$K \to \pi\nu\bar{\nu}$ decays on the lattice

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Phenomenological background

Three decay modes in $K \to \pi\nu\bar{\nu}$

Top-quark (short-distance) contributions dominate in $K \to \pi\nu\bar{\nu}$

- $K^+ \to \pi^+\nu\bar{\nu}$
  - t-quark contribution much larger than c-quark, but $\lambda_t \ll \lambda_c$
  - this compensation makes c-quark important in $A(K^+ \to \pi^+\bar{\nu}\nu)$

$$A^t : A^c : A^u = 68\% : 29\% : 3\%$$ [Cirigliano et.al. Rev. Mod. Phys.]

- $K_L \to \pi^0\nu\bar{\nu}$, CP violating decay
  - $\text{Im}\,\lambda_c \ll \text{Re}\,\lambda_c \Rightarrow A^c$ suppressed, $\text{Im}\,\lambda_u = 0 \Rightarrow A^u = 0$
- $K_S \to \pi^0\nu\bar{\nu}$, similar as K^+ decay, but too difficult in experiment

Only $K^+ \to \pi^+\nu\bar{\nu}$ might be interesting for a non-perturbative calculation

Charm quark contribution, LD or SD dominated?

- A^c is about 29%, if LD dominated, then very interesting for us
- unfortunately, not likely the case
  - Inami & Lim's function for charm quark contribution

$$X^B(x_c) = x_c(-1 - \ln(x_c)), \quad X^Z(x_c) = x_c(-3/4 - \ln(x_c)/4), \quad x_c = \frac{m_c^2}{M_W^2}$$

  - $m_c \ll M_W \Rightarrow \ln(x_c) = -8.28$, log term dominated
  - in log term, most contribution from SD?
- QCD perturbation theory [hep-ph/0603079], charm quark contribution P_c, LO $\rightarrow$ NNLO, uncertainty from varying μ_c 26% $\rightarrow$ 9.8% $\rightarrow$ 2.4%

$$P_c = 0.369 \pm 0.036_{\text{theory}} \pm 0.033_{m_c} \pm 0.009_{\alpha_s} \quad \text{NLO}$$
$$P_c = 0.375 \pm 0.009_{\text{theory}} \pm 0.031_{m_c} \pm 0.009_{\alpha_s} \quad \text{NNLO}$$

Charm quark contribution is likely SD dominated

SM error budget for $K^+ \to \pi^+ \nu\bar{\nu}$

- SM branching ratio for $K^+ \to \pi^+ \nu\bar{\nu}$ [Brod, et.al. 1009.0947]

$$\mathrm{Br}[\mathrm{K}^+ \to \pi^+ \nu\bar{\nu}] = (7.81^{+0.80}_{-0.71\text{para}} \pm 0.29_{\text{theory}}) \times 10^{-11}$$

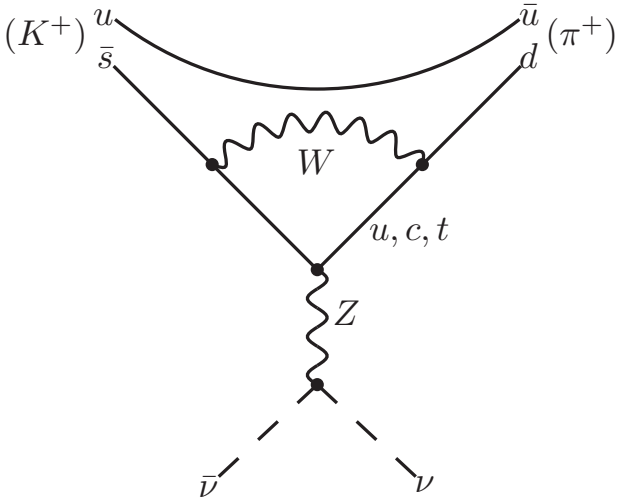
- parametric error $\sim 10\%$ comes from the input parameters, including

$$\mathrm{CKM}: \ (|V_{cb}| : 56\%,\ \bar{\rho} : 21\%,\ \bar{\eta} : 4\%) \quad \mathrm{others} : 19\%$$

