# $K \to \pi$ matrix elements of the chromomagnetic operator on the lattice


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See also a poster

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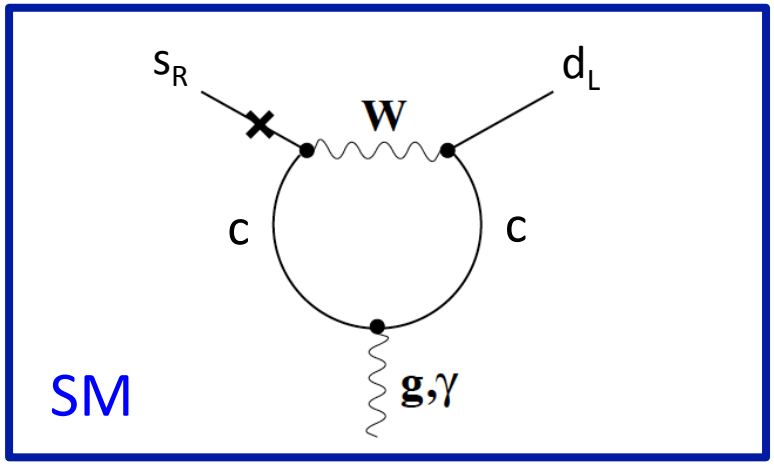
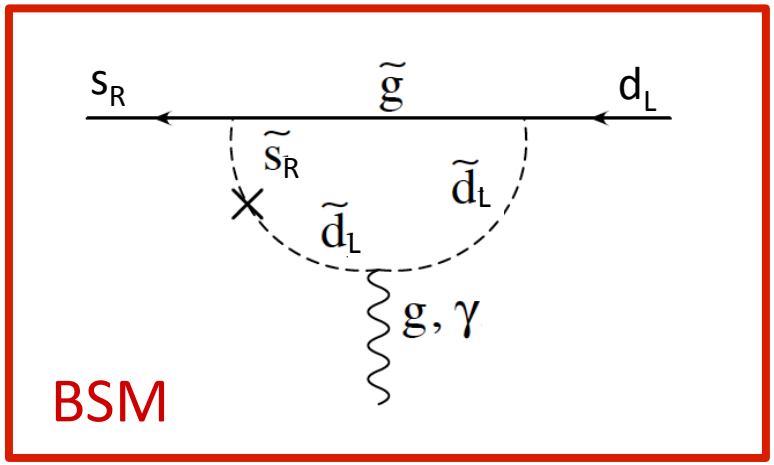
The effective $\Delta S=1$ Hamiltonian of dim=5 contains four magnetic operators:

$$H_{d=5}^{\Delta S=1} = \sum_{i=\pm}\left(C_\gamma^i Q_\gamma^i + C_g^i Q_g^i\right) + h.c.$$

$$Q_\gamma^\pm = \frac{Q_d e}{16\pi^2}\left(\bar{s}_L \sigma^{\mu\nu} F_{\mu\nu} d_R \pm \bar{s}_R \sigma^{\mu\nu} F_{\mu\nu} d_L\right)$$

$$Q_g^\pm = \frac{g}{16\pi^2}\left(\bar{s}_L \sigma^{\mu\nu} G_{\mu\nu} d_R \pm \bar{s}_R \sigma^{\mu\nu} G_{\mu\nu} d_L\right)$$




| | $C_{SM}$ | $C_{BSM}$ | For $M_{NP}$ ~ 1 TeV : |
|---|---|---|---|
| • Dim = 5 ➡ | $\sim 1/M_W$ | $\sim 1/M_{NP}$ | $\sim 10^{-3} / 10^{-2} \sim 10^{-1}$ |
| • $\Delta F \neq 0$ ➡ | $\sim \alpha_w(M_W)$ | $\sim \alpha_s(M_{NP})$ | $\sim 0.09 / 0.03 \sim 3$ |
| • LR chirality ➡ | $\sim m_s / M_W$ | $\sim \delta_{LR}$ | Model dep. / $10^{-3} \sim 1$ ? |

# The EMO

The matrix element is proportional to the tensor form factor, parameterized as:

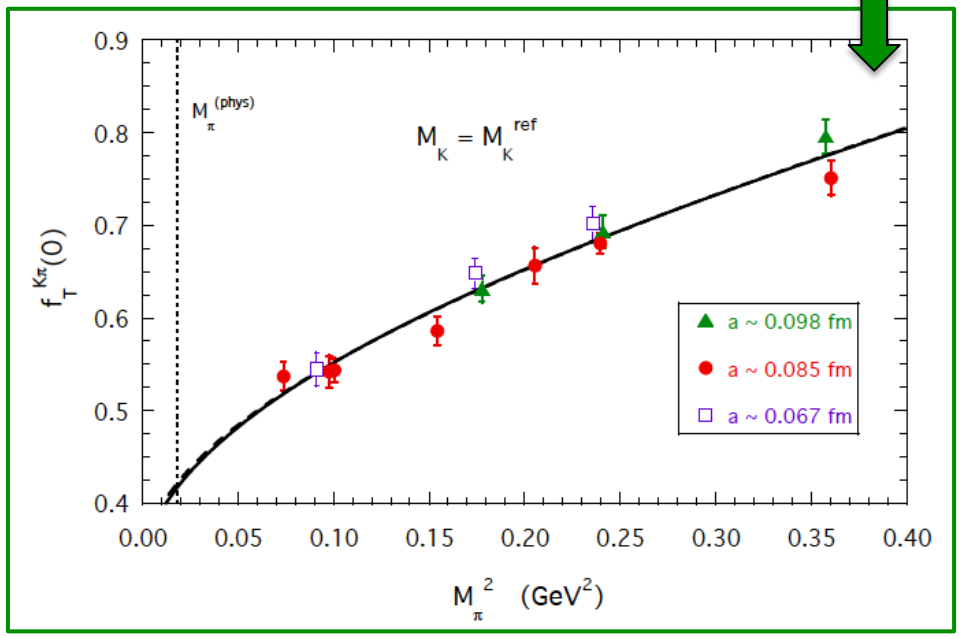
$$\langle \pi^0 | Q_\gamma^+ | K^0 \rangle = \frac{Q_d e}{16\pi^2} F_{\mu\nu} \langle \pi^0 | \bar{s}\sigma^{\mu\nu} d | K^0 \rangle = i \frac{Q_d e \sqrt{2}}{16\pi^2 M_K} p_\pi^\mu p_K^\nu F_{\mu\nu} B_T R_T(q^2)$$

- It has been already computed with **Lattice QCD**

Nf=0: Becirevic, V.L., Martinelli, Mescia, PLB 501 (2001) 98

Nf=2: ETMC, Baum, V.L., Martinelli, Orifici, Simula, PRD84 (2011) 074503

$$B_T = 0.655(24)$$




- It contributes to the rare kaon decay

$BR(K_L \to \pi^0 e^+ e^-)_{EMO} \sim (C_\gamma B_T)^2$

# The CMO

Several matrix elements are of phenomenological interest:

- $\langle \pi^0 | Q_g^+ | K^0 \rangle = -\frac{11}{32\sqrt{2}\pi^2} \frac{M_K^2 (p_K \cdot p_\pi)}{m_s + m_d} B_{CMO}^{K\pi}$
- $\langle \pi^+\pi^- | Q_g^- | K^0 \rangle = i \frac{11}{32\pi^2} \frac{M_K^2 M_\pi^2}{f_\pi (m_s + m_d)} B_{CMO}^{K2\pi}$
- $\langle \pi^+\pi^+\pi^- | Q_g^+ | K^+ \rangle = -\frac{11}{16\pi^2} \frac{M_K^2 M_\pi^2}{f_\pi^2 (m_s + m_d)} B_{CMO}^{K3\pi}$

Relevant for:

$K^0 - \overline{K}^0$ mixing (long dist.)
X.-G. He et al., PRD61 (2000) 071701

$\varepsilon'/\varepsilon$, $\Delta I=1/2$
Buras et al., NPB 566 (2000) 3

$\not{CP}$ in $K \to 3\pi$
D'Ambrosio, Isidori, Martinelli, PLB 480 (2000) 164

At LO in ChPT, the various B-parameters are all equal:

$$Q_g^\pm = \frac{11}{256\pi^2} \frac{f_\pi^2 M_K^2}{m_s + m_d} B_{CMO} \left[ U(D_\mu U^\dagger)(D^\mu U) \pm (D_\mu U^\dagger)(D^\mu U) U^\dagger \right]_{23}$$

