The Multiverse and Scale-Factor Cutoff Measure

— Alan Guth —
MIT
Massachusetts Institute of Technology

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Successes of Inflation

Alan Guth
Massachusetts Institute of Technology

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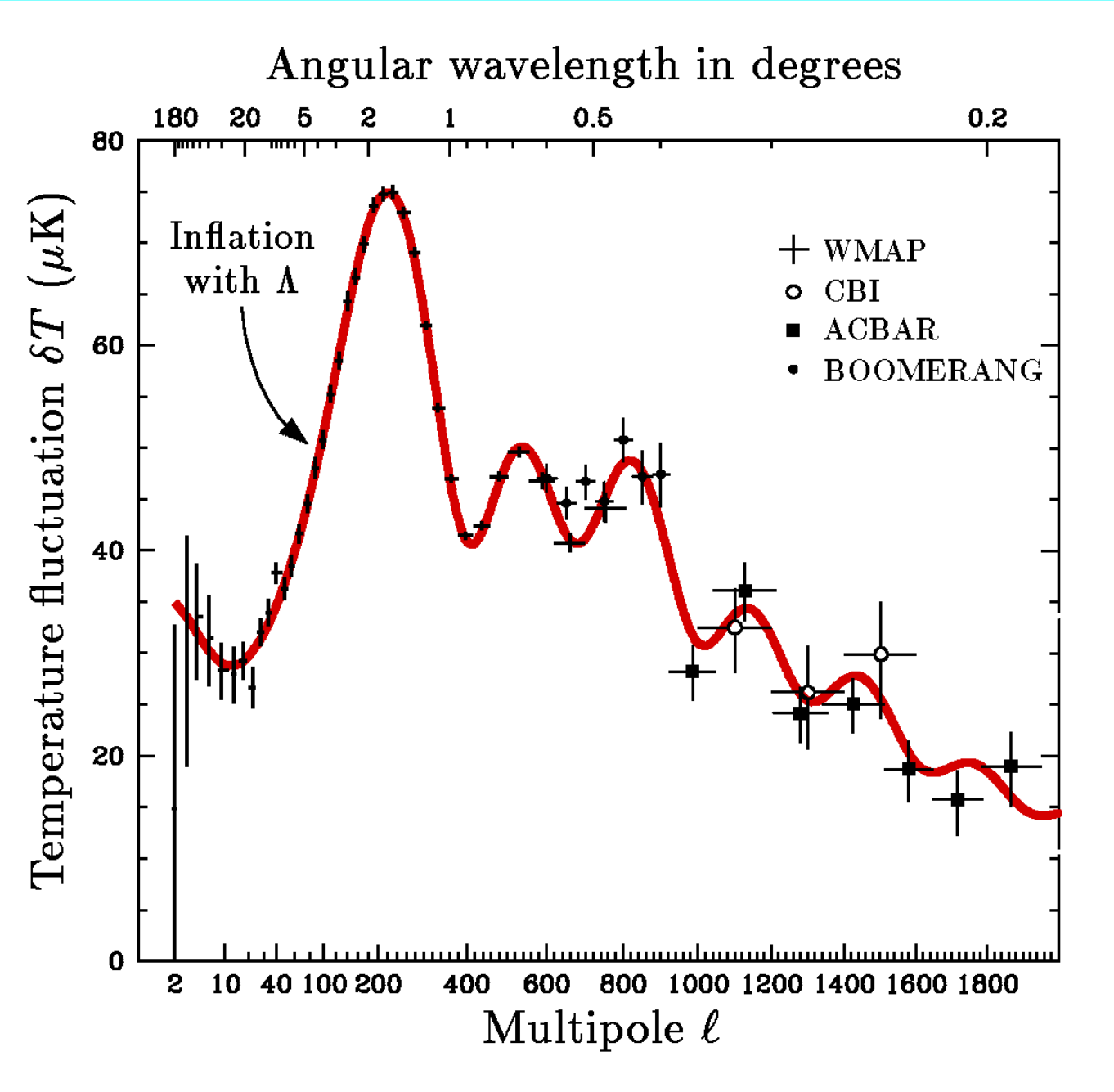
6) Nearly scale invariant, adiabatic, Gaussian density perturbations.