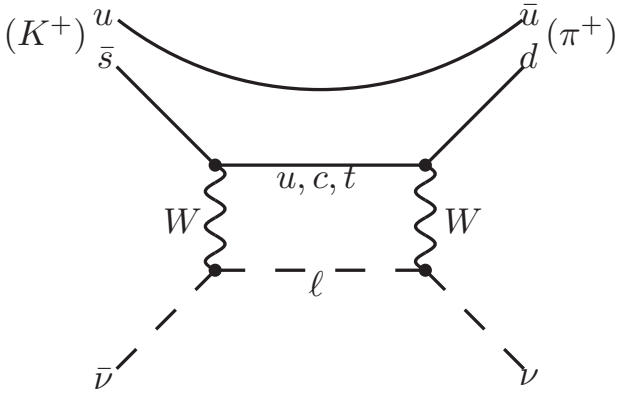
- theory error $\sim 4\%$ dominated by LD contribution $\delta P_{c,u}$
- χPT [hep-ph/0503107] $\delta P_{c,u} = 0.04 \pm 0.02 \Rightarrow$ Br enhanced by 6%
- 6% is comparable to 10%, especially in the future uncertainties in CKM matrix elements may be reduced

Important to determine LD contribution accurately

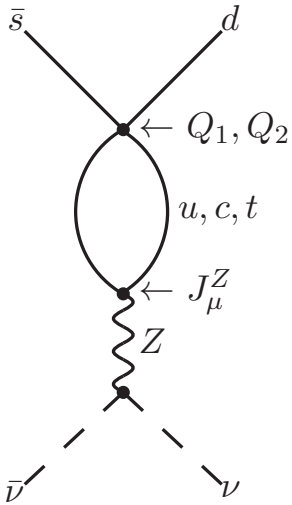
Lattice methodology



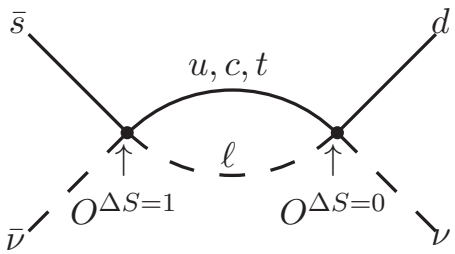
Z-exchange



W-box



$\int d^4x \, \langle \pi | T[Q_{1,2}(x) J_\mu^Z(0)] | K \rangle$



$\int d^4x \, \langle \pi\nu\bar{\nu} | T[O^{\Delta S=1}(x) O^{\Delta S=0}(0)] | K \rangle$

Non-local matrix element

We are going to calculate the non-local matrix element

$$\mathcal{T}^{W,\ell} = \int d^4x \, \langle \pi^+, \nu, \bar{\nu} | \, T[\overbrace{\bar{s}\gamma^\alpha(1-\gamma_5)q \times \bar{\nu}_\ell \gamma_\alpha (1-\gamma_5)\ell(x)}^{O^{\Delta S=1}} \underbrace{\bar{\ell}\gamma^\beta(1-\gamma_5)\nu_\ell \times \bar{q}\gamma_\beta(1-\gamma_5)d(0)}_{O^{\Delta S=0}}] | K^+ \rangle \Big|_{q=u-c}$$

The (anti-)neutrino states are given by

$$\langle \nu_l(\mathbf{p}_\nu) | \bar{\nu}_l(x) | 0 \rangle = \bar{u}(p_\nu) e^{ip_\nu x}, \quad \langle \bar{\nu}_l(\mathbf{p}_{\bar{\nu}}) | \nu_l(0) | 0 \rangle = v(p_{\bar{\nu}}) e^{ip_{\bar{\nu}} 0}$$

Separate the hadronic and leptonic parts in $T^{W,\ell}$

$$\mathcal{T}^{W,\ell} = \int d^4x \, \overbrace{\langle \pi^+ | \, T \, [\bar{s}\gamma_\alpha(1-\gamma_5)q(x) \, \bar{q}\gamma_\beta(1-\gamma_5)d(0)] \, | K^+ \rangle}^{H_{\alpha\beta}\text{: hadronic part}} \times \underbrace{\bar{u}(p_\nu)\gamma^\alpha(1-\gamma_5)S_\ell(x,0)\gamma^\beta(1-\gamma_5)v(p_{\bar{\nu}}) \, e^{ip_\nu x}}_{\bar{u}(p_\nu)\Gamma^{\alpha\beta}v(p_{\bar{\nu}})\text{: leptonic part}}$$

Lorentz structure

For simplicity we may write

$$\mathcal{T}^{W,\ell} = H_{\alpha\beta}\, \bar{u}(p_\nu)\Gamma^{\alpha\beta} v(p_{\bar{\nu}}) \tag{1}$$

Due to the chiral property of the spinors $\bar{u}(p_\nu)$ and $v(p_{\bar{\nu}}) \Rightarrow$

$$\mathcal{T}^{W,\ell} = T^\mu(p_K, p_\nu, p_{\bar{\nu}})\, \bar{u}(p_\nu)\gamma_\mu(1-\gamma_5)v(p_{\bar{\nu}})$$

