QCD at small and not so small x:

NLO BFKL and combined HERA data

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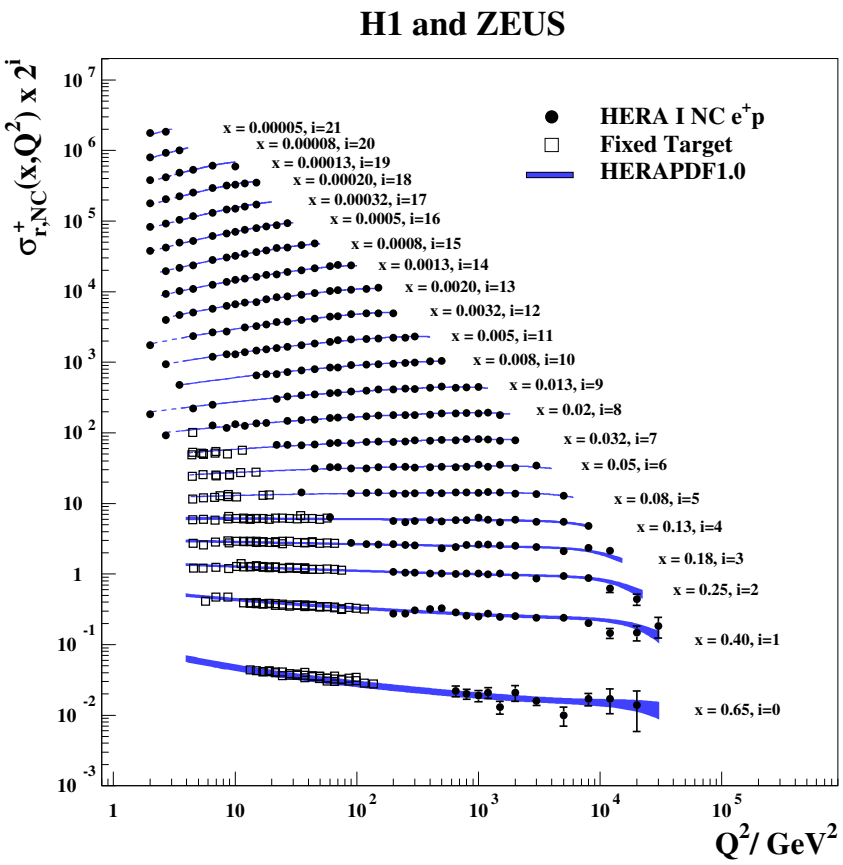
Based on:

- **arXiv:1209.1353**
  [PRL 110 (2013) 041601]
- **arXiv:1301.5283**
  [PRD (2013) xxx]

Brain circulation kick off workshop, March 21, 2013



Deep Inelastic Scattering at small x

collinear factorization: excellent description

$$F_{2,L}(x,Q^2) = \text{coeff. funct.} \otimes \text{pdfs}$$

Q^2 dependence: DGLAP evolution

rise at small $x \simeq \dfrac{Q^2}{s}$

- (perturbative) DGLAP splitting functions
- (non-pert.) x-dependence of pdfs at initial scale

Question: Can we understand this x-dependence entirely from perturbative QCD?

Perturbative analysis of DIS at small x

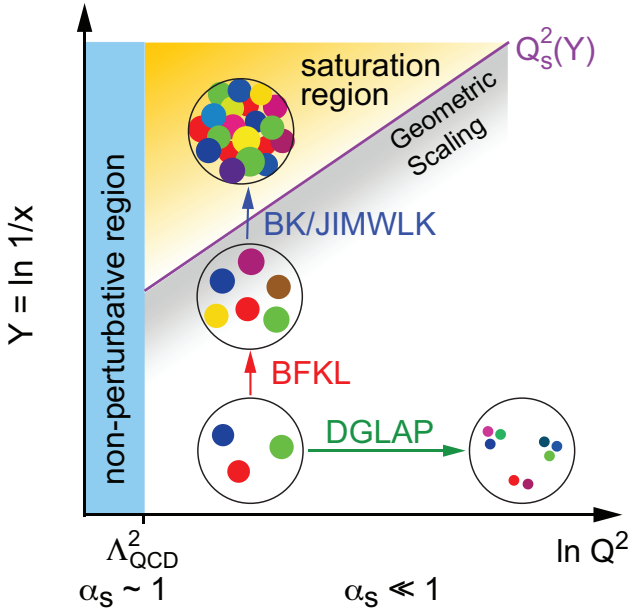
- $x \simeq \dfrac{Q^2}{s} \ll 1$
- $\alpha_s \ln(1/x) \sim 1$

→ resummation: BFKL equation

- LL $\sum_n \left(\alpha_s \ln s\right)^n$ [Fadin, Kuraev, Lipatov (1977)], [Balitsky, Lipatov (1978)]

- NLL $\sum_n \alpha_s \left(\alpha_s \ln s\right)^n$ [Fadin, Lipatov (1998)], [Ciafaloni, Gamici (1998)]

Life beyond BFKL: high parton densities

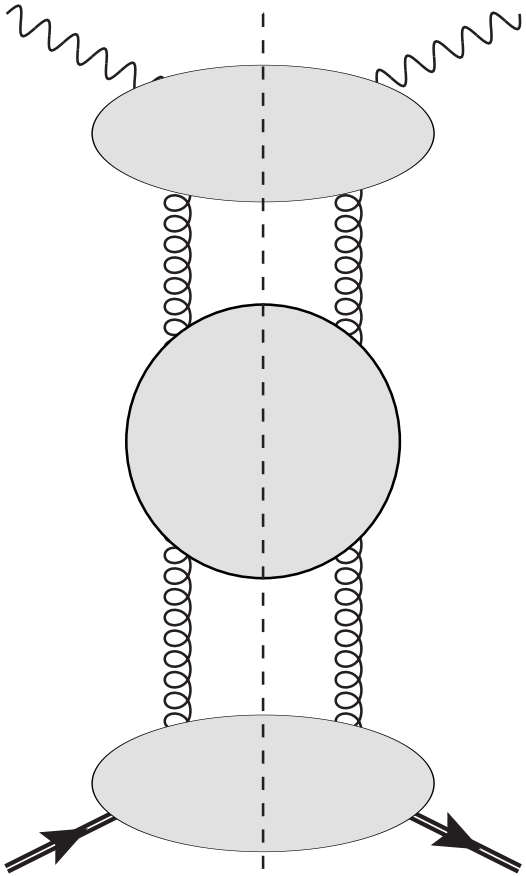


[EIC White Paper, arXiv:1212.1701]

here:

- restrict to BFKL at NLO
  no saturation effects considered
- breakdown at ultra small $x \equiv$
  evidence for saturation/CGC
- interesting: DIS on a heavy ion

high energy factorization (up to NLL) of F_2



photon impact factor

$$F_2 = \int \frac{d\boldsymbol{q}^2}{\boldsymbol{q}^2} \frac{d\boldsymbol{p}^2}{\boldsymbol{p}^2} \Phi_{\gamma^*}\left(\frac{\boldsymbol{q}^2}{Q^2}, \mu^2, s_0\right) f_{\text{BFKL}}\left(\frac{s}{s_0}, \boldsymbol{q}^2, \boldsymbol{p}^2, \mu^2\right) \times \Phi_{\text{proton}}\left(\frac{\boldsymbol{p}^2}{Q_0^2}, s_0\right)$$

