Asymptotic Safety on the lattice: The nonlinear O(N) Sigma model

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Research Training Group
Quantum and Gravitational Fields

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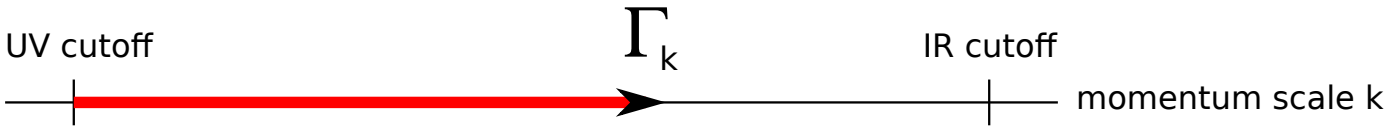
RG approach to QFT

- theory at scale k described by effective average action Γ_k
- upper (ultraviolet) cutoff Λ: $\Gamma_k = S_{mic}$
- use RG flow to integrate out fluctuations until lower cutoff $k = \lambda$
- fundamental theory valid on all scales, limit $\Lambda \to \infty$ and $\lambda \to 0$ exists



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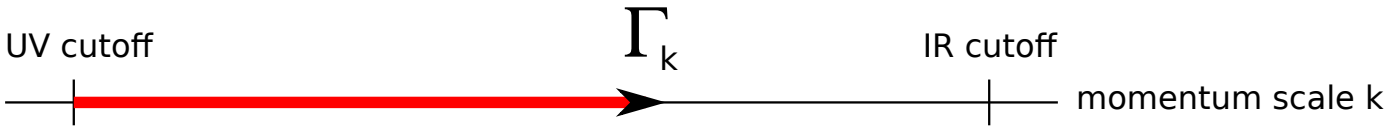

Application to Classical Gravity

- unified theory requires quantization of spacetime metric
- at IR cutoff: Einstein-Hilbert action
- perturbative approach leads to severe divergences
- theory not renormalizable in perturbative way

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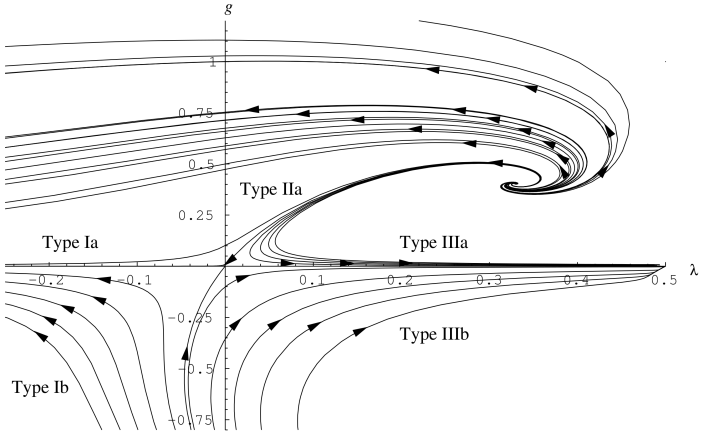
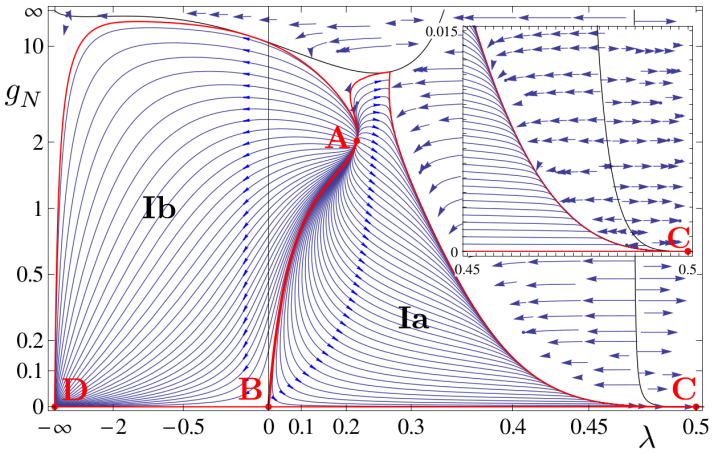
Asymptotic Safety

- Asymptotic Safety Scenario [Weinberg '80] : gravity non-perturbatively renormalizable
- needs nongaussian (ultraviolet) fixed point with finite number of relevant directions to protect UV from unphysical divergences
- asymptotic freedom (QCD): theory approaches gaussian fixed point for $k \to \infty$



[cf. Reuter, Saueressig '02]



[cf. Christiansen, Litim, Pawlowski, Rodigast '12]

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Setting the stage

- nontrivial UV fixed point suspected in ($D > 2$) nonlinear sigma models by perturbation theory and functional RG calculations [Codello, Percacci '08] [Flore, Wipf, Zanusso '12]

O(N) nonlinear Sigma model

$$S = \frac{1}{2g^2} \int d^D x \; \partial_\mu \vec{\phi} \partial^\mu \vec{\phi}, \quad \text{where } \vec{\phi}^2 = 1$$

- Flow diagram determinable from the lattice via Monte Carlo Renormalization Group (MCRG) techniques

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Our Setup

- local HMC with O(N)-valued fields to generate Markov Chain
- blockspin transformation to integrate out fluctuations
- canonical demon method to determine effective couplings [Hasenbusch, Pinn, Wieczerkowski '95]

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RG picture on the lattice

- (discrete) lattice momenta cut off by inverse lattice spacing a^{-1} and inverse linear box size $(aN)^{-1}$
- lattice simulation equivalent to integrating out all fluctuations inbetween
- correlation functions determined by direct measurement

- Blockspin transformation : $a \to 2a$, $N \to N/2$

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Observables

- (discrete) beta function $\tilde{\beta}_i = \tilde{\beta}(g_i) = g_i^{blocked} - g_i$
- (discrete) stability matrix $S_{ij} = \frac{\partial \tilde{\beta}_i}{\partial g_j}$

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Systematic Errors

- finite volume effects not visible for largest lattices considered (32^3)
- discretisation errors small near critical line
- effective action in demon method leads to truncation errors
- half group property of RG transformation R_s (blockspin + demon) violated

$$R_s \circ R_s \neq R_{s^2}$$

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Option 1: Add more operators!

- follow derivative expansion ; up to four operators, i.e. all possible operators up to fourth order in the momenta

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Option 1: Add more operators!

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Option 2: Use improved blockspin transformation!

$$\Phi_{\tilde{x}} \propto P\Big(\exp\Big\{C \cdot \Phi_{\tilde{x}} \sum_{x \in \Lambda_{\tilde{x}}} \phi_x\Big\}\Big)$$

[Hasenfratz, Hasenfratz, Heller, Karsch '84]

- parametrize $C = \sum_i c_i g_i$ such that for $g \to \infty$, $C \to \infty$ holds

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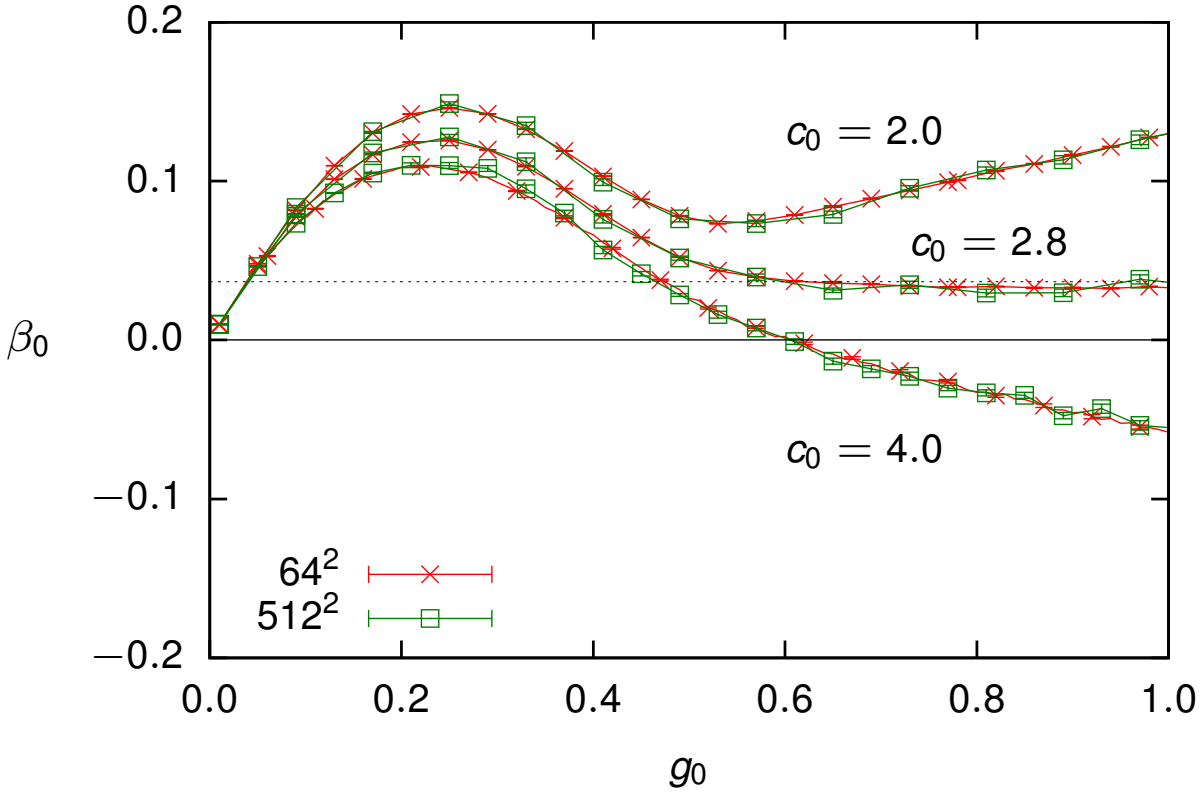
$D=2$: Asymptotic Freedom

1-Parameter effective action



- lattice beta function does not depend on the lattice size (in this truncation)
- considerable variation w.r.t. the optimization parameter c_0
- crucial implications : artificial fixed points emerge / true fixed points vanish due to truncation

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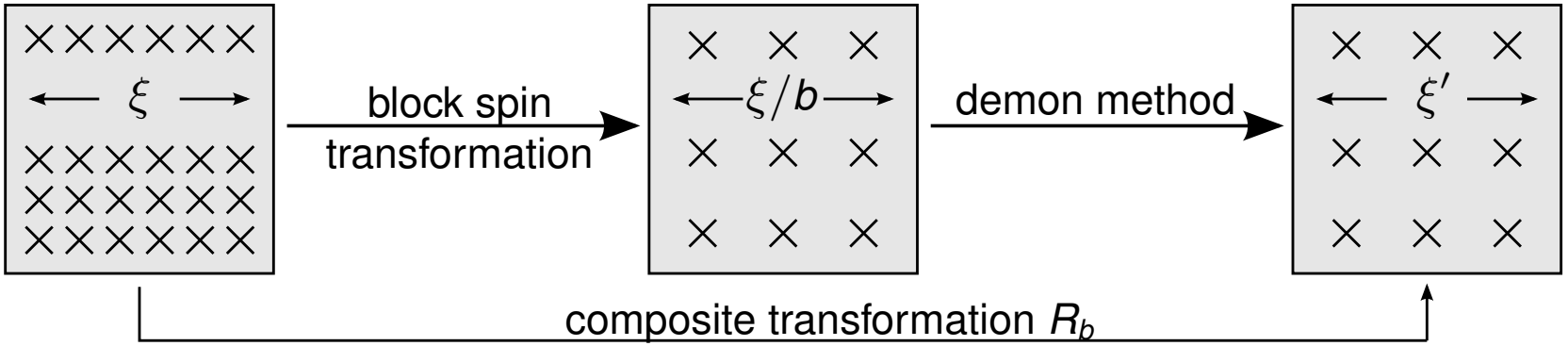
Optimization of the RG transformation I