Massless neutrinos smplifies the structure of $T^\mu = F^\ell(p_K, p_\nu, p_{\bar{\nu}})p_K^\mu$

$$\mathcal{T}^{W,\ell} = F^\ell(p_K, p_\nu, p_{\bar{\nu}})\, \bar{u}(p_\nu)\not{p}_K(1-\gamma_5)v(p_{\bar{\nu}}) \tag{2}$$

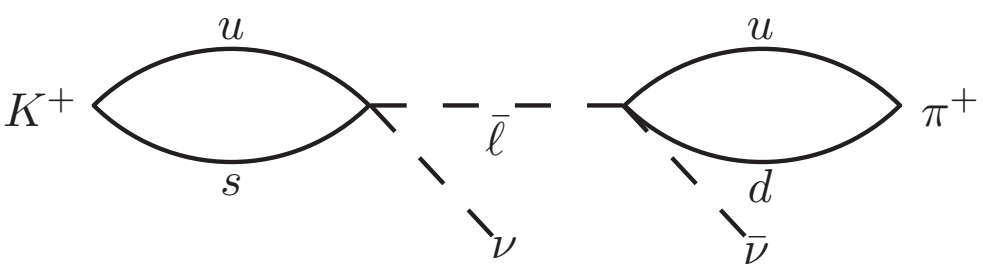
Combining Eq. (1) and (2), we have

$$H_{\alpha\beta}\, \bar{u}(p_\nu)\Gamma^{\alpha\beta} v(p_{\bar{\nu}}) = F^\ell(p_K, p_\nu, p_{\bar{\nu}})\bar{u}(p_\nu)\not{p}_K(1-\gamma_5)v(p_{\bar{\nu}})$$

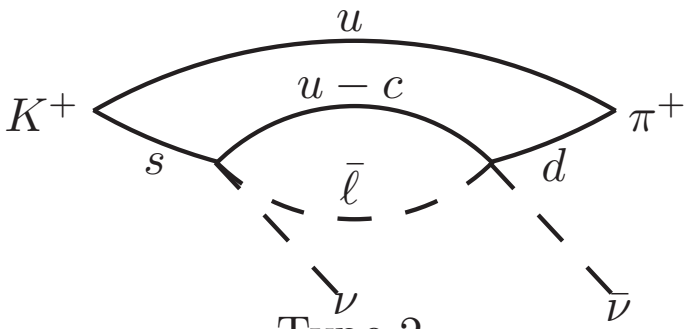
It can be shown that

$$F^\ell(p_K, p_\nu, p_{\bar{\nu}}) = \frac{H_{\alpha\beta}\, \mathrm{Tr}[\Gamma^{\alpha\beta}\not{p}_{\bar{\nu}}\not{p}_K(1-\gamma_5)\not{p}_\nu]}{\mathrm{Tr}[\not{p}_K(1-\gamma_5)\not{p}_{\bar{\nu}}\not{p}_K(1-\gamma_5)\not{p}_\nu]}$$

Short summary of procedures



Type 1



Type 2

- hadronic $H_{\alpha\beta}$ can be calculated from a 4-point correlation function
- leptonic $\Gamma^{\alpha\beta}$ involves a lepton propagator, here we use overlap fermion
- then form factor $F^{\ell}(p_K, p_\nu, p_{\bar{\nu}})$ can be determined
- the decay amplitude for the W-box diagrams can be written as

$$A = G_F^2 V_{us}^* V_{ud} \sum_{\ell=e,\mu,\tau} F^{\ell}(p_K, p_\nu, p_{\bar{\nu}}) p_K^{\mu} \bar{u}(p_\nu)\gamma_\mu(1-\gamma_5)v(p_{\bar{\nu}})$$

Lattice results

Lattice setup

- $16^3 \times 32 \times 16$ domain wall fermion on Iwasaki gauge action
- $m_\pi = 420$ MeV, $m_K = 540$ MeV
- $a^{-1} = 1.729$ GeV, m_c chosen so that $m_c^{\overline{MS}}(2\text{ GeV})$=863 MeV
- every 10 of 8000 configurations are measured
- collinear decay: momentum $\mathbf{p}_K = (0,0,0)$, $\mathbf{p}_\pi = (-p,-p,-p)$, $\mathbf{p}_\nu = \mathbf{p}_{\bar{\nu}} = (p/2, p/2, p/2)$, p is chosen to satisfy $m_K = E_\pi + E_\nu + E_{\bar{\nu}}$

