Ac Dipole Based Chromatic Beta Function Measurement

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Outline

- Beta function measurement via ac dipole:
  - A possible new method to extract unbiased optical parameters from driven oscillation.

- Chromatic beta function measurement and correction via sextupoles.

Extracting Unbiased Optical Parameters from Driven Oscillation

- Existing method need to drive on both sides of betatron tune to cancel systematic errors.

- It has limitation that the driving tune can not be too far away from betatron tune.

- Initial ac dipole phase may have effects on the result.

$$x_s(k) = A_m\{\cos(\psi_m)[\cos(2\pi Q_x^{ac}k) - r\,\cos(2\pi Q_x^{ac}i - 2\pi Q_x^{ac} + 2\chi_x^{ac})] - \sin(\psi_m)[\sin(2\pi Q_x^{ac}k) + r\,\sin(2\pi Q_x^{ac}k - 2\pi Q_x^{ac} + 2\chi_x^{ac})]\}.$$

$$r = \frac{\sin(\pi(Q_x^{ac} + Q_x))}{\sin(\pi(Q_x^{ac} - Q_x))}$$

Parameters for Driven Oscillation

Fitting formula for driven oscillation is

$$\boxed{x_k(s) = A_d \sqrt{\beta_d(s)} \cos\left[2\pi\nu_d k + \psi_d(s) \pm \chi_{acd}\right]}$$

$$A_d \equiv \frac{\theta_{acd}}{4\sin(\pi\delta_d)} \sqrt{(1-\lambda_d^2)\beta_{acd}} \qquad \psi_d(s) = \int_{s_{acd}}^{s} \frac{ds}{\beta_d(s)} \qquad \delta_d = \nu_d - \nu \qquad \lambda_d = \frac{\sin(\nu_d - \nu)}{\sin(\nu_d + \nu)}$$

Scaling of β_d can be determined by fitting Courant-Snyder invariance

$$A_d^2 = \gamma_d(s) x_d(s)^2 + 2\alpha_d(s) x_d(s) x_d'(s) + \beta_d(s) x_d'(s)^2.$$

Relations with twiss parameters for free oscillation are

$$\beta_d = \frac{1 + \lambda_d^2 - 2\lambda_d \cos[2(\psi - \pi\nu)]}{1 - \lambda_d^2} \beta \qquad \psi(s) = \int_{s_{acd}}^{s} \frac{ds}{\beta(s)}$$

$$\tan(\psi_d - \pi\nu_d) = \frac{1 + \lambda_d}{1 - \lambda_d} \tan(\psi - \pi\nu)$$

REFERENCES

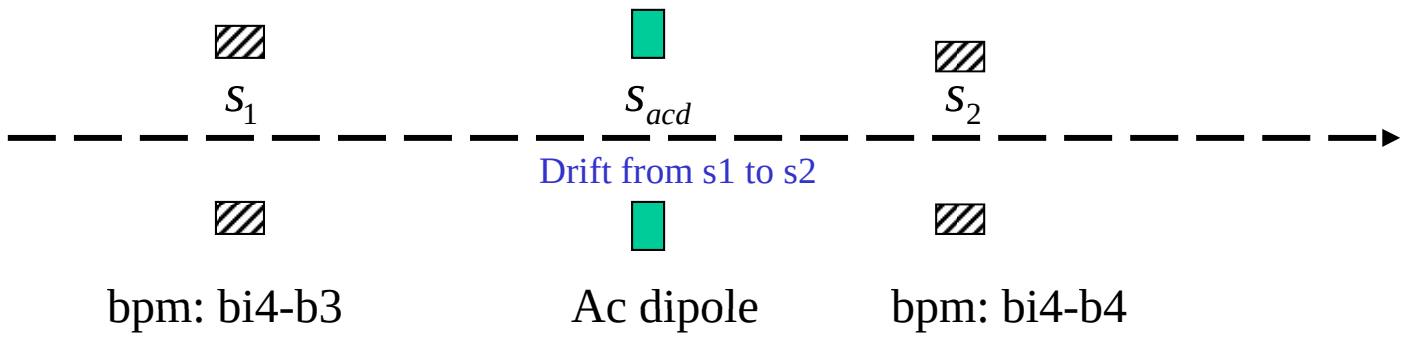
[1] R. Miyamoto, S. E. Kopp, A. Jansson, et al., Physical Review Special Topics - Accelerators and Beams **11**, 084002 (2008).

