Running Coupling Studies of SU(2) Gauge Theories with Many Fermions

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December 6, 2013

Lattice Strong Dynamics (LSD) Collaboration

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Motivation

- A new sector, described by a strongly interacting gauge theory, could play a key role in physics beyond the Standard Model.
- Knowing the extent of the conformal window and the behavior of theories in it and near it could be crucial for building a successful model of BSM physics.
- With the recent discovery of a 125 GeV Higgs-like scalar, $SU(2)$ vector-like gauge theories provide attractive candidates.
- Study of the two-color fundamental conformal window especially interesting and provides evidence for a gauge theory that exhibits novel IR behavior.

- The fermionic part of the Lagrangian given by

$$\mathcal{L} = \bar{\psi} \not{D} \psi = i \begin{pmatrix} \psi_L^* \\ \psi_R^* \end{pmatrix}^{\mathrm{T}} \begin{pmatrix} \sigma_\nu D_\nu & 0 \\ 0 & -\sigma_\nu^\dagger D_\nu \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix},$$

where $\sigma_\nu = (-i, \sigma_k)$ and D_ν is the gauge covariant derivative.

- Defining $\tilde{\psi}_R = \sigma_2 \tau_2 \psi_R^*$, where σ_2 and τ_2 are the second Pauli matrix acting on spinor and color indices respectively, we may rewrite the above Lagrangian

$$\mathcal{L} = i \begin{pmatrix} \psi_L^* \\ \tilde{\psi}_R^* \end{pmatrix}^{\mathrm{T}} \begin{pmatrix} \sigma_\nu D_\nu & 0 \\ 0 & \sigma_\nu D_\nu \end{pmatrix} \begin{pmatrix} \psi_L \\ \tilde{\psi}_R \end{pmatrix} = i \Psi^\dagger \sigma_\nu D_\nu \Psi.$$

- The Lagrangian evidently has an enhanced $U(2N_f)$ (reduced to $SU(2N_f)$ once axial anomaly taken into account) global chiral symmetry. This enhanced symmetry is fundamentally a consequence of the pseudo-reality of the fundamental representation of $SU(2)$.

[1] Peskin

[2] Kogut, Stephanov, Toublan, Verbaarschot and Zhitnitsky

Peculiarities of $SU(2)$ Gauge theory II

- Suppose global chiral symmetry broken by condensate

$$\langle \bar{\psi}\psi \rangle = \frac{1}{2}\Psi^{\mathrm{T}}\sigma_2\tau_2 \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \Psi + \text{h.c.}.$$

- Only a $Sp(2N_f)$ subgroup of $SU(2N_f)$ leaves the condensate invariant.
- Have $\left[(2N_f)^2 - 1\right] - \left[\frac{1}{2}(2N_f)(2N_f - 1)\right] = N_f(2N_f - 1) - 1$ NGBs. This differs significantly from the usual $N_f^2 - 1$ NGBs formed in the case of a complex representation as in QCD.
- Looking for strongly coupled behavior, qualitatively different from QCD. Two-colors seems particularly promising.
- Additionally several papers use this novel pattern of symmetry breaking to write down an effective field theories of scalar and pseudo-scalar NGBs[3] [4].