- MC simulation allows to directly measure lattice correlation functions which are free from truncation errors
- blockspin transformation does not change the IR physics
- idea : simulate the ensemble with truncated couplings
- systematic truncation effects show as differences in the correlation functions of the original and truncated ensemble

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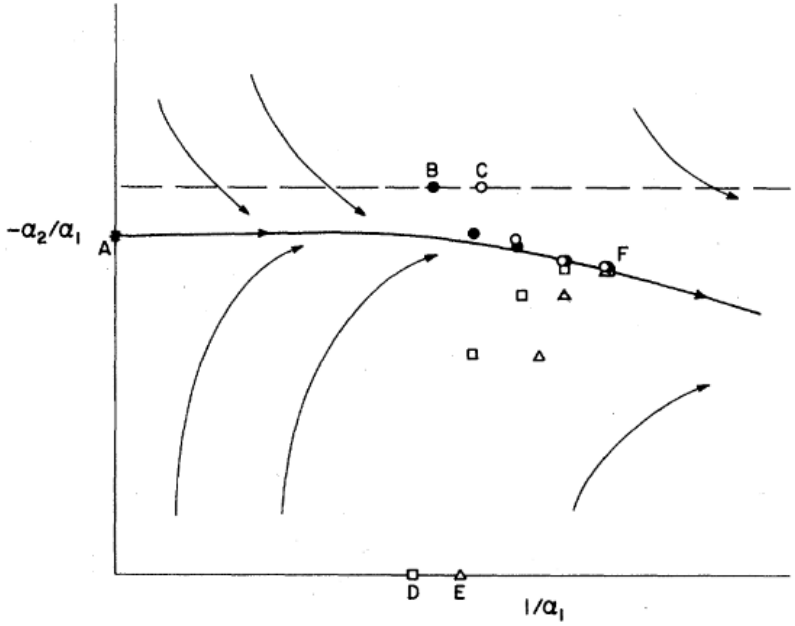
Optimization of the RG transformation II



[cf. Shenker, Tobochnik '80]

- RG picture: trajectories of the RG flow are attracted by the renormalized trajectory , which connects the fixed points of the RG flow
- idea: location of the renormalized trajectory depends on RG scheme
- optimization equivalent to an RG scheme where the renormalized trajectory is closest to a given truncation

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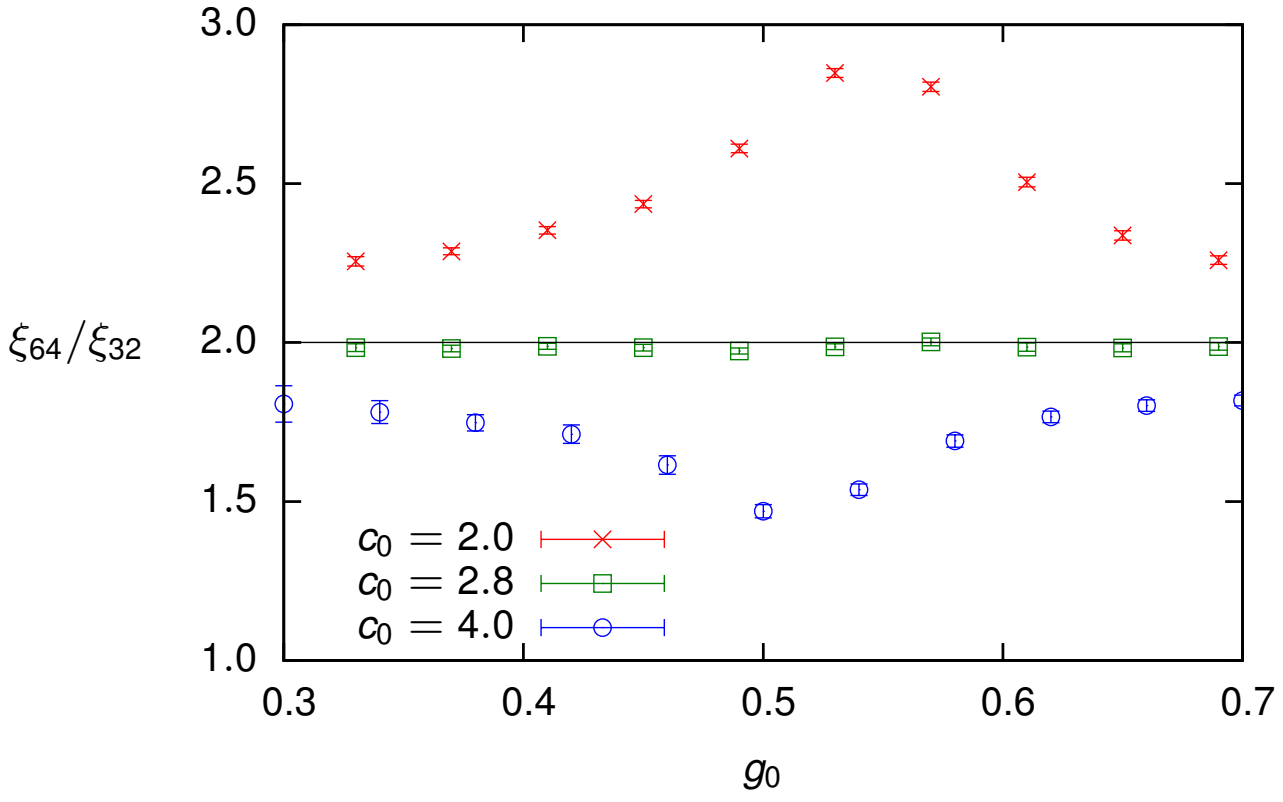
1-Parameter effective action



- compare two-point correlation lengths, ignore all higher correlation functions
- significant deviations from $\xi_{64}/\xi_{32} = 2$ for large and small optimization parameter
- choose $c_0^{opt} = 2.8$ which is closest to the expected value

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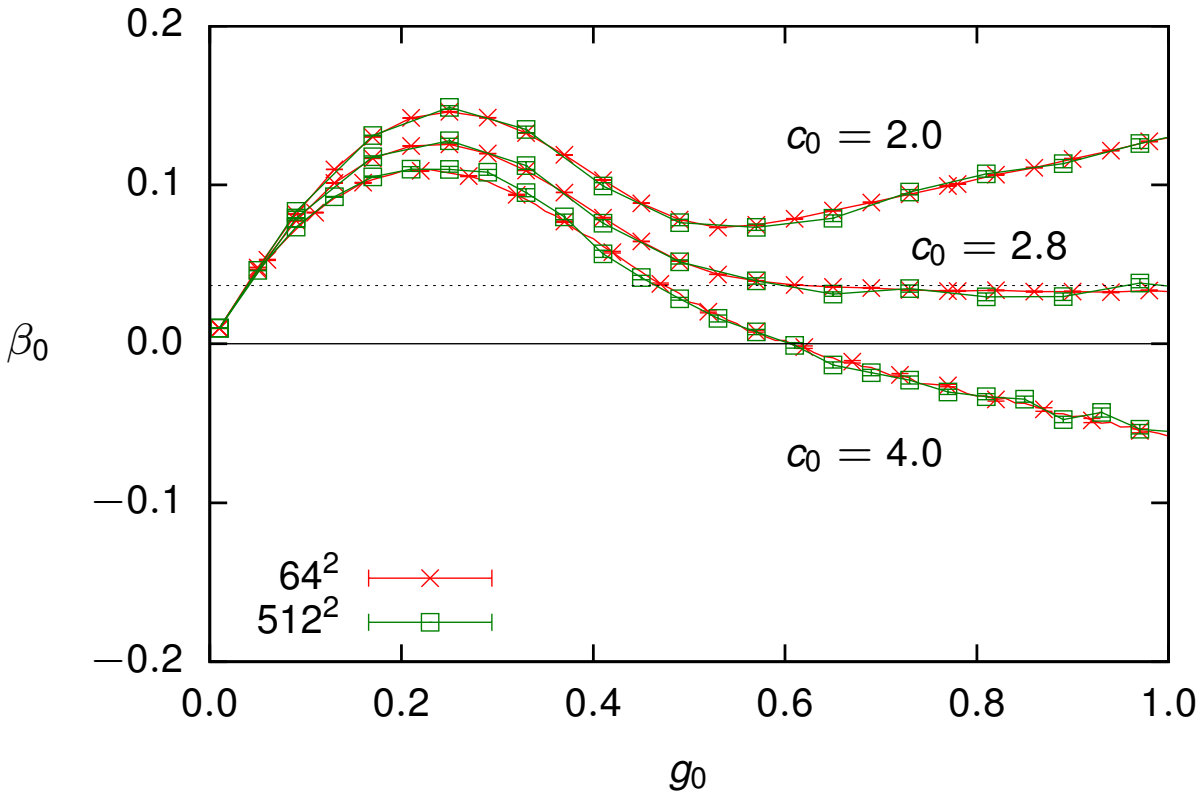
1-Parameter effective action



- beta function for $c_0^{opt} = 2.8$ approaches the known large-N value $ln(2)/6\pi$
- additional zero crossing is an artifact of the truncation
- high-temperature fixed point at zero coupling and low-temperature fixed point at infinite coupling

