TMD-Factorization with Evolution

T. C. Rogers

C.N. Yang Institute for Theoretical Physics, SUNY Stony Brook

- What is needed.
- Summary of TMD-factorization.
- Recent results.
- Summary of work in progress.

Workshop on Forward Physics at RHIC – July 31, 2012

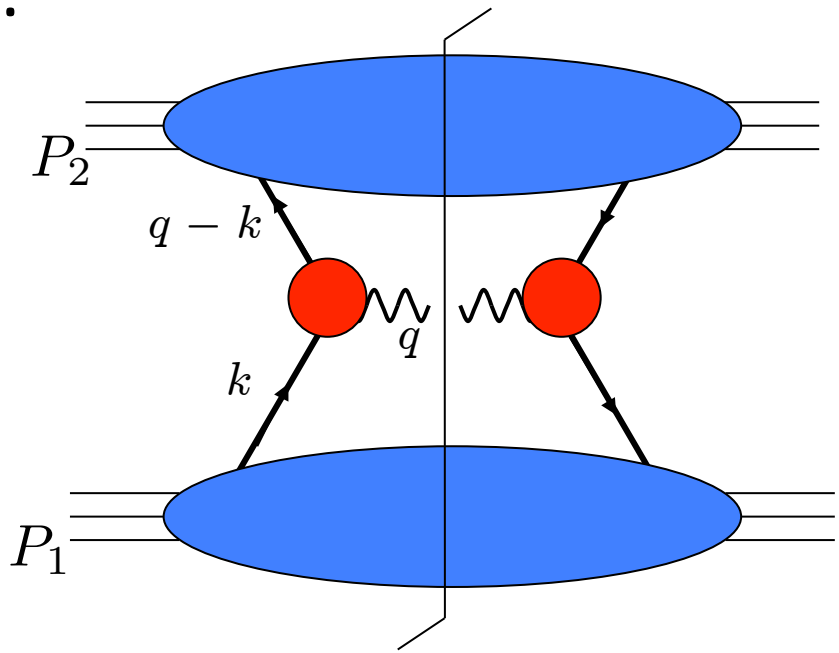
Example:

- Drell-Yan q_T distribution.

Get $\dfrac{d\sigma}{d\mathbf{q}_T \cdots}$

For <u>all</u> q_T.

$$q^2 = Q^2 \gg \Lambda^2_{QCD}$$

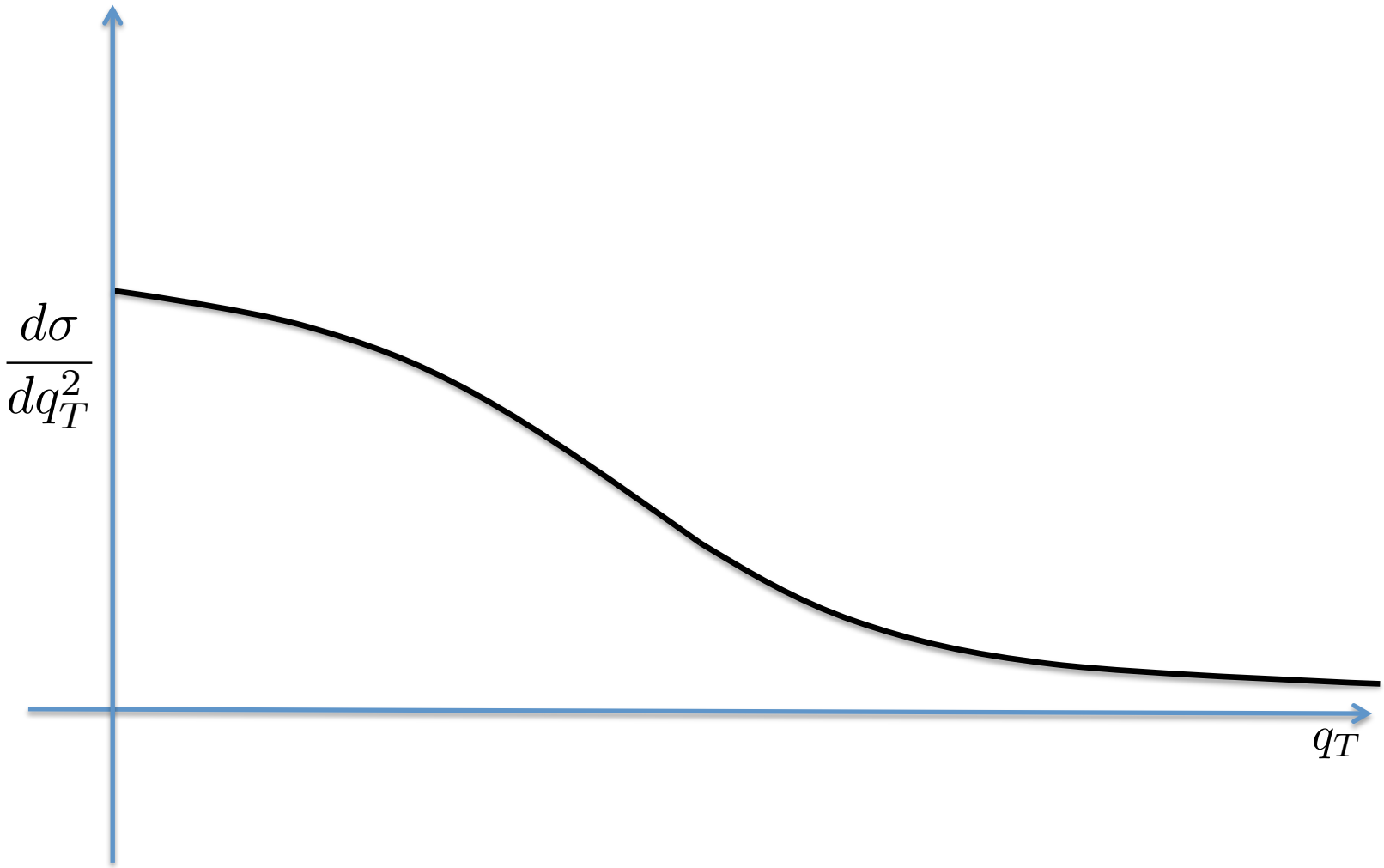


$$k_1 = k$$

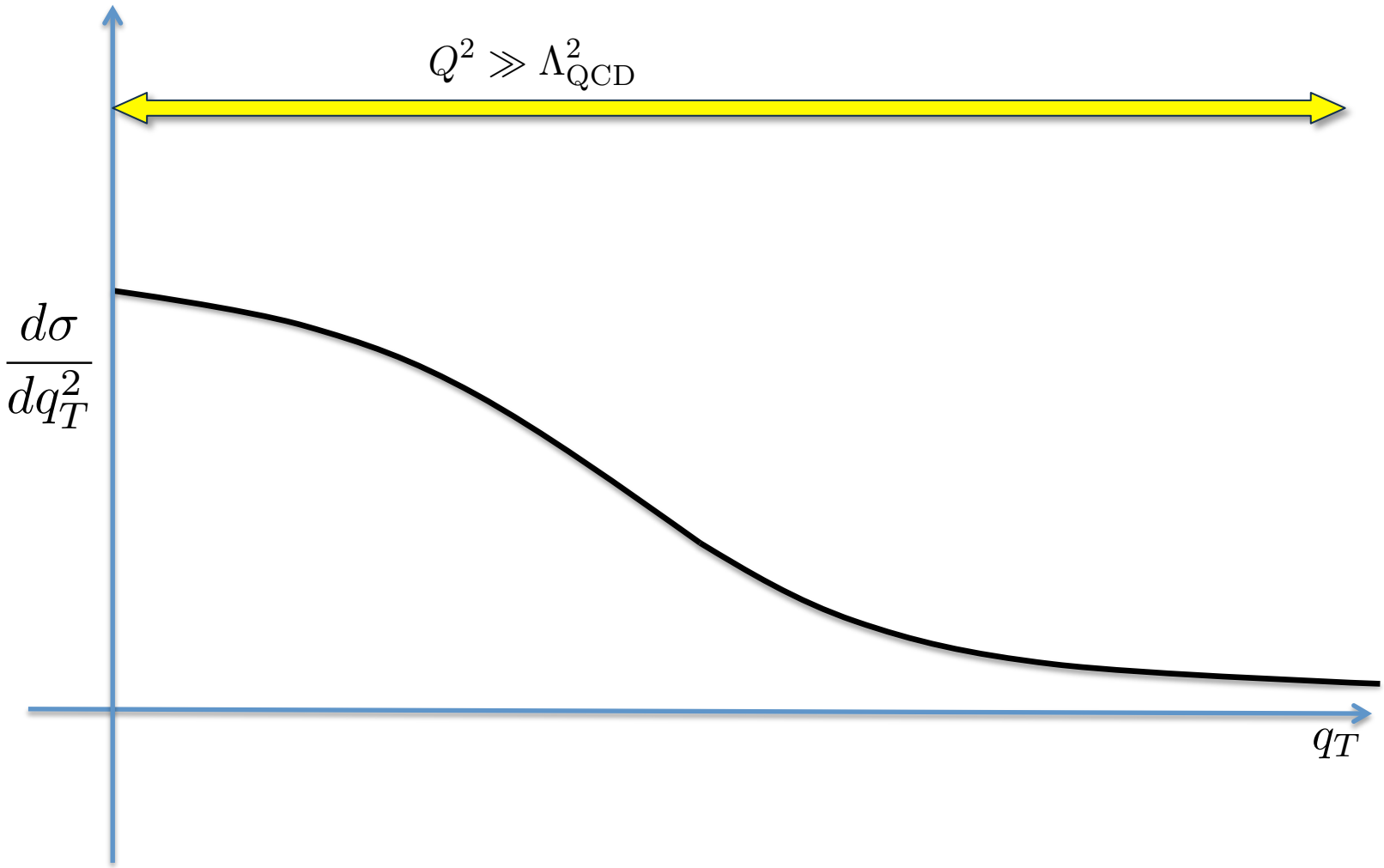
$$k_2 = q - k$$

$$\mathbf{k}_{1T} + \mathbf{k}_{2T} = \mathbf{q}_T$$

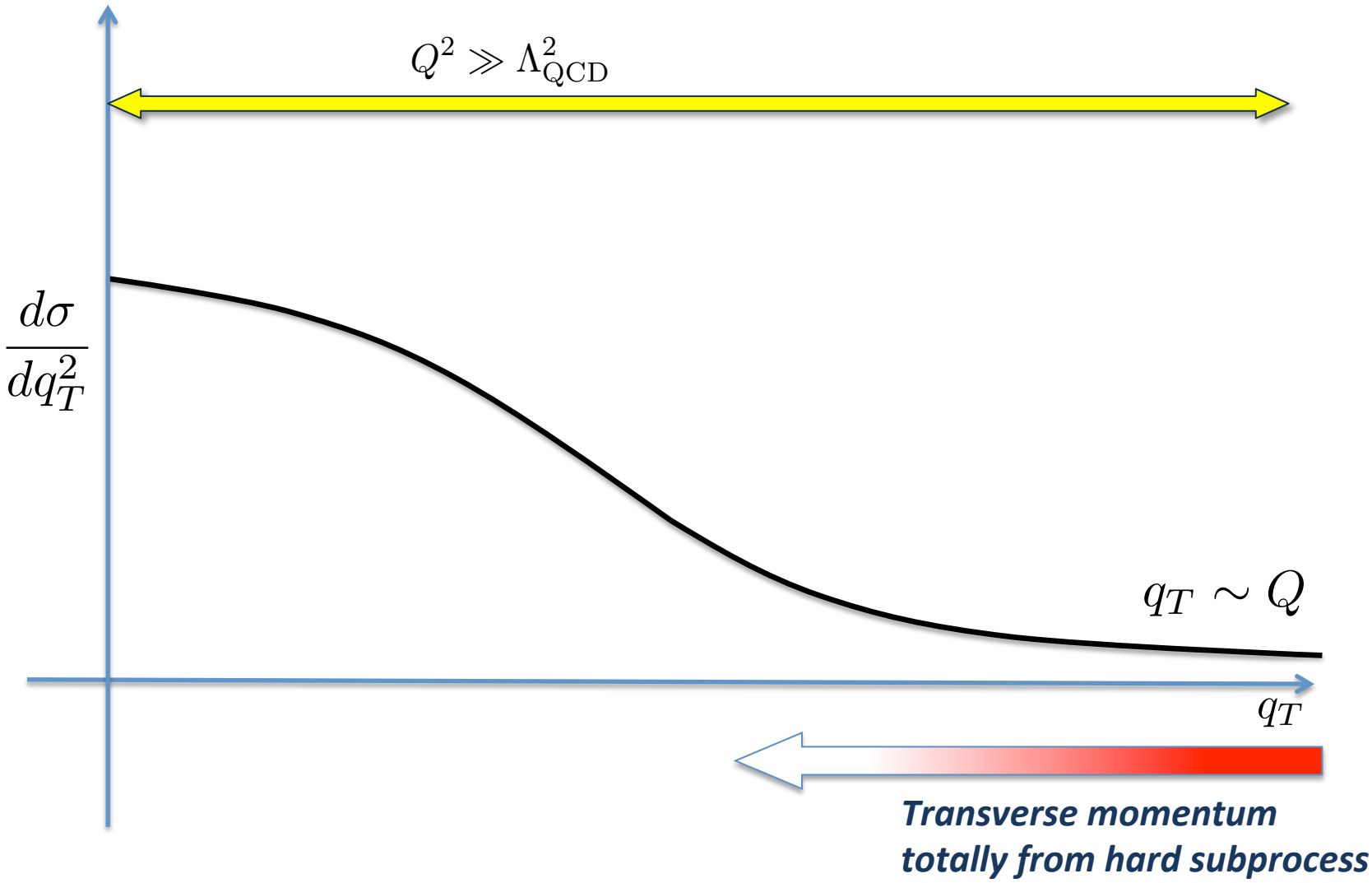
Regions of Transverse Momentum:



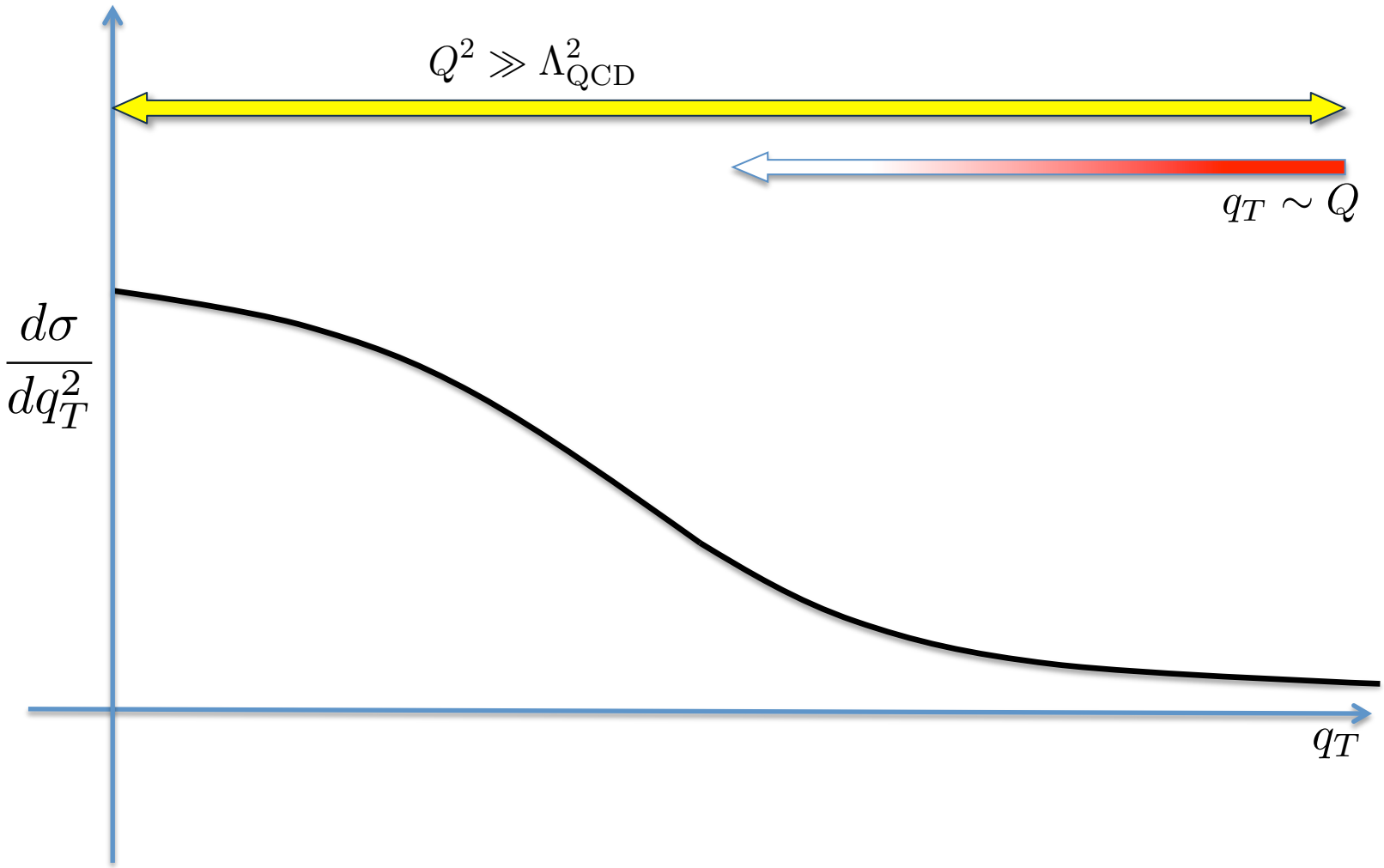
Regions of Transverse Momentum:



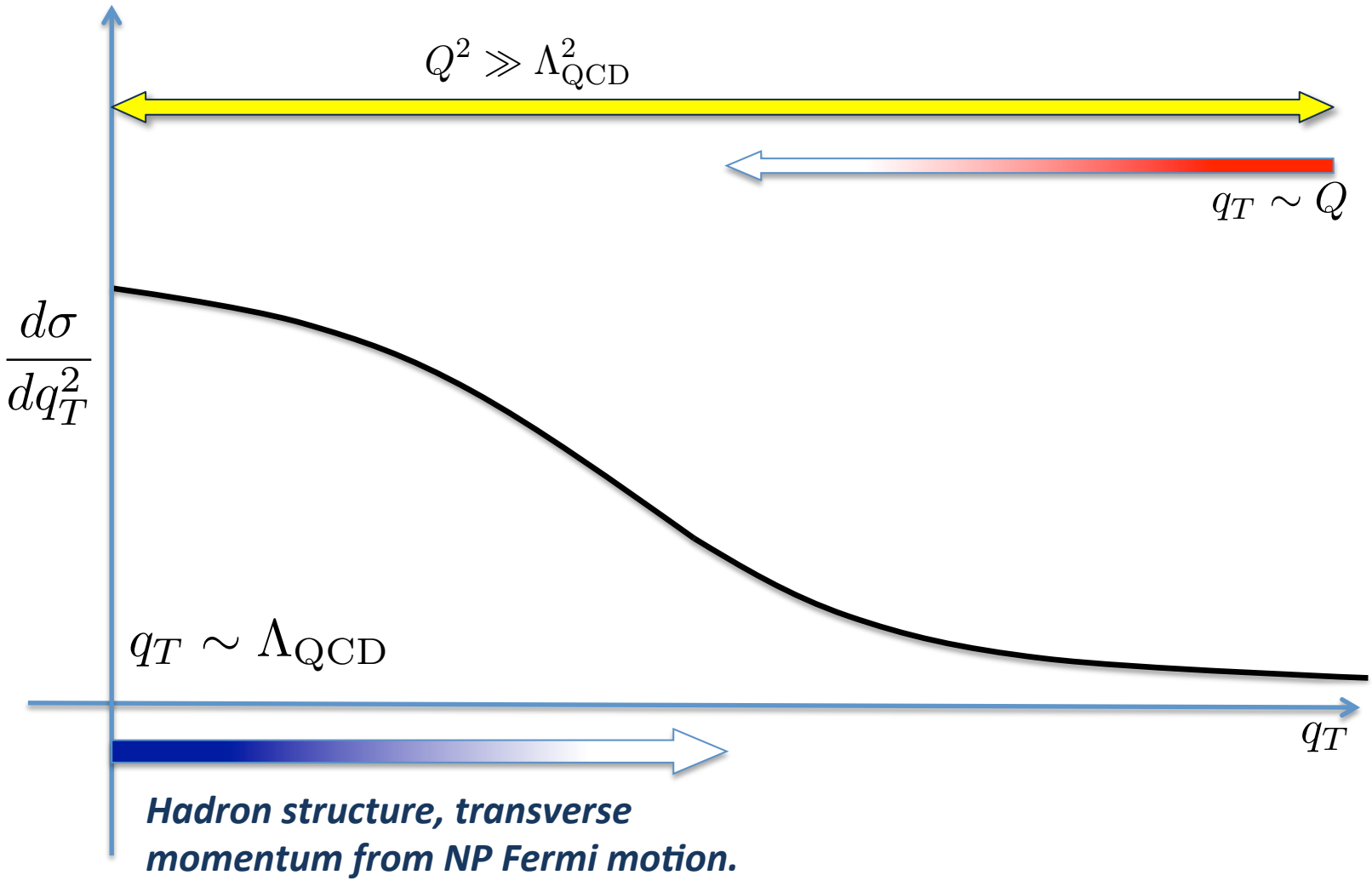
Regions of Transverse Momentum:



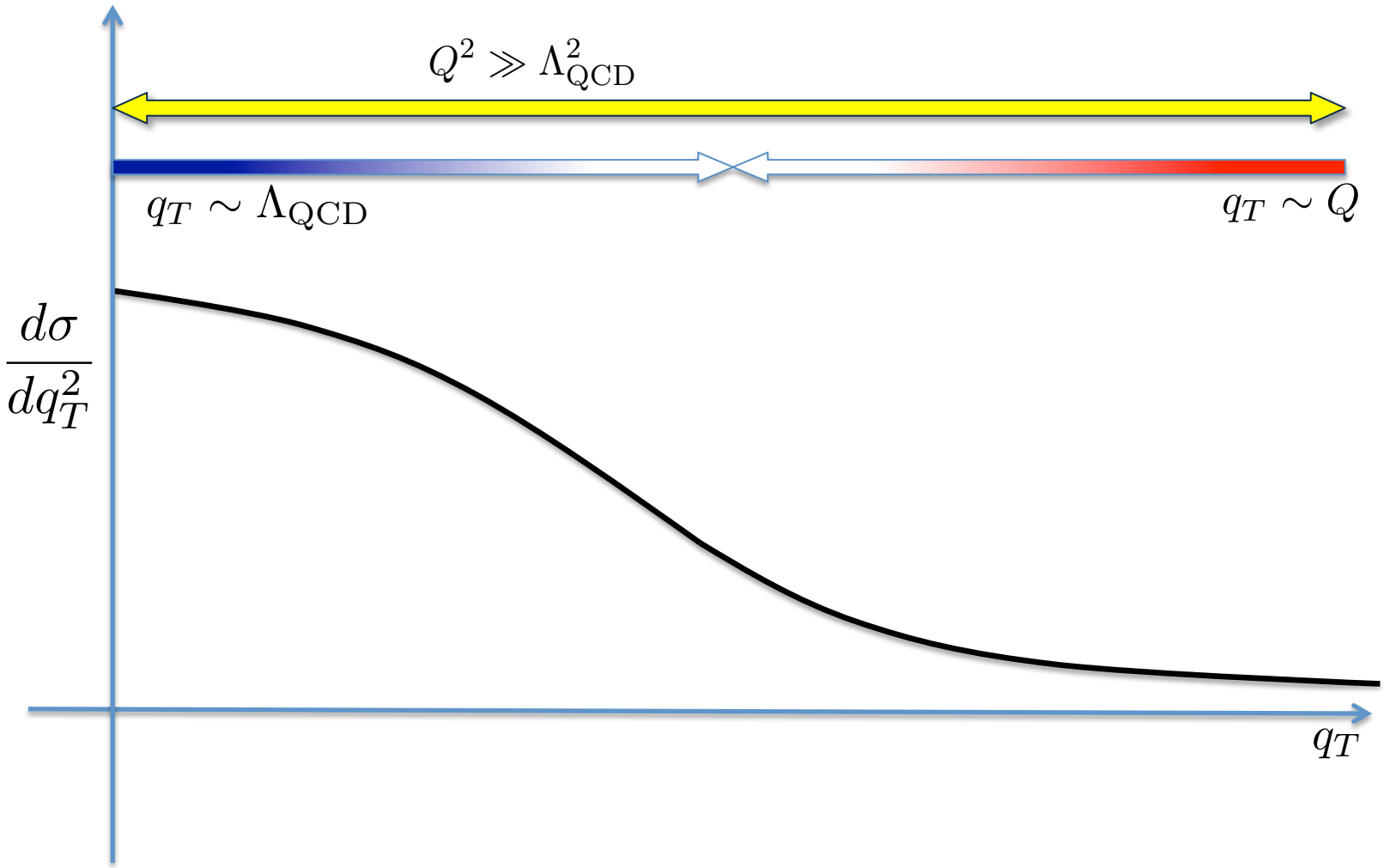
Regions of Transverse Momentum:



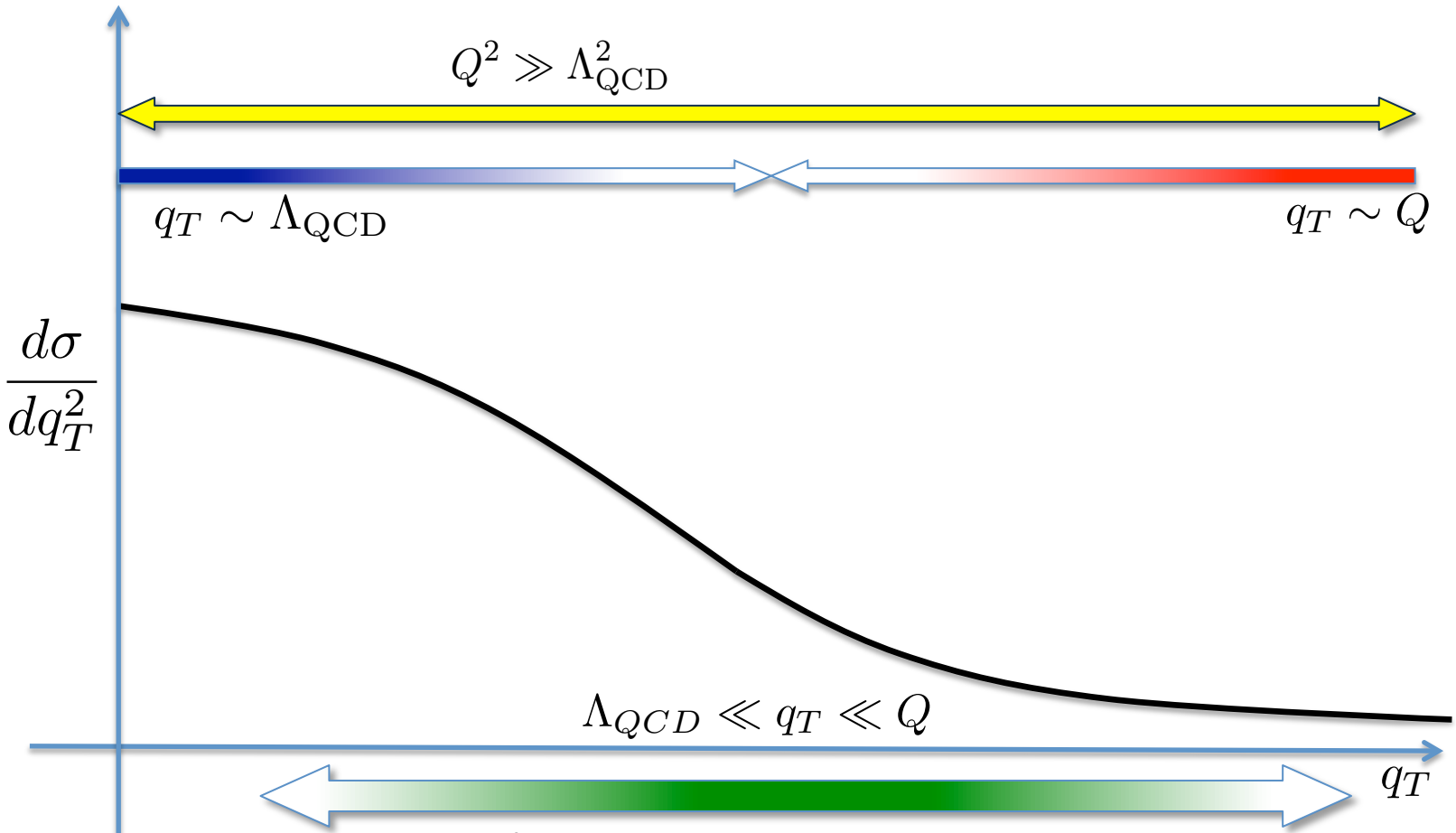
Regions of Transverse Momentum:



Regions of Transverse Momentum:



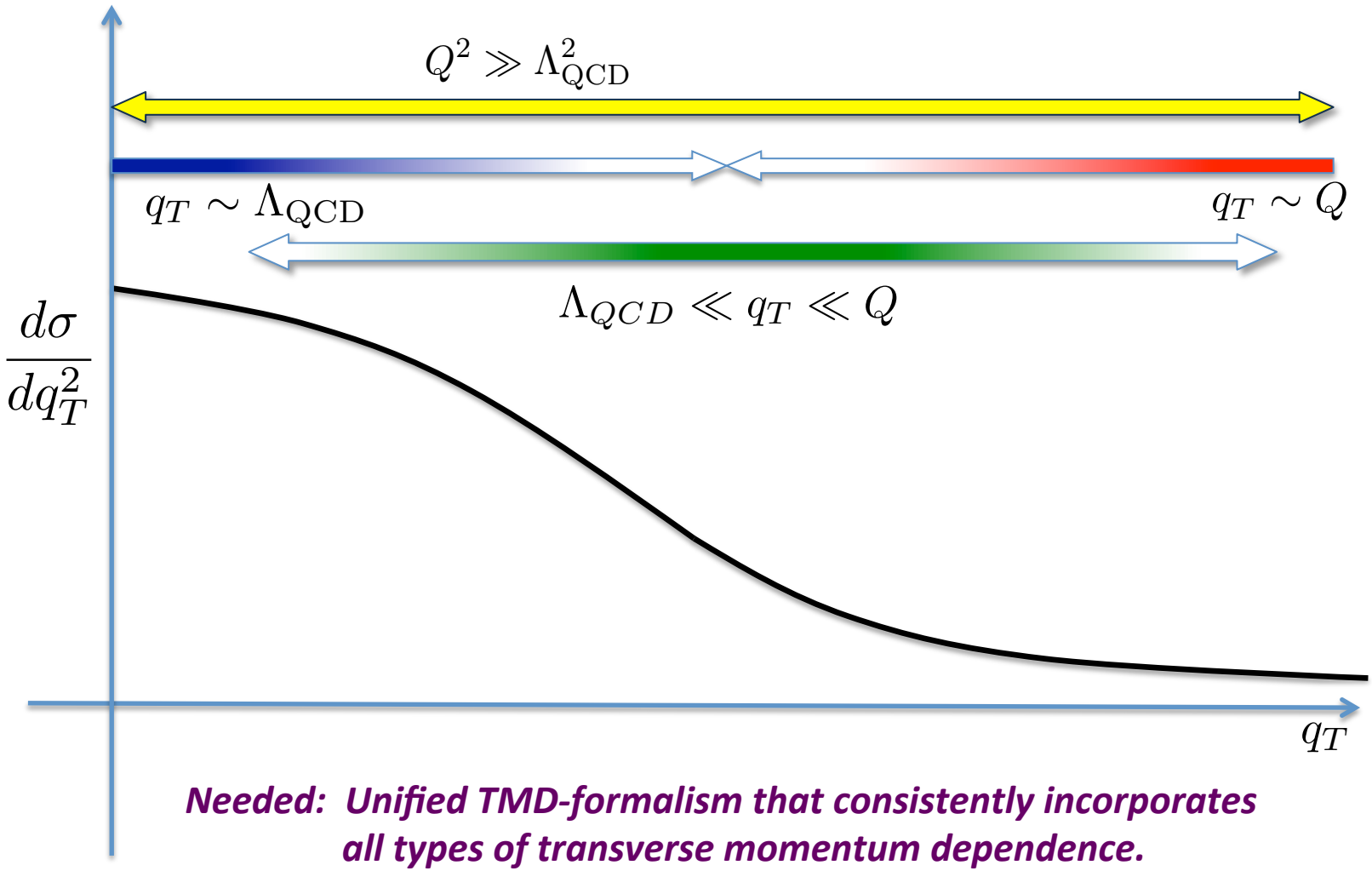
Regions of Transverse Momentum:



<u>Intermediate transverse momentum:</u>

- Transverse momentum dependence calculable from collinear fact. using hard scale q_T.
- Transverse momentum resummation.

Regions of Transverse Momentum:



Needed: Unified TMD-formalism that consistently incorporates all types of transverse momentum dependence.

What is Needed from TMD-Factorization?

- q_T-distribution calculable in pQCD (small coupling and finite coefficients) for all q_T.

- TMD PDFs describing intrinsic properties of hadrons at low q_T.
  - Hard part calculable at $q_T \longrightarrow 0$.

- TMD PDFs separately well-defined with:
  - Universality (akin to collinear PDFs) or,
  - Non-Universality predicted by factorization derivation (e.g., Sivers sign flip) &,
  - Calculable Non-Universality from Evolution (evolution equations).

- Maximum perturbative input:
  - Transverse momentum dependence calculable using collinear factorization at intermediate q_T.
  - Match to totally collinear factorization description at large q_T.

- Error is Λ/Q suppressed point-by-point over full range of q_T.

TMD-Factorization

$$W_{DY}^{\mu\nu} = \sum_f |\mathcal{H}_f(Q;\mu/Q)|^{\mu\nu}$$

Each point and each term to be discussed in turn in rest of talk…

$$\times \int d^2\mathbf{k}_{1T}\, d^2\mathbf{k}_{2T}\, F_{f/P_1}(x_1, \mathbf{k}_{1T}; \mu; \zeta_1)\; F_{f/P_2}(x_2, \mathbf{k}_{2T}; \mu; \zeta_2)\, \delta^{(2)}(\mathbf{k}_{1T} + \mathbf{k}_{2T} - \mathbf{q}_T)$$

$$+ Y(q_T, Q)$$

$$+ \mathcal{O}\left(\left(\frac{\Lambda}{Q}\right)^a\right)$$

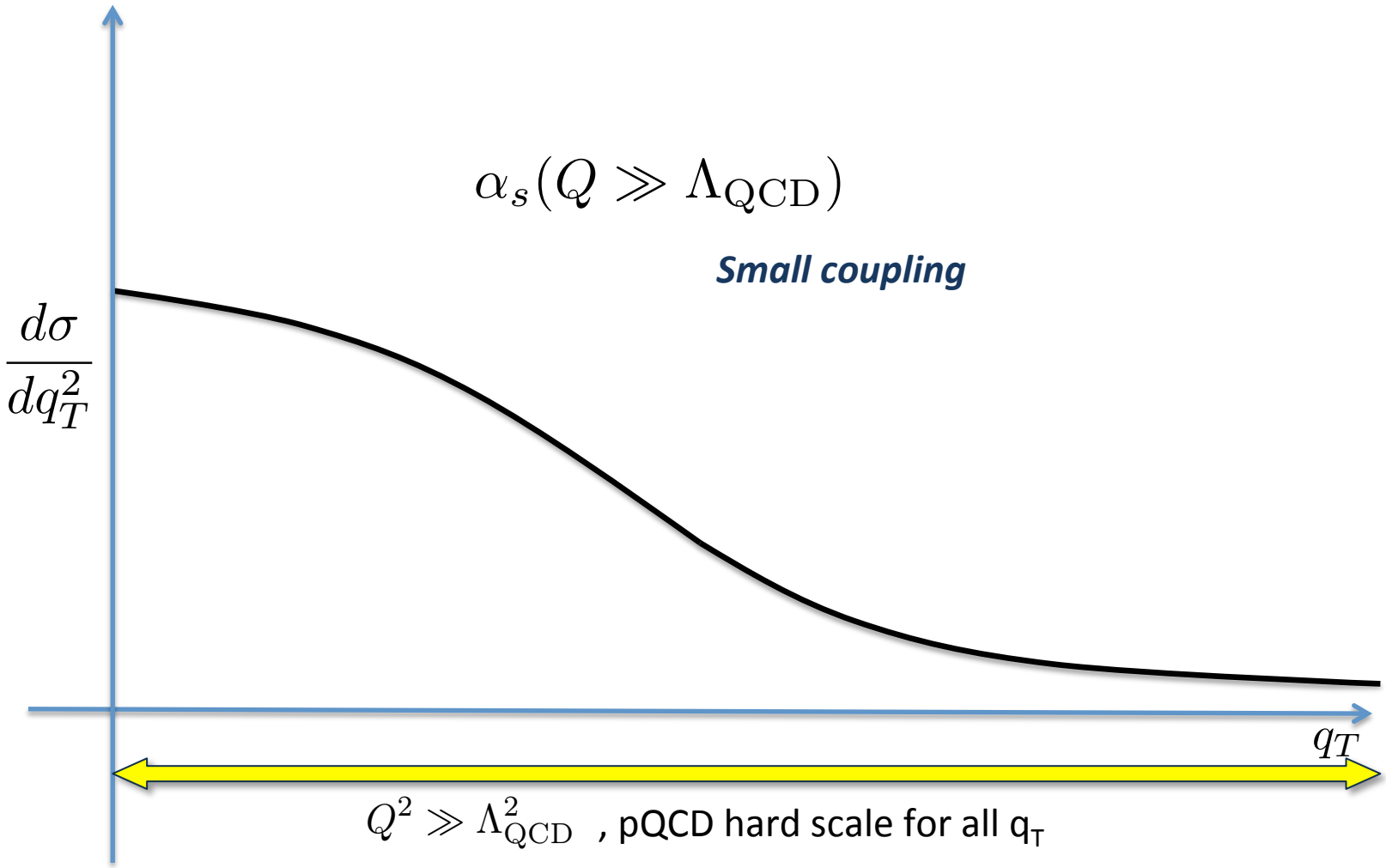
(Collins (2011) Chapters 10,13,14)

(Collins, Soper, Sterman (1982,1983))
(Ji, Ma, Yuan (2004))
…

What is Needed from TMD-Factorization?

- q_T-distribution calculable in pQCD (small coupling and finite coefficients) for all q_T.

Regions of Transverse Momentum:



What is Needed from TMD-Factorization?

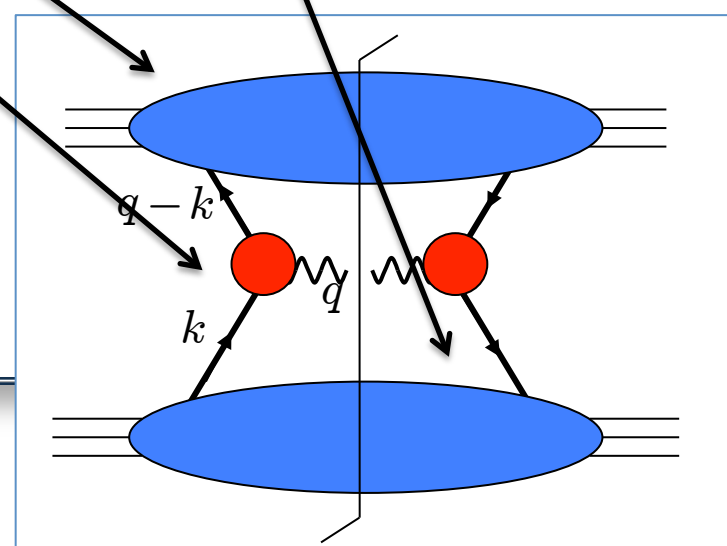
- q_T-distribution calculable in pQCD (small coupling and finite coefficients) for all q_T.

- TMD PDFs describing intrinsic properties of hadrons at low q_T.
  - Hard part calculable at $q_T \longrightarrow 0$.

TMD-Factorization

$$W_{DY}^{\mu\nu} = \sum_f |\mathcal{H}_f(Q;\mu/Q)|^{\mu\nu}$$

$$\times \int d^2\mathbf{k}_{1T}\, d^2\mathbf{k}_{2T}\, F_{f/P_1}(x_1, \mathbf{k}_{1T}; \mu; \zeta_1)\; F_{f/P_2}(x_2, \mathbf{k}_{2T}; \mu; \zeta_2)\, \delta^{(2)}(\mathbf{k}_{1T} + \mathbf{k}_{2T} - \mathbf{q}_T)$$



What is Needed from TMD-Factorization?

- q_T-distribution calculable in pQCD (small coupling and finite coefficients) for all q_T.

- TMD PDFs describing intrinsic properties of hadrons at low q_T.
  - Hard part calculable at $q_T \longrightarrow 0$.

What is Needed from TMD-Factorization?

- q_T-distribution calculable in pQCD (small coupling and finite coefficients) for all q_T.

- TMD PDFs describing intrinsic properties of hadrons at low q_T.
  - Hard part calculable at $q_T \longrightarrow 0$.

- TMD PDFs separately well-defined with:
  - Universality (akin to collinear PDFs) or,
  - Non-Universality predicted by factorization derivation (e.g., Sivers sign flip) &,
  - Calculable Non-Universality from Evolution (evolution equations).

TMD-Factorization

$$W_{DY}^{\mu\nu} = \sum_f |\mathcal{H}_f(Q;\mu/Q)|^{\mu\nu}$$

Similar to generalized TMD parton model

$$\times \int d^2\mathbf{k}_{1T}\, d^2\mathbf{k}_{2T}\, \underbrace{F_{f/P_1}(x_1, \mathbf{k}_{1T}; \mu; \zeta_1)}_{\text{TMD PDF for hadron 1.}}\, \underbrace{F_{f/P_2}(x_2, \mathbf{k}_{2T}; \mu; \zeta_2)}_{\text{TMD PDF for hadron 2.}}\, \delta^{(2)}(\mathbf{k}_{1T} + \mathbf{k}_{2T} - \mathbf{q}_T)$$

TMD part: $q_T \ll Q$

What is Needed from TMD-Factorization?

- q_T-distribution calculable in pQCD (small coupling and finite coefficients) for all q_T.

- TMD PDFs describing intrinsic properties of hadrons at low q_T.
  - Hard part calculable at $q_T \longrightarrow 0$.

- TMD PDFs separately well-defined with:
  - Universality (akin to collinear PDFs) or,
  - Non-Universality predicted by factorization derivation (e.g., Sivers sign flip) &,
  - Calculable Non-Universality from Evolution (evolution equations).

- Maximum perturbative input:
  - Transverse momentum dependence calculable using collinear factorization at intermediate q_T.

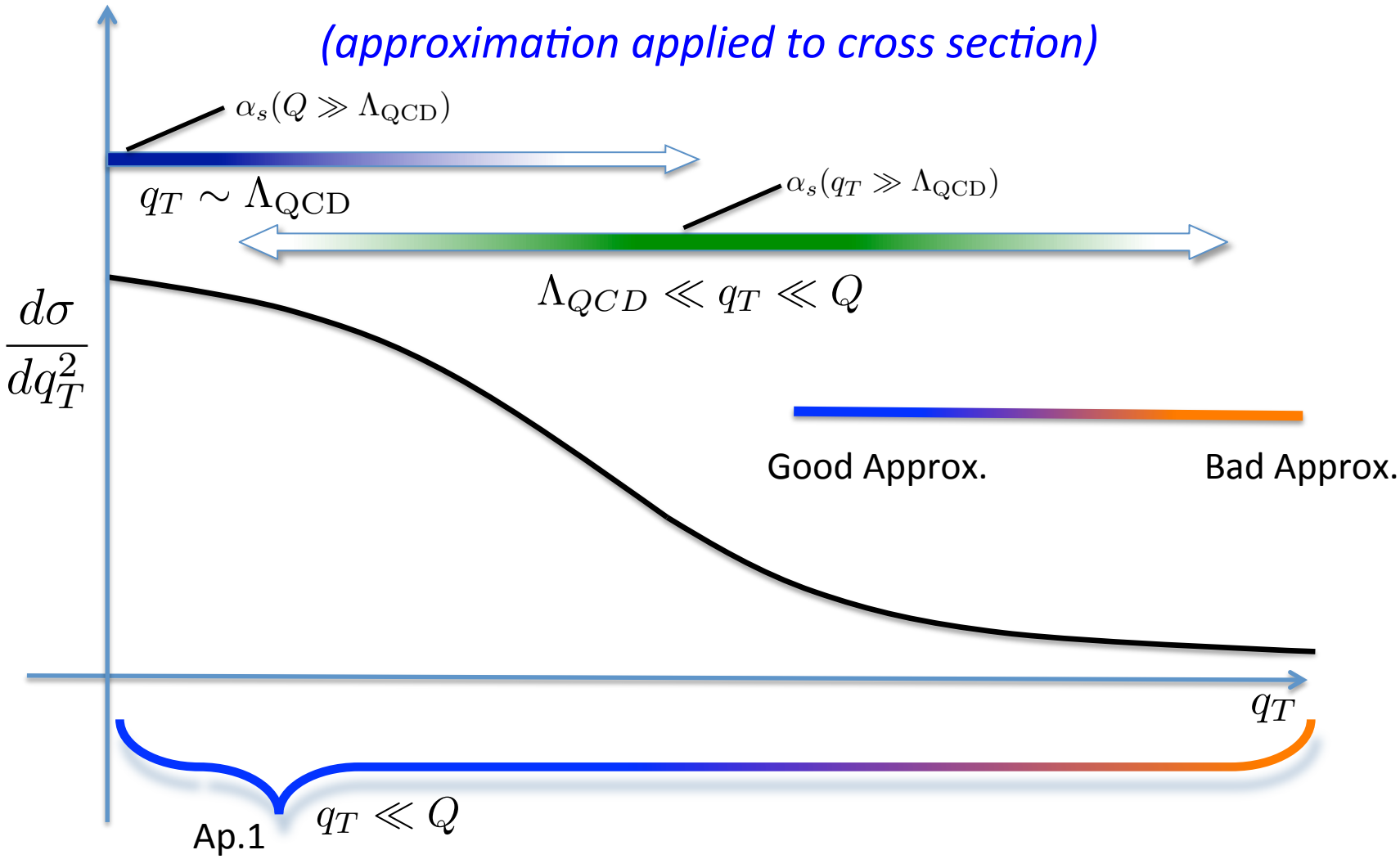
TMD-Factorization

$$W_{DY}^{\mu\nu} = \sum_f |\mathcal{H}_f(Q;\mu/Q)|^{\mu\nu}$$

$$\times \int d^2\mathbf{k}_{1T}\, d^2\mathbf{k}_{2T}\, F_{f/P_1}(x_1, \mathbf{k}_{1T}; \mu; \zeta_1)\; F_{f/P_2}(x_2, \mathbf{k}_{2T}; \mu; \zeta_2)\, \delta^{(2)}(\mathbf{k}_{1T} + \mathbf{k}_{2T} - \mathbf{q}_T)$$

$$F_{f/P_1}(x_1, \mathbf{k}_{1T}; \mu; \zeta_1) = \underbrace{\mathcal{C}_{f/f'}(x/x', \mathbf{k}_{1T}, \zeta_1, \mu, \alpha_s(\mu))}_{\textit{Perturbatively calculable coefficient functions}} \otimes \underbrace{f_{f'/P_1}(x'; \mu)}_{\textit{Collinear PDFs}} + \underbrace{\mathcal{O}\left(\frac{\Lambda_{QCD}}{k_{1T}}\right)}_{\textit{Error}}$$

TMD part


(approximation applied to cross section)
$\alpha_s(Q \gg \Lambda_{\rm QCD})$
$q_T \sim \Lambda_{\rm QCD}$
$\alpha_s(q_T \gg \Lambda_{\rm QCD})$
$\Lambda_{QCD} \ll q_T \ll Q$
$\frac{d\sigma}{dq_T^2}$
Good Approx.
Bad Approx.
q_T
Ap.1
$q_T \ll Q$

q_T regions:

- TMD part: **(q_T << Q)**

$$\frac{d\sigma}{dq_T} - \mathrm{Ap.1}\left(\frac{d\sigma}{dq_T}\right) = \mathcal{O}\left(\frac{q_T}{Q}\right) \times \frac{d\sigma}{dq_T}$$

What is Needed from TMD-Factorization?

