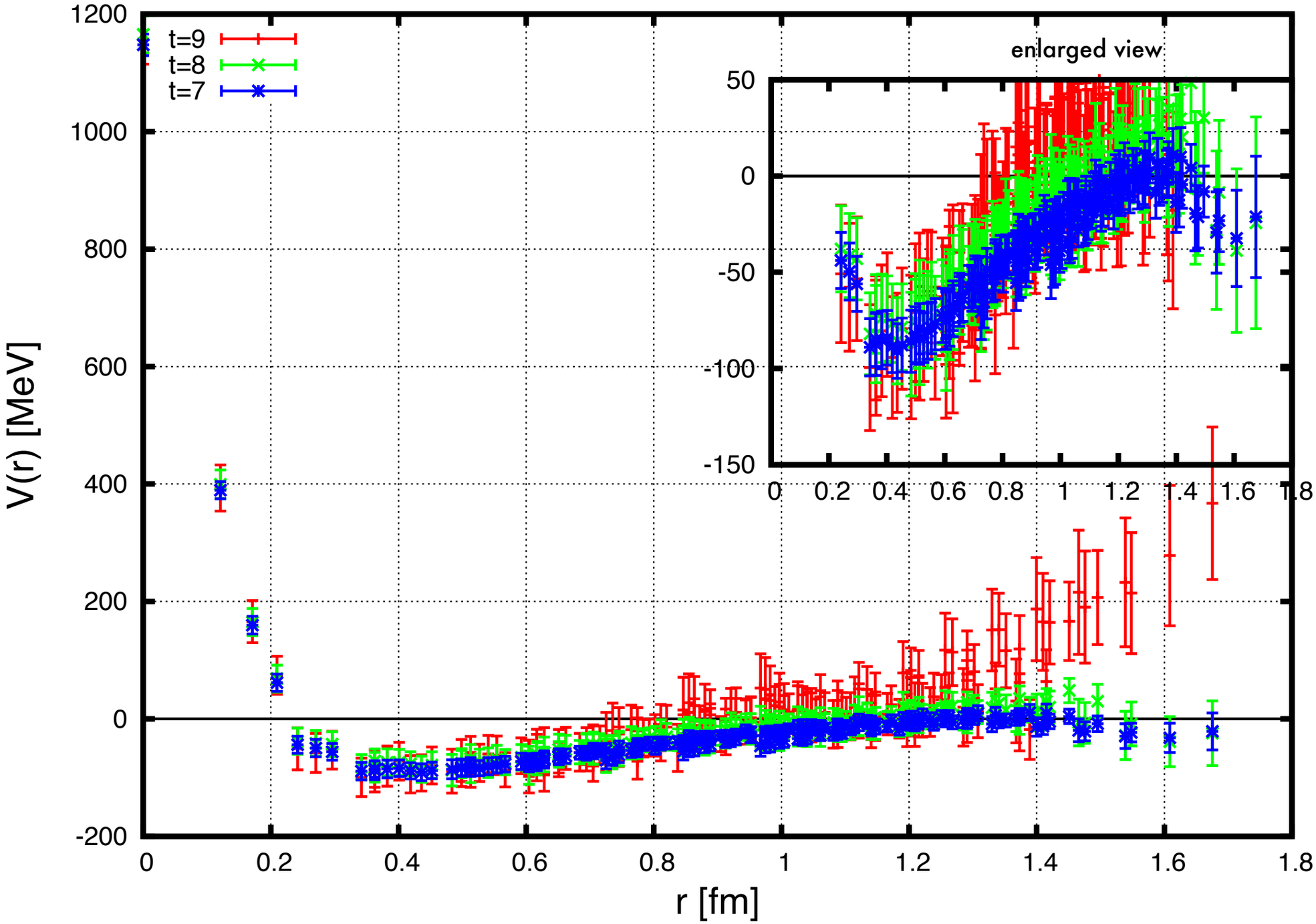
CP-PACS/JLQCD Collaborations: [T. Ishikawa, et al.,Phys. Rev. D78 (2008) 011502(R)]

PACS-CS Collaboration: [S. Aoki, et al.,Phys. Rev. D79 (2009) 034503]

Experiment value of Ω mass is [1672 MeV]

Results of the small volume

Ω-Ω Potential (Small volume) @lattice2013

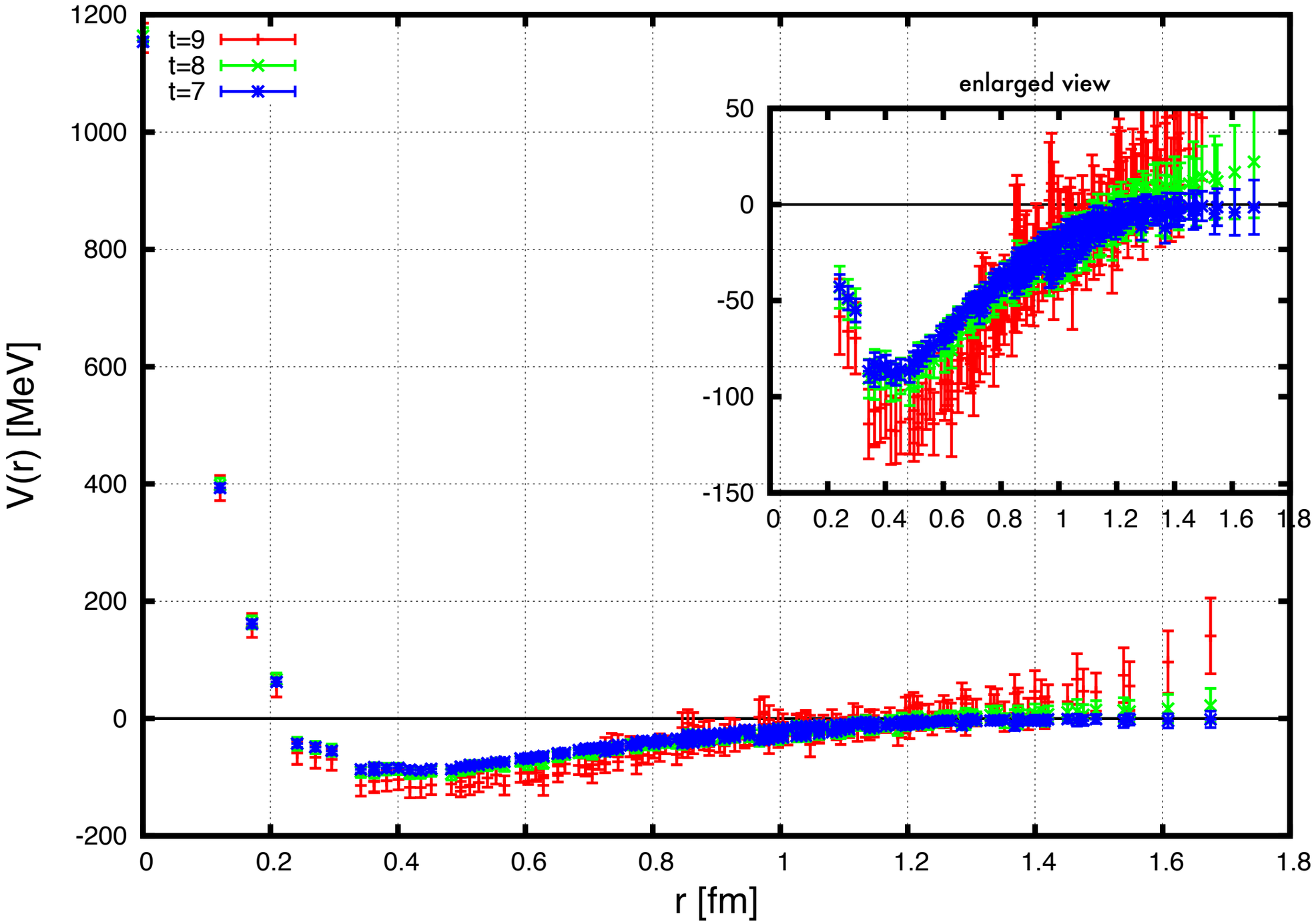


t is relative time between source and sink

$$t \equiv t_1 - t_0$$

$$C_{\Omega\Omega}(\vec{x}, \vec{y}, t_1, t_0) \equiv \langle 0 | \, \Omega(\vec{x}, t_1)\Omega(\vec{y}, t_1)\overline{\Omega}(0, t_0)\overline{\Omega}(0, t_0) \, | 0 \rangle$$

Ω-Ω Potential (Small volume) update



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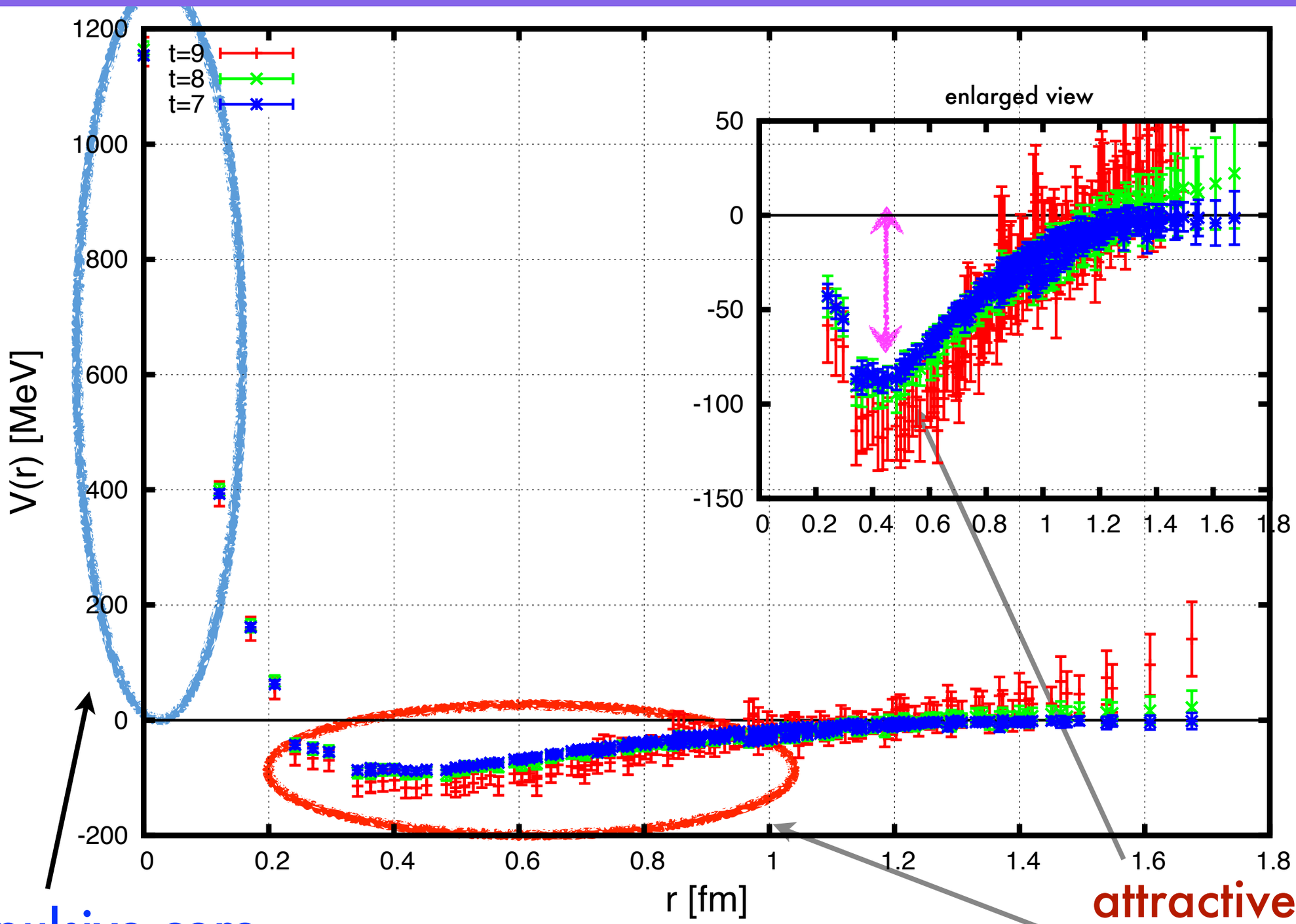

