LATTICE QCD FOR NUCLEAR PHYSICS

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NPLQCD Collaboration

Silas R. Beane
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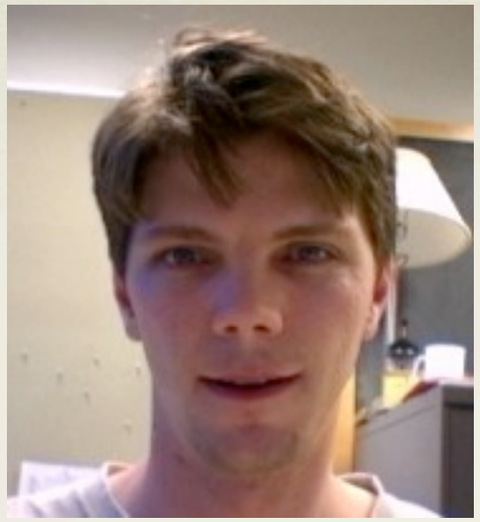
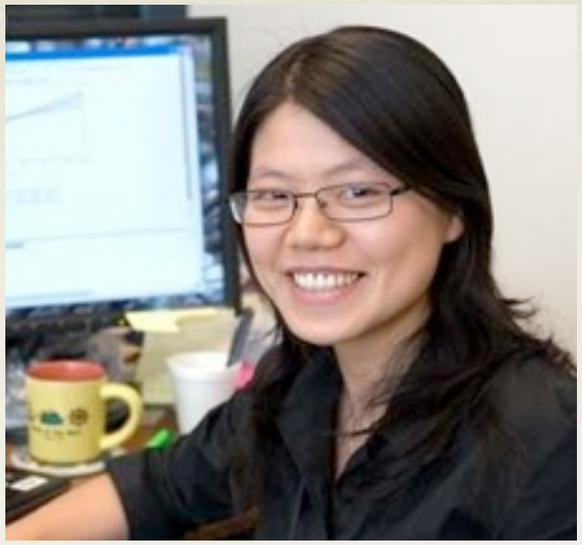


William Detmold
William & Mary



Huey-Wen Lin
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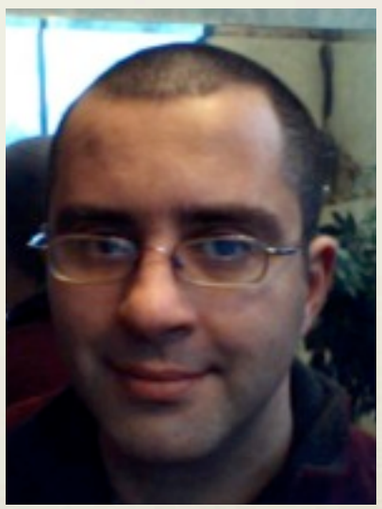
U of Washington



Aaron Torok
Indiana

André Walker-Loud
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Hadron Interactions

Goals:

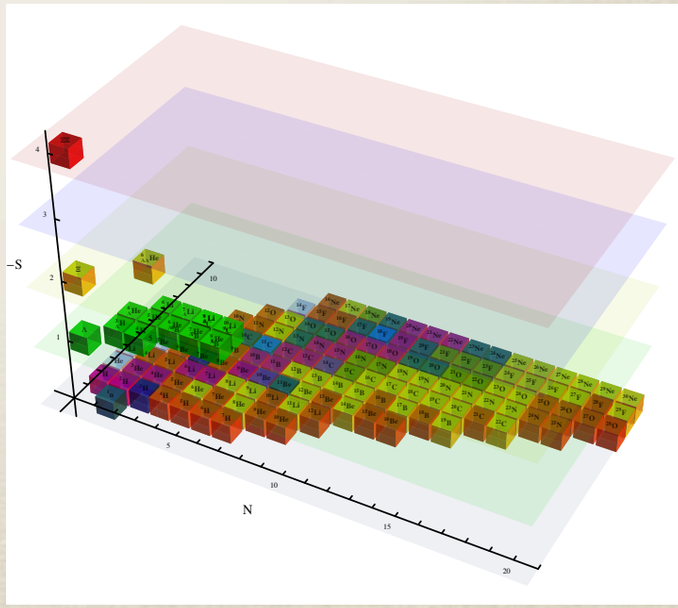
- Challenge: Compute properties of nuclei from QCD
  - Spectrum and structure
- Confirm well know experimental observation for two nucleon systems
- Explore the largely unknown territory of hyper-nuclear physics
- Provide input for the equation of state for nuclear matter in neutron stars
- Provide input for understanding the properties of multi-baryon systems

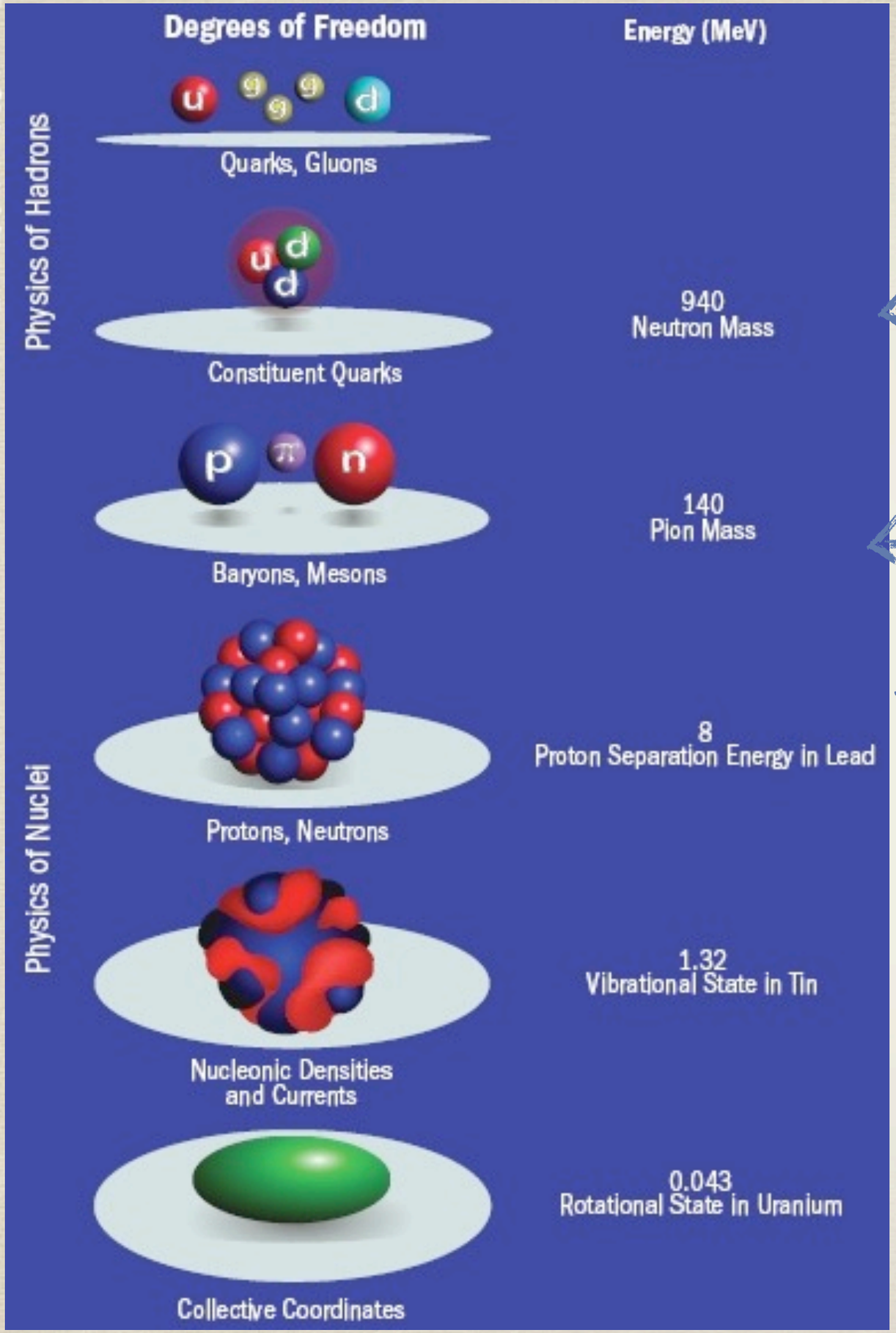
Jefferson Lab

FAIR

J-PARC

KEK
HIGH ENERGY ACCELERATOR RESEARCH ORGANIZATION





QCD

Hadron structure and spectrum

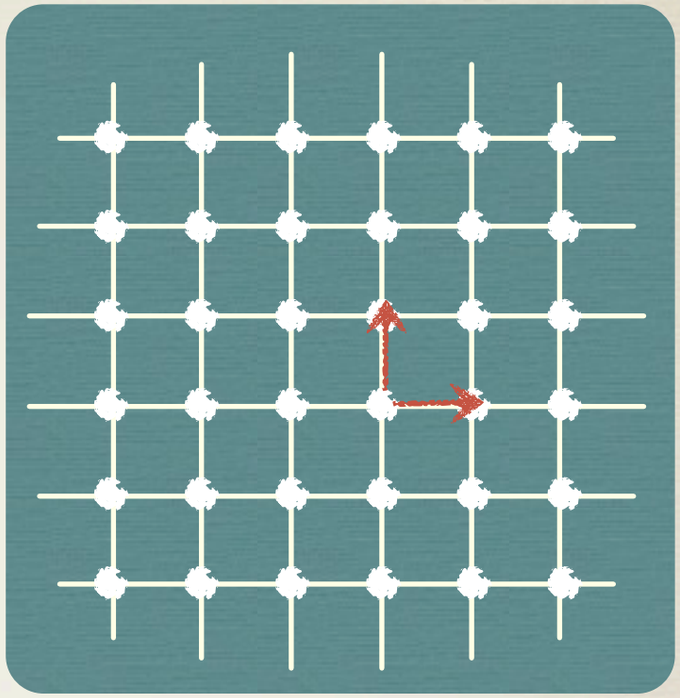
Hadronic Interactions

Nuclear physics

Figure by W. Nazarewicz

What does it take?