CMB: Comparison of Theory and Experiment



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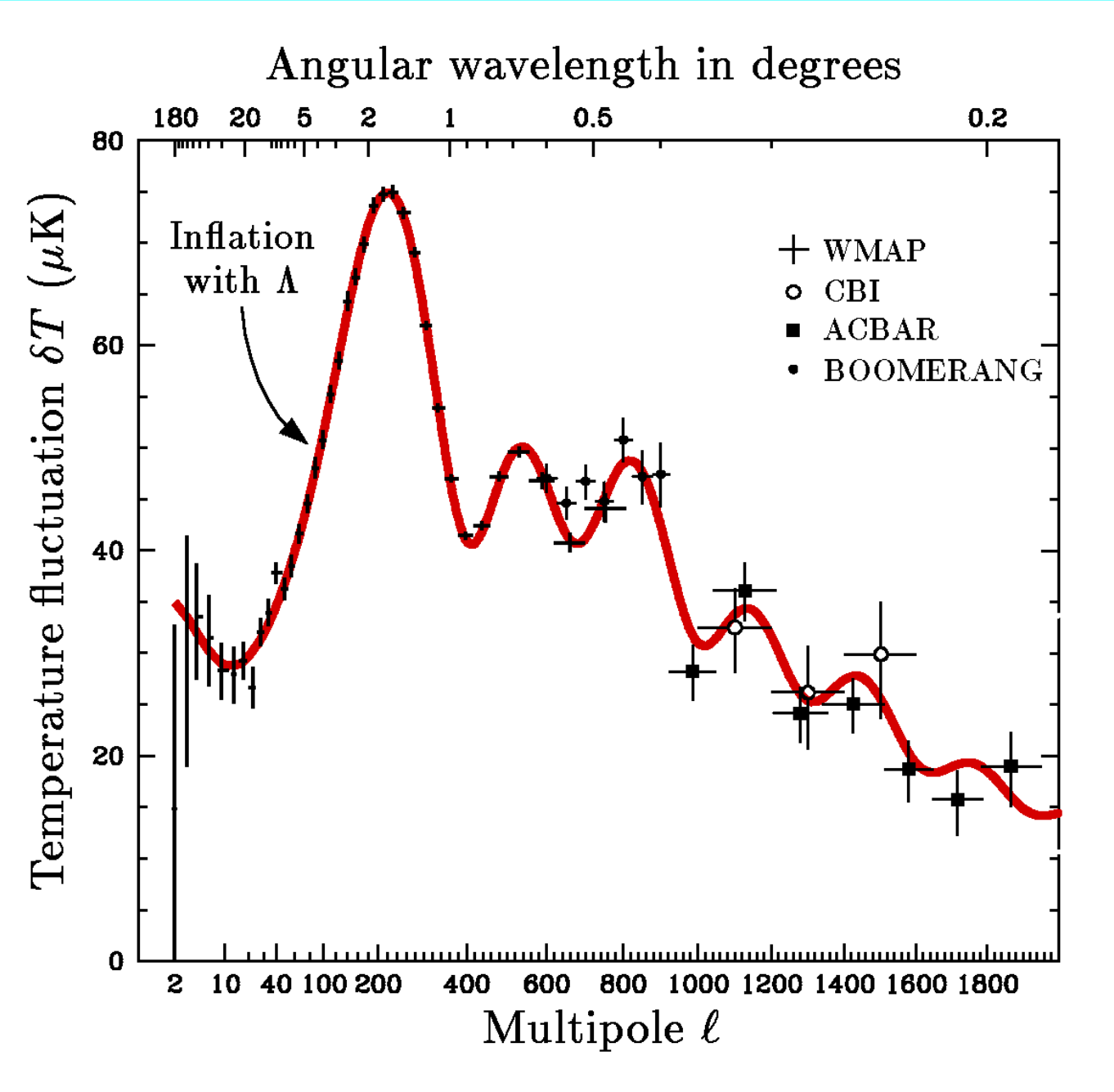