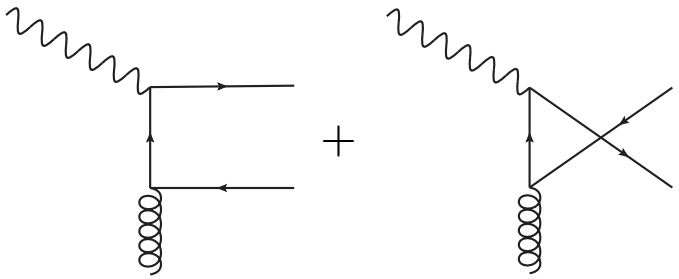
gluon Green's function proton impact factor

convolution in k_T space

s_0: reggeization scale
μ: renormalization scale

photon impact factor

photon at LO



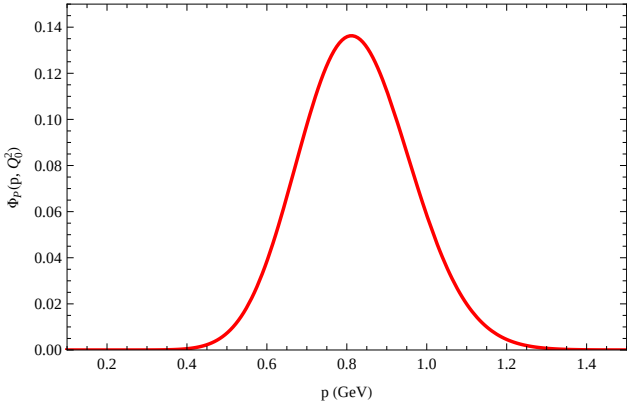
- strict LO [Balitsky, Lipatov (1978), ...]
- DGLAP improved kinematics [Kwiecinski, Martin, Stasto (1997)], [Bialas, Navelet, Peschanski (2001)]

proton impact factor

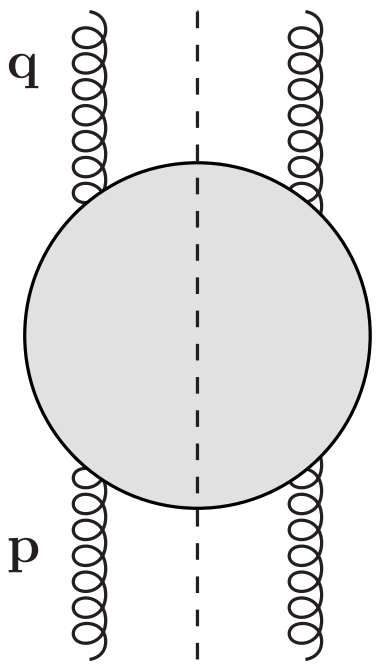
k_T in proton: Poisson distribution

$$\Phi_P\left(p, Q_0^2\right) = \frac{\mathcal{C}}{\Gamma(\delta)}\left(\frac{p^2}{Q_0^2}\right)^{\delta} e^{-\frac{p^2}{Q_0^2}}$$

Normalization $\mathcal{C}$ and parameters Q_0^2, δ to be determined from fit to data



BFKL gluon Green's function



scale choices:

- $s/s_0 = 1/x_g$ - suggested by DGLAP
- $\mu^2 = Q \cdot Q_0$ - both hard and soft scale

scale invariant NLO resummed eigenvalue

$$f_{\text{BFKL}}\left(\frac{1}{x_g}, \boldsymbol{q}^2, \boldsymbol{p}^2, QQ_0\right) = \int \frac{d\gamma}{2\pi^2 i} \frac{1}{\boldsymbol{q}^2} \left(\frac{\boldsymbol{q}^2}{\boldsymbol{p}^2}\right)^{\gamma} x_g^{-\chi(\gamma)} \left[1 - \ln\left(\frac{1}{x_g}\right) \frac{\bar{\alpha}_s^2 \beta_0 \chi_0(\gamma)}{4N_c} \ln \frac{|\boldsymbol{q}||\boldsymbol{p}|}{QQ_0}\right]$$

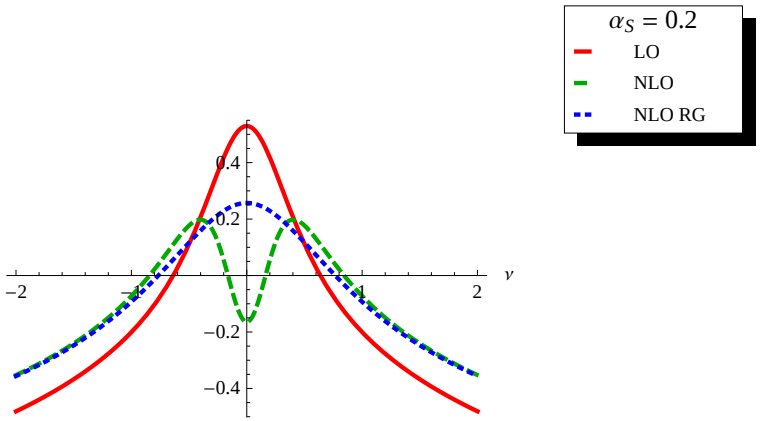
scale dependent NLO corrections

scale invariant resummed NLO eigenvalue

LO BFKL NLO BFKL

$$\chi(\gamma) = \bar{\alpha}_s \chi_0(\gamma) + \bar{\alpha}_s^2 \chi_1(\gamma) - \frac{1}{2}\bar{\alpha}_s^2 \chi_0{}'(\gamma)\chi_0(\gamma) + \chi_{\mathsf{RG}}(\bar{\alpha}_s, \gamma, a, b),$$

NLO BFKL



$\mathcal{O}(\alpha_s^3)$ collinear resummation term [Andersson, Gustafson, Samuelsson (1996)], [Salam (1998)], [Sabio Vera (2005)], [MH, Salas, Sabio Vera (2012)]

$$\chi_{\mathsf{RG}}(\bar{\alpha}_s, \gamma, a, b) = \bar{\alpha}_s(1 + a\bar{\alpha}_s)\left(\psi(\gamma) - \psi(\gamma - b\bar{\alpha}_s)\right) - \frac{\bar{\alpha}_s^2}{2}\psi''(1-\gamma) - b\bar{\alpha}_s^2 \frac{\pi^2}{\sin^2(\pi\gamma)}$$
$$+ \frac{1}{2}\sum_{m=0}^{\infty}\left(\gamma - 1 - m + b\bar{\alpha}_s - \frac{2\bar{\alpha}_s(1 + a\bar{\alpha}_s)}{1 - \gamma + m} + \sqrt{(\gamma - 1 - m + b\bar{\alpha}_s)^2 + 4\bar{\alpha}_s(1 + a\bar{\alpha}_s)}\right).$$

Running coupling corrections

- Observation: 'natural' description, using [Brodsky, Fadin, Kim, Lipatov, Pivovarov (1999)]:

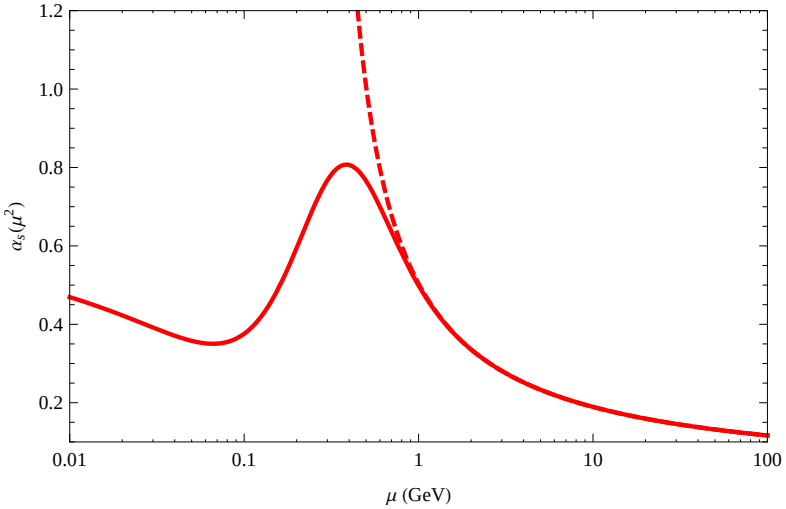
  - non-Abelian physical renormalization scheme ('MOM-scheme')
  - absorb entire (and sizeable) β_0 dependent terms of $\chi_1(\gamma)$ into α_s

$$\alpha_s\left(QQ_0\right) \to \alpha_s\left(QQ_0,\gamma\right) = \frac{4N_c}{\beta_0\left[\log\left(\frac{QQ_0}{\Lambda^2}\right) + \frac{1}{2}\chi_0(\gamma) - \frac{5}{3} + 2\left(1+\frac{2}{3}Y\right)\right]}$$

- Parametrization of the running coupling [Webber (1998)] ➡ small Q^2 region
  - compatible with power corrections to jet observables

$$\alpha_s\left(\mu^2\right) = \frac{4\pi}{\beta_0 \ln\frac{\mu^2}{\Lambda^2}} + f\left(\frac{\mu^2}{\Lambda^2}\right)$$