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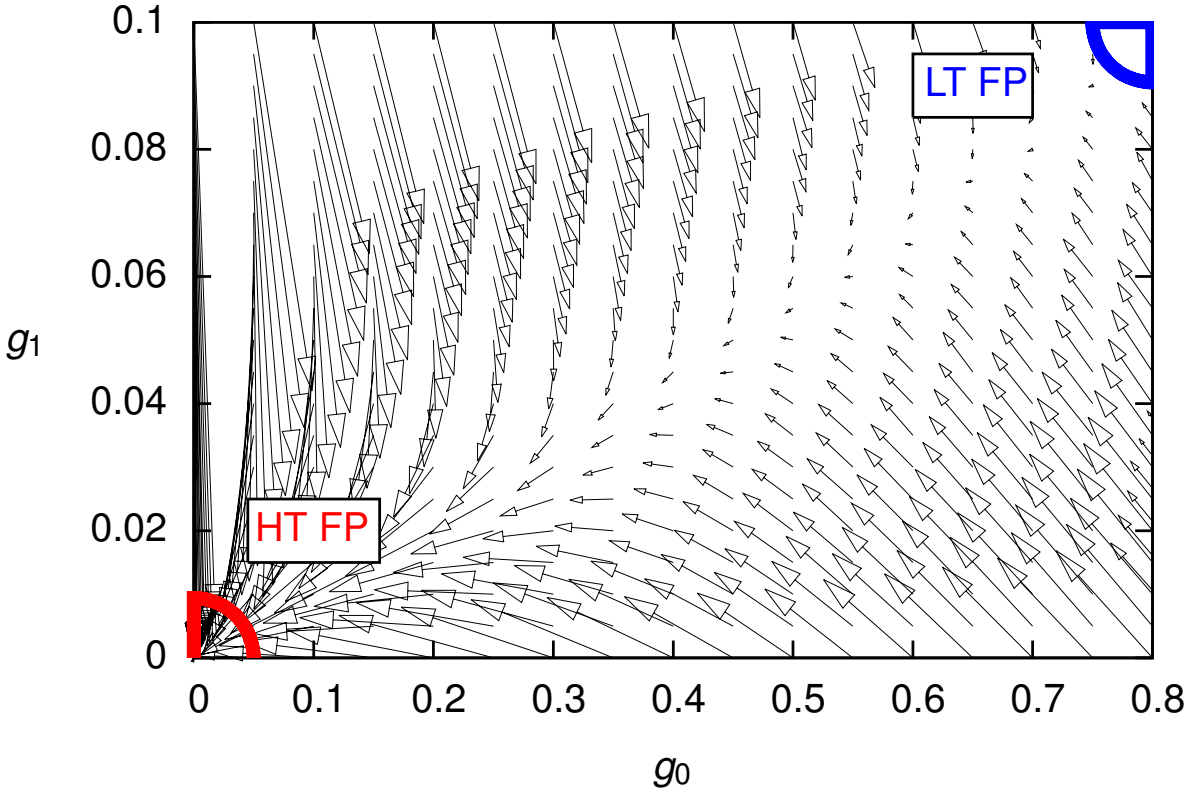
2-Parameter effective action



- trajectories flow from the low-temperature fixed point (LT FP) in the UV to the high-temperature fixed point (HT FP) in the IR
- both operators are IR-irrelevant
- low-temperature FP corresponds to Gaußian FP (asymptotic freedom)

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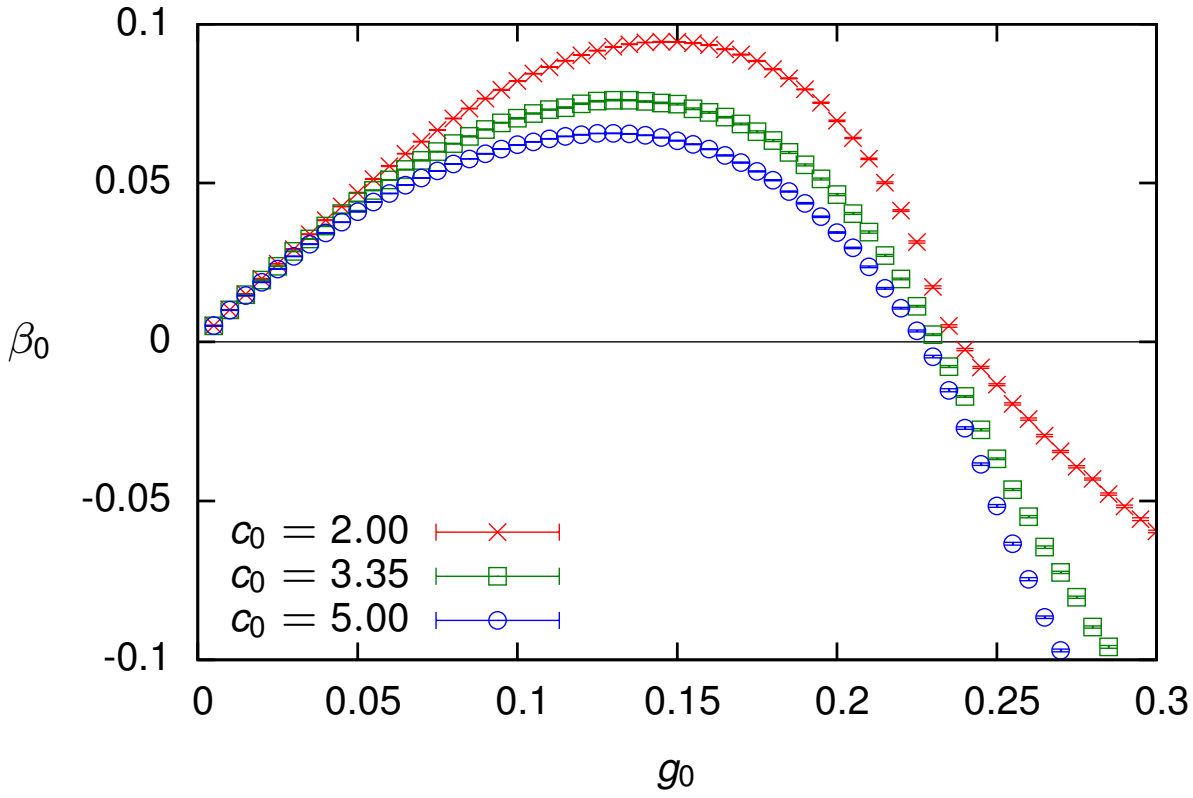
$D = 3$: Asymptotic Safety

1-Parameter effective action



- zero crossing at finite coupling with a UV-attractive direction indicates non-Gaußian fixed point

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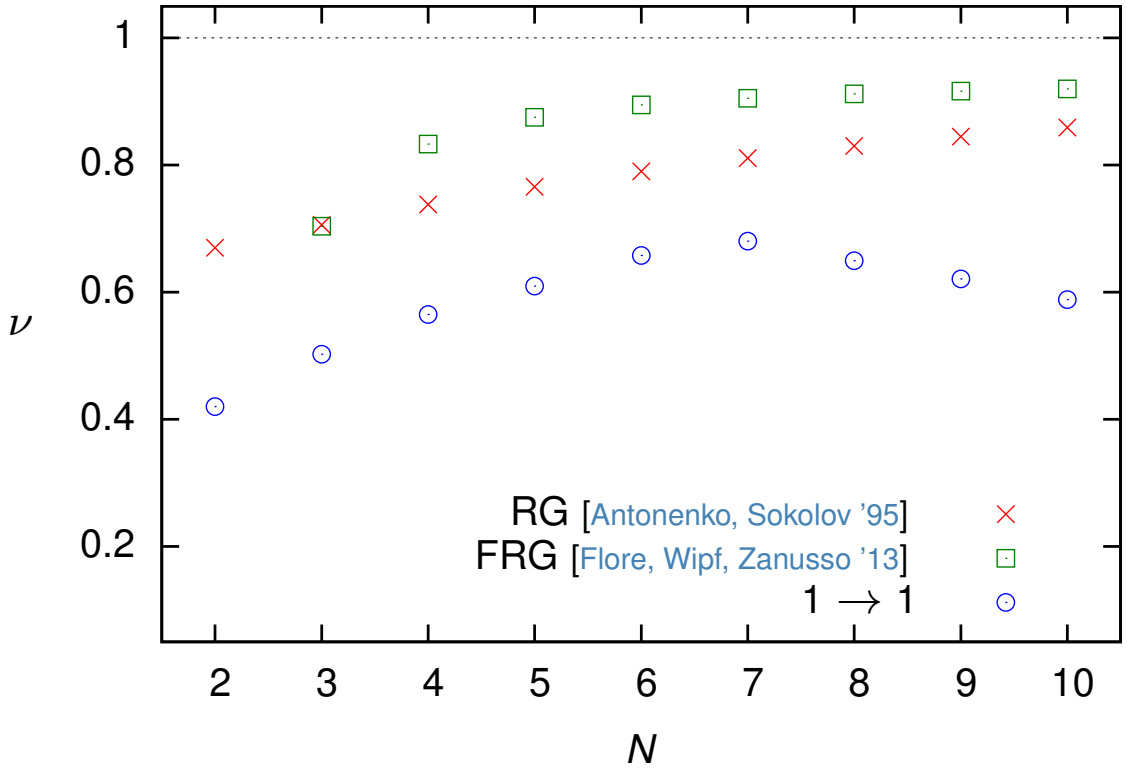
1-Parameter effective action



- critical exponent ν of the correlation length extracted from slope of the beta function
- ν deviates from expected behaviour for $N > 6$

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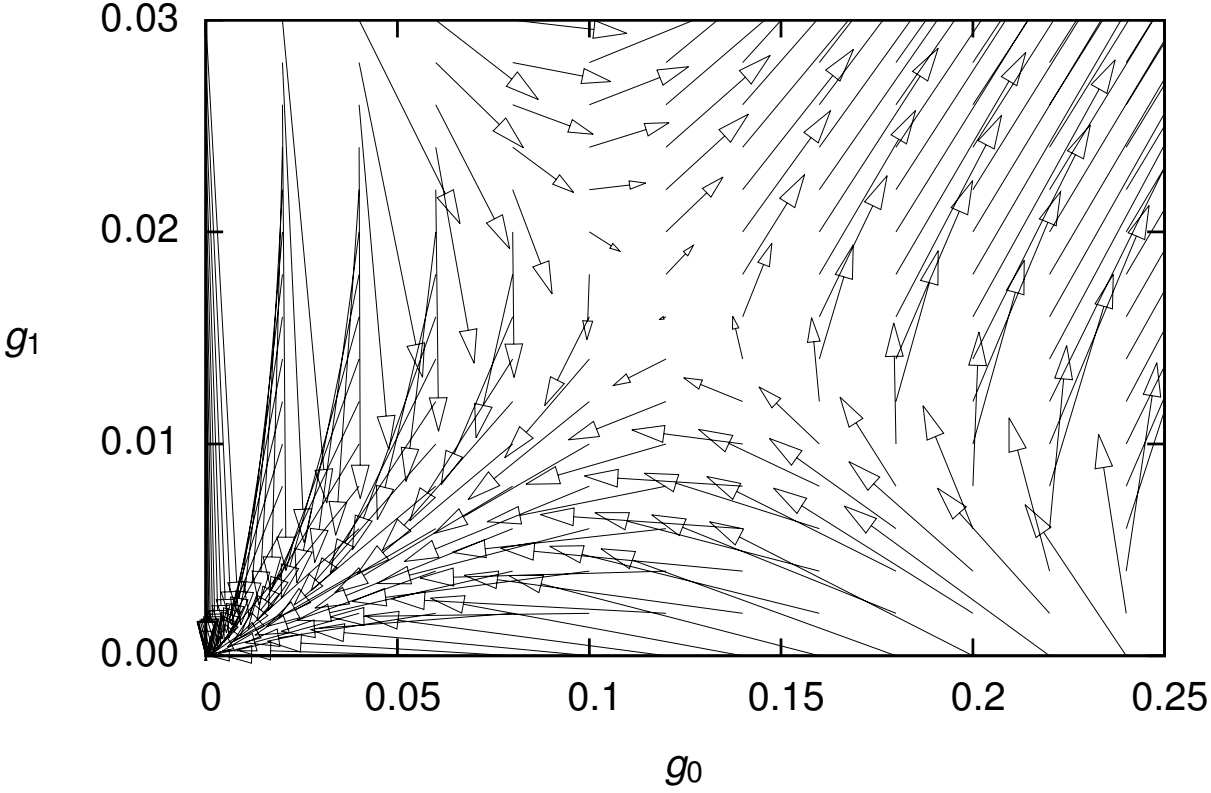
2-Parameter effective action