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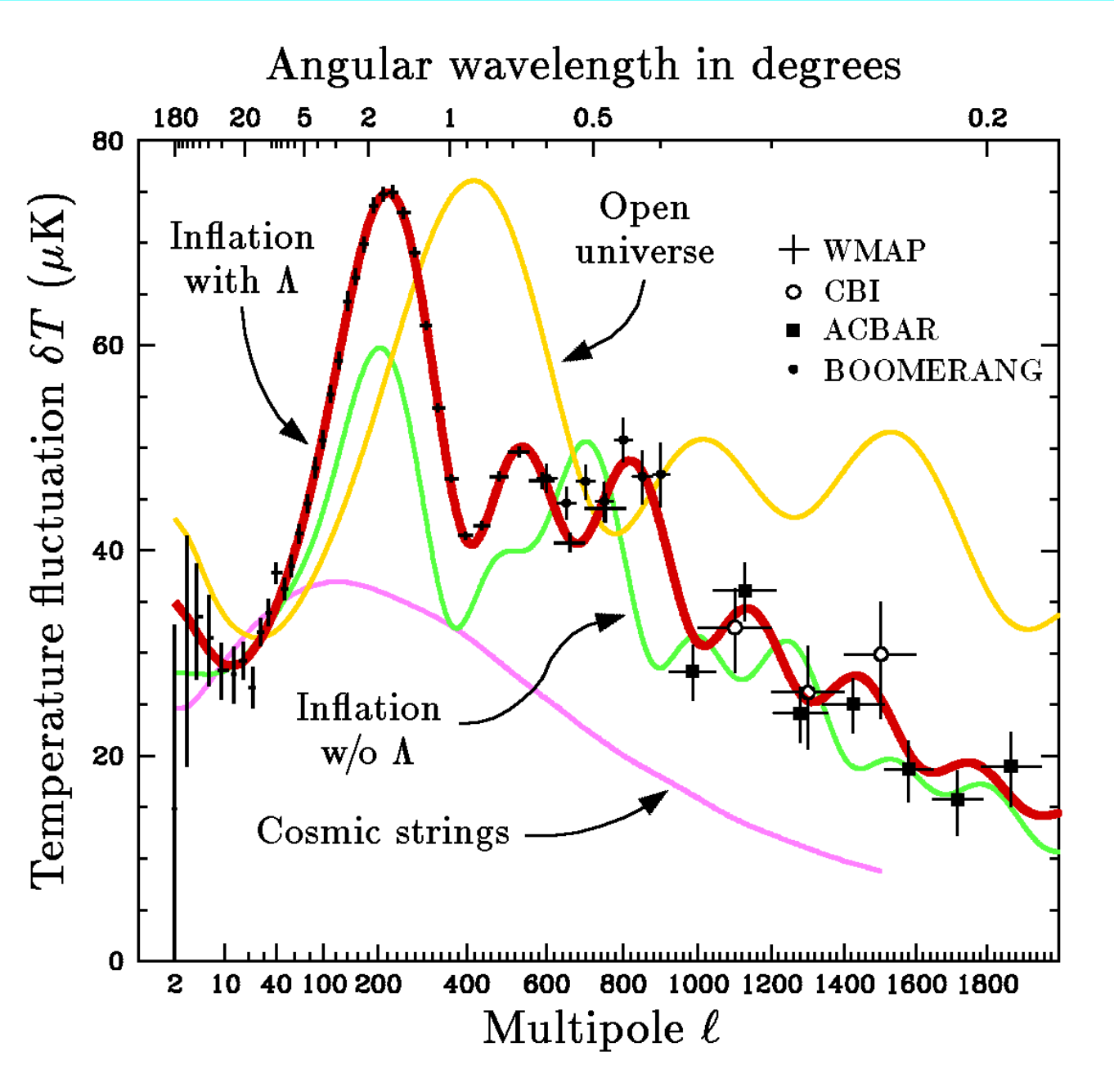