- q_T-distribution calculable in pQCD (small coupling and finite coefficients) for all q_T.

- TMD PDFs describing intrinsic properties of hadrons at low q_T.
  - Hard part calculable at $q_T \longrightarrow 0$.

- TMD PDFs separately well-defined with:
  - Universality (akin to collinear PDFs) or,
  - Non-Universality predicted by factorization derivation (e.g., Sivers sign flip) &,
  - Calculable Non-Universality from Evolution (evolution equations).

- Maximum perturbative input:
  - Transverse momentum dependence calculable using collinear factorization at intermediate q_T.
  - Match to totally collinear factorization description at large q_T.

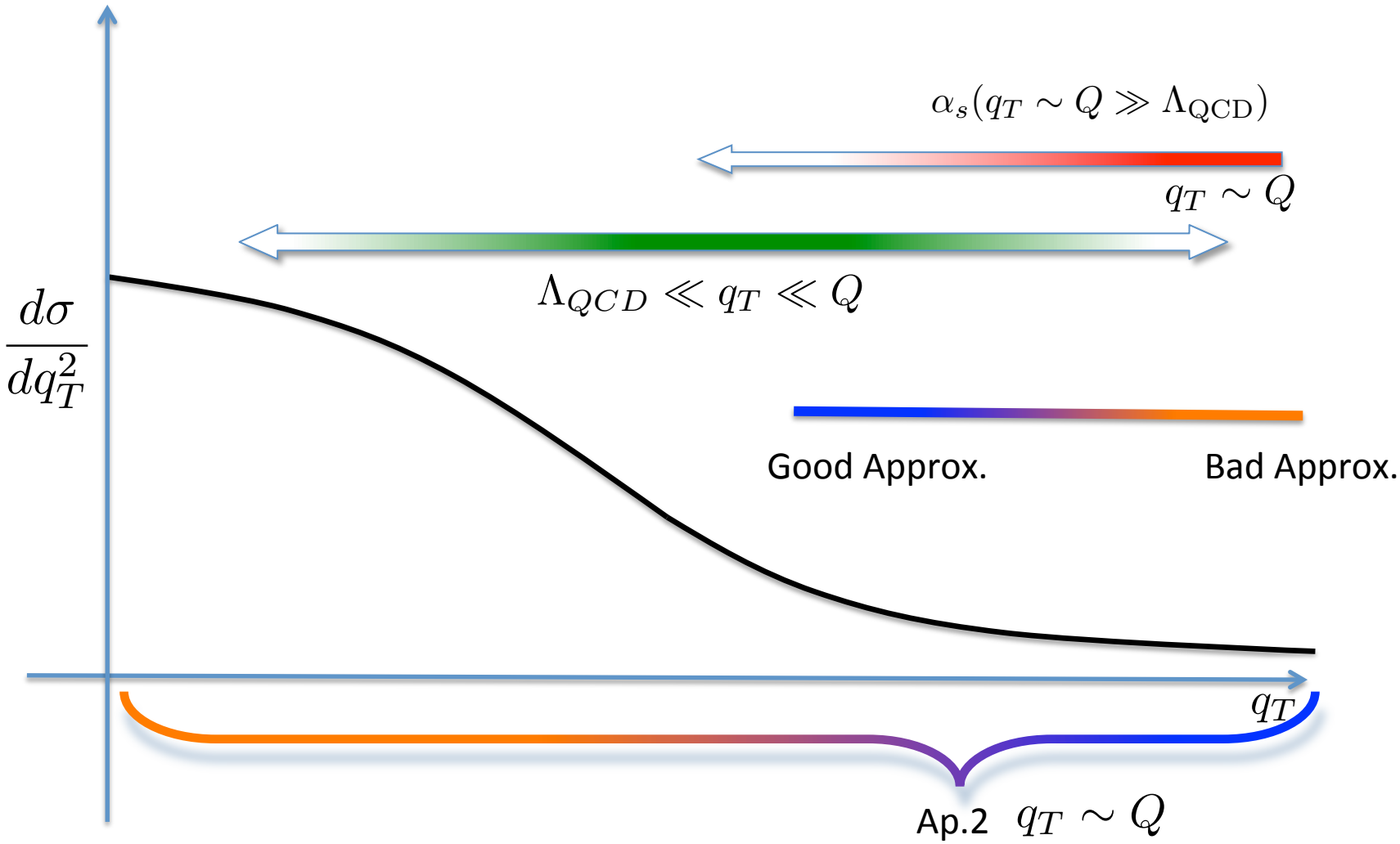
TMD-Factorization

$$W_{DY}^{\mu\nu} = \sum_f |\mathcal{H}_f(Q;\mu/Q)|^{\mu\nu}$$

$$\times \int d^2\mathbf{k}_{1T}\, d^2\mathbf{k}_{2T}\, F_{f/P_1}(x_1, \mathbf{k}_{1T}; \mu; \zeta_1)\; F_{f/P_2}(x_2, \mathbf{k}_{2T}; \mu; \zeta_2)\, \delta^{(2)}(\mathbf{k}_{1T} + \mathbf{k}_{2T} - \mathbf{q}_T)$$

$$+ Y(q_T, Q)$$

Largest q_T:


$\alpha_s(q_T \sim Q \gg \Lambda_{\rm QCD})$
$q_T \sim Q$
$\Lambda_{QCD} \ll q_T \ll Q$
$\frac{d\sigma}{dq_T^2}$
Good Approx.
Bad Approx.
q_T
Ap.2 $q_T \sim Q$

Approximations:

- TMD part: **(q_T << Q)**

$$\frac{d\sigma}{dq_T} - \mathrm{Ap.1}\left(\frac{d\sigma}{dq_T}\right) = \mathcal{O}\left(\frac{q_T}{Q}\right) \times \frac{d\sigma}{dq_T}$$

- Y-term: **(Remainder)**

$$\begin{aligned}
&\mathrm{Ap.2}\left(\frac{d\sigma}{dq_T} - \mathrm{Ap.1}\left(\frac{d\sigma}{dq_T}\right)\right) \\
&\qquad = \left(\frac{d\sigma}{dq_T} - \mathrm{Ap.1}\left(\frac{d\sigma}{dq_T}\right)\right) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{q_T}\right) \times \left(\frac{d\sigma}{dq_T} - \mathrm{Ap.1}\left(\frac{d\sigma}{dq_T}\right)\right) \\
&\qquad = \left(\frac{d\sigma}{dq_T} - \mathrm{Ap.1}\left(\frac{d\sigma}{dq_T}\right)\right) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{\cancel{q_T}}\right) \times \mathcal{O}\left(\frac{\cancel{q_T}}{Q}\right) \times \frac{d\sigma}{dq_T}
\end{aligned}$$

Approximations:

- TMD part: **(q_T << Q)**

$$\frac{d\sigma}{dq_T} - \mathrm{Ap.1}\left(\frac{d\sigma}{dq_T}\right) = \mathcal{O}\left(\frac{q_T}{Q}\right) \times \frac{d\sigma}{dq_T}$$

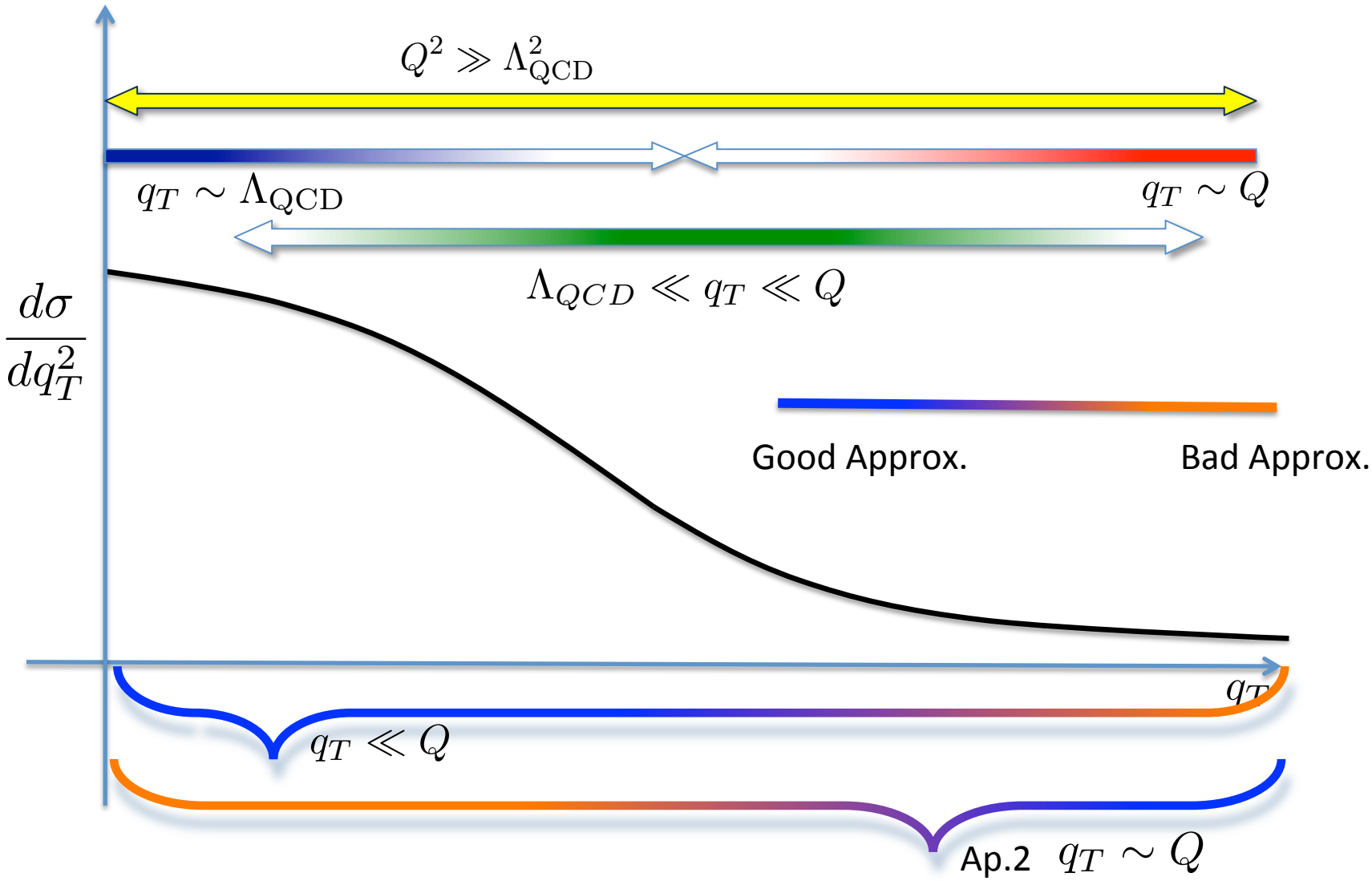
- Y-term: **(Remainder)**

$$\mathrm{Ap.2}\left(\frac{d\sigma}{dq_T} - \mathrm{Ap.1}\left(\frac{d\sigma}{dq_T}\right)\right)$$
$$= \left(\frac{d\sigma}{dq_T} - \mathrm{Ap.1}\left(\frac{d\sigma}{dq_T}\right)\right) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{q_T}\right) \times \left(\frac{d\sigma}{dq_T} - \mathrm{Ap.1}\left(\frac{d\sigma}{dq_T}\right)\right)$$
$$= \left(\frac{d\sigma}{dq_T} - \mathrm{Ap.1}\left(\frac{d\sigma}{dq_T}\right)\right) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{\cancel{q_T}}\right) \times \mathcal{O}\left(\frac{\cancel{q_T}}{Q}\right) \times \frac{d\sigma}{dq_T}$$

- Total:

$$\frac{d\sigma}{dq_T} = \mathrm{Ap.1}\left(\frac{d\sigma}{dq_T}\right) + \left(\frac{d\sigma}{dq_T} - \mathrm{Ap.1}\left(\frac{d\sigma}{dq_T}\right)\right)$$
$$= \underbrace{\mathrm{Ap.1}\left(\frac{d\sigma}{dq_T}\right)}_{\textit{TMD part}} + \underbrace{\mathrm{Ap.2}\left(\frac{d\sigma}{dq_T} - \mathrm{Ap.1}\left(\frac{d\sigma}{dq_T}\right)\right)}_{\textit{Y-term}} + \underbrace{\mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{Q}\right) \times \frac{d\sigma}{dq_T}}_{\textit{Error}}$$

Regions of Transverse Momentum:



TMD-factorization

$$W_{DY}^{\mu\nu} = \sum_f |\mathcal{H}_f(Q;\mu/Q)|^{\mu\nu}$$

$$\times \int d^2\mathbf{k}_{1T}\, d^2\mathbf{k}_{2T}\, F_{f/P_1}(x_1, \mathbf{k}_{1T}; \mu; \zeta_1)\; F_{f/P_2}(x_2, \mathbf{k}_{2T}; \mu; \zeta_2)\, \delta^{(2)}(\mathbf{k}_{1T} + \mathbf{k}_{2T} - \mathbf{q}_T)$$

$$+ Y(q_T, Q)$$

$$\boxed{+ \mathcal{O}\left(\left(\frac{\Lambda}{Q}\right)^a\right)}$$

What is Needed from TMD-Factorization?

- q_T-distribution calculable in pQCD (small coupling and finite coefficients) for all q_T.

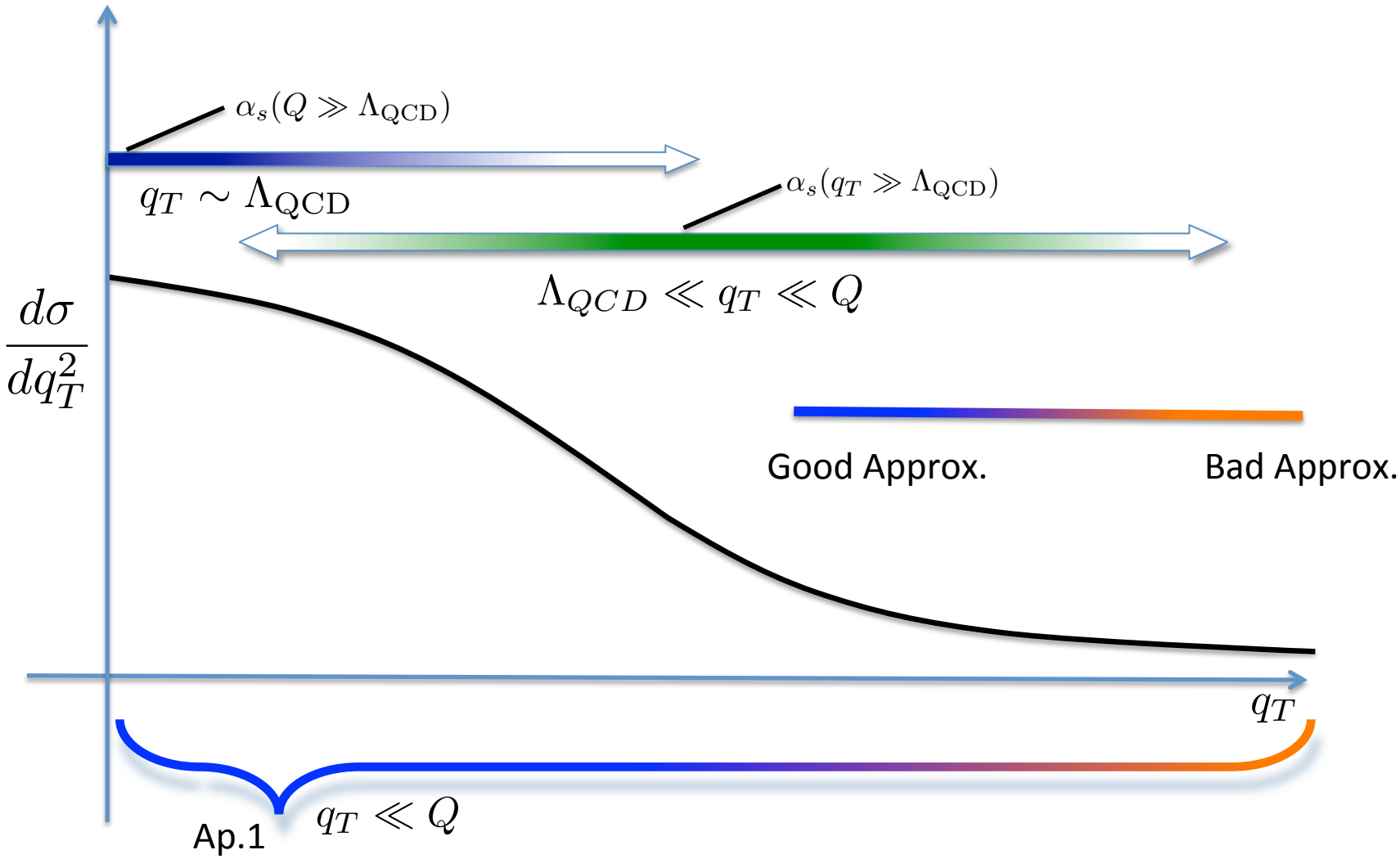
- TMD PDFs describing intrinsic properties of hadrons at low q_T.
  - Hard part calculable at $q_T \longrightarrow 0$.

- TMD PDFs separately well-defined with:
  - Universality (akin to collinear PDFs) or,
  - Non-Universality predicted by factorization derivation (e.g., Sivers sign flip) &,
  - Calculable Non-Universality from Evolution (evolution equations).

- Maximum perturbative input:
  - Transverse momentum dependence calculable using collinear factorization at intermediate q_T.
  - Match to totally collinear factorization description at large q_T.

- Error is Λ/Q suppressed point-by-point over full range of q_T.

Focus: TMD part



What is Needed from TMD-Factorization?

- q_T-distribution calculable in pQCD (small coupling and finite coefficients) for all q_T.

- TMD PDFs describing intrinsic properties of hadrons at low q_T.
  - Hard part calculable at $q_T \longrightarrow 0$.

- TMD PDFs separately well-defined with:
  - Universality (akin to collinear PDFs) or,
  - Non-Universality predicted by factorization derivation (e.g., Sivers sign flip) &,
  - Calculable Non-Universality from Evolution (evolution equations).

- Maximum perturbative input:
  - Transverse momentum dependence calculable using collinear factorization at intermediate q_T.
  - Match to totally collinear factorization description at large q_T.

- Error is Λ/Q suppressed point-by-point over full range of q_T.

Definitions:

- Integrated PDF:

$$f(x) = \text{F.T.}\, \langle p|\, \bar{\psi}(0, w^-, \mathbf{0}_t) \underline{V_w^\dagger(u_\text{J})} \gamma^+ \underline{V_0(u_\text{J})} \psi(0) |p\rangle$$

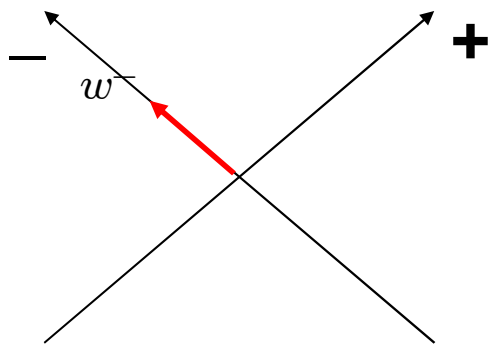
$$u_\text{J} = (0, 1, \mathbf{0}_t)$$

- TMD PDF:

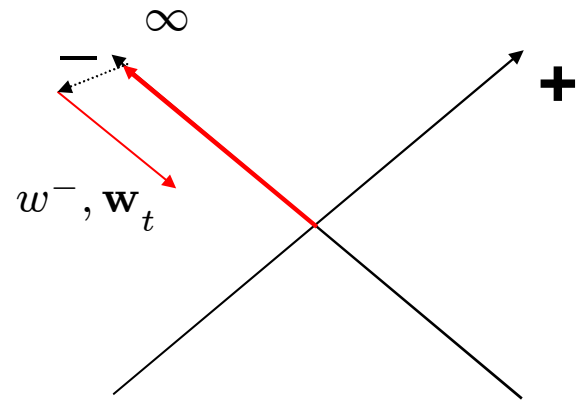
$$F_{f/P_1}(x_1, \mathbf{k}_{1T}) = \quad \text{F.T.}\, \langle p|\, \bar{\psi}(0, w^-, \mathbf{w}_t) \quad \underline{??} \quad \psi(0)\, |p\rangle$$

Definitions:

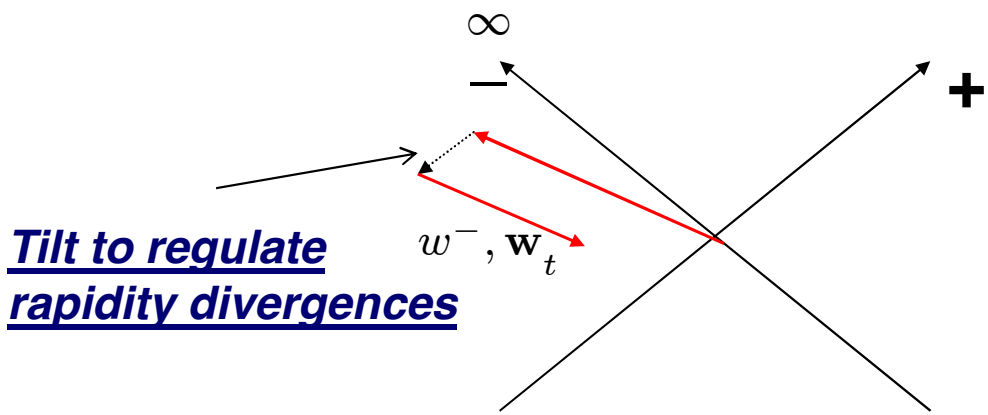
- Paths of Wilson lines in <u>coordinate space</u>:



Standard (Integrated)



Unintegrated First Try



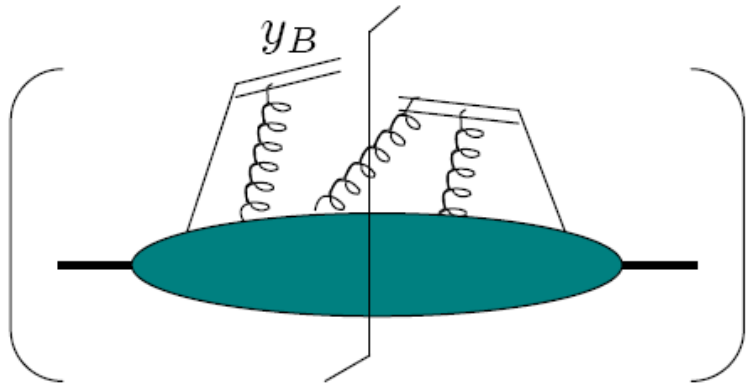
<u>Tilt to regulate rapidity divergences</u>

Unintegrated "tilted" Wilson lines

Definitions:

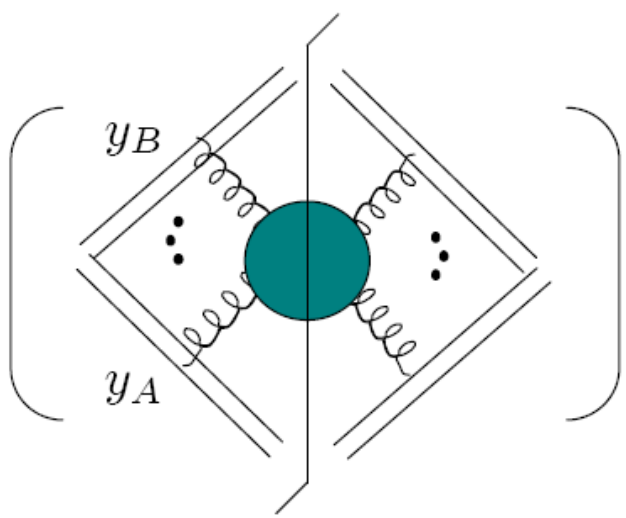
- "Unsubtracted" TMD PDF:

$$\tilde{F}^{\text{unsub}}_{f/P}(x, \mathbf{b}; \mu; y_P - y_B) \sim \text{F.T.}$$



