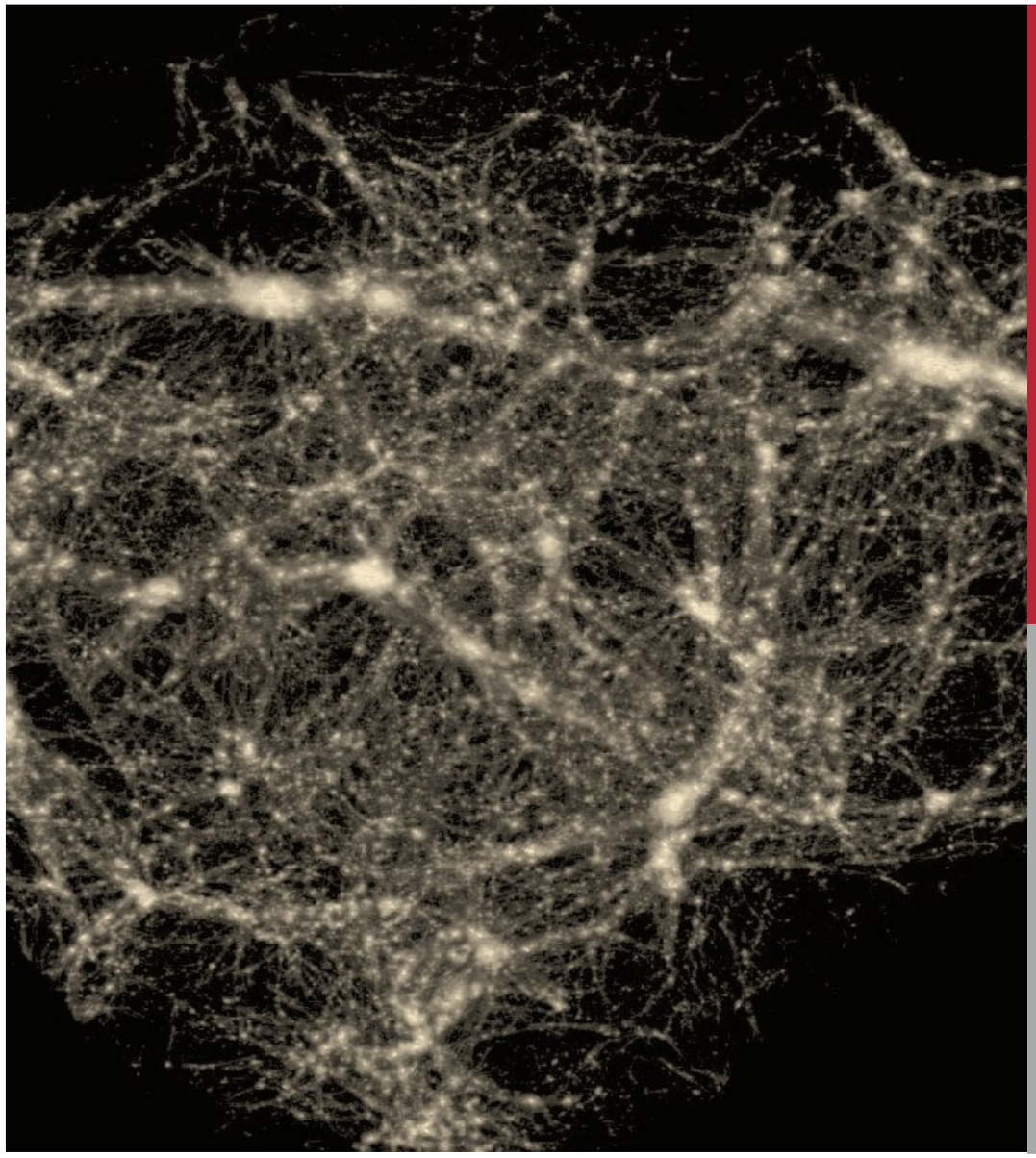

Dark Nuclei

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About

- Based on two recent papers
  - *Dark Nuclei I: Cosmology and Indirect Detection* —1406.2276
  - *Dark Nuclei II: Nuclear Spectroscopy* in Two-Colour QCD — 1406.4116
- Work in collaboration with *Matthew McCullough* & *Andrew Pochinsky*

Two-colour QCD

- Composite nature of baryonic matter motivates consideration of composite models for dark matter
- Here focus on two-colour QCD with two flavours of fundamental fermions
  - Numerically feasible (cheaper than QCD)
  - Recently considered in this context by Lewis et al., Neil & Buckley, Hietanen et al.
- Also investigations of quenched N_c=4 QCD and other theories in this context

Symmetries of two-colour QCD

- Global flavour symmetry $SU(2)_L x SU(2)_R$ enlarges to SU(4)
  - Pseudo-reality of SU(2) - right and left handed quarks can be combined into multiplets

$$\Psi = \begin{pmatrix} u_L \\ d_L \\ -i\sigma_2 C \bar{u}_R^\top \\ -i\sigma_2 C \bar{d}_R^\top \end{pmatrix} \qquad \Psi \xrightarrow{SU(4)} \exp\left(i \sum_{j=1}^{15} \theta_j T_j \right) \Psi$$

- Strong interactions result in condensate that spontaneously breaks the global symmetry: SU(4)→ Sp(4)~SO(5) [Peskin 1980]
- Numerical calculations have significant explicit symmetry breaking: $m_u = m_d \sim \Lambda_{QC_2D}$

Spectrum

- Simplest colour singlets
  - "Pions": $\pi^- \sim \bar{u}\gamma_5 d$, $\pi^0 \sim \bar{u}\gamma_5 u + \bar{d}\gamma_5 d$, $\pi^+ \sim \bar{d}\gamma_5 u$ $J^P = 0^-$
  - (anti-)"Nucleons": ud, $\bar{u}\bar{d}$ $J^P = 0^+$

  } Degenerate SO(5) multiplet
  - "Rhos": $\pi^- \sim \bar{u}\gamma_\mu d$, $\pi^0 \sim \bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d$, $\pi^+ \sim \bar{d}\gamma_\mu u$ $J^P = 1^-$
  - (anti-)"Deltas": $u\gamma_\mu\gamma_5 d$, $\bar{u}\gamma_\mu\gamma_5\bar{d}$ $J^P = 1^+$

  } Degenerate SO(5) multiplet
  - Axial vector, scalar, tensor mesons + associated baryons
- Spectrum studied by [Hietanen et al. 1404.2794]
  - Pion multiplet are Goldstone bosons of χSB: SU(4) → Sp(4)

