2 loop Matching Coefficients in RI/SMOM schemes: Improving the precision of quark masses

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Light Quark Masses in $\overline{\text{MS}}$

Can be determined non-perturbatively through the use of Lattice Simulations of QCD.

Dimensional Regularization is not viable in latttice simulations
Renormalization on the lattice can be done through
regularization independent schemes

This needs to be brought back to MS schemes

We do this by perturbativelly calculating the
Renormalization Constants in both schemes

$$m^{\overline{\text{MS}}} = C_{RI}^{\overline{\text{MS}}} m^{RI} \qquad C_{RI}^{\overline{\text{MS}}} = \frac{Z_m^{\overline{\text{MS}}}}{Z_m^{RI}}$$

Masses and Errors

RBC/UKQCD Coll.

$$m_{ud}^{\overline{\rm MS}}(2GeV) = 3.72(0.16)_{stat}(0.18)_{syst}(0.33)_{ren}MeV$$

$$m_{s}^{\overline{\rm MS}}(2GeV) = 107.3(4.4)_{stat}(4.9)_{syst}(9.7)_{ren}MeV$$

C. Allton et al.

Dominant error comes from renormalization constants. ~10%

More than 60% of the total error.

Regularization Invariant Schemes

RI Momentum Subtraction Scheme (RI/MOM)

Remove UV divergences at certain momentum (subtraction point)

Define the scheme in PT, uniquely fixes renormalization constants for the QCD parameters $(m_R = Z_m^x m_0, \Psi_R = \sqrt{Z_2^x}\Psi_0, \dots)$

RI/MOM

$$\lim_{m_R\to 0} \frac{1}{12 m_R} \mathrm{Tr}[S_R^{-1}(p)]\Big|_{p^2=-\mu^2} = 1 \qquad \lim_{m_R\to 0} \frac{1}{48} \mathrm{Tr}[\gamma^\mu \frac{\partial S_R^{-1}(p)}{\partial p^\mu}]\Big|_{p^2=-\mu^2} = 1$$

RI'/MOM

$$\lim_{m_R\to 0} \frac{1}{12 m_R} \mathrm{Tr}[S_R^{-1}(p)]\Big|_{p^2=-\mu^2} = 1 \qquad \lim_{m_R\to 0} \frac{1}{12 p^2} \mathrm{Tr}[S_R^{-1}(p) \not{p}]\Big|_{p^2=-\mu^2} = 1$$

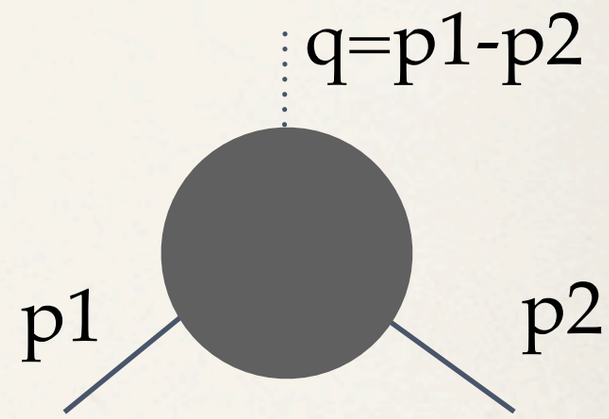
RI/MOM

Ward Identities allow us to relate renorm. condition on S_R to $\Lambda_{O,R}$

$\Lambda_{O,R}$ are the renorm. amputated Greens function for Bilinear Operators

$$O = \bar{q}\Gamma q'$$

$$O_R = Z_O O, \Lambda_{O,R} = \frac{Z_O}{Z_q}\Lambda_{O,b}$$



$$\frac{1}{N}\mathrm{Tr}[\Lambda_{O,R}(p1,p2)D_O]\Big|_{\text{mom. conf.}} = 1$$

We will study... $O = \{\mathbf{1}, \gamma^\mu, \gamma_5, \gamma^\mu\gamma_5\}$

Ward Identities lead to relations among renorm. constants

$$Z_{\gamma^\mu\gamma_5} = Z_{\gamma^\mu} = 1 \qquad Z_{\gamma_5} = Z_{\mathbf{1}} = 1/Z_m$$

