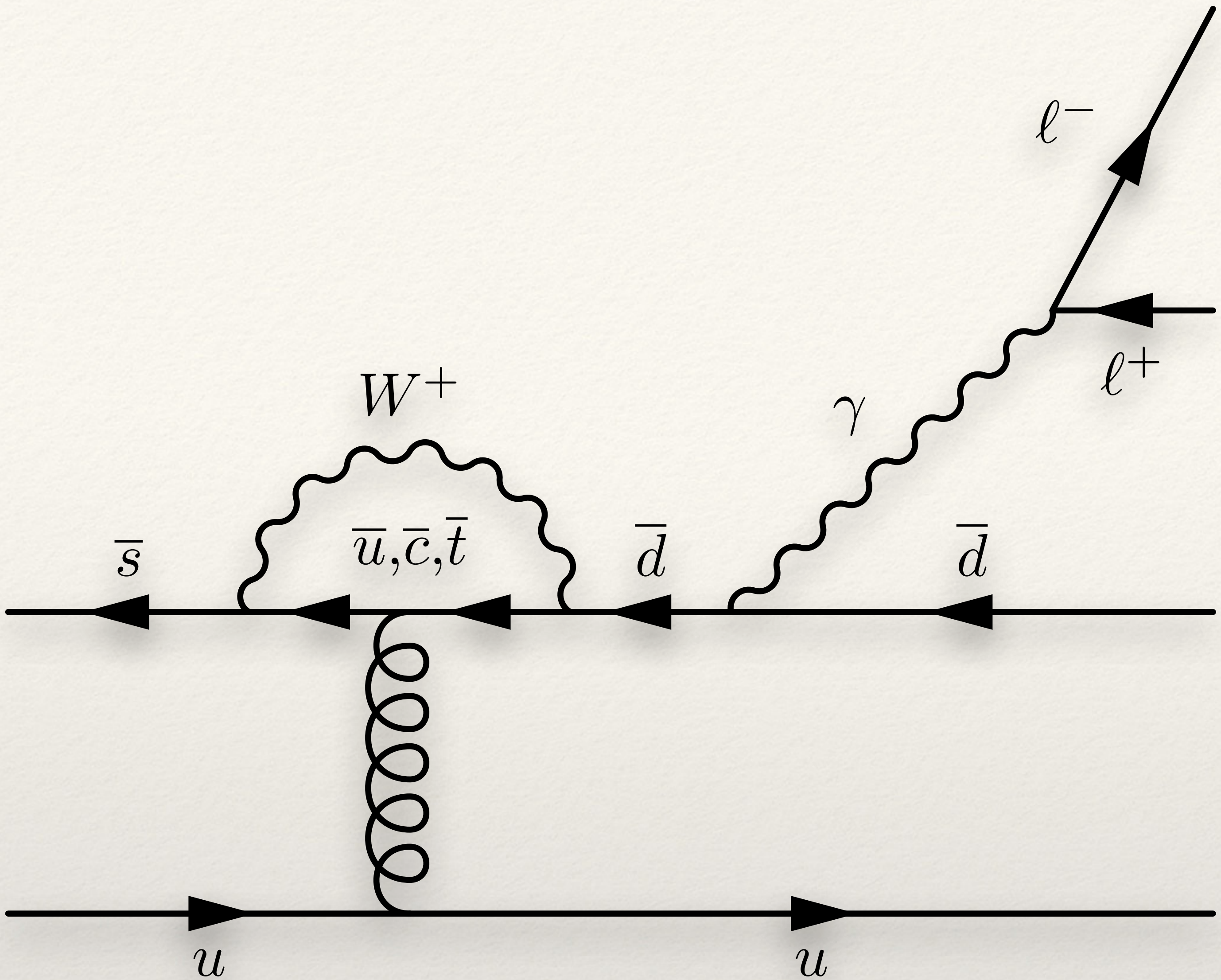

6th of February 2015 — MNME 2015 (BNL, USA)

$K \to \pi\ell^+\ell^-$ decays on the lattice

UNIVERSITY OF Southampton

Antonin J. Portelli
(RBC-UKQCD collaborations)

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- Euclidean formulation
- Ultraviolet & infrared behaviour
- Preliminary lattice results
- Summary & perspectives

Euclidean formulation

Minkowski amplitude

$$\mathscr{A}_\mu^c(q^2) = \int \mathrm{d}^4x \, \langle \pi^c(\mathbf{p}) | \, \mathrm{T}[J_\mu(0) H_W(x)] \, | K^c(\mathbf{k}) \rangle$$

Minkowski amplitude

EM current (weak contribution negligible)

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Spectral representation:

$$\begin{aligned}\mathscr{A}^c_\mu(q^2) = {} & i \int_0^{+\infty} \mathrm{d}E \, \frac{\rho(E)}{2E} \frac{\langle \pi^c(\mathbf{p}) | \, J_\mu(0) \, | E, \mathbf{k} \rangle \langle E, \mathbf{k} | \, H_W(0) \, | K^c(\mathbf{k}) \rangle}{E_K(\mathbf{k}) - E + i\varepsilon} \\ & - i \int_0^{+\infty} \mathrm{d}E \, \frac{\rho_S(E)}{2E} \frac{\langle \pi^c(\mathbf{p}) | \, H_W(0) \, | E, \mathbf{p} \rangle \langle E, \mathbf{p} | \, J_\mu(0) \, | K^c(\mathbf{k}) \rangle}{E - E_\pi(\mathbf{p}) + i\varepsilon}\end{aligned}$$

Euclidean correlation function

$$\Gamma_{\mu}^{(4)\,c}(x,\mathbf{k},\mathbf{p}) = \langle \phi_{\pi^c}(t_\pi,\mathbf{p})\mathrm{T}[J_\mu(0)H_W(x)]\phi_{K^c}(t_K,\mathbf{k})^\dagger \rangle$$

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pion and kaon interpolating operators