Spectrum

- Colour singlets can combine
  - Two-, three-, … particle scattering states
  - "Nuclei" for sufficiently attractive interactions–not *a priori* obvious
- Two "pions" combine to give 25 of states: $\mathbf{5}\otimes\mathbf{5}=\mathbf{1}\oplus\mathbf{10}\oplus\mathbf{14}$
  - J=0 systems, contains B=2,1,0,-1,-2 states
- "pion"+ "rho": J=1 systems with same flavour breakdown

$$\boldsymbol{D}^\mu = \begin{pmatrix} S^\mu_{+} & D^\mu_{2,0} & D^\mu_{1,0} & D^\mu_{1,-1} & D^\mu_{1,1} \\ \overline{D}^\mu_{2,0} & S^\mu_{-} & D^\mu_{-1,0} & D^\mu_{-1,-1} & D^\mu_{-1,1} \\ \overline{D}^\mu_{1,0} & \overline{D}^\mu_{-1,0} & S^\mu_{0} & D^\mu_{0,-1} & D^\mu_{0,1} \\ \overline{D}^\mu_{1,-1} & \overline{D}^\mu_{-1,-1} & \overline{D}^\mu_{0,-1} & S^\mu_{B} & D^\mu_{0,2} \\ \overline{D}^\mu_{1,1} & \overline{D}^\mu_{-1,1} & \overline{D}^\mu_{0,1}+ & \overline{D}^\mu_{0,2} & S^\mu_{\overline{B}} \end{pmatrix}$$

$$\boldsymbol{D}^\mu_{\mathbf{1}} = \mathrm{Tr}(\boldsymbol{D}^\mu)\ ,$$
$$\boldsymbol{D}^\mu_{\mathbf{10}} = \frac{i}{2}\left(\boldsymbol{D}^\mu - \boldsymbol{D}^{\mu T}\right)\ ,$$
$$\boldsymbol{D}^\mu_{\mathbf{14}} = \frac{1}{2}\left(\boldsymbol{D}^\mu + \boldsymbol{D}^{\mu T}\right) - \frac{1}{5}\,\mathrm{Tr}(\boldsymbol{D}^\mu)\mathbb{1}_5$$

- Higher body systems: J=0,1, flavour = $\underbrace{\square\square\square\cdots\square\square}_{n}$, n=2,…,8

Simulations

- Wilson gauge and fermion actions
- HMC using modified **`chroma`**
- 4 β values, 6 masses
- 3 or 4 volumes per choice (β ,m_0)
- Long streams of configurations

Label	β	m_0	$L^3 \times T$	N_{traj}
A	1.8	-1.0890	$12^3 \times 72$	5,000
			$16^3 \times 72$	4,120
			$20^3 \times 72$	3,250
B	2.0	-0.9490	$12^3 \times 48$	10,000
			$16^3 \times 48$	4,000
			$20^3 \times 48$	3,840
			$24^3 \times 48$	2,930
C	2.0	-0.9200	$12^3 \times 48$	10,000
			$16^3 \times 48$	9,780
			$20^3 \times 48$	10,000
D	2.0	-0.8500	$12^3 \times 48$	9,990
			$16^3 \times 48$	5,040
			$16^3 \times 72$	5,000
			$20^3 \times 48$	5,000
			$24^3 \times 48$	5,050
E	2.1	-0.7700	$12^3 \times 72$	5,000
			$16^3 \times 72$	5,000
			$20^3 \times 72$	4,300
F	2.2	-0.6000	$12^3 \times 72$	5,000
			$16^3 \times 72$	5,000
			$20^3 \times 72$	5,000
			$24^3 \times 72$	5,070

Multi-baryon spectrum

- Extract spectrum of multi-baryon states from correlators

$$C_{nN}(t) = \left\langle 0 \left| \left(\sum_{\mathbf{x}} \mathcal{O}_N^{\mathcal{P}}(\mathbf{x}, t) \right)^n \left(\mathcal{O}_N^{\mathcal{S}\dagger}(\mathbf{x}_0, t_0) \right)^n \right| 0 \right\rangle$$

$$C_{nN,\Delta}^{(i,j)}(t) = \left\langle 0 \left| \left(\sum_{\mathbf{x}} \mathcal{O}_N^{\mathcal{P}}(\mathbf{x}, t) \right)^n \sum_{\mathbf{x}} \mathcal{O}_{\Delta_j}^{\mathcal{P}}(\mathbf{x}, t) \left(\mathcal{O}_N^{\mathcal{S}\dagger}(\mathbf{x}_0, t_0) \right)^n \mathcal{O}_{\Delta_i}^{\mathcal{S}\dagger}(\mathbf{x}_0, t_0) \right| 0 \right\rangle$$

$$\mathcal{O}_{\{N,\Delta_i\},s}(\mathbf{x}, t) = \psi_u^\top(\mathbf{x}, t)(-i\sigma_2)C\{1, \gamma_i\gamma_5\}\psi_d(\mathbf{x}, t)$$

  - Local and smeared sources and sinks

- Aim to extract ground state, but need to be careful of thermal effects (use full thermal behaviour of correlator, multiple T's)

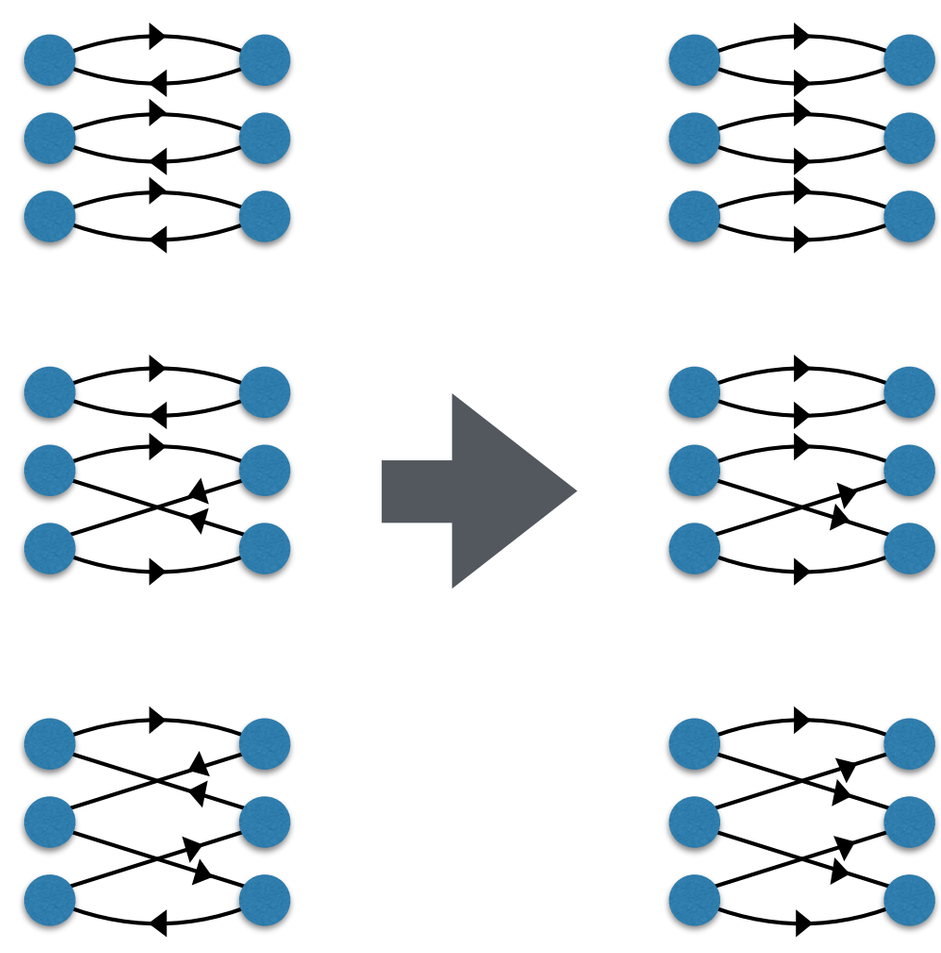
- Final method - correlated single exp fit of shortened time range

Multi-baryon contractions

- SU(2) multi-baryon contractions equivalent to maximal isospin multi-meson contractions
  - Clear from degeneracies but explicitly

