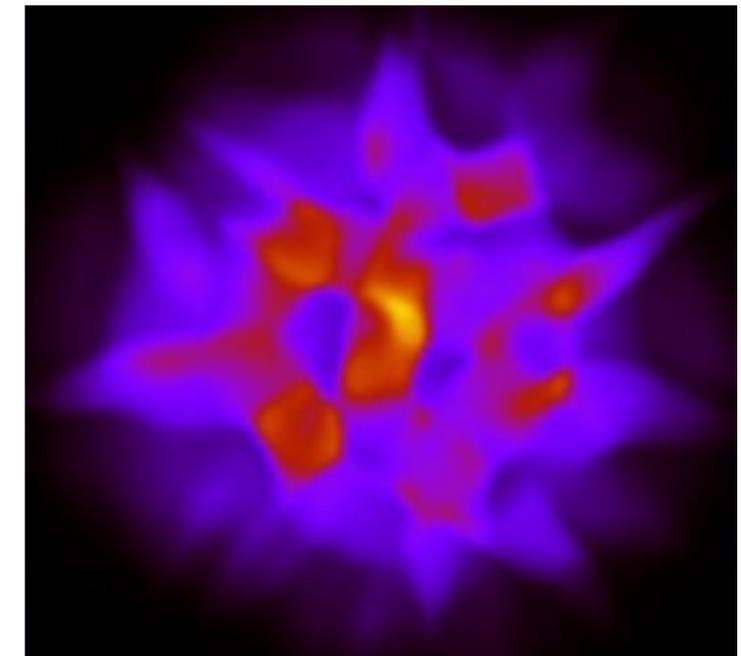
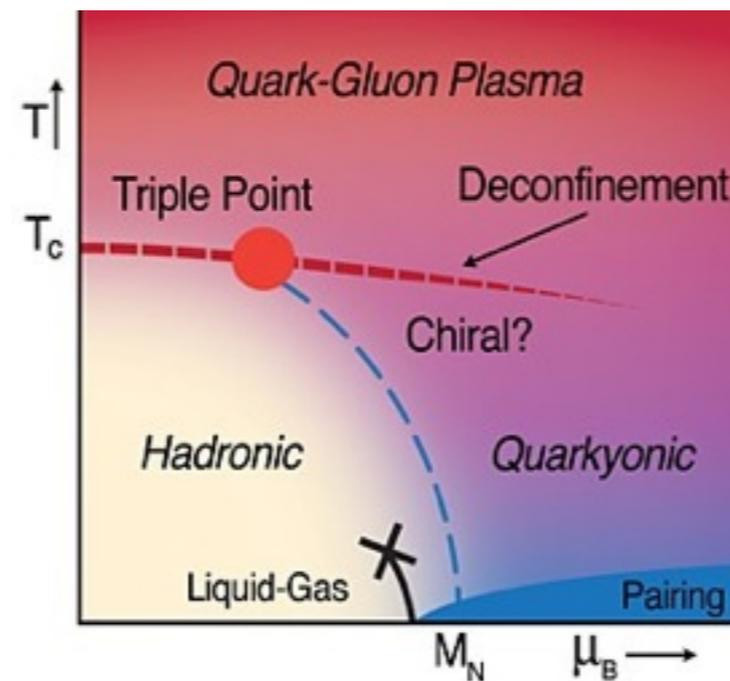
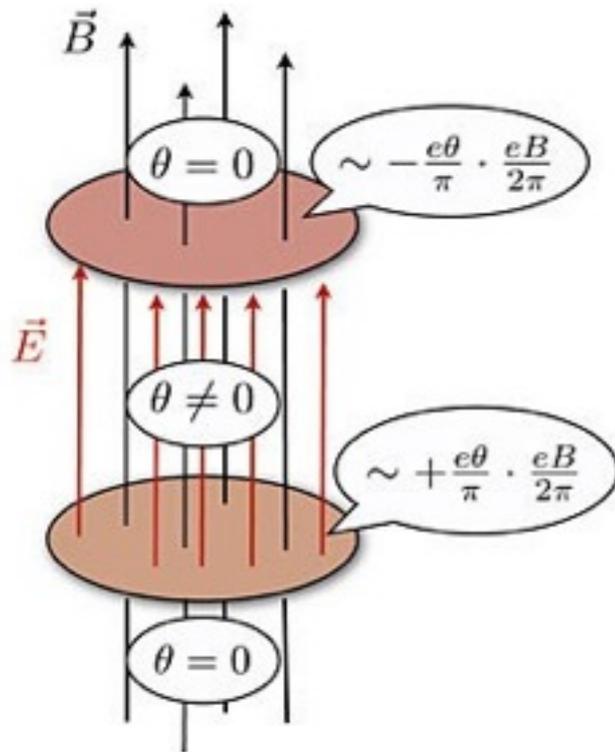


# Special slides for Brain Workshop

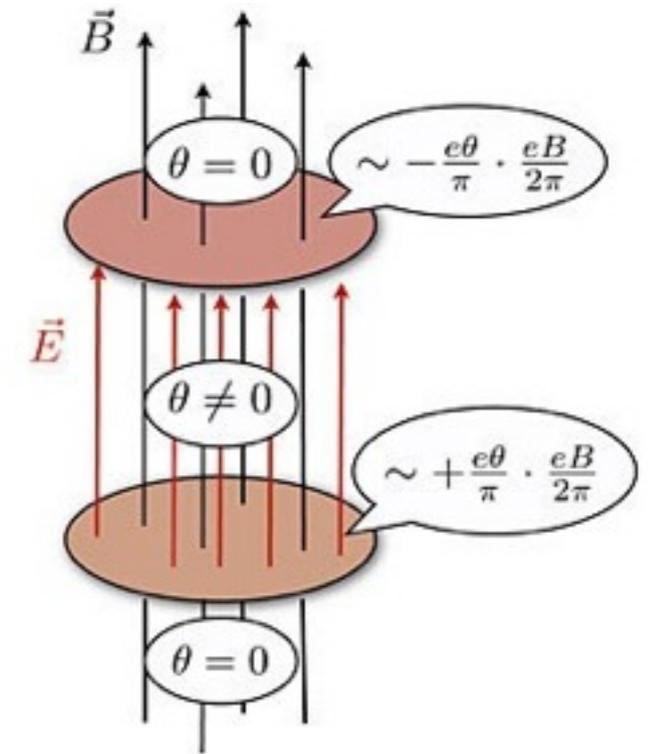
- As the organizer states, the purpose of the presentation “...covering your current interests and activities to encourage discussion and collaboration among us and visitors.”
- For this purpose, I will first give a brief overview of my research interests before returning to today’s topics. Any discussion are more than welcome(my office is at 2-41).
- I would express my gratitude to RBRC as most of my collaborators are or were associated with RBRC(their names will be in blue).