- Soft Factor:

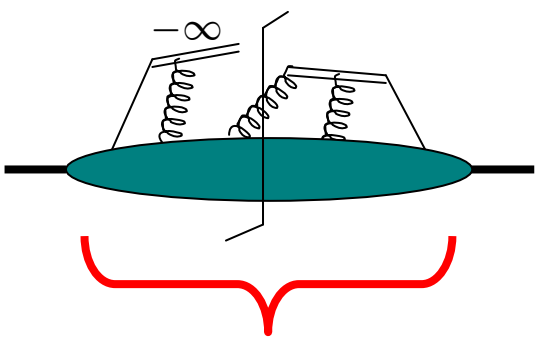
$$\tilde{S}(\mathbf{b}; y_A, y_B) \sim \text{F.T.}$$



Definitions:

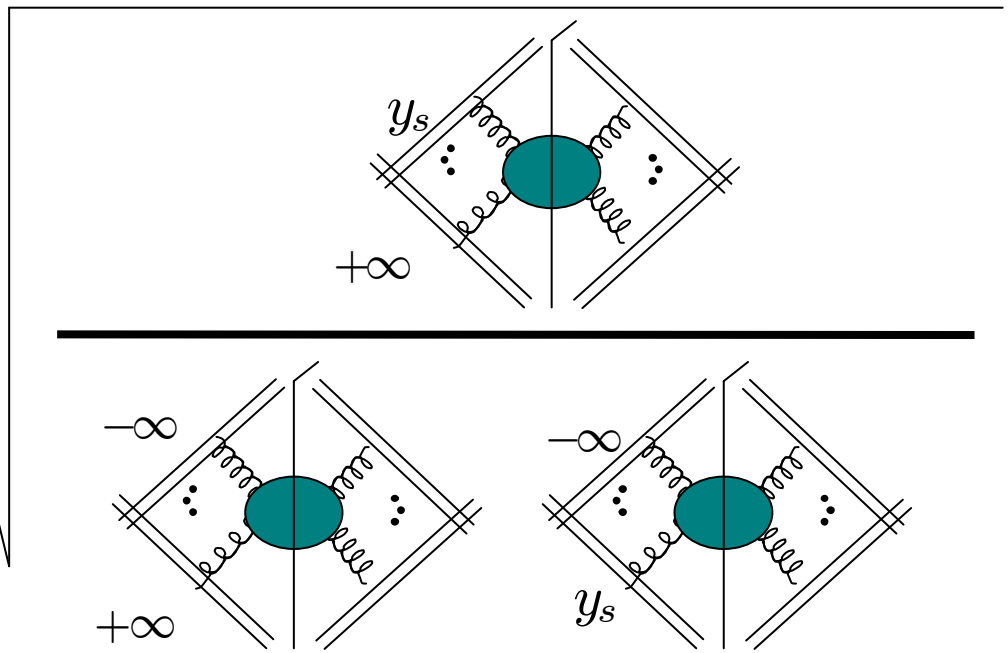
(Dictated by factorization requirements)

$$F_{f/P}(x, b; \mu; \zeta_F) = \qquad \zeta_F = 2M_p^2 x^2 e^{2(y_P - y_s)}$$



"Unsubtracted"

$\times$



(UV and rapidity renormalization needed)

Implements Subtractions/Cancellations

(Collins (2011), chapt. 13)

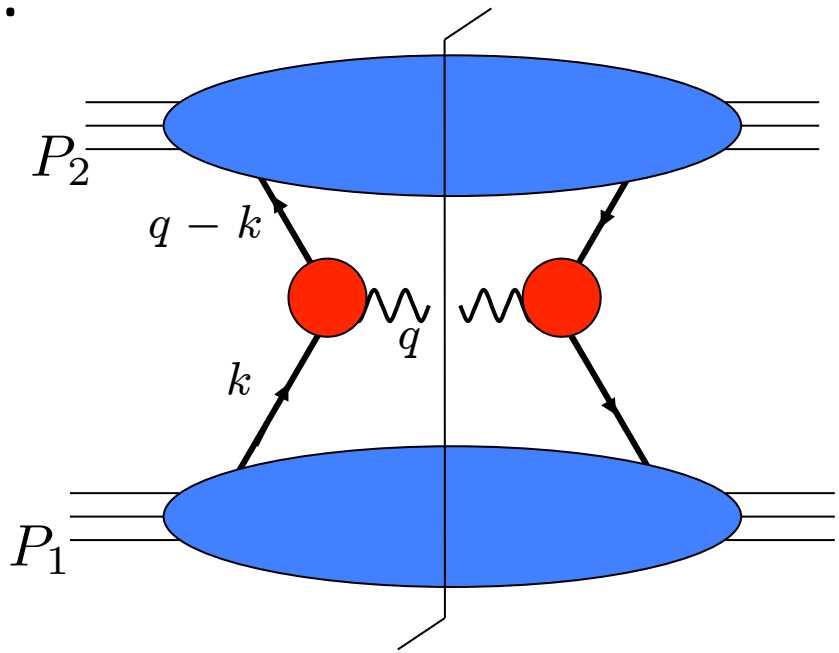
Example:

- Drell-Yan q_T distribution.

Get $\dfrac{d\sigma}{d\mathbf{q}_T \cdots}$

For <u>all</u> q_T.

$$q^2 = Q^2 \gg \Lambda^2_{QCD}$$



$$k_1 = k$$

$$k_2 = q - k$$

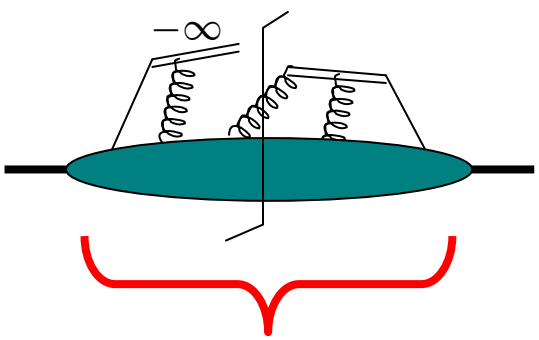
$$\mathbf{k}_{1T} + \mathbf{k}_{2T} = \mathbf{q}_T$$

Definitions:

(Dictated by factorization requirements)

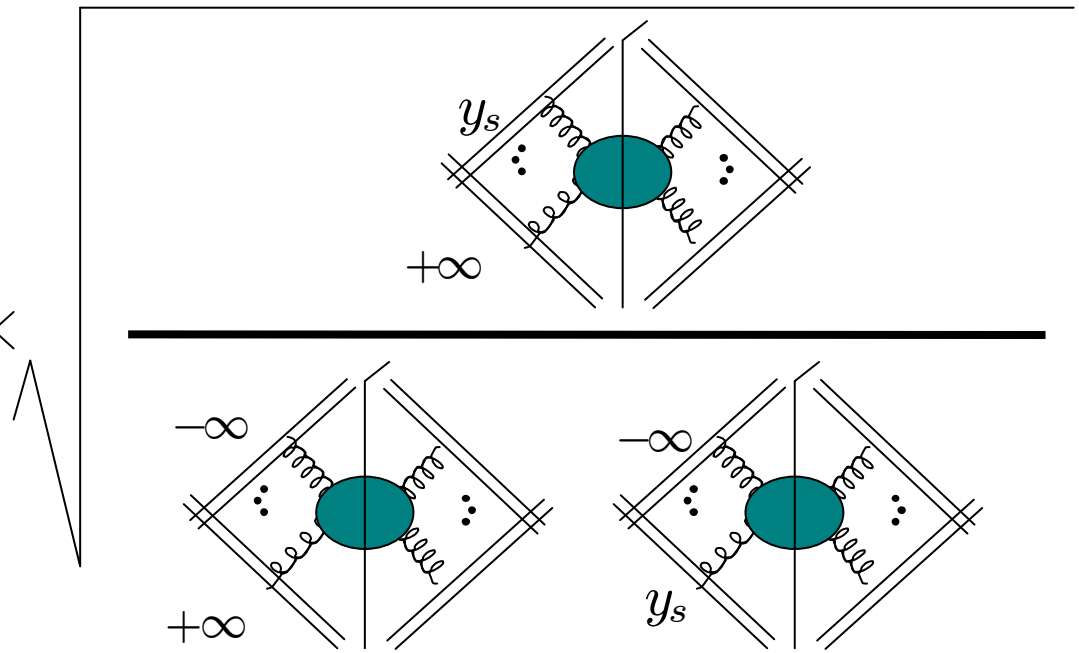
$$\zeta_1\zeta_2 \sim Q^4$$

$$F_{f/P}(x, b; \mu; \zeta_F) = \qquad \zeta_F = 2M_p^2 x^2 e^{2(y_P - y_s)}$$



"Unsubtracted"

$\times$

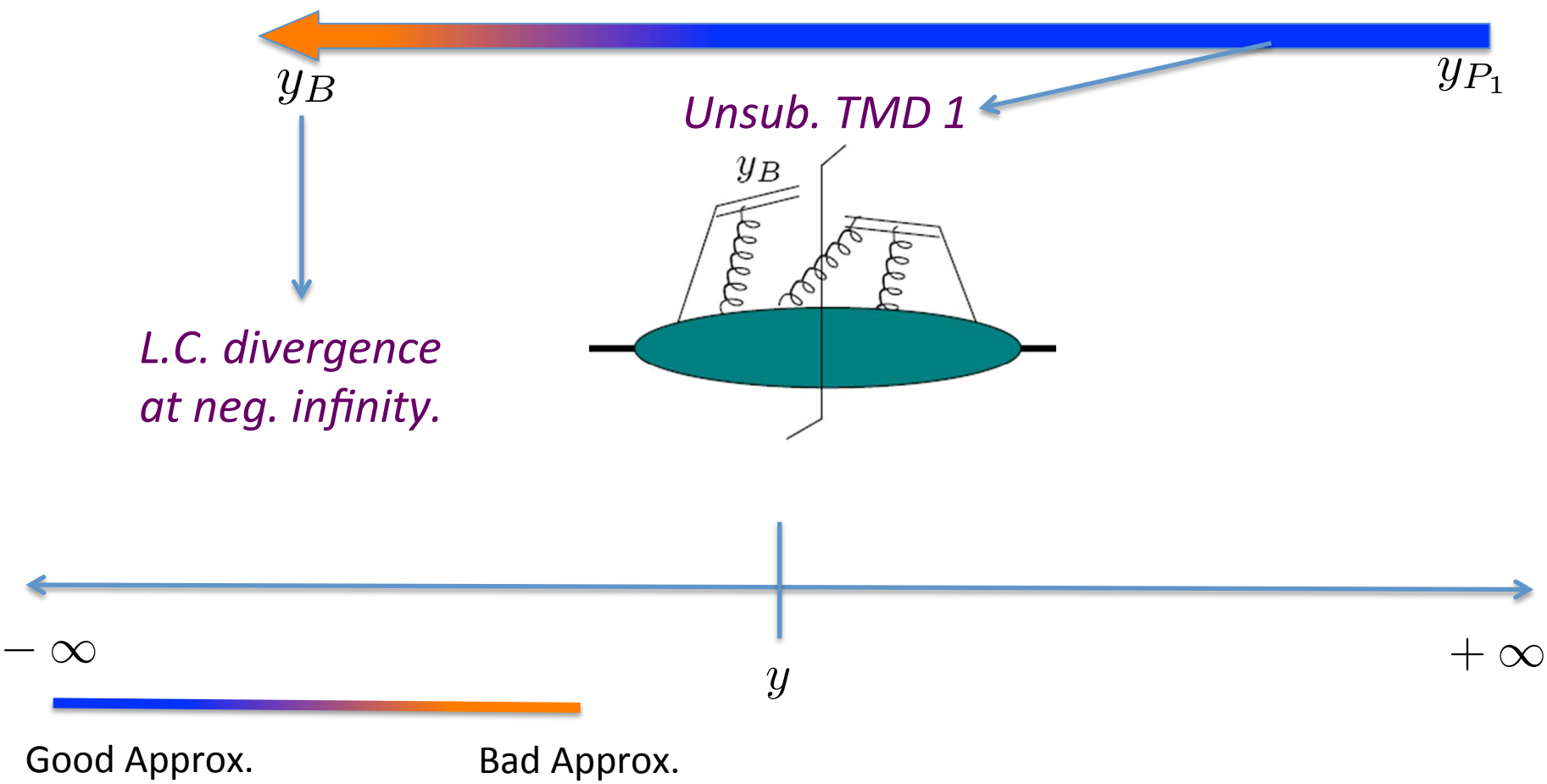


(UV and rapidity renormalization needed)

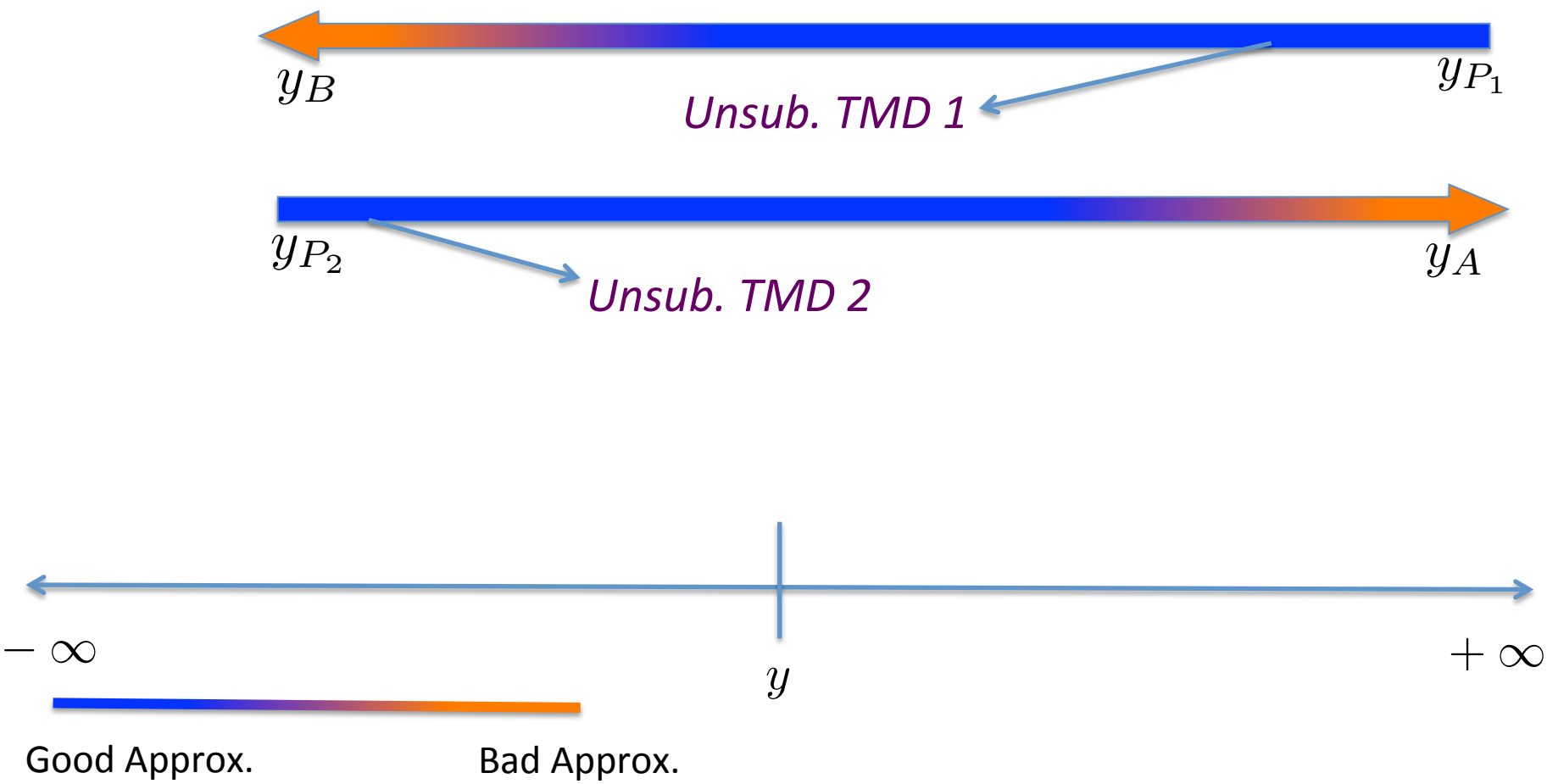
Implements Subtractions/Cancellations

(Collins (2011), chapt. 13)