repulsive core

attractive pocket
depth 95 MeV

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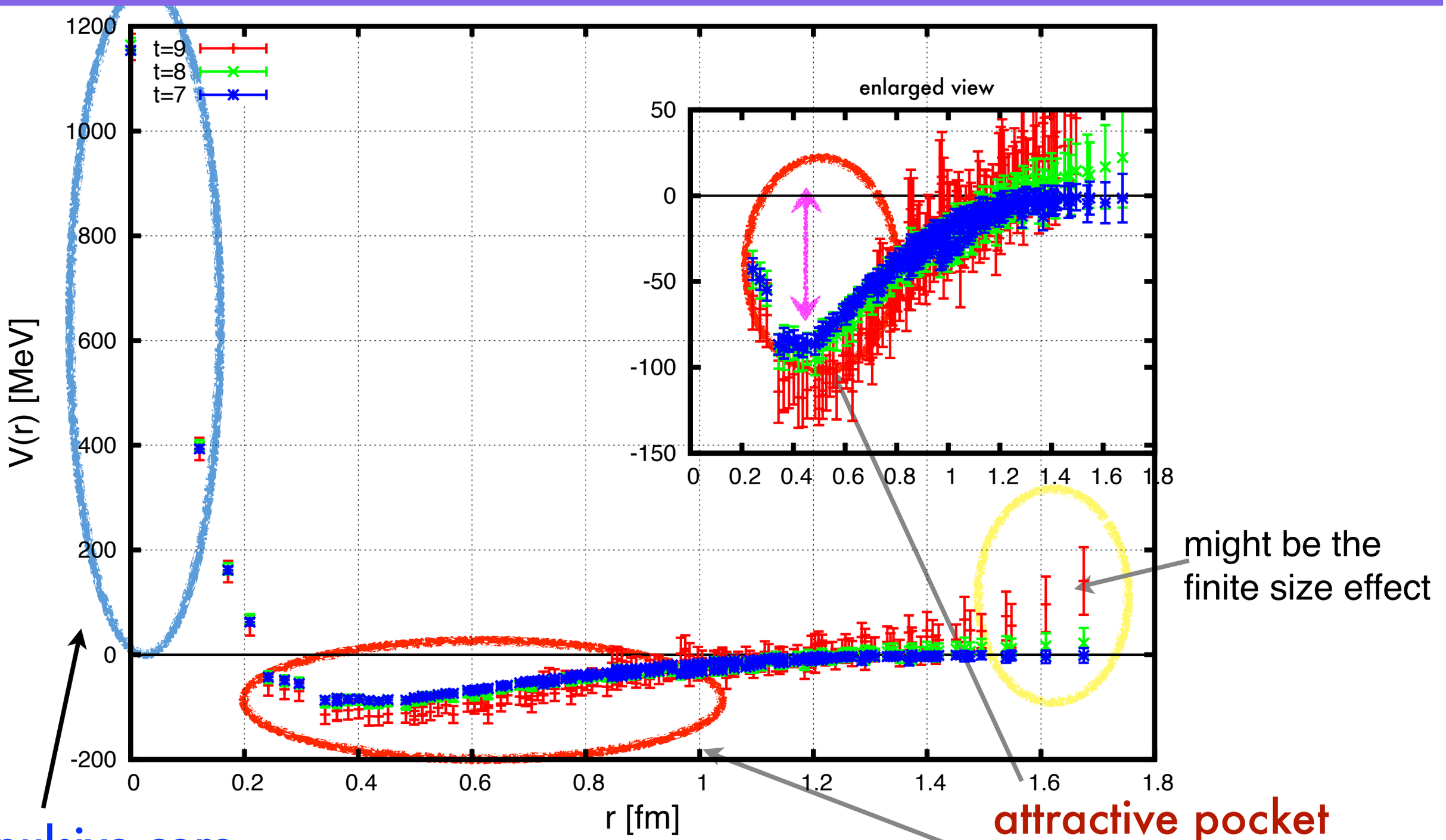

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Phase shift & Binding energy (Small volume)

We found bound states, but binding energies are very small.

	Binding energy	Scattering length
t=7	-0.13 ± 0.28 [MeV]	−13.32 ± 20.34 [fm]
t=8	-0.47 ± 1.00 [MeV]	−7.07 ± 7.70 [fm]
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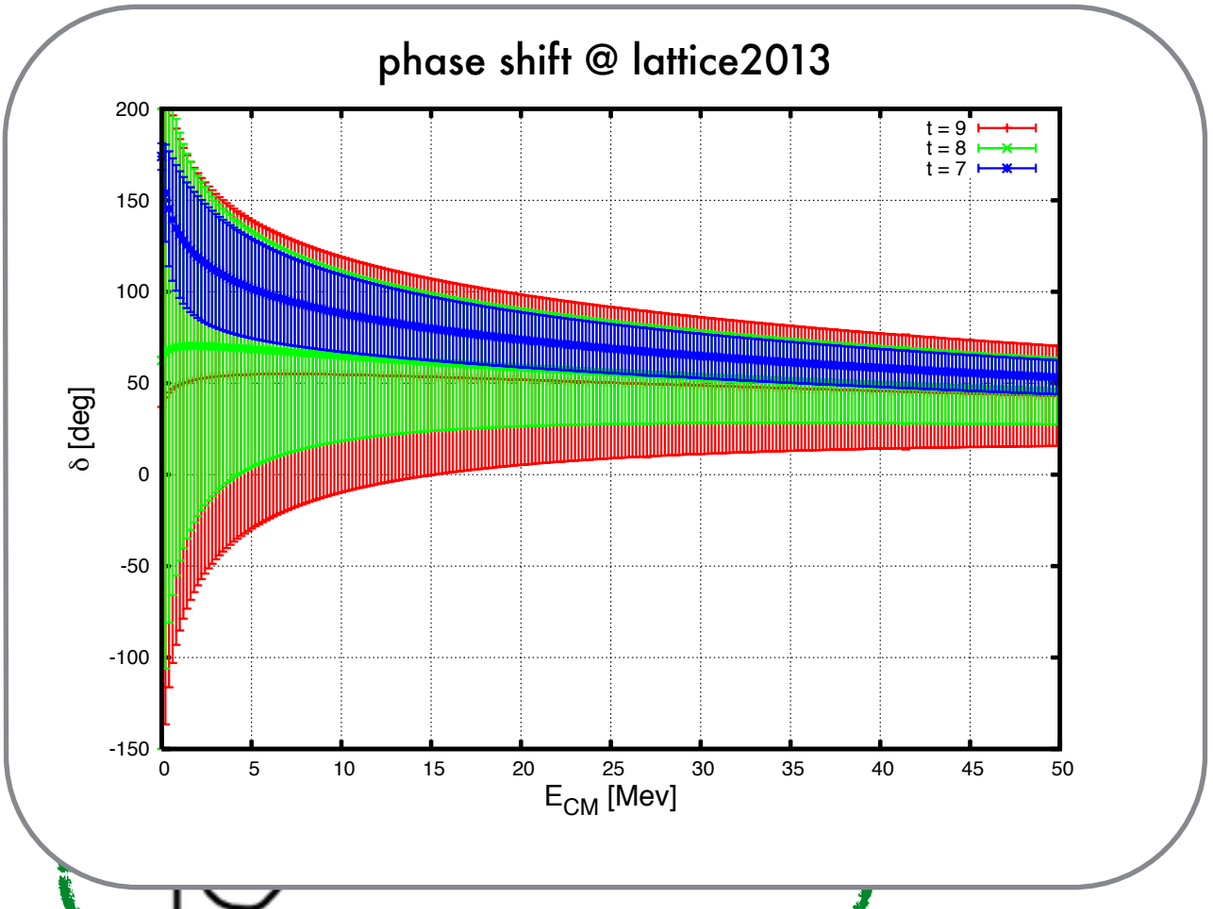