- My research profile can be summarized with three pictures taken from nuclear theory group website: <http://www.bnl.gov/physics/NTG/>

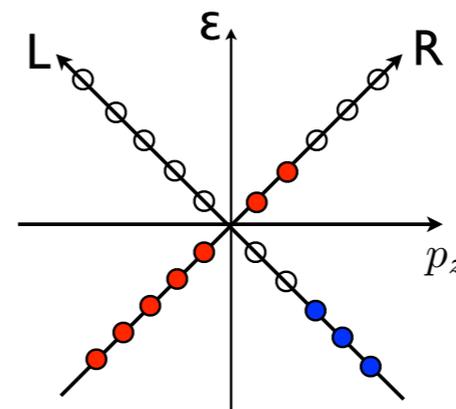


# I. Anomaly induced effects and topological aspects of gauge theories

- A framework to describe chiral charge transport in non-equilibrium environment (chiral kinetic theory). (M. Stephanov and YY, 1207.0747, PRL)
- Its extension and application (M. Stephanov and YY, 1207.0747, PRL, J. Chen, D. Son, M. Stephanov, H. Yee, YY, 1404.5963, M. Stephanov, H. Yee, YY, 1501.00222)



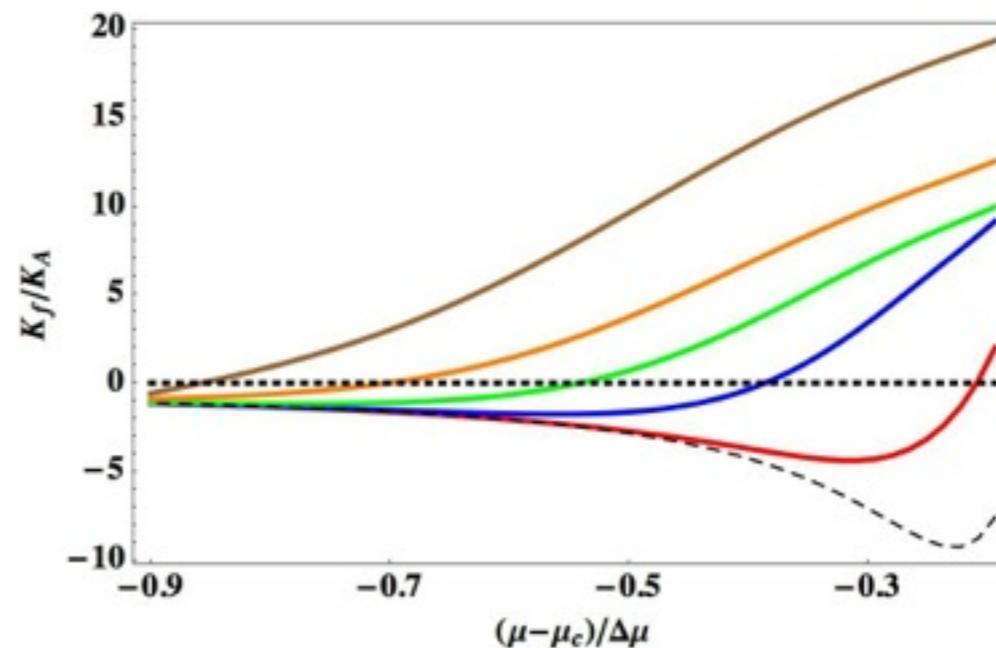
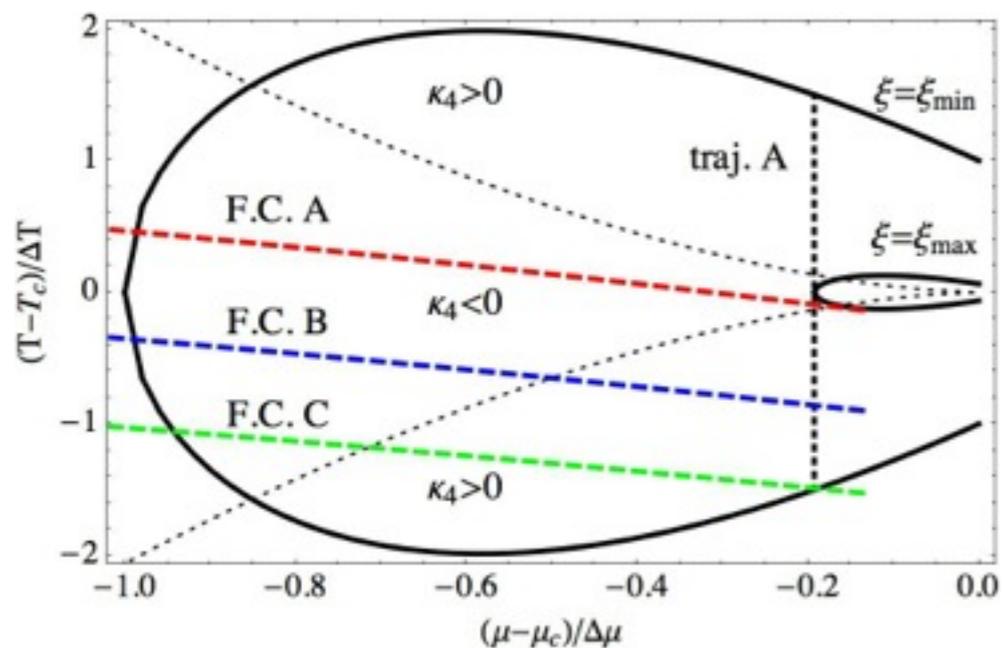
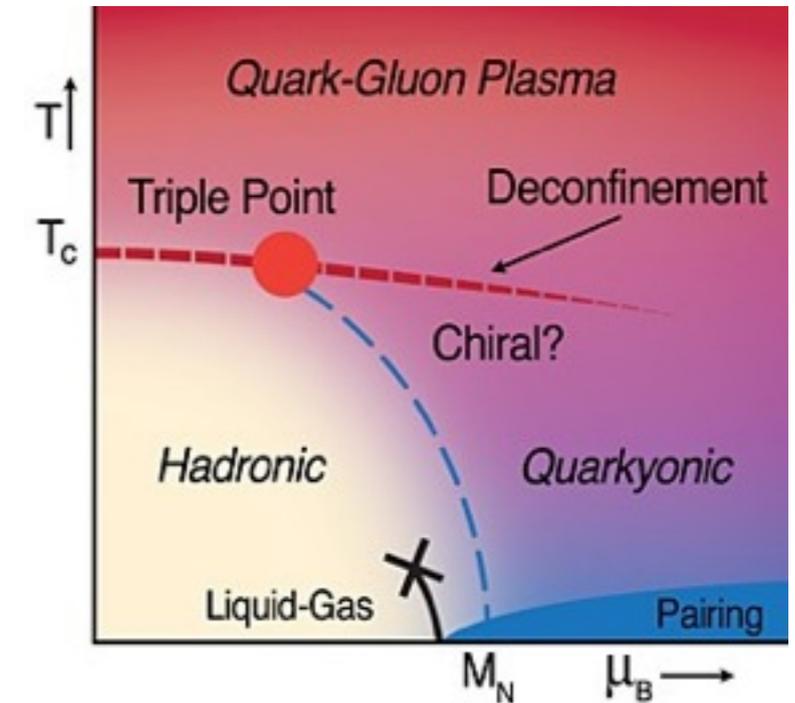
Nothing will come out of nothing.  
—King Lear, Act I.1



How about Berry monopole?