and renormalization condition on $\Lambda_{O,R}$

RI/MOM scheme

Renormalization Conditions on $\Lambda_{O,R}$ for RI/MOM scheme

$$\lim_{m_R\to 0}\frac{1}{48}\mathrm{Tr}[\gamma^\mu\Lambda^\mu_{V,R}(p1,p2)]\Big|_{asym}=1 \qquad \lim_{m_R\to 0}\frac{1}{12}\mathrm{Tr}[\gamma_5\Lambda^\mu_{P,R}(p1,p2)]\Big|_{asym}=1$$

$$\lim_{m_R\to 0}\frac{1}{12}\mathrm{Tr}[\Lambda^\mu_{S,R}(p1,p2)]\Big|_{asym}=1 \qquad \lim_{m_R\to 0}\frac{1}{48}\mathrm{Tr}[\gamma_5\gamma^\mu\Lambda^\mu_{A,R}(p1,p2)]\Big|_{asym}=1$$

Asymmetrical momentum configuration

$$p1^2=p2^2=-\mu^2,\ \ \mu^2>0,\ q=0$$

Matching Factors

$$m_R^{\overline{\rm MS}} = C_m^{\rm RI/MOM} m_R^{\rm RI/MOM}$$

$C_m^{\rm RI/MOM}$ Known up to 3 loops G. Martinelli et al.; Franco, Lubicz; Chetyrkin, Retey; Gracey

Numerically with $n_f = 3 \quad \alpha_s/\pi = 0.1$

$$C_m^{\rm RI/MOM} = 1 - 0.1333.. - 0.0759 - 0.0557$$

$$C_m^{\rm RI'/MOM} = 1 - 0.1333.. - 0.0816.. - 0.0603...$$

Poor Convergence: ~13%, ~8%, ~6%

leads to a large uncertainty in $\overline{\rm MS}$ quark masses

RI/SMOM schemes

Find a more convergent scheme:

Symmetric Subtraction Point more favorable for lattice simullations

$$p1^2 = p2^2 = q^2 = -\mu^2, \quad \mu^2 > 0,$$

Suppresses contamination from unwanted infrared effects

For Asymm. point effects from chiral sym. break. vanish slowly, $1/p^2$

while for Symm. point such effects vanishes much faster, $1/p^6$

It is important to check if these SMOM schemes converge faster...

RI/SMOM schemes

RI/SMOM

$$\lim_{m_R\to 0}\frac{1}{12q^2}\mathrm{Tr}[q^\mu\Lambda^\mu_{V,R}(p1,p2)\not{q}]\Big|_{sym}=1 \qquad \lim_{m_R\to 0}\frac{1}{12}\mathrm{Tr}[\Lambda^\mu_{S,R}(p1,p2)]\Big|_{sym}=1$$

$$\lim_{m_R\to 0}\frac{1}{12}\mathrm{Tr}[\gamma_5\Lambda^\mu_{P,R}(p1,p2)]\Big|_{sym}=1 \qquad \lim_{m_R\to 0}\frac{1}{12q^2}\mathrm{Tr}[q^\mu\Lambda^\mu_{A,R}(p1,p2)\gamma_5\not{q}]\Big|_{sym}=1$$