$B_{CMO} \simeq$ 0.1-0.5 in the chiral quark model [Bertolini et al., NPB 449 (1995) 197 ]

→ We computed $\langle \pi^\pm | Q_g^+ | K^\pm \rangle$

# Renormalization

- A challenging aspect in the study of the CMO is its renormalization pattern, which involves the mixing among 13 operators (off-shell), 2 of which of lower dimension

- For on-shell matrix elements, the mixing allowed by the symmetries of the action is:

$$\hat{O}_{CM} = Z_{CM}\left[ O_{CM}^{bare} - \left( \frac{c_{13}}{a^2} + c_2(m_s^2 + m_d^2) + c_3 m_s m_d \right) S - \frac{c_{12}}{a}(m_s + m_d) P - c_4 O_{DD} \right]$$

Power divergences

where:

$$O_{CM} = g\, \bar{s}\, \sigma_{\mu\nu} G_{\mu\nu} d \quad , \quad S = \bar{s}\, d \quad , \quad O_{DD} = \bar{s}\, \overleftarrow{D}_\mu \overrightarrow{D}_\mu d$$

$$+ \quad P = \bar{s}(i\gamma_5)d$$

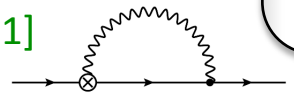
Wrong Parity
[Twisted Mass]

# Renormalization

$$\hat{O}_{CM} = Z_{CM}\left[ O_{CM}^{bare} - \left( \frac{c_{13}}{a^2} + c_2(m_s^2 + m_d^2) + c_3 m_s m_d \right) S - \frac{c_{12}}{a}(m_s + m_d) P - c_4 O_{DD} \right]$$

- A perturbative determination of power divergent coefficients may be not reliable

[Maiani, Martinelli, Sachrajda, NPB 368 (1992) 281]



$$c^{NP} \sim \frac{1}{a} e^{-\frac{1}{\beta_0 g^2}} \sim \Lambda_{QCD}$$

- We must impose non-perturbative subtraction conditions:

1

$$\lim_{m_s, m_d \to 0} \langle \pi(0) | O_{CM}^{sub} | K(0) \rangle = \lim_{m_s, m_d \to 0} \langle \pi(0) | \left( O_{CM}^{bare} - \frac{c_{13}}{a^2} S - c_4^{PT} O_{DD} \right) | K(0) \rangle = 0$$

suggested by the chiral expansion: $\langle \pi^{\pm}(0) | Q_g^{+} | K^{\pm}(0) \rangle \propto p_K \cdot p_\pi = M_K M_\pi$

2

$$\langle 0 | O_{CM}^{sub} | K \rangle = \langle 0 | \left( O_{CM}^{bare} - \frac{c_{13}}{a^2} S - \frac{c_{12}}{a}(m_s + m_d) P \right) | K \rangle = 0$$

← absence of parity violations

# Perturbation theory

[ see Marios Costa's poster ]

$Z_{CM}$ and the $c_{2,3,4}$ of the dim=5 operators are determined in PT

Off shell external particles in order to avoid IR divergences

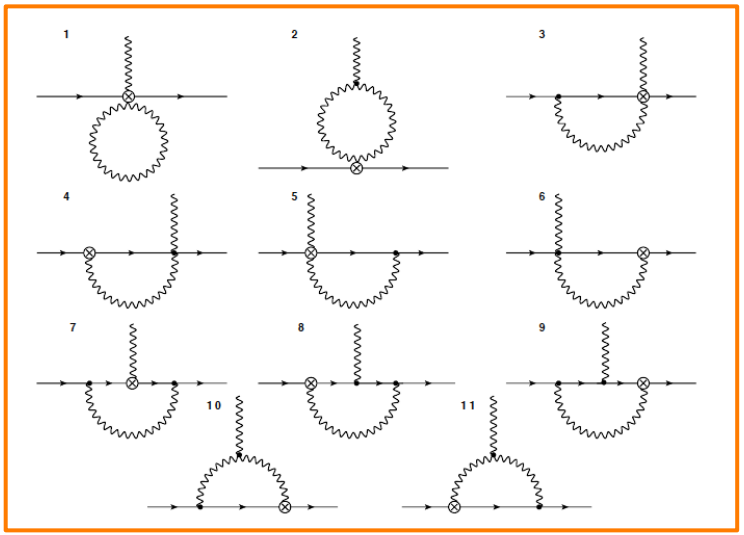
Gauge noninvariant operators, which are BRST invariant and vanish by the equation of motion, must be also taken into account

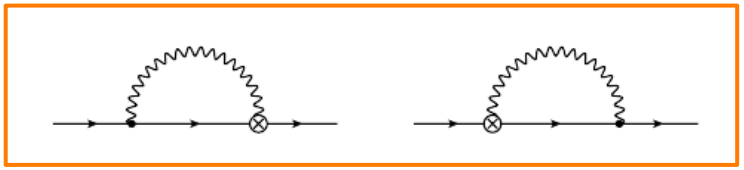
**13 operators**

$$
\begin{aligned}
\mathcal{O}_1 &= g_0\,\overline{\psi}_s\sigma_{\mu\nu}G_{\mu\nu}\psi_d \\
\mathcal{O}_2 &= (m_d^2+m_s^2)\overline{\psi}_s\psi_d \\
\mathcal{O}_3 &= m_d\,m_s\overline{\psi}_s\psi_d \\
\mathcal{O}_4 &= \overline{\psi}_s\overleftarrow{D}_\mu\overrightarrow{D}_\mu\psi_d \\
\mathcal{O}_5 &= \overline{\psi}_s(-\overleftarrow{\not{D}}+m_s)(\overrightarrow{\not{D}}+m_d)\psi_d \\
\mathcal{O}_6 &= \overline{\psi}_s(\overrightarrow{\not{D}}+m_d)^2\psi_d+\overline{\psi}_s(-\overleftarrow{\not{D}}+m_s)^2\psi_d \\
\mathcal{O}_7 &= m_s\overline{\psi}_s(\overrightarrow{\not{D}}+m_d)\psi_d+m_d\overline{\psi}_s(-\overleftarrow{\not{D}}+m_s)\psi_d \\
\mathcal{O}_8 &= m_d\overline{\psi}_s(\overrightarrow{\not{D}}+m_d)\psi_d+m_s\overline{\psi}_s(-\overleftarrow{\not{D}}+m_s)\psi_d \\
\mathcal{O}_9 &= \overline{\psi}_s\overleftarrow{\not{\partial}}(\overrightarrow{\not{D}}+m_d)\psi_d-\overline{\psi}_s(-\overleftarrow{\not{D}}+m_s)\overrightarrow{\not{\partial}}\psi_d \\
\mathcal{O}_{10} &= \overline{\psi}_s\overrightarrow{\not{\partial}}(\overrightarrow{\not{D}}+m_d)\psi_d-\overline{\psi}_s(-\overleftarrow{\not{D}}+m_s)\overleftarrow{\not{\partial}}\psi_d \\
\mathcal{O}_{11} &= i\,r_d\,\overline{\psi}_s\gamma_5(\overrightarrow{\not{D}}+m_d)\psi_d+i\,r_s\,\overline{\psi}_s(-\overleftarrow{\not{D}}+m_s)\gamma_5\psi_d \\
\mathcal{O}_{12} &= i\,(r_d\,m_d+r_s\,m_s)\overline{\psi}_s\gamma_5\psi_d \\
\mathcal{O}_{13} &= \overline{\psi}_s\,\psi_d\,,
\end{aligned}
$$