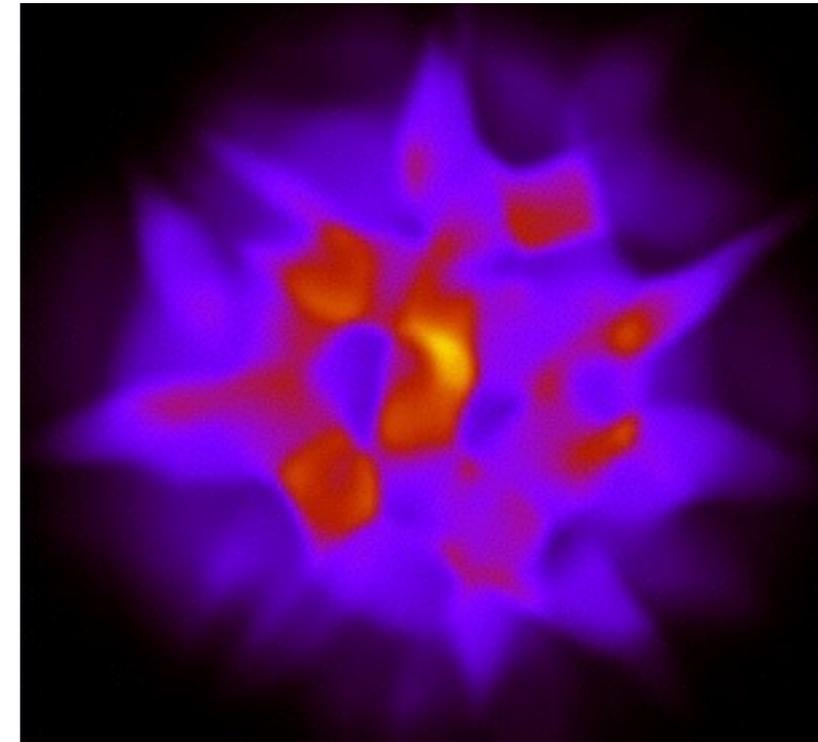
# QCD phase diagram and search for critical point

- A theoretical framework to describe evolution of critical fluctuations in QCD critical regime and its application to locate QCD critical points via Beam Energy Scan(BES) program at RHIC(S. Mukherjee, [R. Venugopalan](#) and YY in preparation)
- Welcome to my talk on Friday at RBRC-BES workshop.



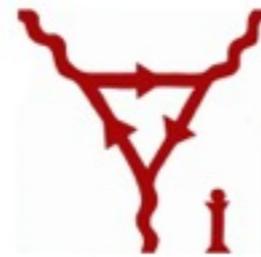
# Hydrodynamics and heavy-ion collisions

- Given hydrodynamic background of heavy-ion collisions, how to explore the properties of quark gluon plasma.
  - e.g. , estimate conductivity of QGP from soft photon production rate(YY, 1312.4434, PRC).
- Novel approaches for hydro. evolution in heavy-ion collisions.
  - e.g. , reconstructing freeze-out surface by data using the maximum entropy method and evolve the hydro. backwards to probe early time profiles of the system(M. Stephanov and YY, 1404.5910).



# The Response of Quark Gluon Plasma to Topological Fluctuations

Yi Yin



*Nuclear Theory Group*

Frontiers of Hadronic Physics: Brains Circulate Three,  
BNL, Feb. 25th

# Outline

- Part I(phenomenological part): Recent results on hydrodynamic evolution of chiral magnetic effect(CME) and proposal for separating background effects([J. Liao](#) and YY, in preparation).
- Part II(theoretical part): New mechanism for generating axial current by topological charge fluctuations(I. Iatrakis, [S. Lin](#), YY, [1411,2863](#)).

# Part I: chiral magnetic effect confronts with the data

# Topological(Charge) Fluctuations in heavy-ion collisions

- In short, bubbles or domains with non-zero winding numbers can be generated in heavy-ion collisions.

$$Q_W = \int d^4x q \quad q \equiv \frac{g^2}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} \text{tr} (G_{\mu\nu} G_{\rho\sigma})$$

- Non-zero topological charge will generate axial(chiral) charge imbalance.

$$\partial_\mu j_A^\mu = -2q$$

# Medium's response: Chiral Magnetic Effects

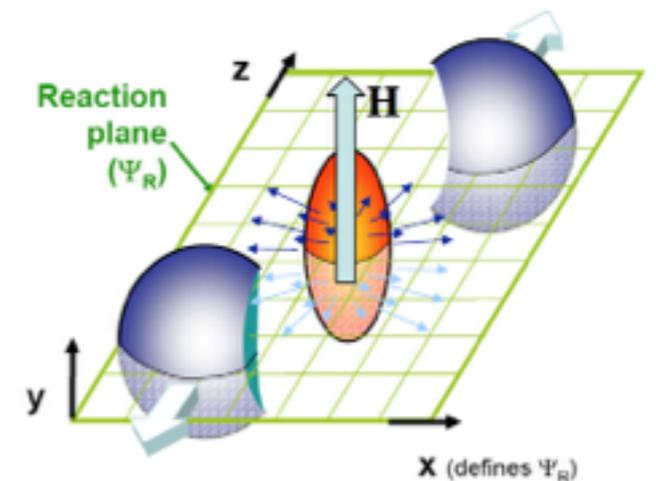
- Chiral magnetic effects: a vector current will be induced by magnetic field for a system with chiral charge imbalance.

$$\Delta J_{V,\text{anom}}^\mu = C_{\text{anom}} \mu_A B^\mu$$

$$\partial_\mu J_A^\mu = C_{\text{anom}} \vec{B} \cdot \vec{E}$$

$$\left( C_{\text{anom}} = \frac{N_c}{2\pi^2} \right)$$

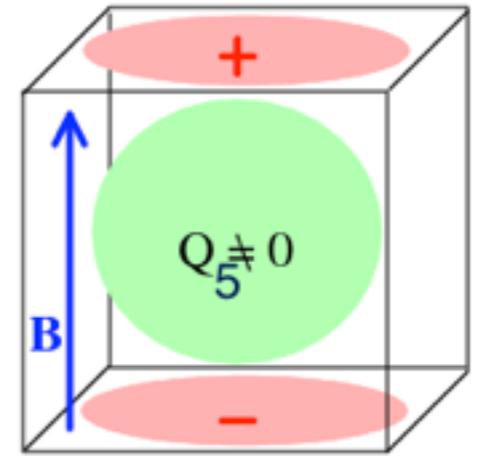
- Experimental relevance: **Strong magnetic field** is created in heavy-ion collisions.



# Observables for CME

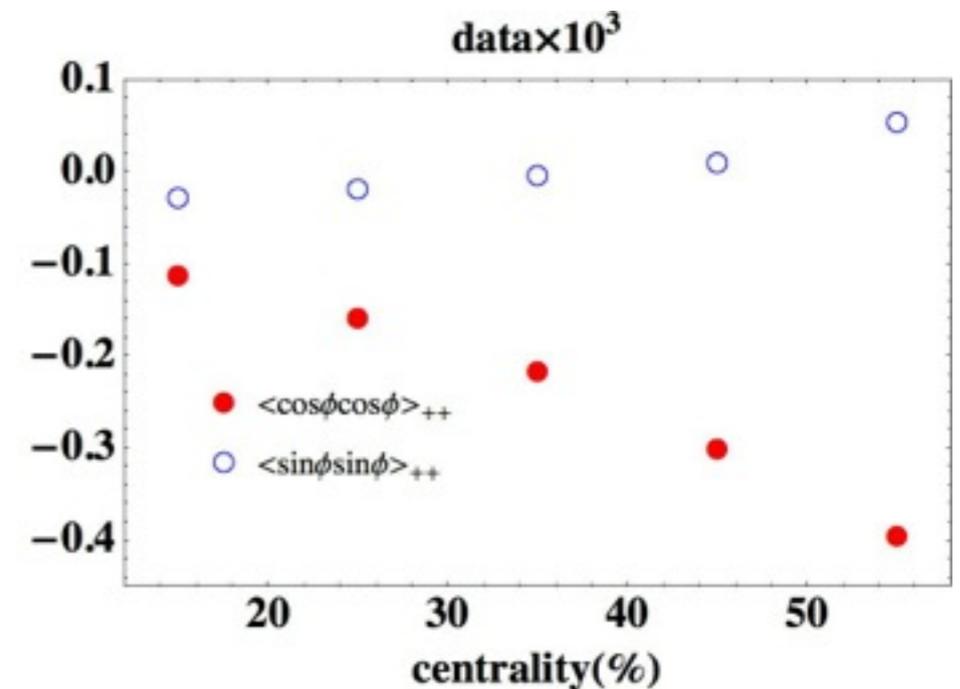
- Axial charge created in heavy-ion collisions will lead to vector current(chiral magnetic effect) thus charge dipole.