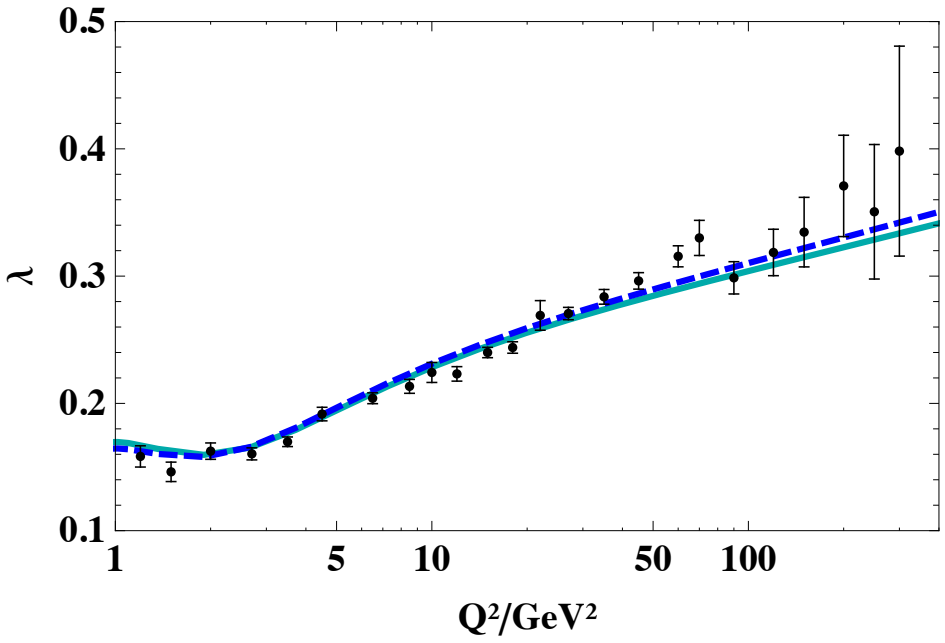
$$f\left(\frac{\mu^2}{\Lambda^2}\right) = \frac{4\pi}{\beta_0}\,\frac{125\left(1+4\frac{\mu^2}{\Lambda^2}\right)}{\left(1-\frac{\mu^2}{\Lambda^2}\right)\left(4+\frac{\mu^2}{\Lambda^2}\right)^4}.$$



Comparison with HERA data - Pomeron intercept $\lambda(Q^2)$

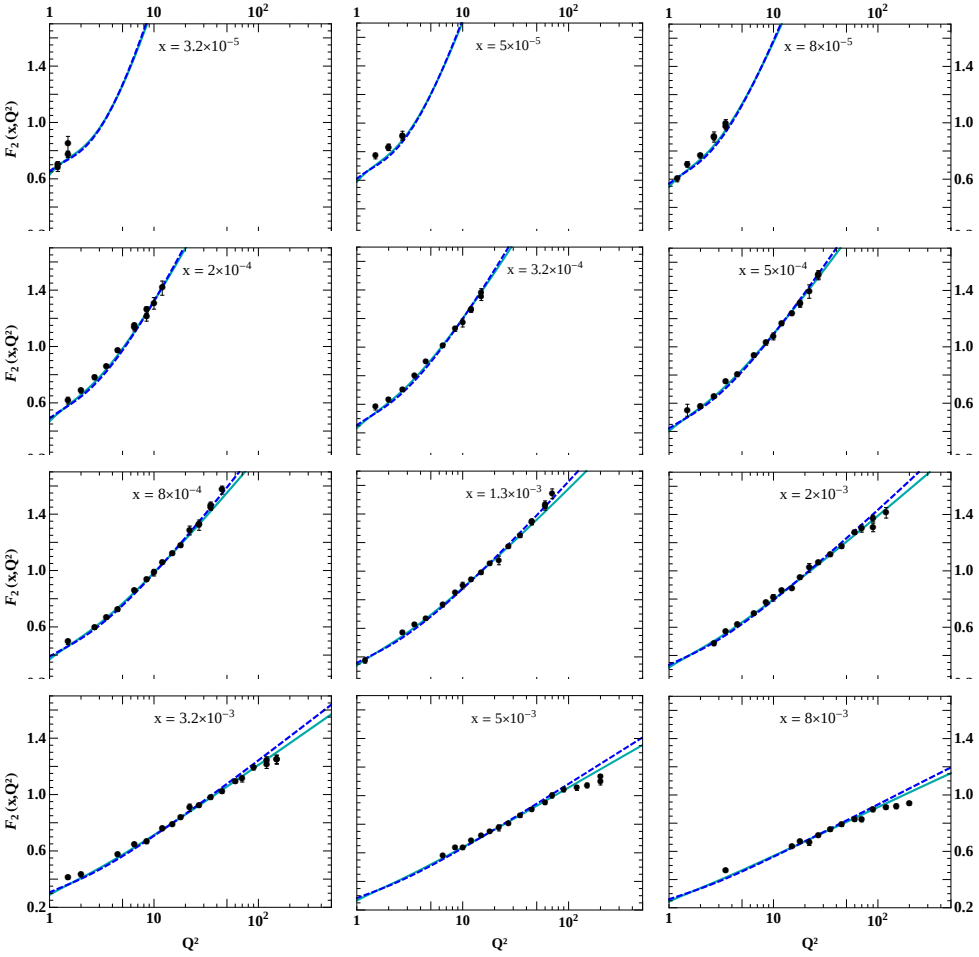
$$\lambda(Q^2) = \left\langle \frac{d \ln F_2}{d \ln 1/x} \right\rangle_x , \quad x < 10^{-2}$$

- MOM scheme with gauge parameter $\xi = 3$ ('Yennie-gauge'), $\Lambda_{\text{QCD}}^{\text{MOM}} = 0.21\text{GeV}$
- LO photon (**straight**): $\delta = 8.4$, $Q_0 = 0.28$ GeV
- photon with improved kinematics: (**dashed**): $\delta = 6.5$, $Q_0 = 0.28$ GeV

