Theory of CNI polarimetry with ^3He

Workshop on Opportunities for polarized ^3He in RHIC and EIC

1. BACKGROUND

About fifteen years ago Yousef Makdisi called me and Elliot Leader with an interesting question: the RHIC polarimeter group was focusing on the use of a coulomb-nuclear interference polarimeter as a promising way to get a precise measurement of the polarization of the proton beams in RHIC. This requires the precise measurement of the up-down or left-right asymmetry in the very small t region of proton-proton elastic scattering. For the ideal case, discussed many years ago by Schwinger, the asymmetry is exactly calculable in terms of the one-photon exchange amplitude given in terms of α and the anomalous magnetic moment of the proton interfering with the nuclear amplitude which is known in terms of the pp total cross section σ_{tot} and ρ the ratio of real to imaginary parts of the strong pp amplitude. This gives a very nice measurable asymmetry and then directly the beam polarization.

This method assumes that the nuclear amplitude does not have a spin-flip part to interfere with the one-photon exchange non-flip amplitude. And this was the crux of Yousef's question: how well do we know that the nuclear spin-flip is small enough to allow this calculation to be used? It was commonly believed and often insisted upon that the high energy nuclear amplitude was spin independent, but Elliot and I were not sure of that. So we organized a little workshop of unbiased experts: Nigel Buttimore, Jacques Soffer and Boris Kopeliovich to join with Elliot and me to look very hard at the question. The conclusion that we came to was that the parameter r_5 that measures the ratio of flip to non-flip scattering was limited at best, both experimentally and theoretically, to 15% while the experiments to be carried out required a much better limit of 5%. Following up on this over the next few years the five of us went off alone or in various subgroups to see what we could do.

We learned a lot from this experience, but the key conclusion we came to was the best way was to measure τ making use of a polarized hydrogen target. You didn't need to do all the work we did to reach this conclusion, but making a suitable hydrogen target was not an easy job and we did not know at the time if one would be available in time for our experimental program.

2. EARLY RESULTS

Over the next several years many measurements were made at BNL, at first all with proton on carbon or proton targets. There was an earlier measurement made at Fermilab E704, but that program was terminated with large errors on A_N, the up-down asymmetry also called the analyzing power. The BNL measurements were made at 24 GeV/c at the AGS and 100 GeV/c at RHIC. The errors steadily improved.

These values of τ fit the data pretty well. The choice of carbon for a target was pragmatic but also very fortunate: it gives us directly the value for the neutron spin-flip coupling complementing the proton target. The formulas we will use are very simple: The proton asymmetry

$$A_N \frac{d\sigma}{dt} = -\frac{8\pi}{s} Im(\phi_+ \phi_5^*) \tag{2.1}$$

and the ^3He asymmetry

$$A'_N \frac{d\sigma}{dt} = \frac{8\pi}{s} \mathrm{Im}(\phi_+ \phi_6^*) \tag{2.2}$$

are given in terms of the two spin-flip amplitudes ϕ_5^* and ϕ_6^*, which are standardly parametrized in terms of the non-flip amplitude ϕ_+ as

$$\phi_5 = \tau_p \frac{\sqrt{-t}]}{m} \phi_+$$

and

$$\phi_6 = -\tau_n \frac{\sqrt{-t}]}{m} \phi_+.$$

Both ϕ_5 and ϕ_6 are complex numbers and are energy dependent.

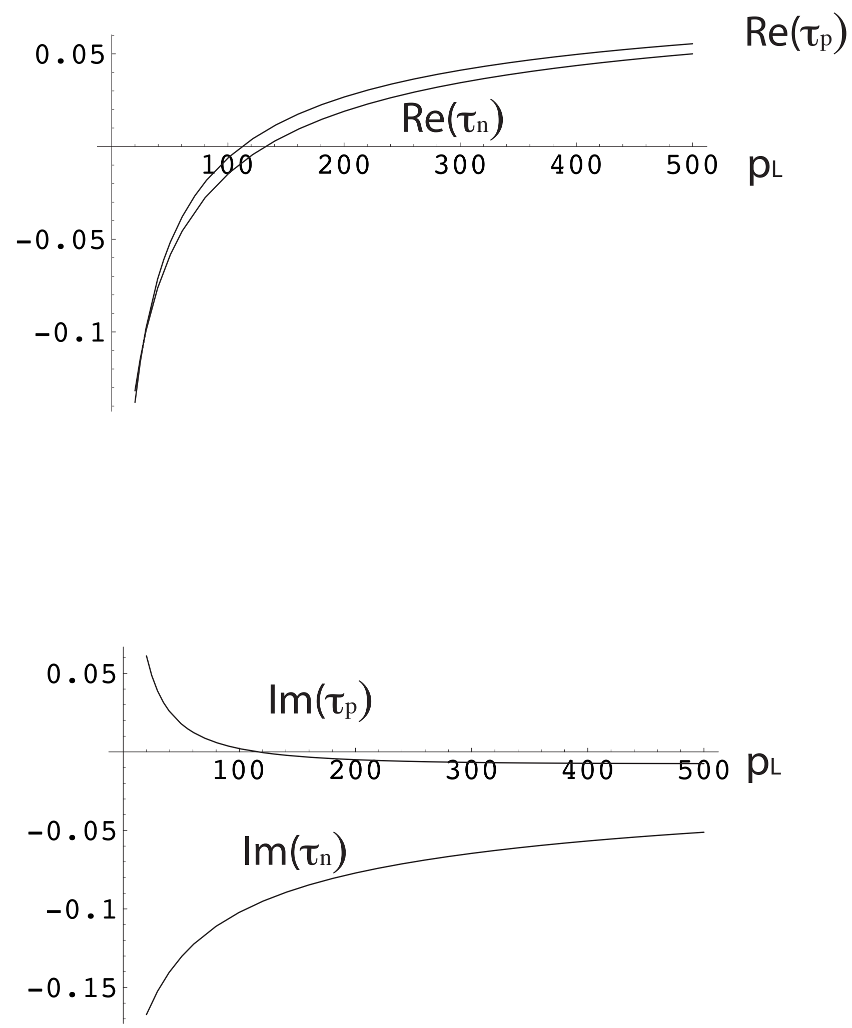
One of the main objectives of my work was to determine the energy dependence of the flip factors which I did using Regge theory. The energy dependence is found to be important, but the t-dependence is argued to be unimportant over the tiny momentum transfer range of the CNI experiments.