$$\frac{dN_{\pm}}{2\pi p_t dp_t d\phi} \propto 1 + 2v_{1,\pm} \cos(\phi) \pm 2a_1(p_t) \sin(\phi) + \dots$$



- The sign of axial charge is random. We need to consider fluctuations.

$$\langle \sin \phi_+ \sin \phi_+ \rangle = \langle a_1 a_1 \rangle + \text{Background effects}$$



- $\langle \cos \phi \cos \phi \rangle$ : **negative** background effects(transverse momentum conservation effects).
- $\langle \sin \phi \sin \phi \rangle$ : **positive** CME + **negative** background effects.

# Quantification of CME in heavy-ion collision I

Similar to elliptic flow  $v_2$  which can be quantified by hydrodynamics evolution,  $a_1$  can be quantified by solving hydrodynamic equations with constitute relation modified by CMe.

PRL 103, 191601 (2009)

PHYSICAL REVIEW LETTERS

week ending  
6 NOVEMBER 2009

## Hydrodynamics with Triangle Anomalies

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(Received 18 August 2009; published 6 November 2009)

We consider the hydrodynamic regime of theories with quantum anomalies for global currents. We show that a hitherto discarded term in the conserved current is not only allowed by symmetries, but is in fact required by triangle anomalies and the second law of thermodynamics. This term leads to a number of new effects, one of which is chiral separation in a rotating fluid at nonzero chemical potential. The new kinetic coefficients can be expressed, in a unique fashion, through the anomaly coefficients and the equation of state. We briefly discuss the relevance of this new hydrodynamic term for physical situations, including heavy-ion collisions.

# Quantification of CME in heavy-ion collision II

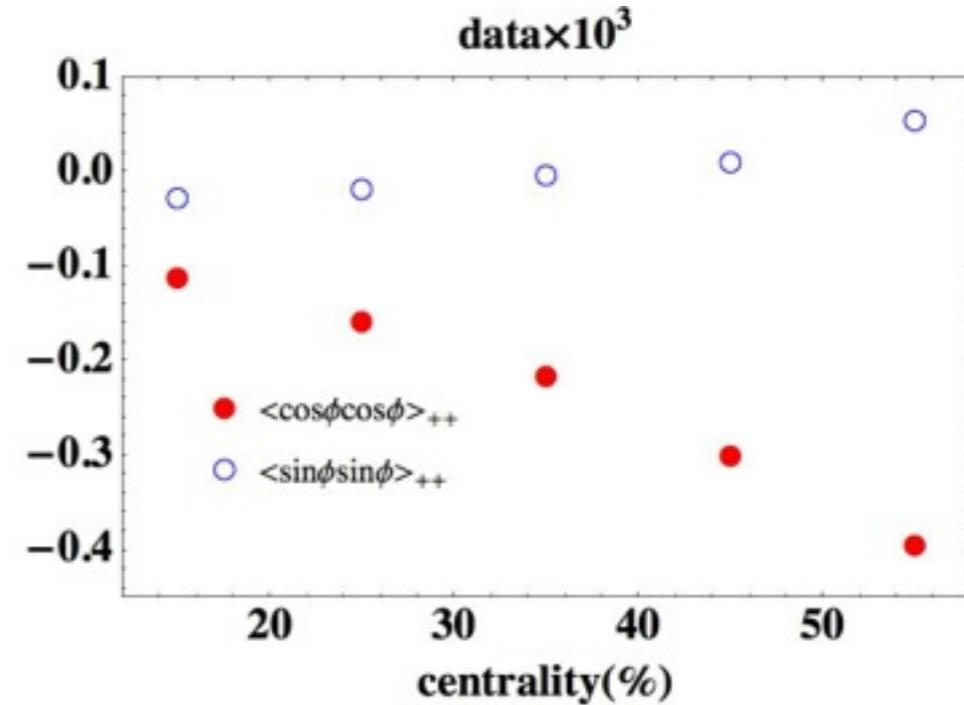
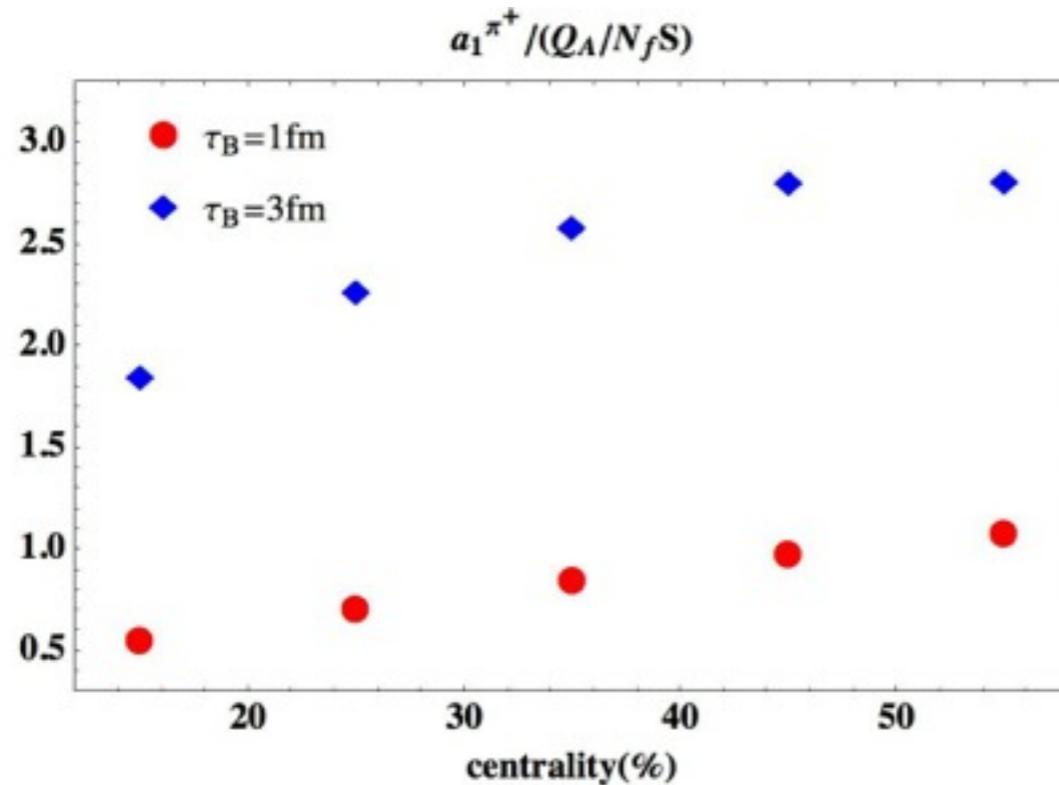
- We solved ideal anomalous hydro. equations linearized around a realistic hydrodynamic background(H. Yee and YY, 1311.2574, PRC, J.Liao and YY, in preparation.).