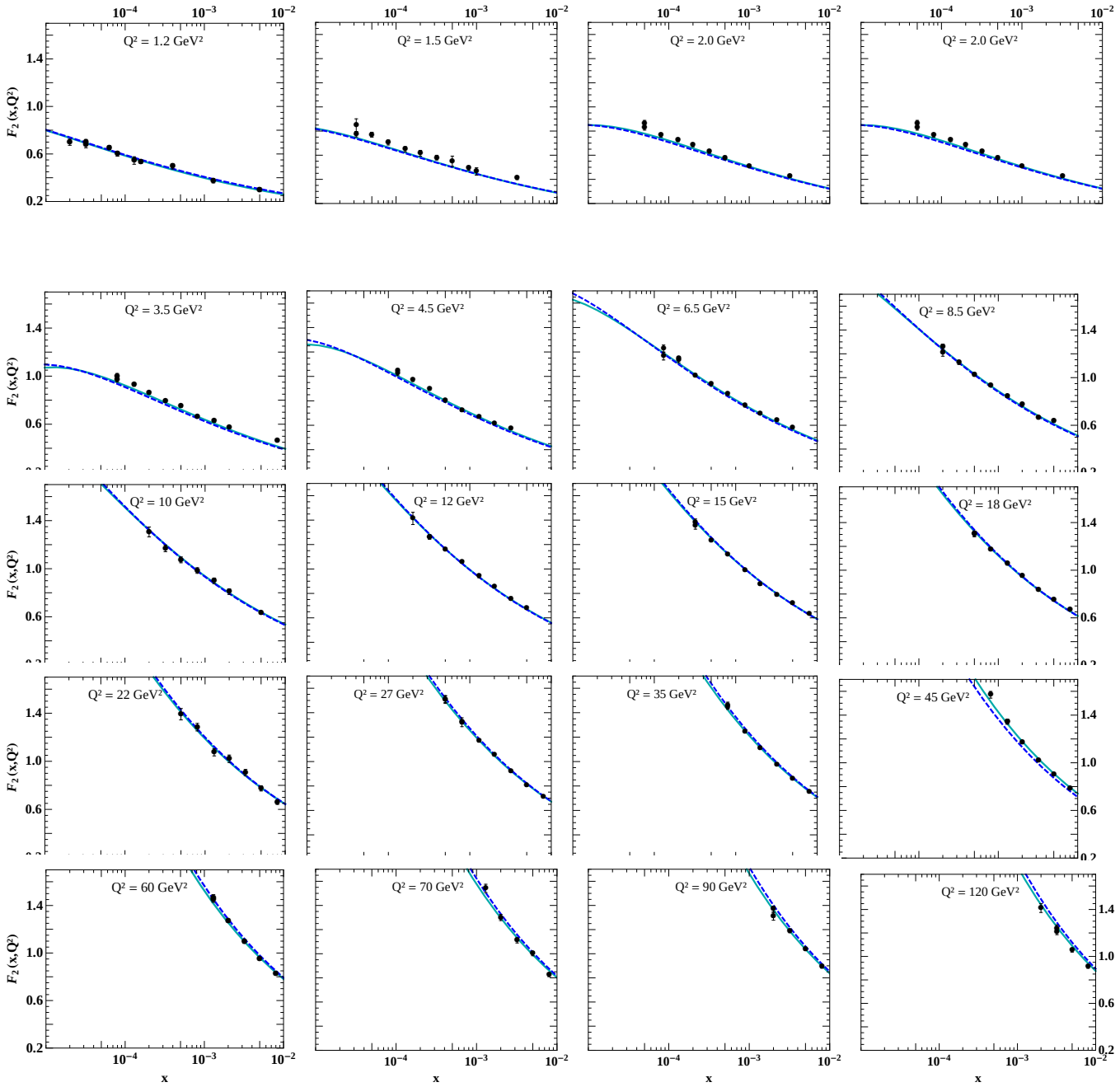


underlying data: [H1 and ZEUS collaboration]

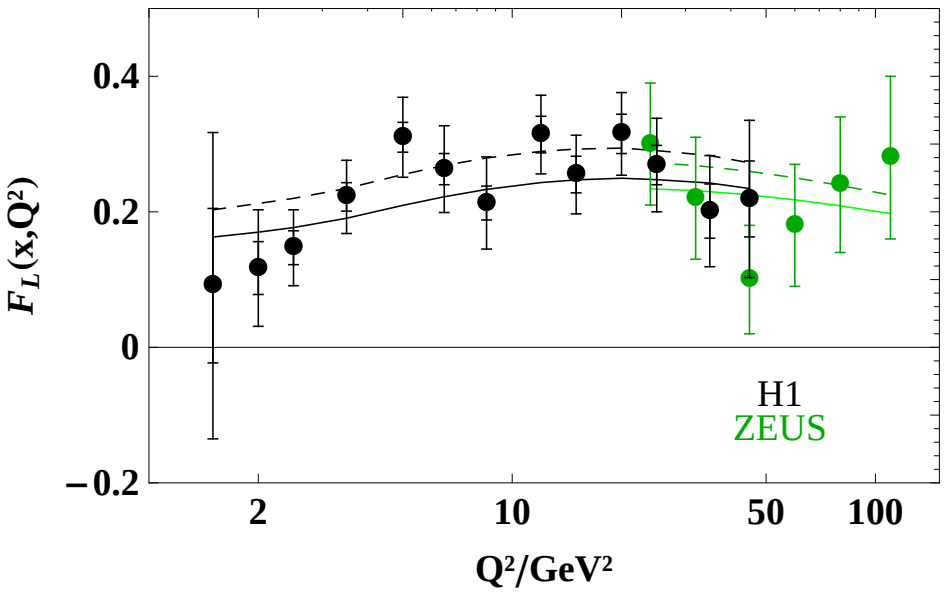
Comparison with HERA data - F_2



- very accurate description of data for both LO photon (**straight**) [$\delta = 8.4$, $Q_0 = 0.28$ GeV, $\mathcal{C} = 1.50$] and photon with improved kinematics (**dashed**) [$\delta = 6.5$, $Q_0 = 0.28$ GeV, $\mathcal{C} = 2.39$]
- find expected deviations at large x and large Q^2

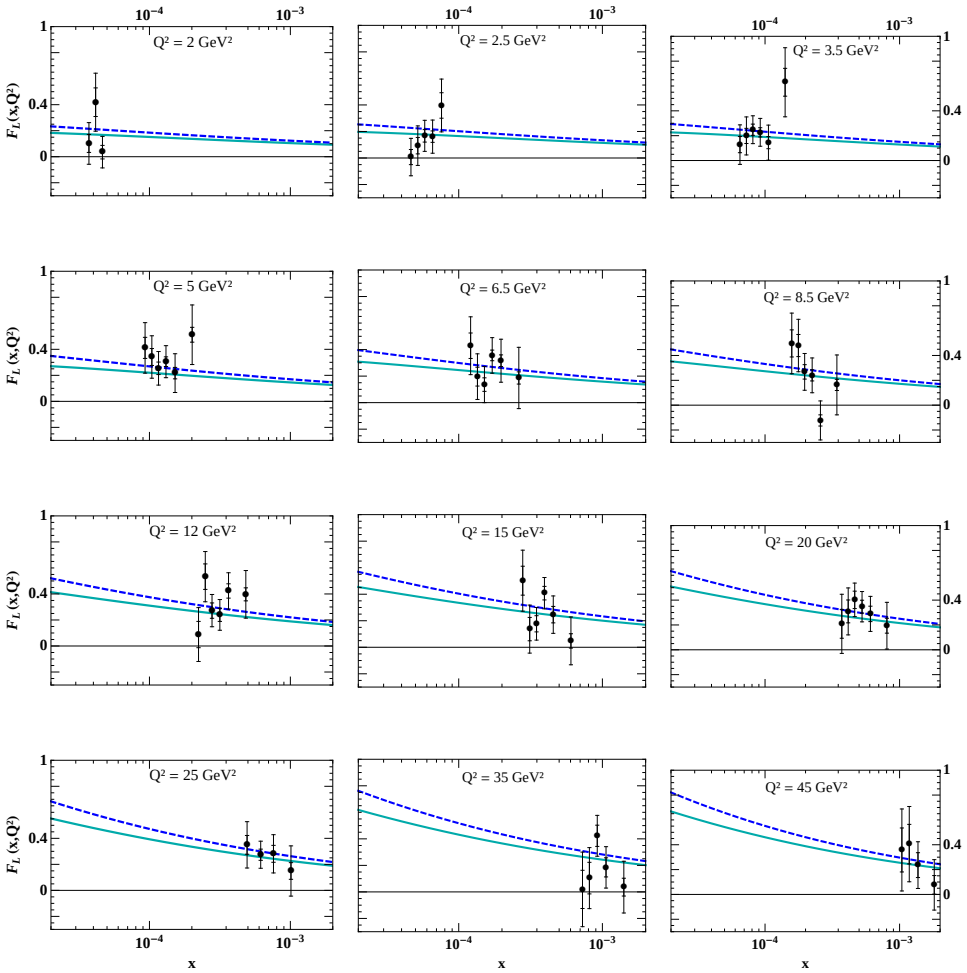

Q² = 1.2 GeV²
Q² = 1.5 GeV²
Q² = 2.0 GeV²
Q² = 2.0 GeV²
Q² = 3.5 GeV²
Q² = 4.5 GeV²
Q² = 6.5 GeV²
Q² = 8.5 GeV²
Q² = 10 GeV²
Q² = 12 GeV²
Q² = 15 GeV²
Q² = 18 GeV²
Q² = 22 GeV²
Q² = 27 GeV²
Q² = 35 GeV²
Q² = 45 GeV²
Q² = 60 GeV²
Q² = 70 GeV²
Q² = 90 GeV²
Q² = 120 GeV²
F₂(x,Q²)
x