We can use these numbers to evaluate the analyzing power of ^3He. The next figure show the calculated energy dependence of real and imaginary parts.

The real parts just shift the analyzing power up or down uniformly while the imaginary part changes the shape of the predicted curve. Explicitly:

$$A_N(s,t) = F_N(s,t)(\kappa_p/2 - Re\tau_p(s) - \rho(s)Im\tau_p(s)(t_c/t)(1 + Im\tau_p(s))(1 + \rho(s)^2)$$

$$A'_N(s,t) = F_N(s,t)(\kappa_n/2 - Re\tau_n(s) - \rho(s)Im\tau_n(s)(t_c/t)(1 + Im\tau_n(s))(1 + \rho(s)^2)$$



Energy dependence of spin-flip factors predicted by Regge model

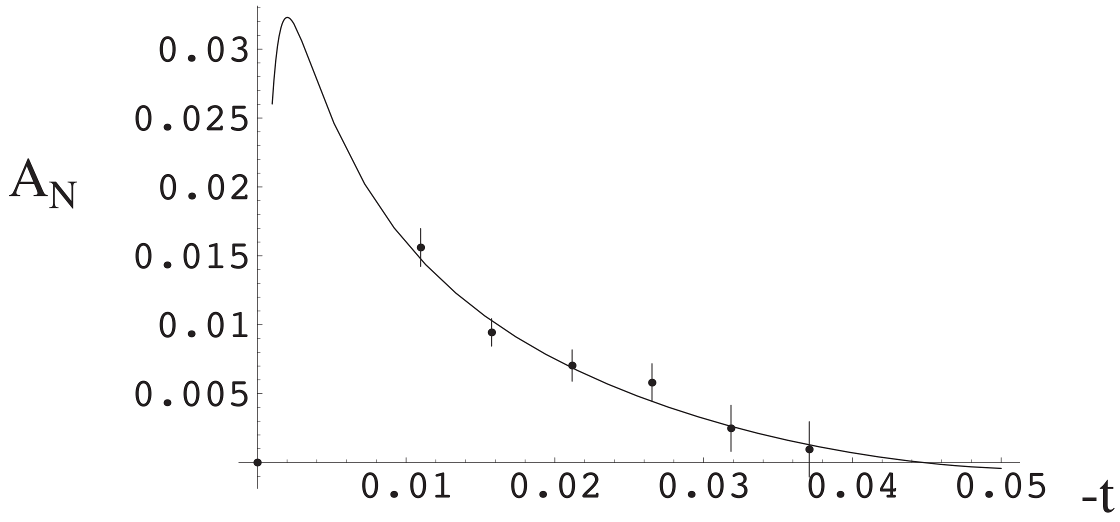
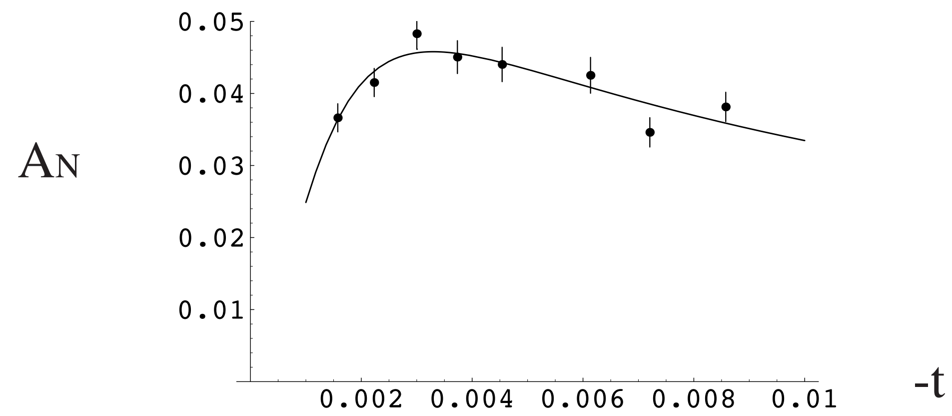


Figure 2: The best fit to the measured analyzing power as given at the Spin 2000 conference.

it is definitely not zero, and is negative.

New p-jet data



τ from p-jet fit = -0.0625 -0.011 i

τ from E704 fit = 0.185 + 0.024 i

The Model

- is based on Regge fit to pp scattering over wide energy range (cf. Cudell et al) which fixes non-flip parameters for the Pomeron (simple or multiple pole), a $C = -1$ vector meson (mainly ω) and a $C = +1$ tensor meson (mainly f_2).