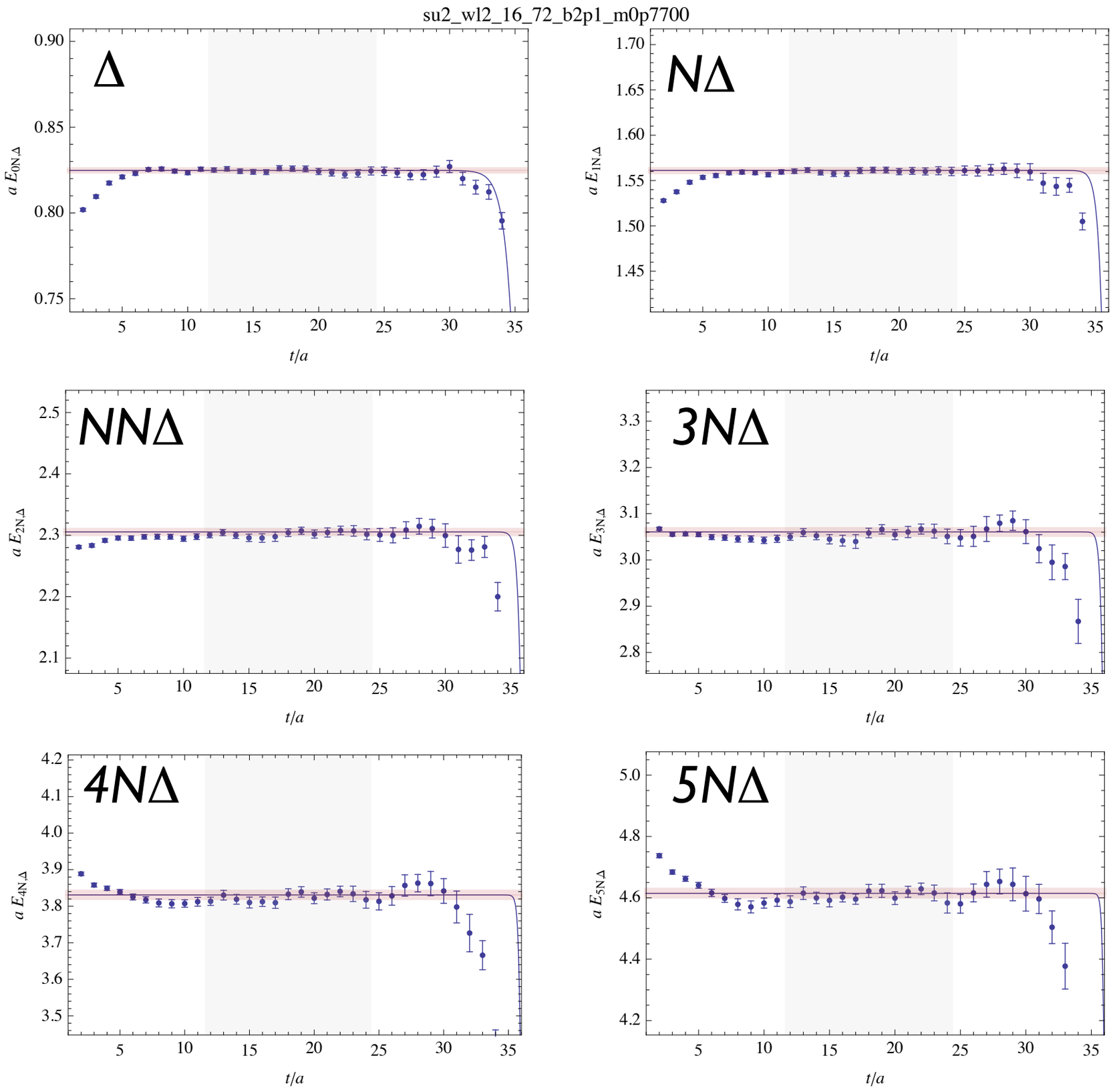
$$S(y,x) = C^\dagger(-i\sigma_2)^\dagger S(x,y)^T(-i\sigma_2)C$$
$$S(y,x) = \gamma_5 S^\dagger(x,y)\gamma_5$$

- Use algorithms from N_c=3 QCD
  [WD & Savage 2011, WD Orginos, Shi 2012]
- (n-1)$N\Delta$ ~ mixed pion-kaon contractions
  [WD & Smigielski 2011]