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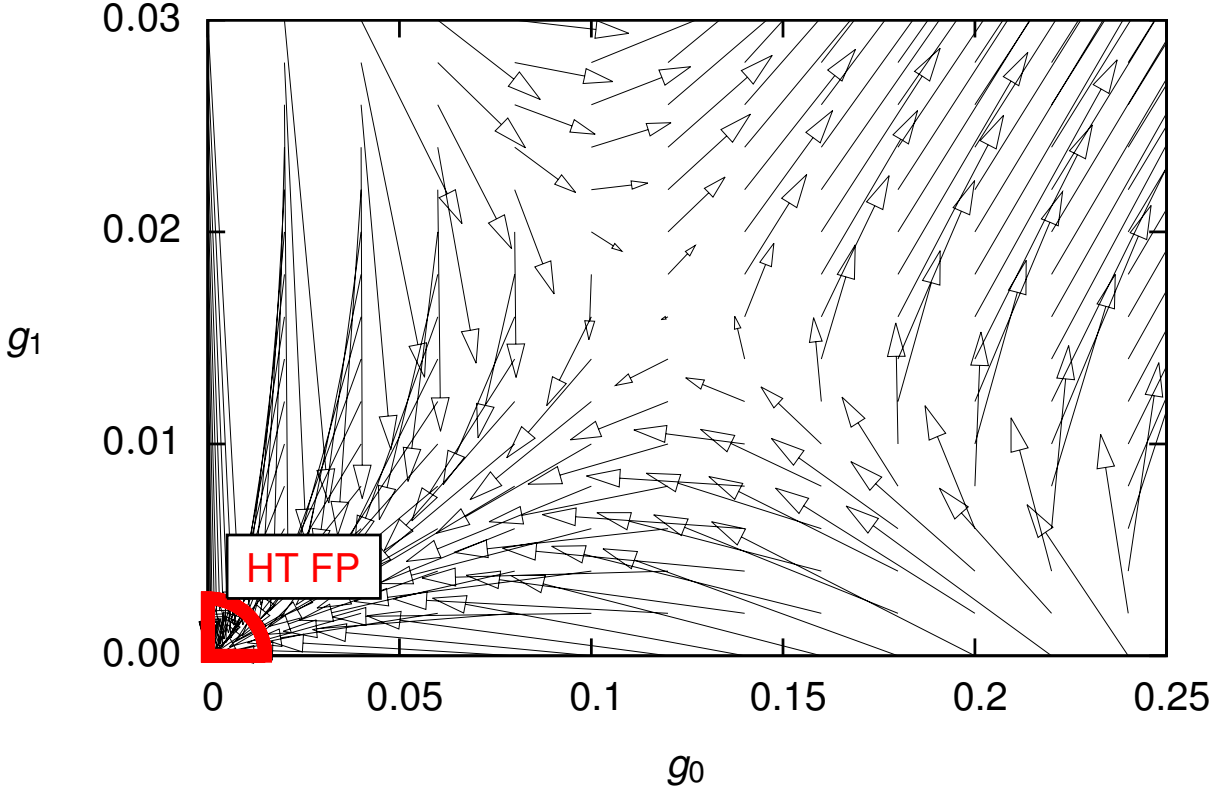

high temperature FP : spins randomly aligned, absolute disorder

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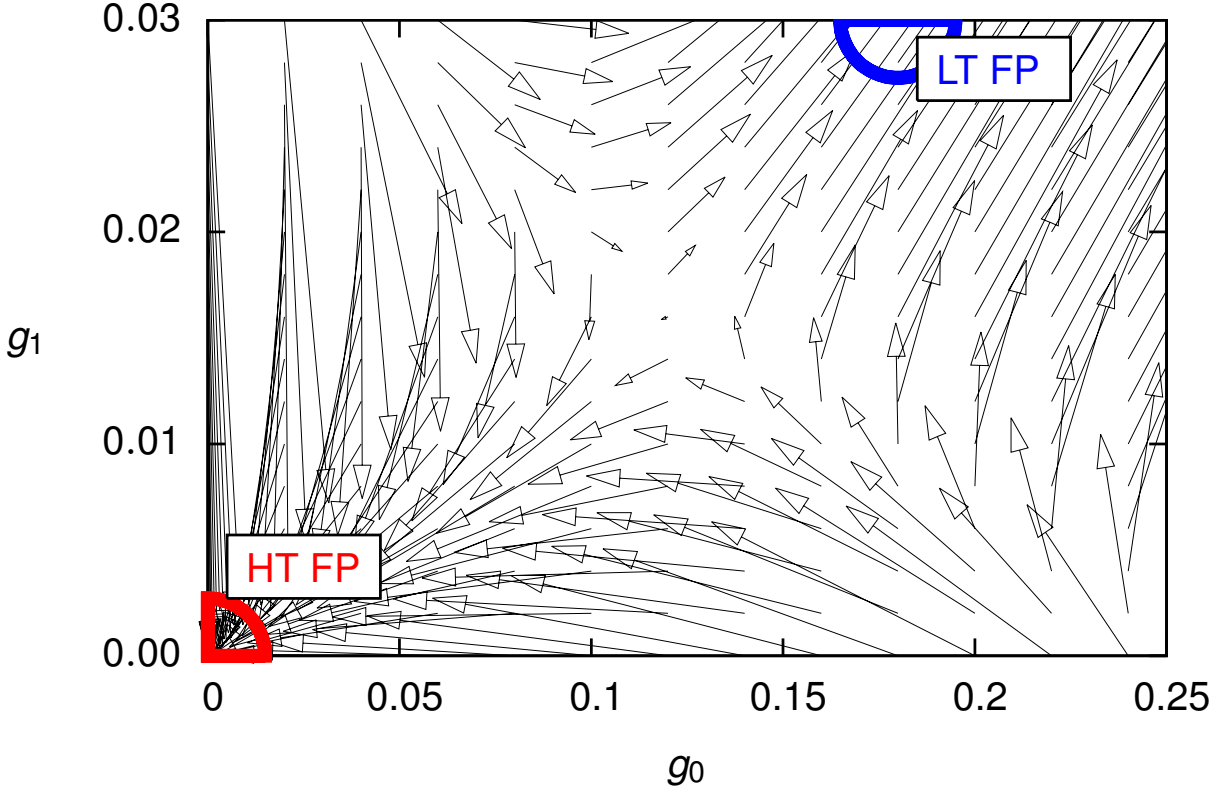

high temperature FP : spins randomly aligned, absolute disorder

low temperature FP : spins uniformly aligned, absolute order

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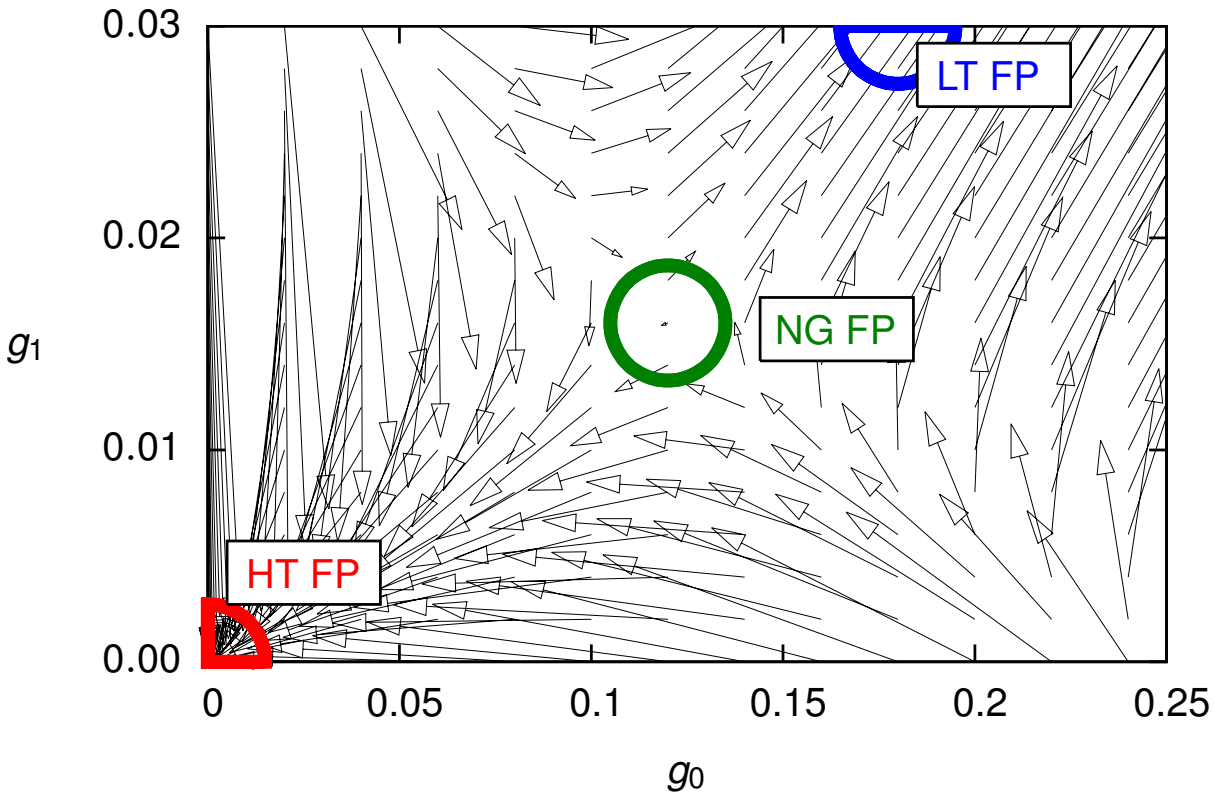

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non-Gaussian FP : 1 IR-relevant, 1 IR-irrelevant direction