Minkowski vs Euclidean

Generic feature of non-local matrix element in 2nd order weak interaction

$$\begin{aligned}\mathcal{T}^M &= i\int dt\,\langle f|T[O^{\Delta S=1}(t)O^{\Delta S=0}(0)]|K\rangle \\ &= \sum_{n_s}\frac{\langle f|O^{\Delta S=1}|n_s\rangle\langle n_s|O^{\Delta S=0}|K\rangle}{E_{n_s}-E_f+i\epsilon}-\sum_{n}\frac{\langle f|O^{\Delta S=0}|n\rangle\langle n|O^{\Delta S=1}|K\rangle}{E_K-E_n+i\epsilon}\end{aligned} \tag{3}$$

In Euclidean space

$$\begin{aligned}\mathcal{T}^E &= \sum_{t=-T_a}^{T_b}\langle f|T[O^{\Delta S=1}(t)O^{\Delta S=0}(0)]|K\rangle \\ &= \sum_{n_s}\frac{\langle f|O^{\Delta S=1}|n_s\rangle\langle n_s|O^{\Delta S=0}|K\rangle}{E_{n_s}-E_f}\left(1-e^{(E_f-E_{n_s})T_b}\right) \\ &\quad -\sum_{n}\frac{\langle f|O^{\Delta S=0}|n\rangle\langle n|O^{\Delta S=1}|K\rangle}{E_K-E_n}\left(1-e^{(E_K-E_n)T_a}\right)\end{aligned} \tag{4}$$

Removing exp growing contamination, $\mathcal{T}^E \Rightarrow \mathcal{T}^M$ (see A. Portelli's talk)

Double integration

Double integration as shown in $K_L - K_S$ mass diff (N. Christ's talk)

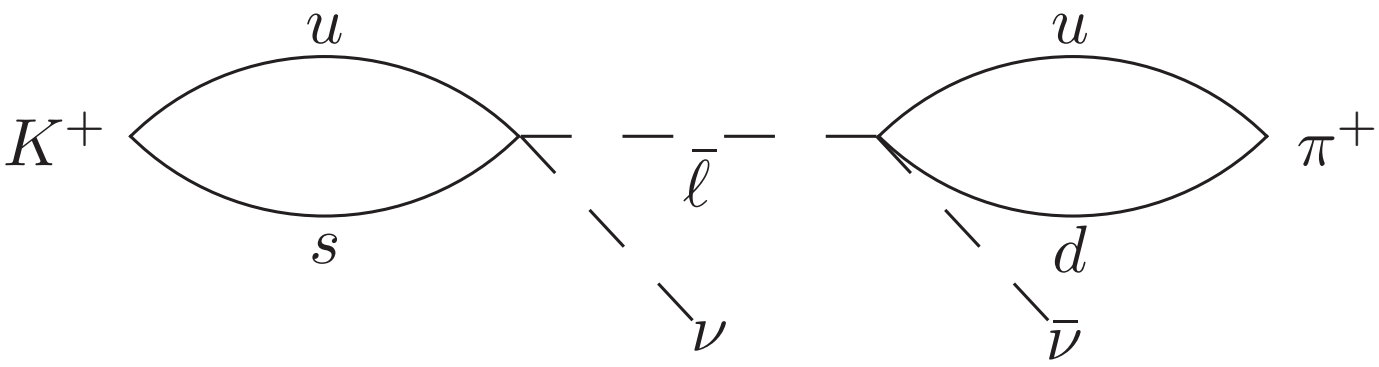
$$\sum_{t_1=t_a}^{t_b}\sum_{t_2=t_a}^{t_b}\langle f|T[O^{\Delta S=1}(t_1)O^{\Delta S=0}(t_2)]|K\rangle\, e^{m_K t_2}e^{-m_f t_2}$$
$$= \sum_{n_s}\frac{\langle f|O^{\Delta S=1}|n_s\rangle\langle n_s|O^{\Delta S=0}|K\rangle}{E_{n_s}-E_f}\left(T-\frac{1-e^{(E_f-E_{n_s})T}}{E_{n_s}-E_f}\right)$$
$$-\sum_{n}\frac{\langle f|O^{\Delta S=0}|n\rangle\langle n|O^{\Delta S=1}|K\rangle}{E_K-E_n}\left(T+\frac{1-e^{(E_K-E_n)T}}{E_K-E_n}\right) \quad (5)$$