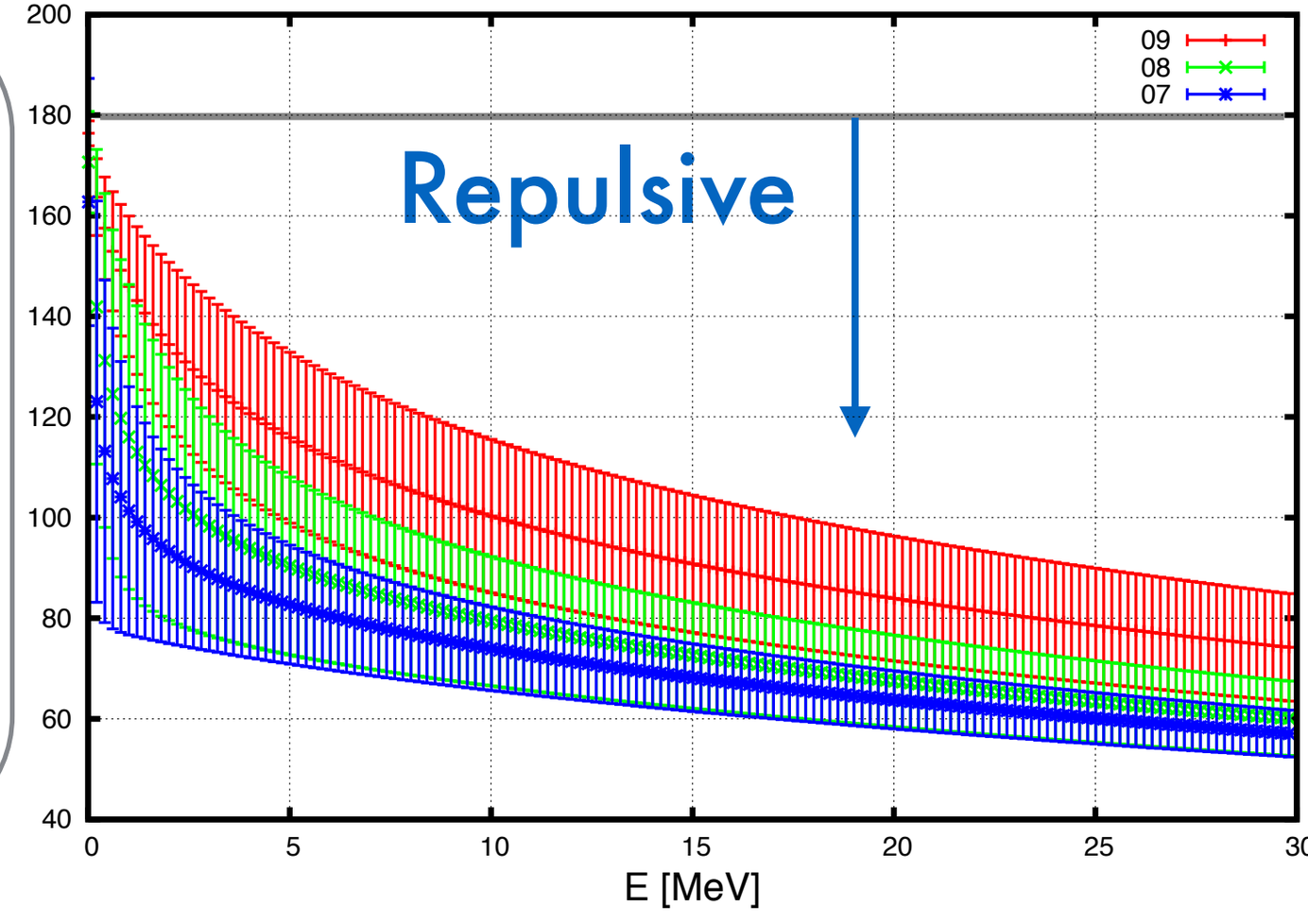

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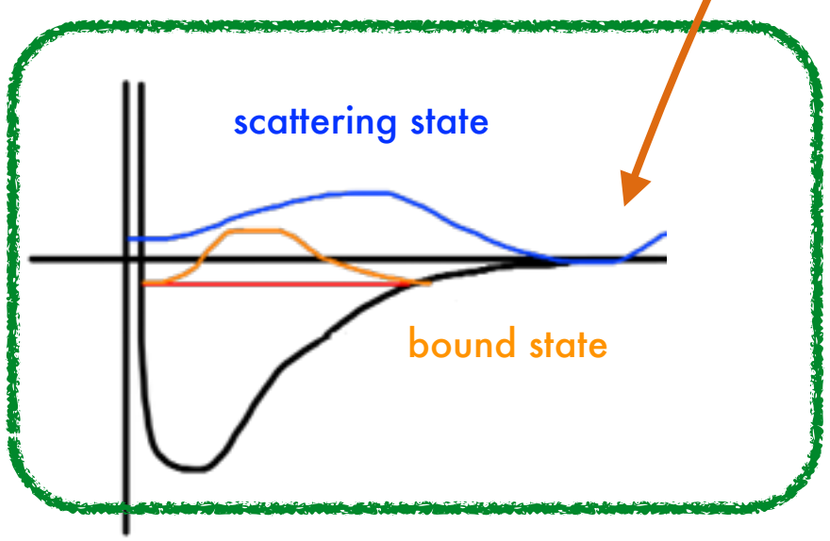
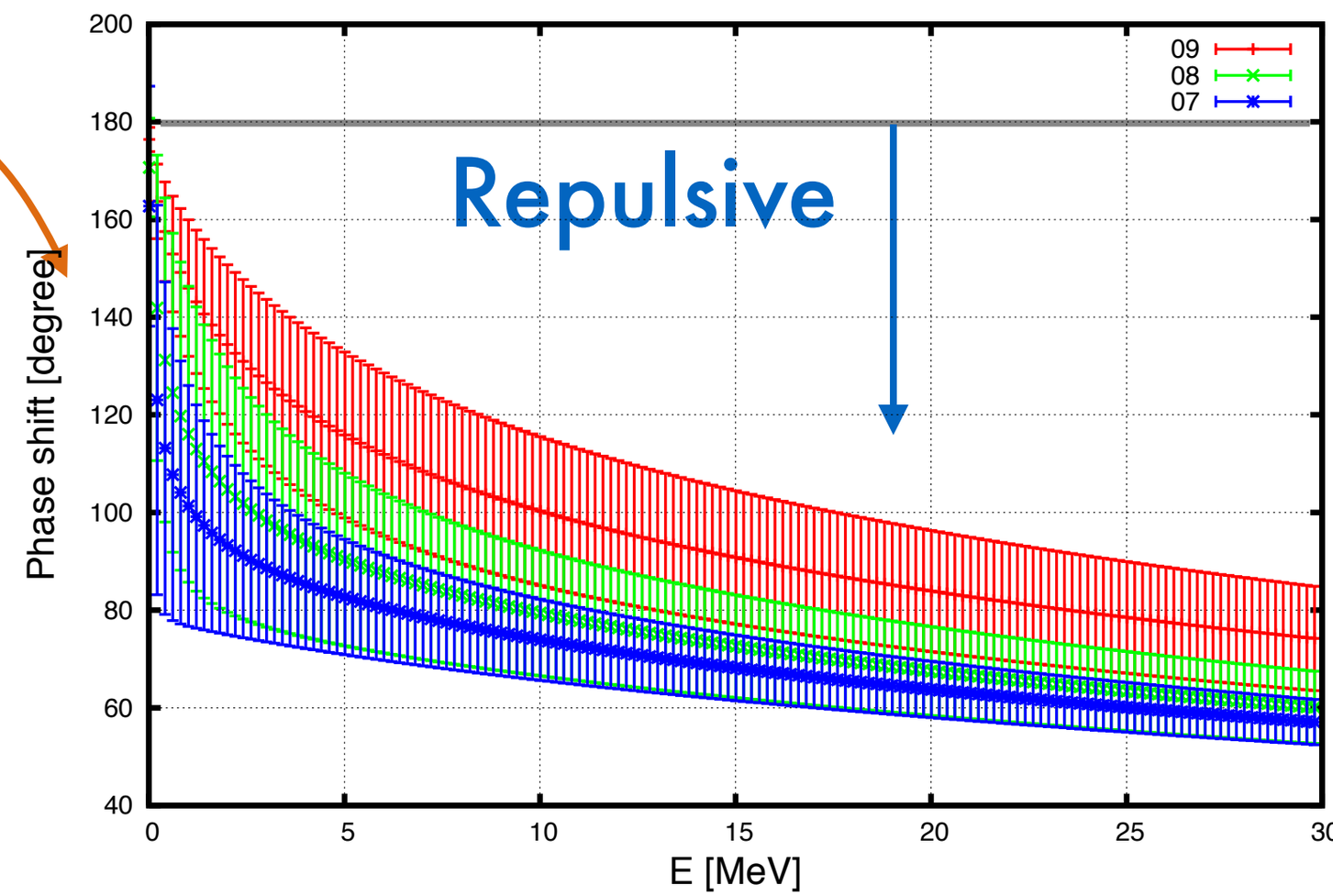
This is the phase shift of the scattering state.

because there is a negative state (bound state).





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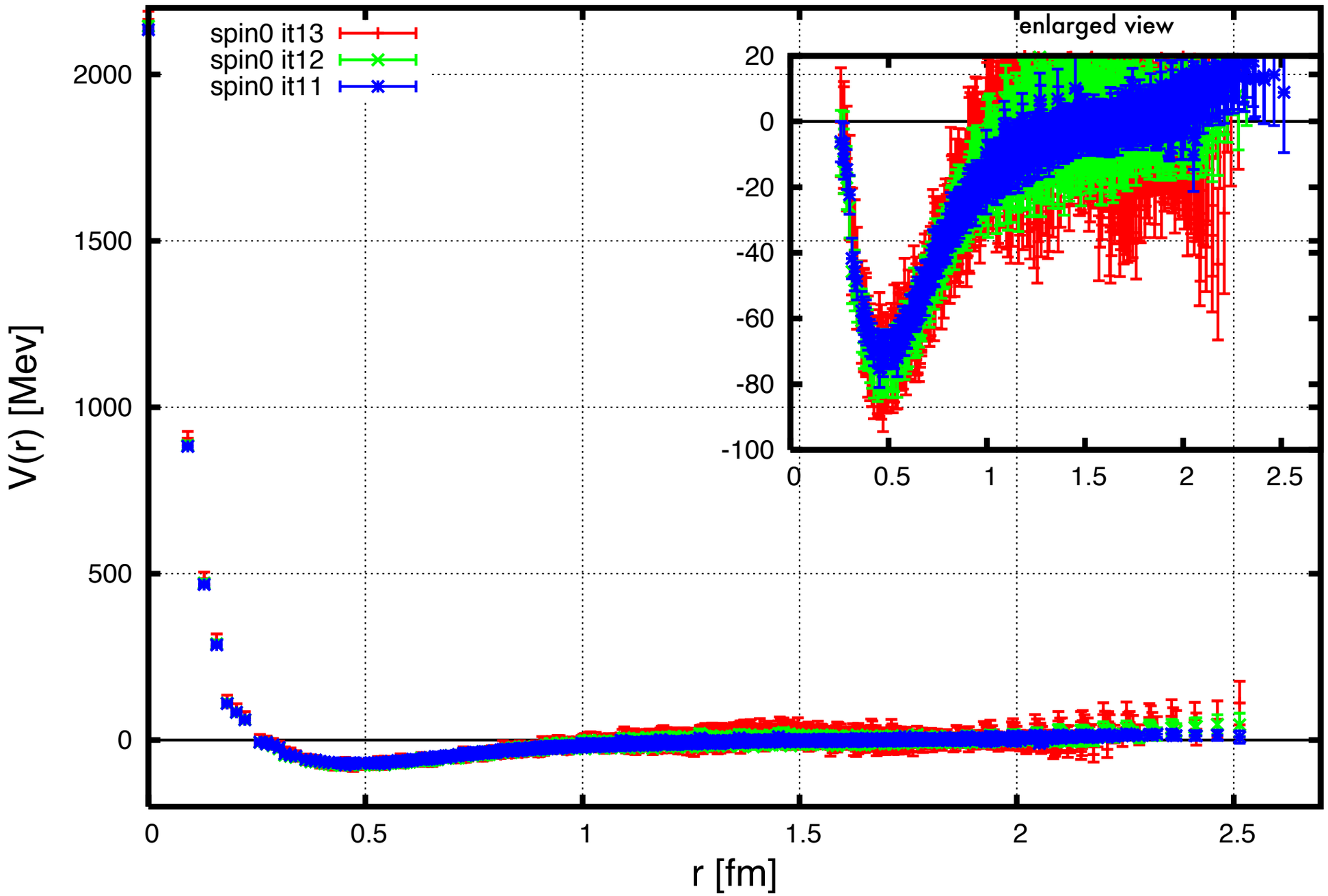

