The origin of axial anomaly in the high temperature phase of QCD

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Reference:

V. Dick, F. Karsch, E. Laermann, S. Mukherjee and S.Sharma, arxiv: 1502.06190.

Outline

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The $U_A(1)$ puzzle

- Origin:
  Anomalous $U_A(1)$ not an exact symmetry of QCD yet may the order of phase transition for $N_f = 2$ [Pisarki & Wilczek, 83].

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  Disconnected contribution to two point-functions in the chiral symmetry restored phase $\rightarrow 0$ as $m \rightarrow 0$ [Cohen, 96] $\Rightarrow U_A(1)$ is restored!

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- Contribution of fermion zero modes neglected. It may result in non-vanishing $U_A(1)$ even when chiral symmetry is restored [Lee & Hatsuda, 96].
  Need a careful lattice study!

Implications for QCD: Critical point

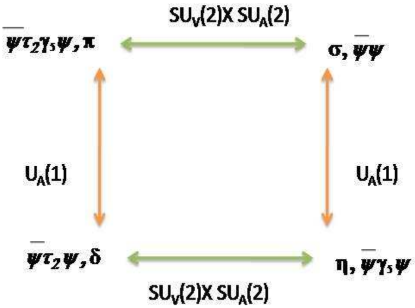
- The existence of critical point crucially depends on number of light quarks.
- A moderately heavy strange quark may shift its position but not its existence.
- Associated with chiral symmetry restoration:
  Need fermions with exact chiral symmetry at finite μ on the lattice.
  [Narayanan & Sharma, 11 , Gavai & Sharma, 12]
- Need to maintain the axial anomaly on the lattice and study its T-dependence.

Constituents of the hot QCD medium

- At $T = 0$, anomaly effects related to instantons [t'Hooft, 76].
- Near T_c, a medium consisting of interacting instantons can explain chiral symmetry breaking $\Rightarrow$ Instanton Liquid Model [Shuryak, 82]
- At $T >> T_c$, medium is like a dilute gas of instantons [Gross, Pisarski & Yaffe, 81].
- What is the medium made up of for $T_c \leq T \leq 2T_c$?

Not an exact symmetry → what observables to look for?

Degeneracy of the correlators with specific quantum numbers in meson channels [Shuryak, 94]



Routes to solve the puzzle

- Either look at the difference of the integrated correlators

$$\chi_\pi - \chi_\delta = \int d^4x \ \left[\langle i\pi^+(x) i\pi^-(0)\rangle - \langle \delta^+(x)\delta^-(0)\rangle\right]$$

- Equivalently study $\rho(\lambda, m_f)$ of the Dirac operator.

$$\chi_\pi - \chi_\delta \overset{V\to\infty}{\to} \int_0^\infty d\lambda \frac{4m_f^2\ \rho(\lambda, m_f)}{(\lambda^2+m_f^2)^2} \ , \ \langle\bar\psi\psi\rangle \overset{V\to\infty}{\to} \int_0^\infty d\lambda \frac{2m_f\ \rho(\lambda, m_f)}{(\lambda^2+m_f^2)}$$

- If chiral symmetry restored: $\langle\bar\psi\psi\rangle = 0 \Rightarrow \lim_{m_f\to 0}\lim_{V\to\infty}\rho(0, m_f) \to 0$.
- A gap in the infrared spectrum $\Rightarrow U_A(1)$ restored
- chiral symmetry restored $+ U_A(1)$ broken if:
$\lim_{\lambda\to 0}\rho(\lambda, m_f) \to \delta(\lambda) m_f^\alpha \ , 1 < \alpha < 2$

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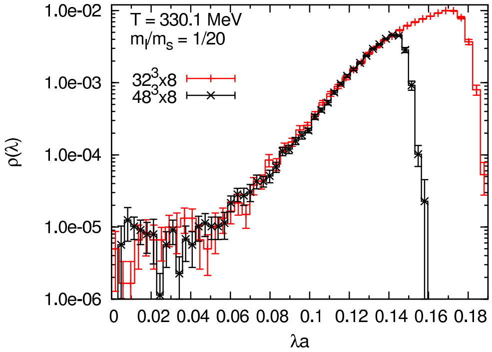
More puzzles from lattice?