Here $T = t_b - t_a + 1$ is defined as size of the integral window

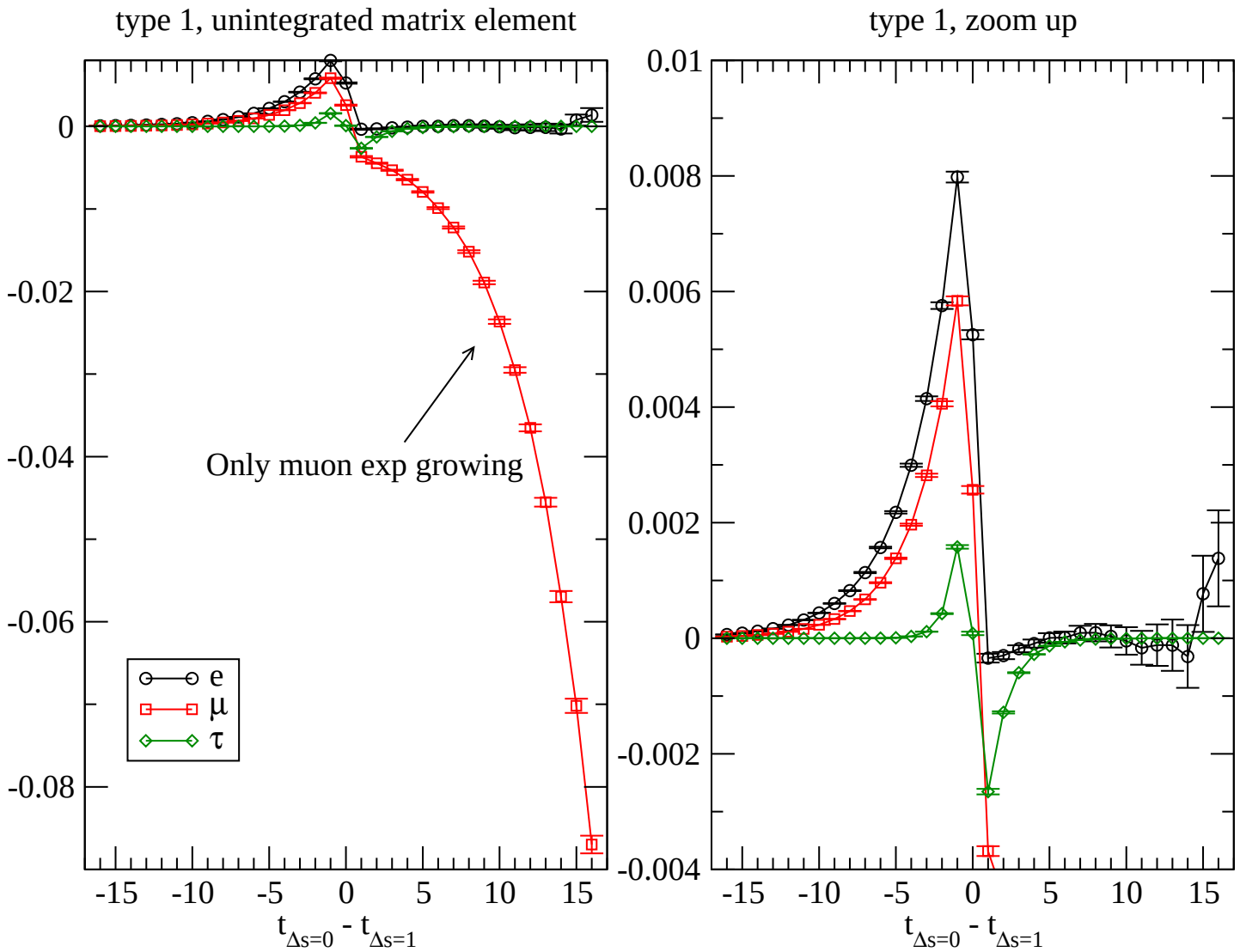
Remove exp., and fit with $a + bT$, the slope b is what we want