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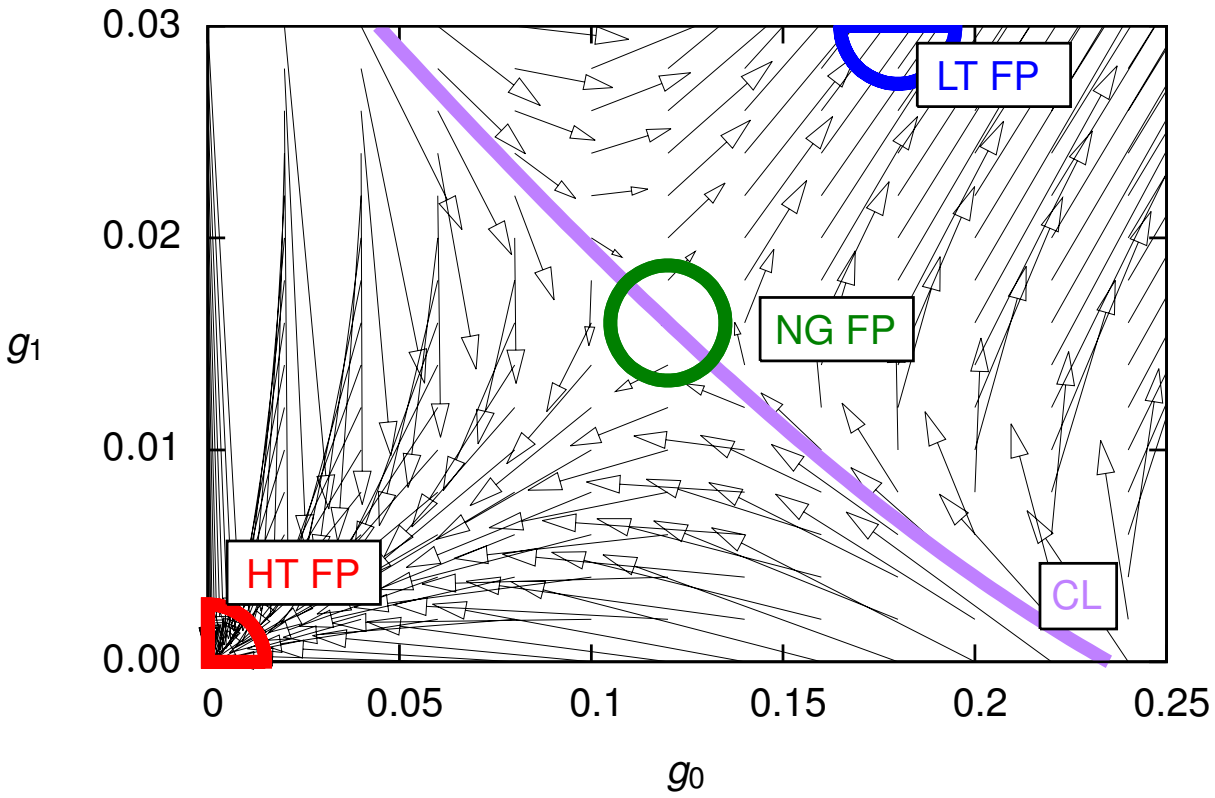

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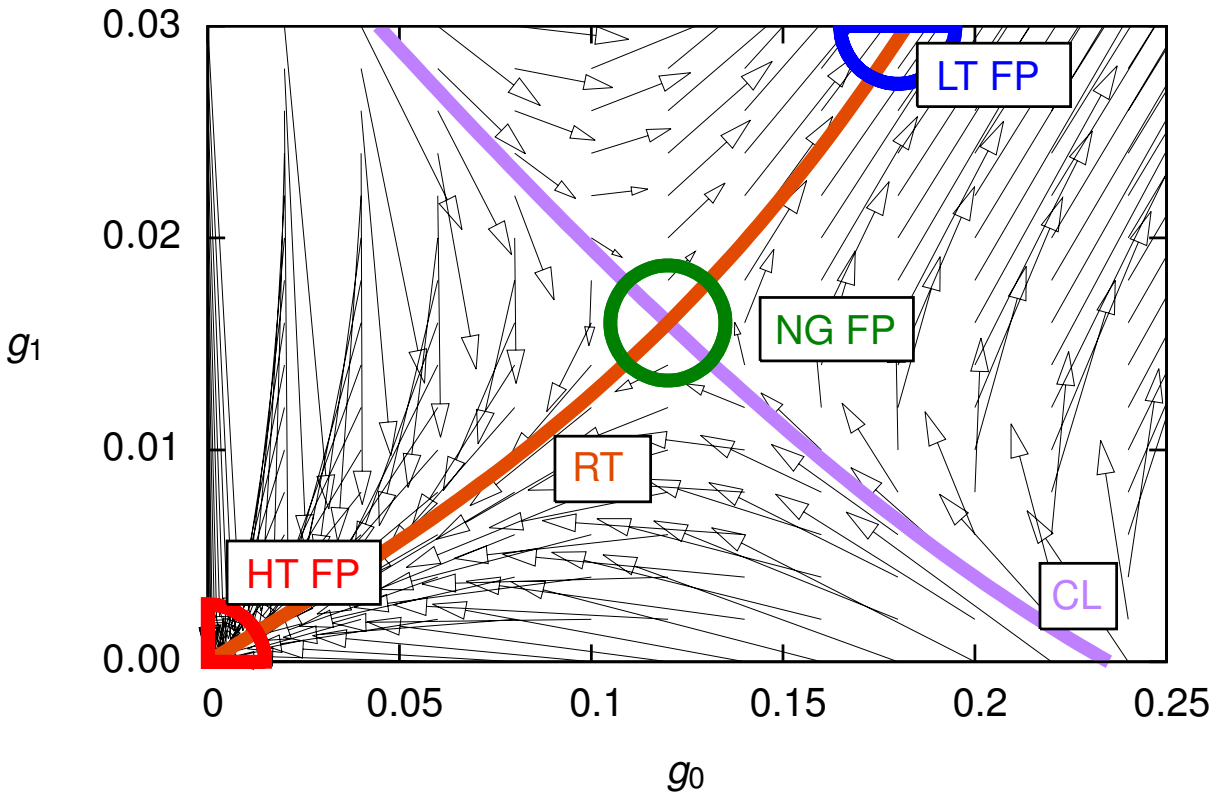

high temperature FP : spins randomly aligned, absolute disorder
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renormalized trajectory : attractor for the RG trajectories

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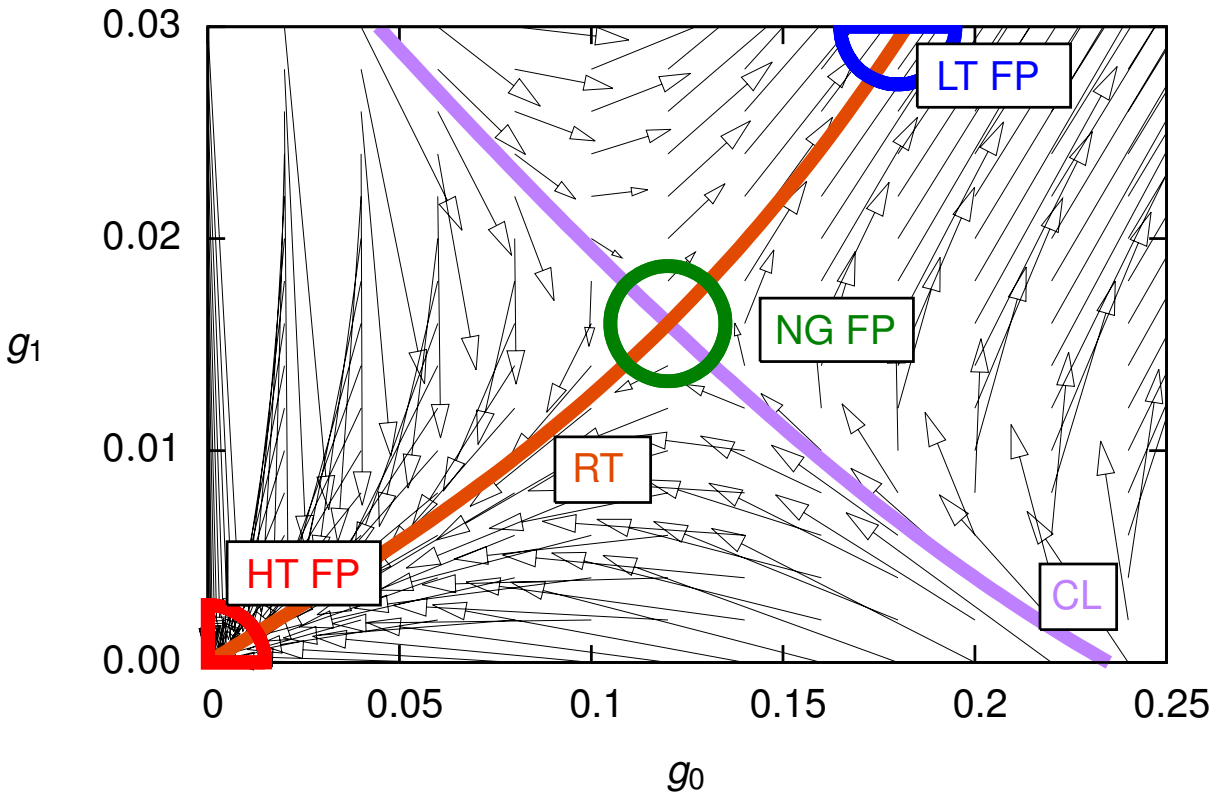

- starting from the usual Hamiltonian for the Heisenberg ferromagnet there are three possible IR fixed points of the RG flow
- universality : non-gaussian FP corresponds to Wilson-Fisher FP of the linear $O(N)$ sigma model

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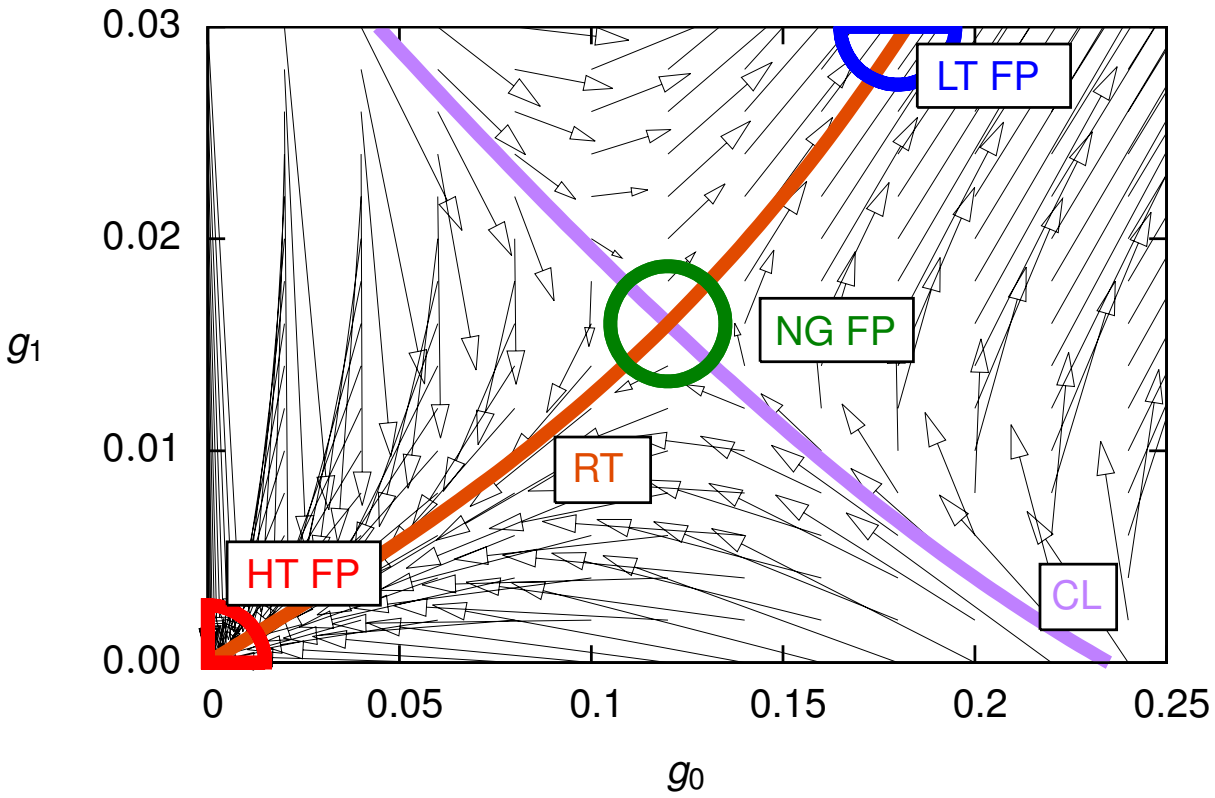
IR or UV fixed point?