Similarly we can use a different set of projectors

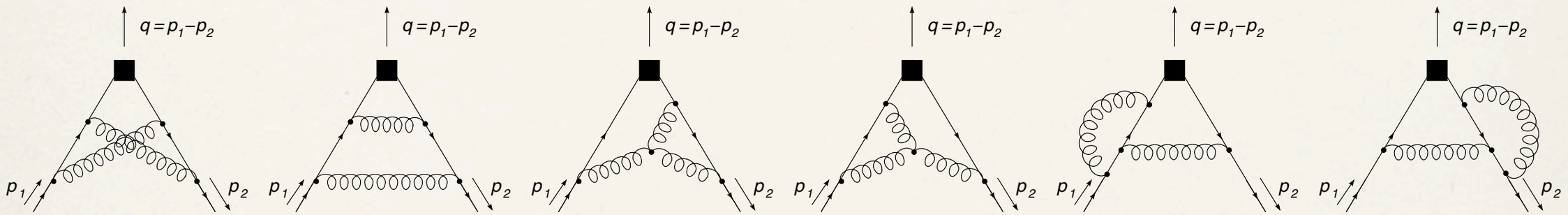
RI/SMOM$_{\gamma^\mu}$

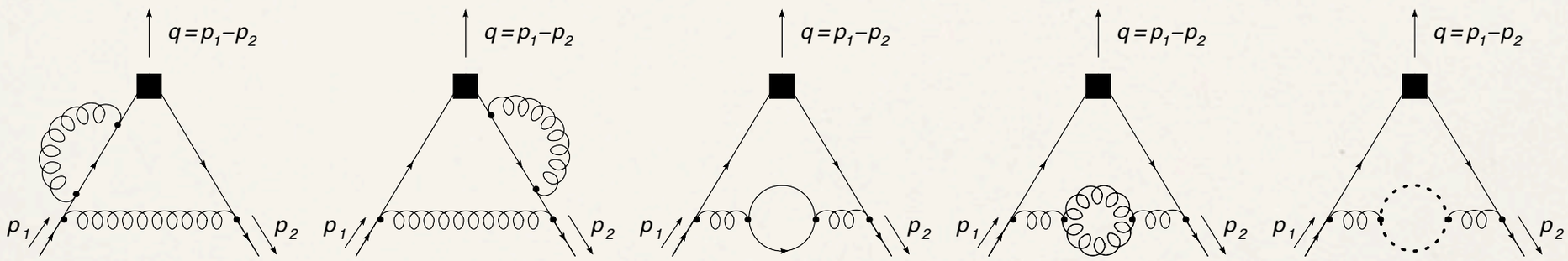
$$\lim_{m_R\to 0}\frac{1}{48}\mathrm{Tr}[\Lambda^\mu_{V,R}(p1,p2)\gamma_m]\Big|_{sym}=1 \qquad \lim_{m_R\to 0}\frac{1}{48}\mathrm{Tr}[\Lambda^\mu_{A,R}(p1,p2)\gamma_5\gamma_m]\Big|_{sym}=1$$

NNLO Calculation for RI/SMOM

LO, NLO 1 diagram each (C. Sturm et al 09)

NNLO: 12 diagrams





Calculation: Completely Automated

IBP/Laporta's algorithm

reduction to 7 master integrals

Computation of master Scalars Integrals

Results at NNLO

Analytical results are rather lengthy

Numerical results with $nf = 3 \quad \alpha_s/\pi = 0.1$

$$C_m^{\mathrm{RI/SMOM}} = 1 - 0.0161380... - 0.00660442...$$

in agreement with Jaeger, Gorbahn

$$C_m^{\mathrm{RI/SMOM}_{\gamma_\mu}} = 1 - 0.049471... - 0.0228421...$$

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$$C_m^{\mathrm{RI/MOM}} = 1 \quad 0.1333... \quad 0.0759... \quad 0.0557...$$

Conclusions

Matching Factors are now known up NNLO for the two RI/SMOM schemes

This results can be used to convert quark masses to MS scheme once one has masses in RI/SMOM schemes

We also computed the mass anomalous dimensions to three loops in RI/SMOM which allows one to evolve the masses to any scale

The use of the RI/SMOM schemes will reduce the systematic error and improve precision of light quark mass determinations from lattice simulations