Equations to Extract Parameters for Free Oscillation

$$\beta(\chi_{acd}) = \frac{1+\lambda_d^2 + 2\lambda_d \cos 2(\tilde{\psi}_d - \chi_{acd} - \pi\nu_d)}{1-\lambda_d^2}\beta_d$$

$$\psi(\chi_{acd}) = \tan^{-1}\left[\frac{1-\lambda_d}{1+\lambda_d}\tan(\tilde{\psi}_d - \chi_{acd} - \pi\nu_d)\right] + \pi\nu$$

The variables obtained directly from fitting turn by turn data are β_d and $\tilde{\psi}_d \equiv \psi_d + \chi_{acd}$. Hence, we need χ_{acd} .



Extract initial ac dipole phase

Beta function for a drift section is $\beta(s) = \beta^* + \frac{(s - s^*)^2}{\beta^*}$

where

$$\beta^*(\chi_{acd}) = \frac{2(s_1 - s_2)^2}{(\beta_1(\chi_{acd}) - \beta_2(\chi_{acd}))^2 + 4(s_1 - s_2)^2}\left[\frac{\beta_1(\chi_{acd}) + \beta_2(\chi_{acd})}{2} - \sqrt{\beta_1(\chi_{acd})\beta_2(\chi_{acd}) - (s_1 - s_2)^2}\right]$$

$$s^*(\chi_{acd}) = \frac{s_1 + s_2}{2} - \frac{(\beta_1(\chi_{acd}) - \beta_2(\chi_{acd}))\cdot(s_1 - s_2)}{(\beta_1(\chi_{acd}) - \beta_2(\chi_{acd}))^2 + 4(s_1 - s_2)^2}\left[\frac{\beta_1(\chi_{acd}) + \beta_2(\chi_{acd})}{2} - \sqrt{\beta_1(\chi_{acd})\beta_2(\chi_{acd}) - (s_1 - s_2)^2}\right]$$

Thus, the phase advance from ac dipole to bpm-2 is $\psi_2(\chi_{acd}) - \psi(s_{acd}) = \int_{s_{acd}}^{s_2} \frac{ds}{\beta(s)}$

By definition, $\psi(s_{acd}) = 0$ and the integration is given by

$$\int_{s_{acd}}^{s_2} \frac{ds}{\beta(s)} = \tan^{-1}\left(\frac{s_2 - s^*}{\beta^*}\right) - \tan^{-1}\left(\frac{s_{acd} - s^*}{\beta^*}\right)$$

We arrive the following equation for the one unknown variable χ_{acd}

$$f_1(\chi_{acd}) = \tan^{-1}\left(\frac{s_2 - s^*(\chi_{acd})}{\beta^*(\chi_{acd})}\right) - \tan^{-1}\left(\frac{s_{acd} - s^*(\chi_{acd})}{\beta^*(\chi_{acd})}\right) - \psi_2(\chi_{acd}) = 0$$

Example of calculating χ_{acd}

$$\tilde{\psi}_{d2}(s) \equiv \chi_{acd} + \int_{s_{acd}}^{s_2} \frac{ds}{\beta_d(s)}$$