- Heisenberg ferromagnet is an effective theory that is defined at some finite UV cutoff
- initial goal : theory that is IR- and UV-complete

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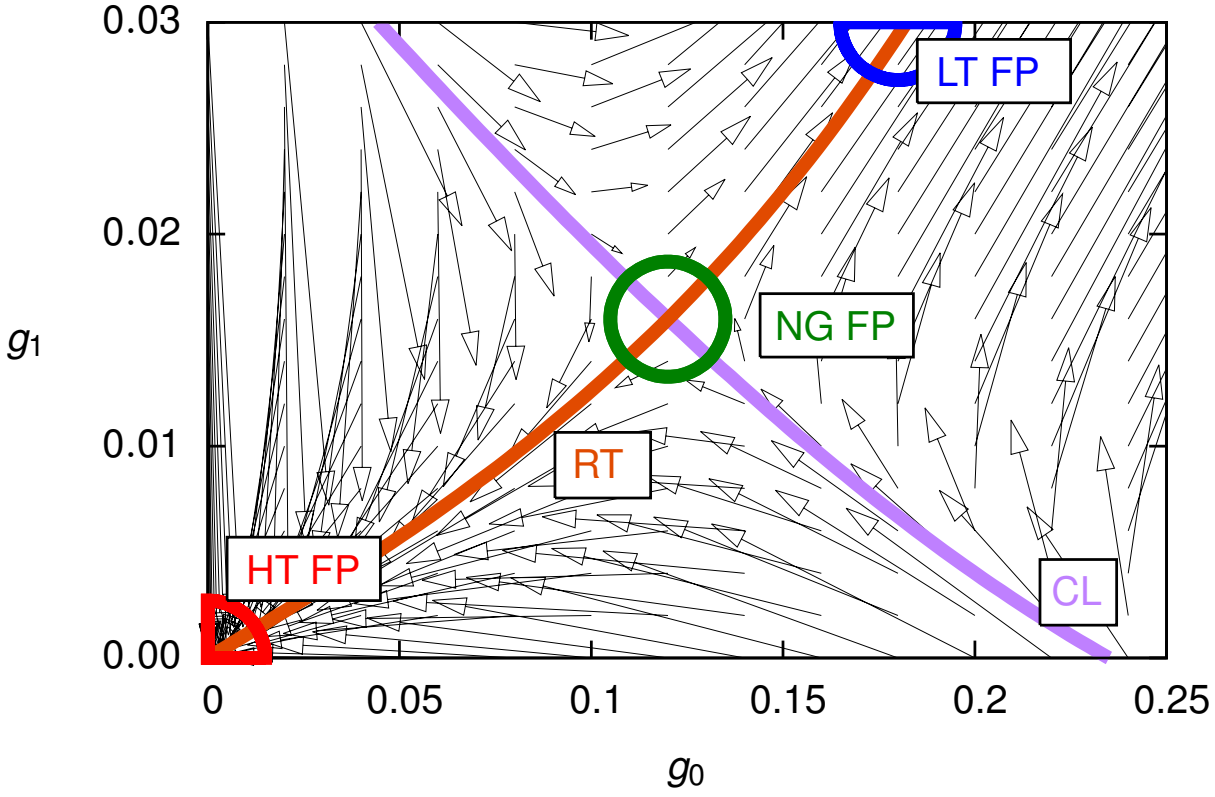
IR or UV fixed point?



- complete theory "lives" on the renormalized trajectory
- non-gaussian fixed point acts as a UV fixed point with one relevant direction
- asymptotic safety scenario realized (in this truncation ...)

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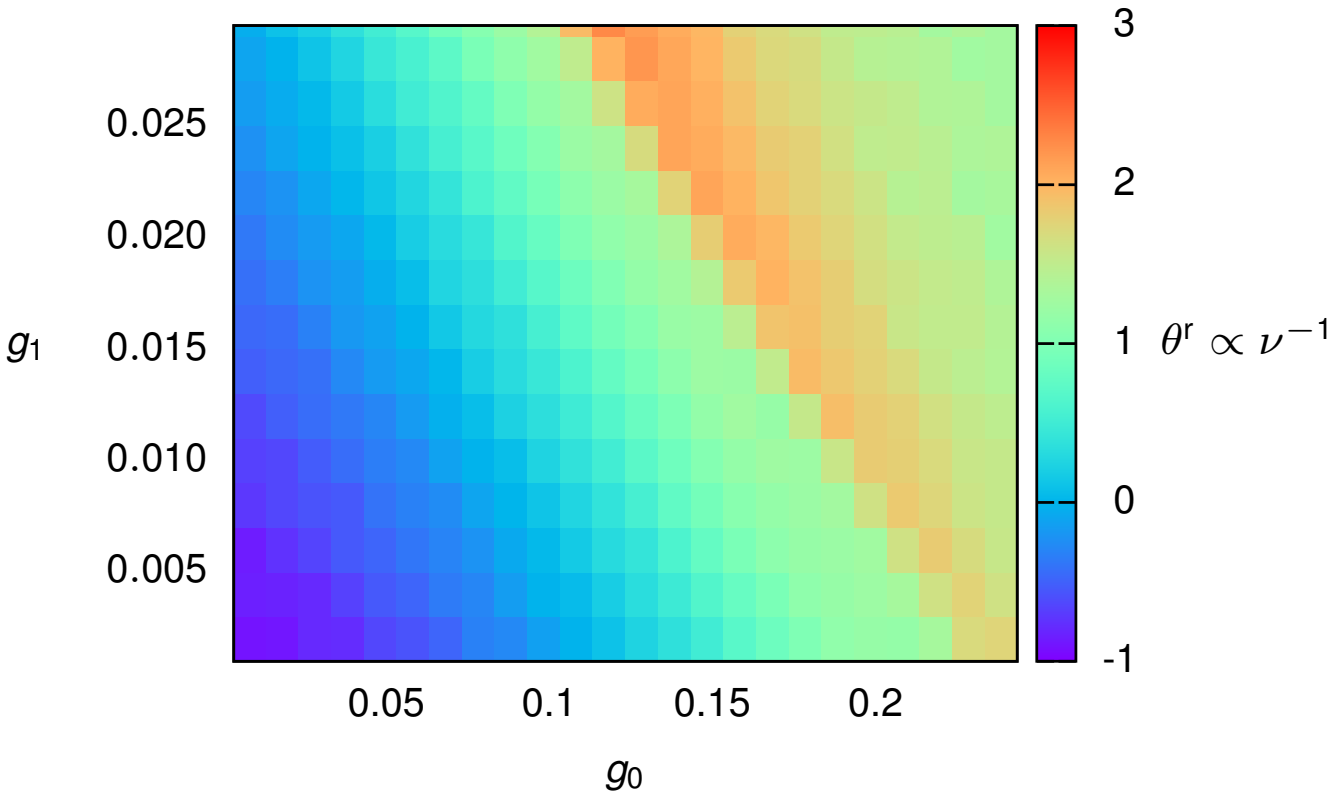
2-Parameter effective action



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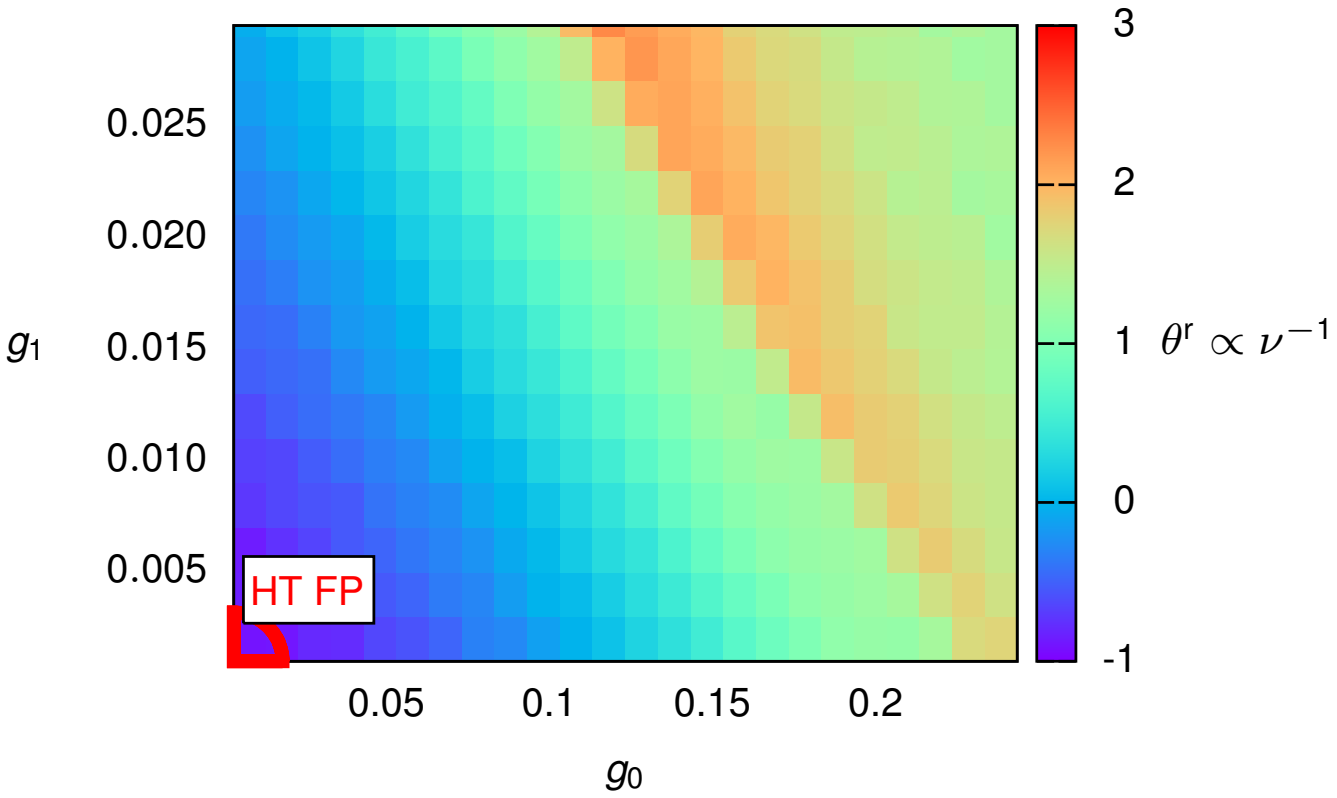