- Hadronic Scale: 1fm ~ 1×10^{-13} cm
- Lattice spacing << 1fm
  - take a=0.1fm
- Lattice size La >> 1fm
  - take La = 3fm
- Lattice 32^4
- Gauge degrees of freedom: $8 \times 4 \times 32^4 = 3.4 \times 10^7$



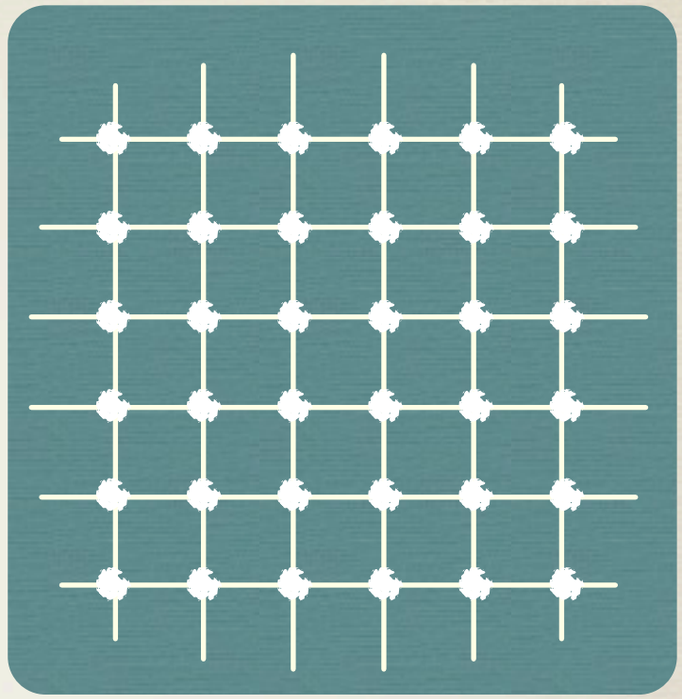
color

dimensions

sites

The pion mass is an additional small scale

$\sim 1/m_\pi \sim 1.4\text{fm}$



Volume corrections $\sim e^{-m_\pi L}$

6 -7 fm boxes might be needed at the physical point

Smaller scale: Binding momentum of the deuteron ~ 40MeV

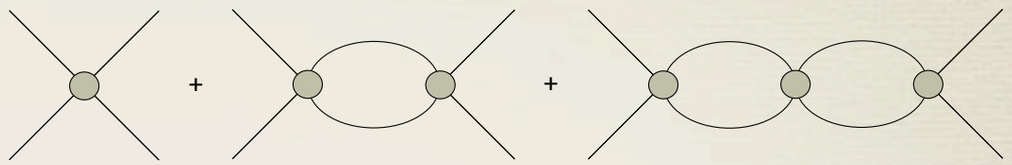
Nuclear energy level splittings are a few MeV

Box sizes of about 10fm will be needed

Scattering on the Lattice

Luscher Comm. Math. Phys 105, 153 '86

Elastic scattering amplitude (s-wave):

$A(p) =$  $+ \cdots$

$$A(p) = \frac{4\pi}{m} \frac{1}{p\,cot\delta \; - \; i\,p}$$

At finite volume one can show:

$$E_n = 2\sqrt{p_n^2 + m^2}$$

$$p \cot \delta(p) \;=\; \frac{1}{\pi L}\, \mathbf{S}\left(\frac{p^2 L^2}{4\pi^2}\right) \qquad \mathbf{S}(\eta) \;\equiv\; \sum_{\mathbf{j}}^{|\mathbf{j}|<\Lambda} \frac{1}{|\mathbf{j}|^2 - \eta} \;-\; 4\pi\Lambda$$

Small p:

$$p \cot \delta(p) = \frac{1}{a} + \frac{1}{2} r p^2 + ...$$

a is the scattering length

Bound States

Luscher Comm. Math. Phys 104, 177 '86

$$E_b = \sqrt{p^2 + m_1^2} + \sqrt{p^2 + m_2^2} - m_1 - m_2 \qquad p^2 < 0$$

$$E_b \approx \frac{p^2}{2\mu} = -\frac{\kappa^2}{2\mu} \qquad \kappa = |p|$$

κ is the "binding momentum" and μ the reduced mass

Finite volume corrections:

$$\Delta E_b = -3|A|^2 \frac{e^{-\kappa L}}{\mu L} + \mathcal{O}\left(e^{-\sqrt{2}\kappa L}\right)$$

cubic group irrep: A_1^+

Two Nucleon spectrum

free nucleons

free 2 particle spectrum

M_n

3fm box

24^3 lattice

anisotropy factor 3.5

$M_\pi = 390$ MeV

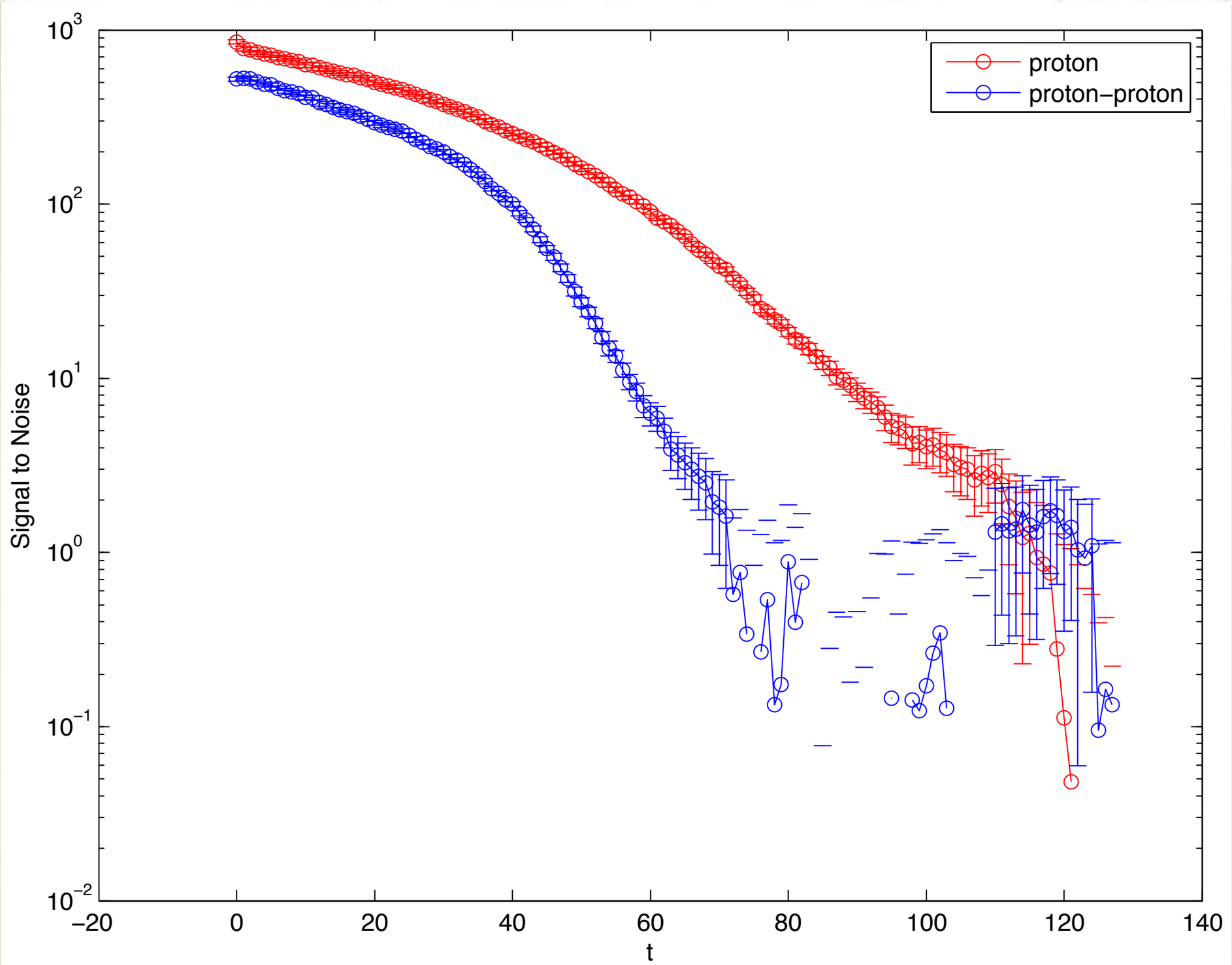
Signal to Noise ratio for correlation functions

$$C(t) = \langle N(t)\bar{N}(0)\rangle \sim Ee^{-M_N t}$$

$$var(C(t)) = \langle N\bar{N}(t)N\bar{N}(0)\rangle \sim Ae^{-2M_N t} + Be^{-3m_\pi t}$$

$$StoN = \frac{C(t)}{\sqrt{var(C(t))}} =\sim Ae^{-(M_N - 3/2m_\pi)t}$$

* The signal to noise ratio drops exponentially with time
* The signal to noise ratio drops exponentially with decreasing pion mass
* For two nucleons: StoN(2N) = StoN(1N)^2



Signal to Noise

$32^3 \times 256$

M_π=390MeV

anisotropy factor 3.5

NPLQCD data

We need to fit for several low lying states for reliable estimation of the ground state of the two particle system in a finite box.