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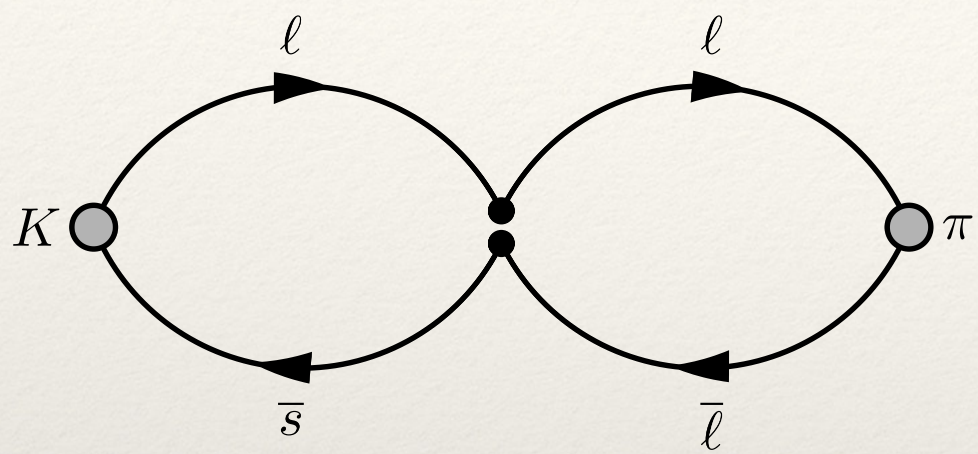
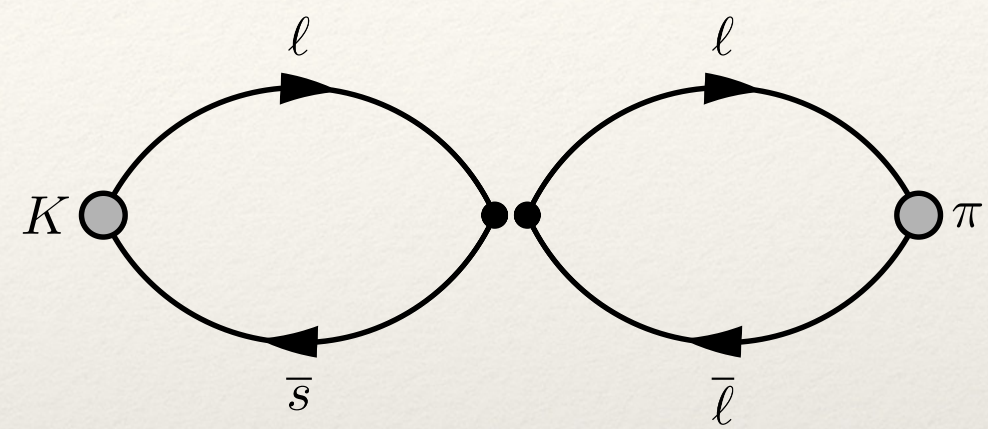
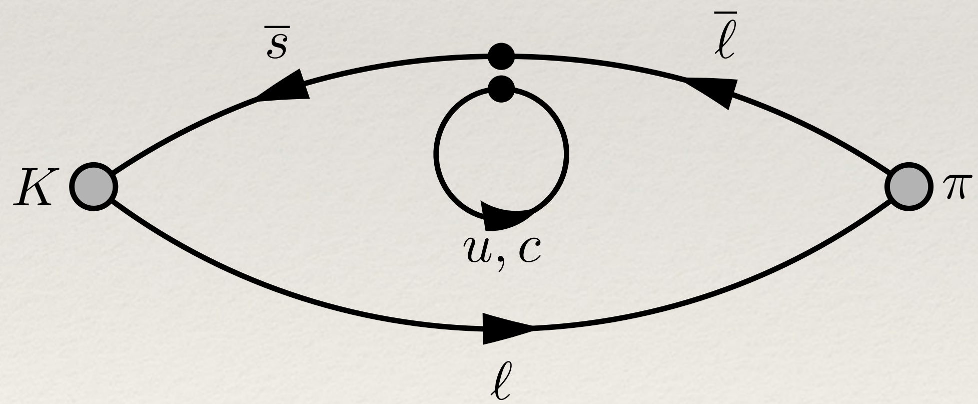
Quark Wick contractions



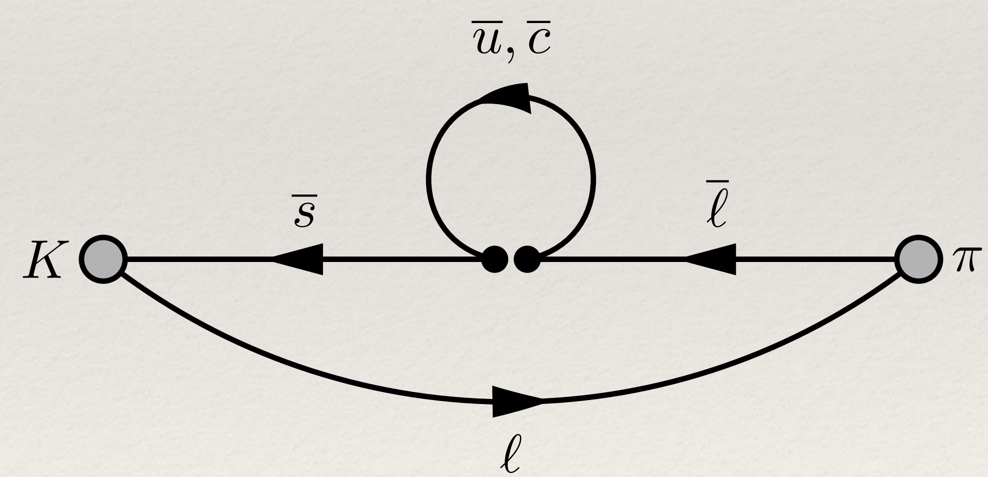
C: "Connected"



W: "Wing"



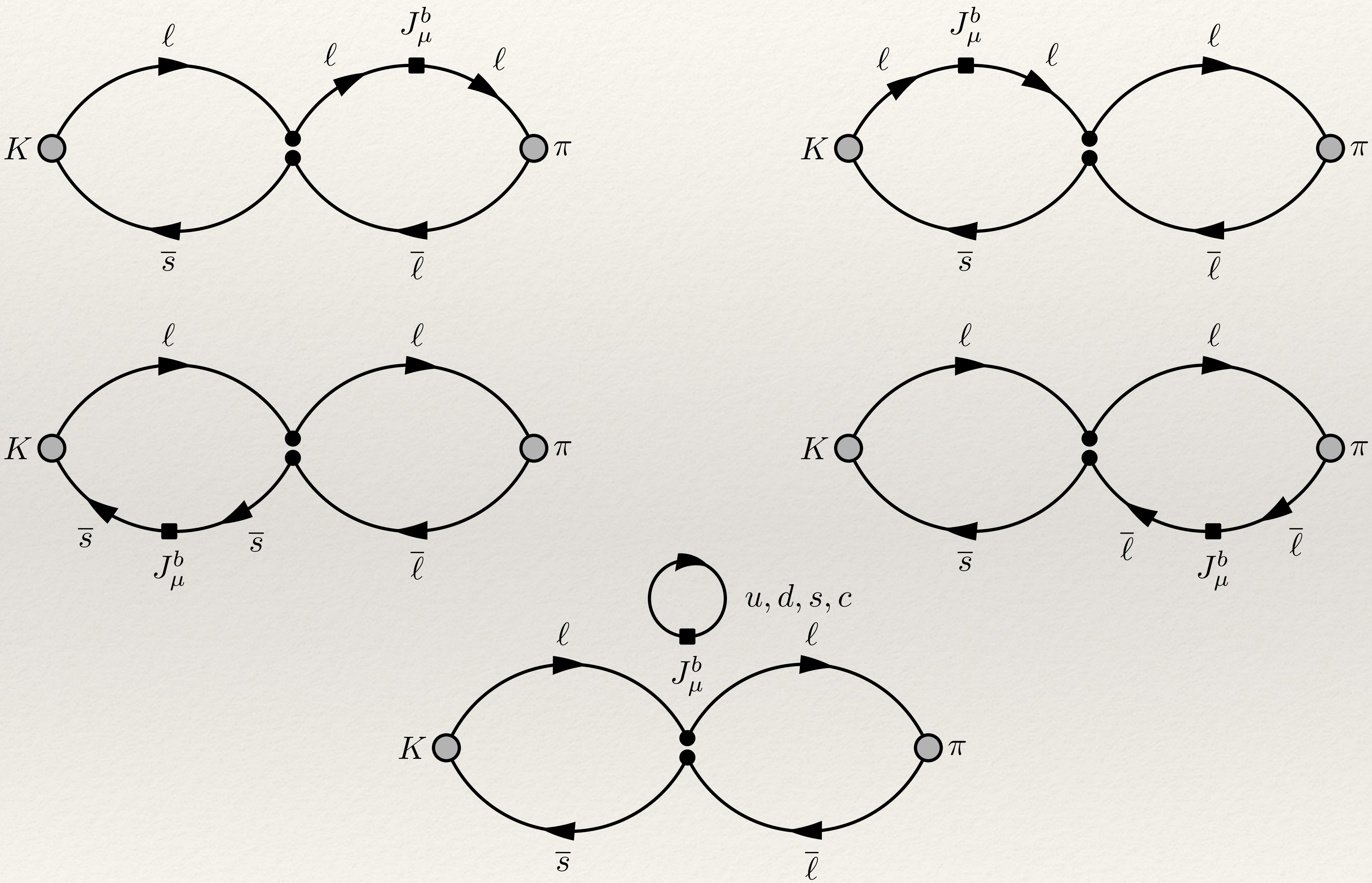
E: "Eye"