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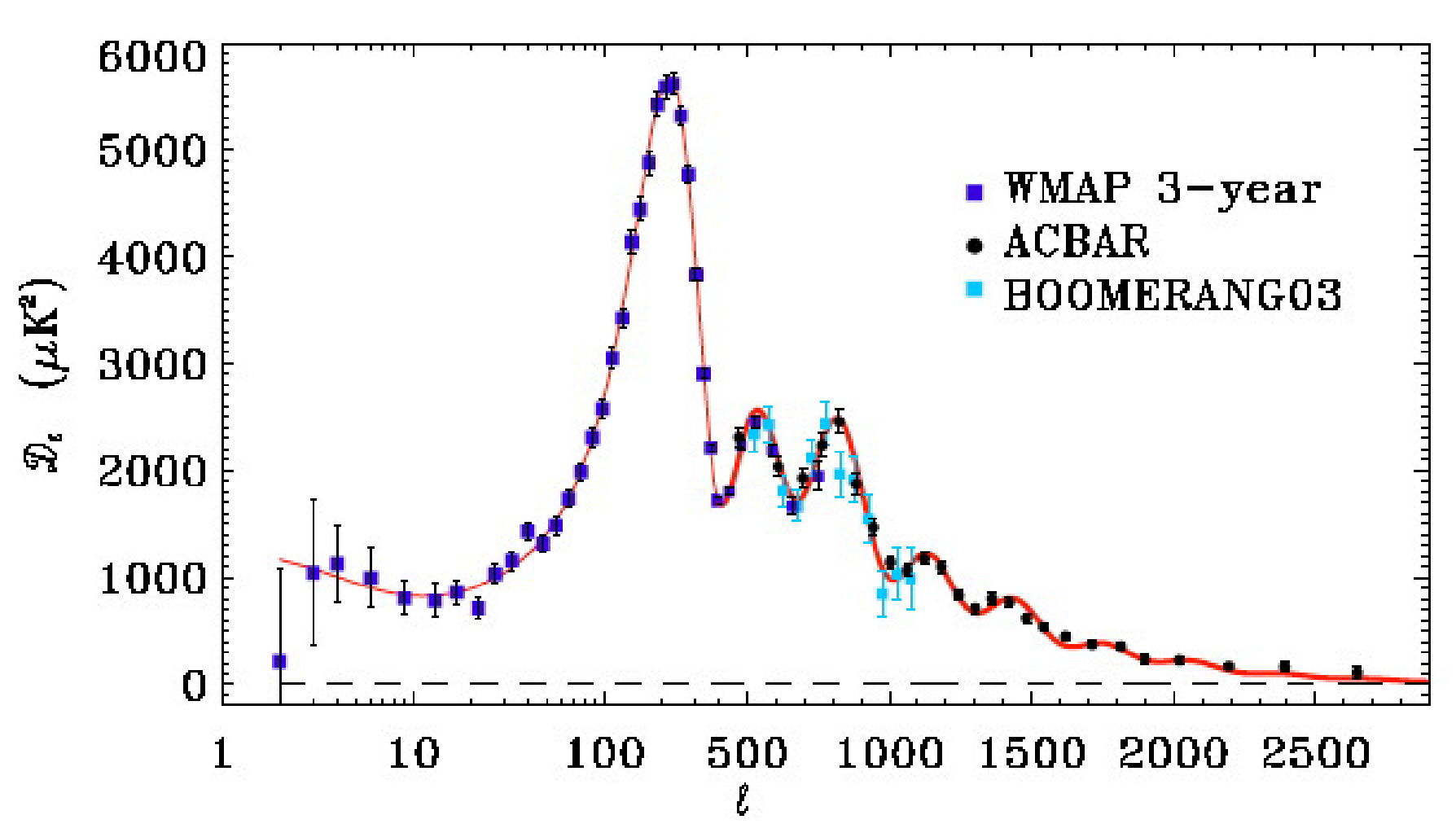


Eternal Inflation

- Essentially all models of inflation lead to (future) eternal inflation.
- Crude explanation: inflating false vacuum is metastable, but decay rate $\ll$ expansion rate. Therefore the volume of false vacuum increases exponentially with time.
- Once inflation starts, it never stops. The inflating region never disappears, but pieces of it undergo decay and produce "pocket universes" ad infinitum.