- The non-flip amplitude is

$$g_0(s, 0) = g_P(s) + g_f(s) + g_\omega(s),$$

where the functions $g_R(s)$ have enery dependence and phase determined by standard Regge theory.

- The corresponding flip amplitude is determined by three real, energy independent constants

$$\begin{aligned} g_5(s,t) &= \tau(s)\frac{\sqrt{-t}}{m} g_0(s,t) \\ &= \frac{\sqrt{-t}}{m}\{\tau_P\, g_P(s) + \tau_f\, g_f(s) + \tau_\omega\, g_\omega(s)\}. \end{aligned}$$

Parameters of the model

The model uses the parameters from one of the fits of Cudell et al (hep-ph/9908218) to total cross section and ρ for pp scattering. Others could be used. The different Regge pole contributions are given as

$$
\begin{aligned}
g_P(s) &= 1.09\,(\frac{1+e^{-i\pi\alpha_P}}{\sin\pi\alpha_P})s^{\alpha_P-1} \\
g_f(s) &= 63.0\,(\frac{1+e^{-i\pi\alpha_f}}{\sin\pi\alpha_f})s^{\alpha_f-1} \\
g_\omega(s) &= 36.2\,(\frac{1-e^{-i\pi\alpha_\omega}}{\sin\pi\alpha_\omega})s^{\alpha_\omega-1}
\end{aligned}
$$

with

$$
\begin{aligned}
\alpha_P &= 1.09 \\
\alpha_f &= 0.64 \\
\alpha_\omega &= 0.44
\end{aligned}
$$

normalized so that $Im[g_0(s)] = \sigma_{\rm tot}$ in millibarns.

so

$$\tau(s) = \{\tau_P\, g_P(s) + \tau_f\, g_f(s) + \tau_\omega\, g_\omega(s)\}/g_0(s,0)$$

- The two constants in $\tau(21.7) = -0.213 - 0.054i$ determines two relations between the three constants $\tau_P, \tau_f, \tau_\omega$

- We need one more measurement to fix their values. If one measures the "shape" of the raw asymmetry over the CNI region *without knowing the value of P at that energy* one can obtain the needed information:

$$S(p_L) = \frac{P Im[\tau(p_L)]}{P(\kappa/2 - Re[\tau(p_L))]} = \frac{Im[\tau(p_L)]}{\kappa/2 - Re[\tau(p_L)]}$$

Asymmetry for CNI elastic collisions of Helium-3 on Ions

The normal single spin asymmetry A_{N} of a spin half fermion (proton or helion) of mass m_{f} and charge $Z_{\mathrm{f}}\, e$ scattering elastically off a target ion of charge $Z_{\mathrm{A}}\, e$ with any spin may be estimated in the electromagnetic hadronic interference region using an approximate high energy small angle helicity nonflip amplitude without Bethe phase

$$\frac{Z_{\mathrm{f}}\, Z_{\mathrm{A}}\, \alpha}{t} \;+\; i\,\frac{\sigma_{\mathrm{tot}}(\mathrm{fA})}{8\,\pi}$$

where $\sigma_{\mathrm{tot}}(\mathrm{fA})$ refers to the total hadronic cross section for fermion ion scattering. If the magnetic moment of the incident spin half fermion is μ_{f} in units of the nuclear magneton involving proton mass m_{p}, the approximate high energy fermion helicity flip amplitude, neglecting the hadronic part, is the electromagnetic helicity flip amplitude

$$\frac{Z_{\mathrm{f}}\, Z_{\mathrm{A}}\, \alpha}{t}\;\left(\frac{\mu}{Z_{\mathrm{f}}} \;-\; \frac{m_{\mathrm{p}}}{m_{\mathrm{f}}}\right)\frac{\sqrt{-t}}{2\,m_{\mathrm{p}}}\,.$$

Noting that the ratio of electromagnetic flip to electromagnetic nonflip amplitudes is at most 3%, that is, 0.1% for squared amplitudes, the fermion asymmetry A_{N} has an extremum at approximate squared momentum transfer

$$-t_{\mathrm{ext}} \;=\; \frac{8\pi\alpha}{\sigma_{\mathrm{tot}}(\mathrm{fA})}\;|Z_{\mathrm{f}}\, Z_{\mathrm{A}}\,|\,\sqrt{3}$$

the extreme value of the fermion single spin asymmetry (either a maximum or minimum depending on the sign of the coefficient in parentheses) at $-t_{\mathrm{ext}}$ being

$$A_{\mathrm{N}}^{\mathrm{ext}}(\mathrm{hA}) \;=\; \left(\frac{\mu_{\mathrm{f}}}{Z_{\mathrm{f}}} \;-\; \frac{m_{\mathrm{p}}}{m_{\mathrm{f}}}\right)\frac{\sqrt{-3\,t_{\mathrm{ext}}}}{4m_{\mathrm{p}}}\,.$$

As an example, the ratio of the asymmetry extrema for helion nucleus and proton nucleus scattering is

$$\frac{A_{\mathrm{N}}^{\mathrm{ext}}(\mathrm{hA})}{A_{\mathrm{N}}^{\mathrm{ext}}(\mathrm{pA})} \;=\; \frac{\mu_{\mathrm{h}}/Z_{\mathrm{h}} - m_{\mathrm{p}}/m_{\mathrm{h}}}{\mu_{\mathrm{p}} - 1}\;\left(\frac{\sigma_{\mathrm{tot}}(\mathrm{hA})/Z_{\mathrm{h}}}{\sigma_{\mathrm{tot}}(\mathrm{pA})}\right)^{-1/2} \;=\; -0.6366\,\sqrt{\frac{3\sigma_{\mathrm{tot}}(\mathrm{pA})}{\sigma_{\mathrm{tot}}(\mathrm{hA})}}\,.$$

The masses and magnetic moments of proton and helium-3 nuclei (helion) are

$$m_{\mathrm{h}}/m_{\mathrm{p}} = 2.99315, \quad \mu_{\mathrm{p}} = 2.79285 \text{ and } \mu_{\mathrm{h}} = -2.1275 \text{ nuclear magnetons}$$

See N. H. Buttimore, Spin 2002 BNL, editor Yousef I. Makdisi et al., page 844.