S: "Saucer"

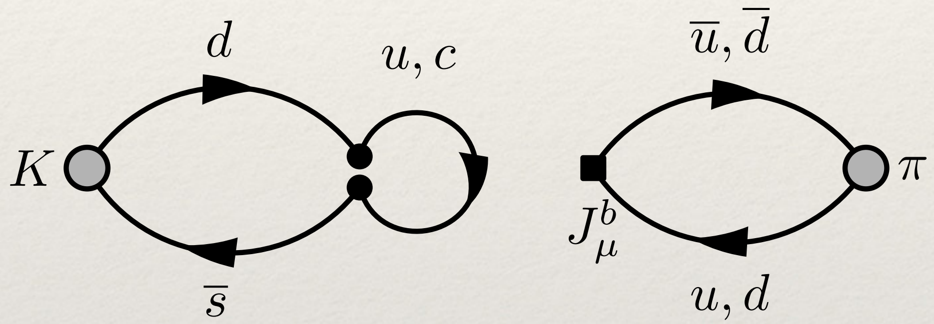
Names: E. Goode

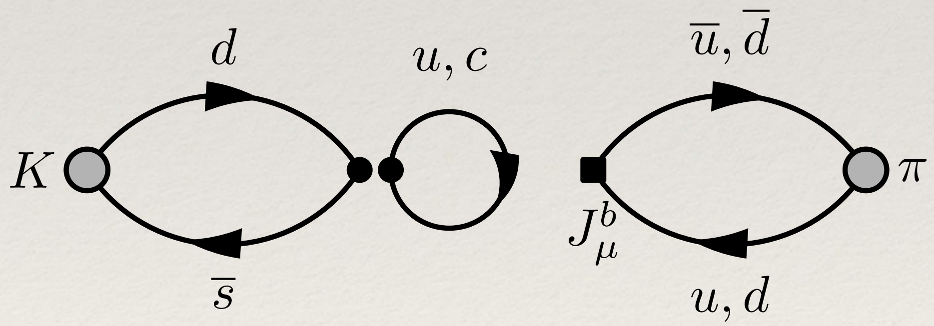
Quark Wick contractions



Quark Wick contractions

Neutral case additional diagrams:





Euclidean spectral representation

Integrated correlator on a finite time interval $[-T_a, T_b]$:

$$\int \mathrm{d}^3\mathbf{x} \int_{-T_a}^{T_b} \mathrm{d}t\, \tilde{\Gamma}_{\mu}^{(4)\,c}(t,\mathbf{x},\mathbf{k},\mathbf{p}) = -\int_0^{+\infty} \mathrm{d}E\, \frac{\rho(E)}{2E} \frac{\langle \pi^c(\mathbf{p})|\, J_\mu(0)\, |E,\mathbf{k}\rangle \langle E,\mathbf{k}|\, H_W(0)\, |K^c(\mathbf{k})\rangle}{E_K(\mathbf{k}) - E} \times \left(1 - e^{[E_K(\mathbf{k})-E]T_a}\right) + \int_0^{+\infty} \mathrm{d}E\, \frac{\rho_S(E)}{2E} \frac{\langle \pi^c(\mathbf{p})|\, H_W(0)\, |E,\mathbf{p}\rangle \langle E,\mathbf{p}|\, J_\mu(0)\, |K^c(\mathbf{k})\rangle}{E - E_\pi(\mathbf{p})} \times \left(1 - e^{-[E-E_\pi(\mathbf{p})]T_b}\right)$$

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- generated by 1, 2 and 3-pion intermediate states

Removing the single-pion divergence

1. Reconstruct the divergent single-pion term by computing J_μ and H_W matrix elements for $\pi \to \pi\gamma^*$ and $K \to \pi$ transitions

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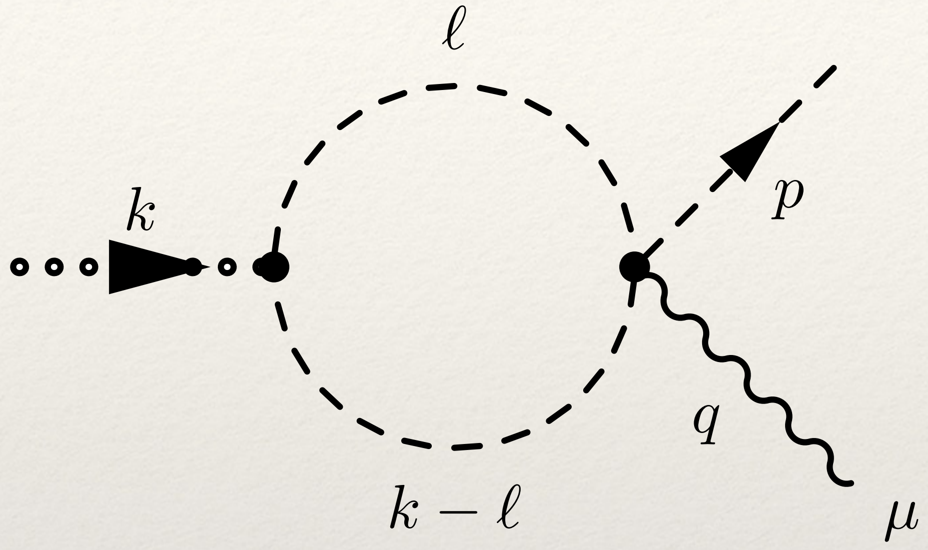
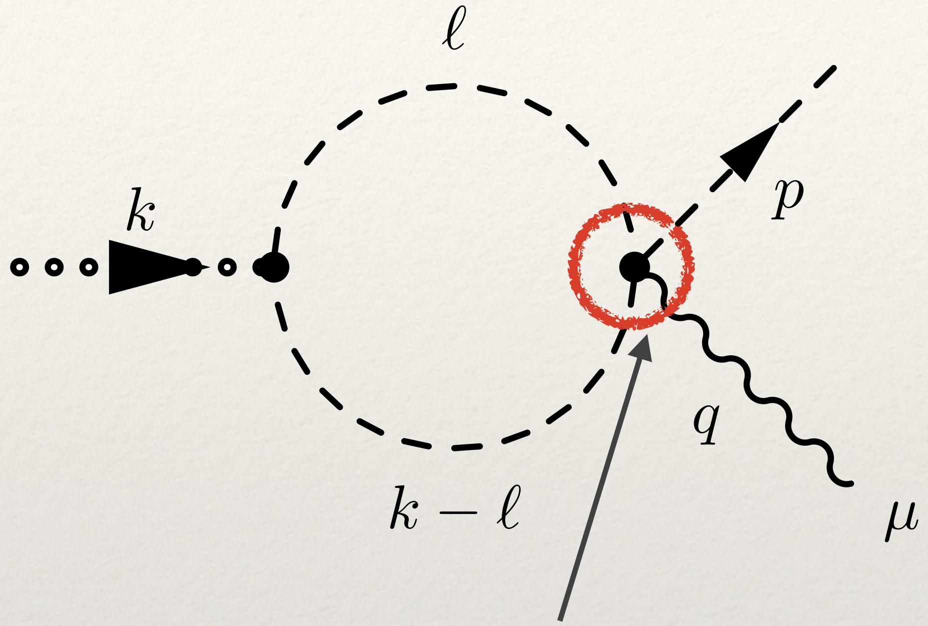
2. One can show that the physical amplitude is invariant under $H_W \mapsto H_W + c_S \bar{s}d\,,$ c_S can be tuned to cancel the $K \to \pi$ matrix element