Use “variational methods”

We need very high statistics to be able to resolve excited state contamination

We need to fit for several low lying states for reliable estimation of the ground state of the two particle system in a finite box.

Use "variational methods"

We need very high statistics to be able to resolve excited state contamination

We really need better algorithms to deal with an exponentially hard problem

1. Hyperon-Hyperon Interactions

2. Hyperon-Nucleon Interactions

H-dibaryon

R. L. Jaffe, Phys. Rev. Lett. 38, 195 (1977)

- Proposed by R. Jaffe 1977
  - Perturbative color-spin interactions are attractive for (uuddss)
  - Diquark picture of scalar diquarks (ud)(ds)(su)

$S=-2$, $B=2$, $J^P=0^+$

- Experimental searches of the H have not found it
  - BNL RHIC (+model): Excludes the region [-95, 0] MeV

A. L. Trattner, PhD Thesis, LBL, UMI-32-54109 (2006).

  - KEK: Resonance near threshold

C. J. Yoon et al., Phys. Rev. C 75, 022201 (2007).

- Several Lattice QCD calculations have been addressing the existence of a bound H

Lattice QCD

- P. B. Mackenzie and H. B. Thacker, Phys. Rev. Lett. 55,2539 (1985)
  - Did not find it
- Y. Iwasaki, T. Yoshie and Y. Tsuboi, Phys. Rev. Lett. 60, 1371 (1988)
  - Found it
- A. Pochinsky, J. W. Negele and B. Scarlet, Nucl. Phys. Proc. Suppl. 73, 255 (1999)
  - Concluded H is not bound in the infinite volume limit
- I. Wetzorke and F. Karsch Nucl. Phys. Proc. Suppl. 119, 287 (2003)
  - Concluded the H is not bound

Lattice QCD

* NLPQCD Phys.Rev.Lett.106.162001 2011
* HALQCD Phys.Rev.Lett.106.162002 2011
* Luo, Loan and Liu arXiv:1106.1945
  * Anisotropic Clover (quenched)
  * Multiple lattice spacings and volumes
  * Result: $B_H = 70(11)(15)\text{MeV}$

NPLQCD: lattice set up

- Anisotropic 2+1 clover fermion lattices

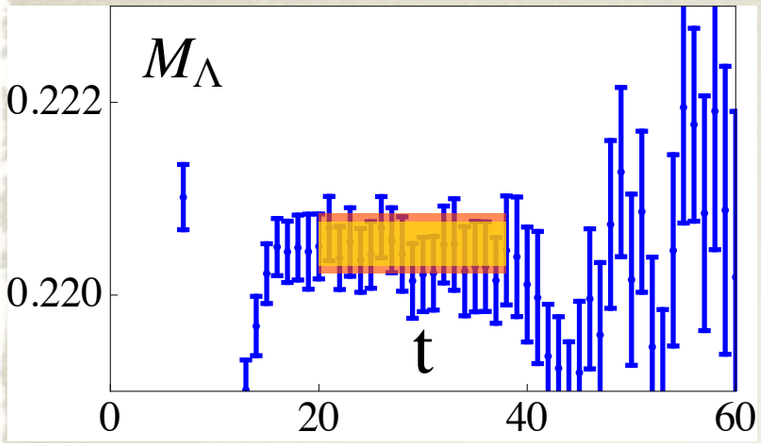
  Hadron Spectrum/JLAB

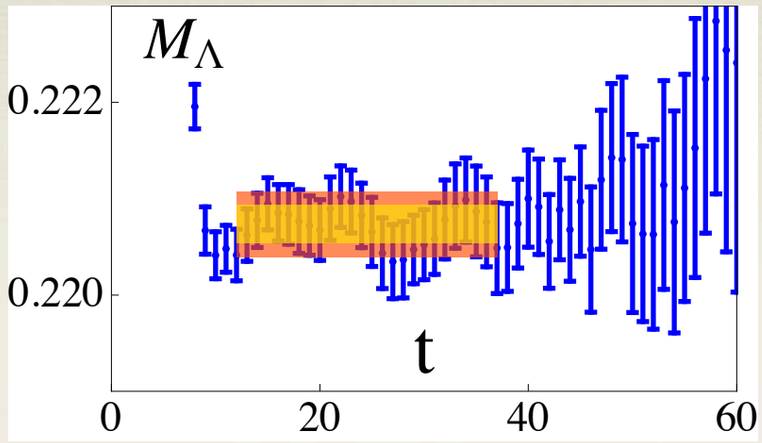
  - a ~ 0.125fm (anisotropy of ~ 3.5)
  - pion mass ~ 390 MeV
  - Volumes 16^3 x 128, 20^3 x 128, 24^3 x 128, 32^3 x 256 — largest box 4fm
  - Smeared source - 3 sink interpolating fields
- Interpolating fields have the structure of s-wave Λ-Λ system
  - I=0, S=-2, A_1 , positive parity

NPLQCD methods

- Compute the ground state energy using Euclidian correlators
  - Use a single source operator with three sinks operator (varied smearing)
- "Diagonalize" the correlators to isolate the ground state
  - Matrix Prony and Multi-exponential fits
- Extract the binding momentum
- Extrapolate to infinite volume using multiple volumes
- Use very high statistics *O*(500K) correlation functions

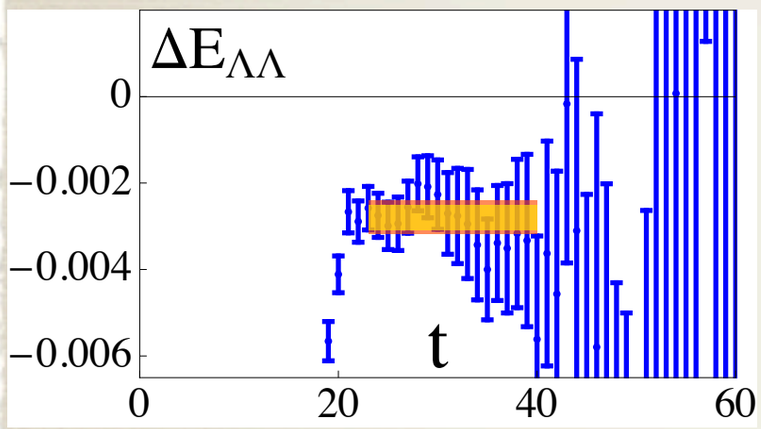
NPLQCD

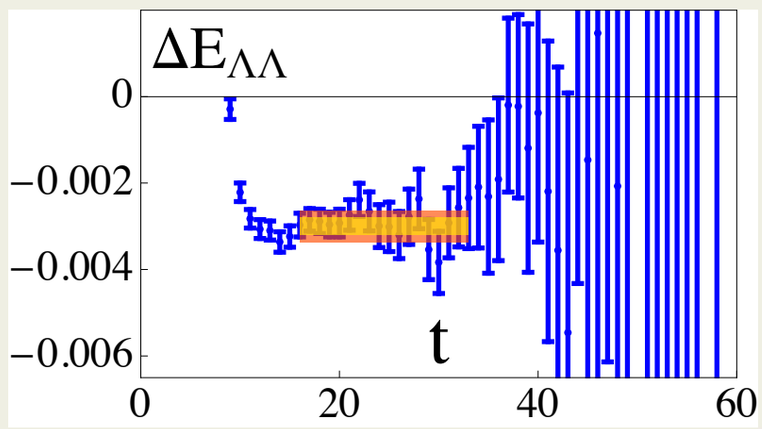




ΛΛ Energy

$$E_{\Lambda\Lambda} = 2\sqrt{p^2 + m_\Lambda^2}$$





binding momentum

$$\kappa = |p|$$

$$\kappa = \gamma + \frac{g_1}{L}\left(e^{-\gamma L} + \sqrt{2}e^{-\sqrt{2}\gamma L}\right)$$

$$B_H = \frac{\gamma^2}{m_\Lambda}$$

NPLQCD: H-dibaryon

$B_H = 16.6 \pm 2.1 \pm 4.6$ MeV

statistical systematic

M_π=390MeV

2+1 Clover anisotropic fermions

NPLQCD: arXiv:1012.3812

Phys. Rev. Lett. **106**, 162001
(Published April 20, 2011)

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Continuum limit?