Comparison with HERA data - F_L



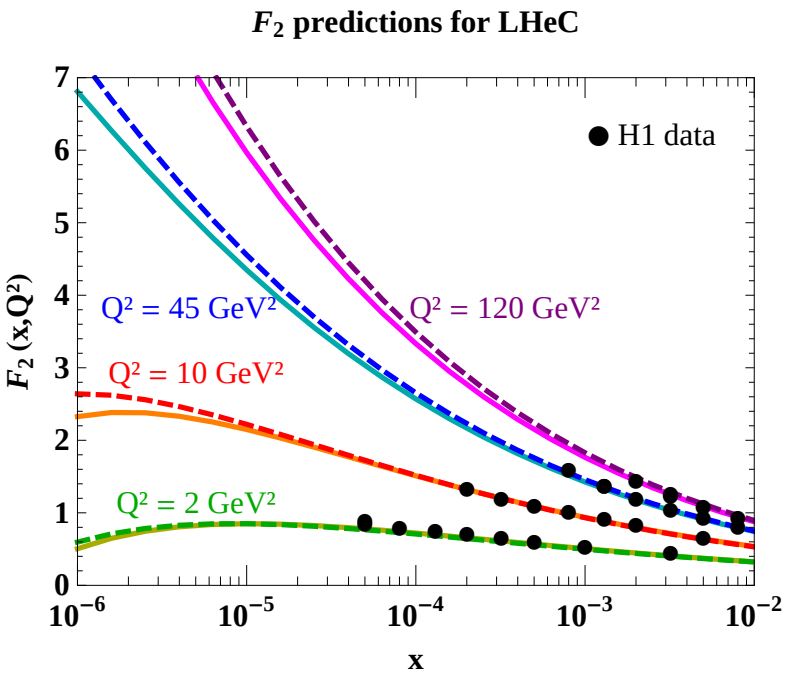
- accuracy of the data considerably diminished w.r.t. F_2
- Inclusive observable: $\langle F_L \rangle_x$
- only change in setup: photon imact factor (quark loop) for long. polarized photon
- LO photon (**straight**) and photon with improved kinematics (**dashed**)

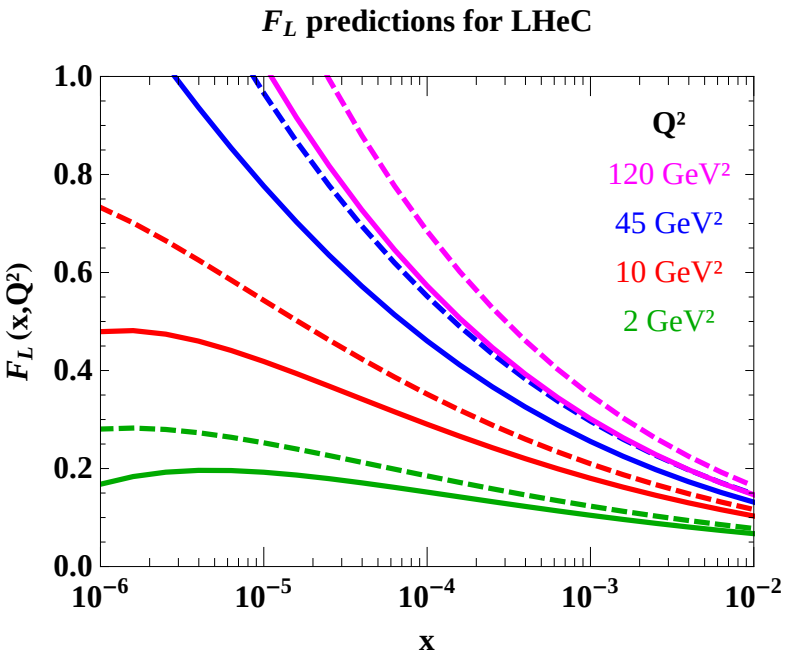
F_L unaveraged



Predictions for an *e.g.* LHeC

Extrapolating our result to proton structure functions at ultra-small x





LO photon (**straight**)

LO photon with improved kinematics (**dashed**)

Summary and Conclusions

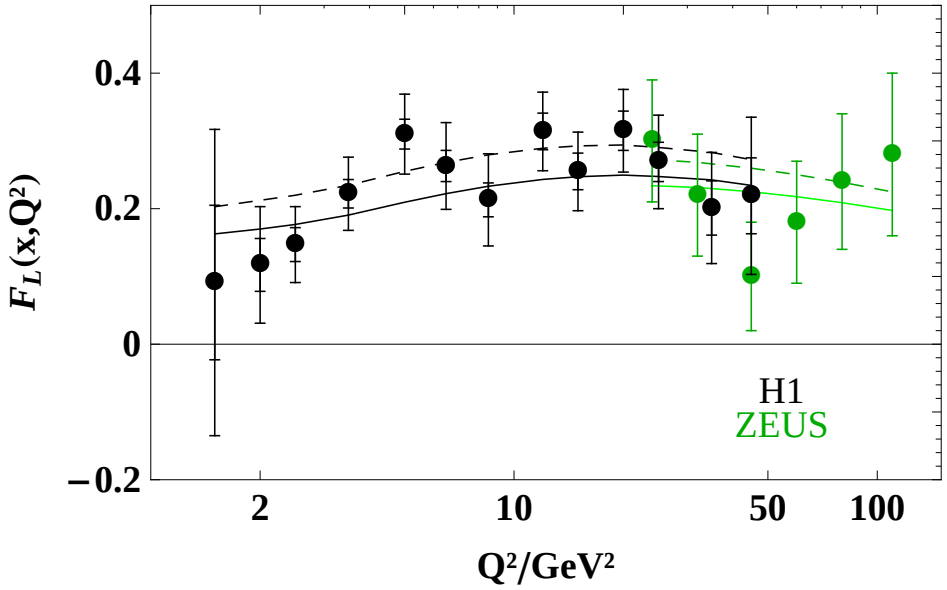
2nd NLO BFKL fit to combined HERA data

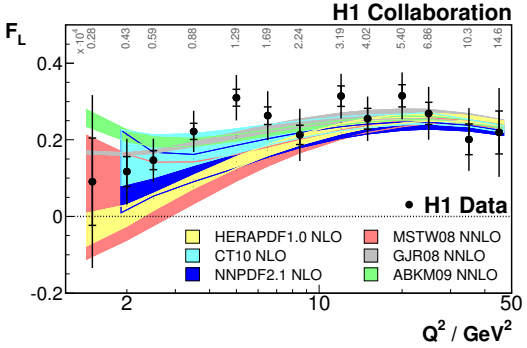
- solution expressed through LO eigenfunctions, (before: discrete Pomeron solution [Ellis, Kowalski, Ross (2008)], [Kowalski, Lipatov, Ross, Watt (2010)])
- use RG improvements + BLM scale setting for Green's function + parametrization of running coupling in the infra-red
- excellent agreement with data in BFKL region, deviations for $Q^2 > 150\text{GeV}^2$, $x > 0.5 \cdot 10^{-2}$ and $Q^2 < 1.2\text{GeV}^2$.
- natural scales for fitted proton parameters: $Q_0 \sim \Lambda_{\text{QCD}}$, normalization $\mathcal{O}(1)$

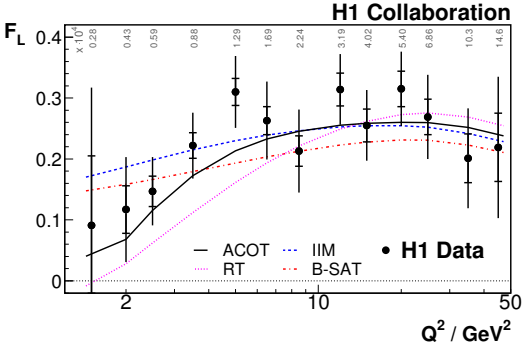
possible improvements mainly for the photon

- NLO corrections [Bartels, Gieseke, Qiao (2000)], [Bartels, Gieseke, Kyrieleis (2001)], [Bartels, Colferai, Gieseke, Kyrieleis (2002)],[Bartels, Kyrieleis (2004)], [Bartles, Chachamis (2006)], [Balitsky, Chirillis (2011), (2013)], [Beuf (2012)]
- quark masses
- treatment of running coupling