Two-pion intermediate states



Two-pion intermediate states



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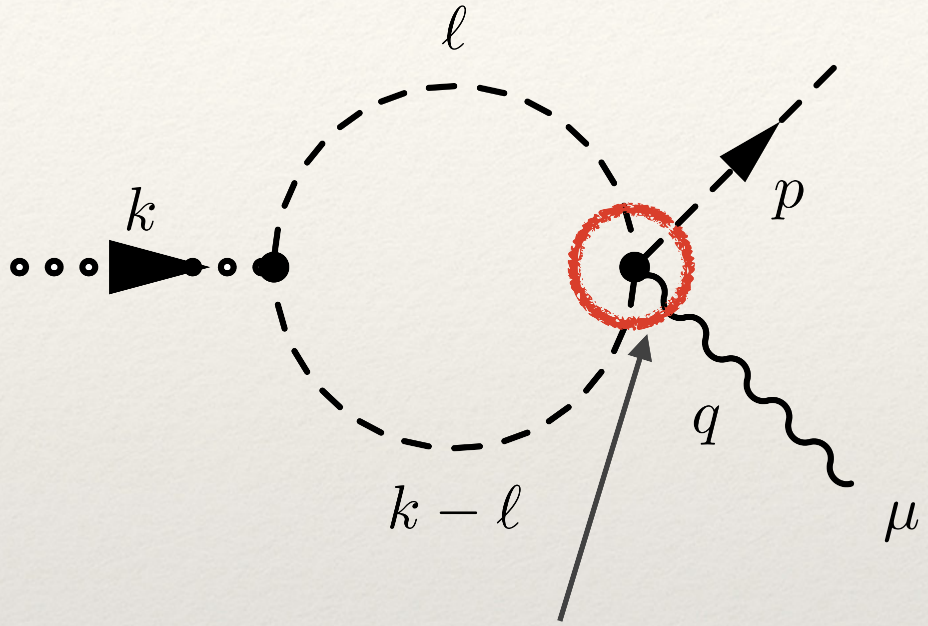

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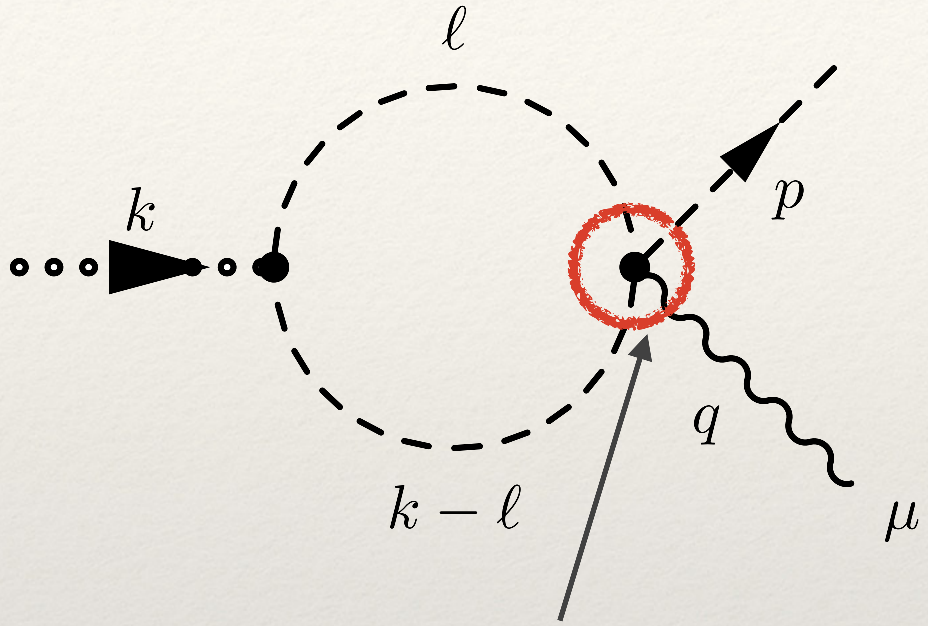

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- Only a problem for pion masses less than ~165 MeV

Ultraviolet & infrared behaviour

Individual operator renormalisation

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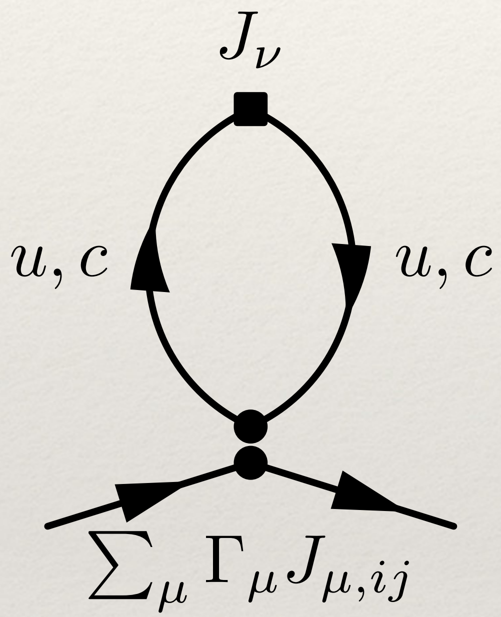
- The renormalisation of the weak hamiltonian is also know and is much more simple with chiral fermions (*cf. e.g.* [Z. Bai, *et al.* PRL, 113(1), p. 112003, 2014]).