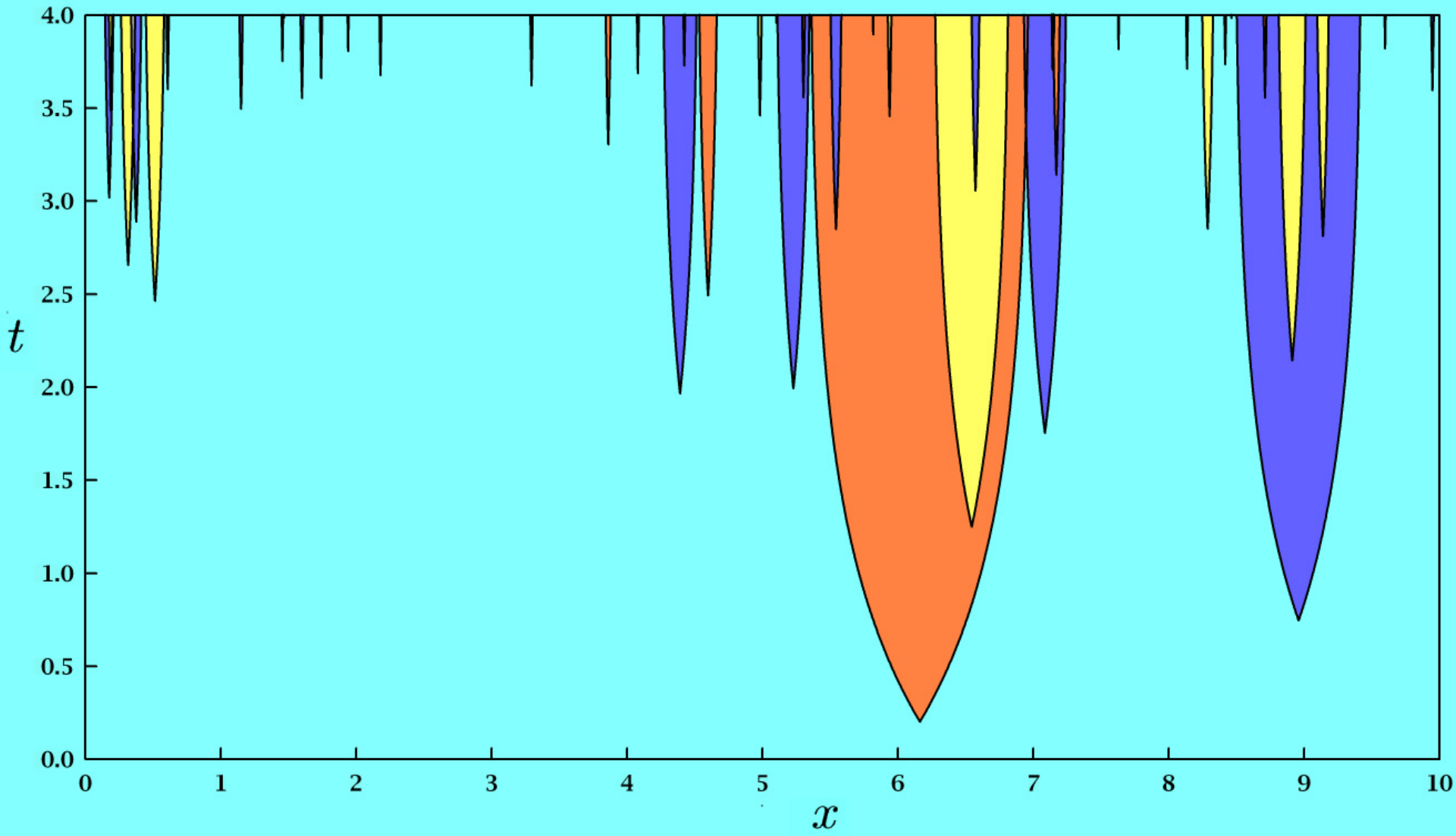
Eternal Inflation and the String Theory Landscape

- ☆ String theorists say that there are $\sim 10^{500}$ long-lived metastable vacua of string theory. Each pocket universe can be filled with a different vacuum.
- ☆ Instead of one universe, inflation produces an infinite number—

A Multiverse

And the multiverse can populate the landscape.

Bubble Nucleation in an Eternally Inflating Universe



Need for a Measure

- Anything that can happen will happen — an infinite number of times!
- To separate the probable from the improbable, we need to compare infinities.
- Example of ambiguity: what fraction of the positive integers are odd? Normal answer: 1/2. BUT, consider listing the integers as $1, 3,\ 2,\ 5, 7,\ 4,\ 9, 11,\ 6\ , \ldots$, always writing two odds and then the next even. Each positive integer appears once and only once, yet it looks like 2/3 of the integers are odd!
- Probabilities can be defined by introducing a cutoff, but in this case the answers can depend strongly on the type of cutoff.

Wave Function of the Universe?

- Formalisms such as the wave function of the universe do not settle this question.
- The formalism defines the probabilities of spacelike slices. But, if eternal inflation occurs, typical slices will each have an infinity of observers on them, with an infinite number observing X and an infinite number observing Y.
- The relative probability of observing X or Y remains undefined.