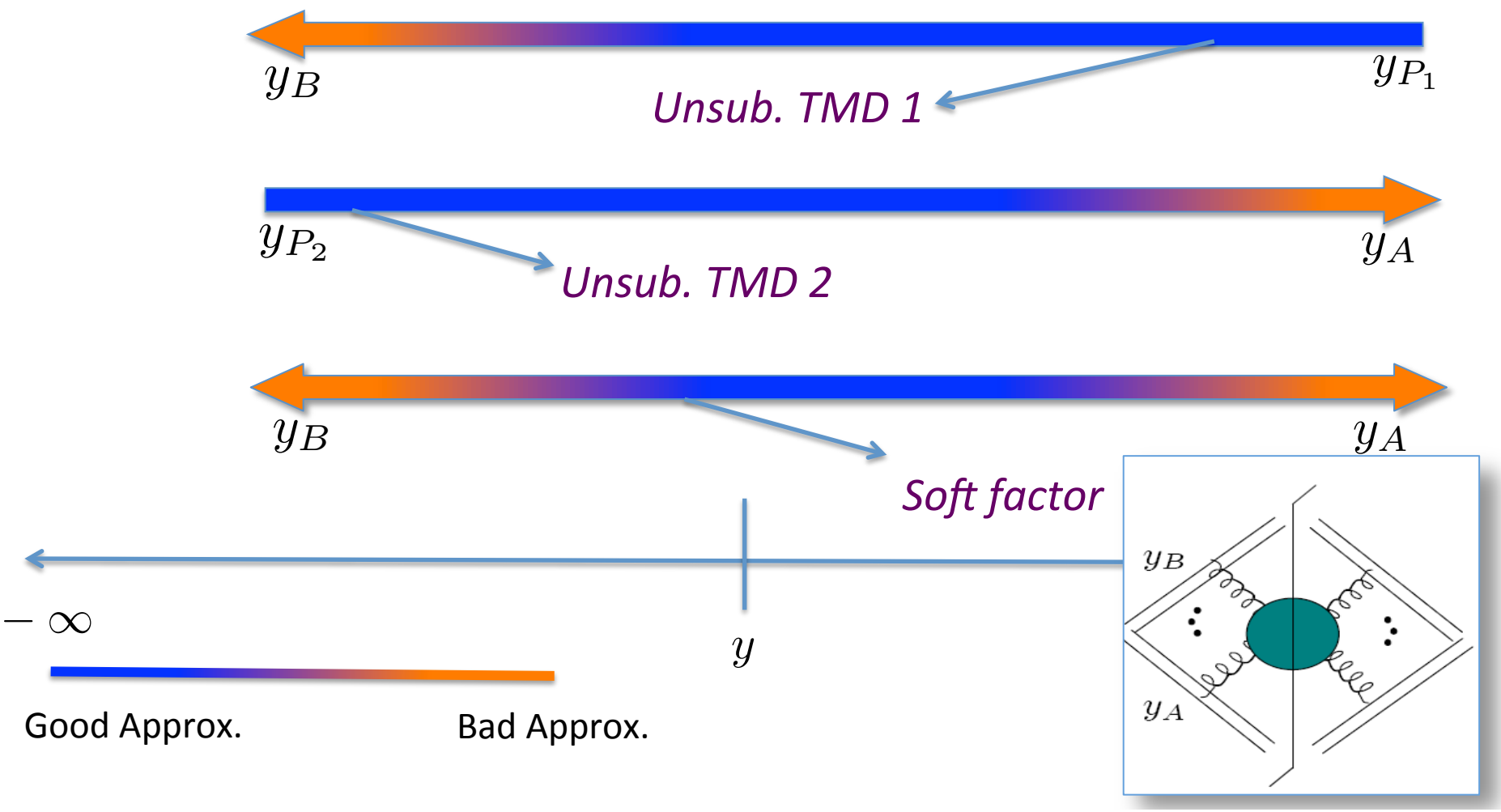
Origin of Square Root



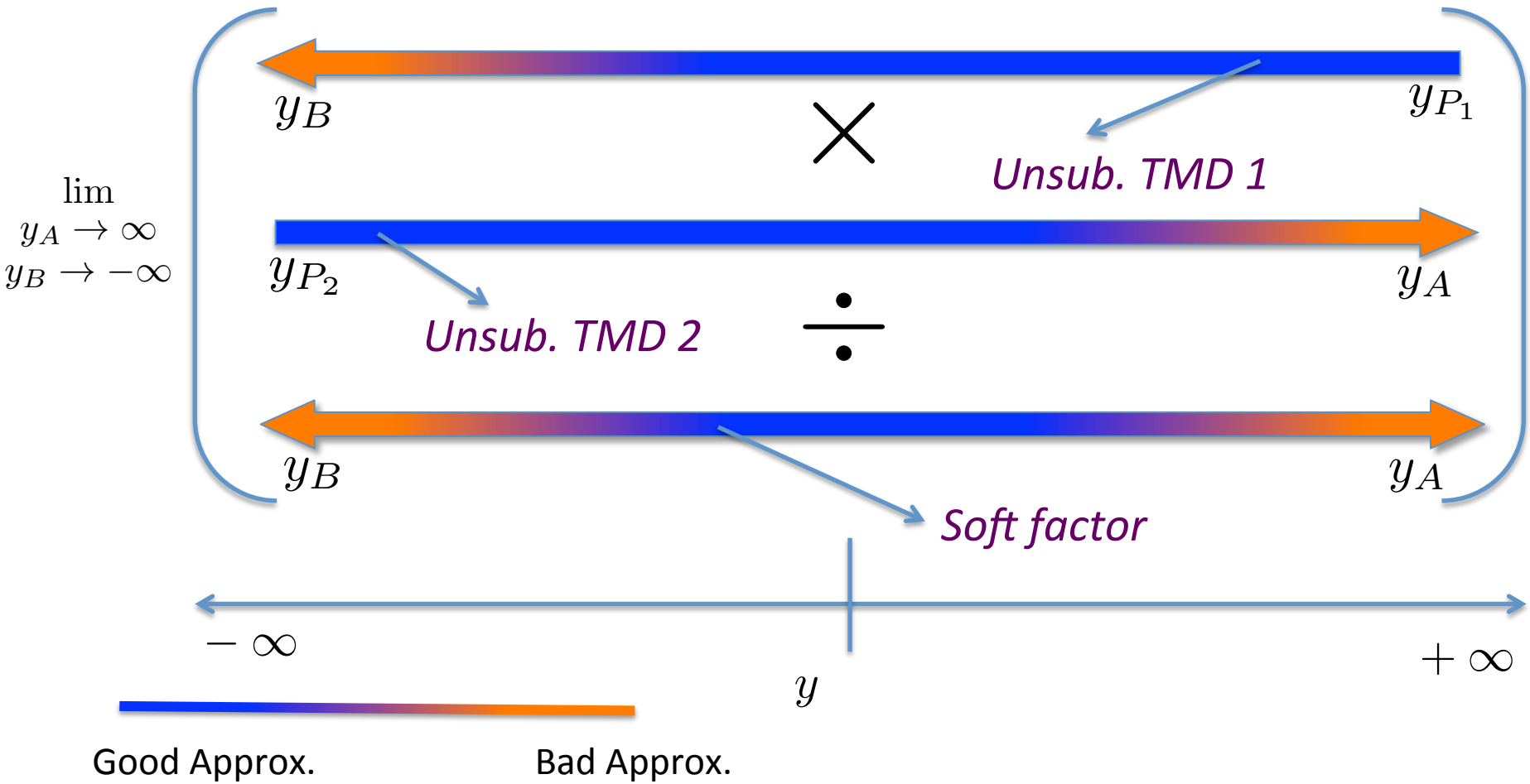
Origin of Square Root



Origin of Square Root



Origin of Square Root

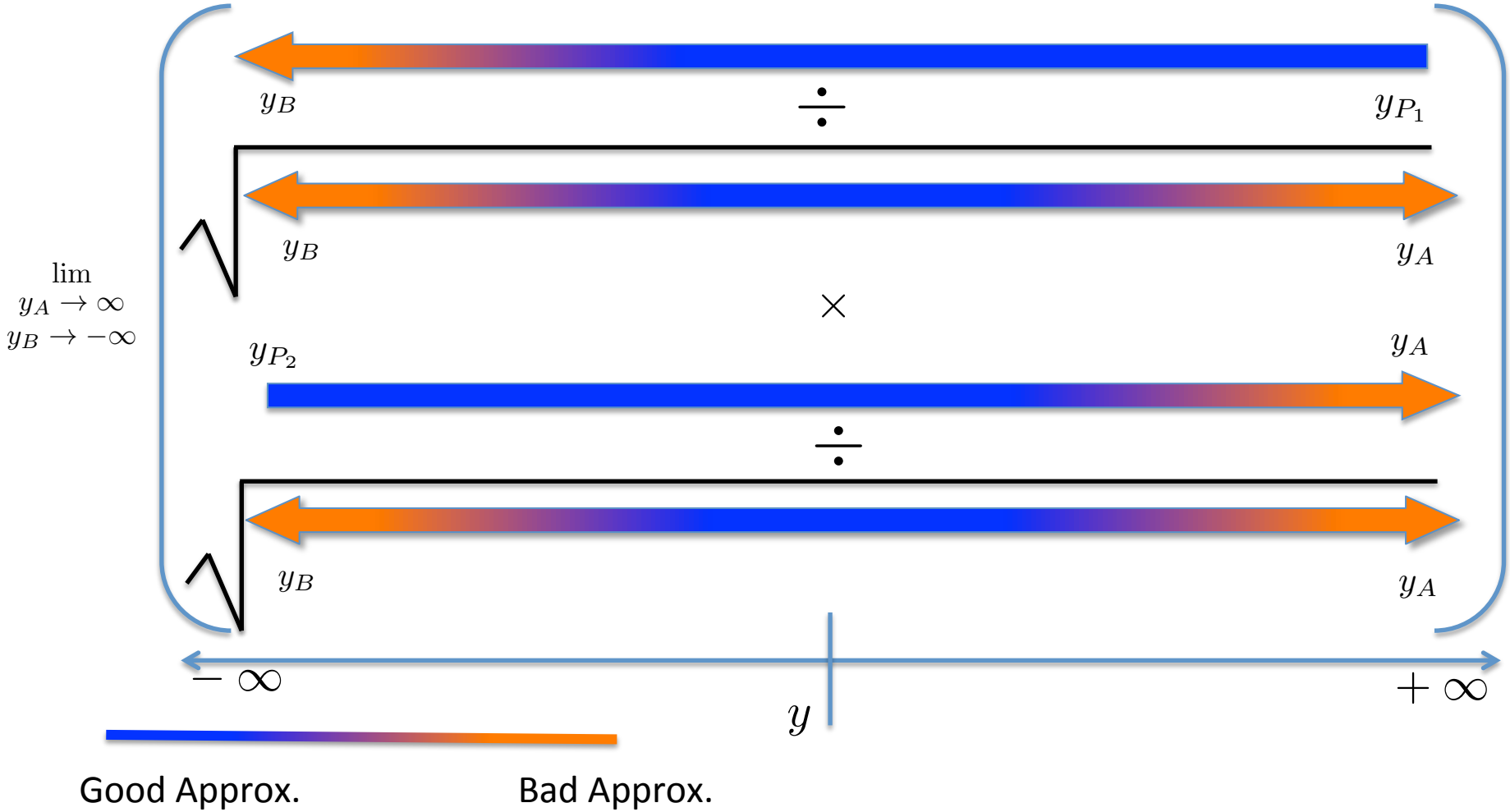


Origin of Square Root

- Start with only the hard part factorized:

$$d\sigma = |\mathcal{H}|^2 \, \frac{\tilde{F}_1^{\text{unsub}}(y_1 - (-\infty)) \times \tilde{F}_2^{\text{unsub}}(+\infty - y_2)}{\tilde{S}(+\infty, -\infty)}.$$

Origin of Square Root



Good Approx. Bad Approx.

Origin of Square Root

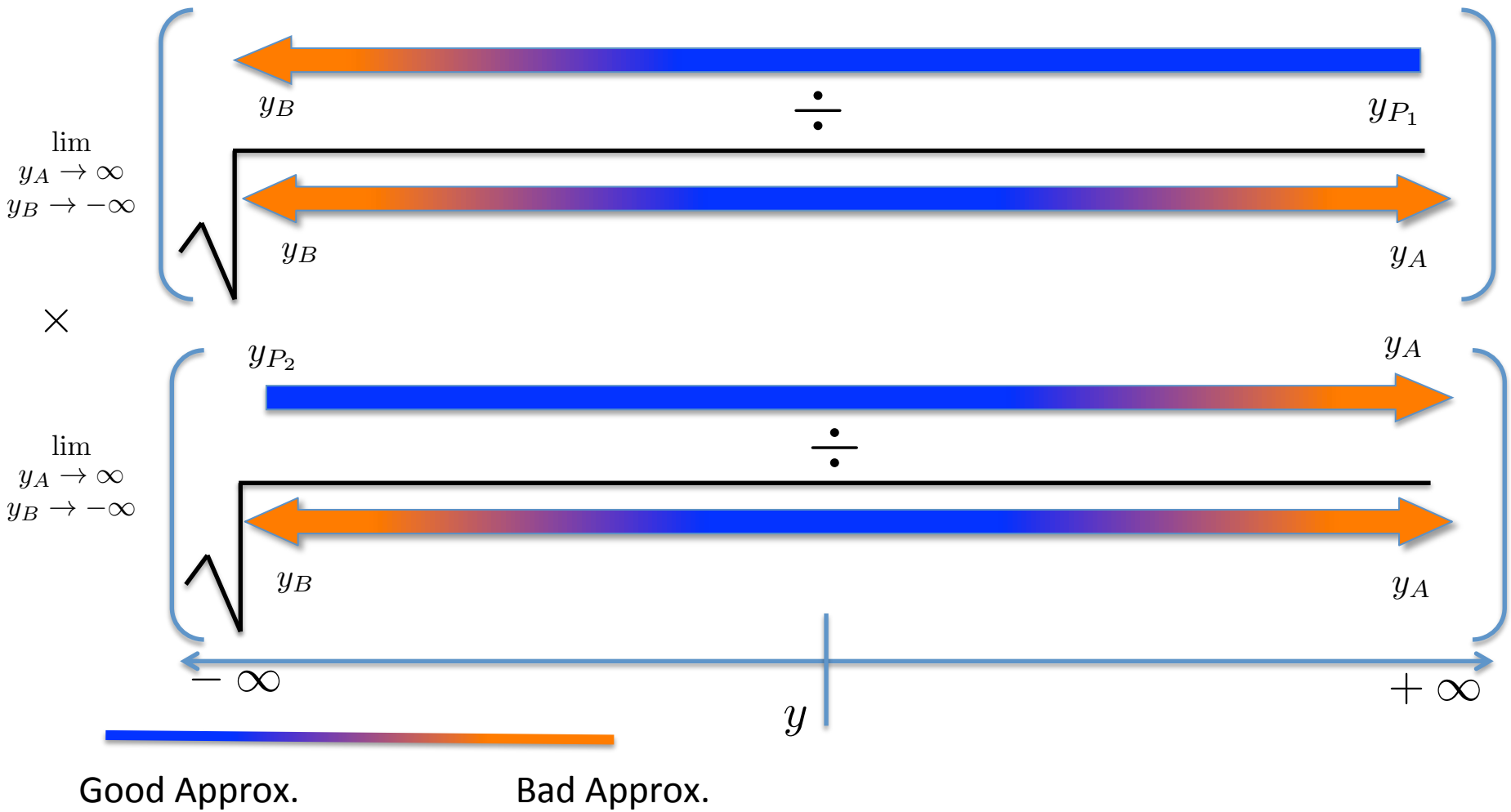
- Start with only the hard part factorized:

$$d\sigma = |\mathcal{H}|^2 \, \frac{\tilde{F}_1^{\text{unsub}}(y_1 - (-\infty)) \times \tilde{F}_2^{\text{unsub}}(+\infty - y_2)}{\tilde{S}(+\infty, -\infty)}.$$

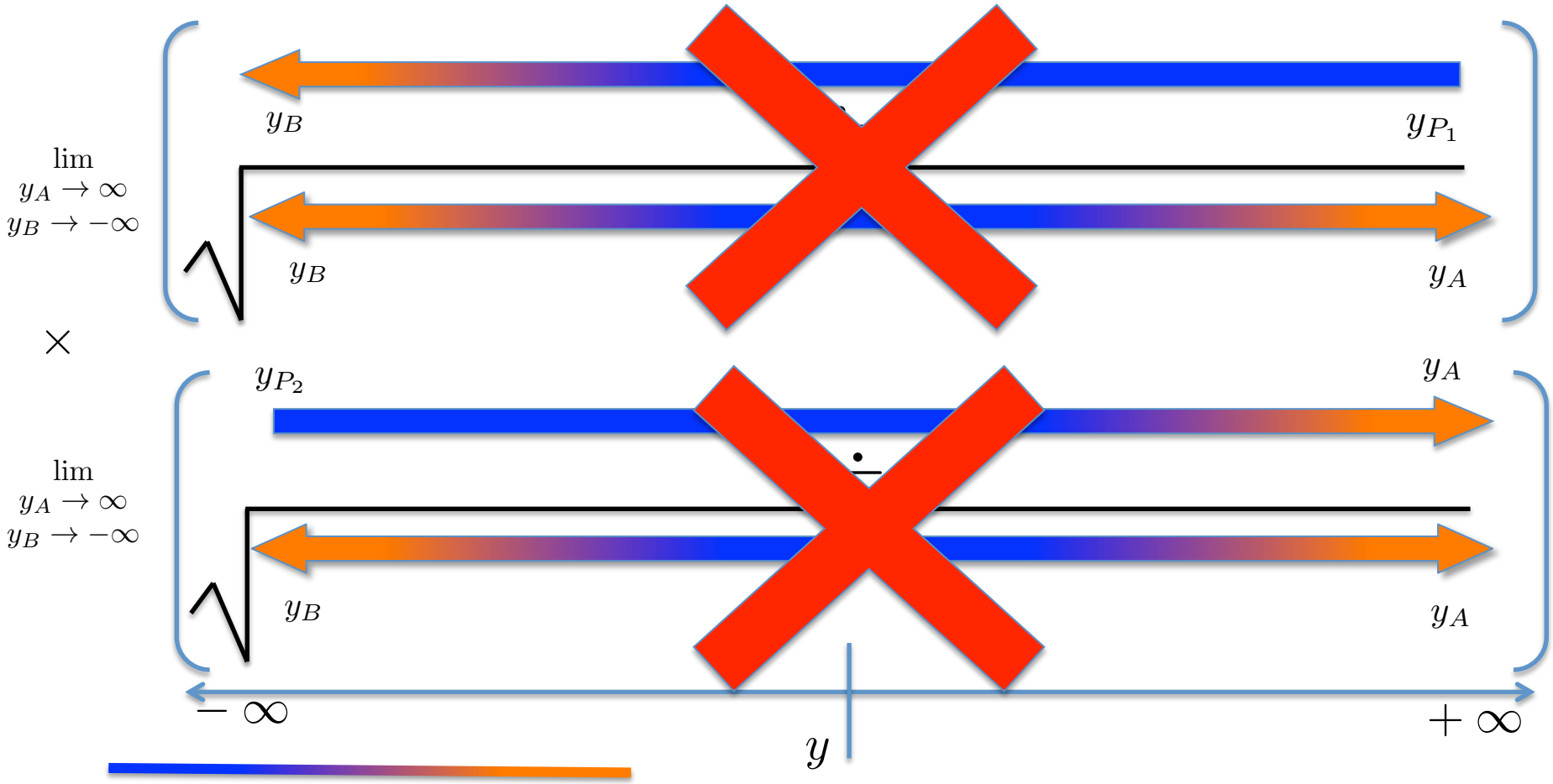
- Separate soft part:

$$d\sigma = |\mathcal{H}|^2 \, \frac{F_1^{\text{unsub}}(y_1 - (-\infty))}{\sqrt{\tilde{S}(+\infty, -\infty)}} \times \frac{\tilde{F}_2^{\text{unsub}}(+\infty - y_2)}{\sqrt{\tilde{S}(+\infty, -\infty)}}.$$

Origin of Square Root

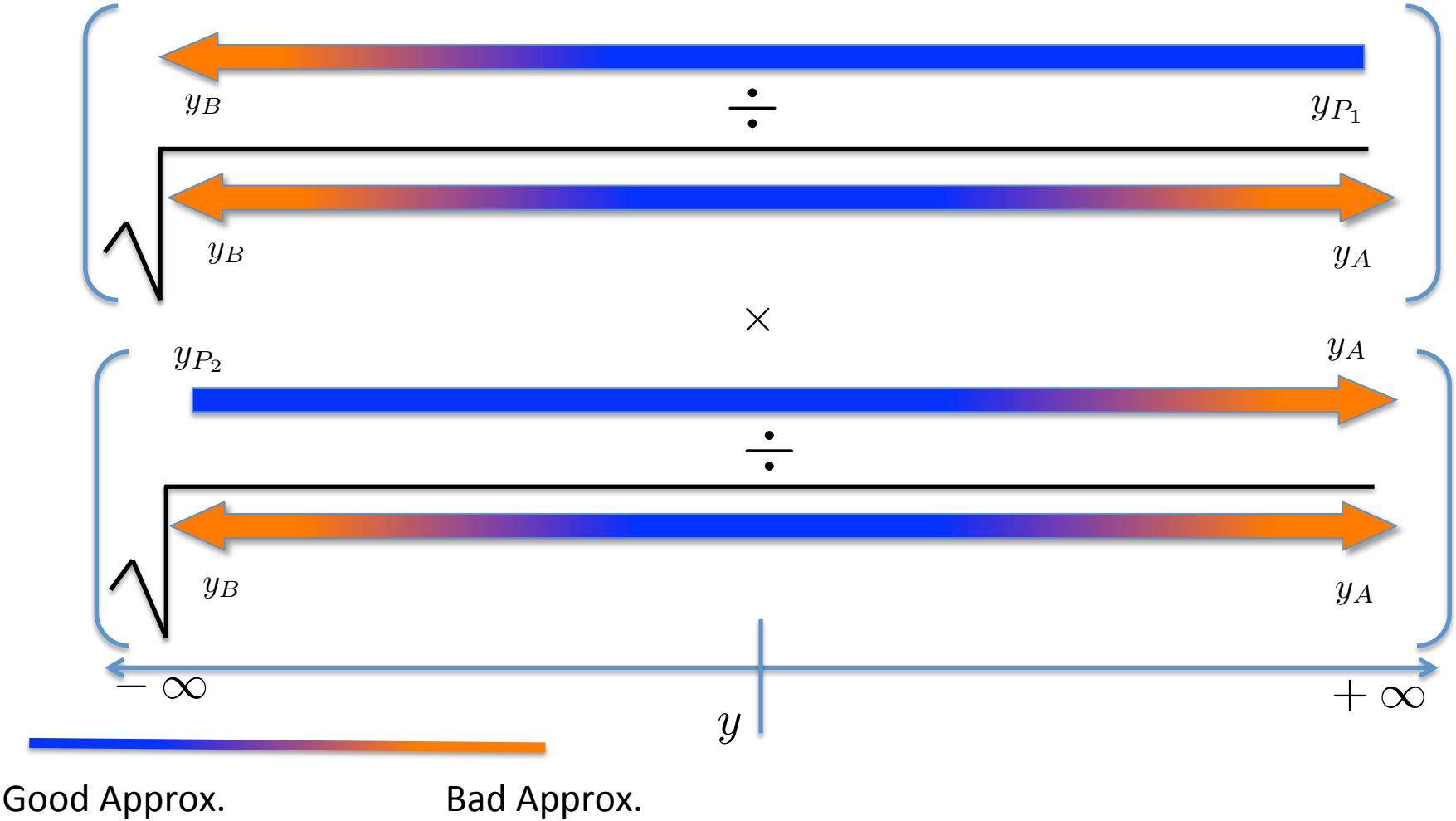


Origin of Square Root



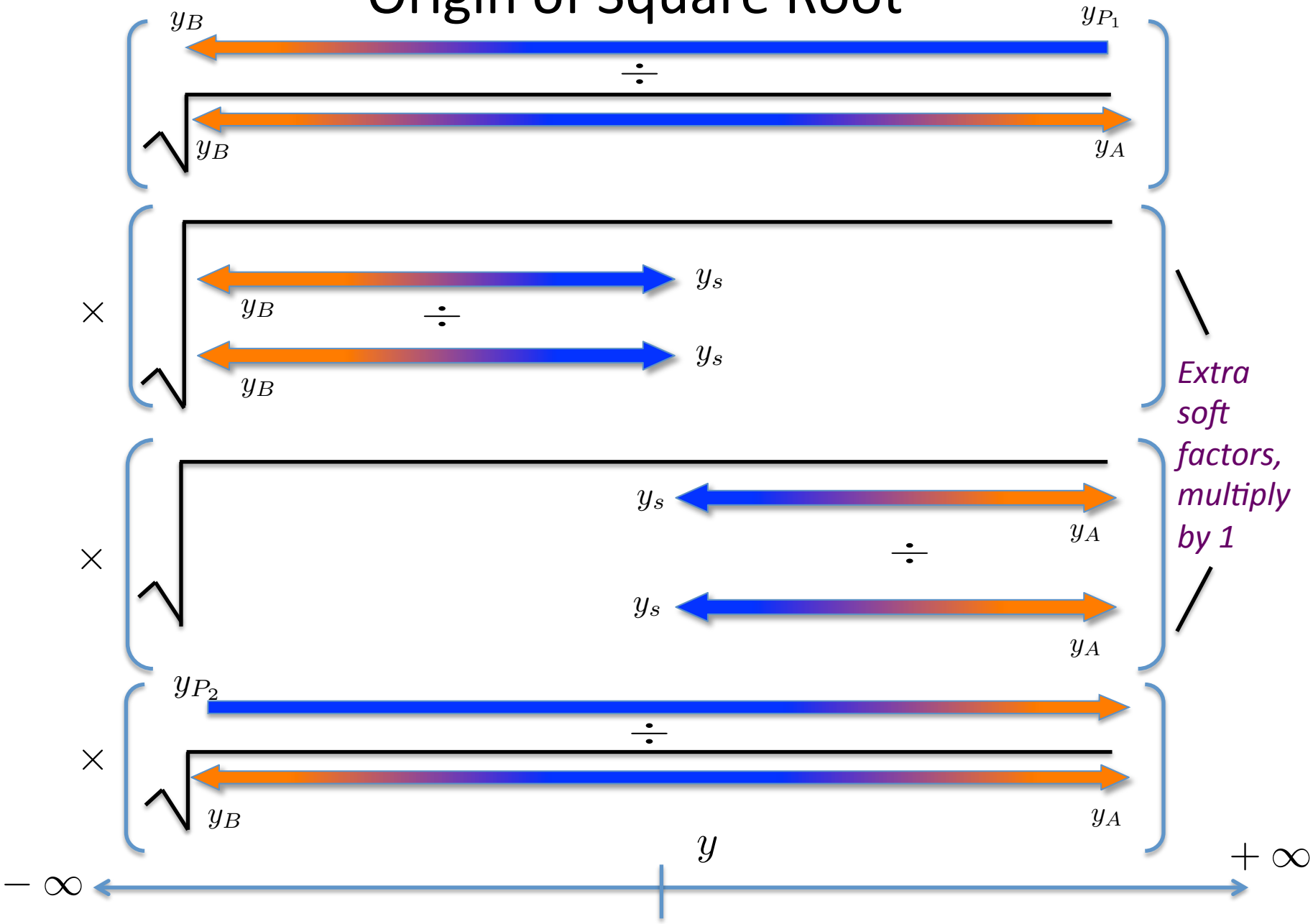
Good Approx. Bad Approx.

Origin of Square Root



Good Approx. Bad Approx.