Short distance operator product

UV divergences may appear in loops between J_μ and H_W :



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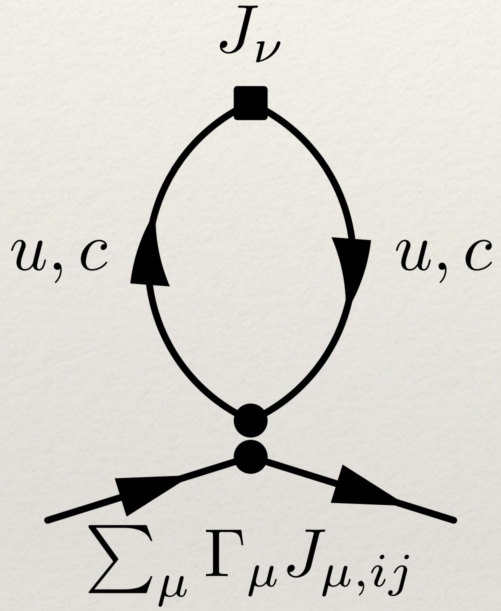

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Same divergence structure than HVP

Short distance operator product

OPE with lattice regularisation:

$$\Pi^{f}_{\mu\nu,ij}(q) \underset{a\to 0}{=} C^{1}_{\mu\nu,ij} + C^{\overline{f}f}_{\mu\nu,ij}\,\langle m_f \overline{f} f\rangle + \ldots$$

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- GIM subtraction cancels mass independent divergences

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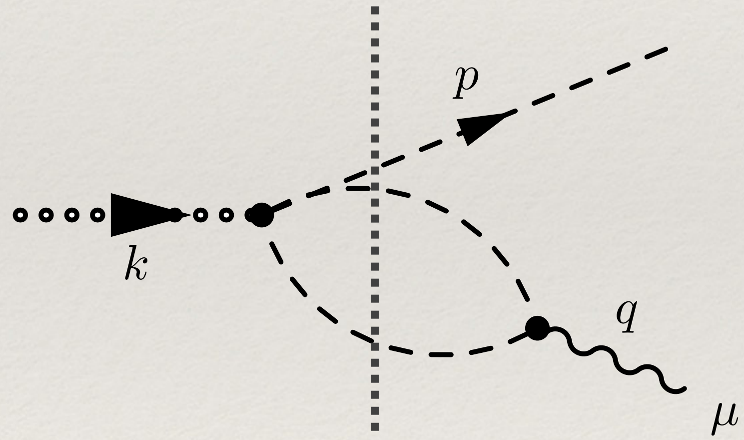

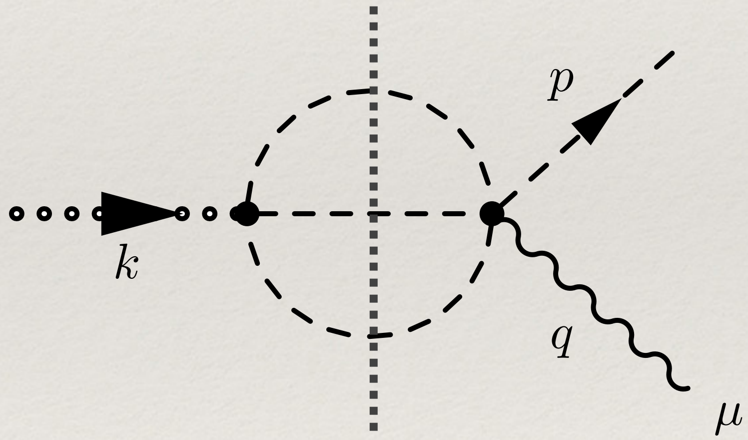

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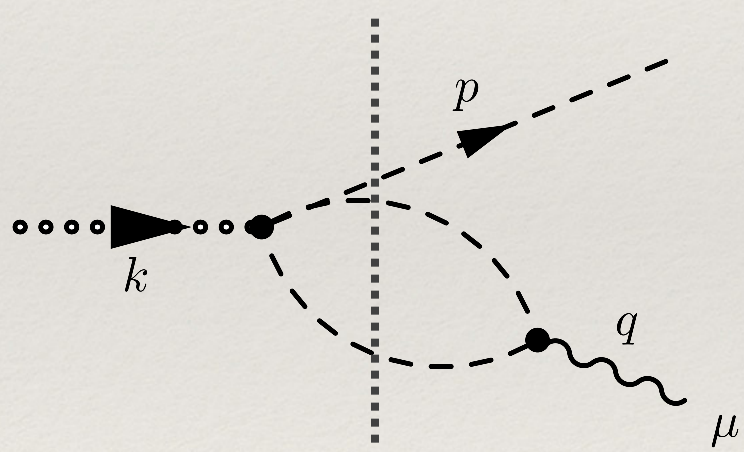