The electromagnetic current matrix element for a spin half fermion of mass m with initial and final four momenta p and p' respectively may be written in two ways

$$\bar{u}' \left(\gamma^\mu F_1 + \frac{1}{2} [\, \gamma^\mu, \gamma^\nu \,] \, \frac{p_\nu - p'_\nu}{2\,m} \, F_2 \right) u \;=\; \bar{u}' \left(\frac{p^\mu + p'^\mu}{2m} \, F_1 + \frac{1}{2} [\, \gamma^\mu, \gamma^\nu \,] \, \frac{p_\nu - p'_\nu}{2\,m} \, G_{\mathrm{M}} \right) u$$

where the electromagnetic form factors $F_1(t)$ and $G_{\mathrm{M}}(t)$, with $t = (p' - p)_\mu (p' - p)^\mu$, have static values related to the charge and magnetic moment of the fermion

$$F_1(0) \;=\; q \,, \qquad \frac{G_{\mathrm{M}}(0)}{2\,m} \;=\; \mu' = \mu \, \frac{e}{2\,m_p} \,,$$

noting here that the magnetic moment μ' is normally quoted as μ when given in terms of the nuclear magneton involving the unit charge e and the mass of a proton m_{p}. Apart from a factor $(2\pi)^3$ for the current, the above analysis is based upon that given on page 454 of “Quantum Field Theory I” by Steven Weinberg with identifications

$$\gamma^\mu = i \gamma^\mu_{\mathrm{W}} \,, \qquad g^{\mu\nu} = -\eta^{\mu\nu}_{\mathrm{W}} = \mathrm{diag}\,(1, \, -1, \, -1, \, -1) \,,$$

$$F_1 = q F_1^{\mathrm{W}}, \qquad F_2 = -q G^{\mathrm{W}} = 2mq F_2^{\mathrm{W}}, \qquad G_{\mathrm{M}} = q F^{\mathrm{W}}.$$

Observe that the Dirac form factor $F_1(t)$ includes the charge q in its normalisation so that expressions also apply to the case of a neutral fermion such as the neutron. The following decomposition for the spinors $\bar{u}' = \bar{u}(p')$ and $u = u(p)$,

$$\bar{u}' \left\{ 2\,m\,\gamma^\mu \;-\; (p' + p)^\mu \;-\; \tfrac{1}{2} [\, \gamma^\mu \,, \, \gamma^\nu \,] \, (p - p')_\nu \right\} u \;=\; 0 \,,$$

leads to a relation for the Sachs magnetic form factor of the spin half particle

$$G_{\mathrm{M}}(t) \;=\; F_1(t) + F_2(t)$$

so that the anomalous magnetic moment of the fermion of electric charge $q = Ze$ is

$$\frac{F_2(0)}{2\,m} \;=\; \mu' - \frac{q}{2m} \;=\; \frac{e}{2} \left(\frac{\mu}{m_p} \;-\; \frac{Z}{m} \right) \,.$$

Note that the magnetic moment μ' has the Dirac value $q/2m$ in the absence of an anomaly, a quantity that is equal to Zm_p/m when expressed in terms of the nuclear magneton $e/2m_p$. With $j = 1/2$ for the spin of a fermion, Weinberg’s (10.6.22) and (10.6.23) provides $\mu^{\mathrm{W}}/j = qF^{\mathrm{W}}(0)/m$. With the normalization condition exhibited after (10.6.18), Weinberg’s (10.6.17) indicates

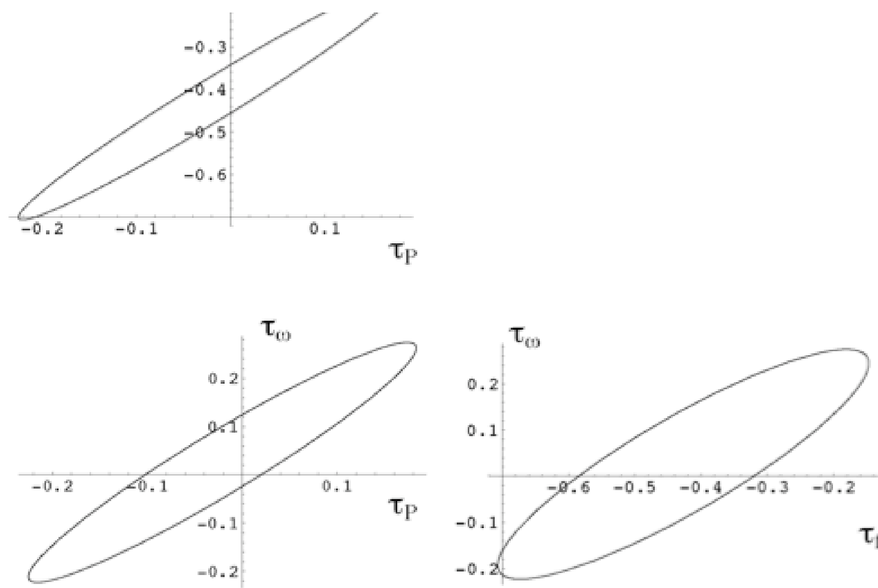
$$2mF_2^{\mathrm{W}}(0) = F^{\mathrm{W}}(0) - F_1^{\mathrm{W}}(0) \;= 2m\mu^{\mathrm{W}}/q - 1 \,; \qquad qF_2^{\mathrm{W}}(0) = \mu^{\mathrm{W}} - q/(2m)$$