In this talk, we use the double integration to gain more precision

Type 1 diagram

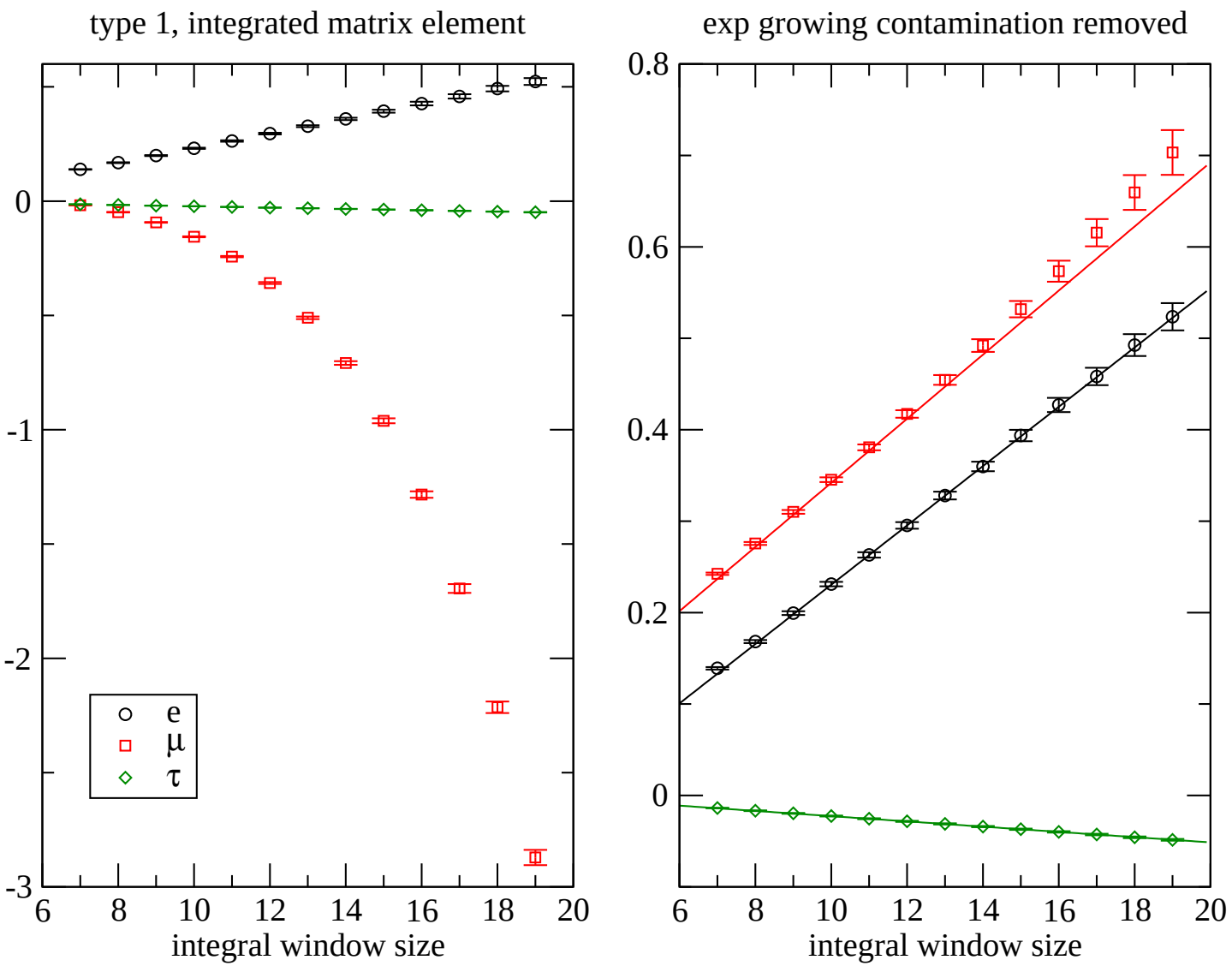


Type 1 diagram, unintegrated matrix element



- e mode: no exp growing contamination due to helicity suppression
- τ mode: no exp growing contamination since tau is heavy

Integrated matrix element



Right figure: the slope of the curve gives the $F^{\ell}(p_K, p_\nu, p_{\bar{\nu}})$

F^ℓ for type 1 diagram

F^ℓ	lattice	model
e	$3.244(90) \times 10^{-2}$	$3.352(12) \times 10^{-2}$
μ	$3.506(77) \times 10^{-2}$	$3.511(13) \times 10^{-2}$
τ	$-2.871(70) \times 10^{-3}$	$-2.836(10) \times 10^{-3}$