- All other finite-size effects: exponentially suppressed

Preliminary lattice results

Lattice setup

- DWF action, 64×24^3 lattice with spacing ~0.12 fm

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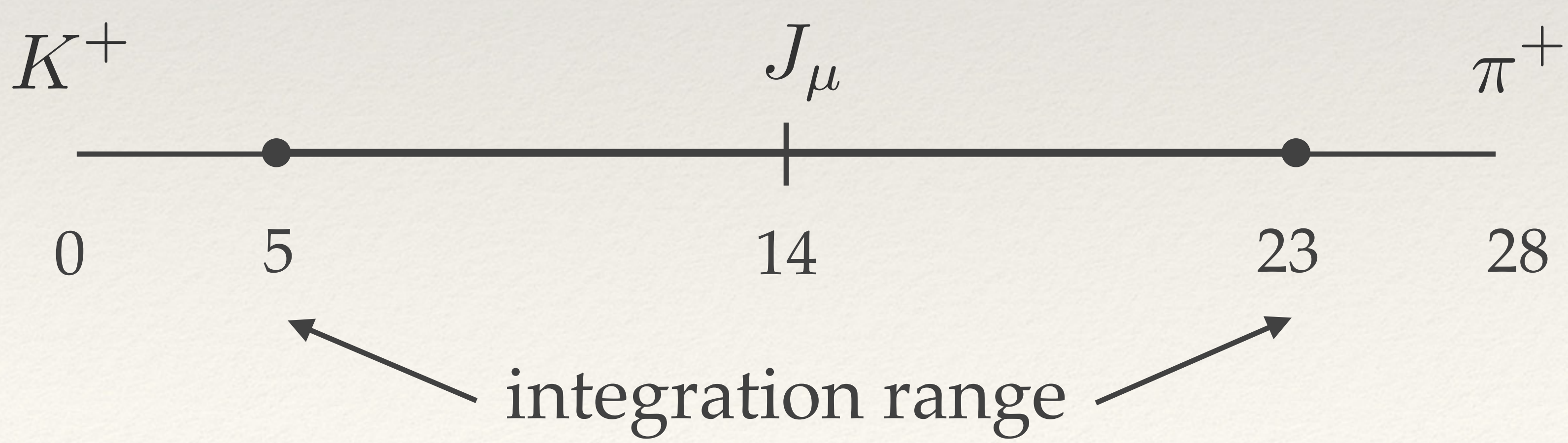
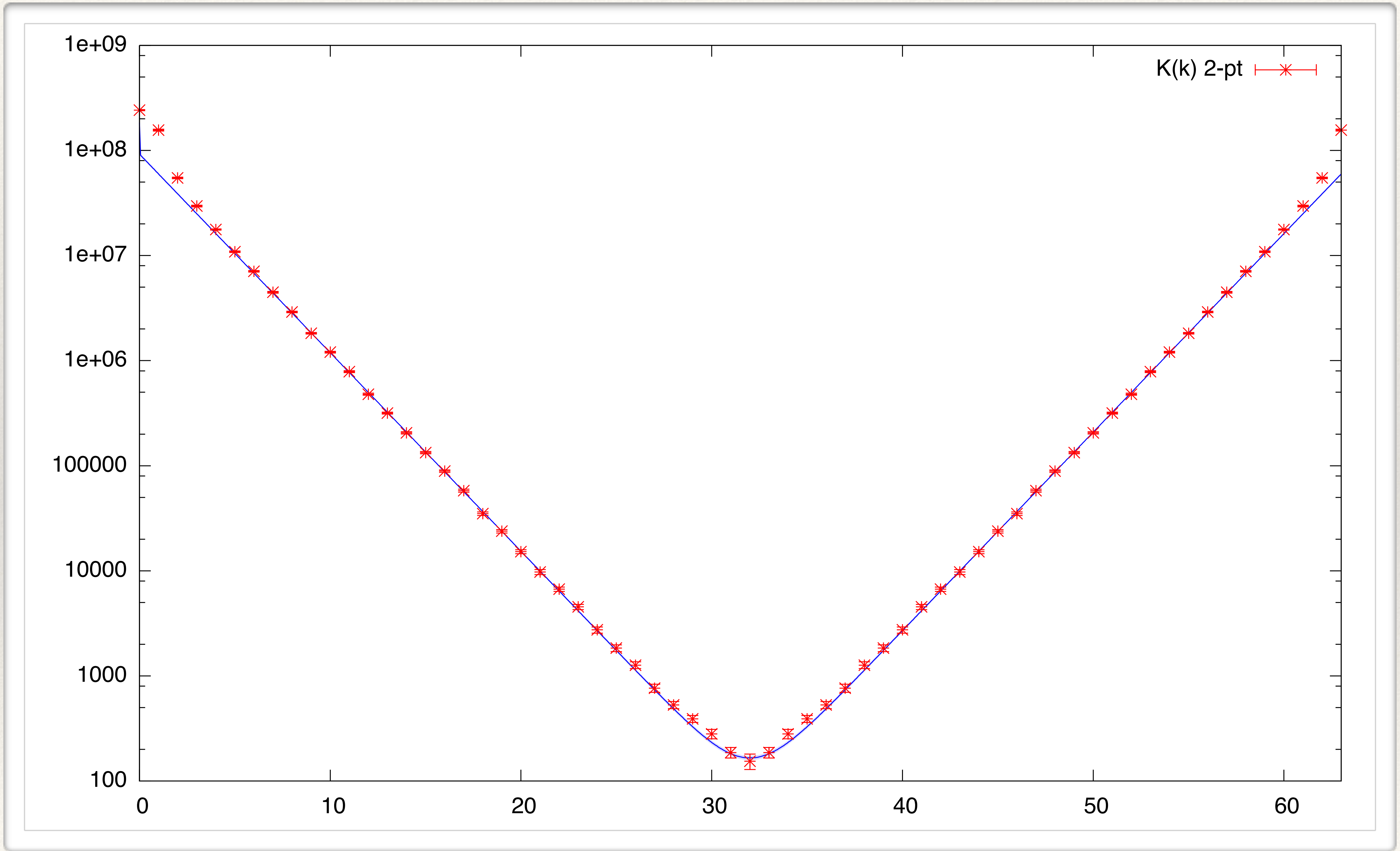
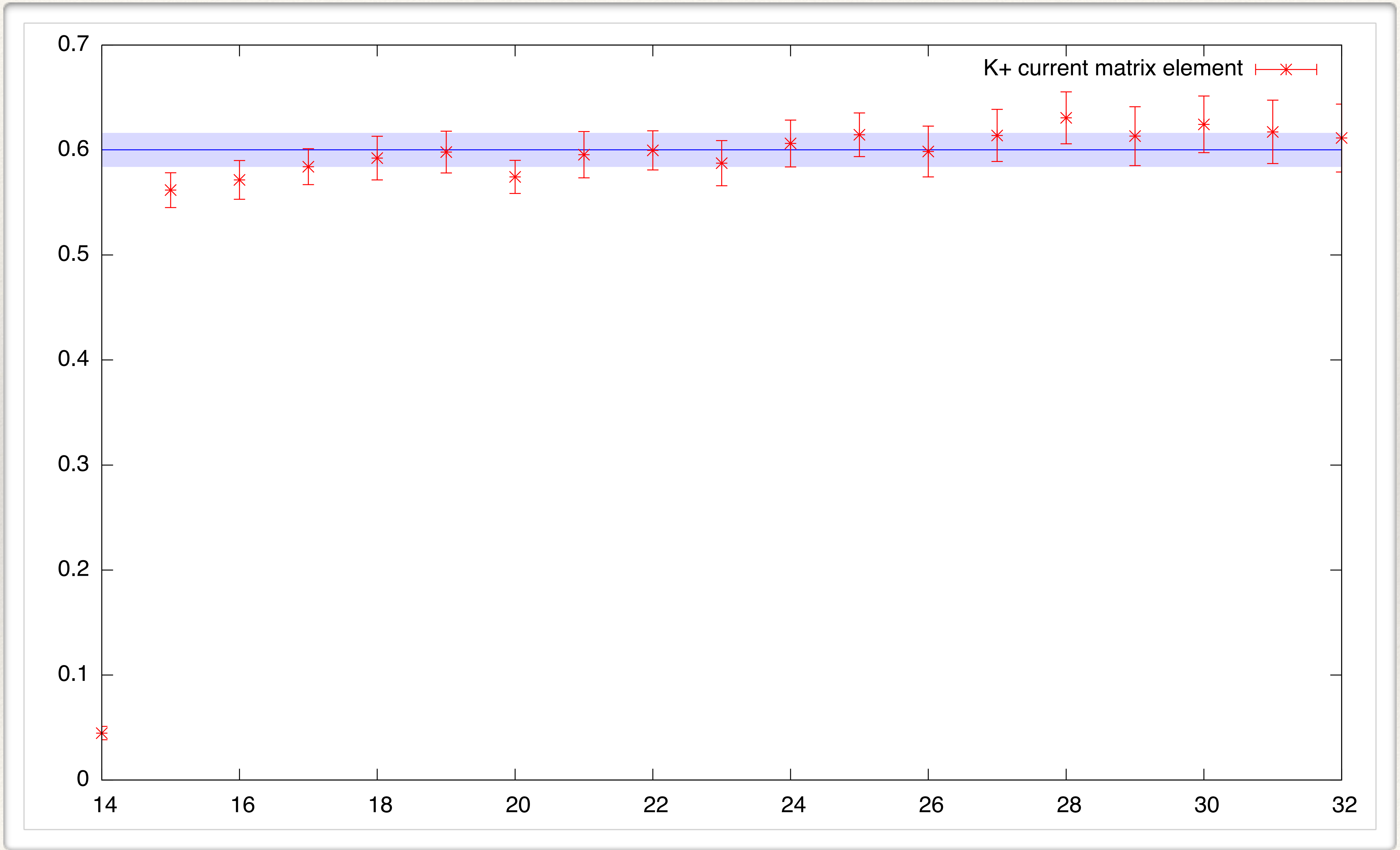
- gauge fixed wall sources, sequential current insertion