Regge spin flip couplings from fitting pC analyzing power A_N at 21.5 and 100 GeV/c

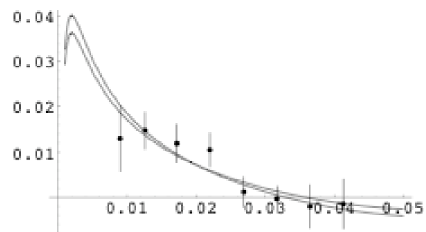
$\tau_P = 0.10$

$\tau_f = -0.79$

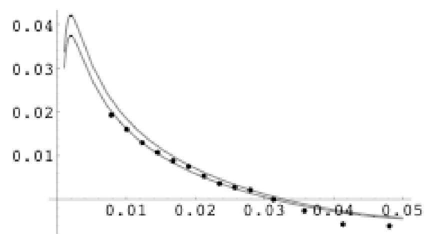
$\tau_\omega = +0.53$



-0.3
-0.4
-0.5
-0.6
-0.2
-0.1
0.1
τ_P
τ_ω
0.2
0.1
-0.1
-0.2
-0.2
-0.1
0.1
τ_P
τ_ω
0.2
0.1
-0.1
-0.2
-0.6
-0.5
-0.4
-0.3
-0.2
τ_f

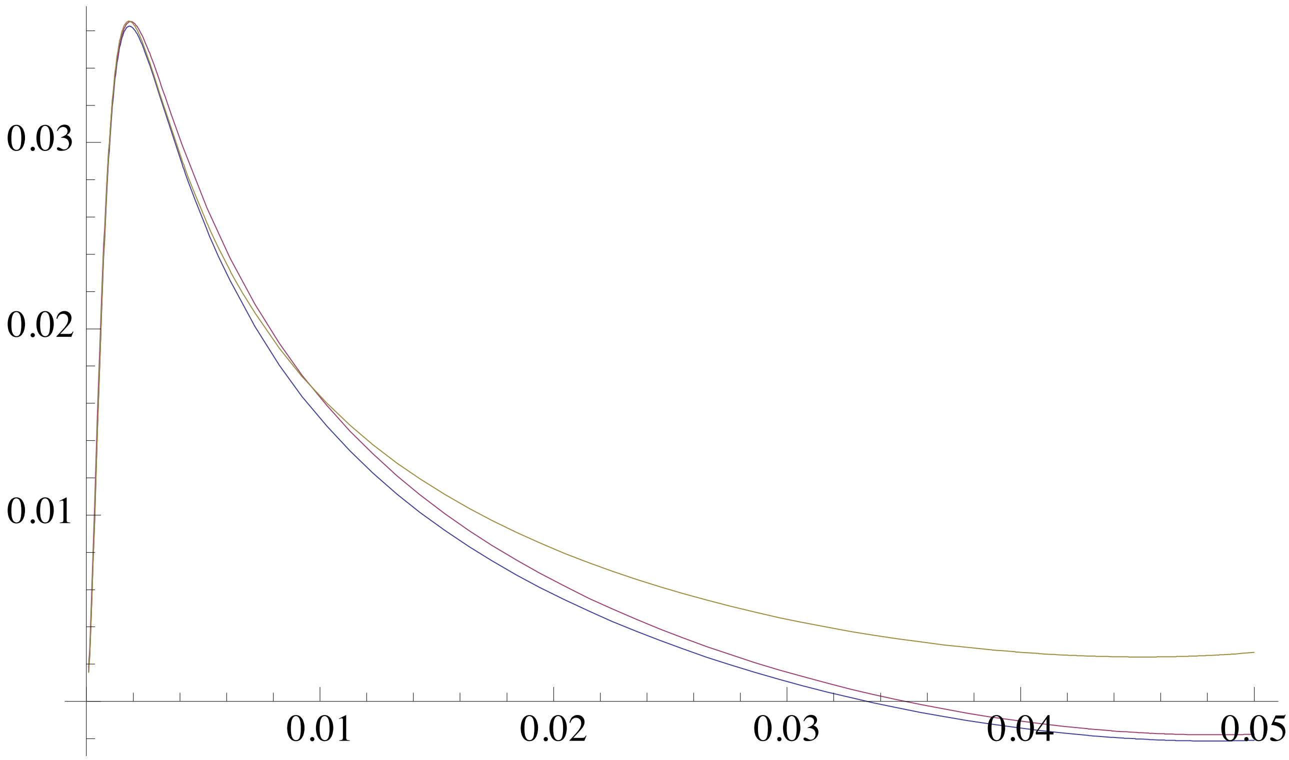


21.7 GeV data and E950 fit with known
P(upper) and Regge prediction based on fit to
100 GeV data and 21.7 GeV shape (lower)