Backup

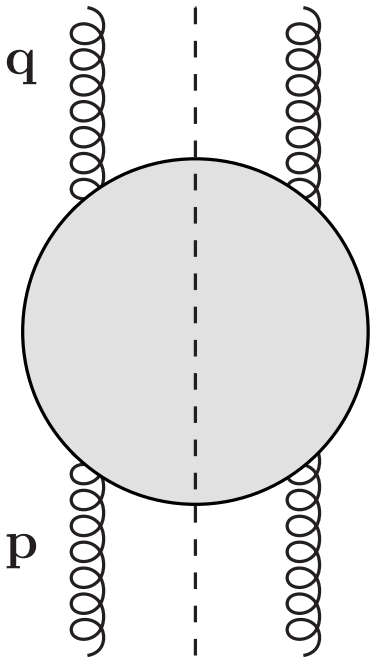

0.4
0.2
0
−0.2
$F_L(x,Q^2)$
2
10
50
100
Q^2/GeV^2
H1
ZEUS


H1 Collaboration
F_L
0.4
0.2
0
-0.2
2
10
50
Q^2 / GeV^2
• H1 Data
HERAPDF1.0 NLO
CT10 NLO
NNPDF2.1 NLO
MSTW08 NNLO
GJR08 NNLO
ABKM09 NNLO


H1 Collaboration
F_L
0.4
0.3
0.2
0.1
0
2
10
50
Q^2 / GeV^2
ACOT
RT
IIM
B-SAT
• H1 Data

The BFKL gluon Green's function - scale choice

$$f_{\mathsf{BFKL}}\left(\frac{s}{s_0}, \boldsymbol{q}^2, \boldsymbol{p}^2, \mu^2\right) = \int \frac{d\omega}{2\pi i}\left(\frac{s}{s_0}\right)^{\omega} f_\omega\left(\boldsymbol{q}^2, \boldsymbol{p}^2, \mu^2\right)$$



symmetric kinematic $\boldsymbol{q}^2 \sim \boldsymbol{p}^2$

$\ln(s/s_0) \longrightarrow \Delta y \simeq \ln \frac{s}{|\boldsymbol{q}||\boldsymbol{p}|}$

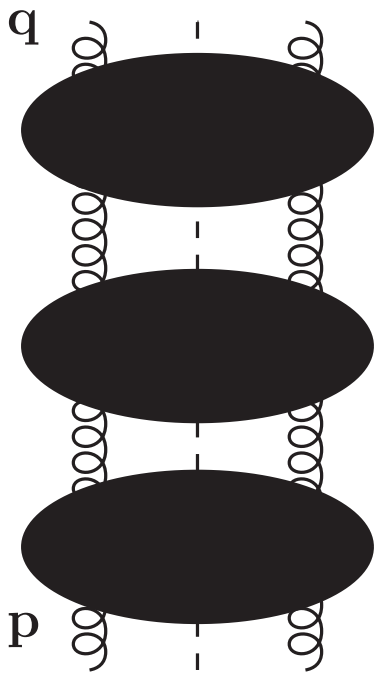
asymmetric DIS kinematic $\boldsymbol{q}^2 \gg \boldsymbol{p}^2$

DGLAP suggests

$\ln(s/s_0) \longrightarrow \ln 1/x_g \simeq \ln s/\boldsymbol{q}^2$

The BFKL gluon Green's function - BFKL equation

$$f_{\mathsf{BFKL}}\left(\frac{s}{s_0}, \boldsymbol{q}^2, \boldsymbol{p}^2, \mu^2\right) = \int \frac{d\omega}{2\pi i}\left(\frac{s}{s_0}\right)^{\omega} f_\omega\left(\boldsymbol{q}^2, \boldsymbol{p}^2, \mu^2\right)$$



BFKL equation

$$\omega f_\omega\left(\boldsymbol{q}^2, \boldsymbol{p}^2\right) = \delta^{(2)}(\boldsymbol{q}^2 - \boldsymbol{p}^2) + \int \frac{d^2\boldsymbol{k}}{\pi} K_{\mathsf{BFKL}}(\boldsymbol{q}, \boldsymbol{k}) f_\omega\left(\boldsymbol{k}^2, \boldsymbol{p}^2\right)$$

BFKL Kernel

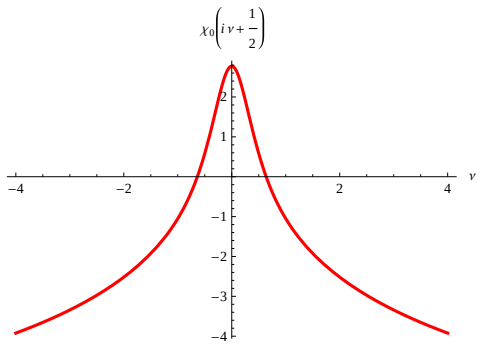
$$K_{\mathsf{BFKL}}(\boldsymbol{q}, \boldsymbol{k}) = \left(\frac{\alpha_s N_c}{\pi}\right) K_{\mathsf{LO}}(\boldsymbol{q}, \boldsymbol{k}) + \left(\frac{\alpha_s N_c}{\pi}\right)^2 K_{\mathsf{NLO}}(\boldsymbol{q}, \boldsymbol{k})$$

LO diagonalized by scale invariant eigenfunctions

$$\int \frac{d^2\boldsymbol{q}}{\pi} K_{\mathsf{LO}}(\boldsymbol{q}, \boldsymbol{p})(\boldsymbol{p}^2)^{\gamma-1} = \frac{\alpha_s N_c}{\pi} \chi_0(\gamma)(\boldsymbol{q}^2)^{\gamma-1}$$

Leading order solution

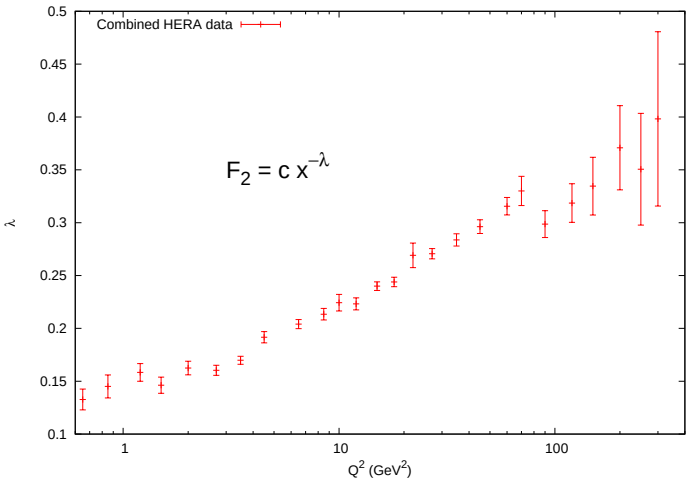
$$f^{\mathsf{LO}}_{\mathsf{BFKL}}\left(\frac{1}{x},\boldsymbol{q}^2,\boldsymbol{p}^2,\mu^2\right)=\frac{1}{\boldsymbol{q}^2}\int\limits_{1/2-i\infty}^{1/2+i\infty}\frac{d\gamma}{2\pi^2 i}\left(\frac{\boldsymbol{q}^2}{\boldsymbol{p}^2}\right)^{\gamma}x^{-\bar{\alpha}_s\chi_0(\gamma)}$$



$$\gamma=\frac{1}{2}+i\nu$$

intercept at saddle point $\nu=0$: $\lambda^{\mathsf{LO}}_{s.p.}=\frac{\alpha_s N_c}{\pi}4\ln 2\simeq .53$ for $\alpha_s=0.2$

$\simeq .40$ for $\alpha_s=0.15$



- order of magnitude correct
- adding naive running of the coupling $\alpha_s(Q^2)$ entirely contradicts data

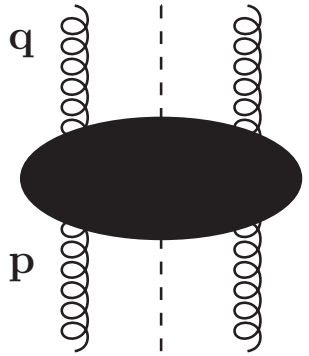
NLO corrections: scale invariant terms

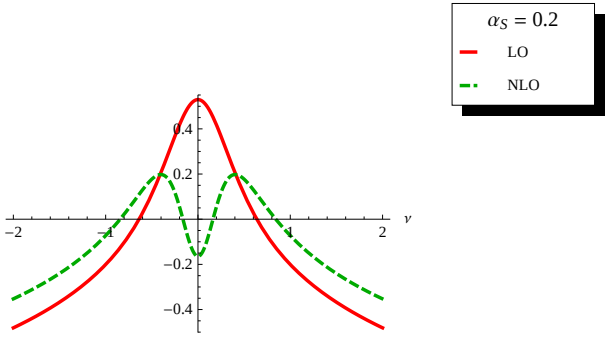
$$\int \frac{d^2\boldsymbol{p}}{\pi}\left(\frac{\boldsymbol{p}^2}{\boldsymbol{q}^2}\right)^{\gamma-1} K^{\text{sc.inv.}}(\boldsymbol{q},\boldsymbol{p}) = \chi^{\text{NLO}}(\gamma,\mu^2) =$$

$$= \bar{\alpha}_s(\mu^2)\left[\chi_0(\gamma) + \bar{\alpha}_s(\mu^2)\left(\chi_1(\gamma) - \frac{1}{2}\chi_0(\gamma)\chi_0'(\gamma)\right)\right]$$