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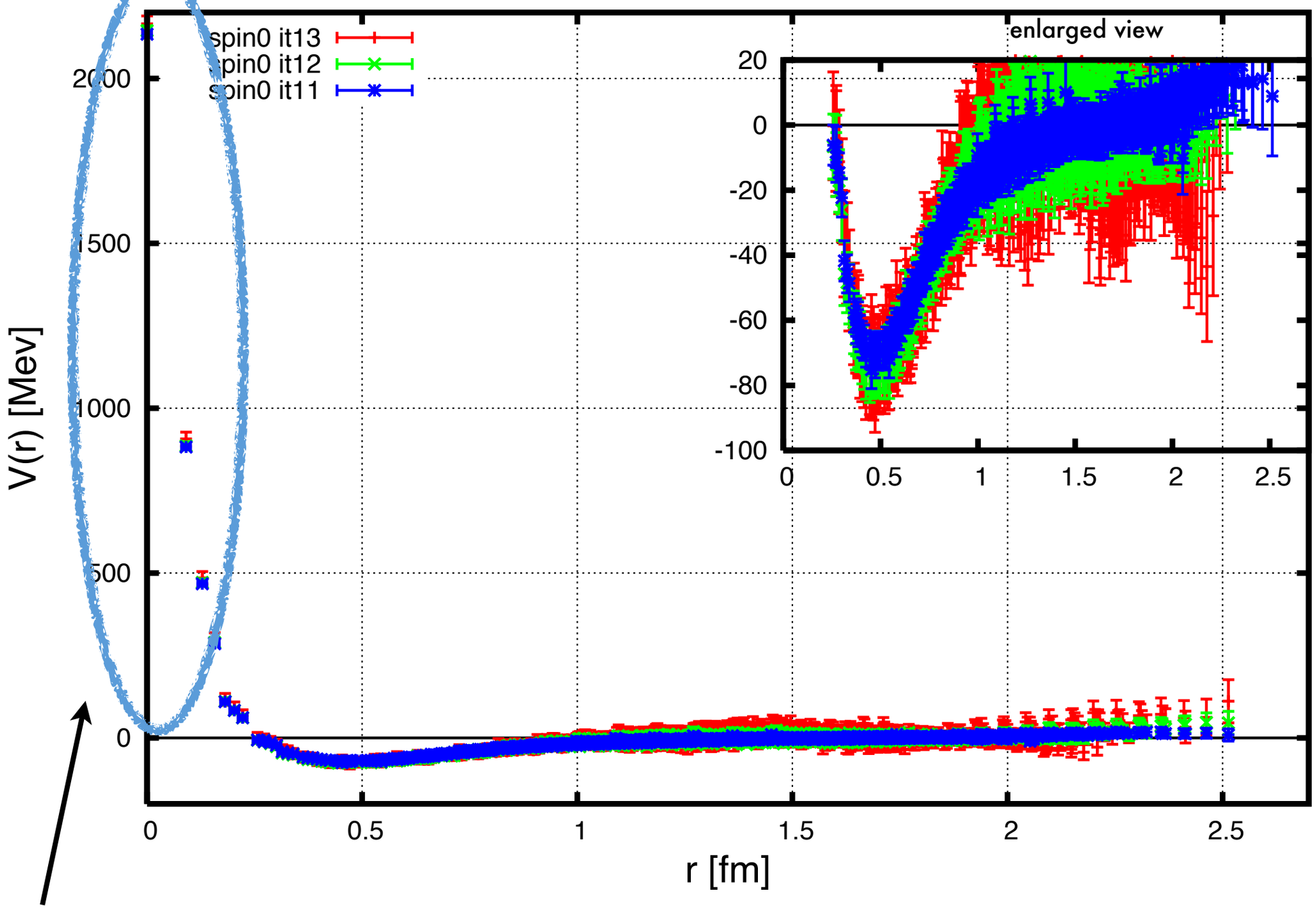

repulsive core grow

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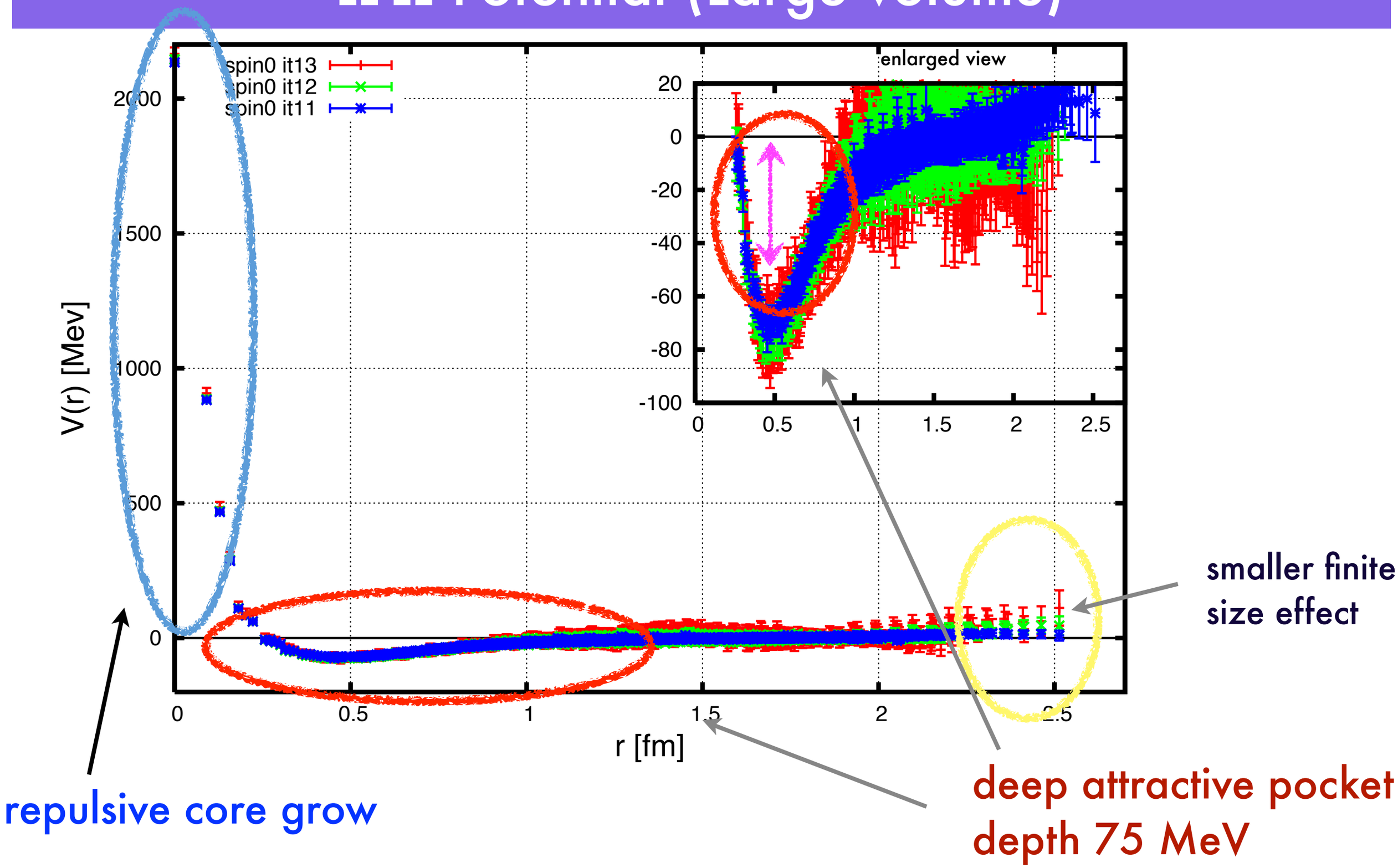
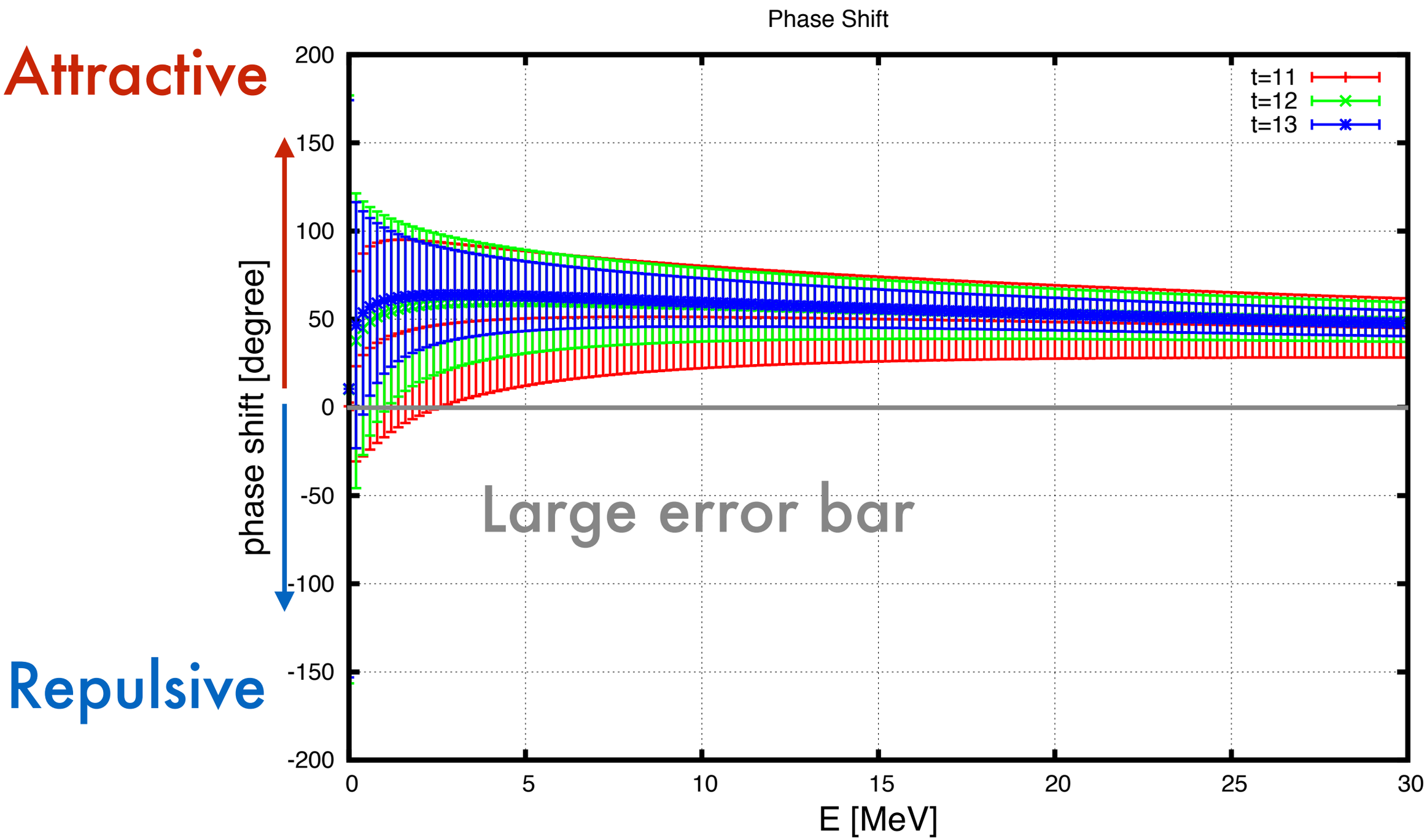
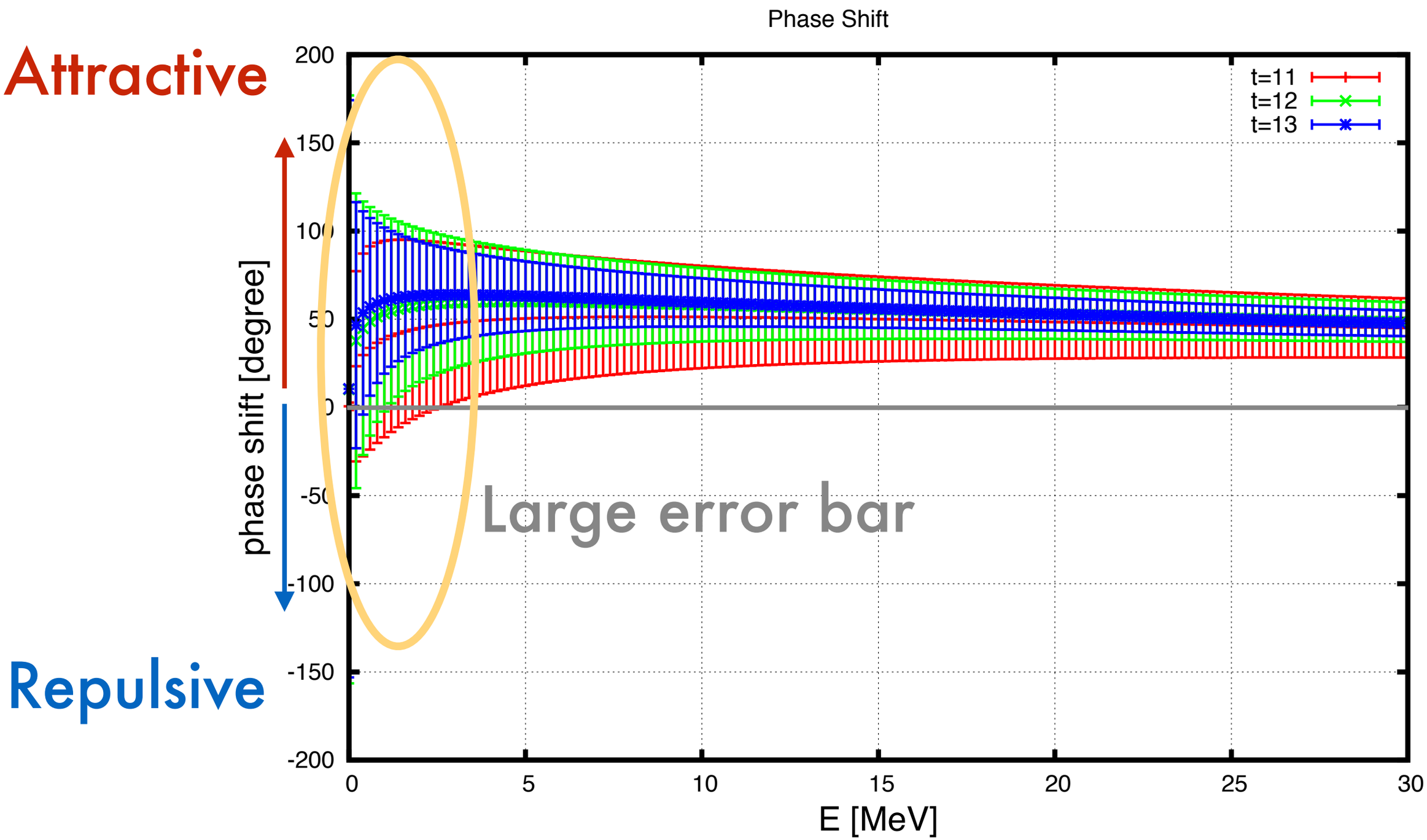