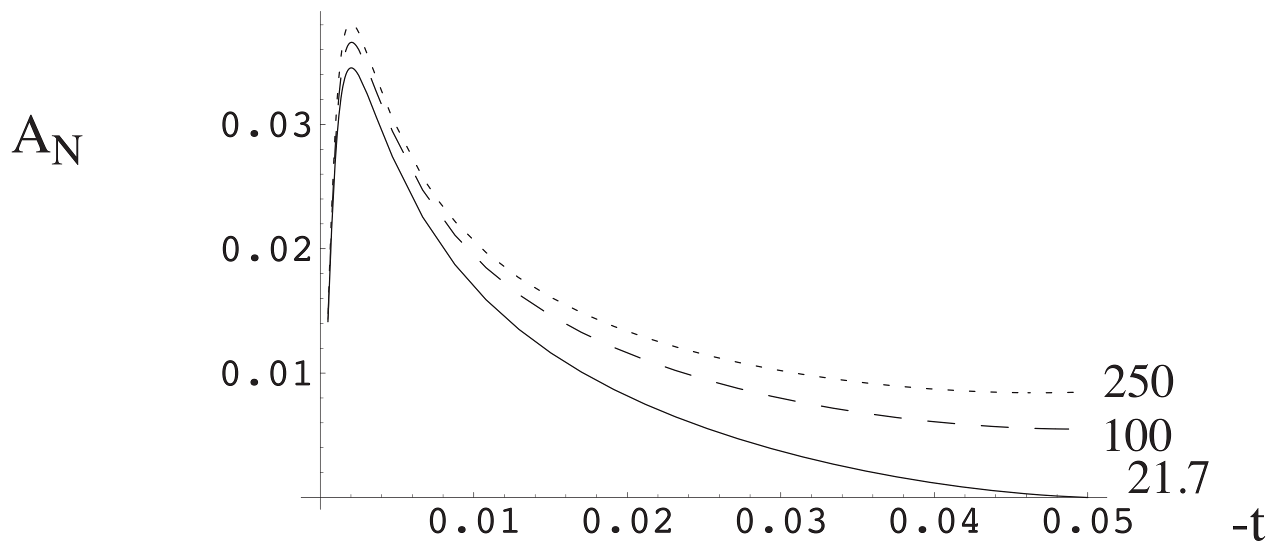
new 100 GeV data, best fit with known
P(lower) and Regge prediction based on E950 fit
and 100 GeV shape (upper)

Energy dependence of analyzing power



A_N for pC scattering in the CNI region. The lowest curve is for a 24 GeV/c beam, the highest for 250 GeV beam and the intermediate curve for 100 GeV/c beam. The lowest curve is actually a best fit

$$-2Im[\tau(s)](\rho_{pC}+\delta_{pC})\Big] + 2\,F_C^h(t)\Big(Im[\tau(s)](1+\rho_{pC}^2)\Big)$$



Analyzing power for pC measured at 21.7 GeV/c lab momentum and predicted at 100 and 250

$I = 1$ couplings and proton-proton elastic scattering

- Because pp scattering involves the exchange of of $I = 1$ Regge poles, the ρ and the a_2 in particular, we cannot simply use the results above to make predictions for this case. But we can use the beautiful new p-jet data and a couple of reasonable assumptions to achive this. We assume (1) at these energies the proton-proton and neutron-proton unpolarized scattering amplitudes are approximately equal and (2) the two $I = 1$ Regge poles are degenerate with the corresponding $I = 0$ Regge pole of the same Charge Conjugation parity, $C = -1$ for ω, ρ and $C = +1$ for f, a_2. Then we can describe pp scattering in terms of 3-parameters: τ_+, τ_- and the pomeron coupling τ_P. Since we already know τ_P, in some sense, from the pC analysis and we can determine two parameters from the real and imaginary parts of τ obtained by fitting the p-jet data, we are in business.

I obtain then

$$\tau_{pp}(100) = -0.065 - 0.0124i$$

and from this and using $\tau_P = 0.09$ we find

$$\begin{aligned} \tau_P &= +.09 \\ \tau_+ &= -0.324 \\ \tau_- &= 1.06 \end{aligned}$$