- In our framework, if initial axial density/entropy is constant,  $a_1$  due to CME factorizes:

$$a_1 \propto (Q_A/S)$$

- For fully anomalous hydro. simulations with axial charge generated by color flux tube as the initial condition, see Yuji's talk at BES workshop.

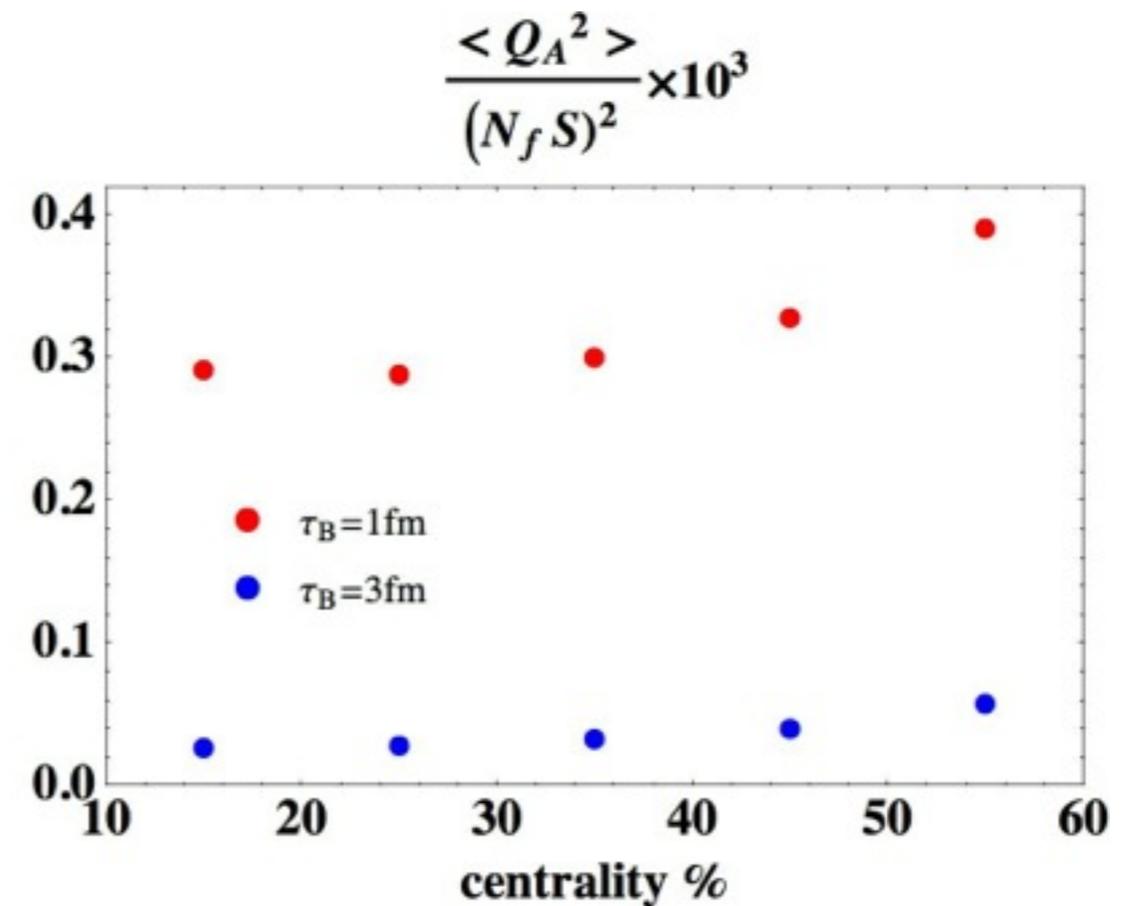
# Results



- Assuming the splitting between  $\langle \sin \sin \rangle$  and  $\langle \cos \cos \rangle$  in the data can be explained by CME, we now want to estimate the topological fluctuations  $\langle Q_A^2 \rangle$  (J. Liao and YY, in preparation).

# The extraction of $\langle Q_A^2 \rangle$

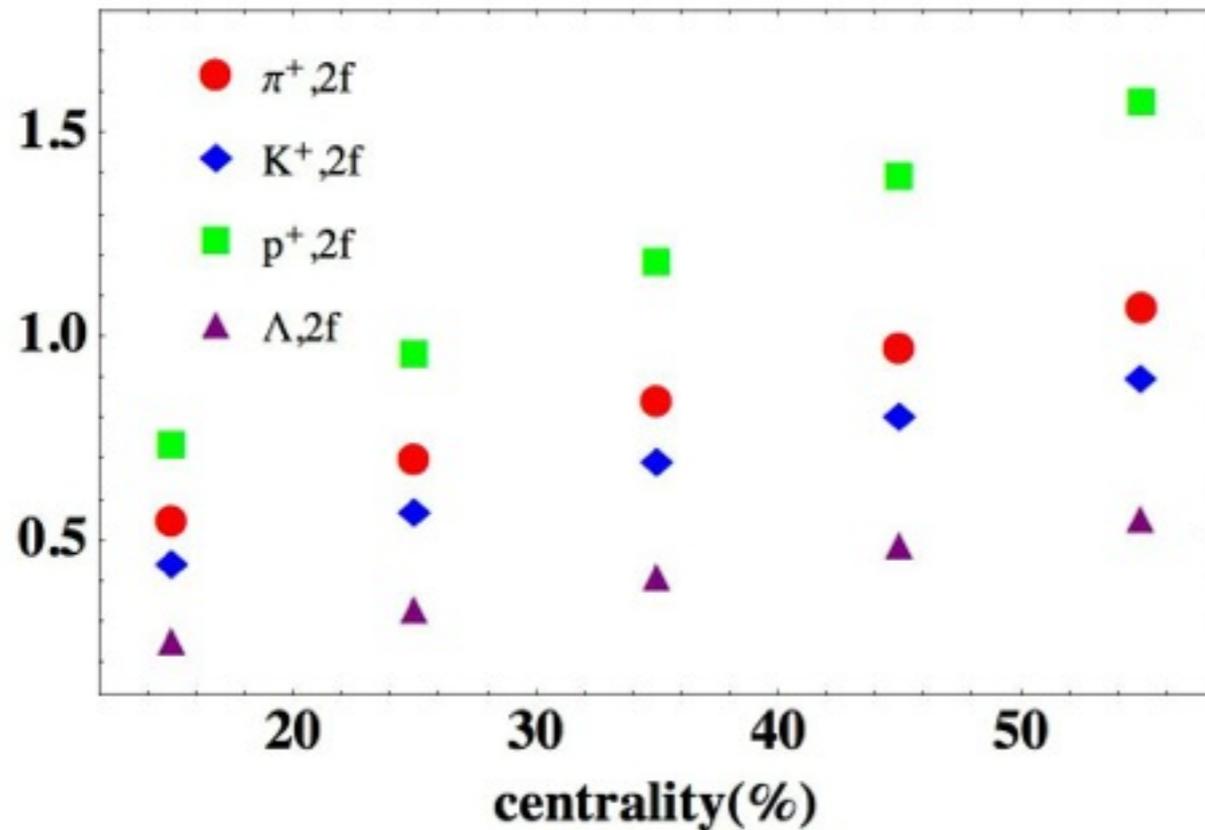
- Small life time of magnetic field can be compensated by large axial charge fluctuations. (different from charge-dependent elliptic flow).