Physical pion mass?

Isospin breaking?

Electromagnetism?

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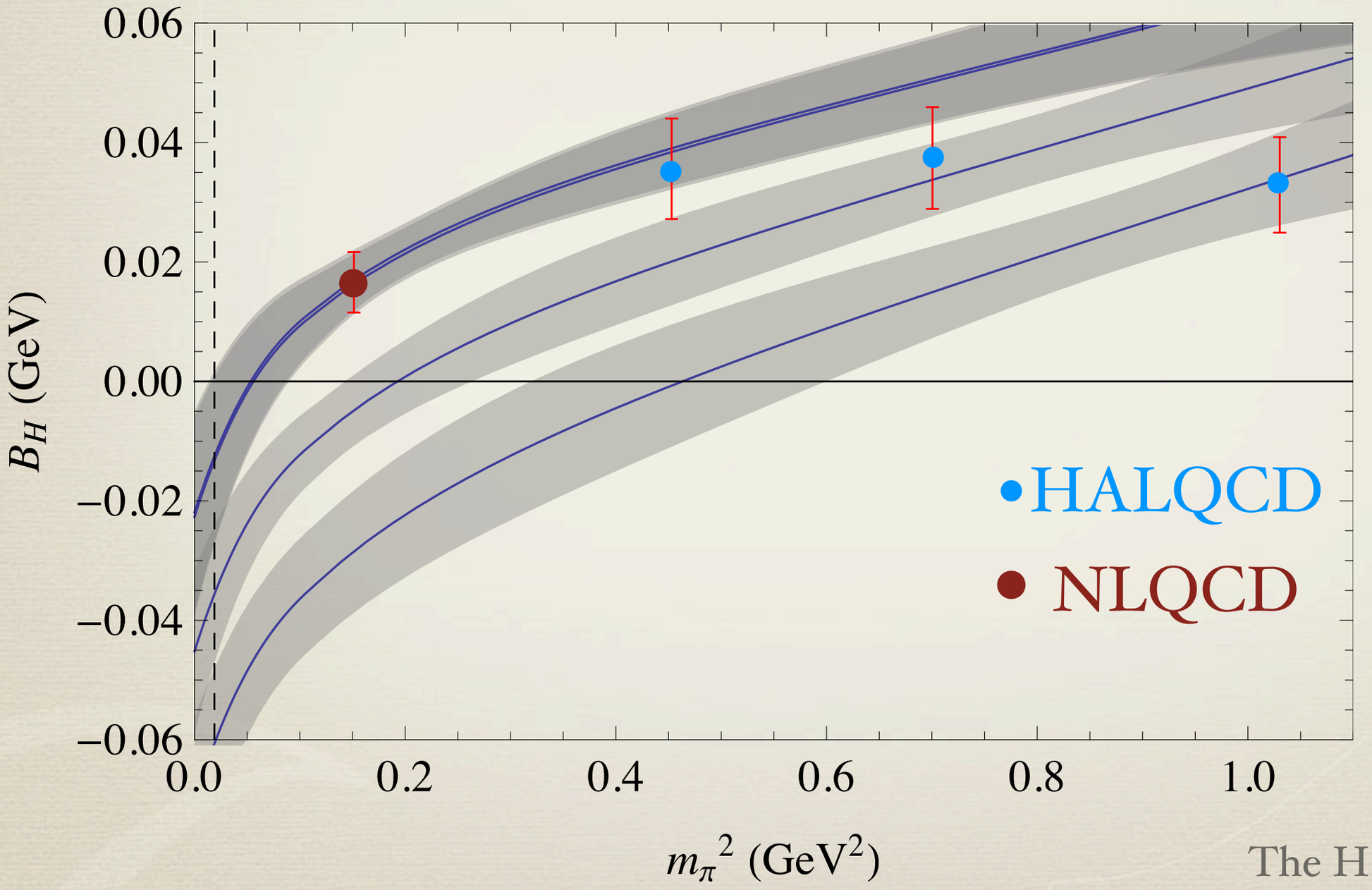
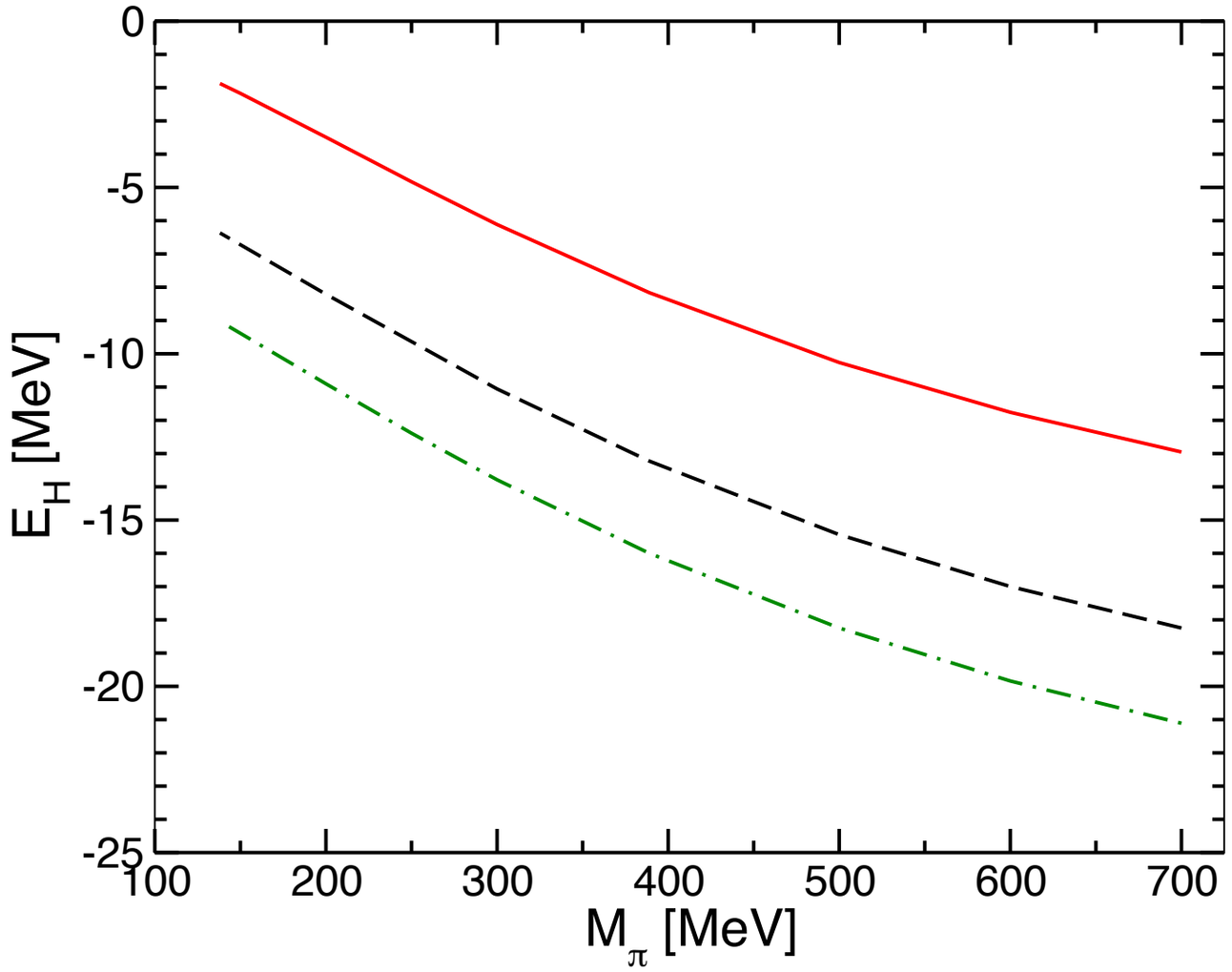
Chiral extrapolation