Phase shift (Large volume)



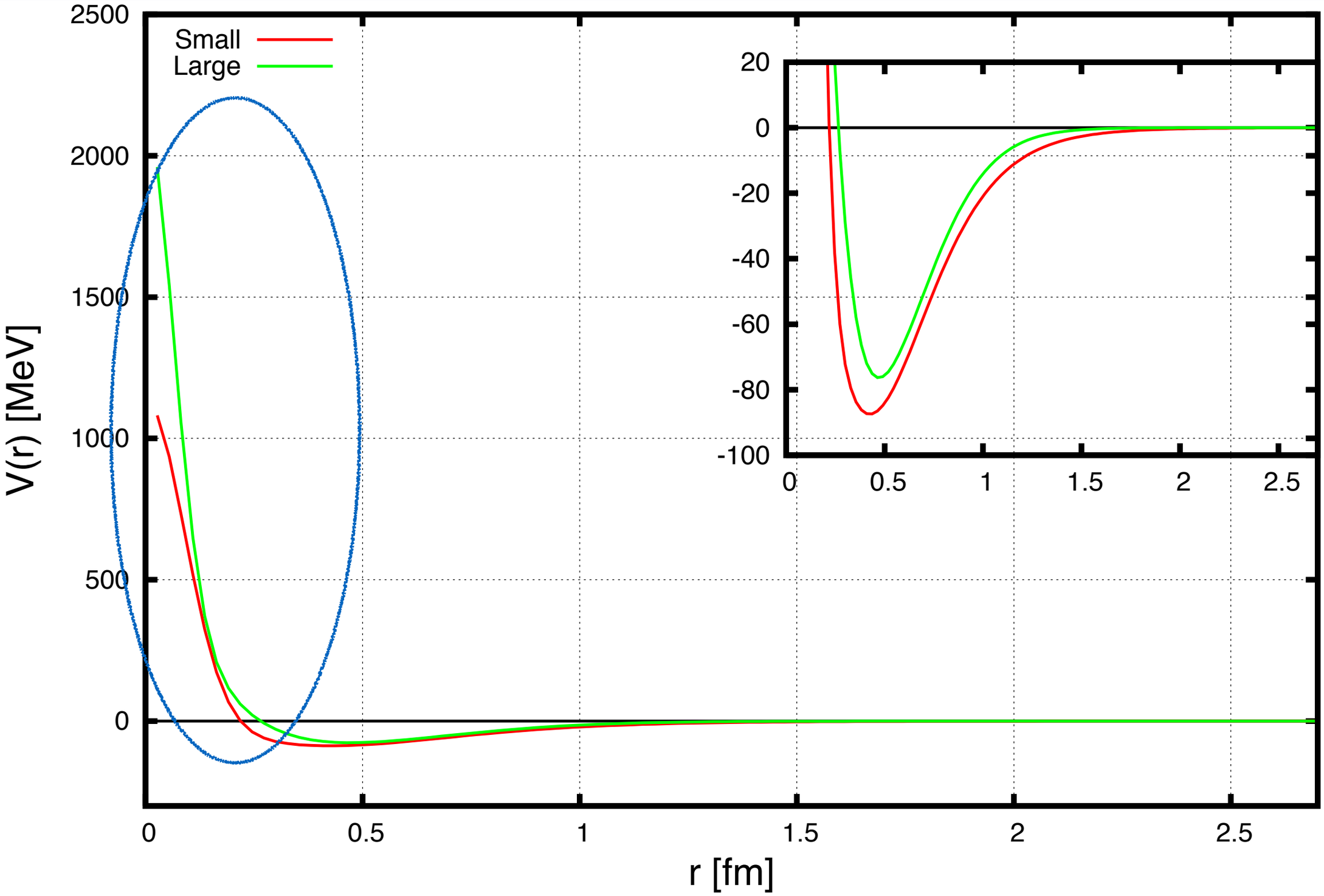
Strongly attractive, but no bound sate exits.

Phase shift (Large volume)



Strongly attractive, but no bound sate exits.

Large vs Small



stronger repulsive core at larger volume

method	potential (our work)	finite volume (Buchoff et al.)
fermion mass	heavy(π=701)	light(π=390)
Lattice volume	2.9[fm]	3.9[fm]
ground sate saturation	not needed	needed
results	strong attraction	weak repulsion(?)

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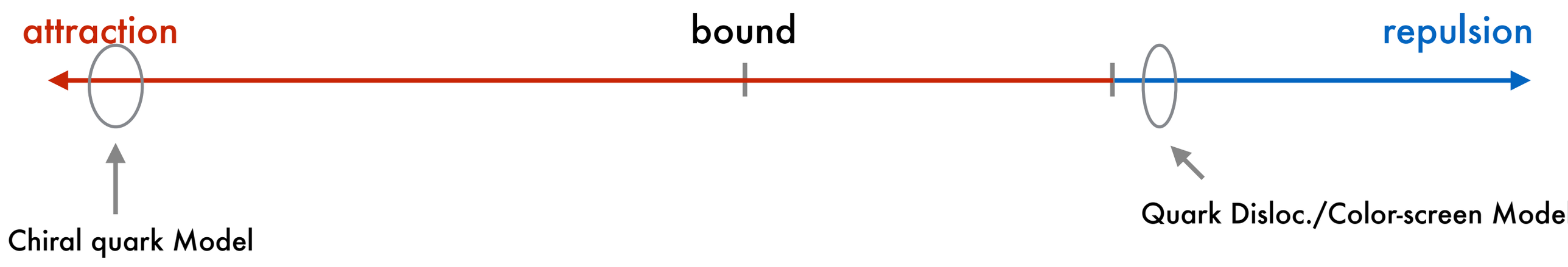

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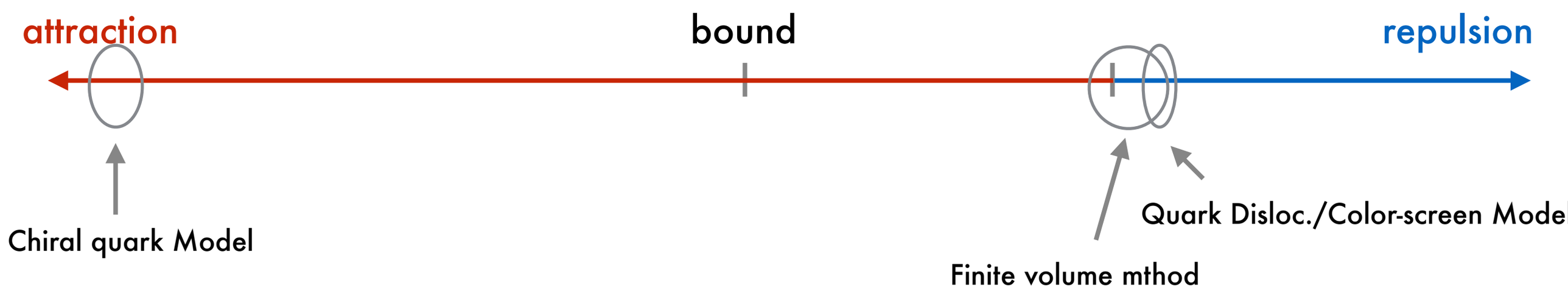