The earlier results on $U_A(1)$ are not conclusive

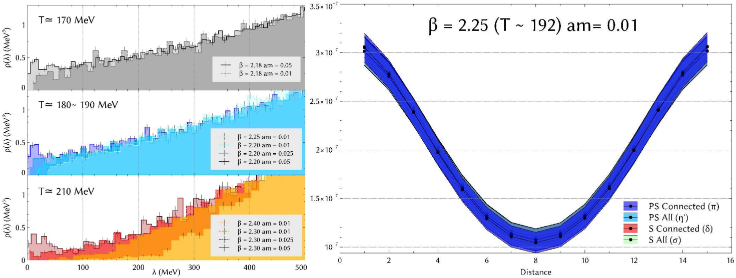


Dynamical (improved)Staggered fermions: First Large volume study $32^3 \times 8 \rightarrow U_A(1)$ broken [Ohno et. al., 12]

Same observation noted earlier for smaller lattice[Chandrasekharan & Christ, 96]

Delicate cut-off effects $\rightarrow$ continuum symmetries not preserved

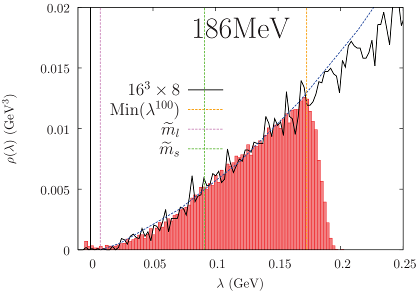
More puzzles from lattice?



Dynamical overlap fermions with exact chiral symmetry on lattice → $U_A(1)$ restored [Cossu et. al, JLQCD collaboration, 11, 12]

Pion mass 220 MeV! Effects of fixing the topology? Thermodynamic equilibrium?

More puzzles from lattice?



Dynamical domain wall fermions with better chiral symmetry $\rightarrow$ $U_A(1)$ broken [Buchoff et. al., 13] Low statistics in the lower end of spectrum?

Optimal domain wall fermions: On small lattice $\rightarrow$ $U_A(1)$ restored [Chiu et. al. 13]

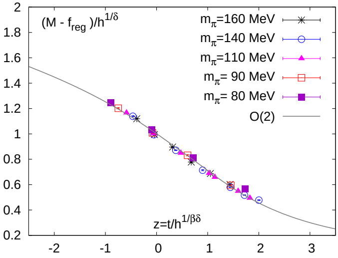
More puzzles from lattice?

Issues need to be addressed

- Sufficient control on the lattice artifacts?
- Is the lattice volume large enough to hold sufficient number of instantons?
- Chiral and continuum limits.

Motivation for our work

- At present improved versions of staggered fermions used extensively for QCD thermodynamics. Continuum limits for T_c, χ_2 are in agreement. [Budapest-Wuppertal collaboration 10, HotQCD collaboration, 11].
- Highly improved staggered quarks(HISQ): minimal taste symmetry breaking$\Rightarrow$ lattice artifacts reduced.
- Hints about the $O(4)$ scaling from these configurations $\rightarrow$ effects of anomaly? [Bielefeld-BNL collaboration, 09, HotQCD collaboration, 11]



- Absence of unique index theorem for staggered fermions makes it difficult!

Index theorem on the lattice

- It is impossible to define chiral fermions on lattice which are (ultra)local.
  [Nielsen & Ninomiya, 82]
- Overlap fermions [Narayanan & Neuberger, 94, Neuberger, 98] have exact chiral symmetry on the lattice.

$$D_{ov} = M(1 + \gamma_5\ \mathbf{sgn}(\gamma_5 D_W(-M)))\ ,\ \mathbf{sgn}(A) = A/\sqrt{A.A}.$$

- It satisfies the Ginsparg-Wilson relation $\{\gamma_5, D_{ov}\} = aD_{ov}\gamma_5 D_{ov}$
  [Ginsparg & Wilson, 82]
- D_{ov} has an exact index theorem like in the continuum $\Rightarrow$ the zero modes of D_{ov} related to topological structures of the underlying gauge field.
  [Hasenfratz, Laliena & Niedermeyer, 98]

Index theorem on the lattice

- We use the overlap as valence operator to probe the infrared spectrum.
- The Highly Improved Staggered Quark(HISQ) configurations form the sea quarks in background.
- We look at the eigenvalue distribution of D_{ov} on the HISQ ensembles.
- Zero modes of D_{ov} related to topological structures of HISQ sea.
- Infrared part of eigenvalue distribution gives us idea about the χ_{SB} , $U_A(1)$ and the topological structures that contribute to them.