P. Shanahan et.al. arXiv:1106.2851



The H may not be bound

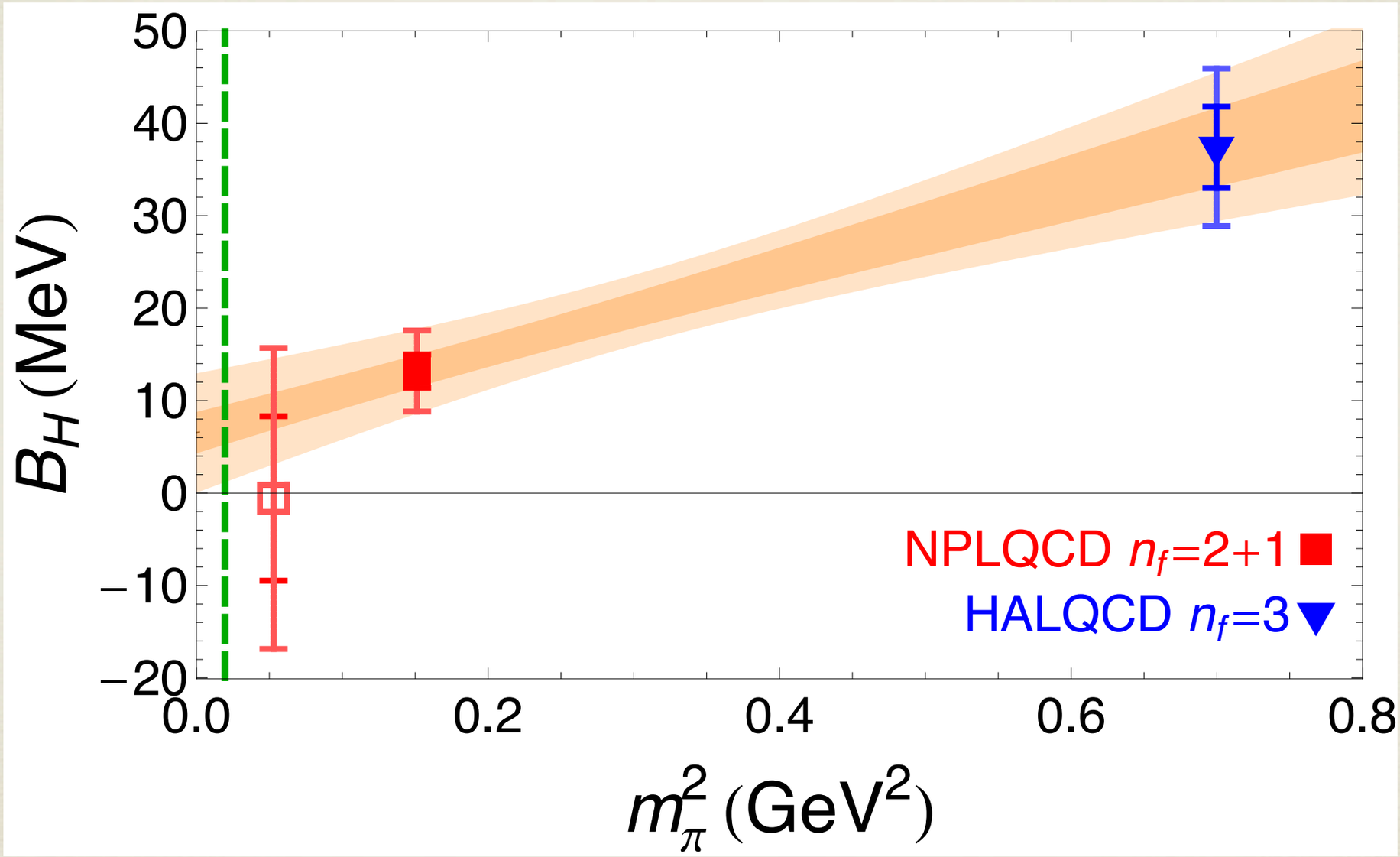
Chiral extrapolation



J. Haidenbauer, Ulf-G. Meisner
arXiv:1109.3590

H-dibaryon: Towards the physical point

[S. Beane et.al. arXiv:1103.2821 Mod. Phys. Lett. A26: 2587, 2011]



HALQCD:Phys.Rev.Lett.106:162002,2011

H-dibaryon:

Is bound at heavy quark masses. May be unbound at the physical point

NPLQCD

H-dibaryon: Towards the physical point

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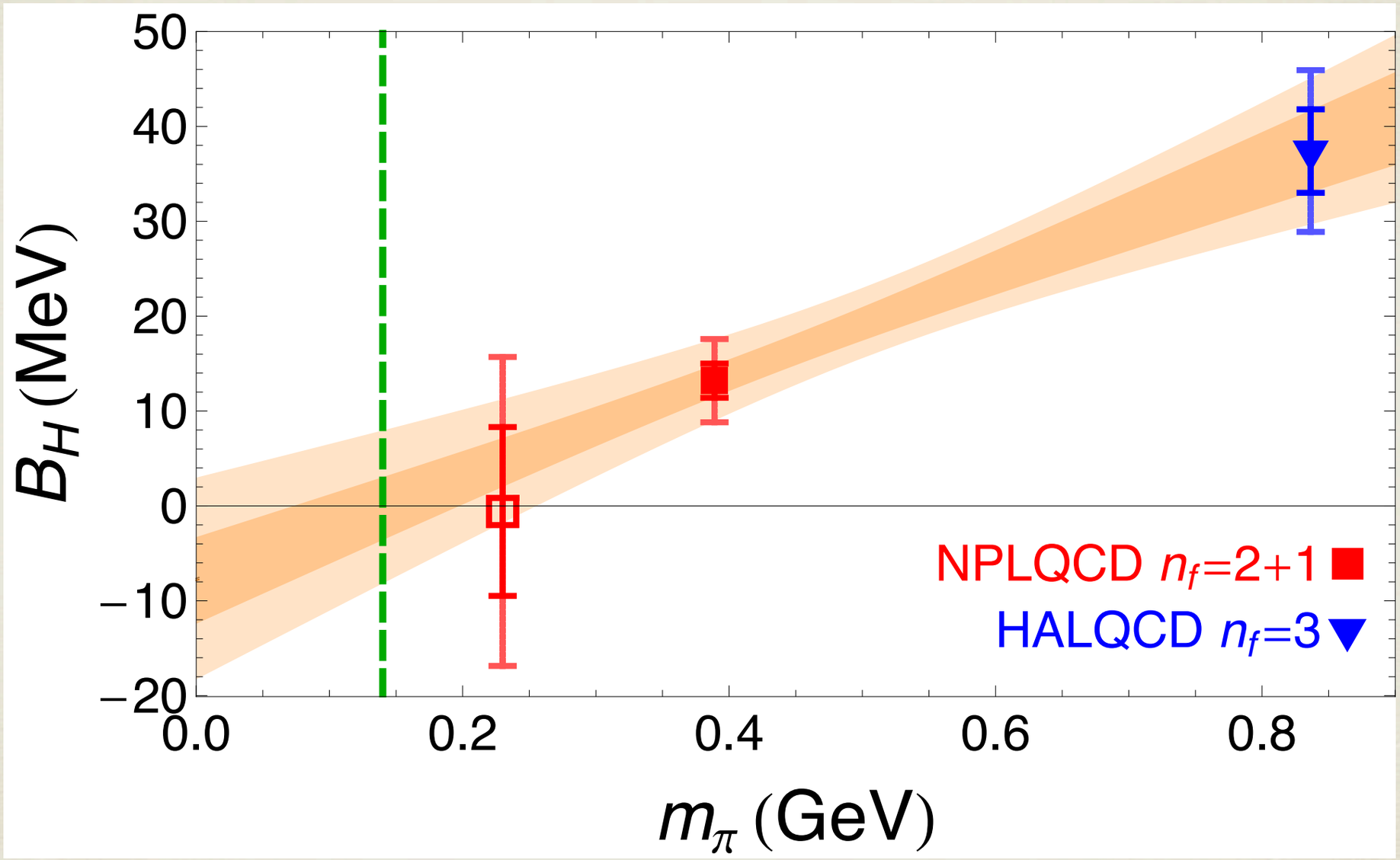