# Perturbation theory

[ see Marios Costa's poster ]



$$Z_1^{L,\overline{\rm MS}} = 1 + \frac{g^2}{16\,\pi^2}\left(N_c\left(-7.9438 + \frac{1}{2}\log\left(a^2\,\bar{\mu}^2\right)\right) + \frac{1}{N_c}\left(4.4851 - \frac{5}{2}\log\left(a^2\,\bar{\mu}^2\right)\right)\right)$$

$$Z_2^{L,\overline{\rm MS}} = \frac{g^2\,C_F}{16\,\pi^2}\left(4.5370 + 6\,\log\left(a^2\,\bar{\mu}^2\right)\right)$$

$$Z_3^{L,\overline{\rm MS}} = 0$$

$$Z_4^{L,\overline{\rm MS}} = 0$$

$$Z_{12}^{L,\overline{\rm MS}} = -\frac{1}{a}\,\frac{g^2\,C_F}{16\,\pi^2}\,(-3.2020)$$

$$Z_{13}^{L,\overline{\rm MS}} = \frac{1}{a^2}\,\frac{g^2\,C_F}{16\,\pi^2}\,(36.0613)$$

$$Z_5^{L,\overline{\rm MS}} = \frac{g^2}{16\,\pi^2}\left(N_c\left(4.2758 - \frac{3}{2}\log\left(a^2\,\bar{\mu}^2\right)\right) + \frac{1}{N_c}\left(-3.7777 + 3\,\log\left(a^2\,\bar{\mu}^2\right)\right)\right)$$

$$Z_6^{L,\overline{\rm MS}} = 0 \qquad Z_7^{L,\overline{\rm MS}} = -\frac{Z_5^{L,\overline{\rm MS}}}{2}$$

$$Z_8^{L,\overline{\rm MS}} = \frac{g^2\,C_F}{16\,\pi^2}\,(-3.7760) \qquad Z_{10}^{L,\overline{\rm MS}} = \frac{g^2\,C_F}{16\,\pi^2}\left(3.7777 - 3\,\log\left(a^2\,\bar{\mu}^2\right)\right)$$

$$Z_9^{L,\overline{\rm MS}} = \frac{Z_5^{L,\overline{\rm MS}}}{2} \qquad Z_{11}^{L,\overline{\rm MS}} = \frac{1}{a}\,\frac{g^2\,C_F}{16\,\pi^2}\,(-3.2020)$$

| | $O_{CM}$ | $(m_s^2 + m_d^2)S$ | $m_s m_d S$ | $O_{DD}$ | $(m_s + m_d)P/a$ | $S/a^2$ |
|---|---|---|---|---|---|---|
| $\beta$ | $Z_{CM}$ | c2 | c3 | c4 | c12 | c13 |
| 1.90 | 1.78 | 0.15 | 0 | 0 | 0.085 | 0.96 |
| 1.95 | 1.75 | 0.10 | 0 | 0 | 0.083 | 0.94 |
| 2.10 | 1.68 | -0.04 | 0 | 0 | 0.077 | 0.87 |

Rather large 1-loop correction…

Red: $g_0^2 = 6/\beta$

Blue: $\tilde{g}^2 = g_0^2 / U_{PL}$

# The numerical simulation

- Nf = 2 + 1 + 1 dynamical quarks
- Twisted mass / Osterwalder-Seiler fermions
- Iwasaki gluon action
- a ≃ 0.089, 0.082, 0.062 fm
- Mpi ≃ 210 - 450 MeV

European Twisted Mass Collaboration

Same setup of

ETM Collaboration,

Carrasco et al.,

arXiv: 1403.4504

[hep-lat]

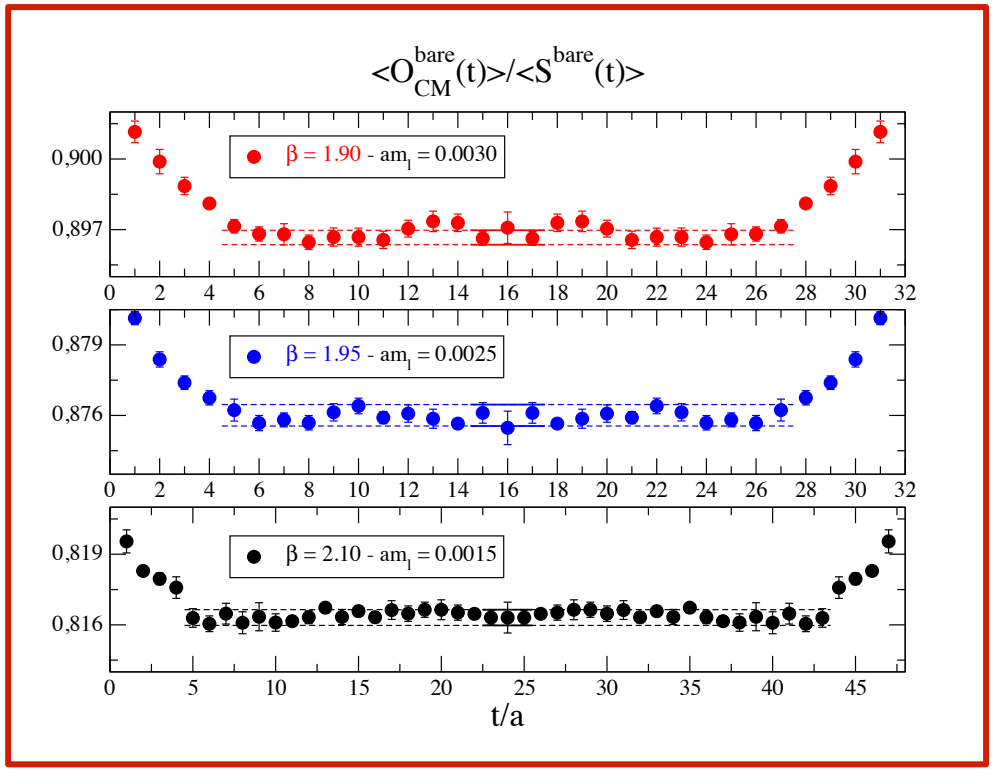
| ensemble | $\beta$ | $V/a^4$ | $a\mu_{sea} = a\mu_\ell$ | $a\mu_\sigma$ | $a\mu_\delta$ | $N_{cfg}$ | $a\mu_s$ |
|---|---|---|---|---|---|---|---|
| $A30.32$ | 1.90 | $32^3 \times 64$ | 0.0030 | 0.15 | 0.19 | 150 | 0.0145, 0.0185, 0.0225 |
| $A40.32$ | | | 0.0040 | | | 100 | |
| $A50.32$ | | | 0.0050 | | | 150 | |
| $A60.24$ | 1.90 | $24^3 \times 48$ | 0.0060 | 0.15 | 0.19 | 150 | |
| $A80.24$ | | | 0.0080 | | | 150 | |
| $A100.24$ | | | 0.0100 | | | 150 | |
| $B25.32$ | 1.95 | $32^3 \times 64$ | 0.0025 | 0.135 | 0.170 | 150 | 0.0141, 0.0180, 0.0219 |
| $B35.32$ | | | 0.0035 | | | 150 | |
| $B55.32$ | | | 0.0055 | | | 150 | |
| $B75.32$ | | | 0.0075 | | | 80 | |
| $B85.24$ | 1.95 | $24^3 \times 48$ | 0.0085 | 0.135 | 0.170 | 150 | |
| $D15.48$ | 2.10 | $48^3 \times 96$ | 0.0015 | 0.12 | 0.1385 | 60 | 0.0118, 0.0151, 0.0184 |
| $D20.48$ | | | 0.0020 | | | 100 | |
| $D30.48$ | | | 0.0030 | | | 100 | |

# NP determination of $c_{13}$

1) Ratio of matrix elements

$$\lim_{m_s,m_d\to 0}\left\langle \pi(0)\left| O_{CM}^{sub}\right| K(0)\right\rangle = \lim_{m_s,m_d\to 0}\left\langle \pi(0)\left|\left( O_{CM}^{bare} - \frac{c_{13}}{a^2} S\right)\right| K(0)\right\rangle = 0$$

$$c_4^{PT-1\ell} = 0$$

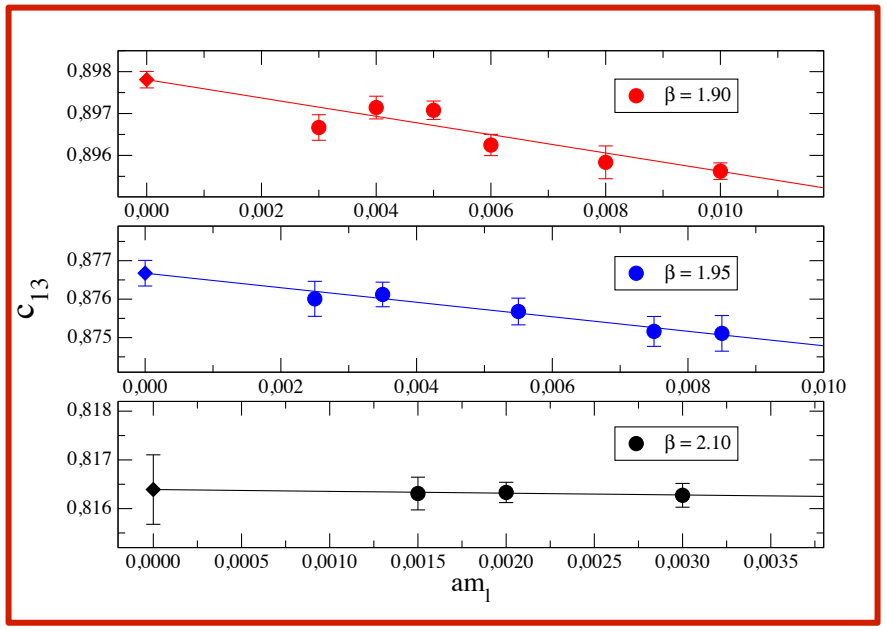



$$\frac{c_{13}}{a^2} = \lim_{m_s,m_d\to 0}\frac{\left\langle \pi(0)\left| O_{CM}^{bare}\right| K(0)\right\rangle}{\left\langle \pi(0)\left| S\right| K(0)\right\rangle}$$