Origin of Square Root



Origin of Square Root

- Start with only the hard part factorized:

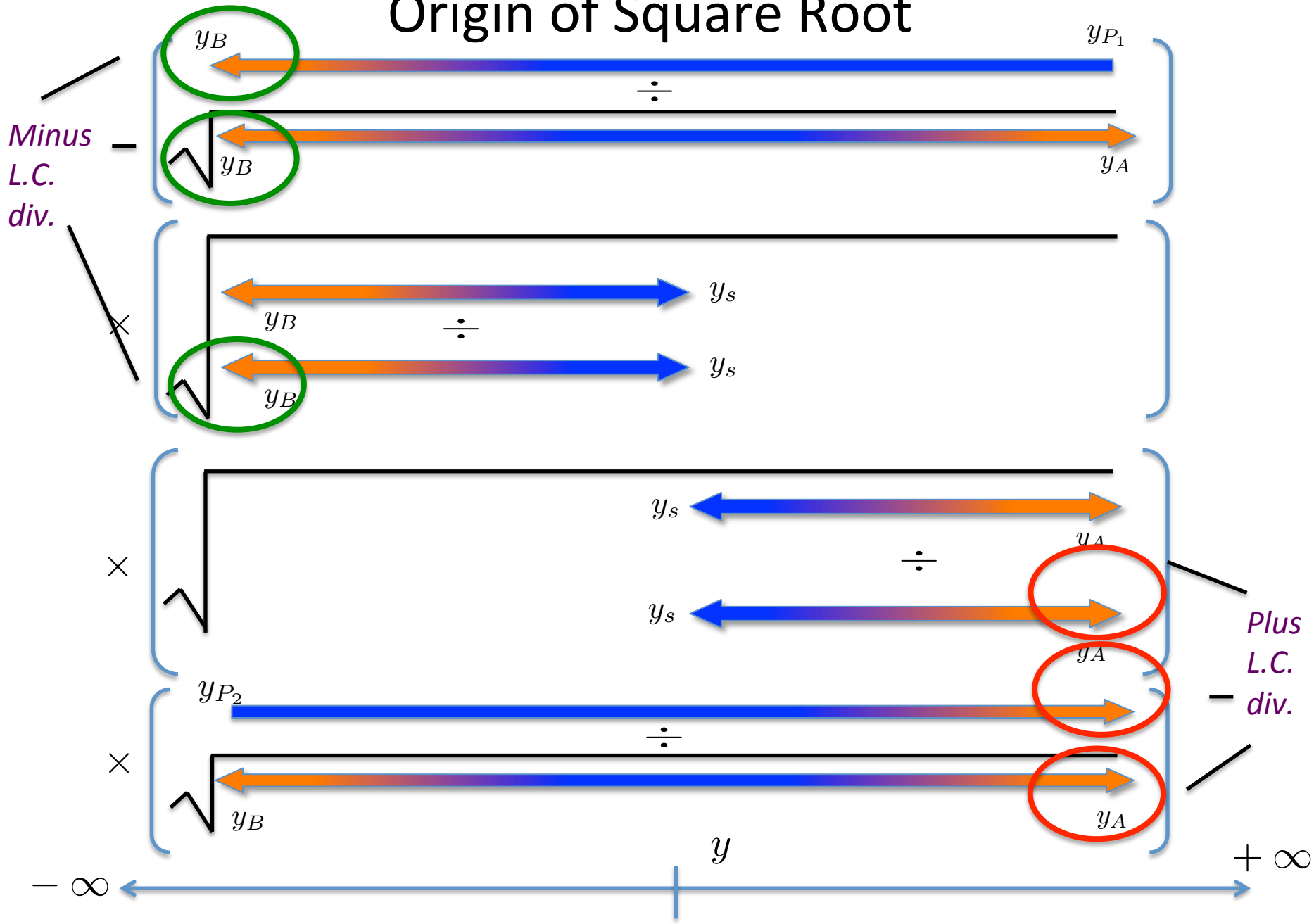
$$d\sigma = |\mathcal{H}|^2 \, \frac{\tilde{F}_1^{\text{unsub}}(y_1 - (-\infty)) \times \tilde{F}_2^{\text{unsub}}(+\infty - y_2)}{\tilde{S}(+\infty, -\infty)}.$$

- Separate soft part:

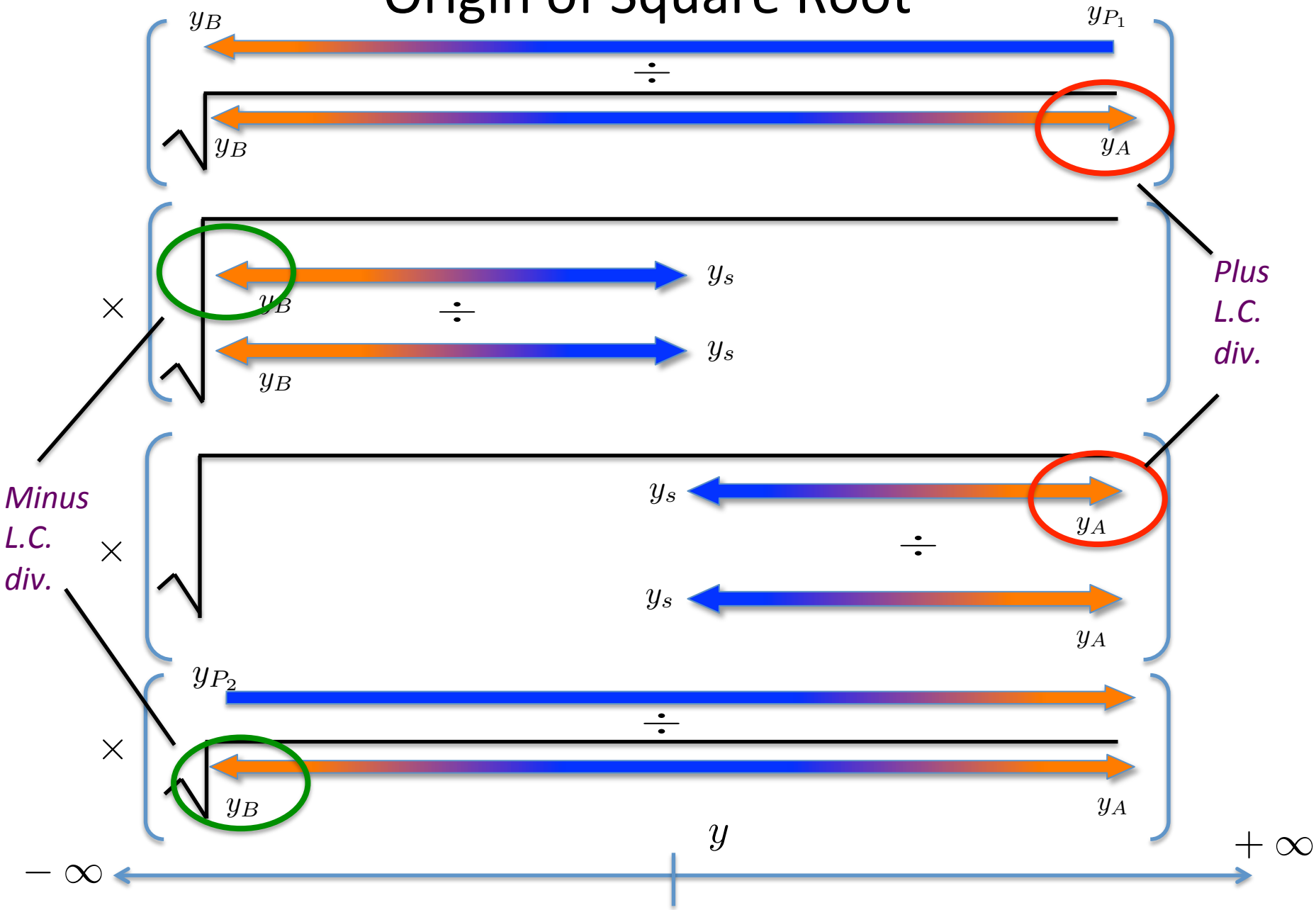
$$d\sigma = |\mathcal{H}|^2 \, \frac{F_1^{\text{unsub}}(y_1 - (-\infty))}{\sqrt{\tilde{S}(+\infty, -\infty)}} \times \frac{\tilde{F}_2^{\text{unsub}}(+\infty - y_2)}{\sqrt{\tilde{S}(+\infty, -\infty)}}.$$

- Multiply by: $\dfrac{\sqrt{\tilde{S}(+\infty, y_s)\, \tilde{S}(y_s, -\infty)}}{\sqrt{\tilde{S}(+\infty, y_s)\, \tilde{S}(y_s, -\infty)}}$

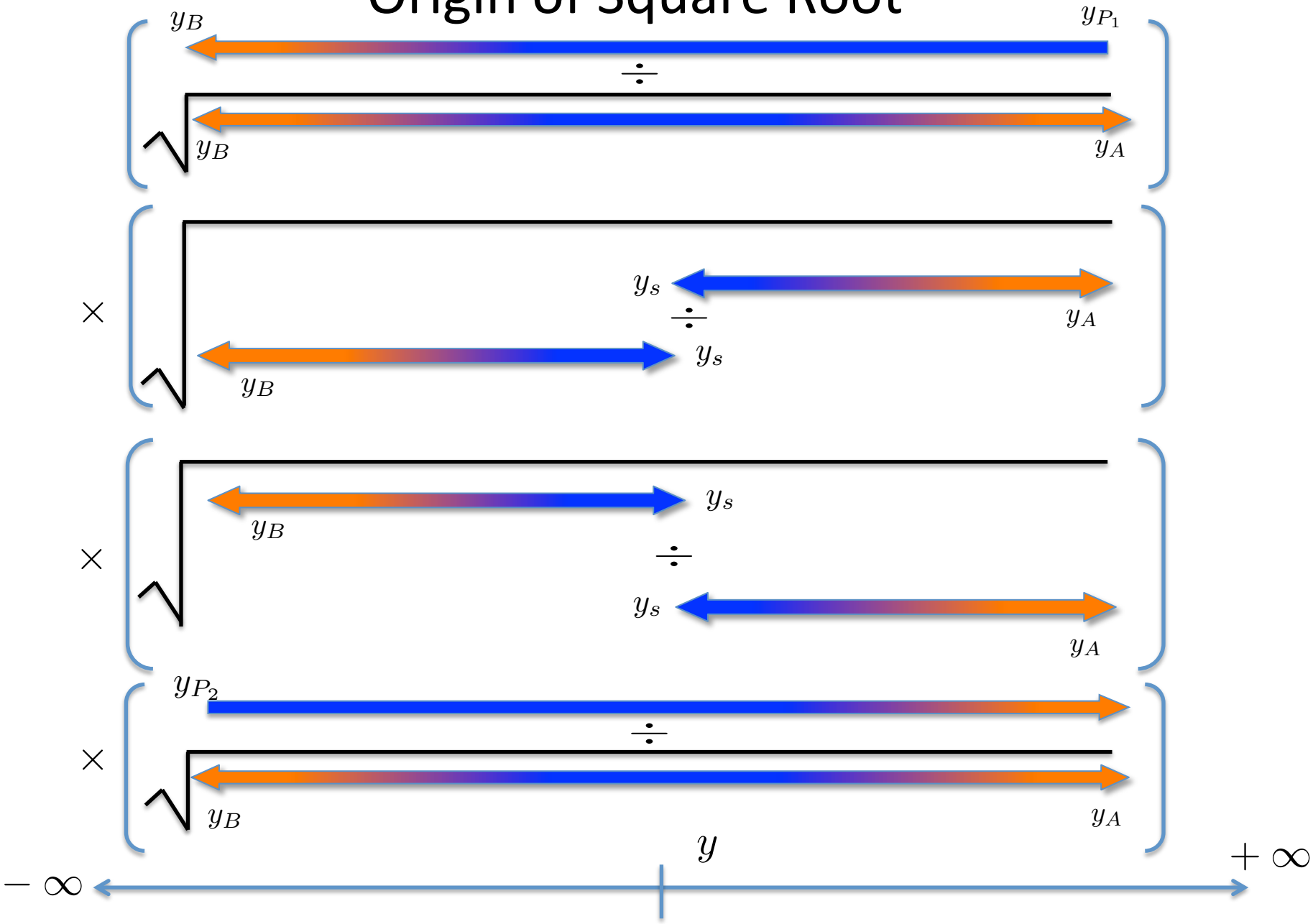
Origin of Square Root



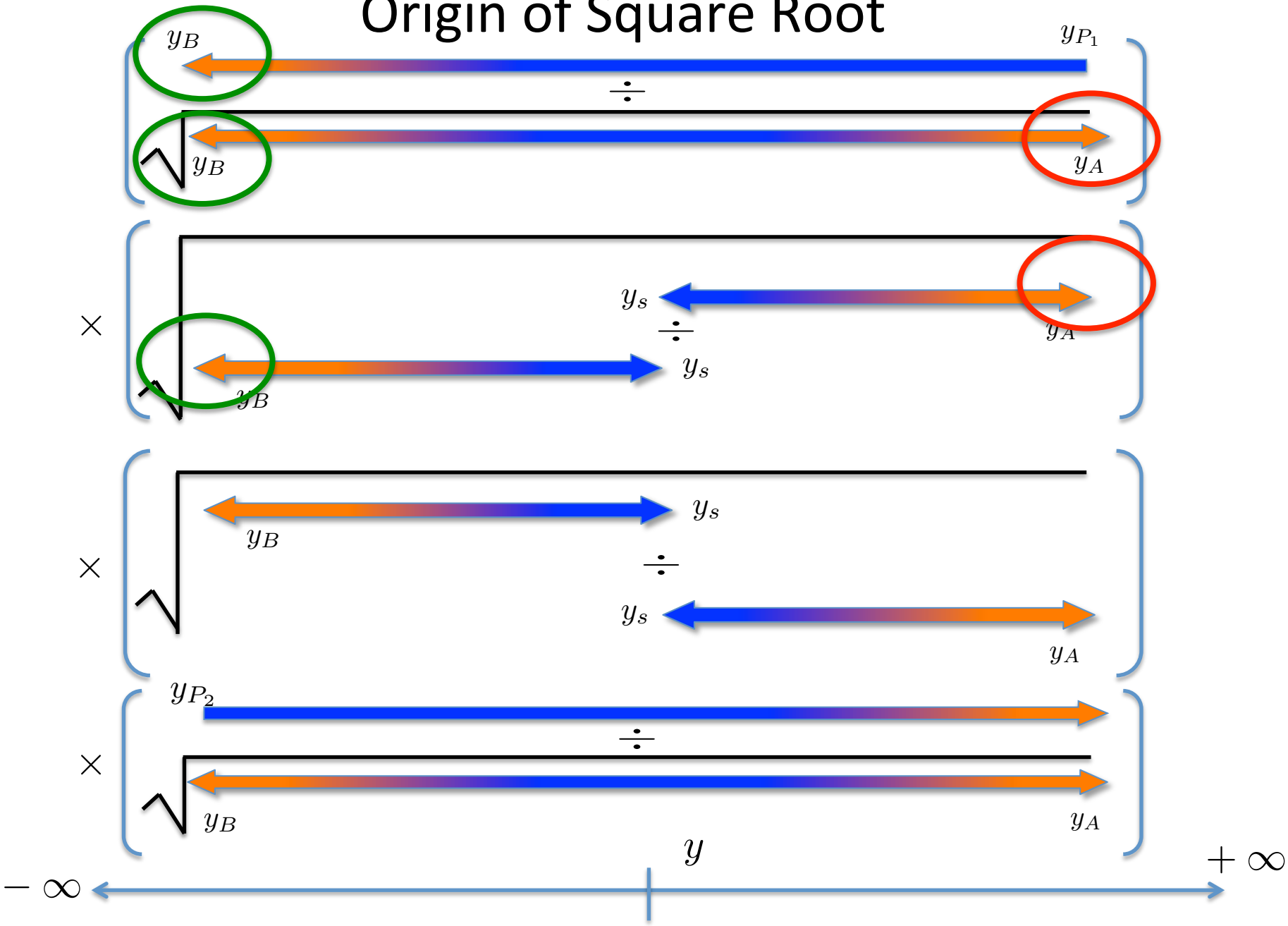
Origin of Square Root



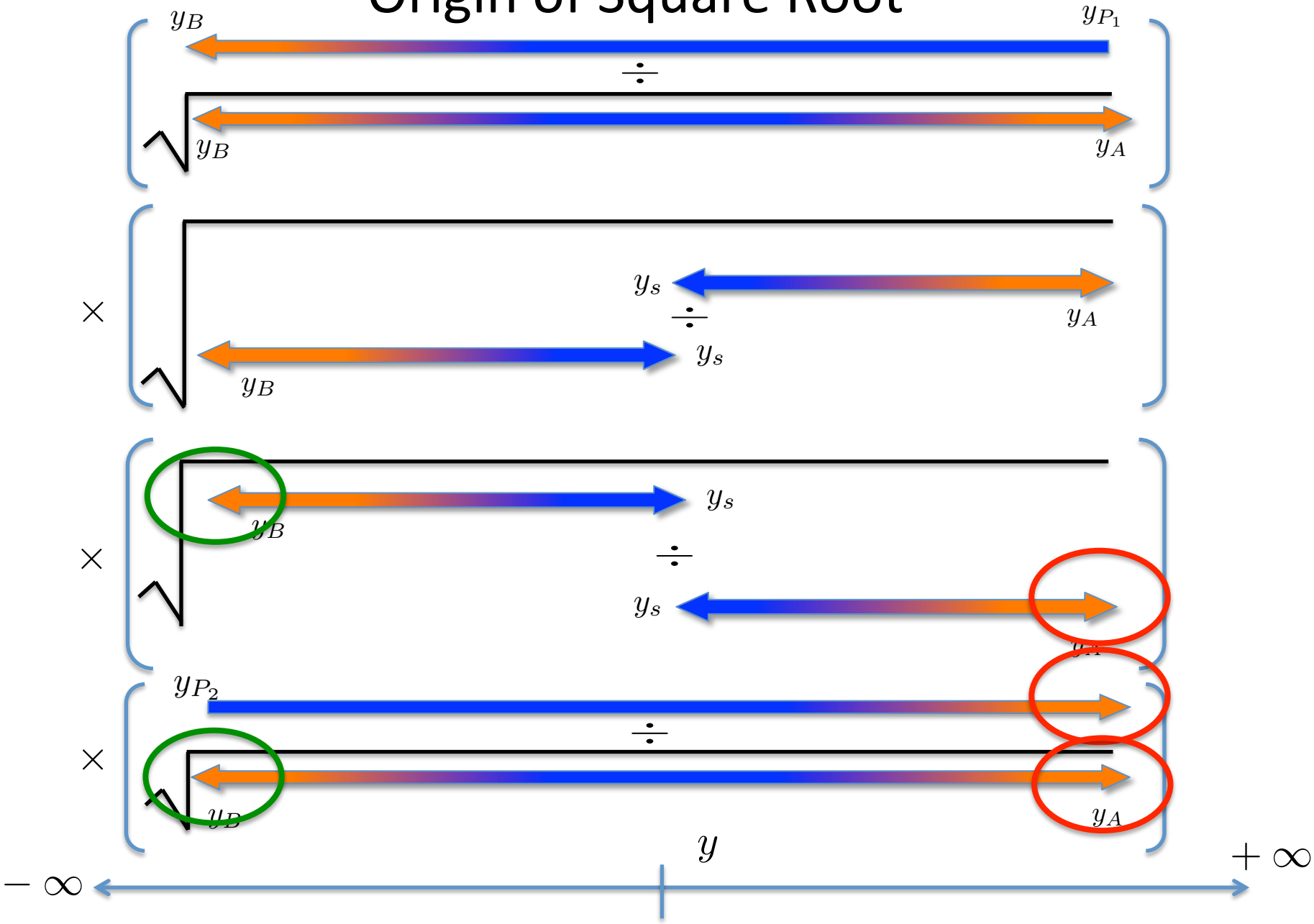
Origin of Square Root



Origin of Square Root



Origin of Square Root



Origin of Square Root

- Start with only the hard part factorized:

$$d\sigma = |\mathcal{H}|^2 \, \frac{\tilde{F}_1^{\rm unsub}(y_1 - (-\infty)) \times \tilde{F}_2^{\rm unsub}(+\infty - y_2)}{\tilde{S}(+\infty, -\infty)}.$$

- Separate soft part:

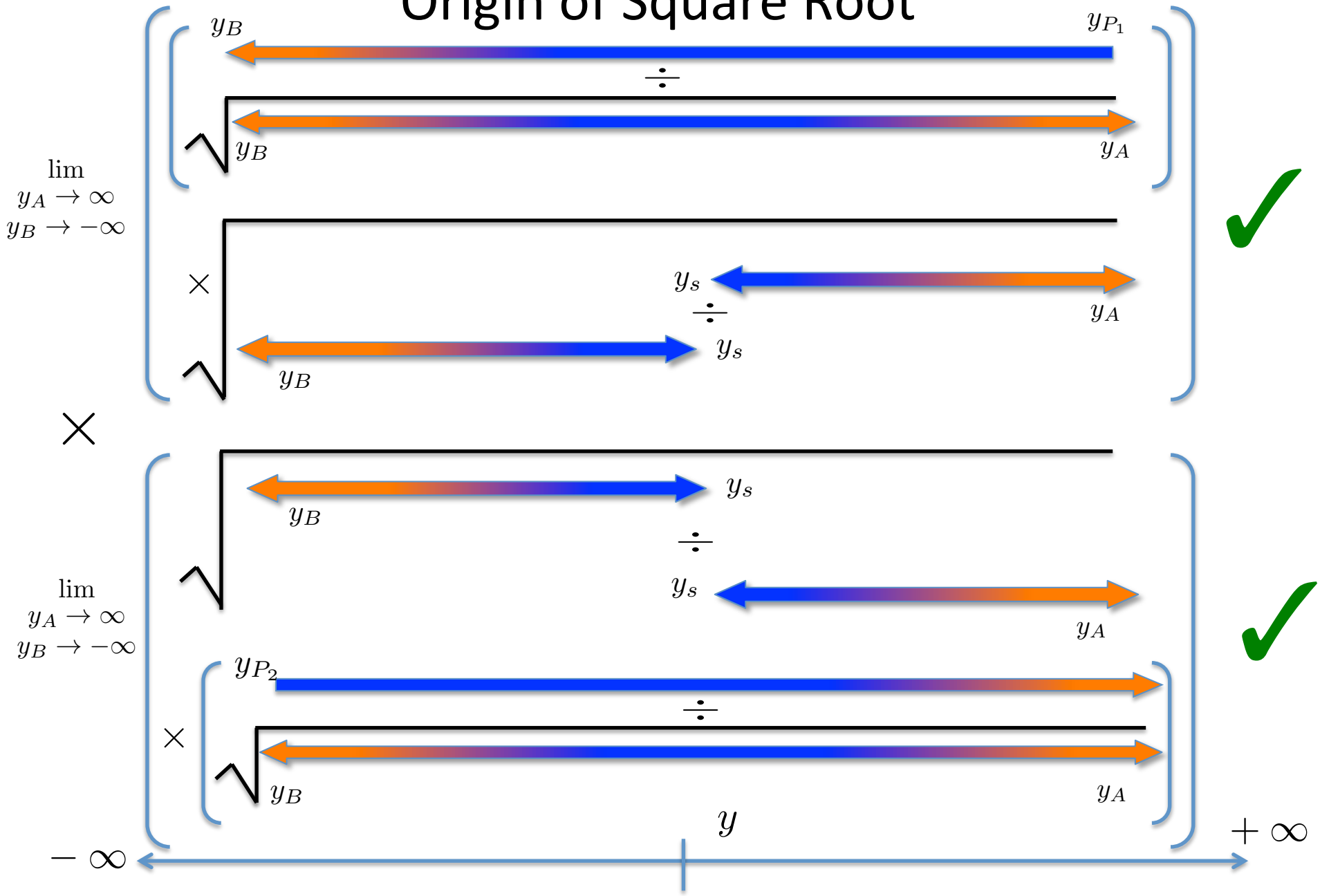
$$d\sigma = |\mathcal{H}|^2 \, \frac{F_1^{\rm unsub}(y_1 - (-\infty))}{\sqrt{\tilde{S}(+\infty, -\infty)}} \times \frac{\tilde{F}_2^{\rm unsub}(+\infty - y_2)}{\sqrt{\tilde{S}(+\infty, -\infty)}}.$$

- Multiply by: $\dfrac{\sqrt{\tilde{S}(+\infty, y_s)\,\tilde{S}(y_s, -\infty)}}{\sqrt{\tilde{S}(+\infty, y_s)\,\tilde{S}(y_s, -\infty)}}$

- Rearrange factors: $d\sigma = |\mathcal{H}|^2 \left\{ F_1^{\rm unsub}(y_1 - (-\infty)) \sqrt{\dfrac{\tilde{S}(+\infty, y_s)}{\tilde{S}(+\infty, -\infty)\tilde{S}(y_s, -\infty)}} \right\}$

$$\times \left\{ \tilde{F}_2^{\rm unsub}(+\infty - y_2) \sqrt{\frac{\tilde{S}(y_s, -\infty)}{\tilde{S}(+\infty, -\infty)\tilde{S}(+\infty, y_s)}} \right\}$$

Origin of Square Root



Origin of Square Root

- Start with only the hard part factorized:

$$d\sigma = |\mathcal{H}|^2 \, \frac{\tilde{F}_1^{\text{unsub}}(y_1 - (-\infty)) \times \tilde{F}_2^{\text{unsub}}(+\infty - y_2)}{\tilde{S}(+\infty, -\infty)}.$$

- Separate soft part:

$$d\sigma = |\mathcal{H}|^2 \, \frac{F_1^{\text{unsub}}(y_1 - (-\infty))}{\sqrt{\tilde{S}(+\infty, -\infty)}} \times \frac{\tilde{F}_2^{\text{unsub}}(+\infty - y_2)}{\sqrt{\tilde{S}(+\infty, -\infty)}}.$$

- Multiply by: $\dfrac{\sqrt{\tilde{S}(+\infty, y_s)\,\tilde{S}(y_s, -\infty)}}{\sqrt{\tilde{S}(+\infty, y_s)\,\tilde{S}(y_s, -\infty)}}$

- Rearrange factors: $d\sigma = |\mathcal{H}|^2 \left\{ F_1^{\text{unsub}}(y_1 - (-\infty)) \sqrt{\dfrac{\tilde{S}(+\infty, y_s)}{\tilde{S}(+\infty, -\infty)\tilde{S}(y_s, -\infty)}} \right\}$

$$\times \left\{ \tilde{F}_2^{\text{unsub}}(+\infty - y_2) \sqrt{\frac{\tilde{S}(y_s, -\infty)}{\tilde{S}(+\infty, -\infty)\tilde{S}(+\infty, y_s)}} \right\}$$

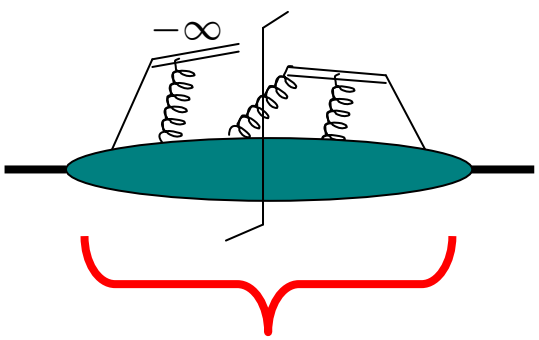
Separately well-defined

Definitions:

(Dictated by factorization requirements)

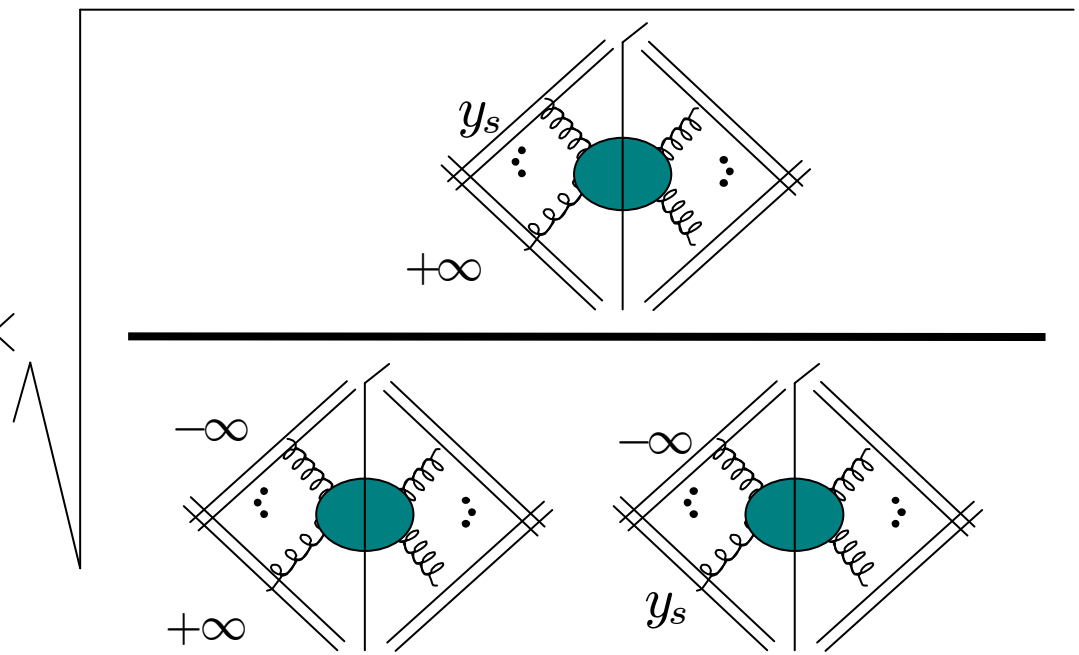
$$\zeta_1\zeta_2 \sim Q^4$$

$$F_{f/P}(x, b; \mu; \zeta_F) = \qquad \zeta_F = 2M_p^2 x^2 e^{2(y_P - y_s)}$$



"Unsubtracted"

$\times$



(UV and rapidity renormalization needed)

Implements Subtractions/Cancellations

(Collins (2011), chapt. 13)

TMD-Factorization

$$W_{DY}^{\mu\nu} = \sum_f |\mathcal{H}_f(Q;\mu/Q)|^{\mu\nu}$$

$$\times \int d^2\mathbf{k}_{1T}\, d^2\mathbf{k}_{2T}\, \underline{F_{f/P_1}(x_1, \mathbf{k}_{1T}; \mu; \zeta_1)}\ \underline{F_{f/P_2}(x_2, \mathbf{k}_{2T}; \mu; \zeta_2)}\, \delta^{(2)}(\mathbf{k}_{1T} + \mathbf{k}_{2T} - \mathbf{q}_T)$$

$$+ Y(q_T, Q)$$

$$+ \mathcal{O}\left(\left(\frac{\Lambda}{Q}\right)^a\right)$$

Evolution

- Collins-Soper Equation:
  - $$\frac{\partial \ln \tilde{F}(x, b_T, \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T; \mu)$$

Perturbatively calculable from definition at small b.

$$\tilde{K}(b_T; \mu) = \frac{1}{2} \frac{\partial}{\partial y_n} \ln \frac{\tilde{S}(b_T; y_n, -\infty)}{\tilde{S}(b_T; +\infty, y_n)}$$

- RG:
  - $$\frac{d\tilde{K}}{d \ln \mu} = -\gamma_K(g(\mu))$$
  - $$\frac{d \ln \tilde{F}(x, b_T; \mu, \zeta)}{d \ln \mu} = -\gamma_F(g(\mu); \zeta/\mu^2)$$

Perturbatively calculable, from definitions

Scales

- UV divergences:

$$\frac{d\tilde{K}}{d\ln\mu} = -\gamma_K(g(\mu))$$

$$\frac{d\ln\tilde{F}(x, b_T; \mu, \zeta)}{d\ln\mu} = -\gamma_F(g(\mu); \zeta/\mu^2)$$

Scales

- UV divergences:

Hard

$$|\mathcal{H}_f(Q;\mu/Q)^2|^{\mu\nu} = e_f^2 |H_0^2|^{\mu\nu}\left(1+\frac{C_F\alpha_s}{\pi}\left[\frac{3}{2}\ln\left(Q^2/\mu^2\right)-\frac{1}{2}\ln^2\left(Q^2/\mu^2\right)-4+\frac{\pi^2}{2}\right]\right)+\mathcal{O}(\alpha_s^2)$$

- No y_s in hard part.
- Pert. theory with: $\mu \sim Q$

Non-Hard

Scales

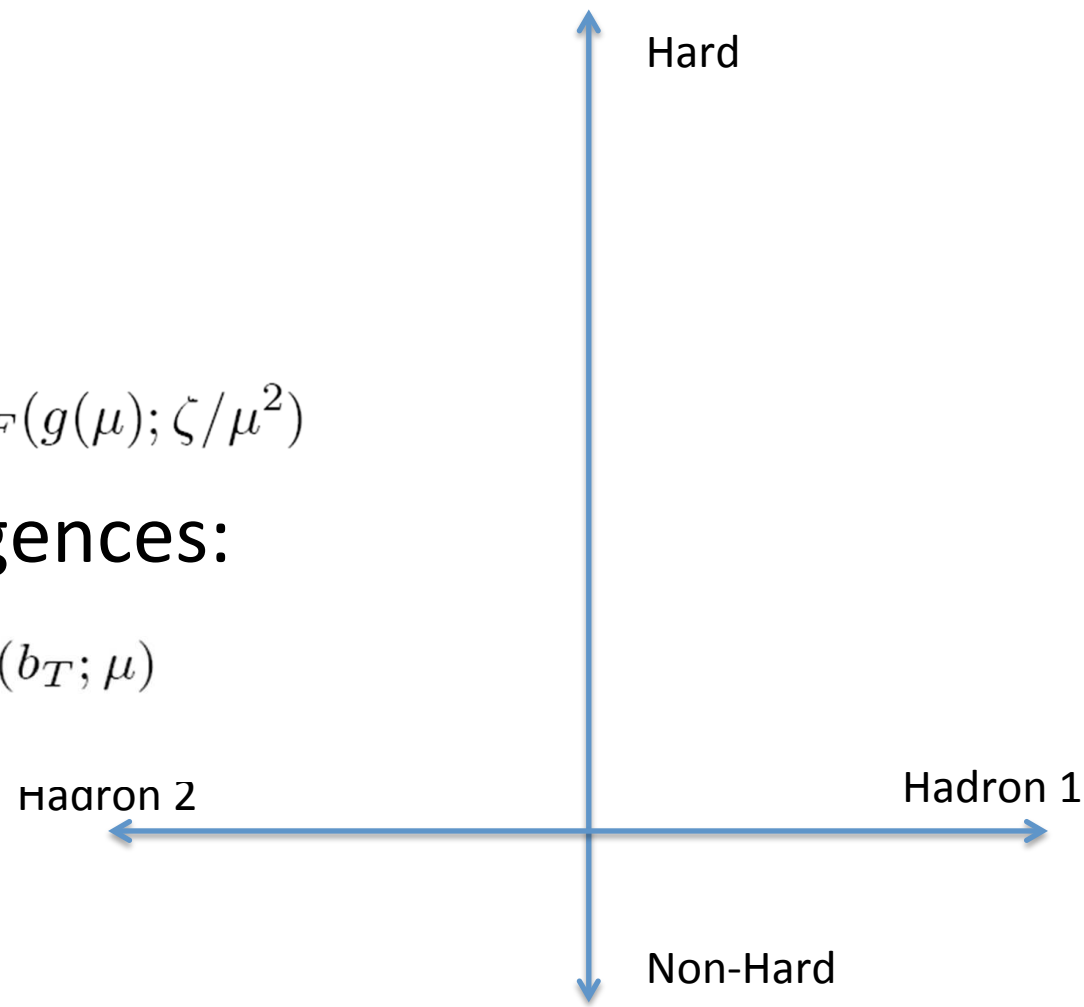
- UV divergences:

$$\frac{d\tilde{K}}{d\ln\mu} = -\gamma_K(g(\mu))$$

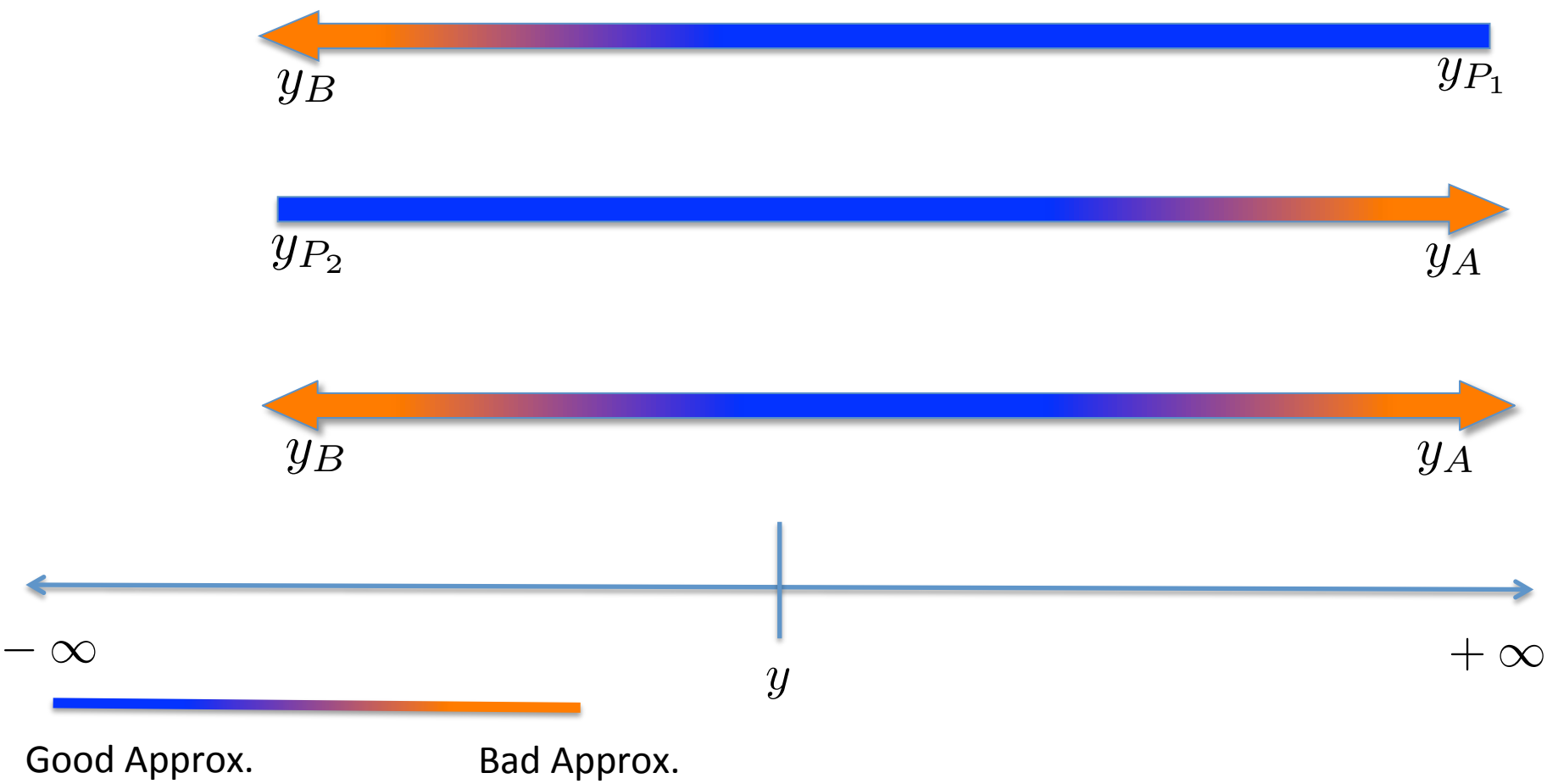
$$\frac{d\ln\tilde{F}(x,b_T;\mu,\zeta)}{d\ln\mu} = -\gamma_F(g(\mu);\zeta/\mu^2)$$

- Light-cone divergences:

$$\frac{\partial\ln\tilde{F}(x,b_T,\mu,\zeta)}{\partial\ln\sqrt{\zeta}} = \tilde{K}(b_T;\mu)$$



Recall: Origin of Square Root



Scales

- UV divergences:

$$\frac{d\tilde{K}}{d\ln\mu} = -\gamma_K(g(\mu))$$

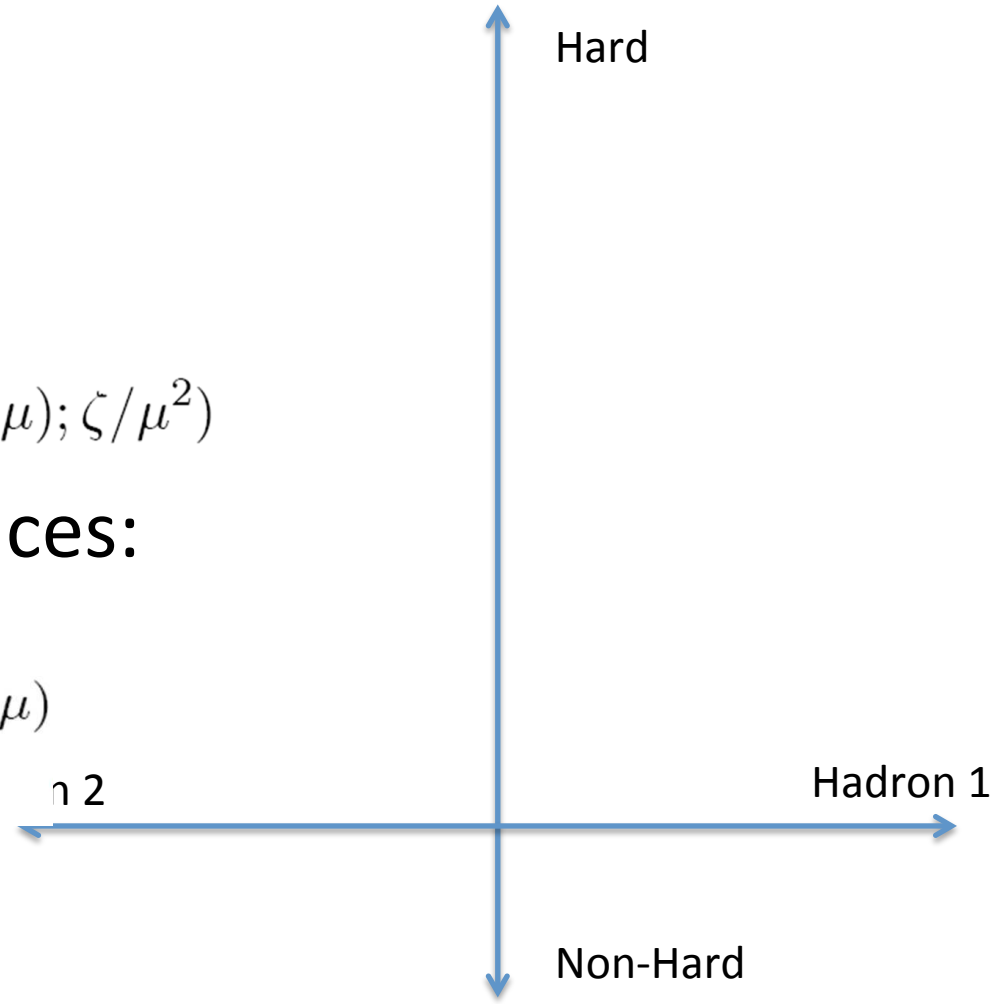
$$\frac{d\ln\tilde{F}(x,b_T;\mu,\zeta)}{d\ln\mu} = -\gamma_F(g(\mu);\zeta/\mu^2)$$

- Light-cone divergences:

$$\frac{\partial\ln\tilde{F}(x,b_T,\mu,\zeta)}{\partial\ln\sqrt{\zeta}} = \tilde{K}(b_T;\mu)$$

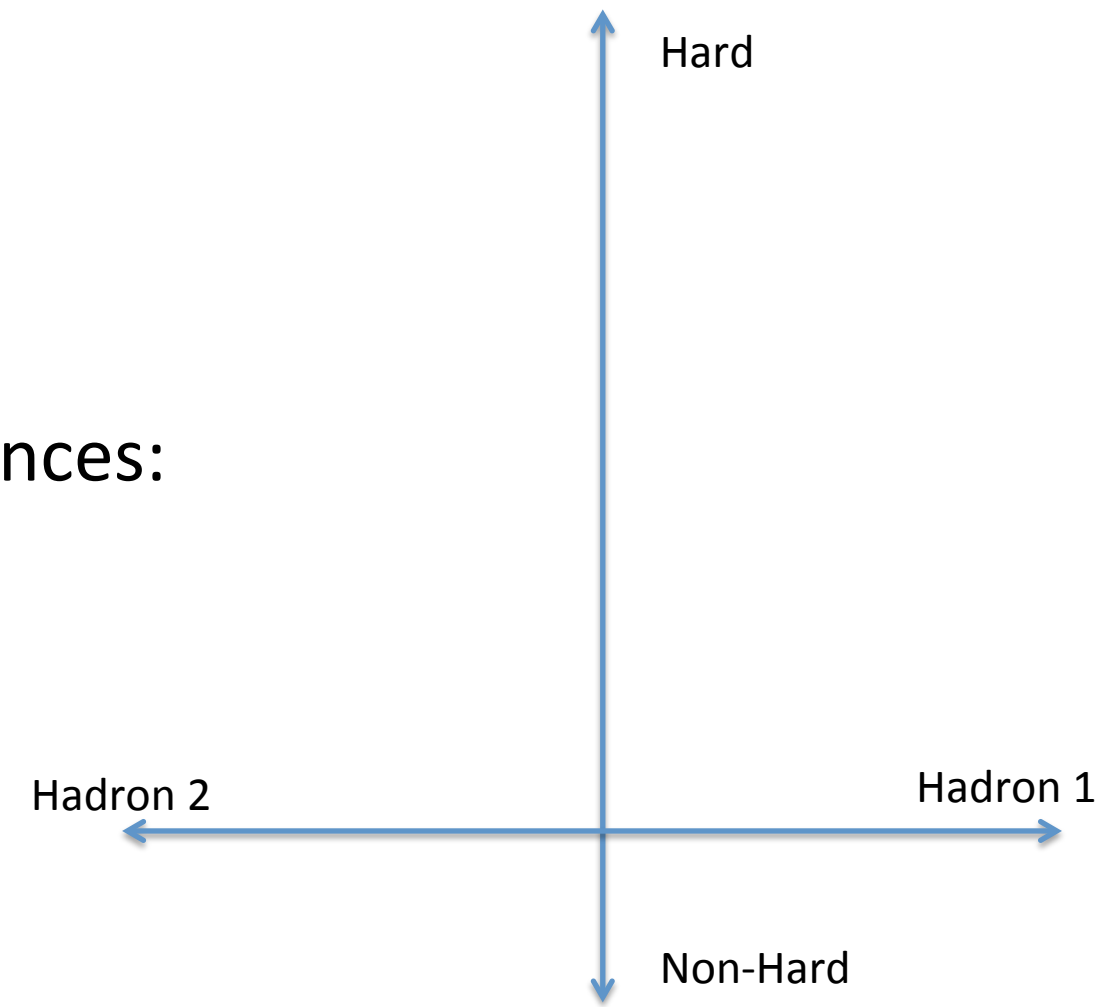
  - Pert. theory requires:

$$\zeta \sim \mu^2$$



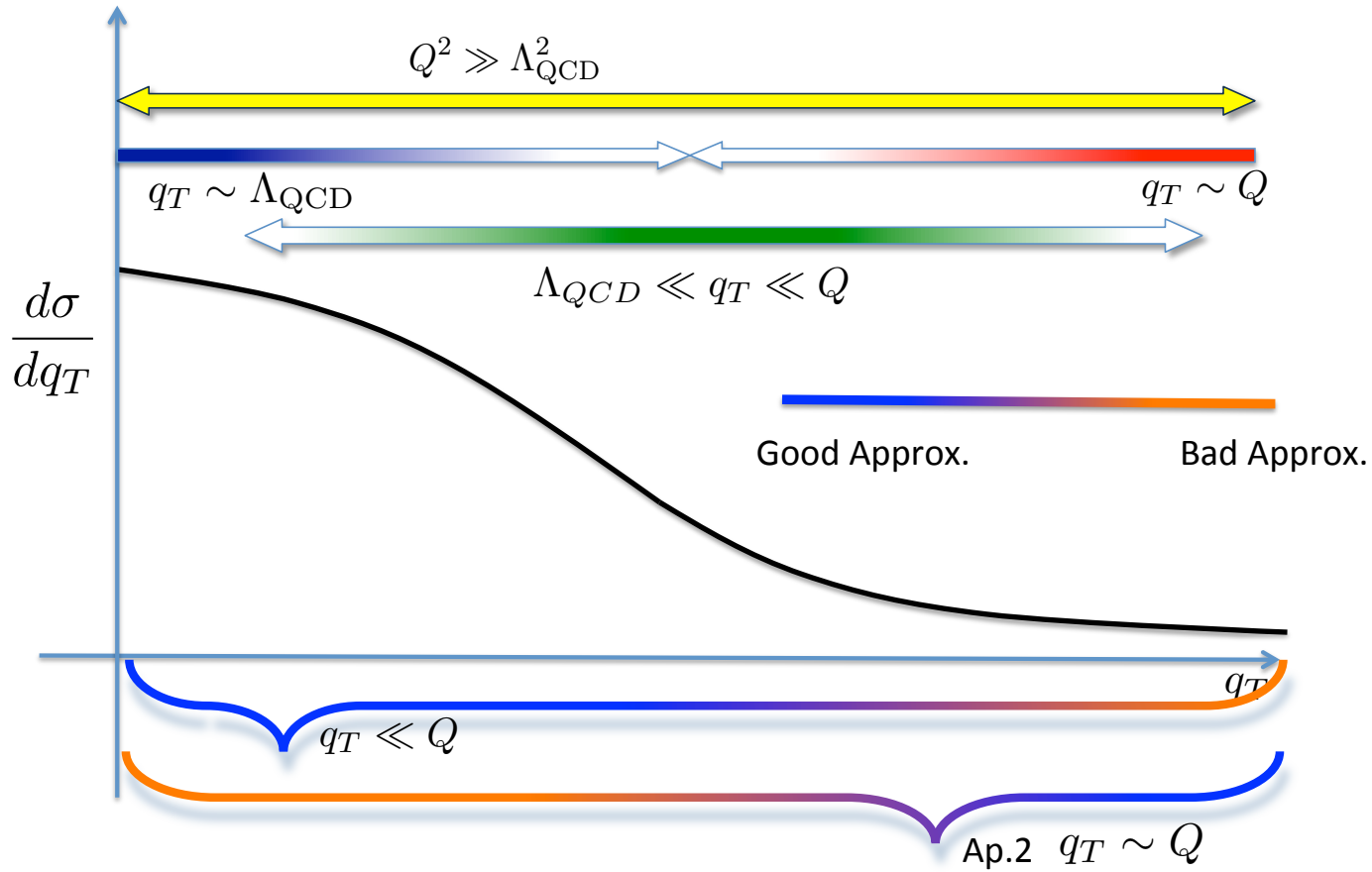
Scales

- UV divergences:
- Light-Cone divergences:
- Maximize perturbative input.