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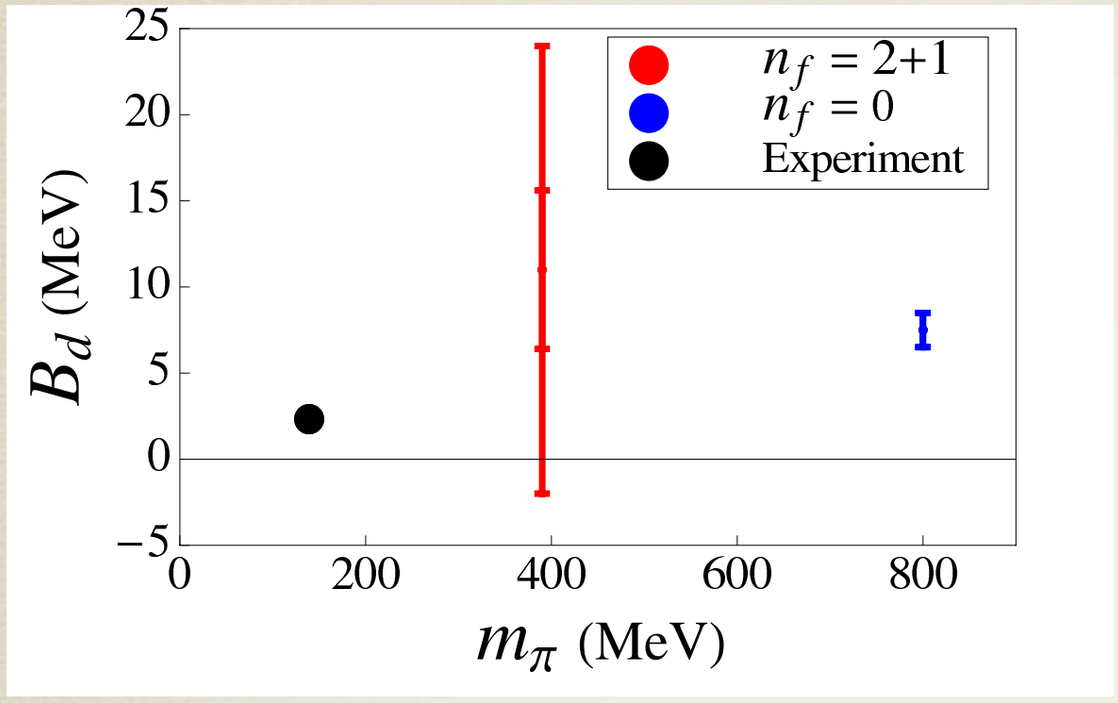
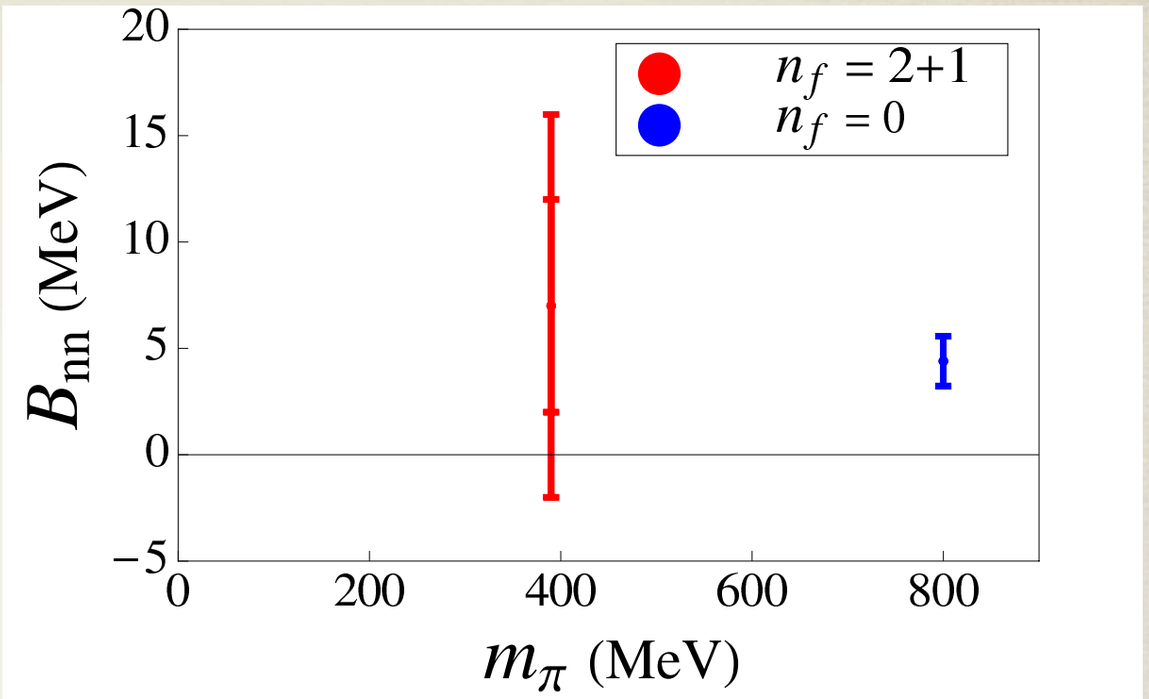
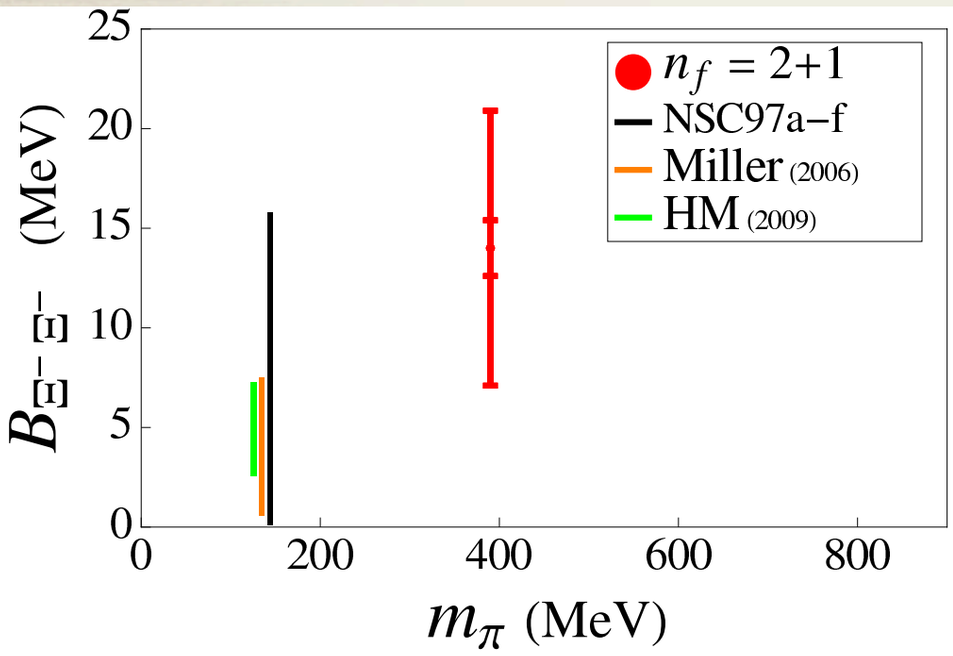
NPLQCD

Two baryon bound states

[S. Beane et.al. arXiv:1108.2889 submitted to Phys.Rev.D]







V. G. J. Stoks and T. A. Rijken
Phys. Rev. C 59, 3009 (1999)
[arXiv:nucl-th/9901028

G. A. Miller,
arXiv:nucl-th/0607006

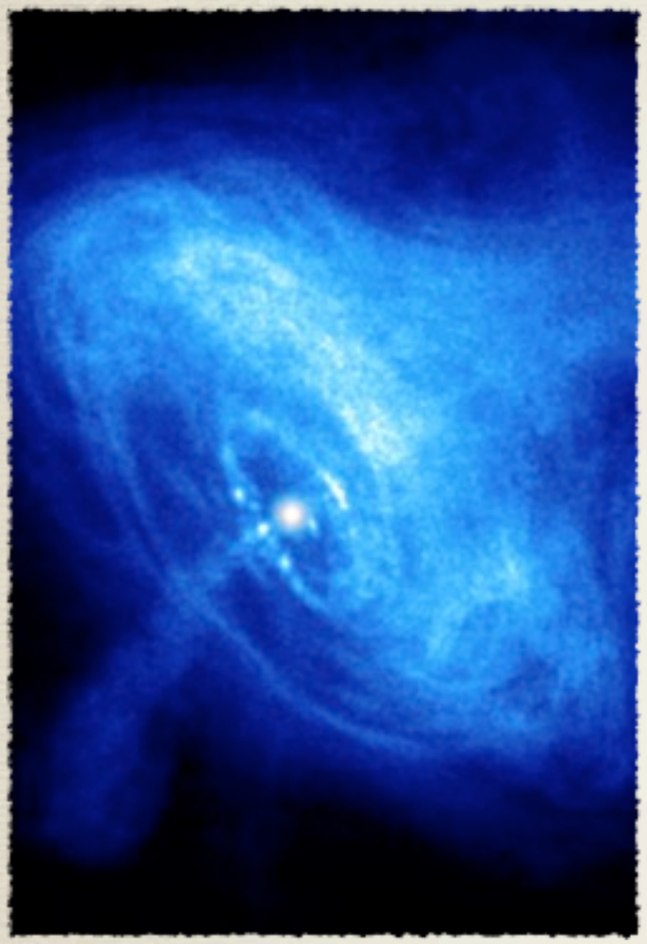
J. Haidenbauer, Ulf-G. Meisner
Phys.Lett.B684,275-280(2010)
arXiv:0907.1395

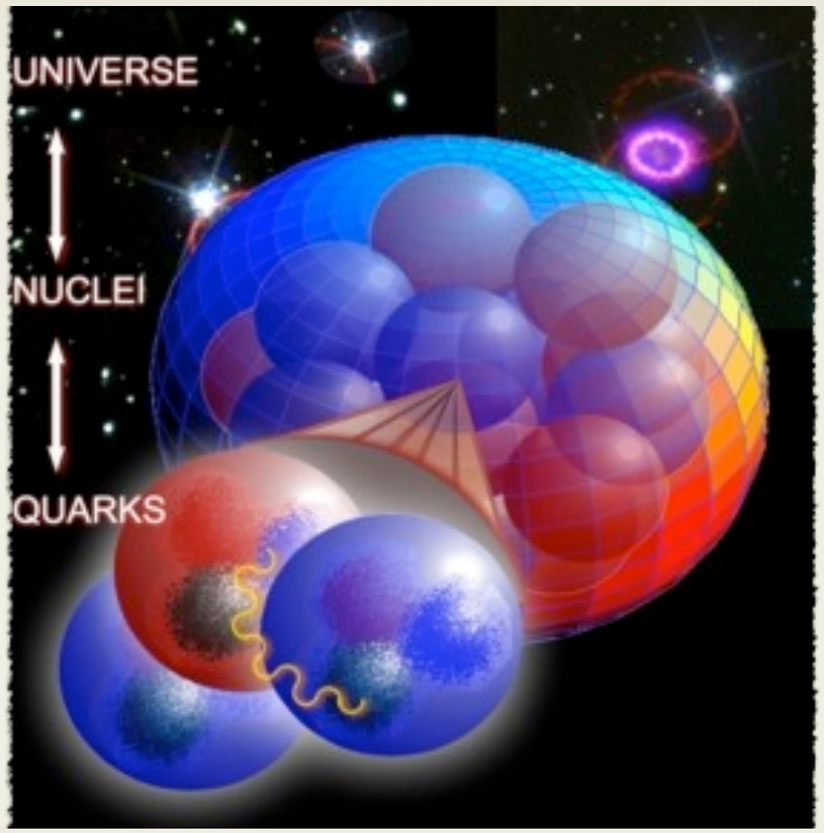
n_f=0:
Yamazaki, Kuramashi, Ukawa
Phys.Rev. D84 (2011) 054506
arXiv: 1105.1418