Better than 1‰ accuracy

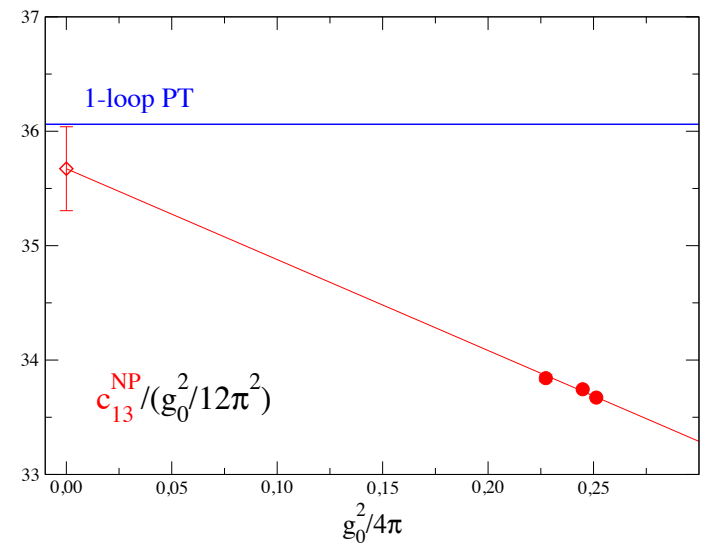
# NP determination of $c_{13}$

## 2) Extrapolation to the chiral limit




$$\frac{c_{13}}{a^2} = \lim_{m_s, m_d \to 0} \frac{\langle \pi(0) | O_{CM}^{bare} | K(0) \rangle}{\langle \pi(0) | S | K(0) \rangle}$$

$$c_{13}^{PT} / \left( g^2 / 12\pi^2 \right) = 36.0613 + \lambda_2 g^2 + \ldots$$




## 3) Results

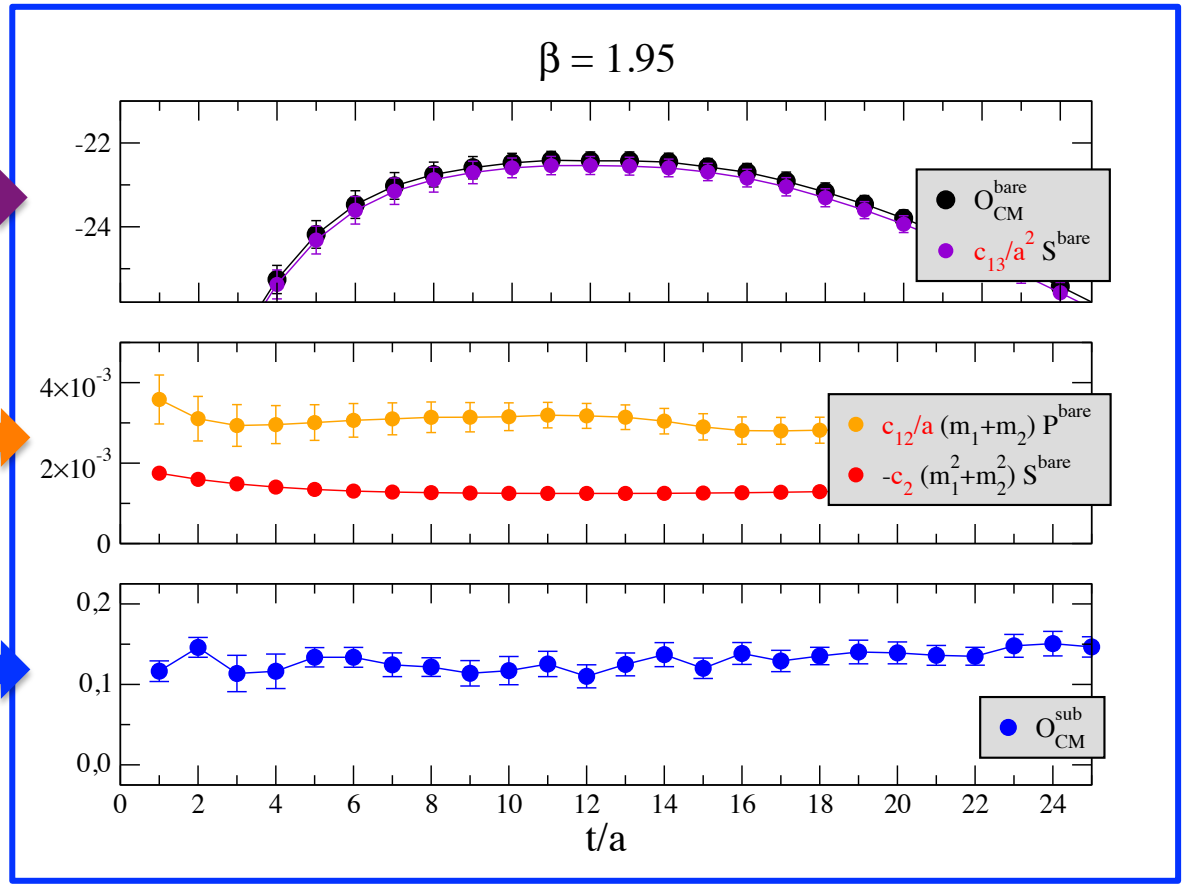
| β | $c_{13}$ [NP] | $c_{13}$ [PT-1$\ell$] |
|---|---|---|
| 1.90 | 0.8978 (2) | 0.96 |
| 1.95 | 0.8768 (3) | 0.94 |
| 2.10 | 0.8164 (7) | 0.87 |

# The "subtracted" CMO

~ 2 x $10^{1}$

~ 2 x $10^{-3}$

~ 2 x $10^{-1}$




$$Z_{CM}\langle\pi|O_{CM}^{sub}|K\rangle = Z_{CM}\langle\pi|\left(O_{CM}^{bare} - c_2(m_s^2+m_d^2)S - \frac{c_{13}}{a^2}S - \frac{c_{12}}{a}(m_s+m_d)P\right)|K\rangle$$