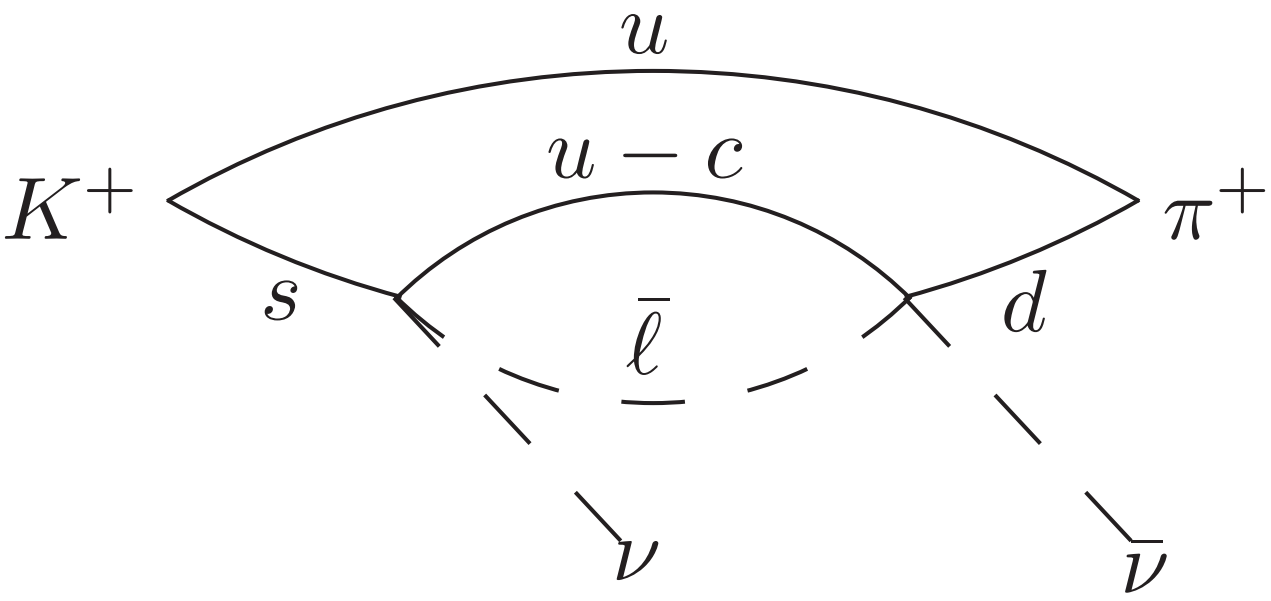
- vacuum state dominance model assuming only single-lepton contribution in the intermediate state

$$f_K f_\pi \bar{u}(p_\nu) \not{p}_K (1-\gamma_5) \frac{\not{q}}{q^2 - m_\ell^2} \not{p}_\pi (1-\gamma_5) v(p_{\bar{\nu}})$$
$$= f_K f_\pi \frac{2q^2}{q^2 - m_\ell^2} \bar{u}(p_\nu) \not{p}_K (1-\gamma_5) v(p_{\bar{\nu}})$$

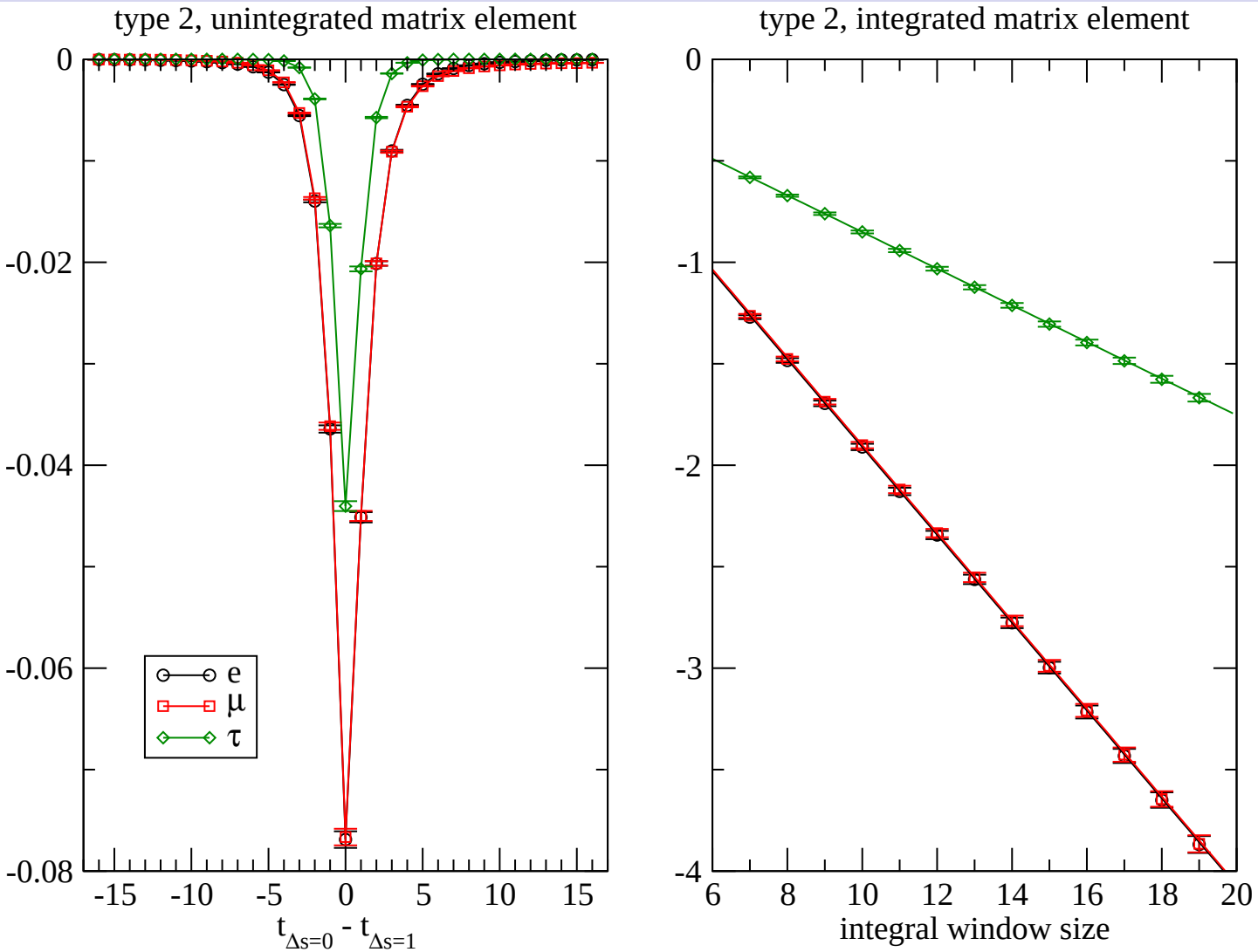
with $q = p_K - p_\nu = p_\pi + p_{\bar{\nu}}$