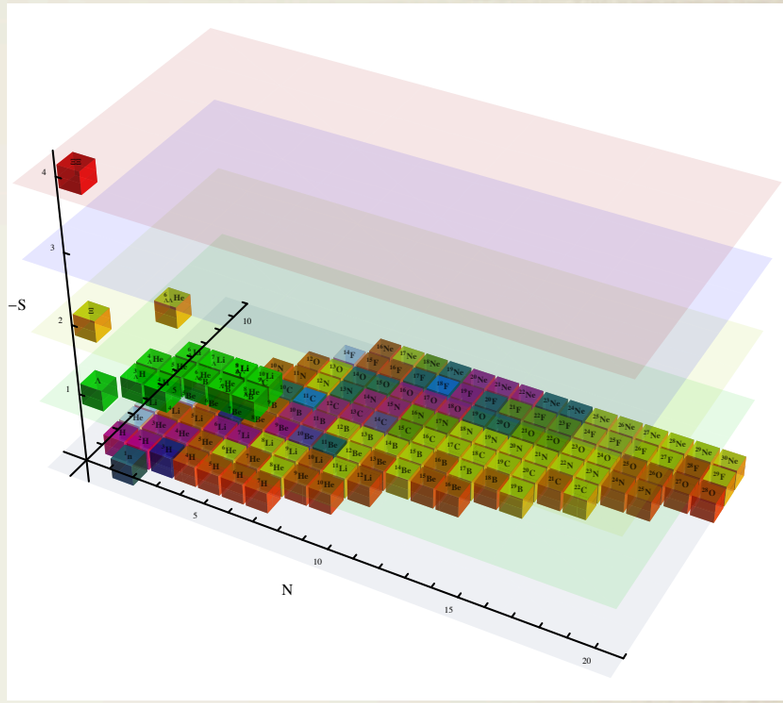
gauge fields 2+1 flavors (JLab)
anisotropic clover m_π~ 390MeV

NPLQCD

Hyperon-Nucleon interactions





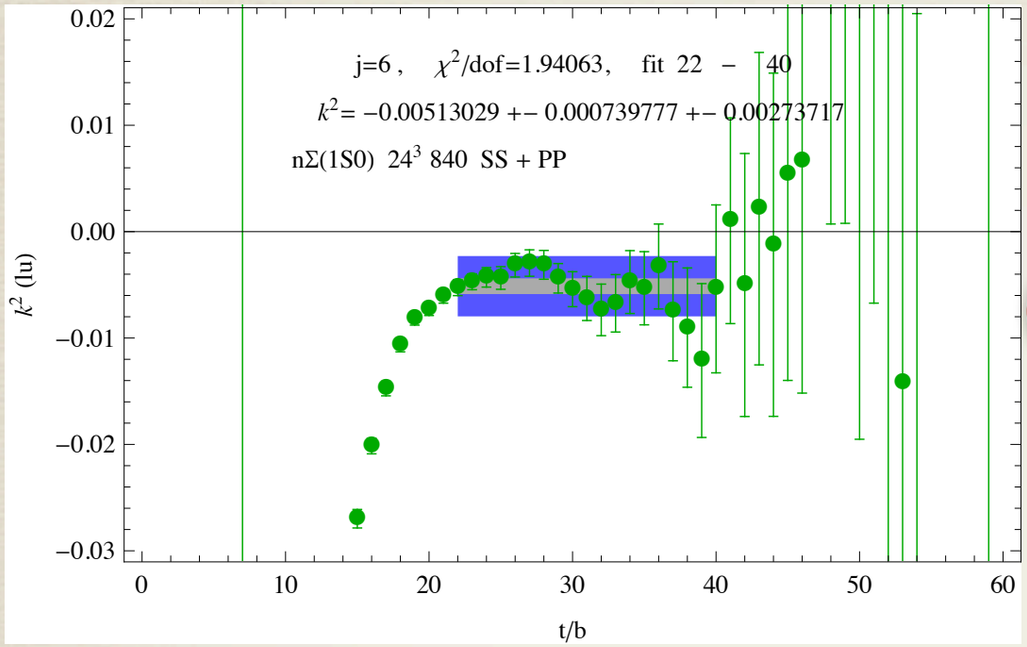


equation of state for nuclear matter in neutron stars

hyper-nuclear physics

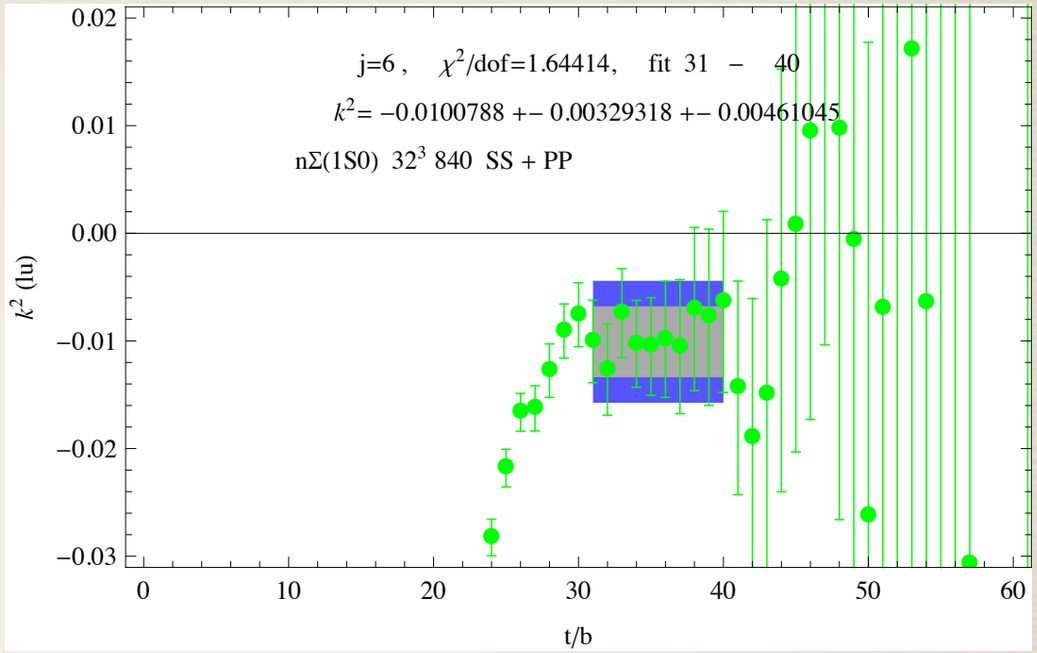
$n\Sigma^-$ interactions

$24^3 \times 128$



$(^1S_0)$

$32^3 \times 256$



Finite volume scaling: $\kappa = \gamma + \frac{g_1}{L}\left(e^{-\gamma L} + \sqrt{2}e^{-\sqrt{2}\gamma L}\right)$

$$B_{n\Sigma} = \frac{\gamma^2}{2\mu_{n\Sigma}} = 25 \pm 9.3 \pm 11 \ \text{MeV}$$