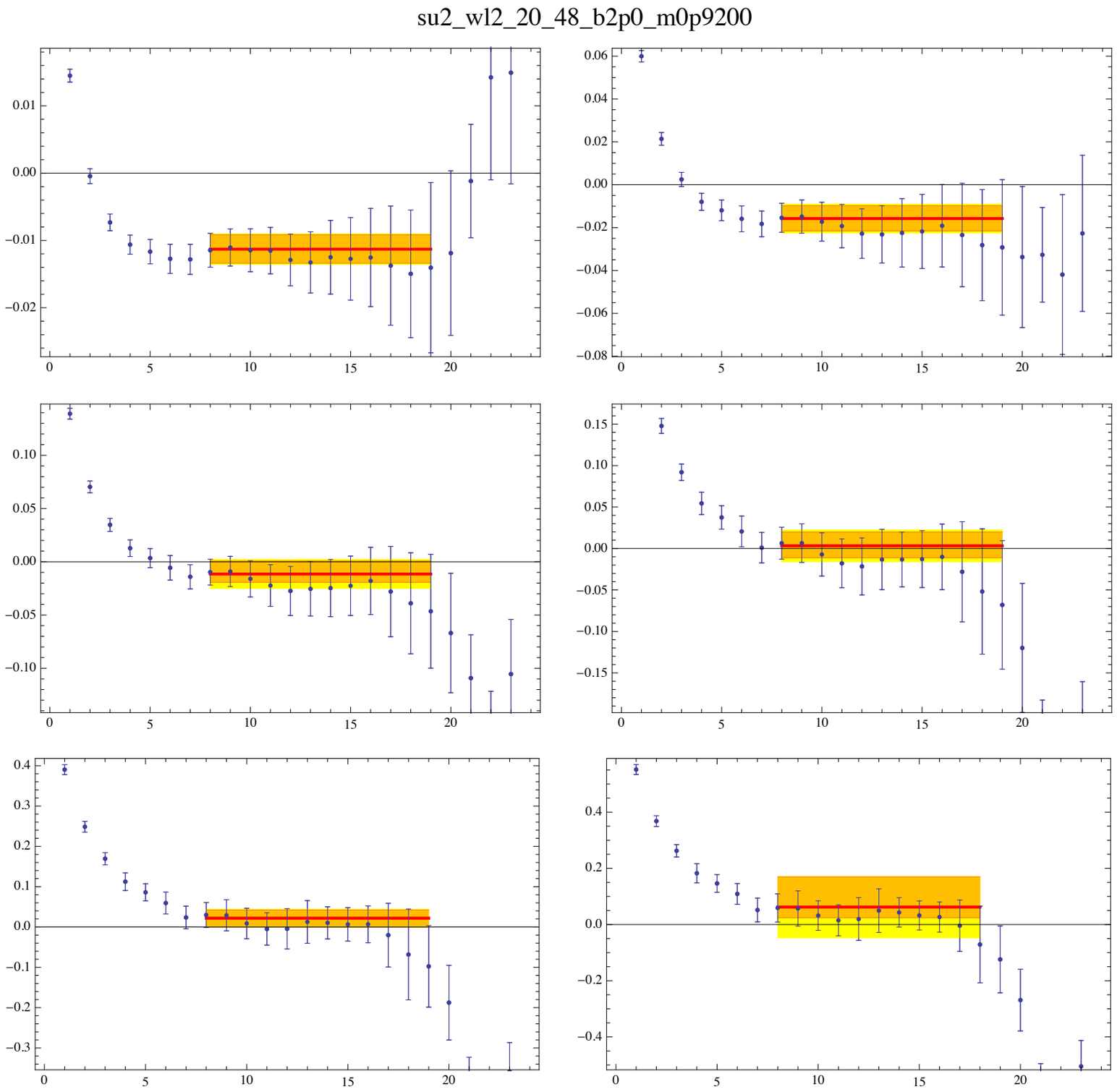


Ex:: three types of contractions for I=3 $\pi\pi\pi$ and NNN

Example effective mass plots

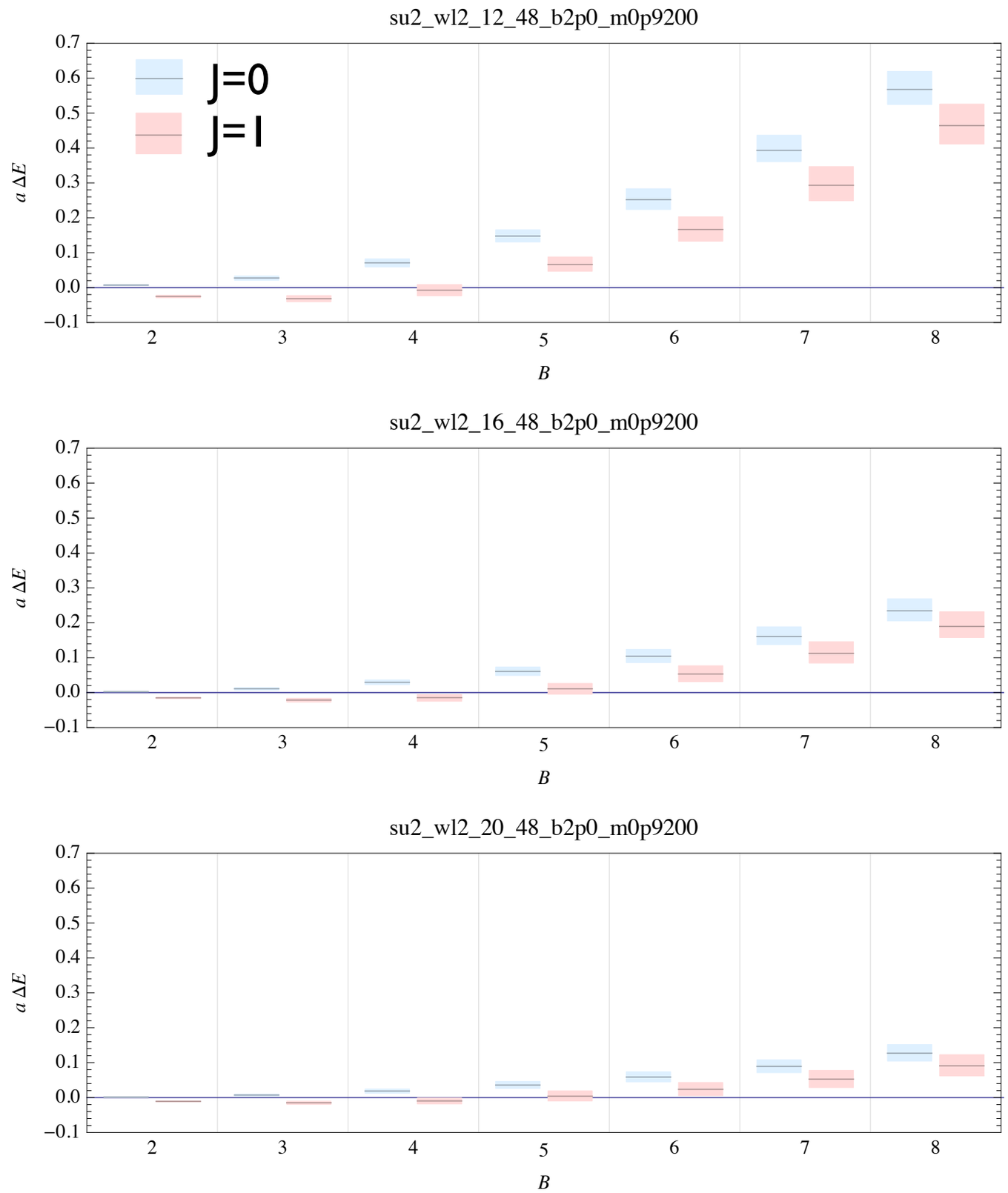


Example effective mass plots



Energy shifts for different volumes

Increasing volume



To be bound or not to be bound…

- If bound/scattering state, expect

$$H_1: \quad \Delta E_{\rm bound}(L) = -\Delta E_\infty \left[1 + C\,\frac{e^{-\kappa L}}{L}\right],$$

$$H_2: \quad \Delta E_{\rm scatter}(L) = \frac{2\pi A}{\mu L^3}\binom{n}{2}\left[1 - \left(\frac{A}{\pi L}\right)\mathcal{I} + \left(\frac{A}{\pi L}\right)^2 [\mathcal{I}^2 + (2n-5)\mathcal{J}]\right] + \frac{B}{L^6}$$

- Assess support for each hypothesis using the Bayes factor*

$$K = \frac{P(D|H_1)}{P(D|H_2)} = \frac{\int P(D|H_1,p_1)P(p_1|H_1)dp_1}{\int P(D|H_2,p_2)P(p_2|H_2)dp_2}$$

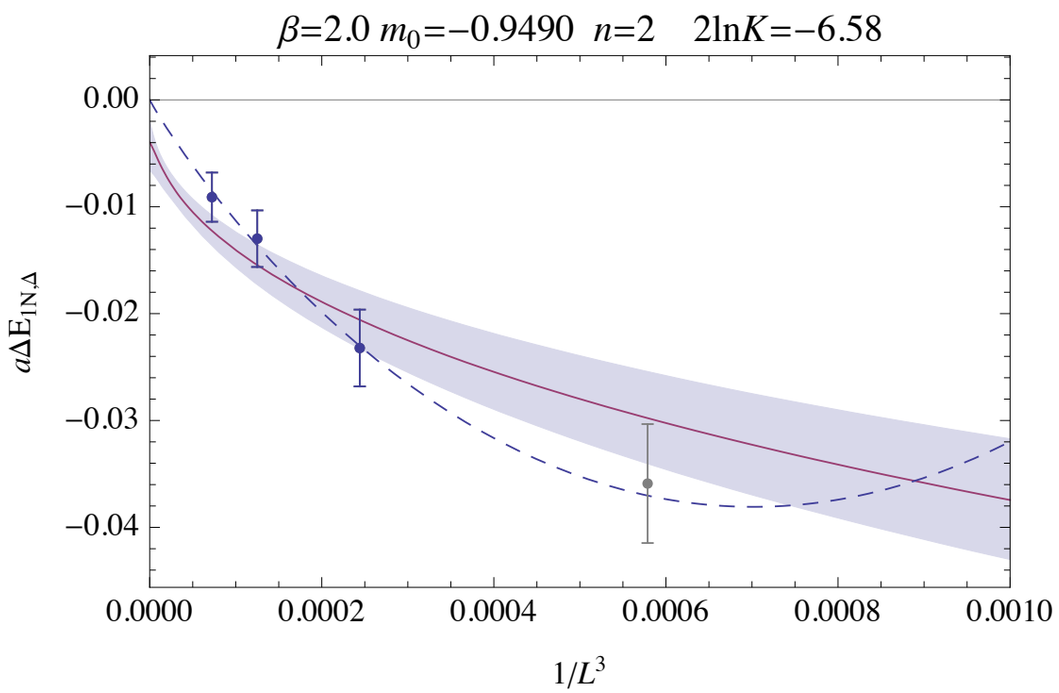
where $\log P(D|H_i,p_i) = -\frac{1}{2}\sum_{j=1}^{N}\frac{[d_j - H_i(x_j;p_i)]^2}{\sigma_j^2}$