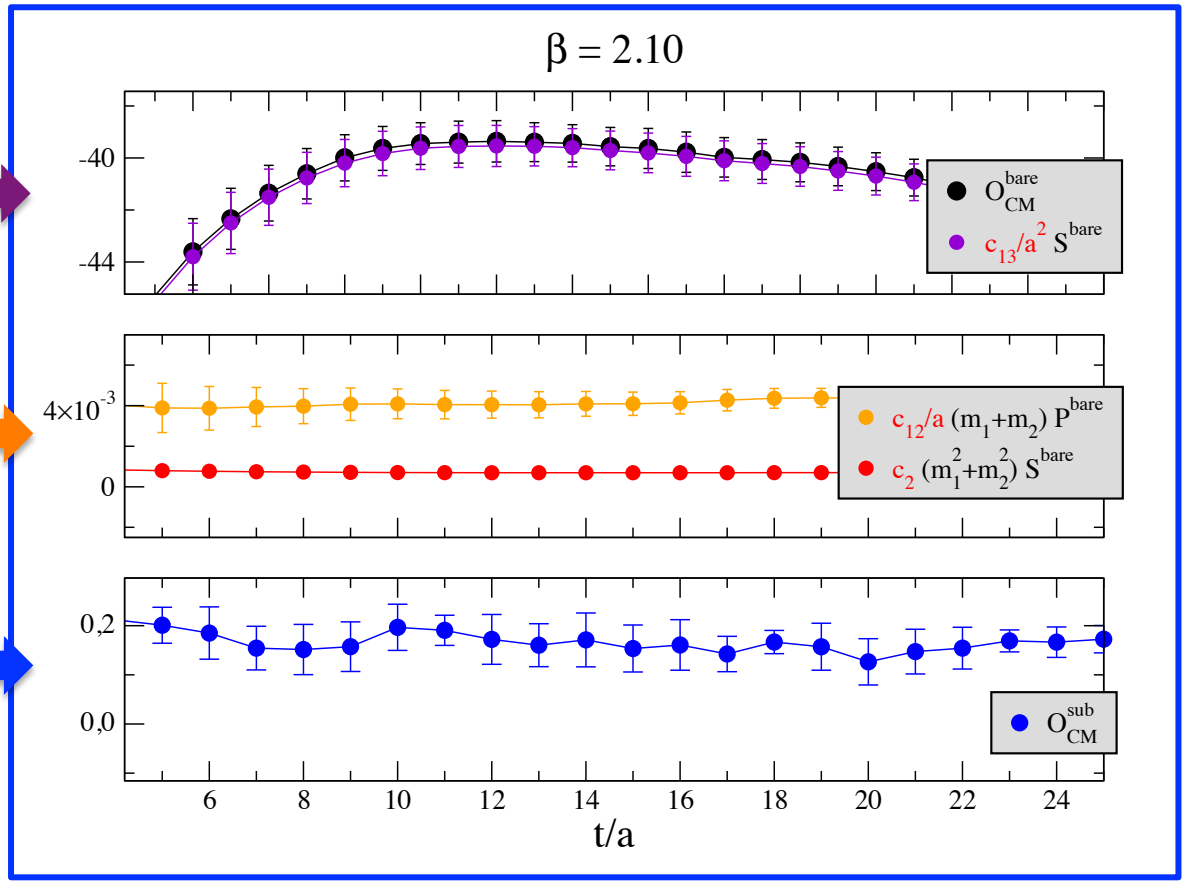
$$c_3^{PT-1\ell} = c_4^{PT-1\ell} = 0$$

# The "subtracted" CMO

~ 4 x $10^{1}$

~ 4 x $10^{-3}$

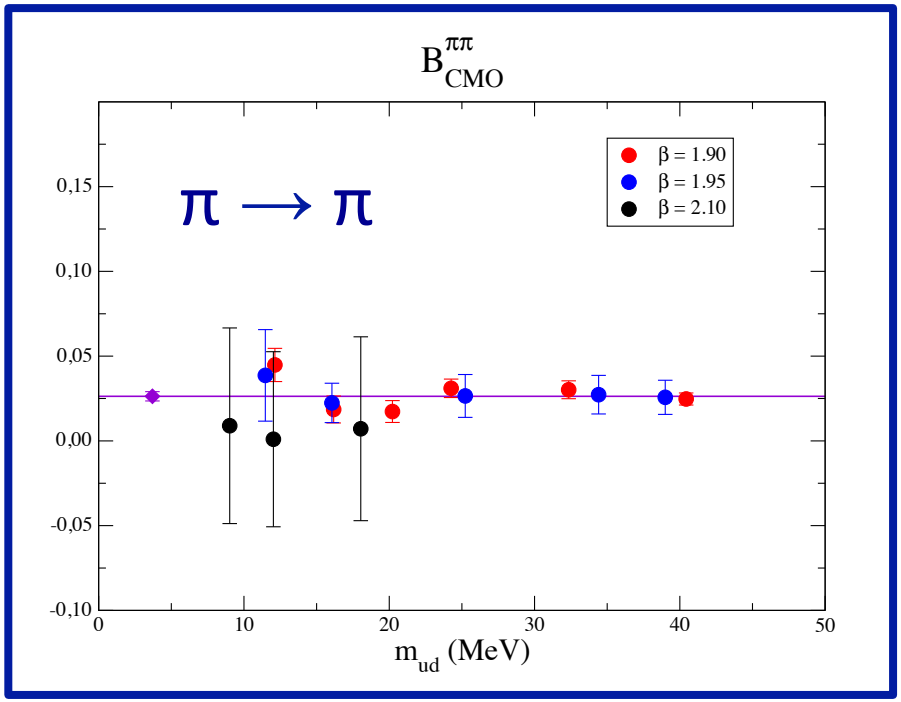
~ 2 x $10^{-1}$

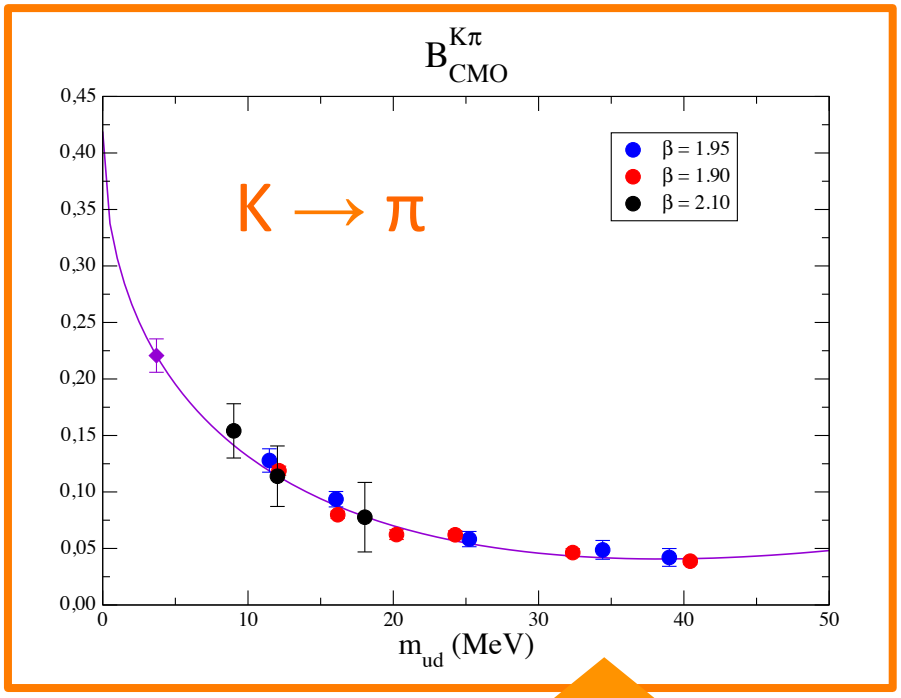



$$Z_{CM}\langle\pi|O_{CM}^{sub}|K\rangle = Z_{CM}\langle\pi|\left(O_{CM}^{bare} - c_2(m_s^2+m_d^2)S - \frac{c_{13}}{a^2}S - \frac{c_{12}}{a}(m_s+m_d)P\right)|K\rangle$$

$$c_3^{PT-1\ell} = c_4^{PT-1\ell} = 0$$

# Chiral extrapolation and results




$$Q_g^+ \propto B_{CMO}\left[U(D_\mu U^\dagger)(D^\mu U)+(D_\mu U^\dagger)(D^\mu U)U^\dagger\right]_{23}$$

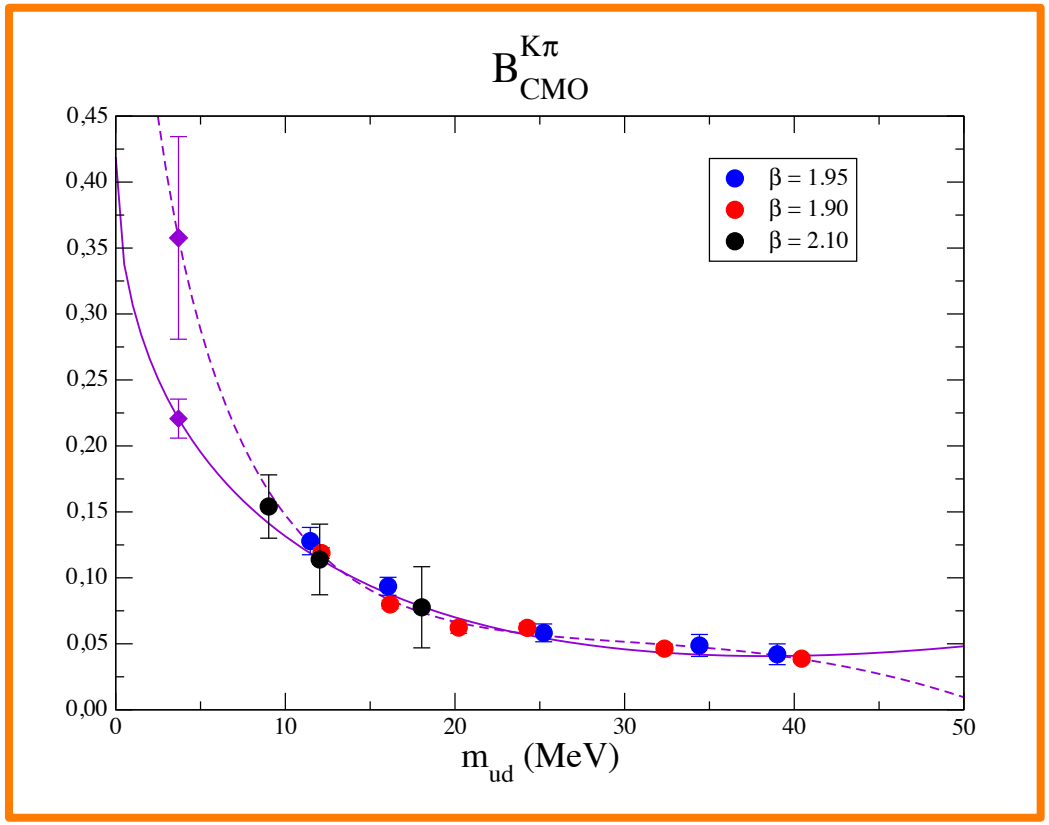
The parameter B is independent of quark masses (and momenta) up to NLO chiral corrections