Recap

Regions of Transverse Momentum:



Recap

Regions of Transverse Momentum:

$$W_{DY}^{\mu\nu} = \sum_f |\mathcal{H}_f(Q;\mu/Q)|^{\mu\nu}$$

Similar to generalized TMD parton model

$$\times \int d^2\mathbf{k}_{1T}\, d^2\mathbf{k}_{2T}\, F_{f/P_1}(x_1, \mathbf{k}_{1T}; \mu; \zeta_1)\; F_{f/P_2}(x_2, \mathbf{k}_{2T}; \mu; \zeta_2)\, \delta^{(2)}(\mathbf{k}_{1T} + \mathbf{k}_{2T} - \mathbf{q}_T)$$

TMD PDF for hadron 1.

TMD PDF for hadron 2.

TMD part: $q_T \ll Q$

Ap.2 $q_T \sim Q$

Recap

Regions of Transverse Momentum:

$$W_{DY}^{\mu\nu} = \sum_f |\mathcal{H}_f(Q;\mu/Q)|^{\mu\nu}$$

Similar to generalized TMD parton model

$$\times \int d^2\mathbf{k}_{1T}\, d^2\mathbf{k}_{2T}\, F_{f/P_1}(x_1,\mathbf{k}_{1T};\mu;\zeta_1)\; F_{f/P_2}(x_2,\mathbf{k}_{2T};\mu;\zeta_2)\, \delta^{(2)}(\mathbf{k}_{1T}+\mathbf{k}_{2T}-\mathbf{q}_T)$$

TMD PDF ***TMD PDF***

$$F_{f/P_1}(x_1,\mathbf{k}_{1T};\mu;\zeta_1) = \underbrace{\mathcal{C}_{f/f'}(x/x',\mathbf{k}_{1T},\zeta_1,\mu,\alpha_s(\mu))}_{\text{Perturbatively calculable coefficient functions}} \otimes \underbrace{f_{f'/P_1}(x';\mu)}_{\text{Collinear PDFs}} + \underbrace{\mathcal{O}\left(\frac{\Lambda_{QCD}}{k_{1T}}\right)}_{\text{Error}}$$

Perturbatively calculable coefficient functions ***Collinear PDFs*** ***Error***

Ap.2 $q_T \sim Q$

Recap

Regions of Transverse Momentum: Evolution

- Collins-Soper Equation:
  - $$\frac{\partial \ln \tilde{F}(x, b_T, \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T; \mu)$$

    Perturbatively calculable from definition at small b.

$$\tilde{K}(b_T; \mu) = \frac{1}{2} \frac{\partial}{\partial y_n} \ln \frac{\tilde{S}(b_T; y_n, -\infty)}{\tilde{S}(b_T; +\infty, y_n)}$$

- RG:
  - $$\frac{d\tilde{K}}{d \ln \mu} = -\gamma_K(g(\mu))$$
  - $$\frac{d \ln \tilde{F}(x, b_T; \mu, \zeta)}{d \ln \mu} = -\gamma_F(g(\mu); \zeta/\mu^2)$$

Perturbatively calculable, from definitions

Implementation (b_T-space)

One physical scale for evolution, dictated by requirements of PT:

$$\boxed{\mu \sim \sqrt{\zeta_1} \sim \sqrt{\zeta_2} \sim Q}$$

$$\tilde{F}_{f/H}(x, b_T, \mu, \zeta) = \sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{f/j}(x/\hat{x}, b_*; \mu_b, g(\mu_b)) f_{j/H}(x, \mu_b) \times \quad \text{A}$$

$$\times \exp\left\{ \ln \frac{\sqrt{\zeta}}{\mu_b} \tilde{K}(b_*; \mu_b) + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F(g(\mu'); 1) - \ln \frac{\sqrt{\zeta}}{\mu'} \gamma_K(g(\mu')) \right] \right\} \times \quad \text{B}$$

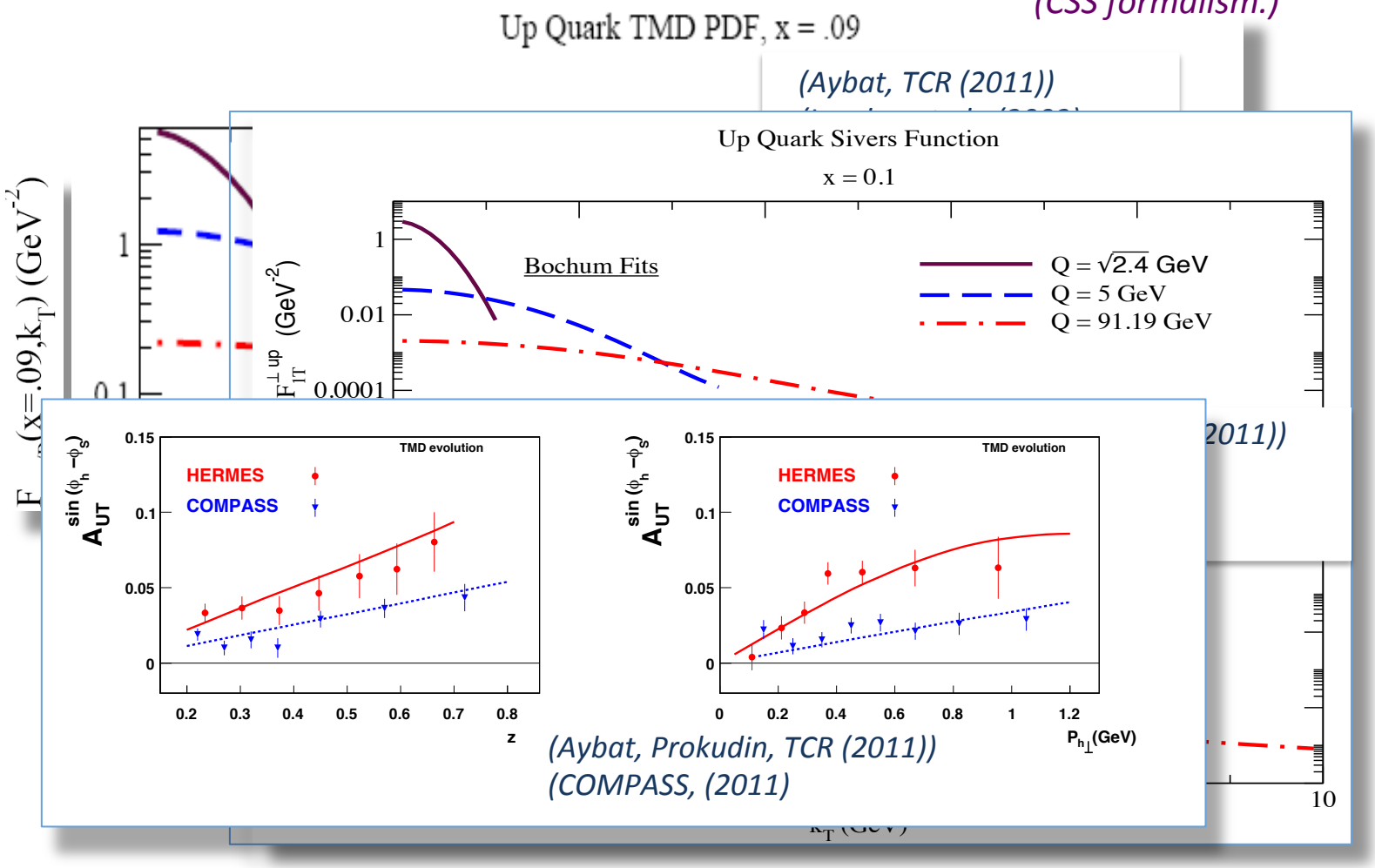
$$\times \exp\left\{ g_{j/H}(x, b_T) + g_K(b_T) \ln \frac{\sqrt{\zeta}}{Q_0} \right\} \quad \text{C}$$

$$\boxed{b_*(b_T) \equiv \frac{b_T}{\sqrt{1 + b_T^2/b_{max}^2}} \qquad \mu_b(b_T) \sim 1/b_*}$$

(CS matching)

Evolved TMD PDFs constructed from old fits:

(CSS formalism.)



(Aybat, TCR (2011))

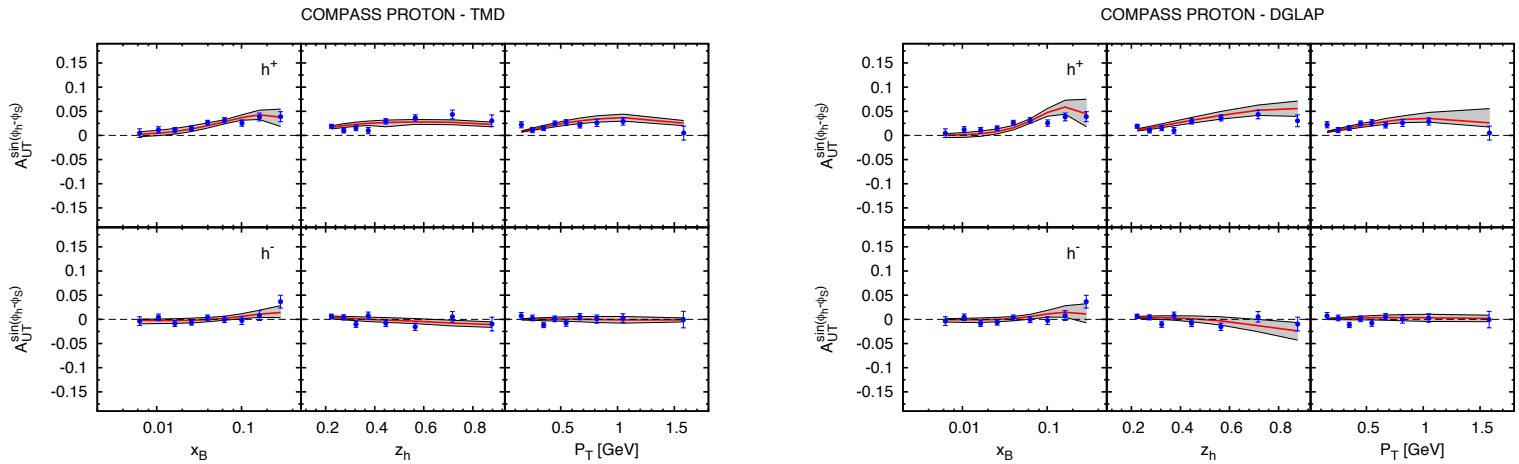
(Aybat, Prokudin, TCR (2011))
(COMPASS, (2011)

https://projects.hepforge.org/tmd/

More recent fitting

- Fixed scale. Sum rules.
  (A. Bacchetta, M. Radici (2012))

- Lowest order evolution:
  (M. Anselmino, M. Boglione, S. Melis (2012))



- Spin dep.
  (Z.-B. Kang,J.-W. Qiu (2012))

- Direct fitting in coordinate space.
  (Boer,Gamberg,Musch,Prokudin (2011))

- Other current fitting activity:
  (Torino-JLab group: A. Prokudin, S. Melis,…)

Testing/Using TMD-Factorization

- New fitting projects, analogous to collinear case.

- Key issue: Maximum amount of data, maximum variety of processes.
  - Cyclic process of fitting and predicting. Analogous to collinear factorization.
    - Much already available:
      - DY, Z/W in pp, pA
      - DY,Z/W in $p\bar{p}$
      - SIDIS
      - e+ e- to back-to-back hadrons.

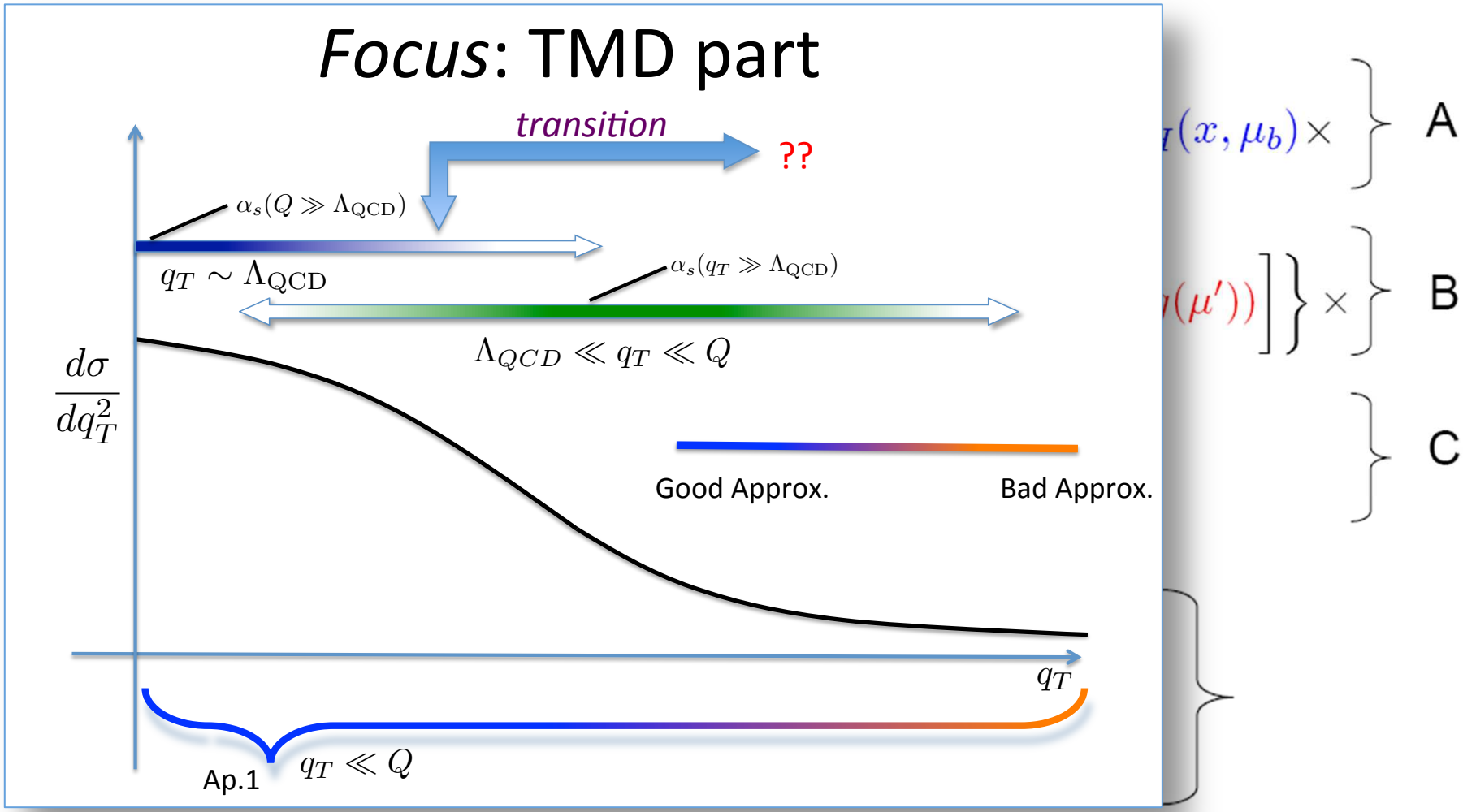
- Isolate interesting non-perturbative contributions.
  - E.g., compare intrinsic transverse momentum of sea and valence quarks.

    (E.g., Compare unpolarized pp and $p\bar{p}$.)

  - Extraction of model parameters.

- Comparing alternative TMD formalisms (e.g. SCET, small-x, etc.)
  - What are the distinctive different phenomenological predictions?
  - What is predicted to be universal vs. non-universal?

Implementation



Momentum Space Evolution

(Coll. with C. Aidala, work in progress)

- Perform all fitting directly in momentum space, with evolution formulated in momentum space.

- Isolate non-perturbative transverse momentum dependent contributions at small q_T.
  - Intrinsic transverse momentum of sea and valence quarks in DY.

(Compare unpolarized pp and $p\bar{p}$.)

Momentum Space Evolution

(Coll. with C. Aidala, work in progress)

$$\tilde{F}_{f/P_1}(x_1, \mathbf{b}_T; \mu, \zeta_1) = \int d^2\mathbf{k}_T \, e^{-i\mathbf{k}_T \cdot \mathbf{b}_T} \, F_{f/P_1}(x_1, \mathbf{k}_T; \mu, \zeta_F)$$

$$\tilde{K}(\mathbf{b}_T; \mu) = \int d^2\mathbf{k}_T \, e^{-i\mathbf{k}_T \cdot \mathbf{b}_T} \, K(\mathbf{k}_T; \mu)$$

$$\frac{\partial}{\partial \ln \sqrt{\zeta_F}} F(x, \mathbf{k}_T; \mu, \zeta_F) = \int d^2\mathbf{q}_T \, K(\mathbf{q}_T; \mu) \, F_{f/P_1}(x_1, \mathbf{k}_T - \mathbf{q}_T; \mu, \zeta_F)$$

$$\frac{d}{d \ln \mu} K(k_T; \mu) = -\gamma_K(g(\mu)) \, \delta(\mathbf{k}_T)$$

$$\frac{d}{d \ln \mu} F_{f/P_1}(x, \mathbf{k}_T; \mu, \zeta_F) = \gamma_F(g(\mu); \zeta_F/\mu^2) \, F_{f/P_1}(x, \mathbf{k}_T; \mu, \zeta_F)$$

Momentum Space Evolution

(Coll. with C. Aidala, work in progress)

- General solutions constructed with a matching defined in k_T space:

$$\mathbf{b}_*(\mathbf{b}_\mathrm{T}) \equiv \frac{\mathbf{b}_\mathrm{T}}{\sqrt{1 + b_T^2/b_\mathrm{max}^2}}$$

$$\mu_*(b_T) = C_1/b_*$$

$$\alpha_s(\mu_*(b_T)) \overset{b_T \to \infty}{=} \alpha_s(C_1/b_{max})$$

(Transverse coord. space)

$$\mathbf{k}_*(\mathbf{k}_T) \equiv \hat{\mathbf{k}}_T \sqrt{k_{min}^2 + k_T^2}$$

$$\mu_*(k_T) \equiv C_1 k_*$$

$$\alpha_s(\mu_*(k_T)) \overset{k_T \to 0}{=} \alpha_s(C_1 k_{min})$$

(Transverse mom. space)

(A. Kulesza, J. Stirling (2001), R. Ellis, S. Veseli (1997))

Momentum Space Evolution

(Coll. with C. Aidala, work in progress)

- General solutions constructed with a matching defined in k_T space:

$$\mathbf{b}_*(\mathbf{b}_\mathrm{T}) \equiv \frac{\mathbf{b}_\mathrm{T}}{\sqrt{1 + b_T^2/b_\mathrm{max}^2}}$$

$$\mu_*(b_T) = C_1/b_*$$

$$\alpha_s(\mu_*(b_T)) \overset{b_T \to \infty}{=} \alpha_s(C_1/b_{max})$$

(Transverse coord. space)

$$\mathbf{k}_*(\mathbf{k}_T) \equiv \hat{\mathbf{k}}_T \sqrt{k_{min}^2 + k_T^2}$$

$$\mu_*(k_T) \equiv C_1 k_*$$

$$\alpha_s(\mu_*(k_T)) \overset{k_T \to 0}{=} \alpha_s(C_1 k_{min})$$

(Transverse mom. space)

(A. Kulesza, J. Stirling (2001), R. Ellis, S. Veseli (1997))

(Study evolution of different transverse momentum regions.)

$$F_{f/p_1}(x_1, \mathbf{k}_t; \mu, \zeta_F) \equiv F_{f/p_1}^{\mathrm{Tail}}(x_1, \mathbf{k}_t; \mu, \zeta_F) + F_{f/p_1}^{\mathrm{Core}}(x_1, \mathbf{k}_t; \mu, \zeta_F)$$

Momentum Space Evolution

(Coll. with C. Aidala, work in progress)

- General solution
  defined in k_T spa

$$\mathbf{b}_*(\mathbf{b}_\mathrm{T}) \equiv \frac{\mathbf{b}_\mathrm{T}}{\sqrt{1+b_T^2/}}$$

$$\mu_*(b_T) = C_1/b_*$$

$$\alpha_s(\mu_*(b_T)) \overset{b_T\to\infty}{=} \alpha_s \qquad in)$$

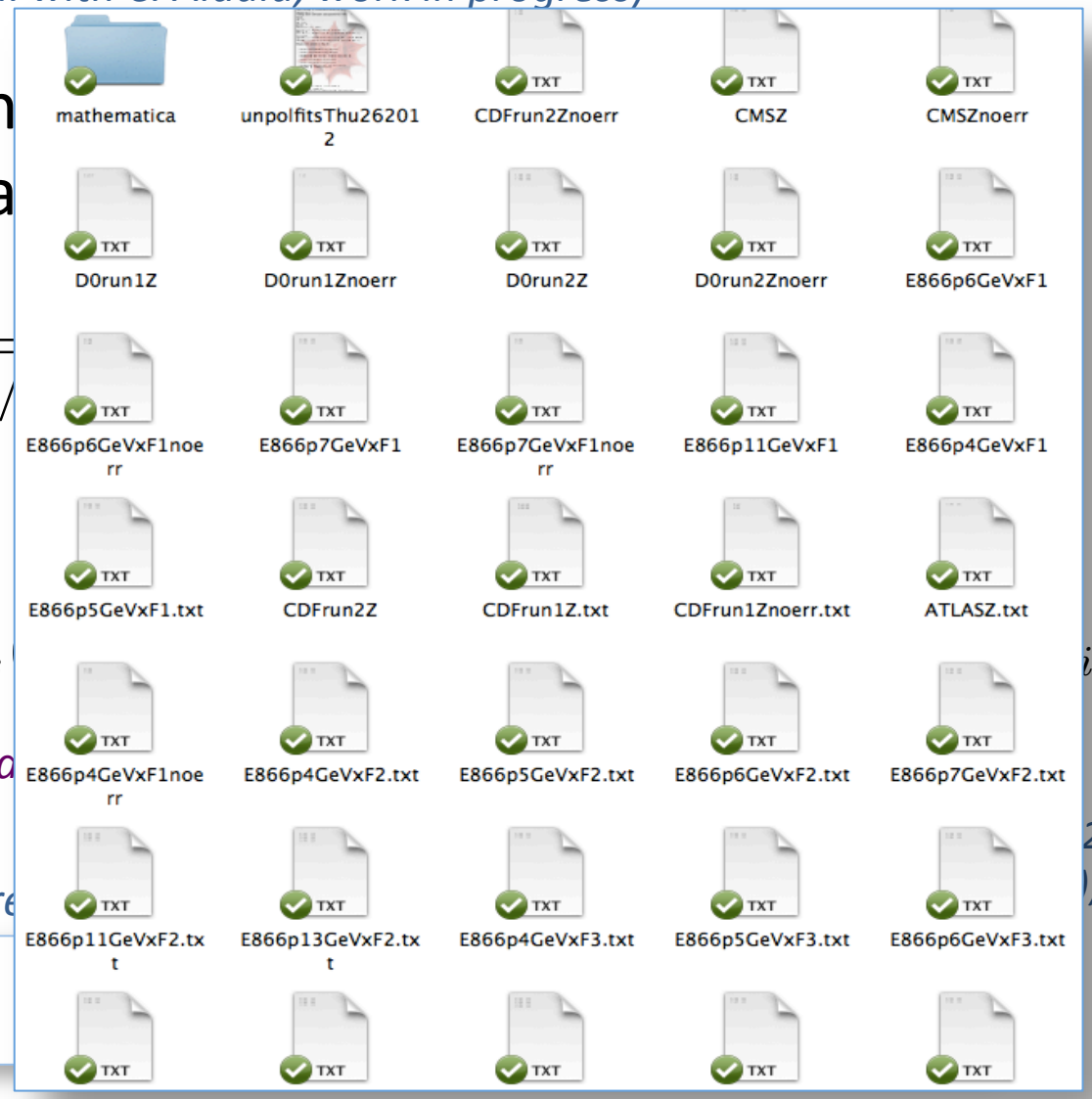
(Transverse coord. spa

2001),

(Study evolution of differe

)

$$F_{f/p_1}(x_1, \mathbf{k}_t; \mu, \zeta_F) \equiv$$



Thanks!