- in the above table, model values are given by $Z_A^{-2} f_K f_\pi \frac{2q^2}{q^2 - m_\ell^2}$

Type 2 diagram



Type 2 diagram, left: unintegrated, right: integrated



intermediate state is given by $\ell + \pi^0$, since pion is heavy, we don't observe significant exponential growing effects

SD divergence in type 2 diagram

- loop integral for type 2 diagram is log divergent (also in Z. Bai's case)
- in the physical world, the SD divergence is cut off by physical M_W
- in the lattice calculation it is cut off by an energy scale $\Lambda_{lat} \sim \frac{1}{a}$
- correction can be made through $A - A_{SD}^{lat} + A_{SD}^{cont} =$

$$\int d^4x \langle f|T\{O_1(x)O_2(0)\}|K\rangle - \langle f|C^{lat}(\mu^2)O_{SD}|K\rangle + \langle f|C^{cont}(\mu^2)O_{SD}|K\rangle \quad (6$$

- $C^{lat}(\mu^2)$ is determined non-perturbatively using RI/MOM approach
- $C^{cont}(\mu^2)$ can be calculated perturbatively, not yet done

Preliminary results

- type 1 diagram

F^ℓ	lattice	model
e	$3.244(90) \times 10^{-2}$	$3.352(12) \times 10^{-2}$
μ	$3.506(77) \times 10^{-2}$	$3.511(13) \times 10^{-2}$
τ	$-2.871(70) \times 10^{-3}$	$-2.836(10) \times 10^{-3}$

- type 2 diagram before subtraction

F^ℓ	lattice
e	$-2.164(31) \times 10^{-1}$
μ	$-2.164(31) \times 10^{-1}$
τ	$-9.03(14) \times 10^{-2}$

- type 2 diagram after subtraction, using $C^{lat}(\mu^2)$

F^ℓ	$\mu^2 = (2\ \text{GeV})^2$	$\mu^2 = (3\ \text{GeV})^2$
e	$-1.400(31) \times 10^{-1}$	$-1.849(31) \times 10^{-1}$
μ	$-1.402(31) \times 10^{-1}$	$-1.850(31) \times 10^{-1}$
τ	$-4.13(14) \times 10^{-2}$	$-6.68(14) \times 10^{-2}$

Conclusion

- so far we focus on the discussion of the W-box diagram
- in the final result, it should be W-box + Z-exchange
- type 1 diagram, single-lepton state dominates the contribution, excited states very highly suppressed (OZI rule)
- type 2 diagram, contribution (before sub.) 6 times larger than type 1, (after sub.) 4-5 times larger
- ChPT [hep-ph/0503107] consider type 1 diagram as $O(p^2)$ contribution and type 2 as $O(p^4)$, thus type 2 $\ll$ type 1
- for type 2 diagram, correct cont. SD contribution need to be included
- since type 2 diagram involve $\ell + \pi^0$, FV effects need to be esitmated