[3] Galloway, Evans, Luty, and Tachi

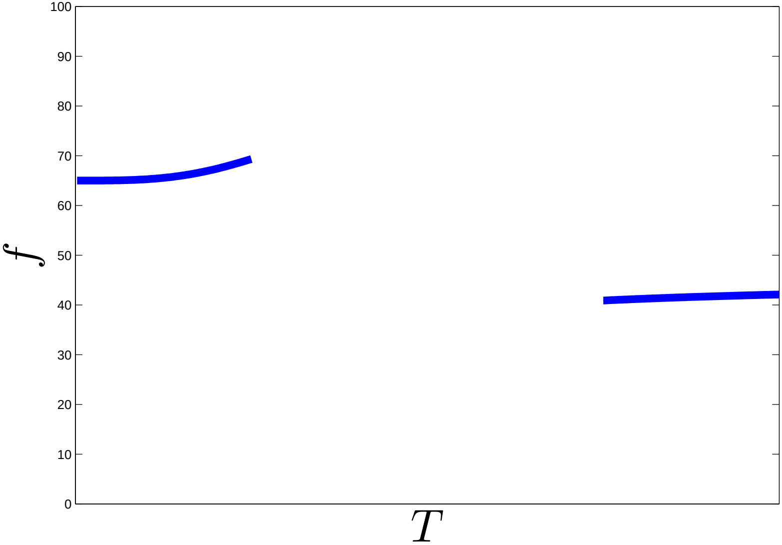
[4] Katz, Nelson, and Walker

Thermal Inequality

- A way to estimate N_f^c is conjectured thermal inequality[5]. Consider $f(T) = 90F(T)/\pi^2 T^4$.
- Basic idea: postulate that the massless degrees of freedom in the UV ($f_{\mathrm{UV}} = f(\infty)$) should be greater than or equal to the massless degrees of freedom in the IR ($f_{\mathrm{IR}} = f(0)$).
- In an asymptotically free theory that undergoes χSB, it's easy to calculate the degrees of freedom:
  - in UV (free theory of gauge bosons and fermions): $f_{\mathrm{UV}} = 2\left(N_c^2 - 1\right) + \frac{7}{2} N_c N_f$.
  - In IR (free theory of NGBs):
    - $N_c \geq 3$: $f_{\mathrm{IR}} = N_f^2 - 1$.
    - $N_c = 2$: $f_{\mathrm{IR}} = 2N_f^2 - N_f - 1$.
- For $N_c \geq 3$ ACS conjecture implies $N_{f,\mathrm{ACS}}^c < \frac{1}{4}\left(7N_c + \sqrt{81N_c^2 - 16}\right)$ and this bound is slightly larger than the LG estimate.
- For $N_c = 2$, thermal inequality implies $N_f^c \lesssim 4.7$.

[5] Appelquist, et al.

What if χSB in $N_f = 6$?



Ladder Gap Analysis

- Ladder gap equation lets one crudely estimate critical value of $\bar{g}_c^2$ required to trigger χSB[6].
- Basic idea: find value of $\bar{g}^2$ at which solutions to the rainbow approximated Schwinger-Dyson equation are consistent with spontaneous χSB.
- $\bar{g}_c^2 = \frac{4\pi^2}{3C_2(R)} \approx 17.5$.
- By combining this estimate with the two-loop IRFP value, can get an estimate on the edge of the conformal window N_f^c with condition $\bar{g}_*^2(N_c, N_f) = \bar{g}_c^2(N_c)$, yielding

$$N^c_{f,\mathrm{LG}} = N_c\left(4 + \frac{6}{15 - 25N_c^2}\right) \approx 8.$$

[6] Cohen and Georgi

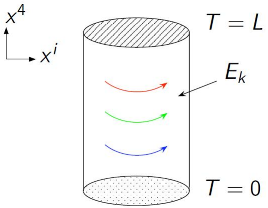
Survey[7] of Running Coupling Results

- Schrödinger Functional study indicates $N_f = 4$ is outside the conformal window.
- Evedince for IRFP and hence inside conformal window:
  - $N_f = 8$ (twisted Polyakov loop scheme)
  - $N_f = 10$ (SF).
- $N_f = 6$, arguable the most interesting case, tackled by many groups (all SF) has remained inconclusive.
- We pursue $N_f = 6$, with stout-smeared Wilson fermion action.

[7] Bursa et al. 2010, Ohki et al. 2010, Karavitra et al. 2011, Voronov 2012 and 2013, and Hayakawa et al. 2013

Schrödinger Functional Running Coupling[8]

- The Schrödinger Functional is defined within a Euclidean box of spatial extent L, with:
- Periodic boundary conditions in spatial directions.
- Dirichlet boundary conditions in time direction, η parametrizes the strength of the constant chromoelectric background field.



- The SF is given by

$$\mathcal{Z}(\eta) = \int D\left[A, \psi, \bar{\psi}\right] e^{-S\left[A,\psi,\bar{\psi};\eta\right]}$$

and can define a non-perturbative running coupling by,

$$\frac{k}{\bar{g}^2\left(g_0^2, L/a\right)} = \frac{\partial}{\partial \eta} \log \mathcal{Z} = \left\langle \frac{\partial S}{\partial \eta} \right\rangle .$$