For K→π higher order chiral corrections are large

# Chiral extrapolation and results

$$\left\langle \pi^+ \middle| \hat{Q}_g^+ \middle| K^+ \right\rangle \propto B_{CMO}^{LO} (p_K \cdot p_\pi) \left[ 1 + \alpha M_K^2 + \beta M_\pi^2 + \gamma (p_\pi \cdot p_K) + \ldots \right] =$$

$$\underset{\vec{p}_K = \vec{p}_\pi = 0}{=} B_{CMO}^{LO} M_K M_\pi \left[ \underbrace{\alpha' + \beta' m_l + \gamma' m_l^{1/2}}_{F1} + \ldots \right]$$




## Different chiral fits:

F1 → $B_{CMO}^{K\pi} = 0.22(2)$

F1 + $m^{3/2}$ → $B_{CMO}^{K\pi} = 0.36(8)$

F1 + $m^2$ → $B_{CMO}^{K\pi} = 0.34(7)$

$$B_{CMO} = 0.29(9)_{chiral}(6)_{PT} = 0.29(11)$$

# Phenomenological implications

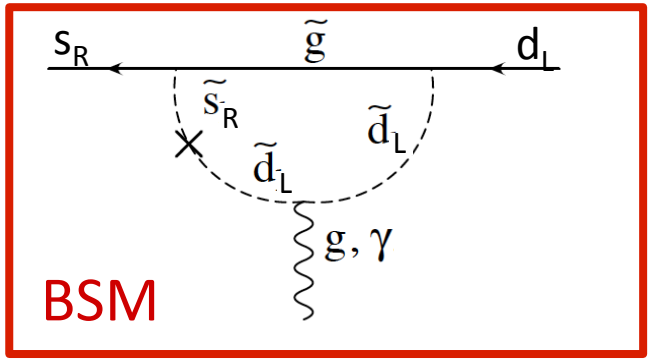
In previous phenomen. analysis
[Buras et al., NPB 566 (2000) 3]

On the lattice we find:

$$B_{CMO} = 1 - 4 \quad \Longleftrightarrow \quad B_{CMO} = 0.29(11)$$

**SM contribution is smaller. NP couplings can be larger**

The Wilson coefficients of the CMO and the EMO are typically closely related.




E.g. in SUSY models:

$$C_\gamma^{\pm}(m_{\tilde{g}}) = \frac{\pi\,\alpha_s(m_{\tilde{g}})}{m_{\tilde{g}}}\left[(\delta_{LR}^D)_{21} \pm (\delta_{LR}^D)_{12}^*\right] F_0(m_{\tilde{g}}^2 / m_{\tilde{q}}^2) \quad \Longleftarrow \quad K_L \to \pi^0 e^+ e^-$$

$$C_g^{\pm}(m_{\tilde{g}}) = \frac{\pi\,\alpha_s(m_{\tilde{g}})}{m_{\tilde{g}}}\left[(\delta_{LR}^D)_{21} \pm (\delta_{LR}^D)_{12}^*\right] G_0(m_{\tilde{g}}^2 / m_{\tilde{q}}^2) \quad \Longleftarrow \quad \varepsilon'/\varepsilon\,,\ K \to 3\pi,\ \ldots$$

Gabbiani, Gabrielli, Masiero, Silvestrini, NPB 477 (1996) 321

# CONCLUSIONS

- The CMO can receive potentially large contributions from New Physics
- We have performed the first lattice QCD calculation of the $K \to \pi$ matrix element:

$$B_{CMO} = 0.29(11)$$

- The main sources of error are the chiral extrapolation and the (large) 1-loop multiplicative renormalization
- The lattice result differs from previous estimates and will provide interesting bounds on BSM physics effects

**A phenomenological analysis is in progress**

# BACKUP SLIDES

# $B_{CMO}$ in the chiral quark model

- The expression of the CMO in ChPT is determined, at LO, up to a single multiplicative low energy constant:

$$Q_g^{\pm} = G_{CMO}\left[U(D_\mu U^\dagger)(D^\mu U) \pm (D_\mu U^\dagger)(D^\mu U)U^\dagger\right]_{23}$$

- In the chiral quark model: [Bertolini, Eeg, Fabbrichesi , NPB 449 (1995) 197 ]

$$G_{CMO}^{\chi QM} = -\frac{11}{128\pi^2}\langle\bar{q}q\rangle_G \equiv -\frac{11}{128\pi^2}\left[-\frac{1}{12M}\left\langle\frac{\alpha_s}{\pi}GG\right\rangle\right]$$

$M \approx \Lambda_{QCD}$

"constituent" quark mass

- The B-parameter is defined as:

$$G_{CMO} = -\frac{11}{128\pi^2}\langle\bar{q}q\rangle B_{CMO} = \frac{11}{256\pi^2}\frac{f_\pi^2 M_K^2}{m_s + m_d} B_{CMO}$$

- Therefore, in the χQM:

$$B_{CMO}^{\chi QM} = \frac{\langle \bar{q}q \rangle_G}{\langle \bar{q}q \rangle} = 2\frac{\langle (\alpha_s/\pi)GG \rangle / (12M)}{f_\pi^2 M_K^2 / (m_s + m_d)}$$

- The value of the gluon condensate is poorly known. Current estimates can be summarized as:

$$\left\langle \frac{\alpha_s}{\pi} GG \right\rangle = 0 \pm 0.012 \text{ GeV}^4$$

Using also $M = 0.1 - 0.4 \text{ GeV}$ one finds:

$$B_{CMO}^{\chi QM} = 0.1 - 0.5$$

# Symmetries

The mixing pattern is determined by the symmetries of the action.

In the TM-OS case the relevant symmetries are:

a) $P \times D_d \times (m \leftrightarrow -m)$

b) $D_d \times R_5$

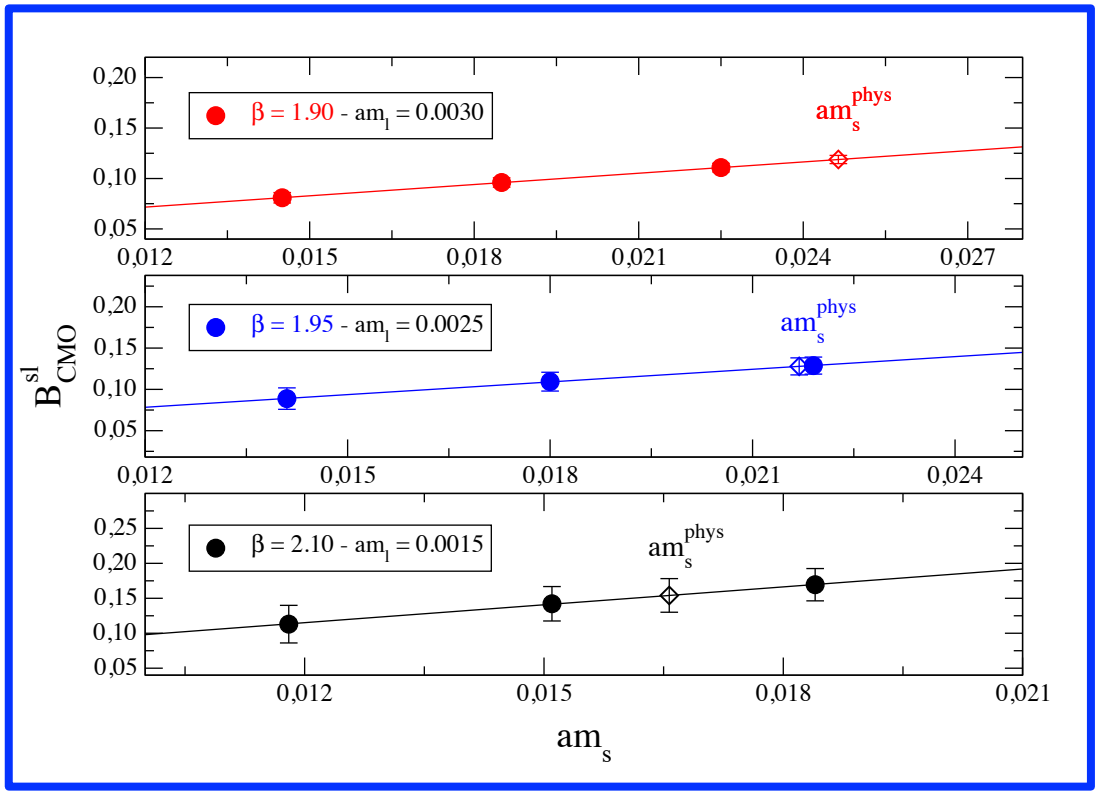
c) $C \times (s \leftrightarrow d)$ or $\tilde{c})$ $C \times P \times (s \leftrightarrow d)$