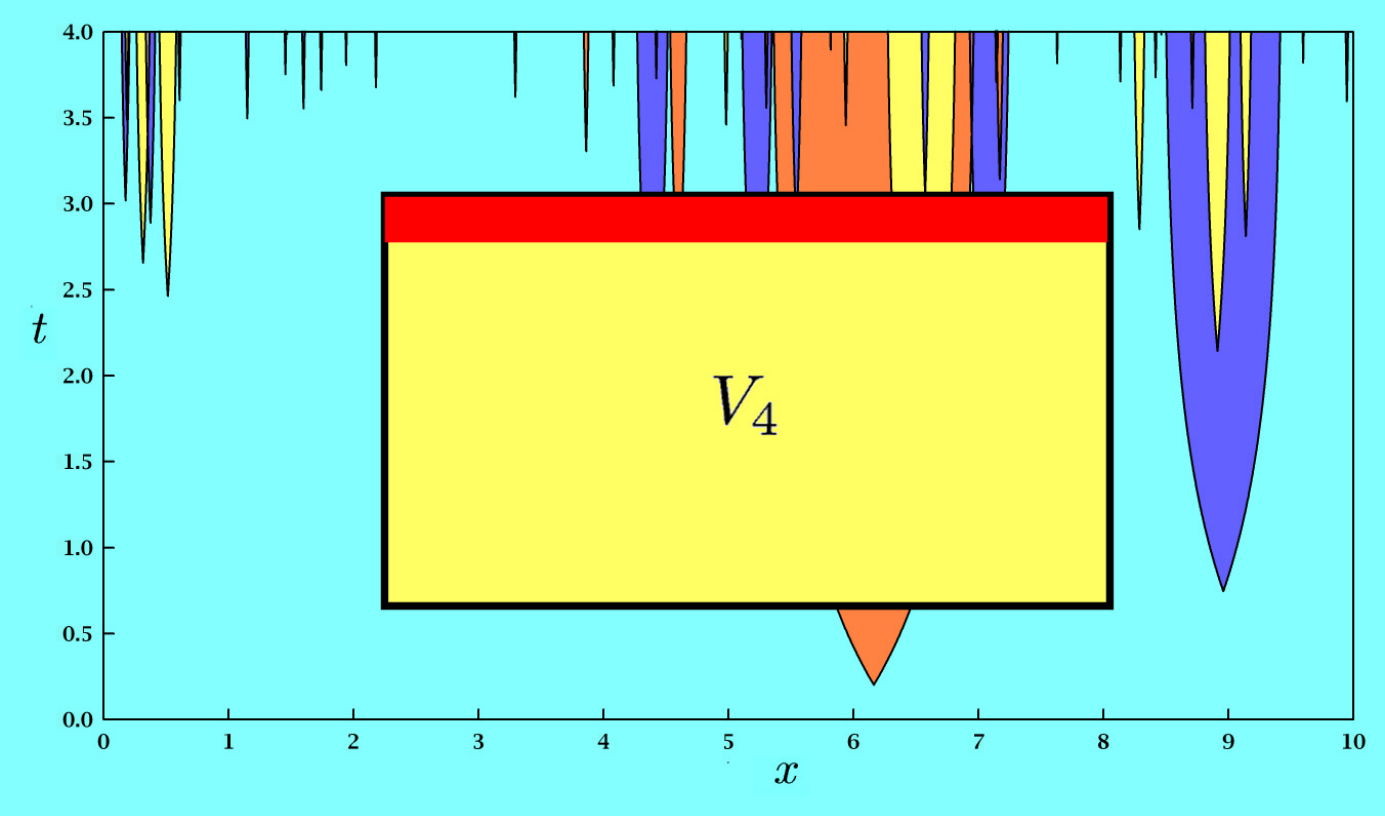
Constructing a Measure

- What is a measure, exactly — It is a prescription for counting the relative number of events of different types.
- For example: of all acceleratores in the multiverse that resemble the LHC in some specified way, what is the fraction that (a) see the standard model Higgs, (b) see no Higgs, or (c) see a Higgs system of some other description?

A Simple Strategy

- Choose a finite spacetime volume V_4, calculate the desired ratio of events in the finite volume, and then take the limit as $V_4 \to \infty$.
- Failure: the resulting answer depends crucially on the method for choosing and enlarging the spacetime volume.

Reason for Difficulty



Due to the exponential expansion, V_4 is always dominated by the final time surface. So the method of selecting the surface is always crucial.

Search for a Recipe

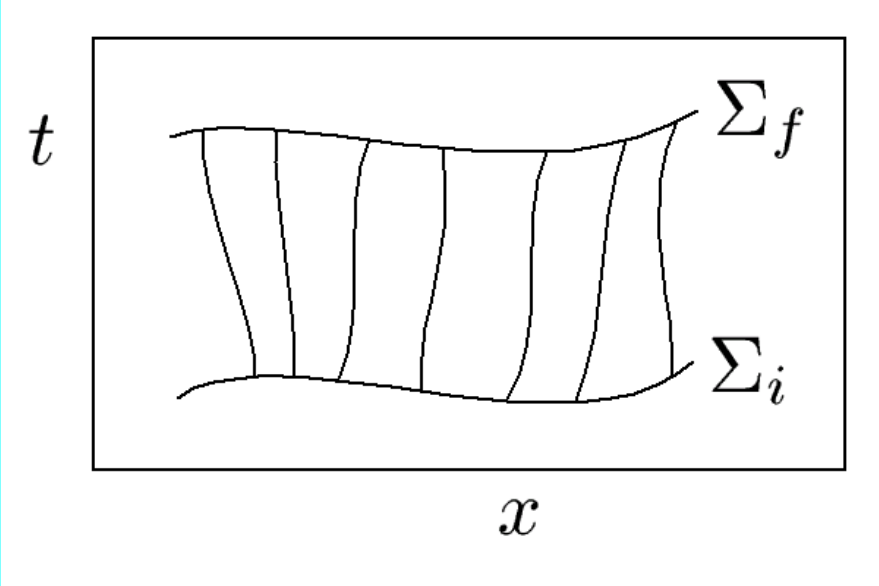
- The problem is ~25 years old. Very frustrating! Even if the evolution of the multiverse is fully described, predictions are still impossible.
- Is there a fundamental principle to define probabilities? **Not yet.** There is some hope offered by Garriga and Vilenkin's *Holographic Multiverse* (0809.4257 hep-th).
- Current Status: try out ideas and reject those that conflict with observation or known physics.