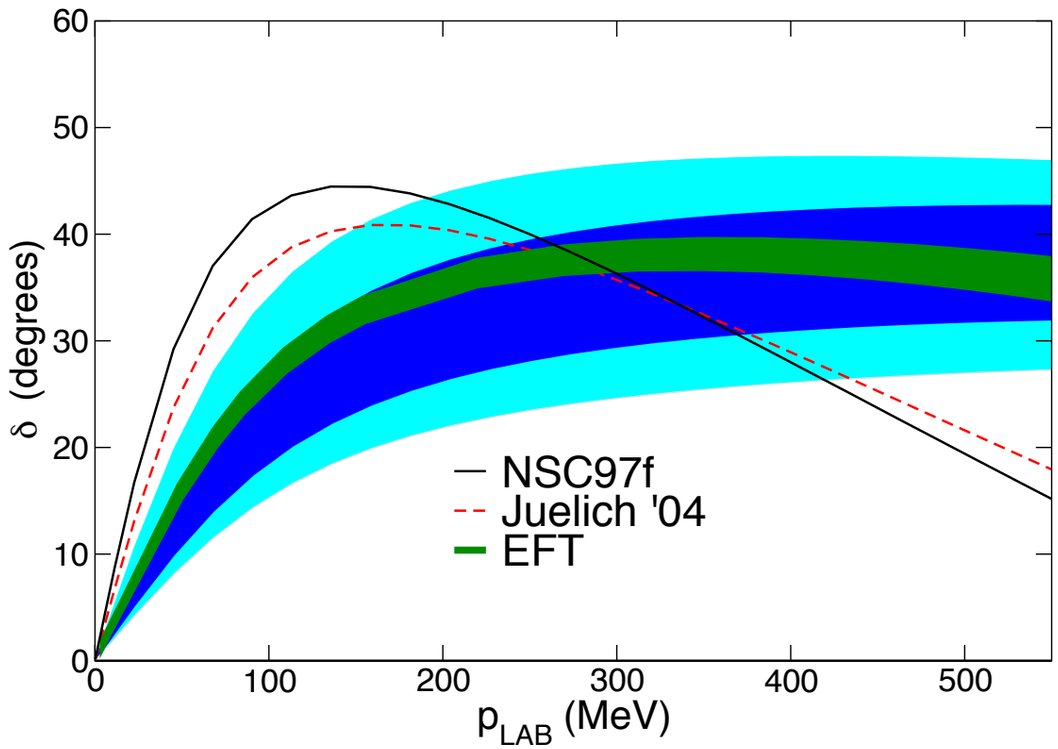
constrain LO YN EFT

gauge fields: 2+1 flavors (JLab), anisotropic clover m_π~ 390MeV

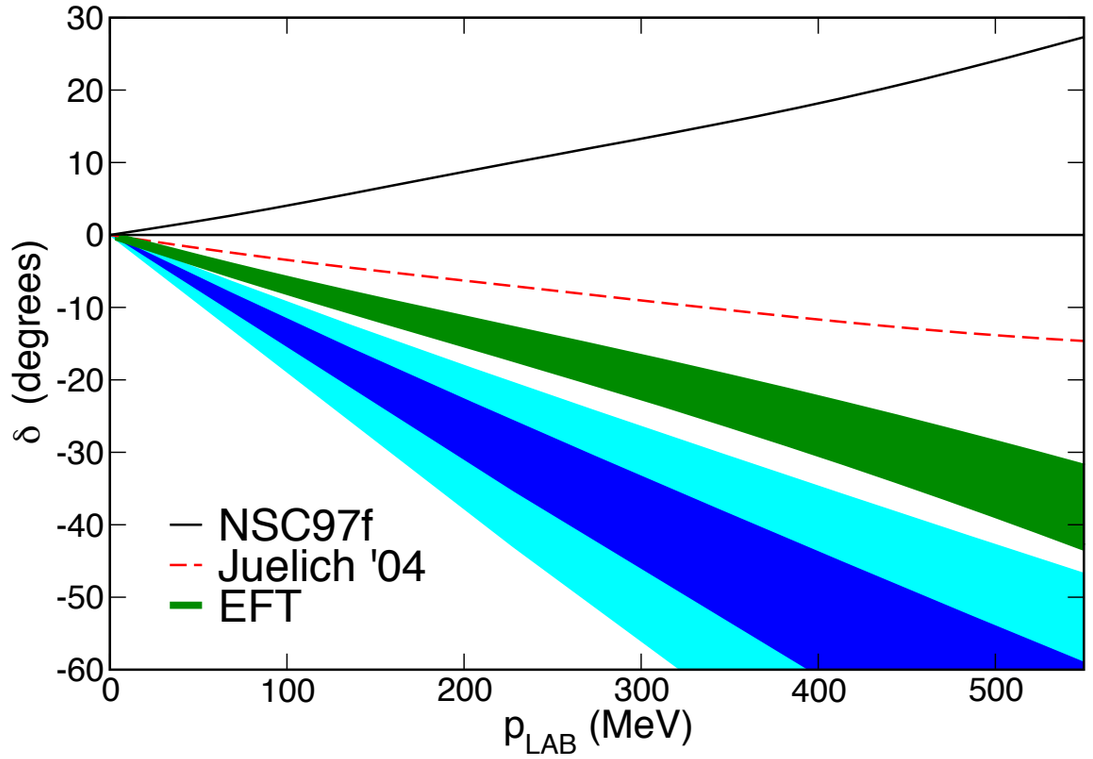
NPLQCD

Hyperon-Nucleon

[S. Beane et.al. arXiv:1204.3606 submitted to Phys. Rev. Lett.]



n-Σ spin singlet



n-Σ spin triplet

gauge fields 2+1 flavors (JLab)
anisotropic clover
m_π~ 390MeV

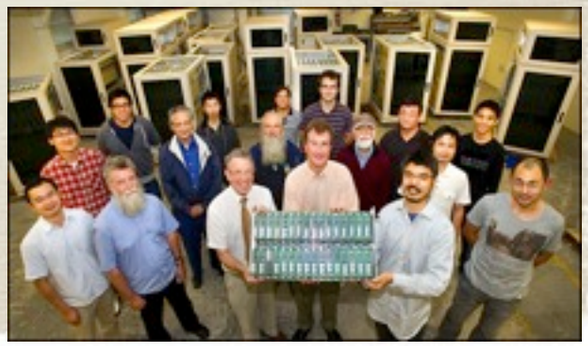
NPLQCD

Conclusions

- Observables related to Hadronic interactions are now being computed
  - Meson-Meson sector: Precision results already exist
  - Progress in the Baryon sector
    - Calculation of deuteron and He binding energy on the way

Yamazaki, Kuramashi, Ukawa (PACS-CS)
Phys.Rev.D81:111504 (2010)
Phys.Rev.D84:054506 (2011)

- Petaflop computing is here
  - Significant aid in achieving precision in the baryon sector
  - Will take us a long way in understanding how the nuclear force emerges from QCD



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way

amashi, Ukawa (PACS-CS)
v.D81:111504 (2010)
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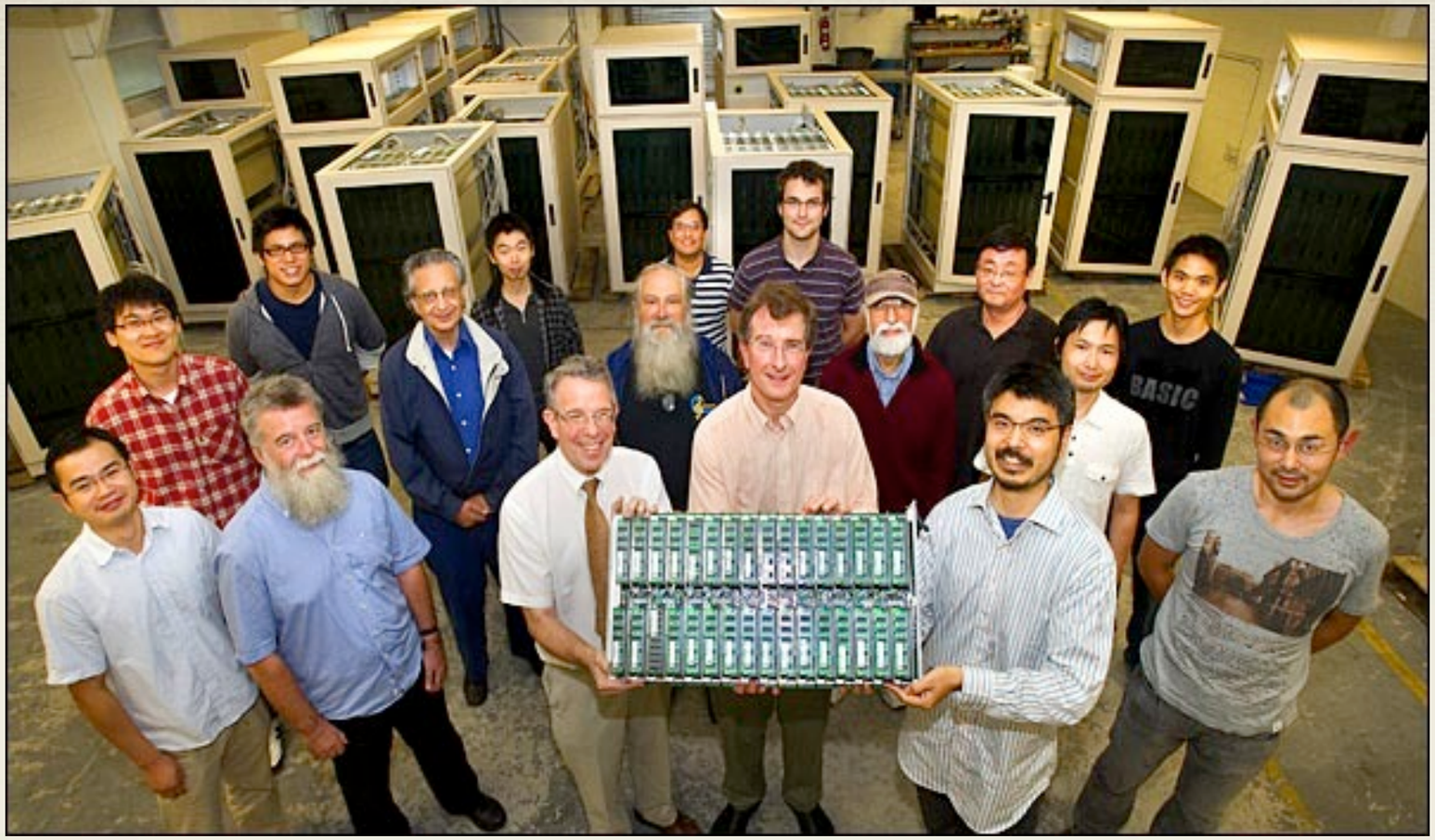
or


BASIC

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