$$\Rightarrow \chi_{acd} \le \tilde{\psi}_{d2}(s)$$

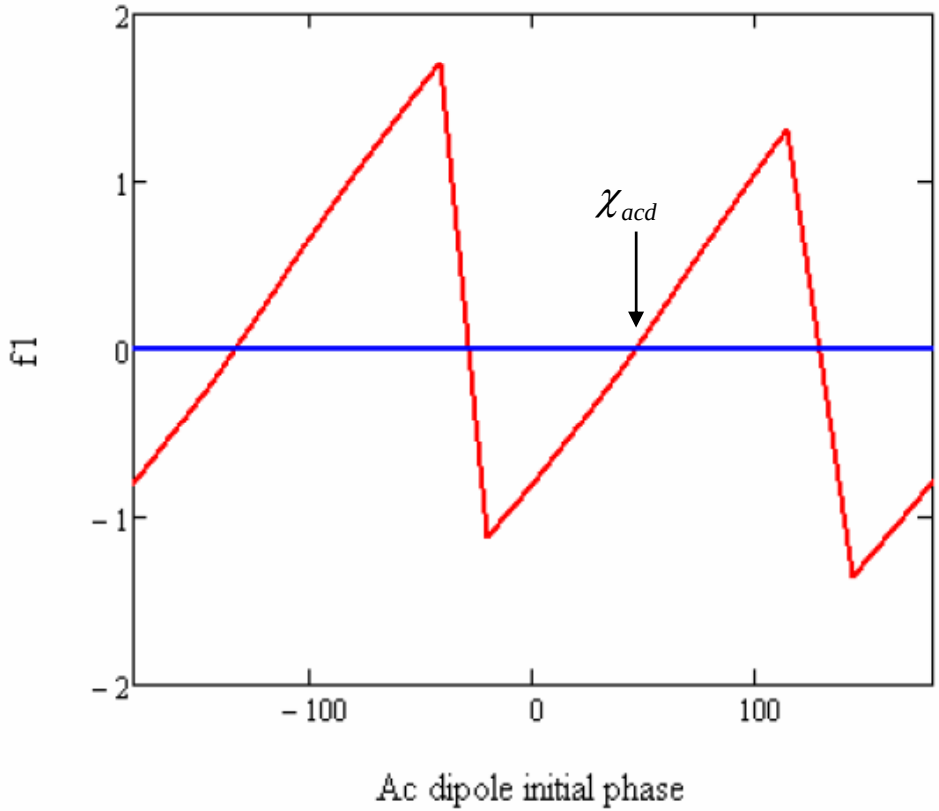
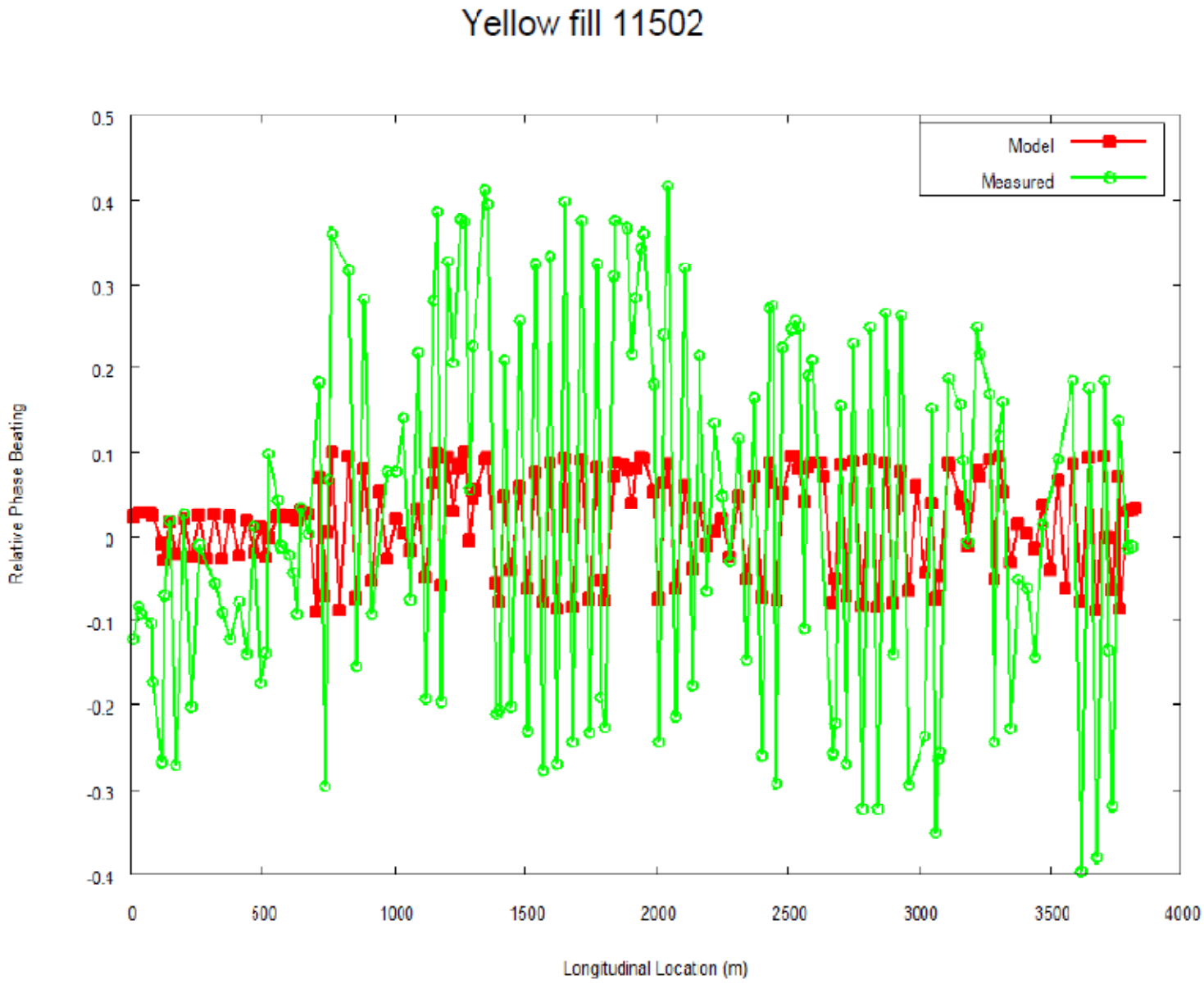