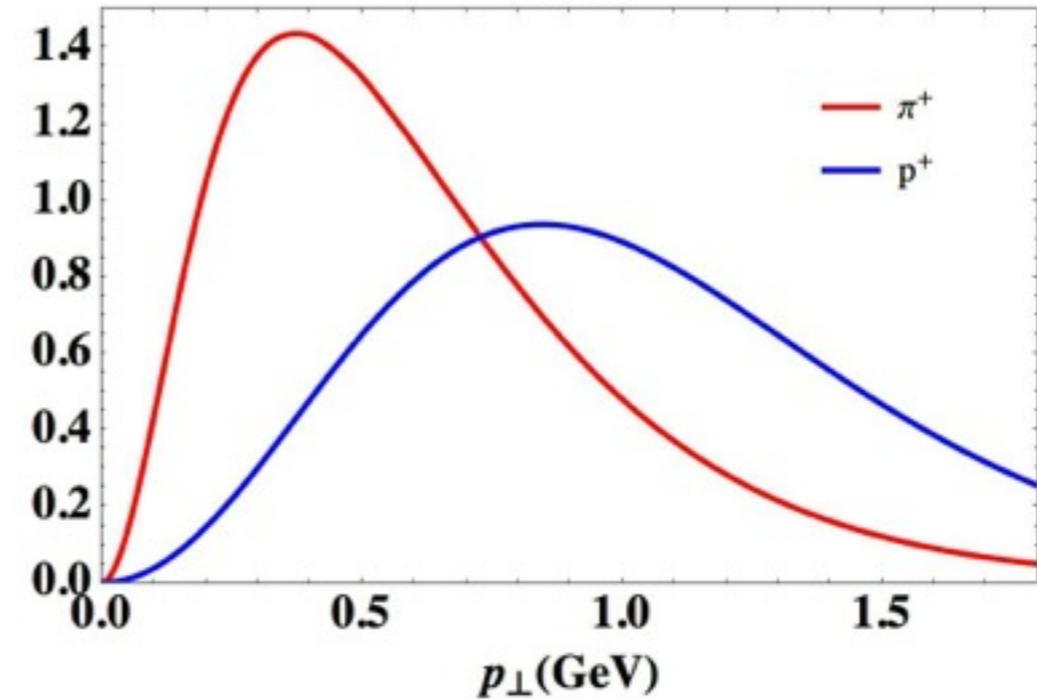
- The magnitude of axial charge fluctuation is inline with the estimation based on sphaleron transition rate (Kharzeev-Krasnitz-Venugoplan: hep-ph/0109253v1).
- Centrality-dependence: smaller system has a larger fluctuations.