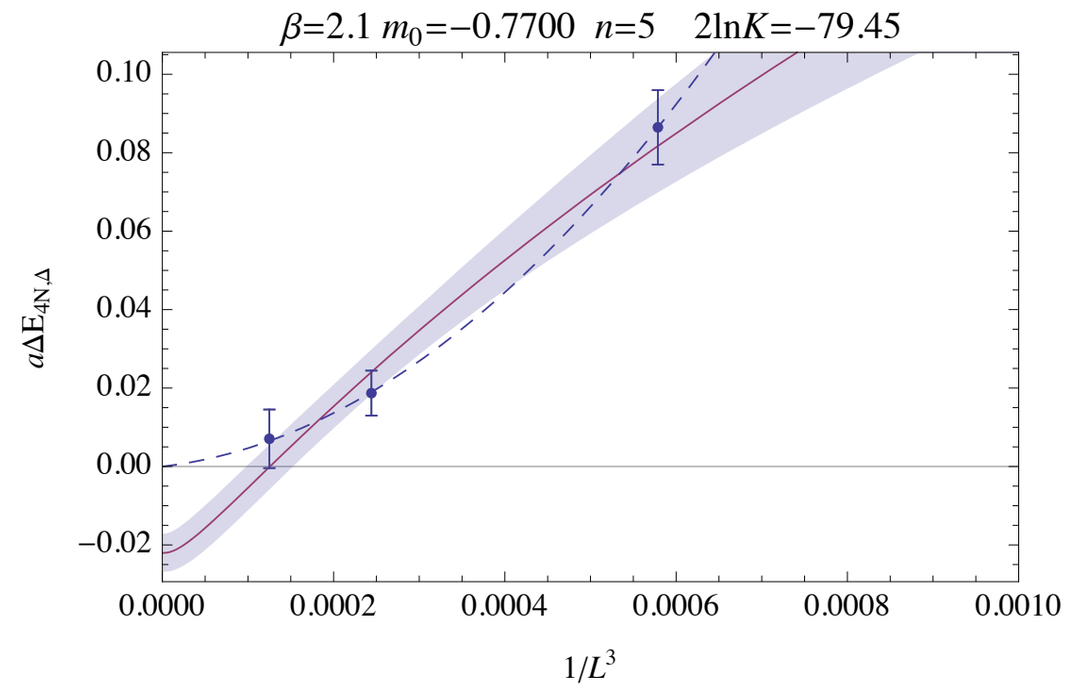
and $P(p_i|H_i)$ are broad prior distributions

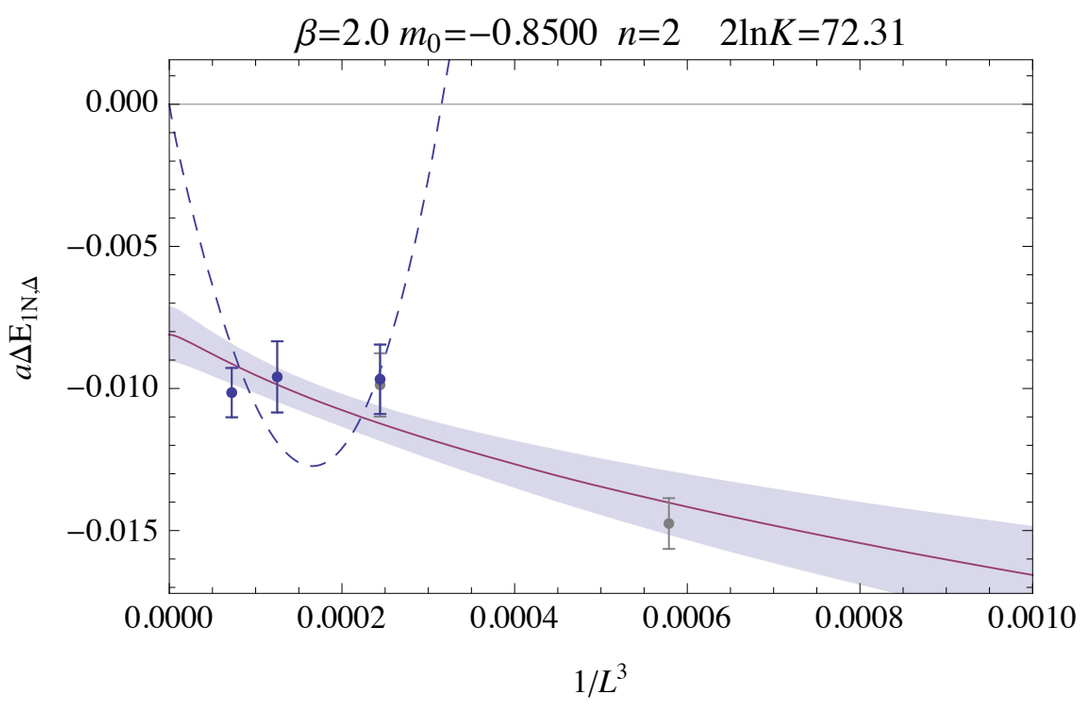
- If *2 ln[K] > 6* :"strong evidence" of preference for H_1 over H_2 then ask what are the bounds on the binding energy

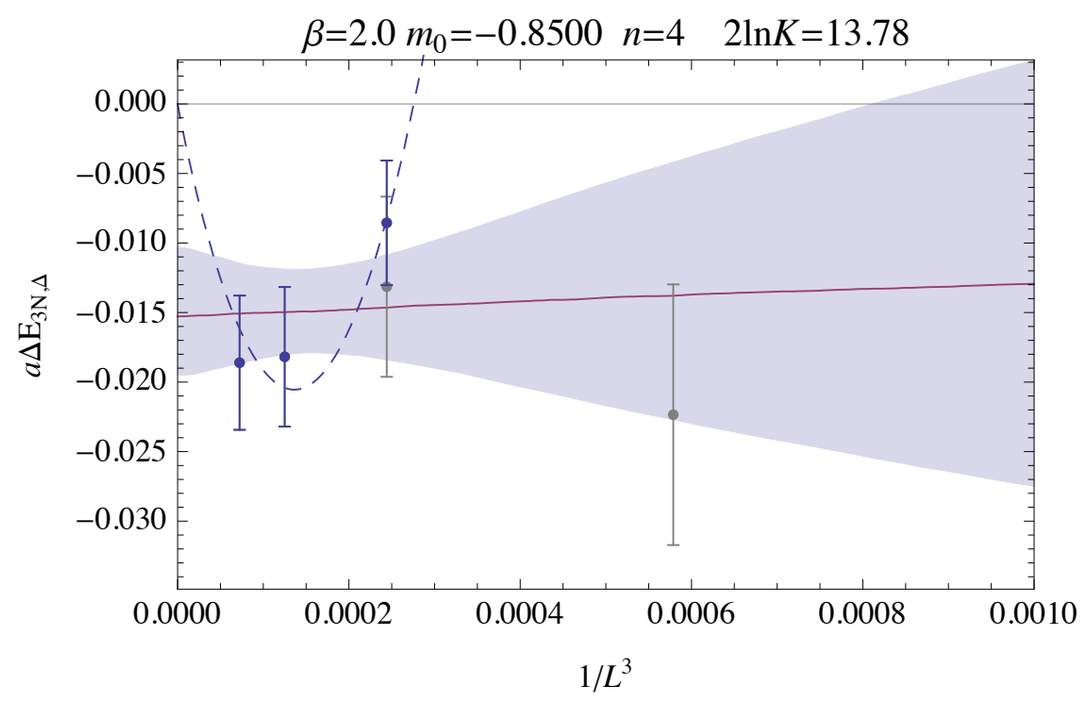
*Kass and Raftery (1995). *Journal of the American Statistical Association* **90** (430): 791.