[8] M. Luscher et al. 1992

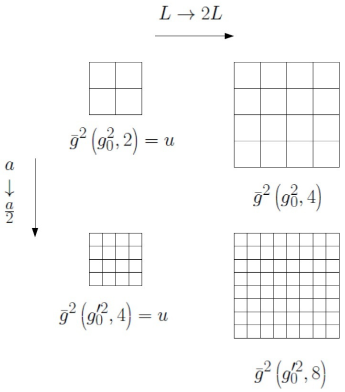
Step-Scaling[9] Function

- The quantity of interest is the continuum step scaling function $\sigma(u,s)$ defined by,

$$\int_{u}^{\sigma(u,s)} \frac{d\bar{g}^2}{\beta(\bar{g}^2)} = 2\log(s)\,.$$

- On the lattice, we have the discrete step scaling function,

$$\Sigma\left(u,s,\frac{a}{L}\right) \equiv \bar{g}^2\left(g_{0_*}^2, \frac{sL}{a}\right)\Bigg|_{\bar{g}^2\left(g_{0_*}^2,\frac{L}{a}\right)=u}\,.$$



- Taking the continuum limit,

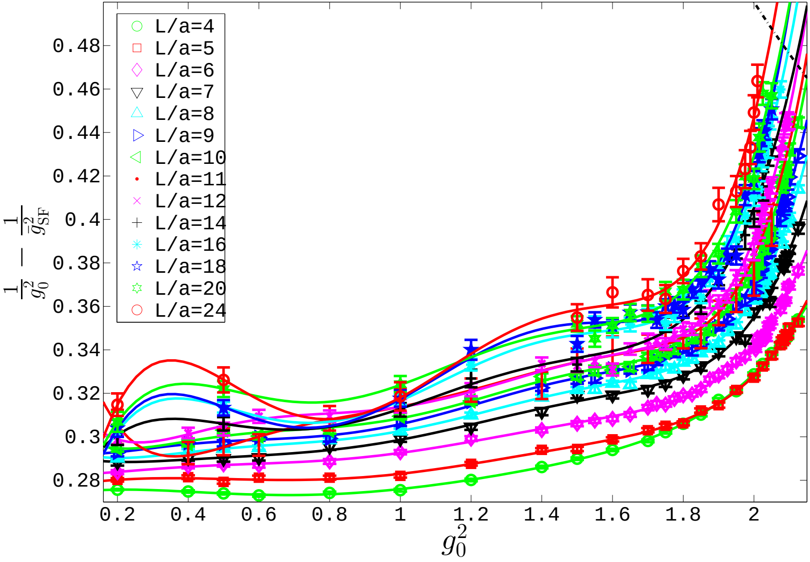
$$\lim_{\frac{a}{L}\to 0} \Sigma\left(u,s,\frac{a}{L}\right) = \sigma(u,s)\,.$$

[9] M. Luscher et al. 1991

Interpolating Functions I

- Motivated by lattice perturbation theory, we fit an interpolating function to
$$\frac{1}{g_0^2} - \frac{1}{\bar{g}_{\mathrm{SF}}^{\,2}}.$$
- Try various fits:
  - Fit each lattice volume with a piecewise linear (connect-the-dots) function.
  - Fit each lattice volume independently to function
$$\frac{1}{g_0^2} - \frac{1}{\bar{g}_{\mathrm{SF}}^{\,2}\left(g_0^2, \frac{a}{L}\right)} = \sum_{i=0}^{N_{L/a}} \alpha_{i,L/a} g_0^2.$$

Interpolating Functions II

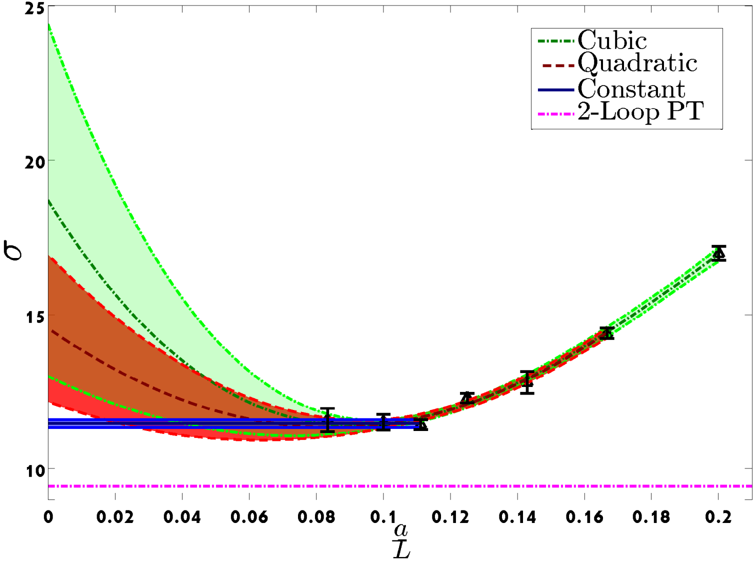


Continuum Extrapolation I

- At each u independently, calculate $\Sigma\left(u, s, a/L\right)$ for $S = 2$ and $L/a = 5$, 6, 7, 8, 9, 10, and 12.
- These data points are fit to a polynomial in a/L:

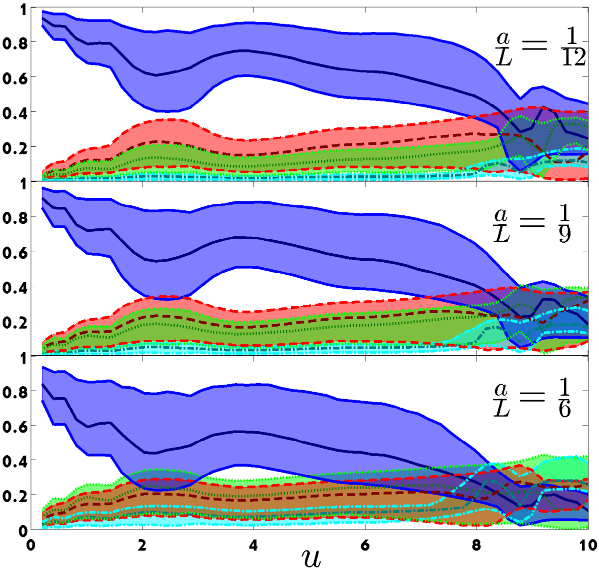
$$p\left(a/L\right) = \sum_{i=0}^{N} \alpha_i \left(\frac{a}{L}\right)^i .$$

- Finally, this fit is extrapolated to the continuum, i.e. $\sigma = p\left(0\right) = \alpha_0$.
- This procedure requires choices:
  - order of polynomial used.
  - subset $\left\{\Sigma\left(u, s, a/L\right)\right\}$ used.

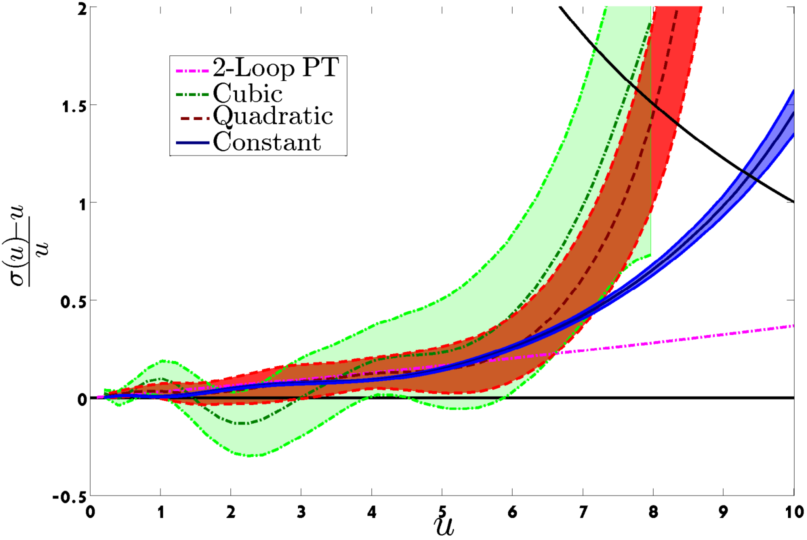


Cubic
Quadratic
Constant
2-Loop PT
25
20
15
10
σ
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2
$\frac{a}{L}$

Continuum Extrapolation III



Continuum Step-Scaling



Conclusions and Outlook

- For an $SU(2)$ gauge theory with six massless fundamental fermions, we find no evidence of an IRFP in the running gauge coupling as defined in the SF scheme.
- Our simulations reach well into a strong-coupling range, $\bar{g}^2 \approx 20$, potentially capable of triggering chiral symmetry breaking and confinement.
- We conclude that this theory either flows to a very strong IRFP, so-far unseen in non-SUSY theories, or it breaks chiral symmetry and confines, producing a large number (65) of NGBs, well above the number of underlying fermionic and gauge degrees of freedom.
- We could in principle probe even larger couplings than presented here, but the computational challenges and lattice-artifact difficulties grow with coupling strength.
- Other approaches, such as the computation of correlation functions and the particle spectrum, will be important to firmly establish the infrared nature of this theory.