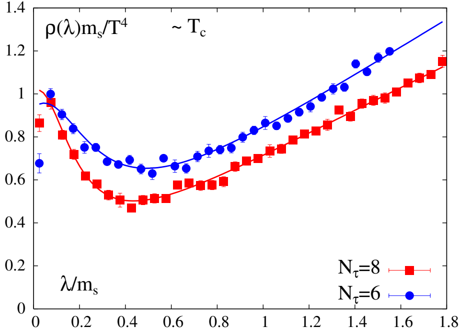
Outline

Numerical details

- Lattice size: 4D hypercube with $N = 32,\ 24$ sites along each spatial dim and $N_\tau = 8,\ 6$ sites along temporal dim.
- Volumes, $V = N^3 a^3$, Temperature, $T = \frac{1}{N_\tau a}$, a is the lattice spacing.
- Box size: $m_\pi \ V^{1/3} > 3$
- 2 light+1 heavy flavour
- Input m_s physical ≈ 100 MeV and $m_s/m_l = 20 \Rightarrow m_\pi = 160$ MeV.
- To study the chiral limit we have another set of $N_\tau = 6$ configurations T_c with lighter than physical $m_\pi = 110$ MeV.

General features: Near zero mode peak +bulk

We fit the eigenvalue density to ansatz: $\rho(\lambda) = \frac{A\epsilon}{\lambda^2+A} + B\lambda^\gamma$



Bulk rises linearly as λ,no gap seen.

If $\rho(\lambda, m)$ analytic in $m^2 \Rightarrow$ signature of $U_A(1)$ restoration as chiral symmetry restored: $\rho \propto \lambda^3$ [Aoki, Fukaya & Taniguchi, 12].

Eigenvalue distribution near T_c

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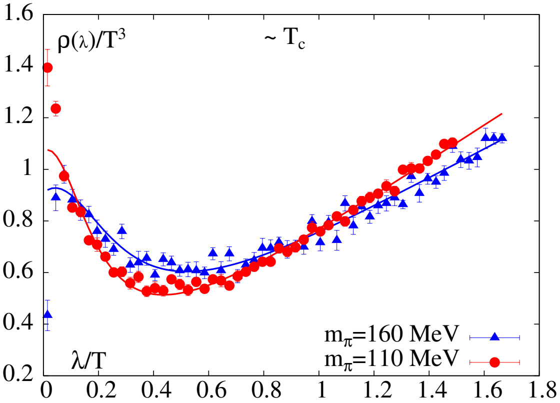
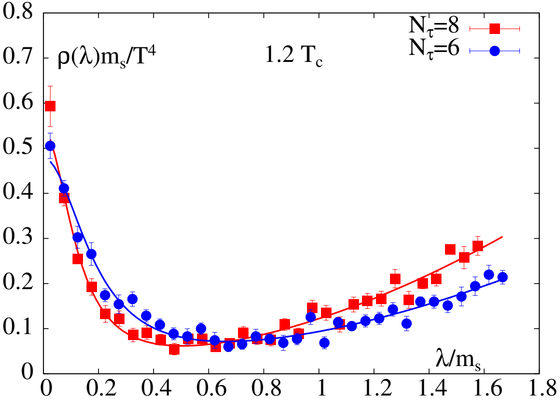


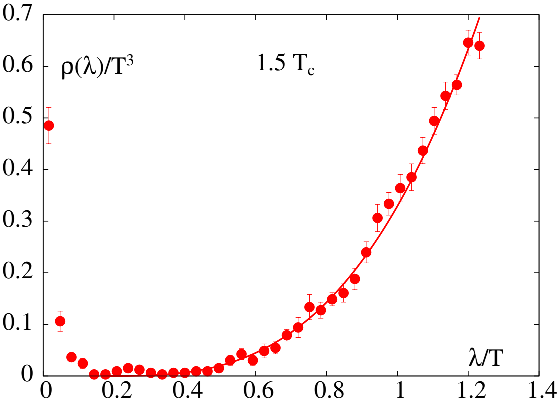
No gap even when quark mass reduced!

At higher temperatures..



Near zero mode peak shows little cut-off dependence. Bulk rises as λ^2.

At higher temperatures..