Infinite volume extrapolations





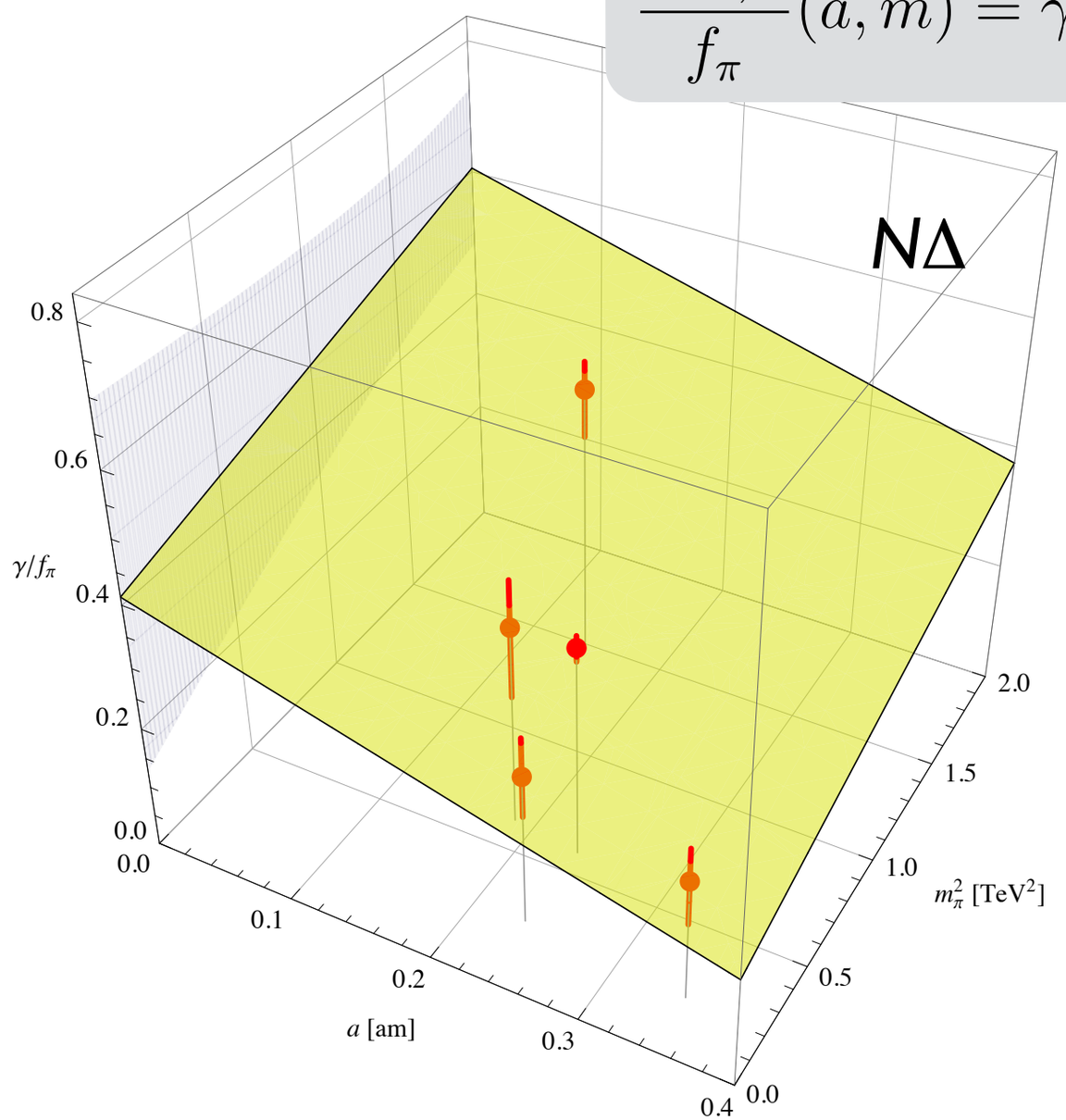


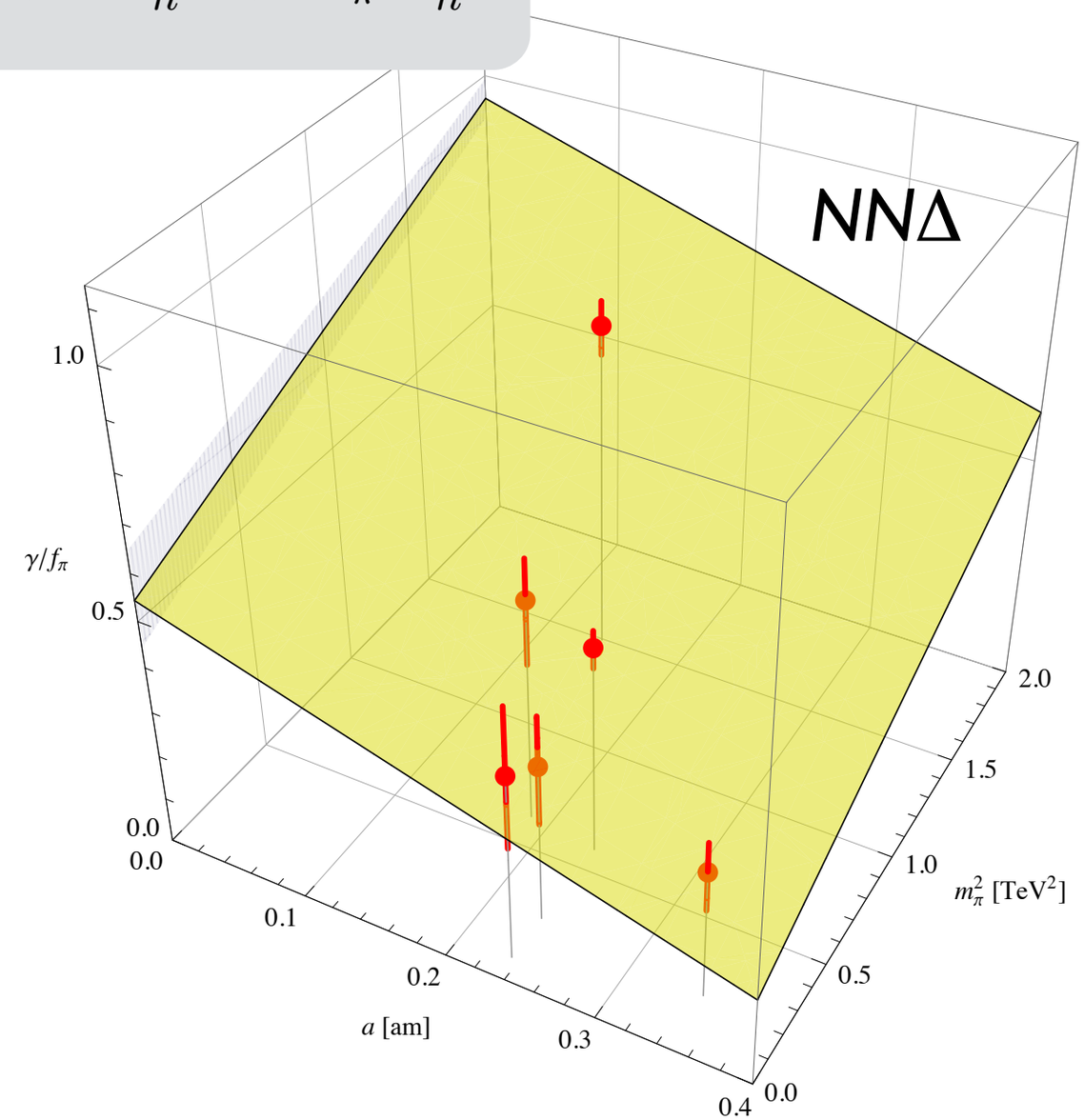


Continuum extrapolations

- Simple continuum limit extrapolation of binding momentum, γ