Example 1: Proper Time Cutoff

1) Choose initial finite spacelike hypersurface Σ_i.

2) Construct family of timelike fiducial geodesics normal to it, projected into the future.

3) Cut off the fiducial geodesics on a final hypersurface Σ_f, defined by proper time $\tau = \tau_c$.



4) Calculate desired ratios, and then let $\tau_c \to \infty$.

☆ If the region includes an eternal worldline, then it can be argued that the limit exists, and is independent of the initial hypersurface.

☆ Strongly favors large amounts of inflation. The fastest inflating state, and its decay products, will dominate.

Example 2: Scale-Factor Cutoff Measure

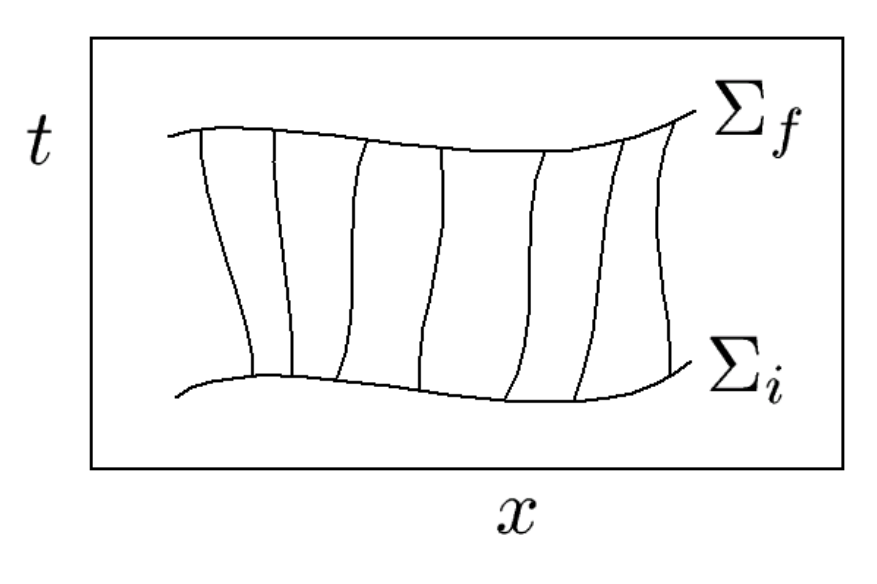
- Same method as proper time cutoff, but use scale-factor time $t_{\rm sf}$ instead of proper time τ.

$$\mathrm{d}t_{\rm sf} = \frac{1}{3}\, u^{\mu}_{;\mu}\, \mathrm{d}\tau \ ,$$

or equivalently

$$\rho \propto e^{-3t_{\rm sf}} \ ,$$

where ρ = density of fiducial worldlines.



- Since volumes grow as $e^{3t_{\rm sf}}$, regardless of inflation, large amounts of inflation are not favored. Instead, the state with the slowest decay rate is favored — called the dominant vacuum. It, and its "decay products," will dominate the volume. "Decay" includes tunneling to a high energy false vacuum.

Example 3: Pocket-Based Measures

1) Tag bubbles to be counted. One method is to project back to Σ_i along fiducial geodesics, and impose a cutoff on the projected volume.

2) Choose a method of specifying an allowed comoving volume in each bubble. Usually chosen to be proportional to e^{3N}, where N is the number of e-folds of slow roll inflation.

Example 4: Causal Patch Measures

1) Follow a single worldline, and define ratios of events in its causal diamond. Even when inflation is eternal, a single observer has an exponentially decreasing probability of continuing to inflate. With probability 1, his wordline will end in a terminal bubble — one with zero or negative vacuum energy.

2) Drawback(?) – one must average over initial conditions, using some theory that determines initial conditions. The attractor nature of inflation is lost.