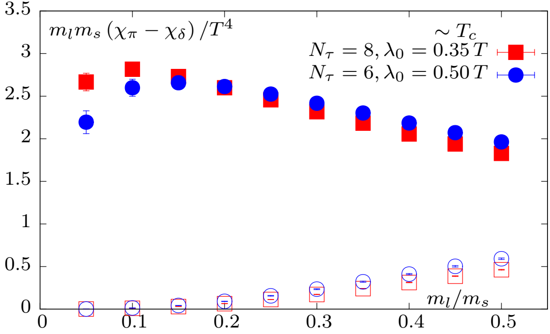


At $1.5\,T_c$, the bulk and the near zero peak decouples completely

Bulk rises as λ^3.

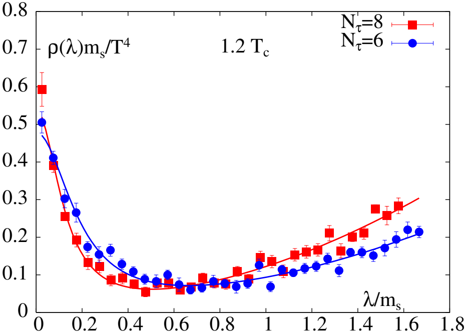
Microscopic origin of $U_A(1)$

- m_s tuned by matching RG invariant combination $m_s^2 \frac{\frac{\langle\bar\psi\psi\rangle_s}{m_s} - \chi^s_{conn}}{T^4}$.
- Renormalized $\chi_\pi - \chi_\delta$ has negligible cut-off dependence near T_c.
- Significant contribution comes from the near zero modes than the bulk $\Rightarrow$ Near zero modes responsible for $U_A(1)$ breaking



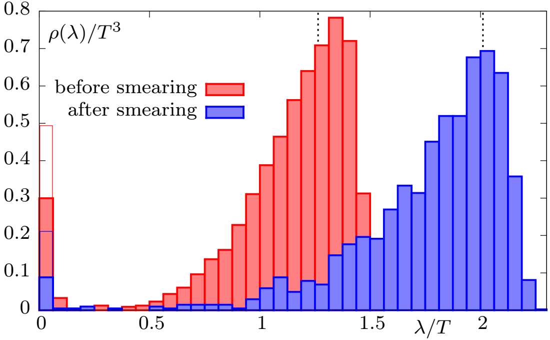
Robustness of near zero modes

- The infrared part could be affected by unphysical dislocations. $\rightarrow$ effect of partial quenching as well as rough configurations.
- These have smaller classical action than instantons. $\rightarrow$ cut-off effect.
- We do not observe any significant cut-off dependence of the renormalized eigen spectrum at high T.



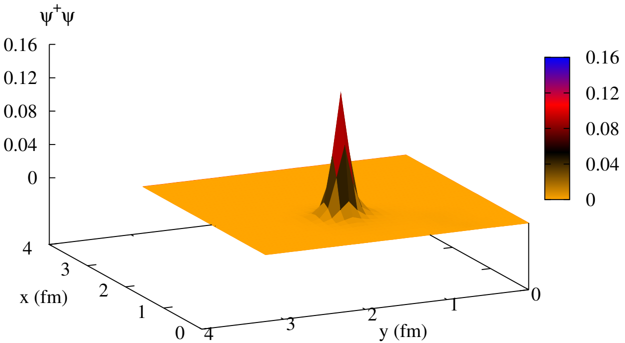
Robustness of near zero modes

- HYP smearing [Hasenfratz & Knechtli, 02] expected to eliminate such small localized structures.



- Smearing does not eliminate the near zero modes.
- Difference? Smearing may suppress small instantons.

Localization properties of zero modes at $1.5\ T_c$



Localized in space.

Fit to $\psi_0^\dagger(x)\psi_0(x) \simeq \frac{\rho^2}{\pi}\frac{1}{(x^2+\rho^2)^3} \to \rho = 0.223(8)$ fm.