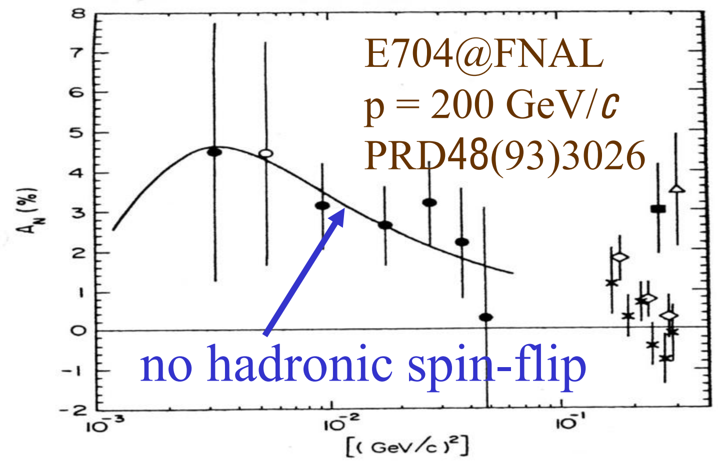
I remind for the $I = 0$ Reggeons

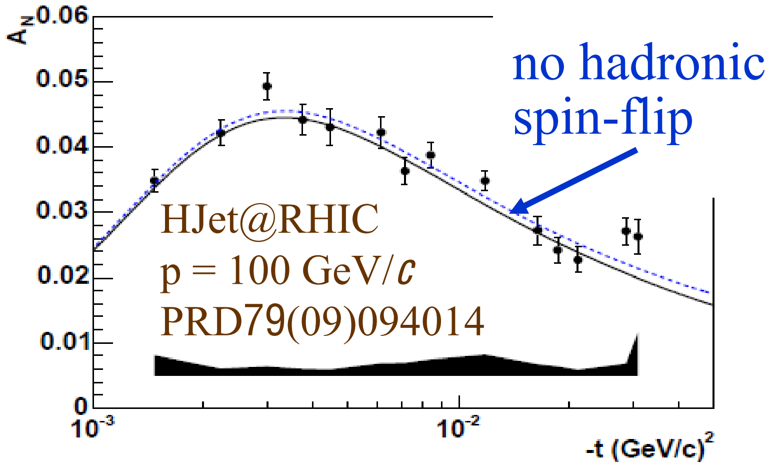
$$\begin{aligned} \tau_P &= +.09 \\ \tau_f &= -0.30 \\ \tau_\omega &= 0.19 \end{aligned}$$

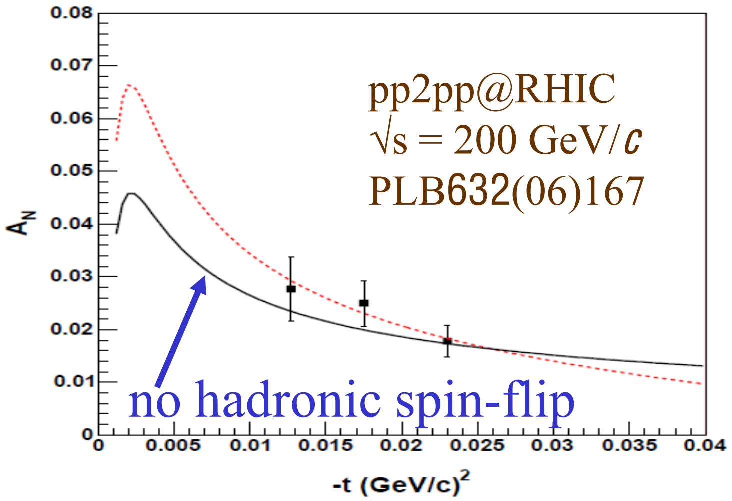
so the f coupling is not much modified by the a_2 but the the $C = -1$ is changed by a factor of 10! This is not too much of a surprise: the ancient fit of Berger et al shows just such a pattern.

A_N measurements in the CNI region

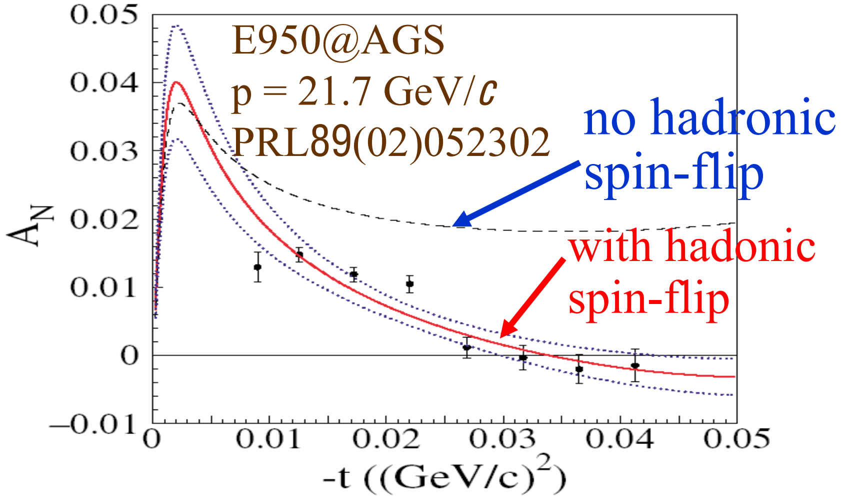
pp Analyzing Power

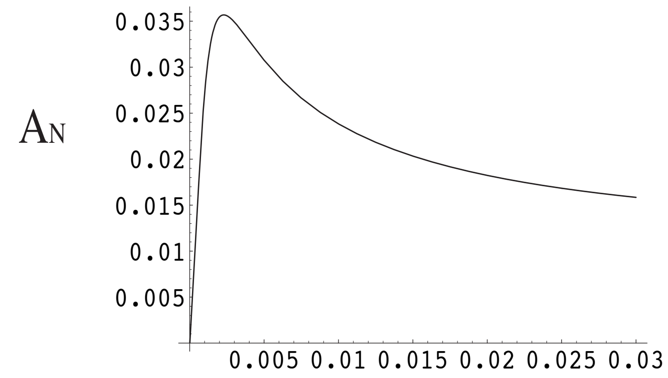






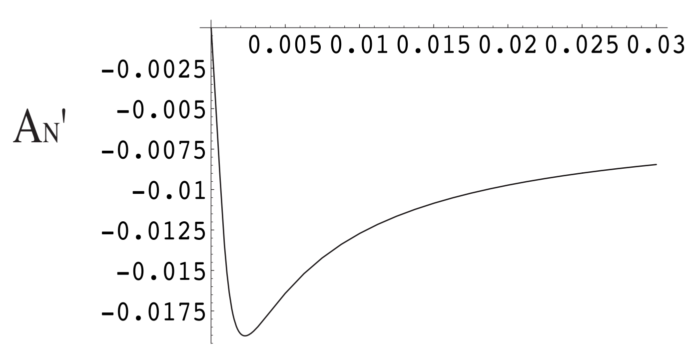
pC Analyzing Power





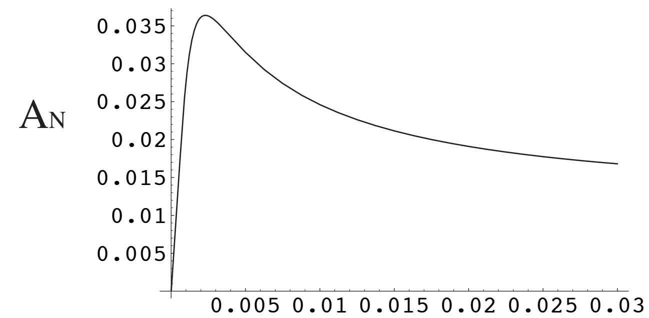
(a)