high temperature FP: $\nu \approx -1$

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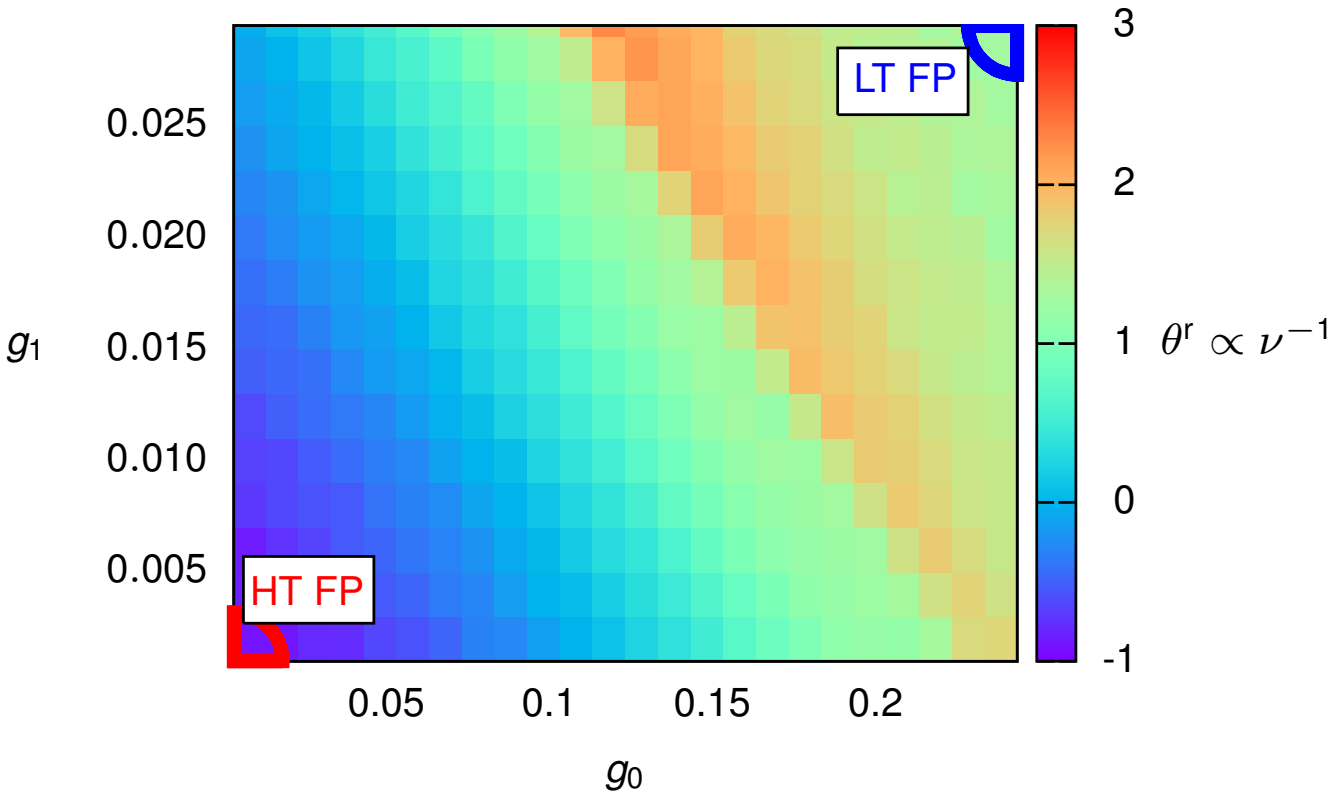

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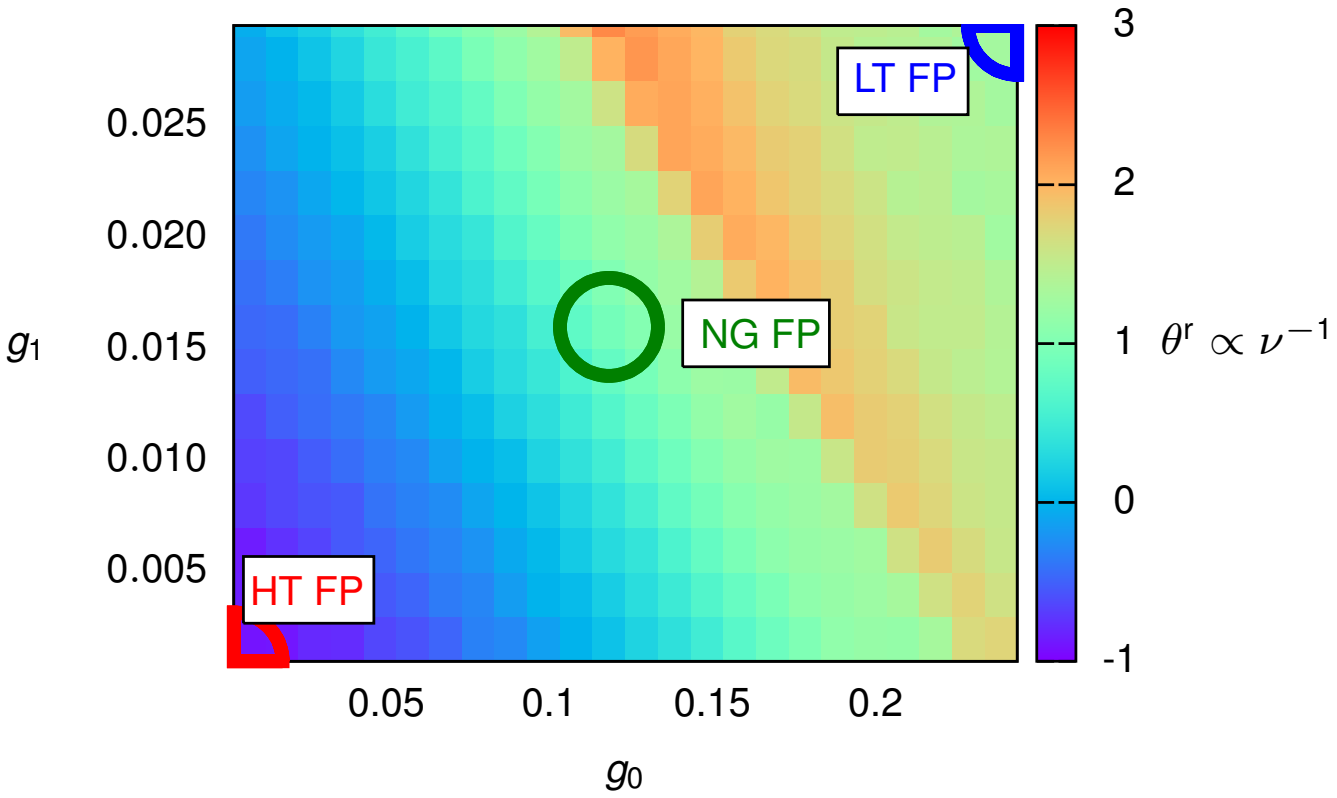

high temperature FP: $\nu \approx -1$

low temperature FP: $\nu \approx 1$

non-Gaussian FP: $\nu = 0.62(3)$

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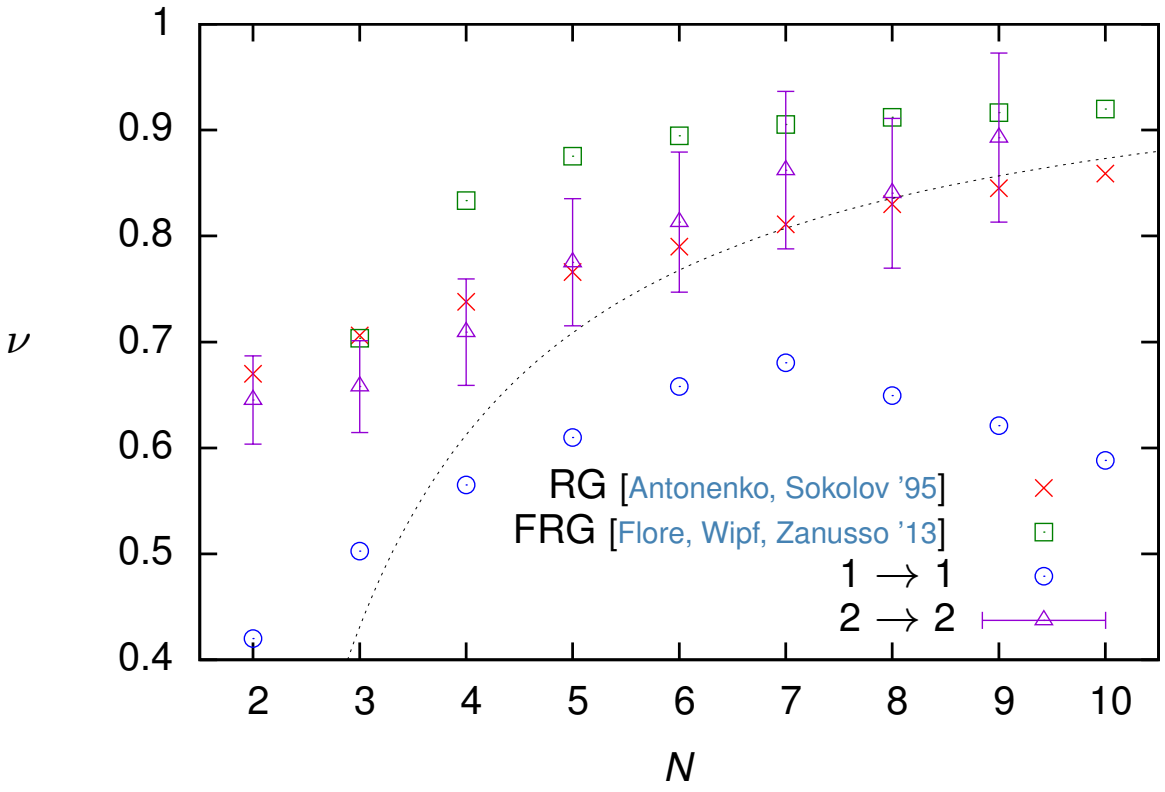
1-Parameter effective action



- significant improvement for the critical exponent ν of the correlation length
- large-N limit inconclusive (numerical effort grows with N)