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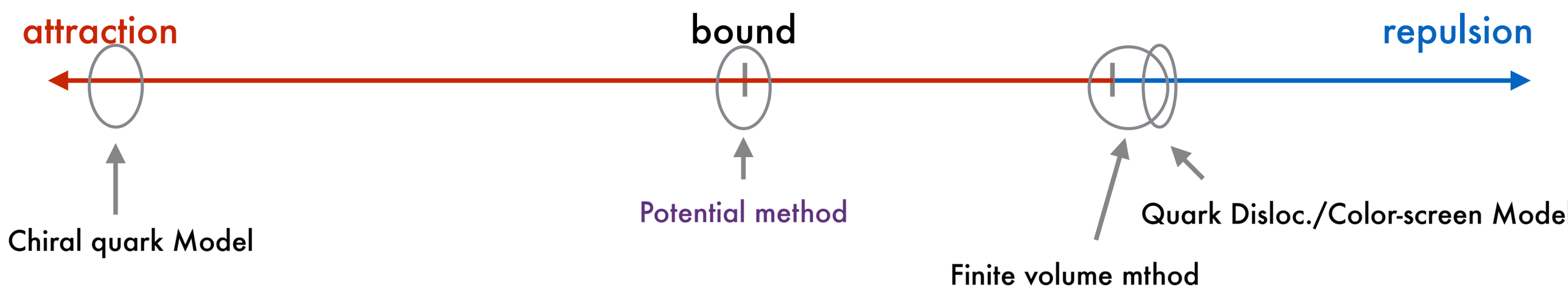

Conclusion

- We extended HAL method to decouplet-decouplet system.
- We showed small volume and large volume results.
- J=0 Omega-Omega interaction is strongly attractive but we can not decide whether the bound state exists or not due to large errors.

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Thank you!

Contact
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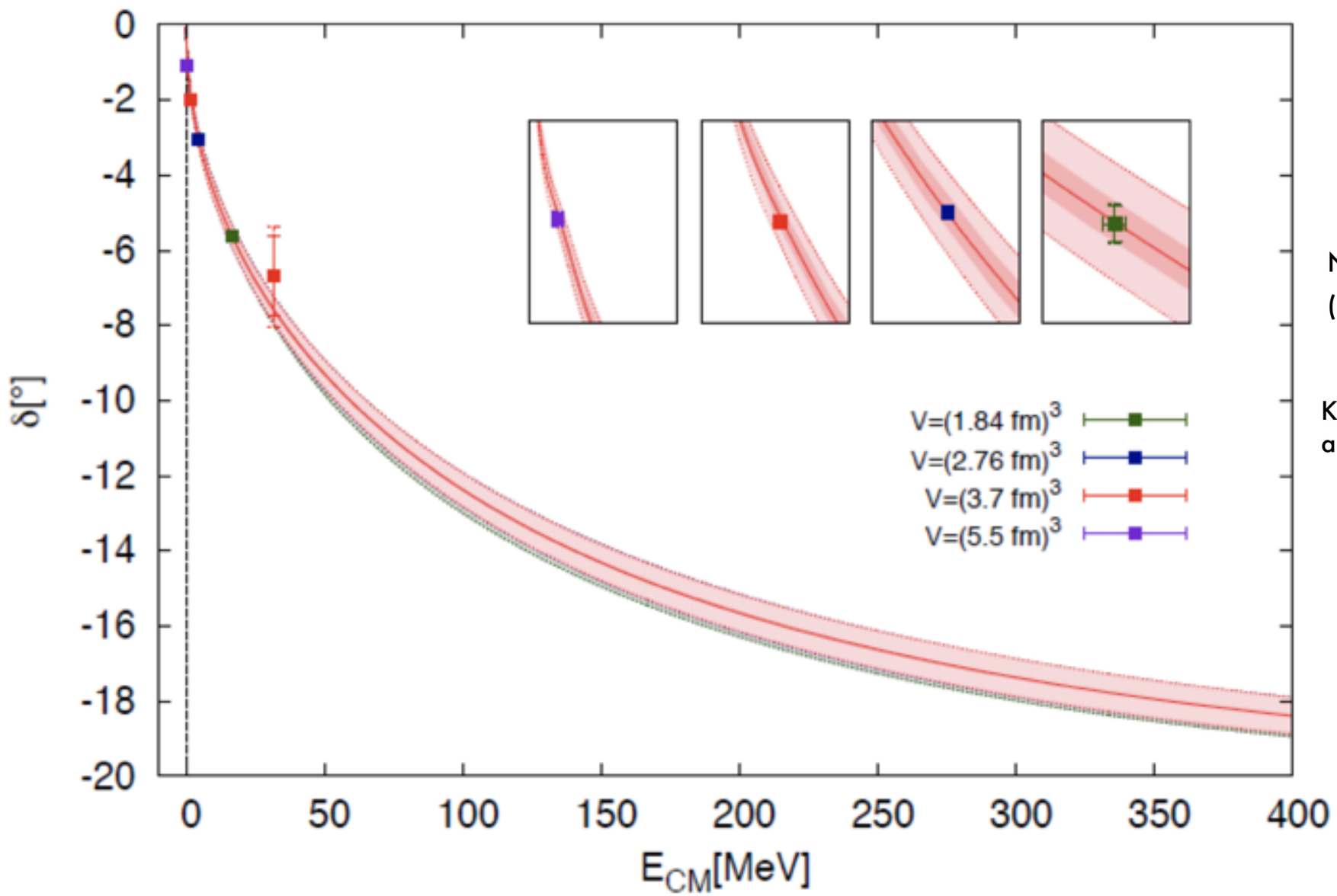
web: http://www-het.ph.tsukuba.ac.jp/~sinyamada/index.html

Back up slide

comparison Luscher method, HAL method(phase shift π-π in I=2 channel)



N_s=16,24,32,48, N_t=128 a=0.115 fm,
(quenched QCD, m_π=940 MeV)

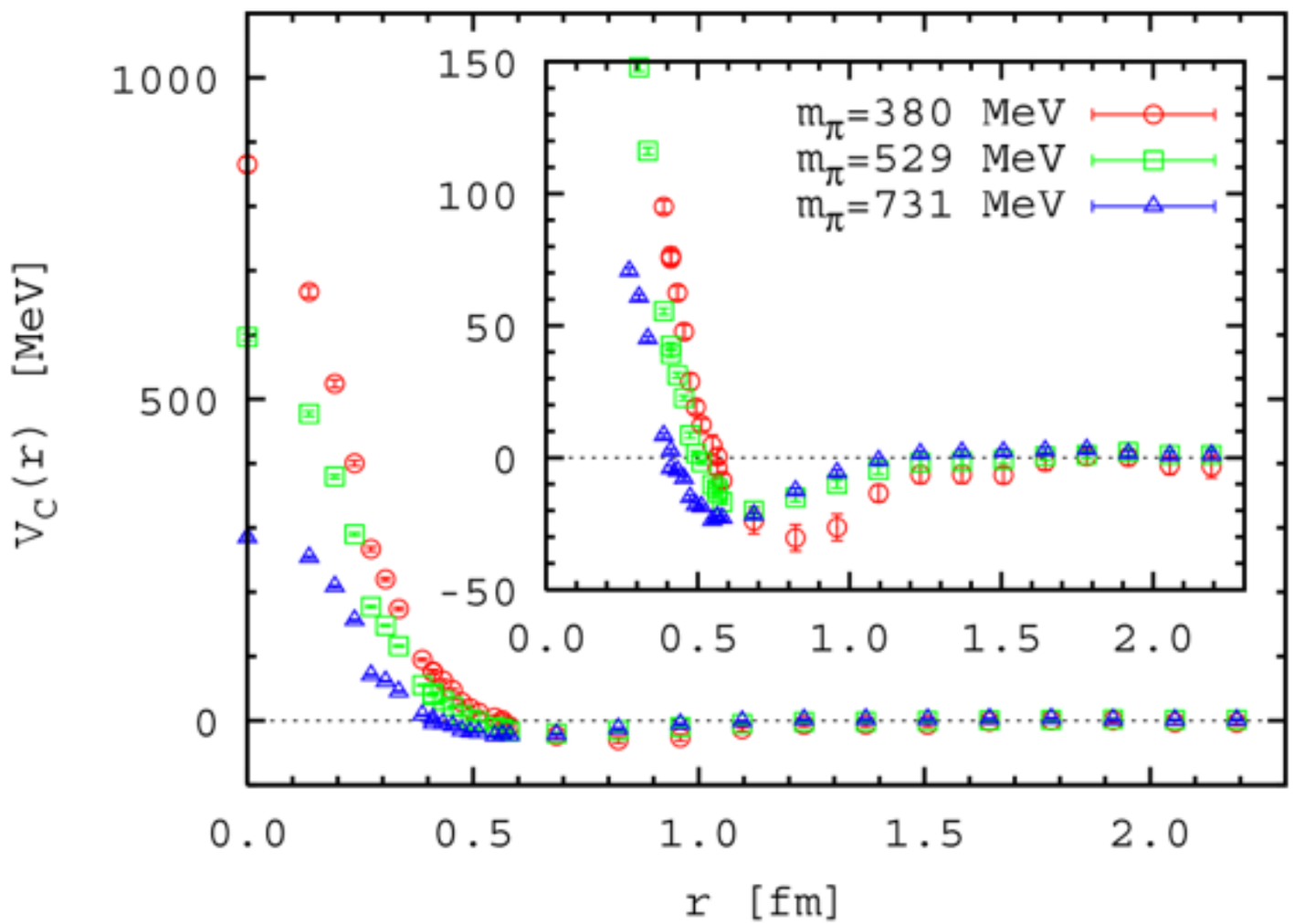
Kurth, Ishii, Doi, Aoki & Hatsuda,
arXiv: 1305.4462 [hep-lat]

The result of phase shift have been found to agree well between the two methods!

It's difficult to compare these methods without calculating finite volume method at large t and more statics!