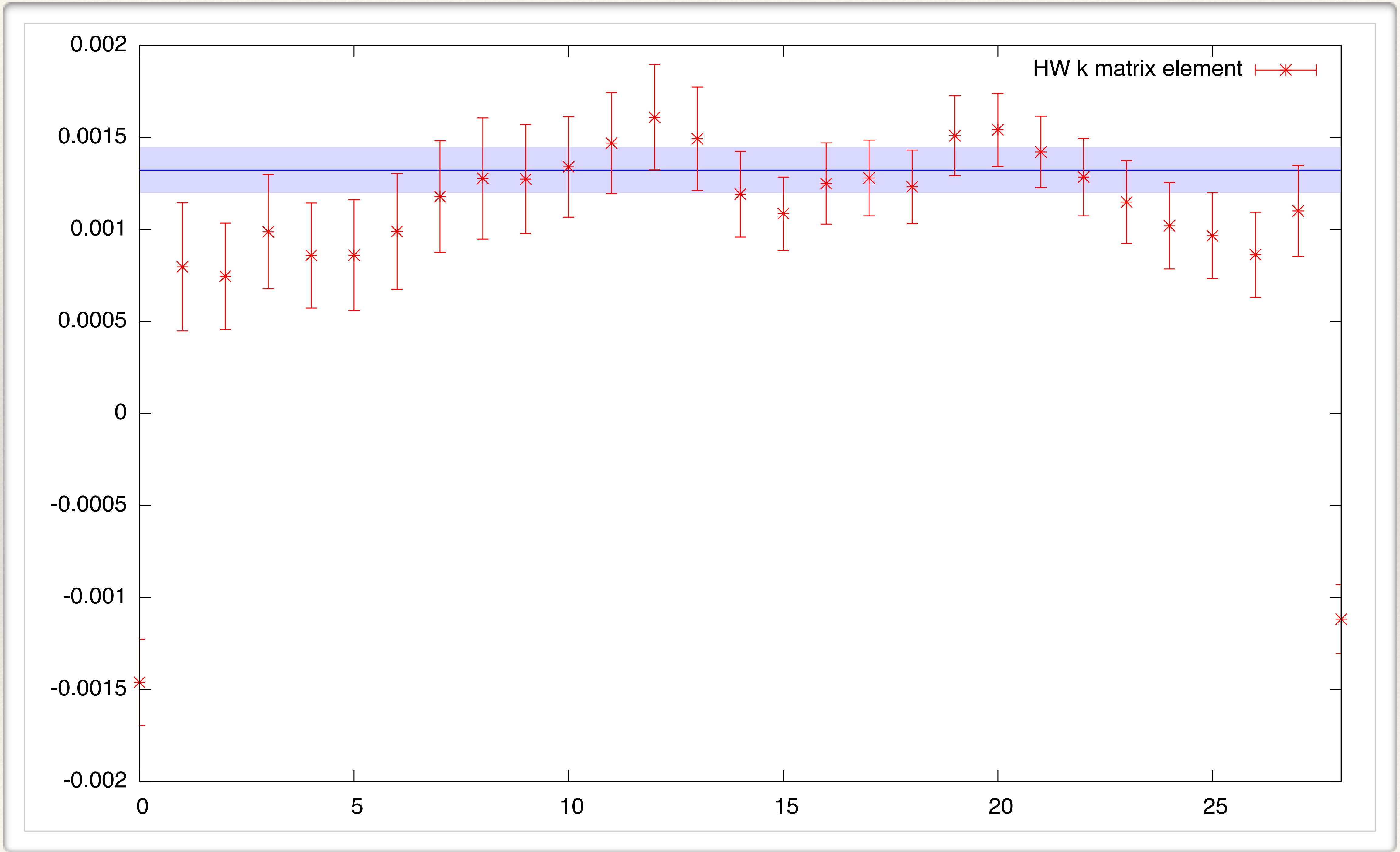
2-point function fit



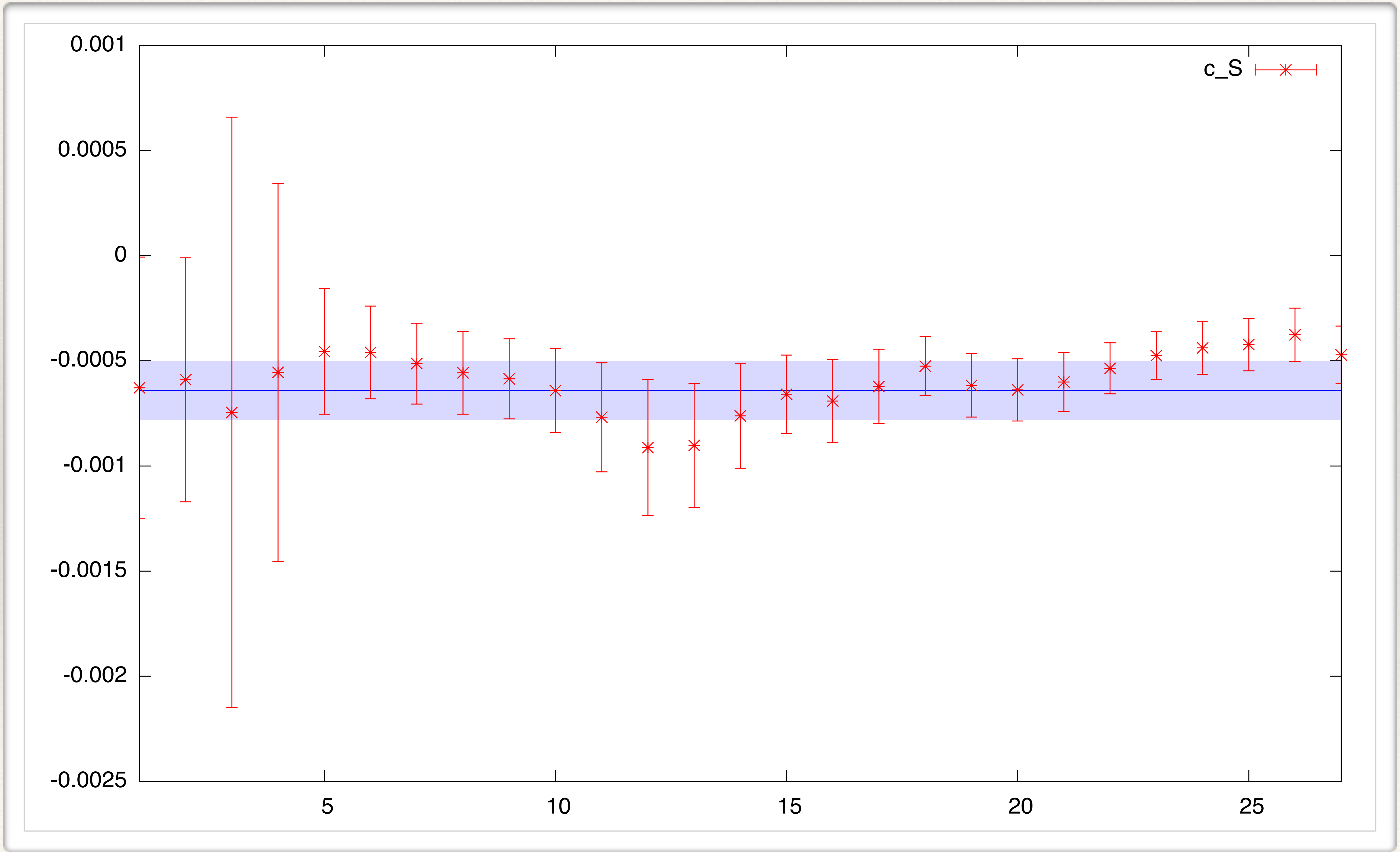
EM current matrix element



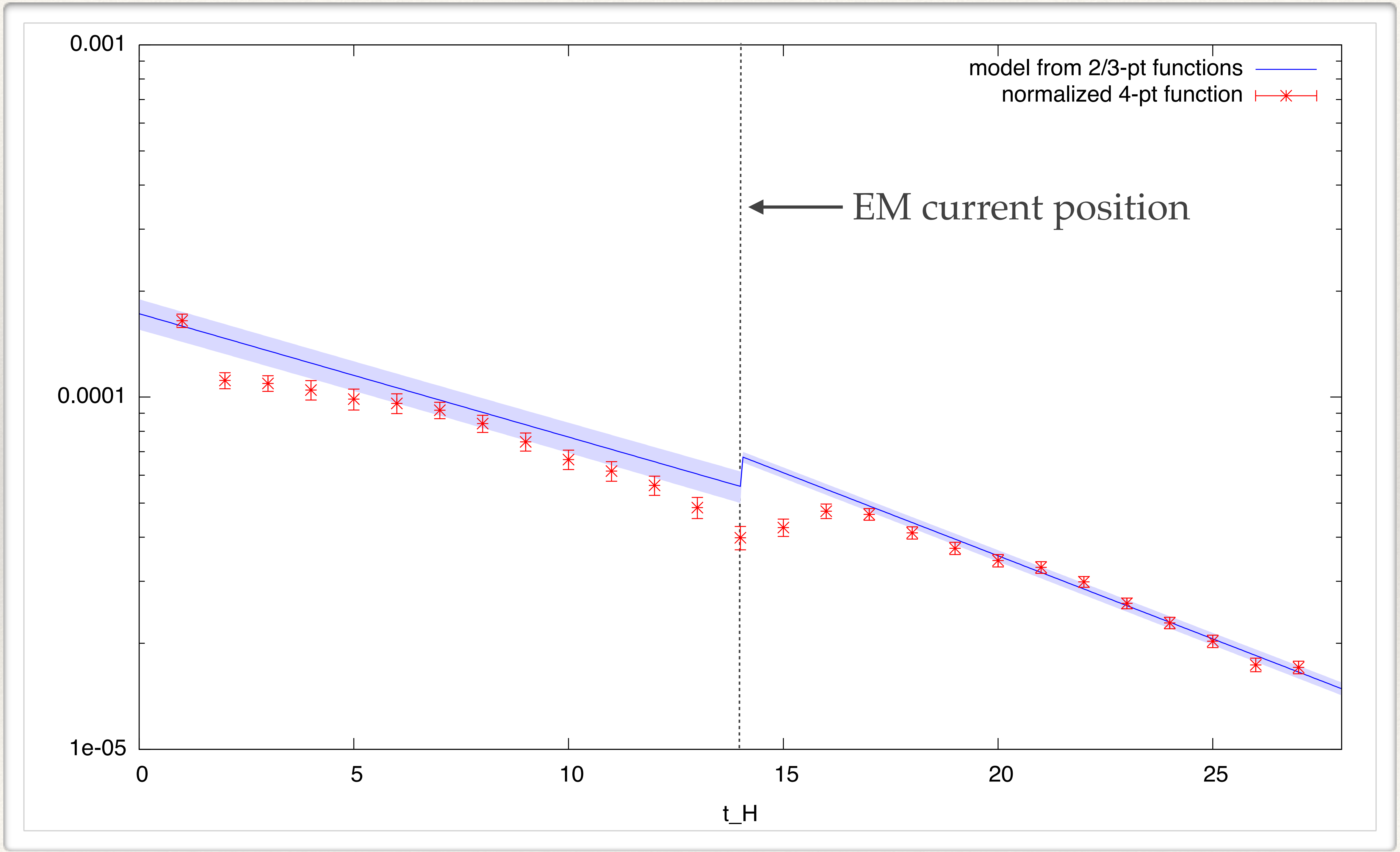
Weak Hamiltonian matrix element



c_S determination