# Mass ordering of CME

$a_1/(Q_A/N_f S)$

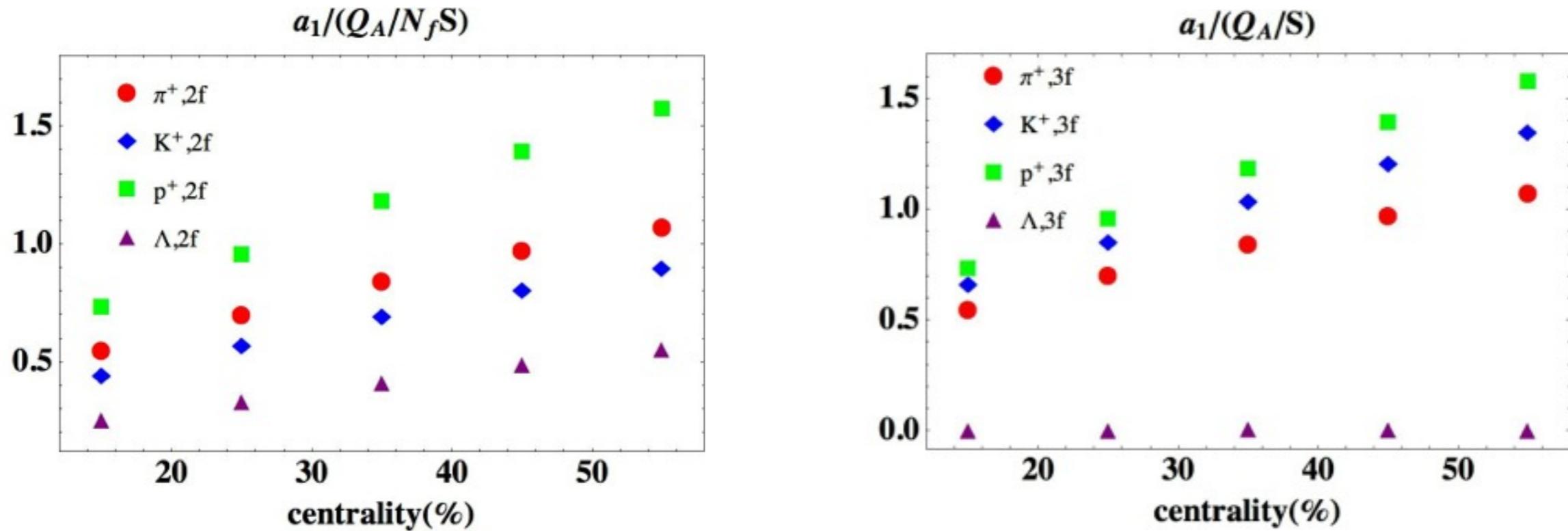


$f_H(p_\perp)$



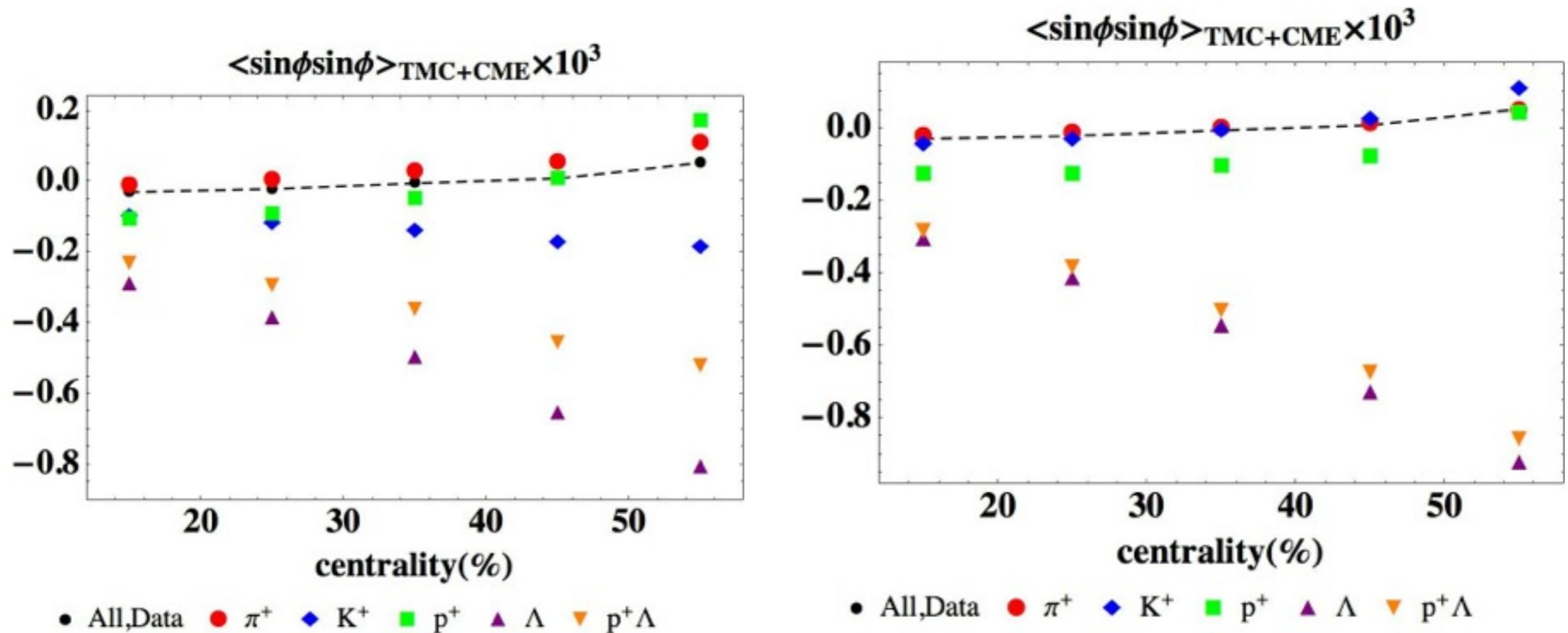
- $a_1^{p^+} > a_1^{\pi^+}$ : the dominate contribution to integrated  $a_1$  is from  $p_\perp$  window,  $p_\perp = p_\perp^*$  which would maximize distribution function:  $p_\perp^2 \exp(-p^\mu u_\mu)$ .
- Heavier the hadron is, the larger  $p_\perp^*$  is.
- $a_1(p_\perp)$  grows with  $p_\perp$ : the integrated  $a_1$  is larger for heavy hadrons.

# Comparison between 2-flavor scenario and 3-flavor scenario



- $a_1^{\pi^+} > a_1^{K^+} > a_1^{\Lambda}$ : if  $s$  quark does not contribute to CME current.  $K^+(u\bar{s})$  behaves as a particle with charge  $2e/3$  and  $\Lambda$  baryon( $uds$ ) behaves as a particle with charge  $e/3$ .
- Potential way to measure quark (fractional)charge in heavy-ion experiments!
- If  $s$  contributes to CME.  $a_1^{\Lambda} \approx 0$  (distinguishing CME and CVE, Kharzeev-Son,2010).

# Predictions for $\langle \sin \phi \sin \phi \rangle_{++}$ of different hadron species



- Would the data reveal the role played by s quark?
- Relevance to BES: role of s could change from top RHIC energy to lower energy.

# Summary of Part I:

- We have extracted the magnitude of topological fluctuations from data by solving anomalous hydro.
- We show mass-order among different hadrons can be used to test CME signal.
- We assume  $Q_{A/S}$  is constant initially. But for each theta-domain, their winding numbers are random and therefore initial axial charge are in-homogeneous.
- What would be the role played by such in-homogeneity?

# Part II: Axial current induced by theta domain

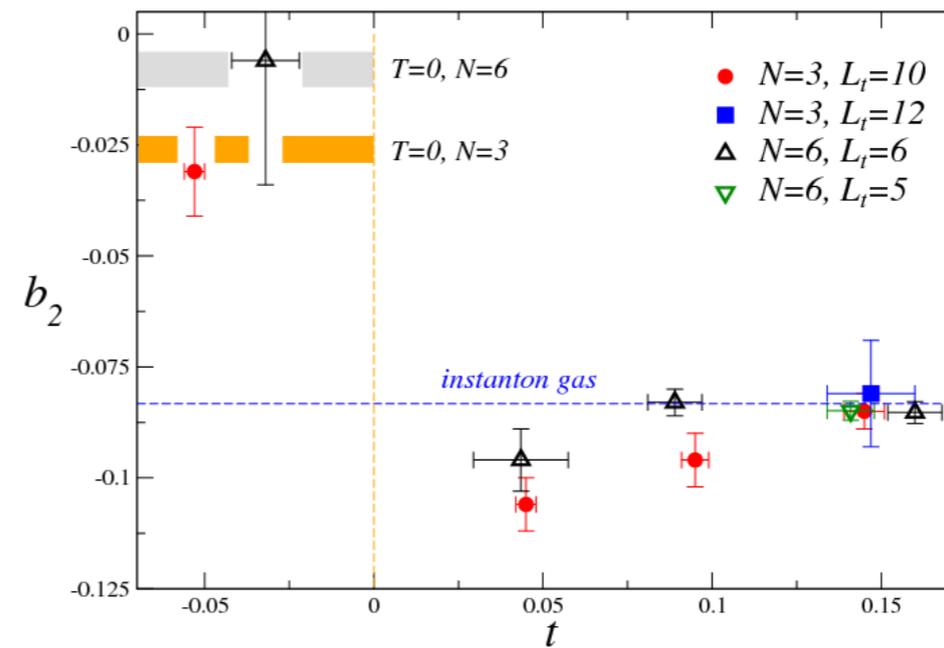
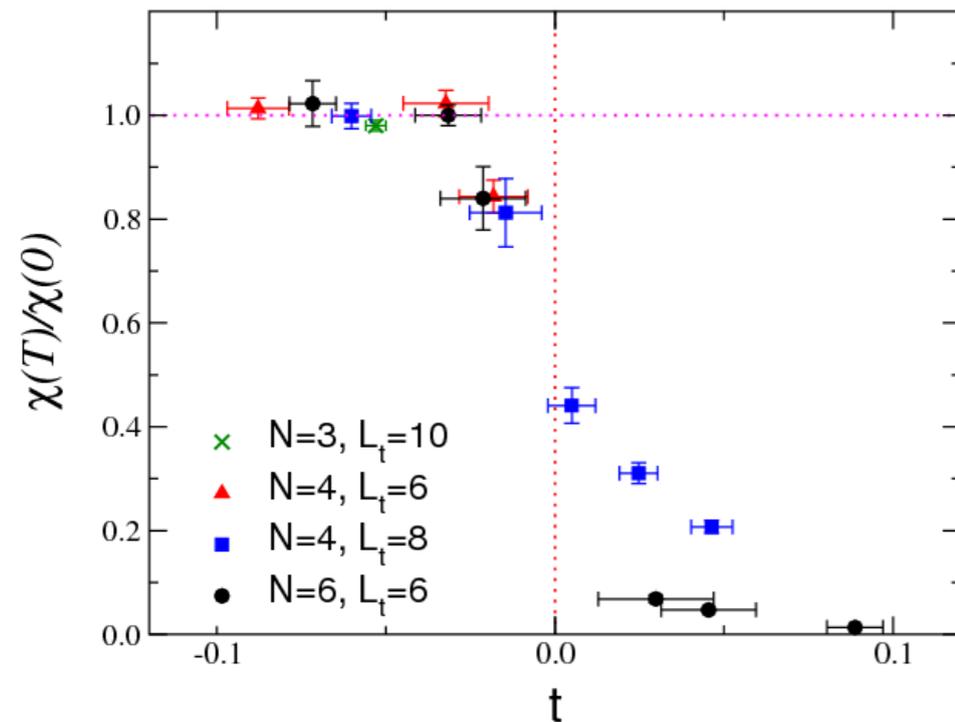
# Free-energy and theta-domain.