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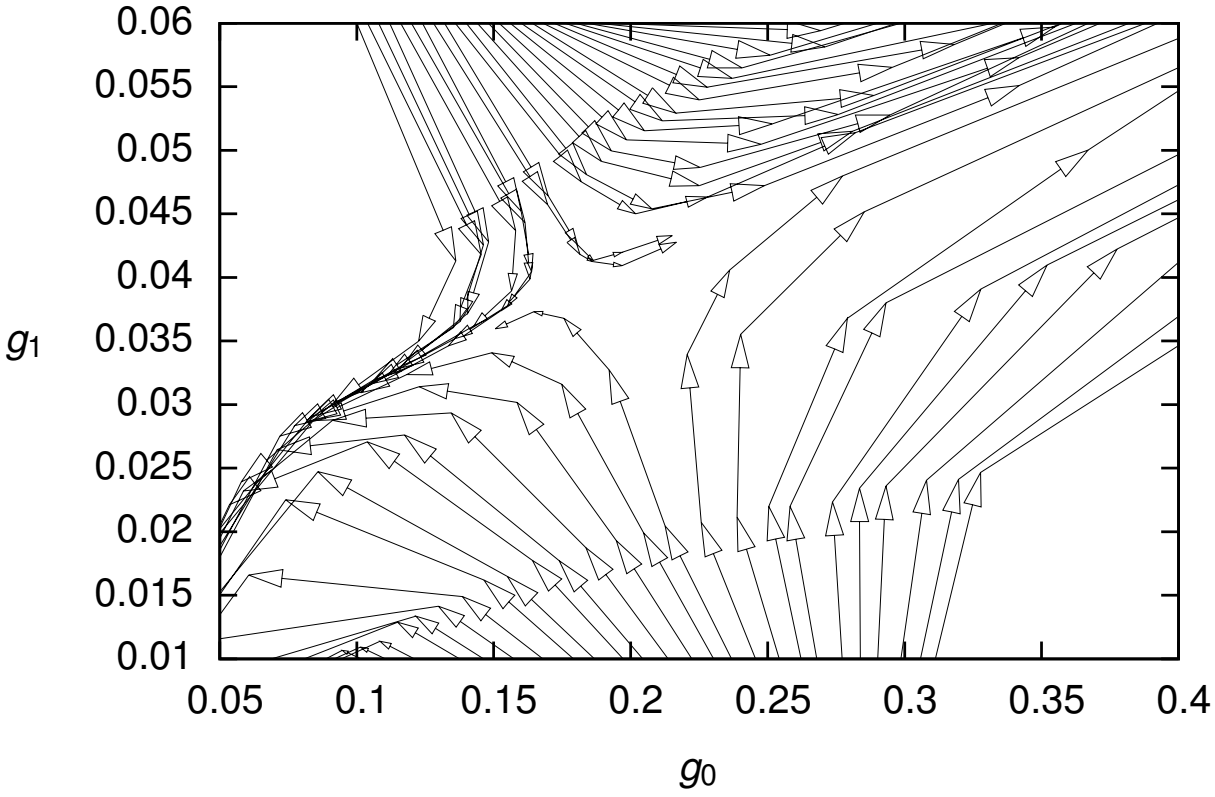
3-Parameter effective action



- only irrelevant coupling is added

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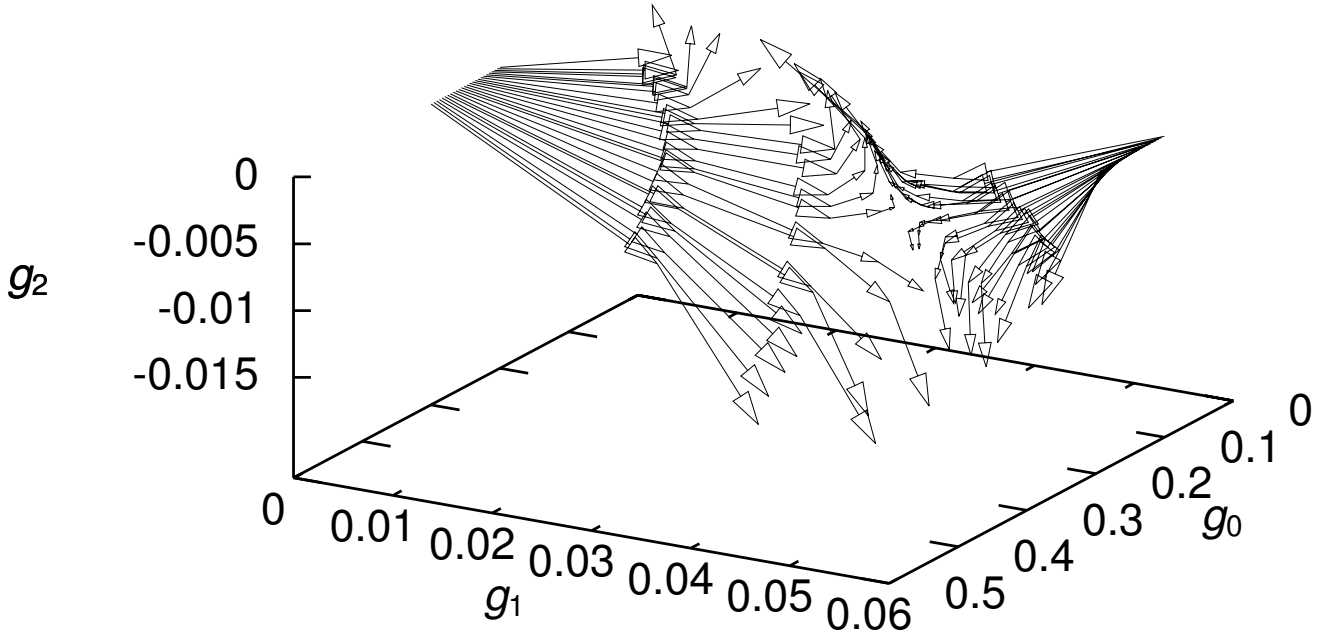
3-Parameter effective action



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Summary

- MCRG+Demon Method suitable to obtain *global* flow diagram
- in principle arbitrarily long trajectories on moderate lattices
- systematic errors of truncation reduced by optimization scheme

D=2

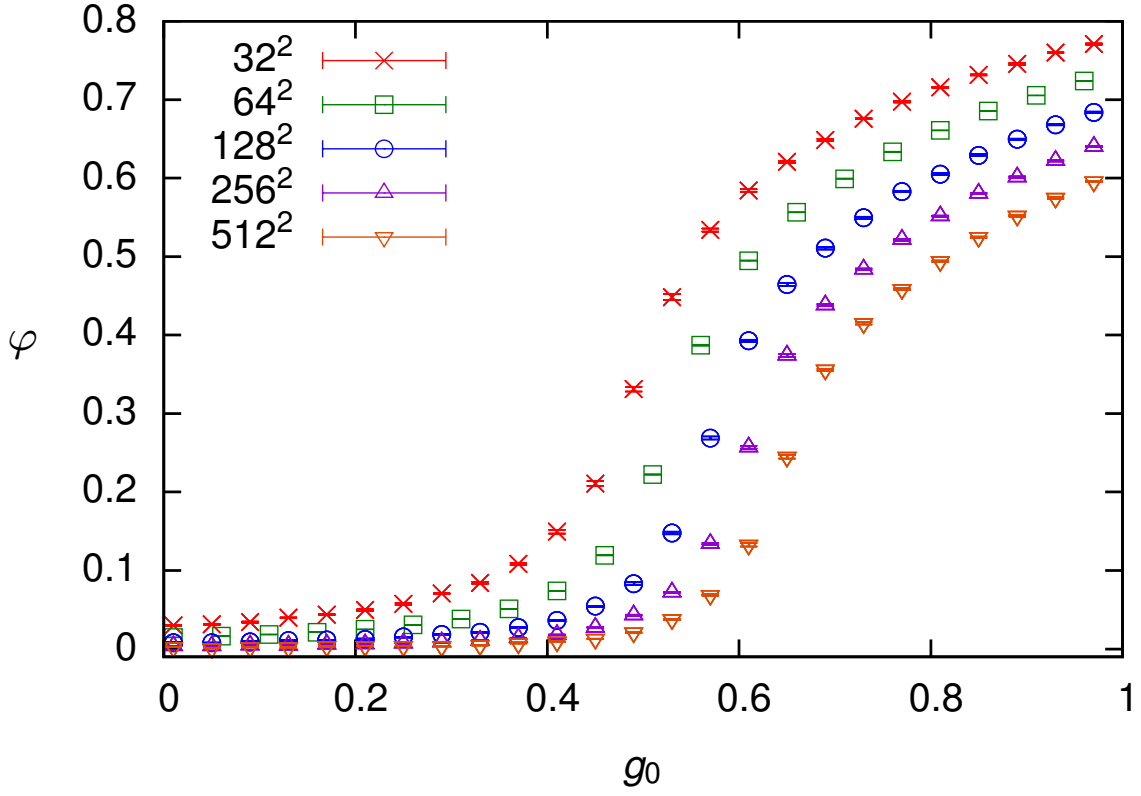
- flow diagram shows low-temperature (Gaußian) FP in the UV and high-temperature FP in the IR (asymptotic freedom reproduced)

D=3

- flow diagram reveals two trivial IR fixed points (absolute order and absolute disorder) and one nontrivial UV fixed point (Wilson-Fisher-FP)
- fixed point structure stable against change of truncation
- only one relevant direction $\rightarrow$ asymptotic safety scenario realized

Outlook

- applicable to fermionic models (e.g. Thirring model)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64^2
128^2
256^2
512^2

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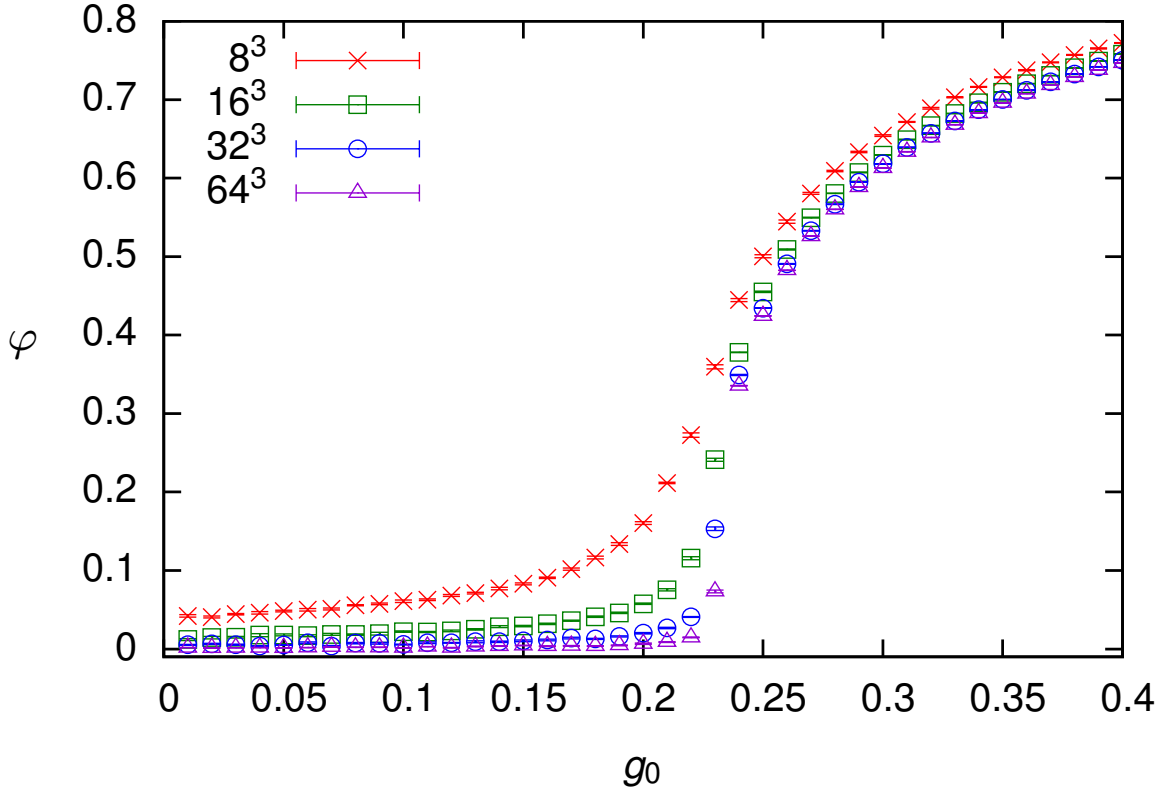
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g_0

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