Figure 1: Plotting of function $f_1(\chi_{acd})$ defined in eq. (16). For the plotting, $\delta_d = 0.01$, $\nu = 0.7324$, $s_1 = 3232.24m$ (bi4-b3), $s_2 = 3268.51m$ (bi4-b4), $s_{acd} = 3235.96m$ (rfac1), $\beta_{d1} = 50m$, $\beta_{d2} = 40m$, $\tilde{\psi}_{d1} = 20^o$ and $\tilde{\psi}_{d2} = 100^o$. The location where the red curve crosses zero gives the initial ac dipole phase χ_{acd}.

Chromatic Beta Function Measurement

- Substantial chromatic phase-beating was observed in yellow ring during one of run 10 APEX session.
- Further measurements are needed for verification.
- It is also important to verify that chromatic beta function are corrected during second order chromaticity correction.

Chromatic Beta Function Measurement in Run 10



Chromatic Beta Function Correction

Formula grabbed from Rhic-ap-note-348 by Y. Luo

$$\frac{1}{\beta_{x,y}(s)}\frac{\partial\beta_{x,y}(s)}{\partial\delta}=\pm\frac{1}{2\sin(2\pi Q_{x,y})}\oint\beta_{x,y}(s')[\mp K_1(s')\pm K_2(s')D_x(s')]\cos\left(2\left|\phi_{x,y}(s)-\phi_{x,y}(s')\right|-2\pi Q_{x,y}\right)ds'$$

$$\xi_{x,y}^{(2)}=-\frac{1}{2}\xi_{x,y}^{(1)}+\frac{1}{4\pi}\oint[\mp K_1(s')\pm K_2(s')D_x(s')]\frac{\partial\beta_{x,y}(s')}{\partial\delta}ds'\pm\frac{1}{4\pi}\oint K_2(s')\beta_{x,y}(s')D_x^{(2)}(s')ds'$$

$$\xi_{x,y}^{(2)}\equiv\frac{1}{2}\frac{\partial^2 Q_{x,y}}{\partial\delta^2}\bigg|_{\delta=0}$$

According to above formula and previous simulations, chromatic beta beating will be corrected along with second order chromaticity correction. We plan to use ac dipole measure and verify the correction in the coming run.

Summary

- A new method to extract twiss parameters from driven oscillation is under investigation.
- More chromatic beta function measurement should be conducted to understand the observed chromatic beta-beating.
- Chromatic beta measurement also helps to verity nonlinear lattice correction.