$\uparrow$

NLO correction due to asymmetric s_0



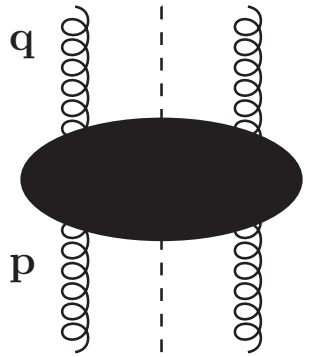


- negative NLO intercept in DIS kinematics
  $\lambda^{\text{NLO}}_{s.p.} \simeq -0.16$ for $\alpha_s = 0.2$
  $\lambda^{\text{NLO}}_{s.p.} \simeq .01$ for $\alpha_s = 0.15$
- numerical instability
- reason: large (double) logs for (anti-) collinear limit of external momoenta
- γ rep.: (triple/double) poles at $\gamma = 0, 1$

(anti-)collinear poles of the NLO eigenvalue

$$\chi^{\text{NLO}}(\gamma) \simeq \frac{\bar{\alpha}_s(1+\alpha_s a)}{\gamma} + \frac{\bar{\alpha}_s^2 b}{\gamma^2} + \frac{\bar{\alpha}_s(1+\alpha_s a)}{1-\gamma} + \frac{\bar{\alpha}_s^2 b}{(1-\gamma)^2} - \frac{\bar{\alpha}_s^2}{(1-\gamma)^3}$$

treatment: collinear factorization



- $\frac{1}{(1-\gamma)^3} \quad \Leftrightarrow \quad \boldsymbol{p}^2 \gg \boldsymbol{q}^2$
- choice $s/s_0 = 1/x_g$ dictated by DGLAP for $\boldsymbol{q}^2 \gg \boldsymbol{p}^2$

  $\boldsymbol{p}^2 \gg \boldsymbol{q}^2$ DGLAP suggests: $(1/x_g)^\omega (\boldsymbol{q}^2/(\boldsymbol{p}^2)^\omega$,

fix at LL : replace $\chi_0(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma)$

$$\rightarrow \quad \tilde{\chi}_0(\gamma,\omega) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma+\omega)$$

[Andersson, Gustafson, Samuelsson (1996)]

expand $\omega = \bar{\alpha}_s \tilde{\chi}_0(\gamma,\omega) = \bar{\alpha}_s \chi_0(\gamma) - \frac{\bar{\alpha}_s^2}{(1-\gamma)^3} + \mathcal{O}(\bar{\alpha}_s^3)$

(anti-)collinear poles of the NLO eigenvalue

$$\chi^{\text{NLO}}(\gamma) \simeq \frac{\bar{\alpha}_s(1+\alpha_s a)}{\gamma} + \frac{\bar{\alpha}_s^2 b}{\gamma^2} + \frac{\bar{\alpha}_s(1+\alpha_s a)}{1-\gamma} + \frac{\bar{\alpha}_s^2 b}{(1-\gamma)^2} - \frac{\bar{\alpha}_s^2}{(1-\gamma)^3}$$

coefficients a, b closely related to high energy limit of (N)LO DGLAP splitting kernels
$\longrightarrow$ resummation [Salam (1998)]

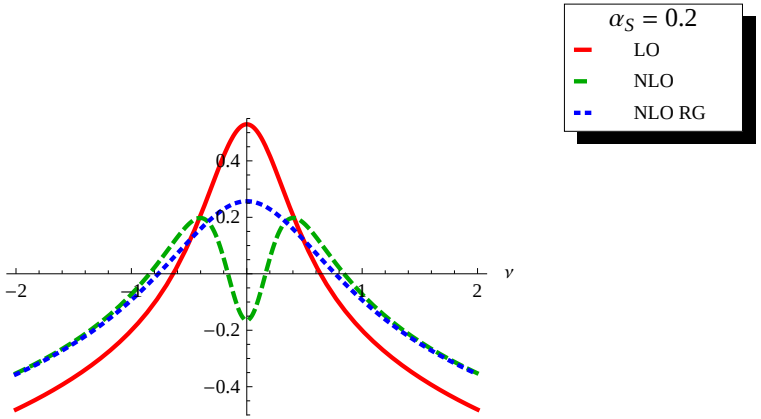
$$\begin{aligned}\omega &= \bar{\alpha}_s(1+A\bar{\alpha}_s)\left(2\psi(1) - \psi\left(\gamma + B\bar{\alpha}_s\right) - \psi\left(1-\gamma+\omega+B\bar{\alpha}_s\right)\right) \\ &= \bar{\alpha}_s(1+A\bar{\alpha}_s)\sum_{m=0}^{\infty}\left(\frac{1}{\gamma+m+B\bar{\alpha}_s} + \frac{1}{1-\gamma+m+\omega+B\bar{\alpha}_s} - \frac{2}{m+1}\right).\end{aligned}$$

disentangle complicated ω-dependence through 'all-pole-approximation' [Sabio Vera (2005)], [MH, Salas, Sabio Vera (2012)] $\longrightarrow$ resummed NLO eigenvalue

Note: do not add new information, only resum NLO BFKL divergenceses (in contrast to [Ciafaloni, Colferai, Salam, Stasto (2003), (2004), (2006), (2007)], [Altarelli, Ball, Forte (2003), (2004), (2005), (2008)])

(anti-)collinear poles of the NLO eigenvalue

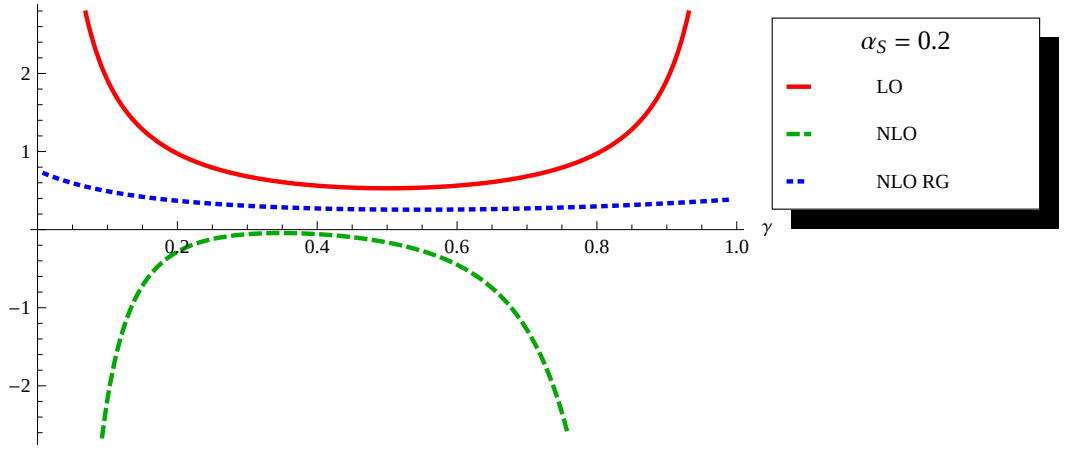
$$\chi(\gamma) = \bar{\alpha}_s \chi_0(\gamma) + \bar{\alpha}_s^2 \chi_1(\gamma) - \frac{1}{2}\bar{\alpha}_s^2 \chi_0{}'(\gamma)\chi_0(\gamma) + \chi_{\mathsf{RG}}(\bar{\alpha}_s, \gamma, a, b),$$