if $r_s = r_d$ if $r_s = -r_d$

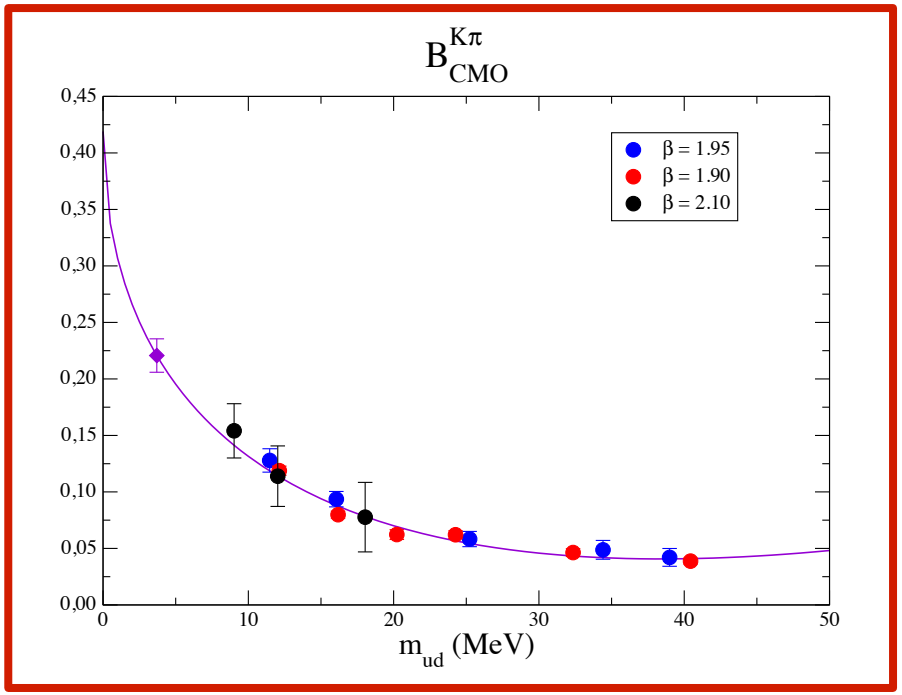
| | Operators | $\mathcal{P}\times\mathcal{D}_d\times$ $(m\to -m)$ | $\mathcal{D}_d\times\mathcal{R}_5$ | $\mathcal{C}\times\mathcal{S}$ if $r_s=r_d$ | $\mathcal{C}\times\mathcal{P}\times\mathcal{S}$ if $r_s=-r_d$ |
|---|---|---|---|---|---|
| Dimension 3 operators | | | | | |
| ✓ | $\overline{\psi}_s\psi_d$ | $-$ | $+$ | $+$ | $+$ |
| | $i\,\overline{\psi}_s\gamma_5\psi_d$ | $+$ | $+$ | $+$ | $-$ |
| Dimension 4 operators | | | | | |
| | $(m_d+m_s)\overline{\psi}_s\psi_d$ | $+$ | $+$ | $+$ | $+$ |
| | $(m_d-m_s)\overline{\psi}_s\psi_d$ | $+$ | $+$ | $-$ | $-$ |
| $(+)$ | $i\,(m_d+m_s)\overline{\psi}_s\gamma_5\psi_d$ | $-$ | $+$ | $+$ | $-$ |
| $(-)$ | $i\,(m_d-m_s)\overline{\psi}_s\gamma_5\psi_d$ | $-$ | $+$ | $-$ | $+$ |
| | $\overline{\psi}_s(\not{D}+m_d)\psi_d+\overline{\psi}_s(-\not{D}+m_s)\psi_d$ | $+$ | $+$ | $+$ | $+$ |
| | $\overline{\psi}_s(\overrightarrow{\not{D}}+m_d)\psi_d-\overline{\psi}_s(-\overleftarrow{\not{D}}+m_s)\psi_d$ | $+$ | $+$ | $-$ | $-$ |
| $(+)$ | $i\,\overline{\psi}_s\gamma_5(\overrightarrow{\not{D}}+m_d)\psi_d+i\,\overline{\psi}_s(-\overleftarrow{\not{D}}+m_s)\gamma_5\psi_d$ | $-$ | $+$ | $+$ | $-$ |
| $(-)$ | $i\,\overline{\psi}_s\gamma_5(\overrightarrow{\not{D}}+m_d)\psi_d-i\,\overline{\psi}_s(-\overleftarrow{\not{D}}+m_s)\gamma_5\psi_d$ | $-$ | $+$ | $-$ | $+$ |
| Dimension 5 operators | | | | | |
| ✓ | $g_0\,\overline{\psi}_s\sigma_{\mu\nu}G_{\mu\nu}\psi_d$ | $-$ | $+$ | $+$ | $+$ |
| | $i\,g_0\,\overline{\psi}_s\gamma_5\sigma_{\mu\nu}G_{\mu\nu}\psi_d$ | $+$ | $+$ | $+$ | $-$ |
| ✓ | $(m_d^2+m_s^2)\overline{\psi}_s\psi_d$ | $-$ | $+$ | $+$ | $+$ |
| | $i\,(m_d^2+m_s^2)\overline{\psi}_s\gamma_5\psi_d$ | $+$ | $+$ | $+$ | $-$ |
| | $(m_d^2-m_s^2)\overline{\psi}_s\psi_d$ | $-$ | $+$ | $-$ | $-$ |
| | $i\,(m_d^2-m_s^2)\overline{\psi}_s\gamma_5\psi_d$ | $+$ | $+$ | $-$ | $+$ |
| ✓ | $m_d\,m_s\overline{\psi}_s\psi_d$ | $-$ | $+$ | $+$ | $+$ |
| | $i\,m_d\,m_s\overline{\psi}_s\gamma_5\psi_d$ | $+$ | $+$ | $+$ | $-$ |