Back up slide

Mass dependence (N-N interaction)



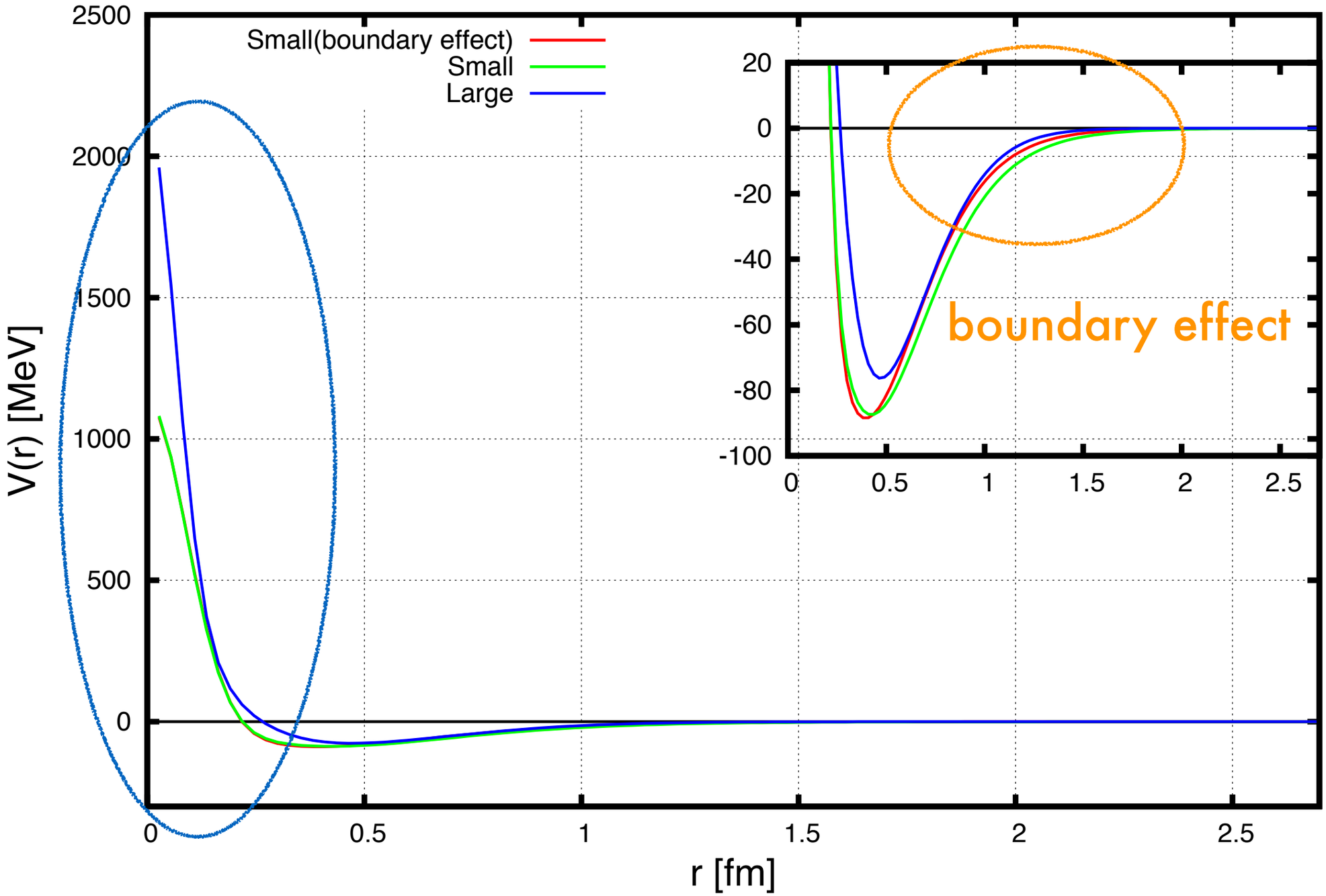
light u,d

Fig. 5. The central potentials in the $^1S_0$ channel for three different quark masses.

We expect Ω-Ω is similar to N-N case

[Sinya Aoki et al. Prog. Theor. Phys. 123 ,89]

Large vs Small



because growth of repulsive core

boundary effect

$$V(\vec{r}) \Rightarrow \tilde{V}(\vec{r}) = \sum_{\vec{n}\in\mathbb{Z}^3} V(\vec{r} + L\vec{n})$$

S=0 ⇐ a special circumstance in Ω-Ω system

- flavor is completely symmetry
- wall source

source operator

$$\overline{\Omega} = \varepsilon^{abc}(\gamma_k C)_{\beta\gamma}\overline{s}^a_\alpha \overline{s}^b_\beta \overline{s}^c_\gamma$$

a,b,c: color index
α,β,γ: spin index

highest state in Ω-Ω (spin3)

$$\overline{s}^a_{\frac{1}{2}}(x)\overline{s}^b_{\frac{1}{2}}(x)\overline{s}^c_{\frac{1}{2}}(x)\overline{s}^a_{\frac{1}{2}}(y)\overline{s}^b_{\frac{1}{2}}(y)\overline{s}^c_{\frac{1}{2}}(y)$$

For simply neglect ε , γC

We can make all state using lowering operator

spin3⇒spin2⇒spin1⇒spin0

For example one term of spin2 state

$$\overline{s}^a_{\frac{1}{2}}(x)\overline{s}^b_{\frac{1}{2}}(x)\overline{s}^c_{\frac{1}{2}}(x)\overline{s}^a_{\frac{1}{2}}(y)\overline{s}^b_{\frac{1}{2}}(y)\overline{s}^c_{-\frac{1}{2}}(y)$$

spin2 term is written by linear combination of these terms.

$$\overline{s}^a_{\frac{1}{2}}(x)\overline{s}^b_{\frac{1}{2}}(x)\overline{s}^c_{\frac{1}{2}}(x)\overline{s}^a_{\frac{1}{2}}(y)\overline{s}^b_{\frac{1}{2}}(y)\overline{s}^c_{-\frac{1}{2}}(y)$$

wall source⇒ $$\left(\sum_x \overline{s}^a_{\frac{1}{2}}(x)\right)\left(\sum_{x'} \overline{s}^b_{\frac{1}{2}}(x')\right)\left(\sum_{x''} \overline{s}^c_{\frac{1}{2}}(x'')\right)\left(\sum_y \overline{s}^a_{\frac{1}{2}}(y)\right)\left(\sum_{y'} \overline{s}^b_{\frac{1}{2}}(y')\right)\left(\sum_{y''} \overline{s}^c_{-\frac{1}{2}}(y'')\right)$$

$$= -\left(\sum_y \overline{s}^a_{\frac{1}{2}}(y)\right)\left(\sum_{x'} \overline{s}^b_{\frac{1}{2}}(x')\right)\left(\sum_{x''} \overline{s}^c_{\frac{1}{2}}(x'')\right)\left(\sum_x \overline{s}^a_{\frac{1}{2}}(x)\right)\left(\sum_{y'} \overline{s}^b_{\frac{1}{2}}(y')\right)\left(\sum_{y''} \overline{s}^c_{-\frac{1}{2}}(y'')\right)$$

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$$= 0$$

Spin2 state should be 0

$$\overline{s}^a_{\frac{1}{2}}(x)\overline{s}^b_{\frac{1}{2}}(x)\overline{s}^c_{\frac{1}{2}}(x)\overline{s}^a_{\frac{1}{2}}(y)\overline{s}^b_{\frac{1}{2}}(y)\overline{s}^c_{-\frac{1}{2}}(y)$$

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fermionic

$$= -\left(\sum_y \overline{s}^a_{\frac{1}{2}}(y)\right)\left(\sum_{x'} \overline{s}^b_{\frac{1}{2}}(x')\right)\left(\sum_{x''} \overline{s}^c_{\frac{1}{2}}(x'')\right)\left(\sum_x \overline{s}^a_{\frac{1}{2}}(x)\right)\left(\sum_{y'} \overline{s}^b_{\frac{1}{2}}(y')\right)\left(\sum_{y''} \overline{s}^c_{-\frac{1}{2}}(y'')\right)$$

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x,y are inner arguments

$$= -\left(\sum_x \overline{s}^a_{\frac{1}{2}}(x)\right)\left(\sum_{x'} \overline{s}^b_{\frac{1}{2}}(x')\right)\left(\sum_{x''} \overline{s}^c_{\frac{1}{2}}(x'')\right)\left(\sum_y \overline{s}^a_{\frac{1}{2}}(y)\right)\left(\sum_{y'} \overline{s}^b_{\frac{1}{2}}(y')\right)\left(\sum_{y''} \overline{s}^c_{-\frac{1}{2}}(y'')\right)$$

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Spin2 state should be 0

Spin0 remain

$$\left(\sum_x \overline{s}^a_{\frac{1}{2}}(x)\right)\left(\sum_{x'} \overline{s}^b_{\frac{1}{2}}(x')\right)\left(\sum_{x''} \overline{s}^c_{\frac{1}{2}}(x'')\right)\left(\sum_y \overline{s}^a_{-\frac{1}{2}}(y)\right)\left(\sum_{y'} \overline{s}^b_{-\frac{1}{2}}(y')\right)\left(\sum_{y''} \overline{s}^c_{-\frac{1}{2}}(y'')\right)$$

Existence of energy independent nonlocal potential

We assume linear independence of NBS wave function
There is a dual bases

$$\int d^3r \tilde{\psi}_{k'}(r)\psi_k(r) = (2\pi)^3\delta^3(k'-k)$$

We define K

$$\begin{aligned} K_k(r) &\equiv (\nabla^2 + k^2)\psi_k(r) \\ &= \int \frac{d^3k'}{(2\pi)^3} K_{k'}(r) \int d^3r' \tilde{\psi}_{k'}(r')\psi_k(r') \\ &= \int d^3r' \left\{ \int \frac{d^3k'}{(2\pi)^3} K_{k'}(r)\tilde{\psi}_{k'}(r') \right\} \psi_k(r') \end{aligned}$$

If we define

$$U(r,r') \equiv \frac{1}{m} \int \frac{d^3k'}{(2\pi)^3} K_{k'}(r)\tilde{\psi}_{k'}(r')$$

Then we have

$$(\frac{k^2}{m} + \frac{1}{m}\nabla^2)\psi_k(r) = \int d^3r' U(r,r')\psi_k(r')$$

Construction of the potential

Extraction of the NBS wave from Lattice QCD

$$C_{\Omega\Omega}(\vec{x}, \vec{y}, t, t_0) \equiv \langle 0| \Omega(\vec{x}, t)\Omega(\vec{y}, t)\overline{\Omega}(0, t_0)\overline{\Omega}(0, t_0) |0\rangle$$

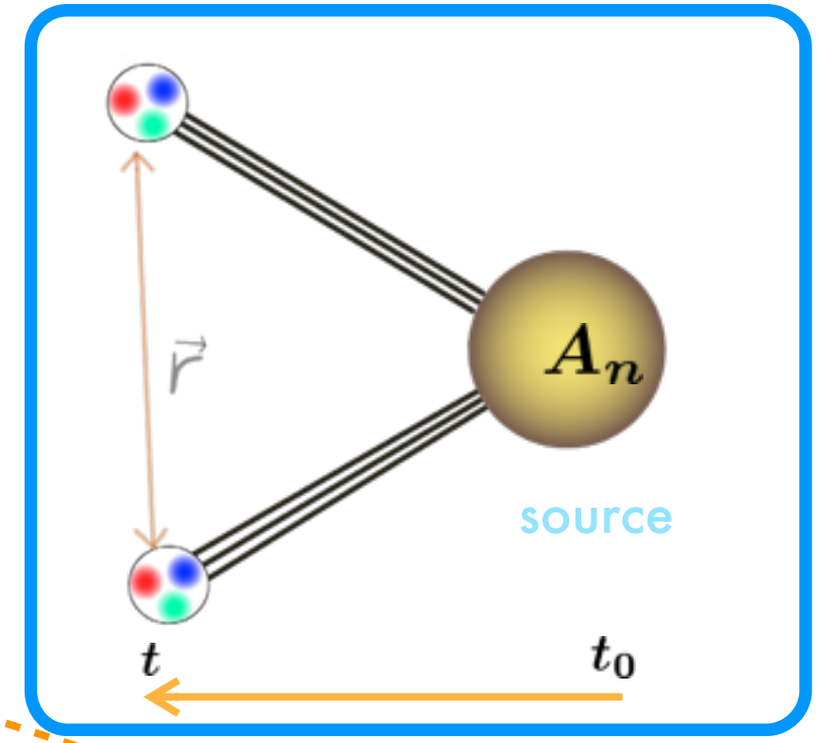
$$= \sum_n \langle 0| \Omega(\vec{x}, t)\Omega(\vec{y}, t) |n\rangle e^{-E_n(t-t_0)} \langle n| \overline{\Omega}(0, t_0)\overline{\Omega}(0, t_0) |0\rangle$$

$$\psi_{k_n}(\vec{x} - \vec{y}, n)$$

$$A_n$$

$$= \sum_n A_n \psi_{k_n}(x - y, n) e^{-E_n(t-t_0)} + \cdots$$

Image



inelastic contributions

Excited states are suppressed exponentially at large $t - t_0$
We can get the NBS wave at ground state