Pitfalls of Measures: Youngness Problem

- Severe problem in the proper time measure.
- Problem: Since the available volume grows $\sim e^{3H\tau}$, where $H^{-1} \approx 10^{-37}$ s (for example), most bubbles in the sample formed within a few Hubble times of the final time cutoff. Older bubbles are **VERY** rare.
- Quantitatively, one can define local Robertson-Walker time τ_p for each spacetime point in any pocket universe. The available spatial volume for a given τ_p obeys

$$V(\tau_p) \propto \frac{a^3_{\rm pock}(\tau_p)\, P^{\rm survival}_{\rm pock}(\tau_p)}{a^3_{\rm parent}(\tau_p)\, P^{\rm survival}_{\rm parent}(\tau_p)} \ .$$

- Example of youngness bias: the "coolness" problem (Tegmark, astro-ph/0410281). I.e., the cosmic microwave background should be hotter. If we could have evolved one billion years earlier, when $T_{\rm CMB} = 2.9$K, $V(\tau_p)$ would be larger by about $\exp\left(10^{60}\right)$.
- Scale-factor cutoff measure has only a very mild (and tolerable) youngness bias.

$$V(t_{\rm sf}) \propto \frac{a^3_{\rm pock}(t_{\rm sf})\, P^{\rm survival}_{\rm pock}(t_{\rm sf})}{a^3_{\rm parent}(t_{\rm sf})\, P^{\rm survival}_{\rm parent}(t_{\rm sf})}$$

$$= \frac{P^{\rm survival}_{\rm pock}(t_{\rm sf})}{P^{\rm survival}_{\rm parent}(t_{\rm sf})} \; .$$

Pitfalls of Measures: Q Catastrophe

- Serious problem in proper time and pocket-based measures.
- If the measure strongly favors slow-roll inflation, then $Q = (\delta\rho/\rho)_{\text{primordial}}$ is affected. It is strongly correlated with the parameters of the inflationary potential. Q is pushed exponentially to very large or very small values. But in our universe, Q is near the center of its anthropically allowed range.

Pitfalls of Measures: Boltzmann Brains

- Serious problem in pocket-based measures, and in other measures not discussed here.
- Consider our own pocket universe, and assume that dark energy = vacuum energy. Assume that our vacuum is stable, or that nucleation rate for other bubbles in our vacuum obeys $\lambda \ll H^4$.
- Then our pocket will be eternal, with infinite de Sitter expansion in the future.
- De Sitter space has a nonzero Gibbons-Hawking temperature $T_{\rm GH} = H/2\pi$. Thermal fluctuations can produce anything, with $P \sim e^{-M/T_{GH}}$.
- Brains like ours can nucleate, with a low rate, but with an infinite spacetime volume. They are called Boltzmann brains (BB's).

Boltzmann Brains: A Thought Experiment

1) Observer closes her eyes. (Idealization: all sensory input is shut off.)

2) While the eyes are shut, the brain has access only to its memory and thought processes — i.e., to stored data and programs.

3) For every normal observer, there are an infinite number of BB's with exactly the same data and programs (by random choice).

4) The observer's brain predicts what will happen when she opens her eyes, based on previous knowledge. But the probability is 1 that the brain is really a BB. Prediction: the next observation will have no logical relationship to the world she remembers.

5) Conclusion: our continued observation of a coherent world gives overwhelming evidence that we do ***not*** live in a world with an infinite ratio of BB's to normal observers.

Boltzmann Brains and Scale-Factor Cutoff Measure

- For scale-factor cutoff measure,

$$V(t_{\rm sf}) \propto \frac{a^3_{\rm pock}(t_{\rm sf})\, P^{\rm survival}_{\rm pock}(t_{\rm sf})}{a^3_{\rm parent}(t_{\rm sf})\, P^{\rm survival}_{\rm parent}(t_{\rm sf})}$$
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- $V(t_{\rm sf})$ falls off exponentially with pocket proper time τ_p, with a very slow time constant, related to decay rates. The exponential convergence is just what is needed to make the number of BB's finite. It is still not certain whether the number is acceptable, because it depends on estimates of decay rates and BB nucleation rates. It is plausible that the rates are okay.

Summary

- ☆ Scale-factor measure looks good. It is my favorite.
- ☆ Causal patch measures may also be okay. They give up the attractor nature of inflation, so they depend on a theory of initial conditions.
- ☆ Pocket-based measures suffer from the Q catastrophe, and from Boltzmann brains.
- ☆ Proper time measure suffers from a very serious youngness paradox.