$$\frac{\gamma_{nN,\Delta}}{f_\pi}(a,m) = \gamma^{(0)}_{nN,\Delta} + a\ \delta^{(a)}_n + {m_\pi}^2 \delta^{(m)}_n$$

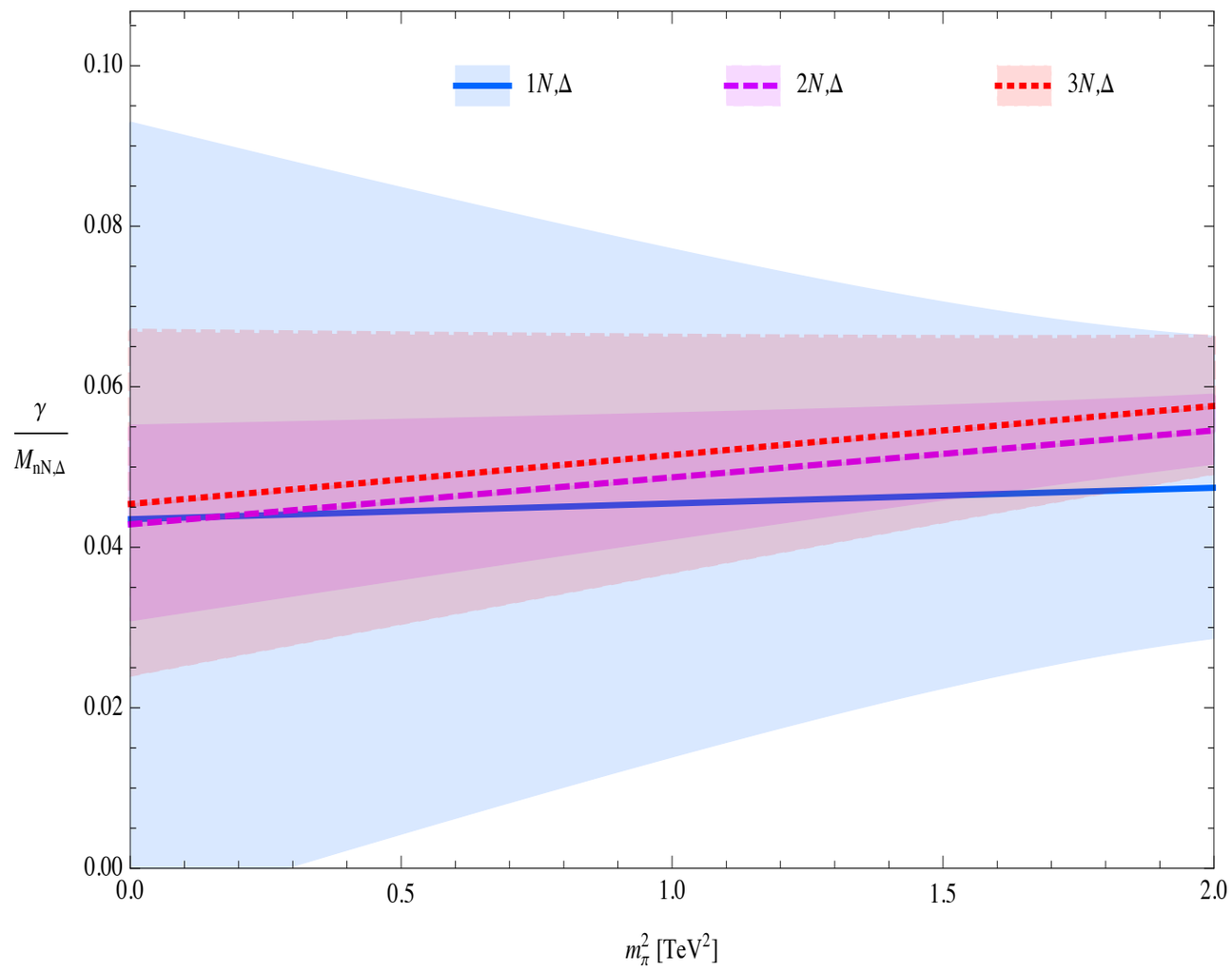




NB: physical scale set by demanding f_π=246 GeV (arbitrary)