proton analyzing power at pL=100 GeV/c



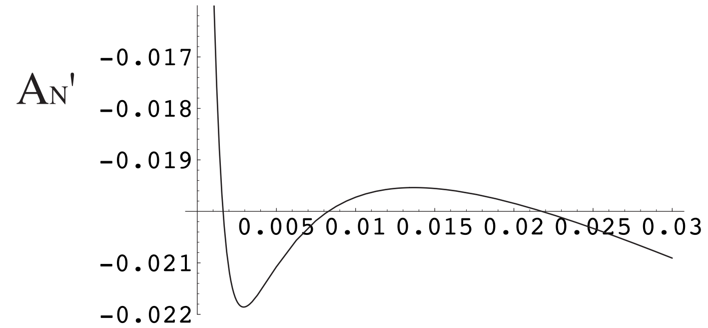
neutron analyzing power at pL=100 GeV/c

(b)



(a)

proton analyzing power at pL=100 GeV/c



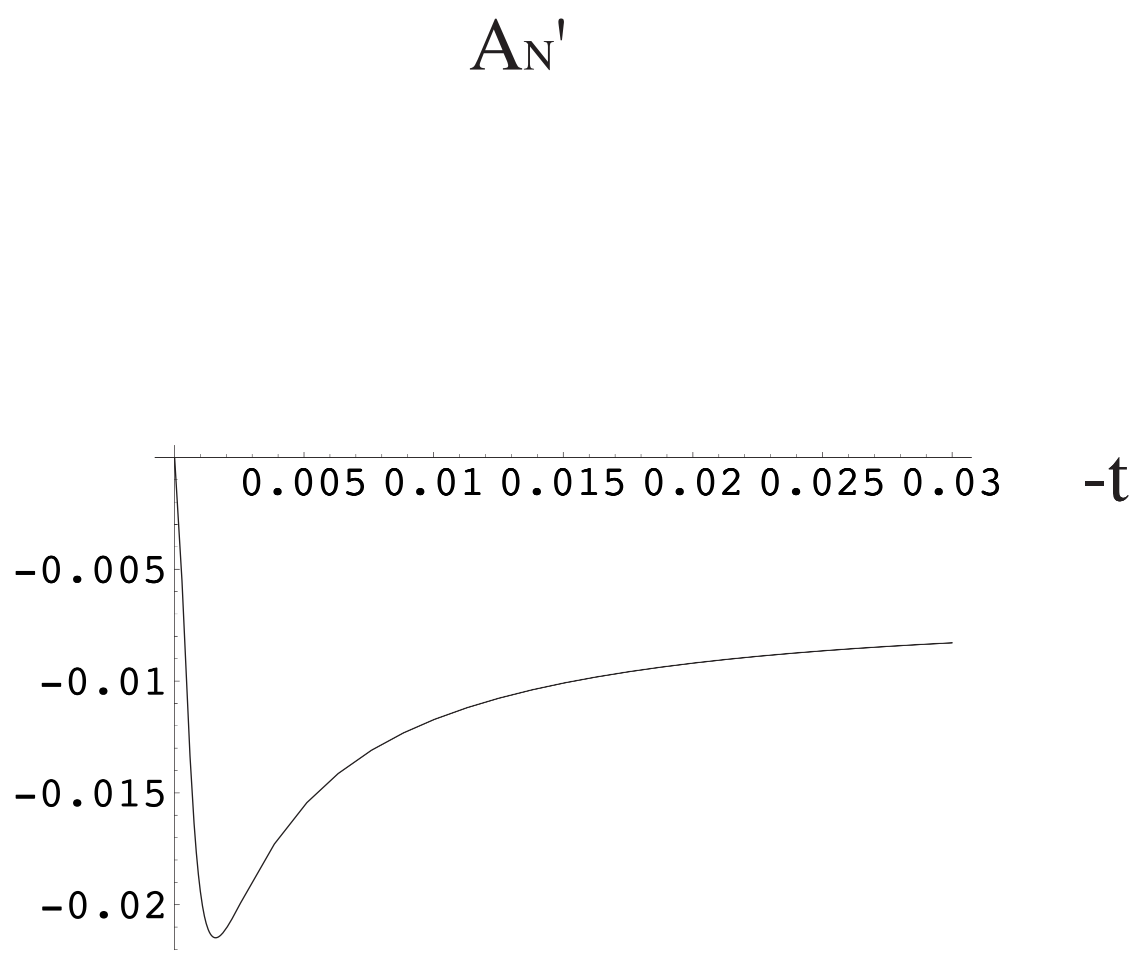
(b)

neutron analyzing power at pL=100 GeV/c

Analyzing powers calculated with

$\tau p = 0.014 + 0.002\, i$

and $\tau n = -0.006 - 0.102 i$



proton -3 Helium colliding beams at 150 GeV/c lab momentum per nucleon