Rare kaon decay correlation function



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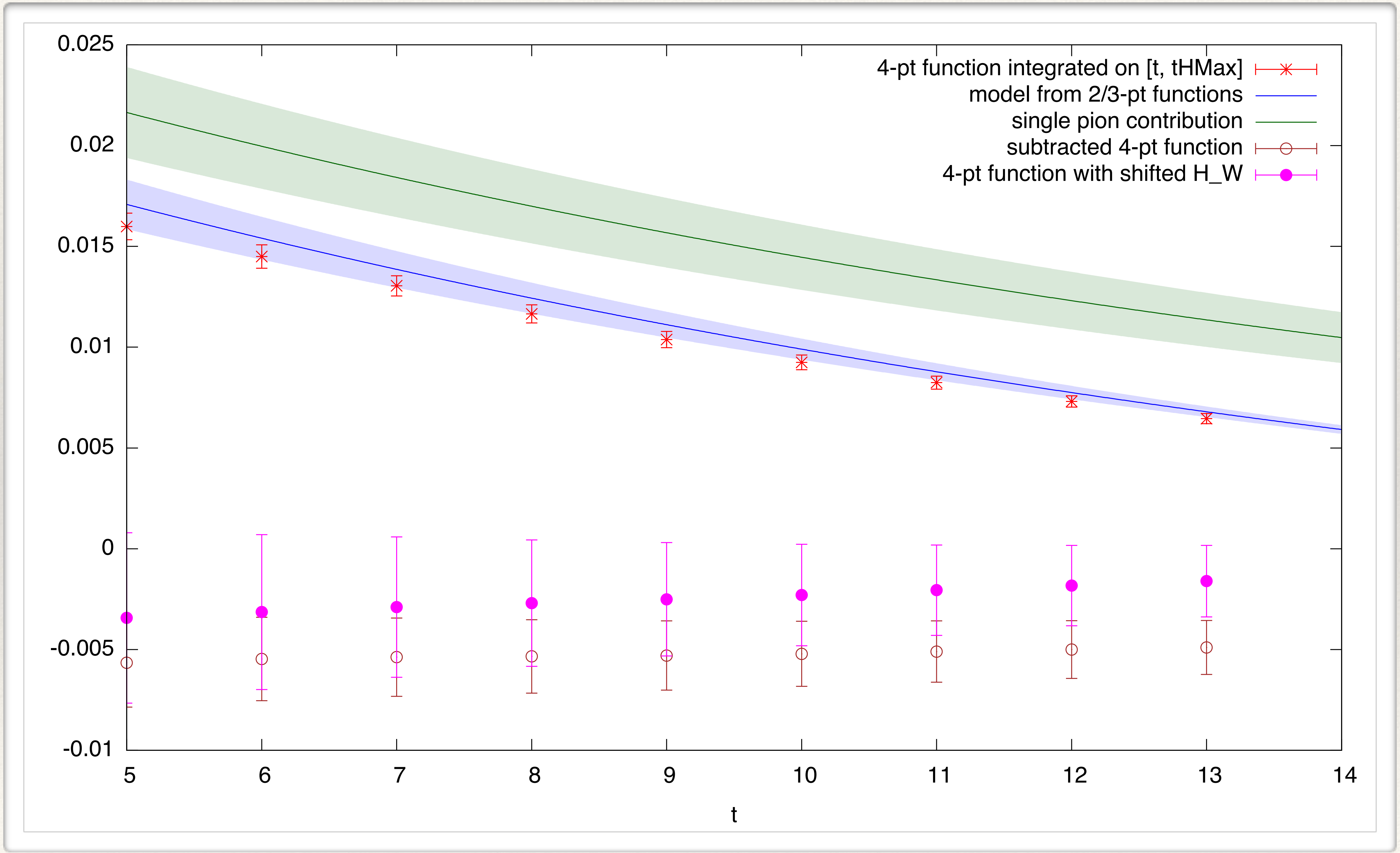

Summary & outlook

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Thank you!