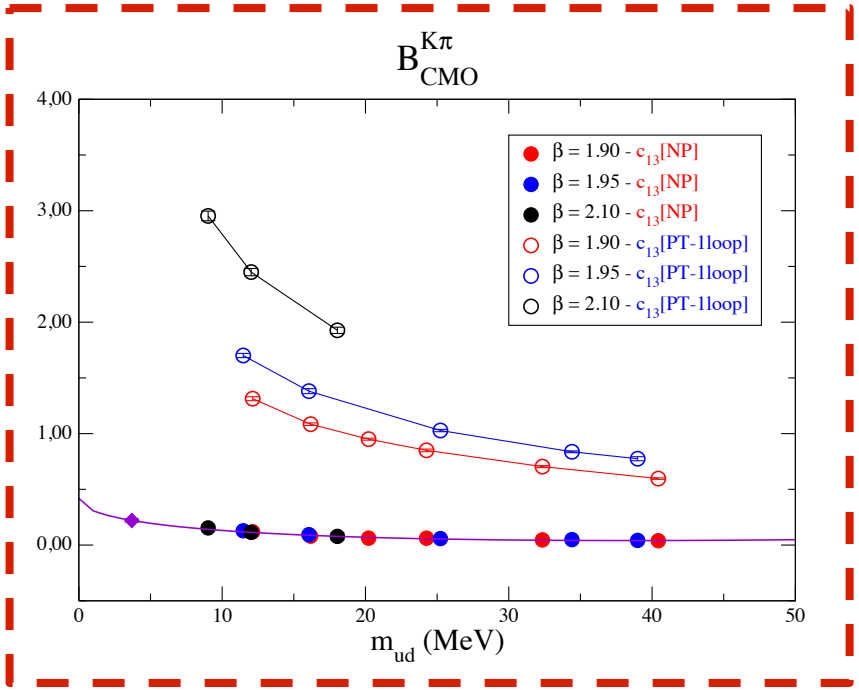
| | Operators | $\mathcal{P}\times\mathcal{D}_d\times$ $(m\to -m)$ | $\mathcal{D}_d\times\mathcal{R}_5$ | $\mathcal{C}\times\mathcal{S}$ if $r_s=r_d$ | $\mathcal{C}\times\mathcal{P}\times\mathcal{S}$ if $r_s=-r_d$ |
|---|---|---|---|---|---|
| ✓ | $m_s\overline{\psi}_s(\overrightarrow{\not{D}}+m_d)\psi_d+m_d\overline{\psi}_s(-\overleftarrow{\not{D}}+m_s)\psi_d$ | $-$ | $+$ | $+$ | $+$ |
| ✓ | $m_d\overline{\psi}_s(\overrightarrow{\not{D}}+m_d)\psi_d+m_s\overline{\psi}_s(-\overleftarrow{\not{D}}+m_s)\psi_d$ | $-$ | $+$ | $+$ | $+$ |
| | $m_s\overline{\psi}_s(\overrightarrow{\not{D}}+m_d)\psi_d-m_d\overline{\psi}_s(-\overleftarrow{\not{D}}+m_s)\psi_d$ | $-$ | $+$ | $-$ | $-$ |
| | $m_d\overline{\psi}_s(\overrightarrow{\not{D}}+m_d)\psi_d-m_s\overline{\psi}_s(-\overleftarrow{\not{D}}+m_s)\psi_d$ | $-$ | $+$ | $-$ | $-$ |
| | $i\,m_s\overline{\psi}_s\gamma_5(\overrightarrow{\not{D}}+m_d)\psi_d+i\,m_d\overline{\psi}_s(-\overleftarrow{\not{D}}+m_s)\gamma_5\psi_d$ | $+$ | $+$ | $+$ | $-$ |
| | $i\,m_d\overline{\psi}_s\gamma_5(\overrightarrow{\not{D}}+m_d)\psi_d+i\,m_s\overline{\psi}_s(-\overleftarrow{\not{D}}+m_s)\gamma_5\psi_d$ | $+$ | $+$ | $+$ | $-$ |
| | $i\,m_s\overline{\psi}_s\gamma_5(\overrightarrow{\not{D}}+m_d)\psi_d-i\,m_d\overline{\psi}_s(-\overleftarrow{\not{D}}+m_s)\gamma_5\psi_d$ | $+$ | $+$ | $-$ | $+$ |
| | $i\,m_d\overline{\psi}_s\gamma_5(\overrightarrow{\not{D}}+m_d)\psi_d-i\,m_s\overline{\psi}_s(-\overleftarrow{\not{D}}+m_s)\gamma_5\psi_d$ | $+$ | $+$ | $-$ | $+$ |
| ✓ | $\overline{\psi}_s(\overrightarrow{\not{D}}+m_d)^2\psi_d+\overline{\psi}_s(-\overleftarrow{\not{D}}+m_s)^2\psi_d$ | $-$ | $+$ | $+$ | $+$ |
| | $\overline{\psi}_s(\overrightarrow{\not{D}}+m_d)^2\psi_d-\overline{\psi}_s(-\overleftarrow{\not{D}}+m_s)^2\psi_d$ | $-$ | $+$ | $-$ | $-$ |
| | $i\,\overline{\psi}_s\gamma_5(\overrightarrow{\not{D}}+m_d)^2\psi_d+i\,\overline{\psi}_s(-\overleftarrow{\not{D}}+m_s)^2\gamma_5\psi_d$ | $+$ | $+$ | $+$ | $-$ |
| | $i\,\overline{\psi}_s\gamma_5(\overrightarrow{\not{D}}+m_d)^2\psi_d-i\,\overline{\psi}_s(-\overleftarrow{\not{D}}+m_s)^2\gamma_5\psi_d$ | $+$ | $+$ | $-$ | $+$ |
| ✓ | $\overline{\psi}_s\overleftarrow{D}_\mu\overrightarrow{D}_\mu\psi_d$ | $-$ | $+$ | $+$ | $+$ |
| | $i\,\overline{\psi}_s\gamma_5\overleftarrow{D}_\mu\overrightarrow{D}_\mu\psi_d$ | $+$ | $+$ | $+$ | $-$ |
| ✓ | $\overline{\psi}_s(-\overleftarrow{\not{D}}+m_s)\,(\overrightarrow{\not{D}}+m_d)\psi_d$ | $-$ | $+$ | $+$ | $+$ |
| | $i\,\overline{\psi}_s(-\overleftarrow{\not{D}}+m_s)\,\gamma_5\,(\overrightarrow{\not{D}}+m_d)\psi_d$ | $+$ | $+$ | $+$ | $-$ |
| ✓ | $\overline{\psi}_s\overleftarrow{\not{\partial}}\,(\overrightarrow{\not{D}}+m_d)\psi_d-\overline{\psi}_s(-\overleftarrow{\not{D}}+m_s)\overrightarrow{\not{\partial}}\,\psi_d$ | $-$ | $+$ | $+$ | $+$ |
| ✓ | $\overline{\psi}_s\overleftarrow{\not{\partial}}\,(\overrightarrow{\not{D}}+m_d)\psi_d-\overline{\psi}_s(-\overleftarrow{\not{D}}+m_s)\overrightarrow{\not{\partial}}\,\psi_d$ | $-$ | $+$ | $+$ | $+$ |
| | $\overline{\psi}_s\overleftarrow{\not{\partial}}\,(\overrightarrow{\not{D}}+m_d)\psi_d+\overline{\psi}_s(-\overleftarrow{\not{D}}+m_s)\overrightarrow{\not{\partial}}\,\psi_d$ | $-$ | $+$ | $-$ | $-$ |
| | $\overline{\psi}_s\overrightarrow{\not{\partial}}\,(\overrightarrow{\not{D}}+m_d)\psi_d+\overline{\psi}_s(-\overleftarrow{\not{D}}+m_s)\overleftarrow{\not{\partial}}\,\psi_d$ | $-$ | $+$ | $-$ | $-$ |
| | $i\,\overline{\psi}_s\overleftarrow{\not{\partial}}\,\gamma_5(\overrightarrow{\not{D}}+m_d)\psi_d-i\,\overline{\psi}_s(-\overleftarrow{\not{D}}+m_s)\gamma_5\overrightarrow{\not{\partial}}\,\psi_d$ | $+$ | $+$ | $+$ | $-$ |
| | $i\,\overline{\psi}_s\overrightarrow{\not{\partial}}\,\gamma_5(\overrightarrow{\not{D}}+m_d)\psi_d-i\,\overline{\psi}_s(-\overleftarrow{\not{D}}+m_s)\gamma_5\overleftarrow{\not{\partial}}\,\psi_d$ | $+$ | $+$ | $+$ | $-$ |
| | $i\,\overline{\psi}_s\overleftarrow{\not{\partial}}\,\gamma_5(\overrightarrow{\not{D}}+m_d)\psi_d+i\,\overline{\psi}_s(-\overleftarrow{\not{D}}+m_s)\gamma_5\overrightarrow{\not{\partial}}\,\psi_d$ | $+$ | $+$ | $-$ | $+$ |
| | $i\,\overline{\psi}_s\overrightarrow{\not{\partial}}\,\gamma_5(\overrightarrow{\not{D}}+m_d)\psi_d+i\,\overline{\psi}_s(-\overleftarrow{\not{D}}+m_s)\gamma_5\overleftarrow{\not{\partial}}\,\psi_d$ | $+$ | $+$ | $-$ | $+$ |

# Interpolation to the physical strange mass



# Importance of a non perturbative determination of $c_{13}/a^2$




| β | $c_{13}$ [NP] | $c_{13}$ [PT-1ℓ] |
|---|---|---|
| 1.90 | 0.8978 (2) | 0.96 |
| 1.95 | 0.8768 (3) | 0.94 |
| 2.10 | 0.8164 (7) | 0.87 |

$c_{13}$ [PT-1ℓ] / $c_{13}$ [NP] - 1 ≃ 6-7 %

With c13 PT-1ℓ