- To describe the dynamics of “ $\theta$  domain”, we introduce a space-time dependent  $\theta$  angle  $\theta(t, \mathbf{x})$ . (One may interpret  $\theta(t, \mathbf{x})$  as an effective axion field creating a “ $\theta$ -domain”. )
- We now ask for a pure Yang-Mills theory in de-confined phase, how would the free-energy depends on  $\theta$  and its gradients  $\nabla\theta(\mathbf{x})$ :

$$F(\theta, \nabla\theta(\mathbf{x}); T > T_c) = \frac{1}{2}\chi_{\text{top}} [1 + b_2\theta^2] \theta^2 + \frac{1}{2}\kappa_{\text{CS}} [\nabla\theta(\mathbf{x})]^2$$

- $\kappa_{\text{CS}}$ : characterizing the energy cost of creating a in-homogeneous profile of  $\theta$ .

# Lattice aspects



- The topological fluctuations as well  $\chi_{\text{Top}}$  as  $b_2$  have been measured on lattice at finite temperature (Bonati et. al, 1309.6059, see also Sayantan's talk).

$$F(\theta, \nabla\theta(\mathbf{x})) = \frac{1}{2} \chi_{\text{top}} [1 + b_2 \theta^2] \theta^2 + \frac{1}{2} \kappa_{\text{CS}} [\nabla\theta(\mathbf{x})]^2$$

- $\kappa_{\text{CS}}$  has not yet been measured on lattice but can be important!

$$\kappa_{\text{CS}} = - \lim_{k \rightarrow 0} \frac{d^2}{dk^2} G_{qq}^{\text{E}}(\omega = 0, k).$$

# Linear response theory

- $\kappa_{CS}$  also enters into low frequency, low momentum behavior of  $G_R^{qq}(\omega, \mathbf{k})$ :

$$G_R^{qq}(\omega, k) = \frac{1}{2} \left( -i \frac{\Gamma_{CS}}{T} \omega - \kappa_{CS} k^2 + \dots \right),$$

$\Gamma_{CS}$ : Chern-Simons diffusive rate. (We consider de-confined phase and will neglect  $\chi_{top}$ )

- The response of  $q(\omega, \mathbf{k})$  to  $\theta(\omega, \mathbf{k})$  is related to Green's function:  $G_{qq}^R(\omega, k) \sim \langle qq \rangle$ .

$$q(\omega, \mathbf{k}) = G_R^{qq}(\omega, \mathbf{k}) \theta(\omega, \mathbf{k}).$$

- The anomaly relation

$$\partial_\mu j_A^\mu = -2q(t, \vec{x}) = - \left( \frac{\Gamma_{CS}}{T} \partial_t - \kappa_{CS} \nabla_x^2 \right) \theta(t, \mathbf{x}).$$

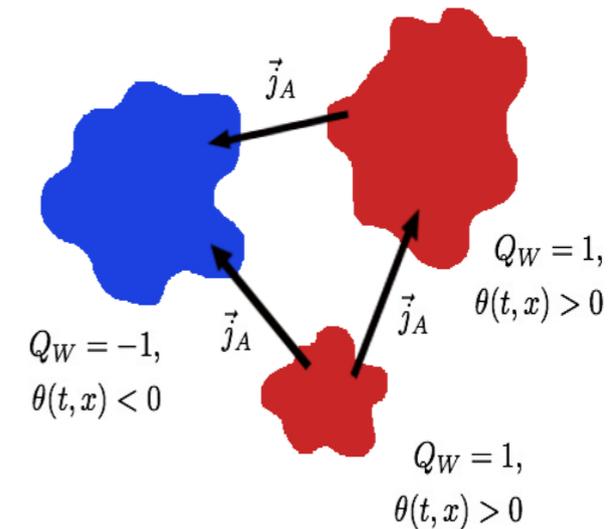
- Static limit:  $\vec{\nabla} \cdot \mathbf{j}_A = \kappa_{CS} \vec{\nabla} \theta(\mathbf{x})$ .
- We have a axial current induced by the gradient of  $\theta(t, \mathbf{x})$

$$\mathbf{j}_A = \kappa_{CS} \vec{\nabla} \theta(\mathbf{x}).$$

# Interpretation

- Axial current will be generated by theta-domain.

$$\mathbf{j}_A = \kappa_{CS} \vec{\nabla} \theta(\mathbf{x}).$$



- Energy argument:  $\theta$  is coupled to  $q$ .

$$2q\theta \sim (\partial_\mu j_A^\mu) \theta \sim j_A^\mu \partial_\mu \theta$$

- Static limit:  $(\vec{\nabla} \mathbf{j}_A) \theta \sim \kappa_{CS} (\vec{\nabla} \theta)^2 / 2$ .
- Substituting our expression for  $j_A$ , the energy carried by  $j_A$  is precisely transferred from the energy of having a  $\vec{\nabla} \theta$  in gluonic sector:  $\kappa_{CS} (\vec{\nabla} \theta)^2 / 2$ .
- In homogeneous limit we also have  $n_A \partial_t \theta$  and therefore  $\partial_t \theta$  is related to  $\mu_A$  (Fukushima-Kharzeev-Warringa, 2008).

# Homogeneous limit

- Indeed, we consider homogeneous limit of the relation:

$$\partial_{\mu} j_A^{\mu} = - \left( \frac{\Gamma_{CS}}{T} \partial_t - \kappa_{CS} \nabla_x^2 \right) \theta(t, \mathbf{x}).$$

- The axial density is now related to  $\theta$ :

$$\partial_t n_A = - \frac{\Gamma_{CS}}{T} \partial_t \theta.$$

- Using the linearized equation of state:  $n_A = \chi \mu_A$ , we have:

$$\mu_A = - \frac{\Gamma_{CS}}{\chi T} \theta = - \frac{\theta}{\tau_{\text{sph}}}.$$

$\theta(t) \sim e^{-t/\tau_{\text{sph}}}$ , one has  $\partial_t \theta = -\theta/\tau_{\text{sph}} = \mu_A$ .

# Summary

- We show, by model-independent argument, that the dynamics of  $\theta$  will induce axial charge density and axial current:

$$\mathbf{j}_A = \kappa_{CS} \vec{\nabla} \theta(\mathbf{x}) \quad \mu_A = -\frac{\theta}{\tau_{\text{sph}}} = \partial_t \theta.$$

Similar to super-fluid (Josephson-type equation), but at finite temperature.

- We have checked our results in a holographic model ( $\kappa_{CS} \sim T^2$ ).
- It would be useful to measure  $\kappa_{CS}$  on lattice.
- A microscopic understanding?
- Would it be an phenomenological footprint of such current?
- In current practice of anomalous hydrodynamics,  $\vec{j}_A$  is assumed to be co-moving with the background fluid velocity  $u^\mu$ . Our results suggest the axial charge might not co-move with the background flow (again, similar to two-fluid picture of superfluid).

# Summary

- We have explore the phenomenological aspects of chiral magnetic effects.
- We proposed a new mechanism for generating axial currents.