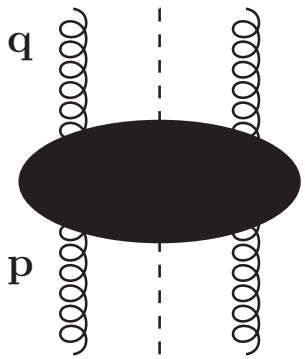


with

$$\chi_{\mathsf{RG}}(\bar{\alpha}_s, \gamma, a, b) = \bar{\alpha}_s(1 + a\bar{\alpha}_s)\left(\psi(\gamma) - \psi(\gamma - b\bar{\alpha}_s)\right) - \frac{\bar{\alpha}_s^2}{2}\psi''(1-\gamma) - b\bar{\alpha}_s^2 \frac{\pi^2}{\sin^2(\pi\gamma)}$$
$$+ \frac{1}{2}\sum_{m=0}^{\infty}\left(\gamma - 1 - m + b\bar{\alpha}_s - \frac{2\bar{\alpha}_s(1 + a\bar{\alpha}_s)}{1 - \gamma + m} + \sqrt{(\gamma - 1 - m + b\bar{\alpha}_s)^2 + 4\bar{\alpha}_s(1 + a\bar{\alpha}_s)}\right).$$

Weak k_T ordering at NLO





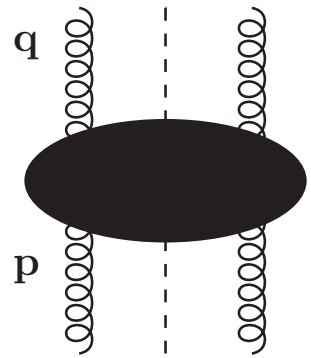
asymmetric scale choice s/s_0 ➡ $K(\boldsymbol{q},\boldsymbol{q}) = \int \frac{d\gamma}{2\pi i} \frac{1}{\boldsymbol{q}^2} \left(\frac{\boldsymbol{q}^2}{\boldsymbol{p}^2}\right)^{\gamma} \chi(\gamma)$:

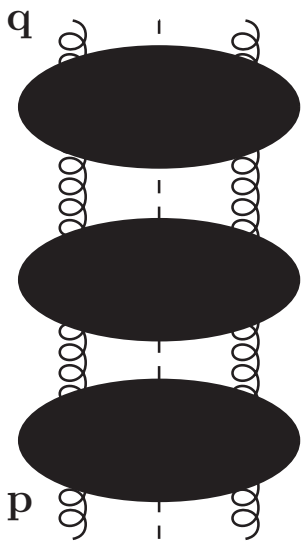
- breaks at NLO the symmetry $\boldsymbol{q} \leftrightarrow \boldsymbol{p}$
- $\boldsymbol{q}^2 > \boldsymbol{p}^2$ in average ➡ weak ordering ➡ IR stability

running coupling corrections with external scale dependence

$$\int \frac{d\gamma}{2\pi i}\frac{1}{\boldsymbol{q}^2}\left(\frac{\boldsymbol{q}^2}{\boldsymbol{p}^2}\right)^{\gamma}\bar{\alpha}_s\chi_0(\gamma)\left(-\frac{\alpha_s\beta_0}{4\pi}\ln\frac{|\boldsymbol{q}||\boldsymbol{p}|}{\mu^2}\right)$$

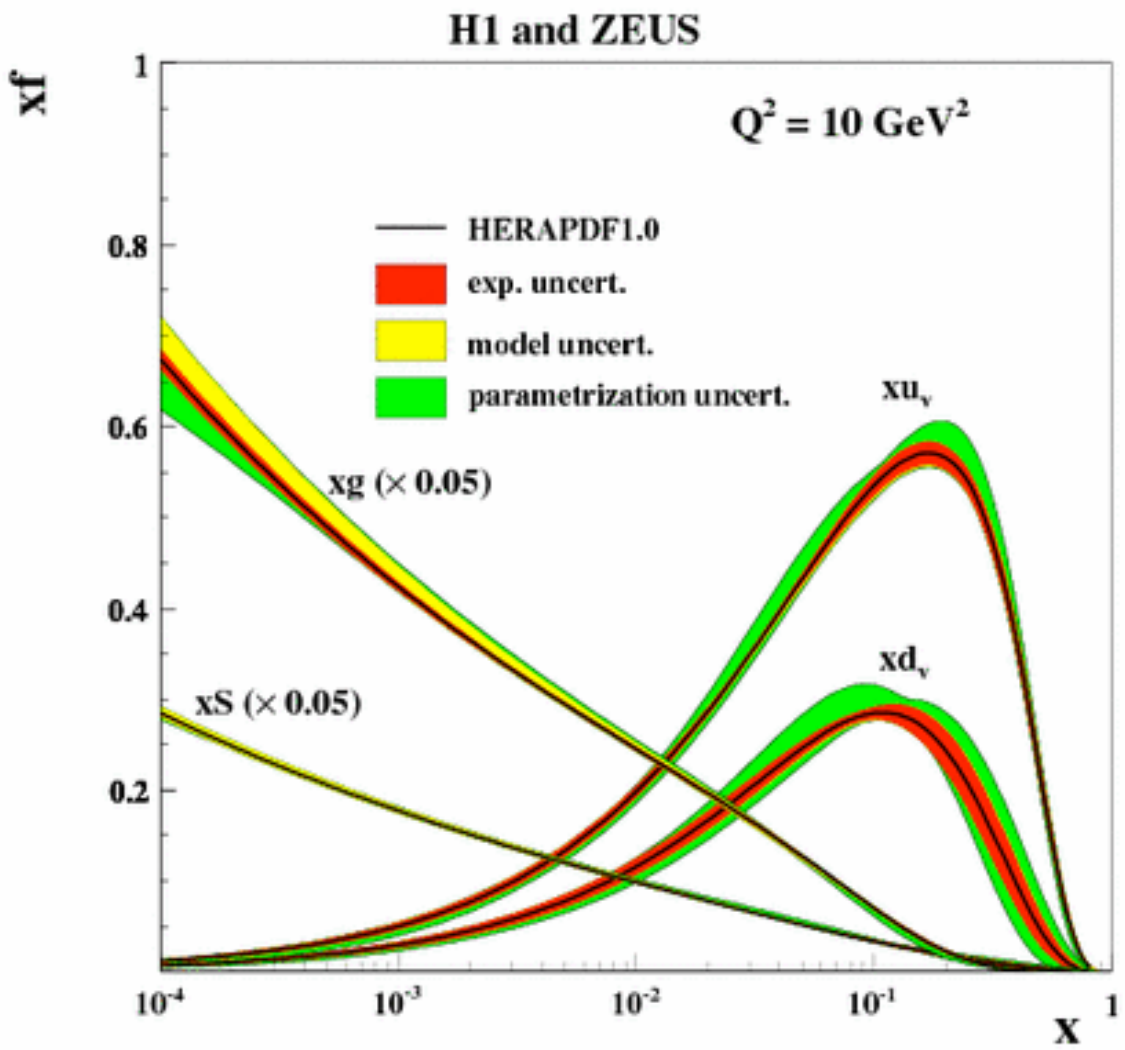
do not exponeniate → treated as pure NLO correction





NLO resummed Green's function:

$$f_{\text{BFKL}}\left(\frac{1}{x_g},\boldsymbol{q}^2,\boldsymbol{p}^2\right)=\int\frac{d\gamma}{2\pi^2 i}\frac{1}{\boldsymbol{q}^2}\left(\frac{\boldsymbol{q}^2}{\boldsymbol{p}^2}\right)^{\gamma}x_g^{-\chi(\gamma)}$$
$$\left[1-\ln\left(\frac{1}{x_g}\right)\frac{\bar{\alpha}_s^2\beta_0\chi_0(\gamma)}{4N_c}\ln\frac{|\boldsymbol{q}||\boldsymbol{p}|}{\mu^2}\right]$$


H1 and ZEUS
xf
$Q^2 = 10\ GeV^2$
HERAPDF1.0
exp. uncert.
model uncert.
parametrization uncert.
xu_v
xd_v
xg (× 0.05)
xS (× 0.05)
1
0.8
0.6
0.4
0.2
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
x