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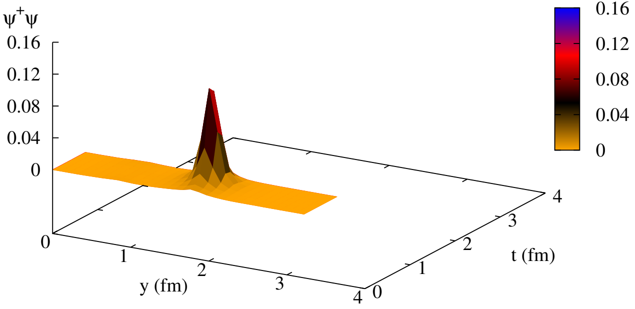
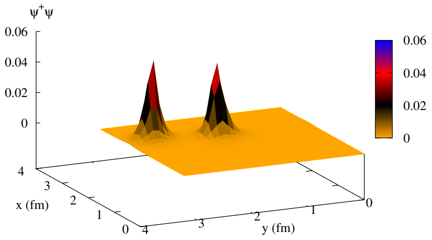
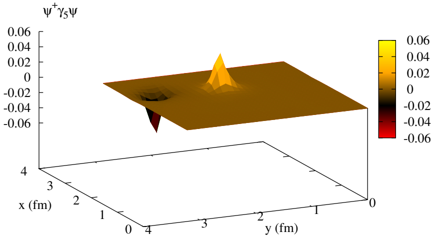


Localized in the temporal direction as well, $\rho < 1/T \rightarrow$ behave as zero-T instantons.

The near-zero modes





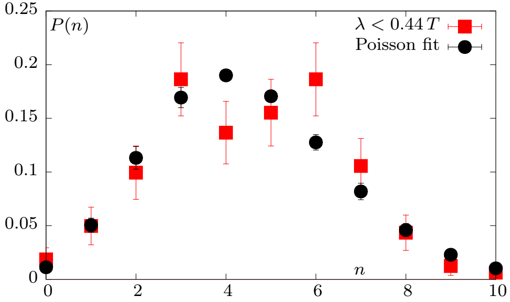
Near-zero modes due to a weakly interacting instanton-antiinstanton pair.

The nature of the infrared modes at $1.5\,T_c$

- If n=total no. of instantons+antiinstantons and form a dilute gas,

$$P(n, \langle n\rangle) = \langle n\rangle^n \mathrm{e}^{-\langle \mathrm{n}\rangle}/\mathrm{n}!$$

- For $\lambda/T < 0.44$, the value of $\langle n\rangle = 4.2 = \langle n^2\rangle \Rightarrow$ density $\simeq 0.147(7) fm^{-4}$. This is much dilute than an instanton liquid with density $1 fm^{-4}$.

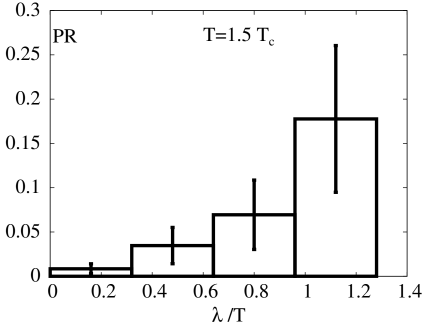


The measures of localization: PR

The Participation Ratio measures the degree of localization

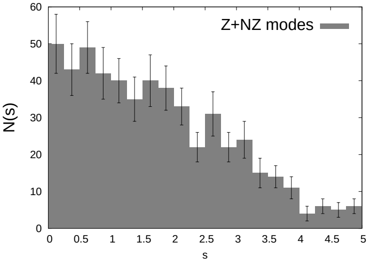
$$PR = \frac{1}{N^3 N_\tau} \left[\sum_x \left(\psi^\dagger(x)\psi(x) \right)^2 \right]^{-1} .$$

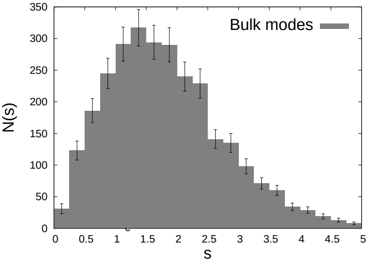
$PR = 1$ (delocalized), $PR = f$ (localized)
The low-lying modes are highly localized unlike the bulk $\rightarrow$ Anderson-Mott localization? [Garcia-Garcia & Osborn, 06, Kovacs et al, 12,13].



The characteristics of the eigen spectrum

- The level spacing between eigenvalues has a system dependent mean+ fluctuations which are universal
- Unfolding or removing the mean gives fluctuations consistent with Poissonian for $\lambda/T < 0.4$.
- Level spacing of bulk eigenvalues seem to be following predictions from Random Matrix Theory $\rightarrow$ delocalized.





Outline

What we studied till now

- On large volume lattice we found that $U_A(1)$ broken for $T \leq 1.5\,T_c$. The fermion near-zero modes are mainly responsible for its breaking.
- Already at 1.5 T_c the QCD medium can be described as a dilute gas of instantons.