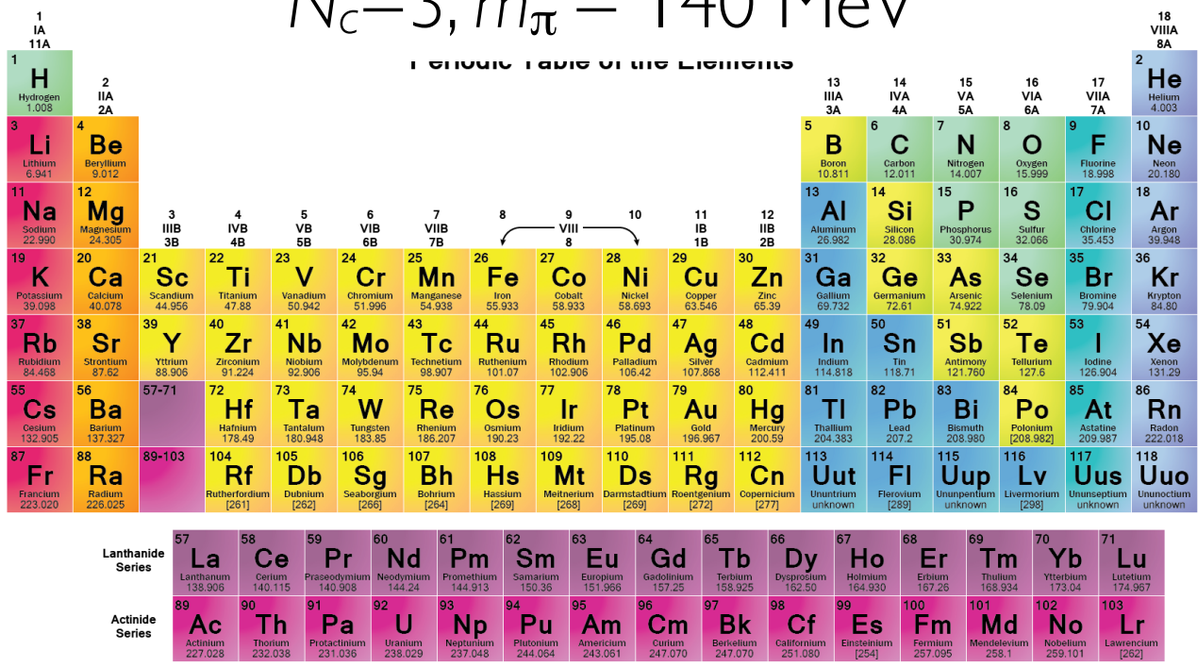
Dark nuclei

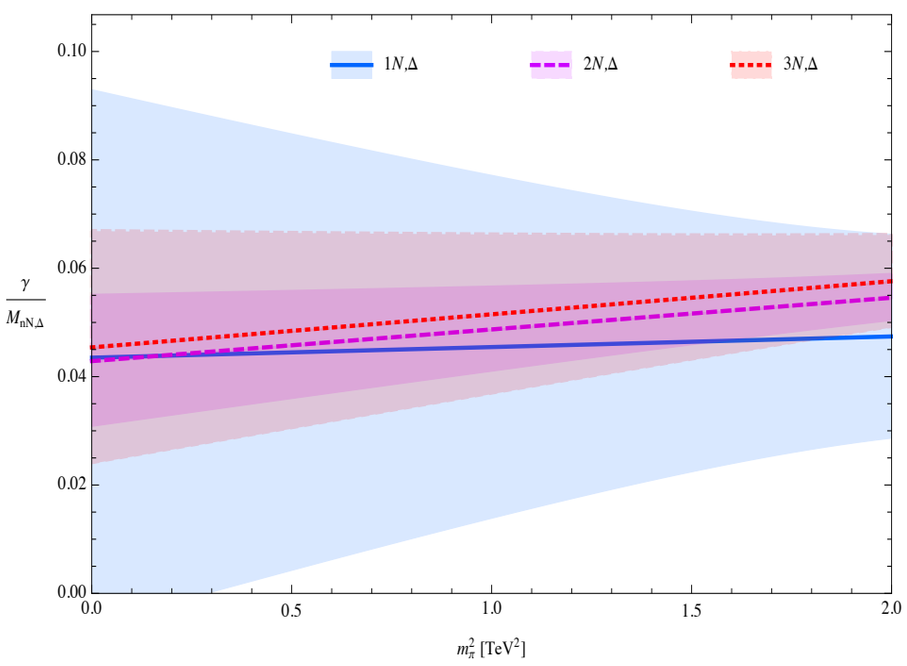
- J=0 nuclei: very likely unbound (small K, all positively shifted)
- J=1, strong evidence for bound states for B=2,3, perhaps 4
  B=5,..,8 seem unbound
  - Bindings decrease with quark mass and increase towards continuum
  - Strength of binding is significant w.r.t. mass
- Nuclear states with other quantum #s may also be bound



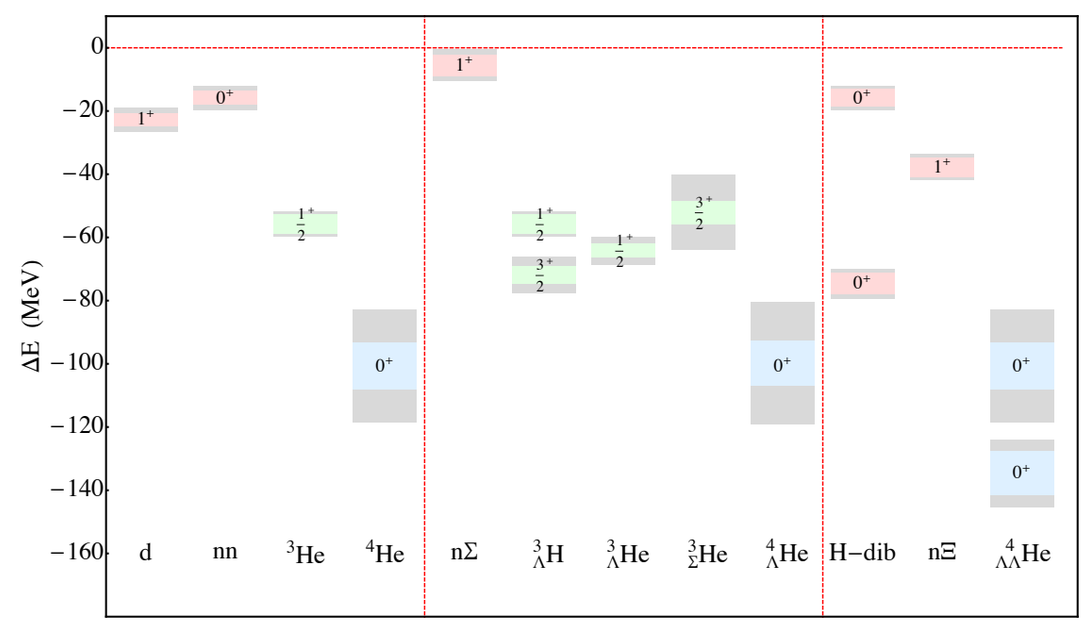
The ubiquity of nuclei?

N_c=3, m_π = 140 MeV

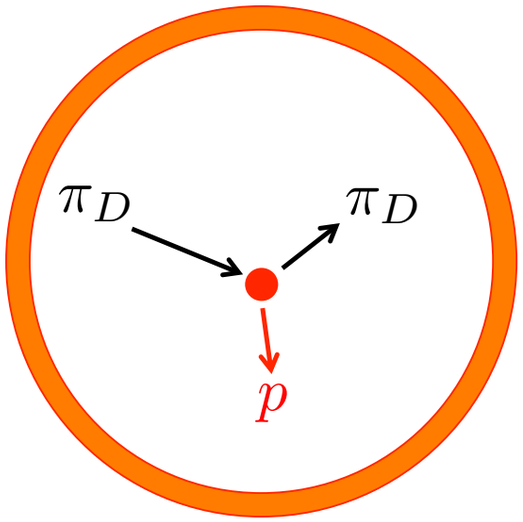
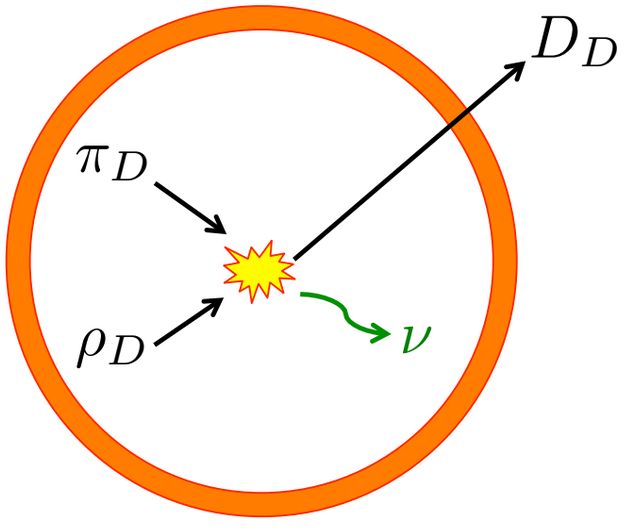




N_c=2, m_π = 1 TeV



N_c=3, m_π = 400–800 MeV

Phenomenology

- Presence of nuclear binding energies: new scale for phenomenology can be significantly different than Λ_{QC^2D}
- New processes in dark sector: dark nucleosynthesis, dark capture processes
  - Modify early universe cosmology (both symmetric & asymmetric scenarios)
- Significant modifications to dark matter capture in astrophysical bodies
- Very rich phenomenology!