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Extraction of the NBS wave from Lattice QCD

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$$1 = \sum_n |n\rangle\langle n|$$

$$= \sum_n \langle 0|\,\Omega(\vec{x},t)\Omega(\vec{y},t)\,|n\rangle\, e^{-E_n(t-t_0)}\,\langle n|\,\overline{\Omega}(0,t_0)\overline{\Omega}(0,t_0)\,|0\rangle$$

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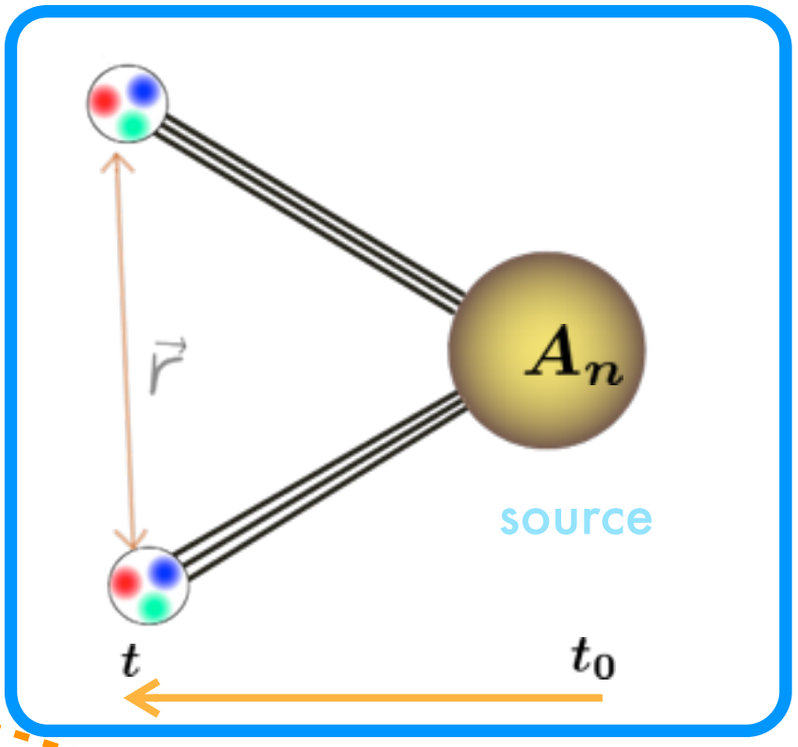
Image



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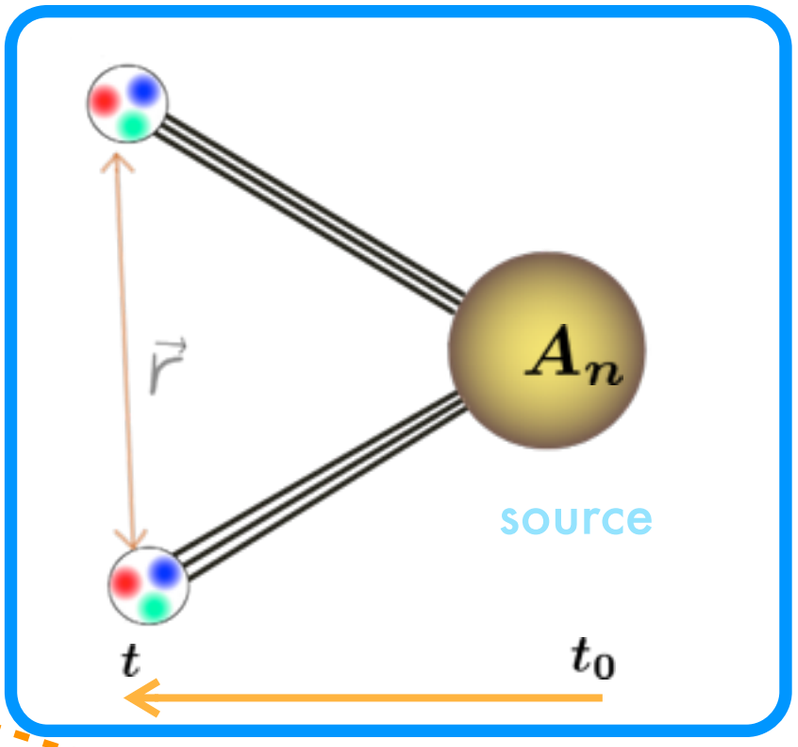

inelastic contributions

Excited states are suppressed exponentially at large $t - t_0$
We can get the NBS wave at ground state

An origin of large statistical errors

HAL QCD method

[N.Ishii et al.,PLB712(2012)437.]

Time dependent Schrodinger-type equation

$$\left(\frac{1}{4m}\frac{\partial^2}{\partial t^2} + \frac{1}{m}\nabla^2 - \frac{\partial}{\partial t}\right)R = \int dr' U(r,r')R$$

HAL QCD method

[N.Ishii et al.,PLB712(2012)437.]

Time dependent Schrodinger-type equation

R-correlator is defined as

$$R \equiv \frac{\Psi(r,t)}{e^{-2mt}} = \sum_n \phi_n(r) e^{-W_n t}$$

$$W_n \equiv 2\sqrt{m^2 + \vec{k}_n^{\,2}} - 2m$$

$$\boxed{\left(\frac{1}{4m}\frac{\partial^2}{\partial t^2} + \frac{1}{m}\nabla^2 - \frac{\partial}{\partial t}\right)R = \int dr' U(r,r') R}$$

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①

$$-\frac{\partial}{\partial t} R = \sum_n W_n \phi_n(r) e^{-W_n t} = \sum_n \left(\frac{\vec{k}_n^{\,2}}{m} - \frac{W^2}{4m}\right)\phi_n(r) e^{-W_n t}$$

$$\left(\frac{1}{4m}\frac{\partial^2}{\partial t^2} + \frac{1}{m}\nabla^2 - \frac{\partial}{\partial t}\right)R = \int dr' U(r,r') R$$

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$$-\frac{\partial}{\partial t} R = \sum_n W_n \phi_n(r) e^{-W_n t} = \sum_n \left(\frac{\vec{k}_n^2}{m} - \frac{W^2}{4m}\right)\phi_n(r) e^{-W_n t}$$

$$\left(\frac{1}{4m}\frac{\partial^2}{\partial t^2} + \frac{1}{m}\nabla^2 - \frac{\partial}{\partial t}\right)R = \int dr' U(r,r') R$$

① From identity

$$W_n = \frac{\vec{k}_n^2}{m} - \frac{W_n^2}{4m}$$

$$\frac{W_n^2}{4m} = \frac{1}{4m}\left(4m^2 + 4\vec{k}_n^2 + 4m^2 - 8m\sqrt{m^2 + \vec{k}_n^2}\right)$$

$$= 2m + \frac{\vec{k}_n^2}{m} - 2\sqrt{m^2 + \vec{k}_n^2}$$

$$= \frac{\vec{k}_n^2}{m} - W_n$$

HAL QCD method

[N.Ishii et al.,PLB712(2012)437.]

Time dependent Schrodinger-type equation

R-correlator is defined as

$$W_n \equiv 2\sqrt{m^2 + \vec{k}_n^2} - 2m$$

$$R \equiv \frac{\Psi(r,t)}{e^{-2mt}} = \sum_n \phi_n(r) e^{-W_n t}$$

①

$$-\frac{\partial}{\partial t} R = \sum_n W_n \phi_n(r) e^{-W_n t} = \sum_n \left(\frac{\vec{k}_n^2}{m} - \frac{W^2}{4m}\right)\phi_n(r) e^{-W_n t}$$

$$= \sum_n \left(\frac{\vec{k}_n^2}{m} - \frac{1}{4m}\frac{\partial^2}{\partial t^2}\right)\phi_n(r) e^{-W_n t}$$

②

$$= \left(-\frac{1}{m}\nabla^2 - \frac{1}{4m}\frac{\partial^2}{\partial t^2}\right)R + \int dr' U(r,r') R$$

$$\left(\frac{1}{4m}\frac{\partial^2}{\partial t^2} + \frac{1}{m}\nabla^2 - \frac{\partial}{\partial t}\right)R = \int dr' U(r,r') R$$

② NBS wave function satisfies Schorodinger eq.

$$\left(\frac{k^2}{m} + \frac{1}{m}\nabla^2\right)\psi_k(r) = \int d^3r' U(r,r')\psi_k(x')$$

Time depend method

[N.Ishii et al.,PLB712(2012)437.]

Time dependent Schrodinger-like equation

$$\left(\frac{1}{4m}\frac{\partial^2}{\partial t^2}+\frac{1}{m}\nabla^2-\frac{\partial}{\partial t}\right)R=\int dr' U(r,r')R$$

Time depend method

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$$\left(\frac{1}{4m}\frac{\partial^2}{\partial t^2} + \frac{1}{m}\nabla^2 - \frac{\partial}{\partial t}\right)R = \int dr' U(r,r')R$$

We can calculate energy independent non-local potential
without relying on the ground state saturation!

fit function dependence

fit function

Gauss + Yukawa

$$f(r) = b_1 e^{-b_2 r^2} + b_3(1 - e^{-b_4 r^2})\frac{e^{-b_5 r}}{r}$$

$$\lim_{r \to 0} f(r) = b_1$$

Gauss + Yukawa^2

$$f(r) = b_1 e^{-b_2 r^2} + b_3(1 - e^{-b_4 r^2})^2(\frac{e^{-b_5 r}}{r})^2$$

$$\lim_{r \to 0} f(r) = b_1$$

2Gauss + Yukawa^2

$$f(r) = b_1 e^{-b_2 r^2} + b_3 e^{-b_4 r^2} + b_5(1 - e^{-b_6 r^2})^2(\frac{e^{-b_7 r}}{r})^2$$

$$\lim_{r \to 0} f(r) = b_1 + b_3$$