

Thermalization process in high-energy heavy-ion collisions

Soeren Schlichting
Brookhaven National Lab

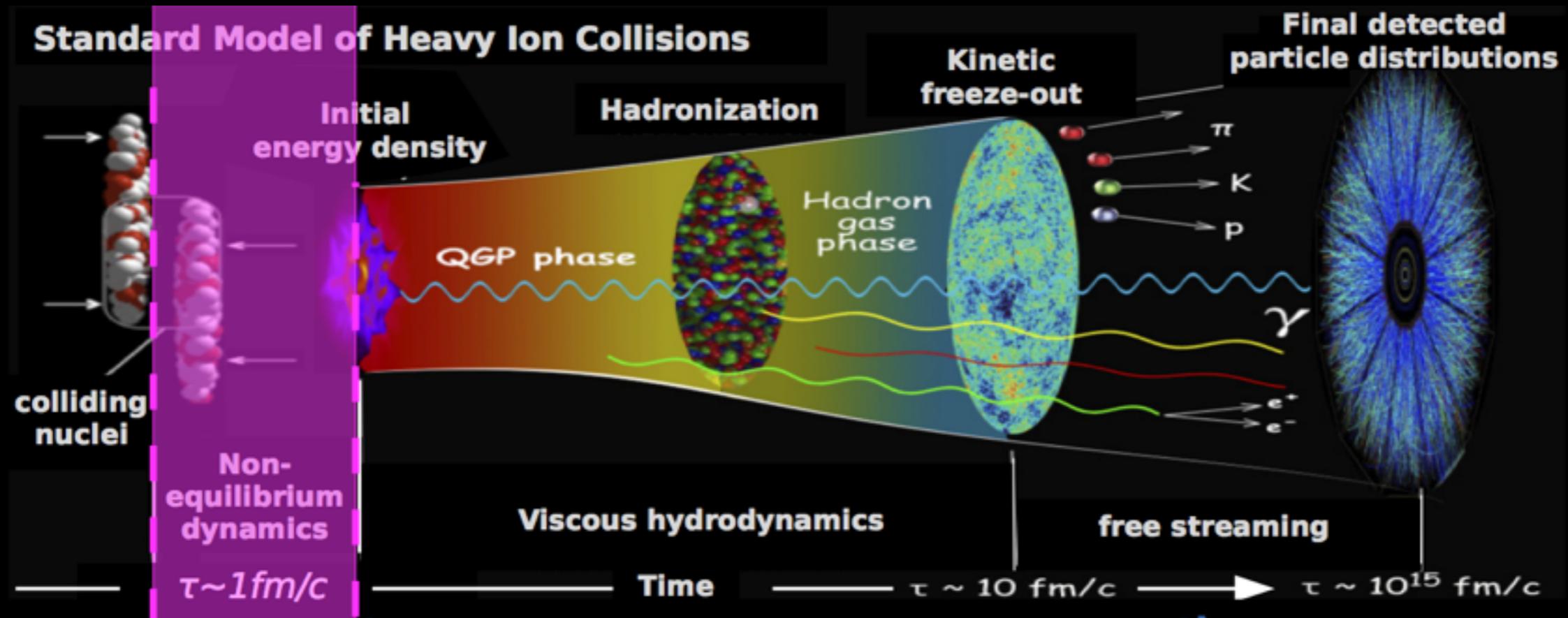
*Based on work in collaboration with
Berges, Boguslavski & Venugopalan*

PRD 89 (2014) 7, 074011 & PRL 114 (2015) 061601

Brain Workshop Feb 2015, Brookhaven



Motivation

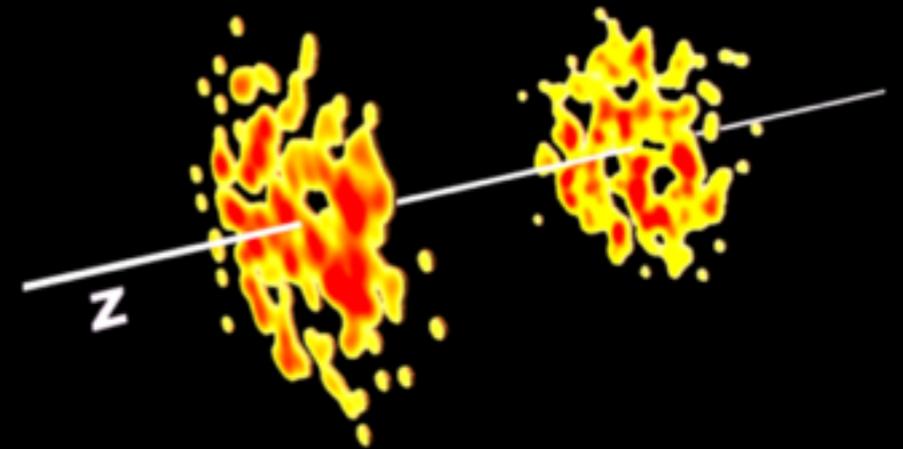


- Well established phenomenology describing the space-time evolution of heavy-ion collisions at later
- Little understanding of how a thermalized quark gluon plasma is formed starting from the collision of heavy nuclei

Weak-coupling picture

Color glass condensate framework

- High-energy nuclei feature a large number of small- x gluons with typical momentum $Q_s \gg \Lambda_{\text{QCD}}$
- Collision of high-energy nuclei leads to a far-from equilibrium state 'Glasma' characterized by a large phase space occupancy of gluons



$$f(p \sim Q_s) \sim 1/\alpha_s$$

-> *Even though the relevant coupling $\alpha_s(Q_s) \ll 1$ is small the system is strongly correlated because of high gluon density*

From the violence of the collision ...to the calm of the quark-gluon fluid



Initial state:
Far from equilibrium



*Non-equilibrium
dynamics*



Final state:
Thermal equilibrium



How is thermal equilibrium achieved?

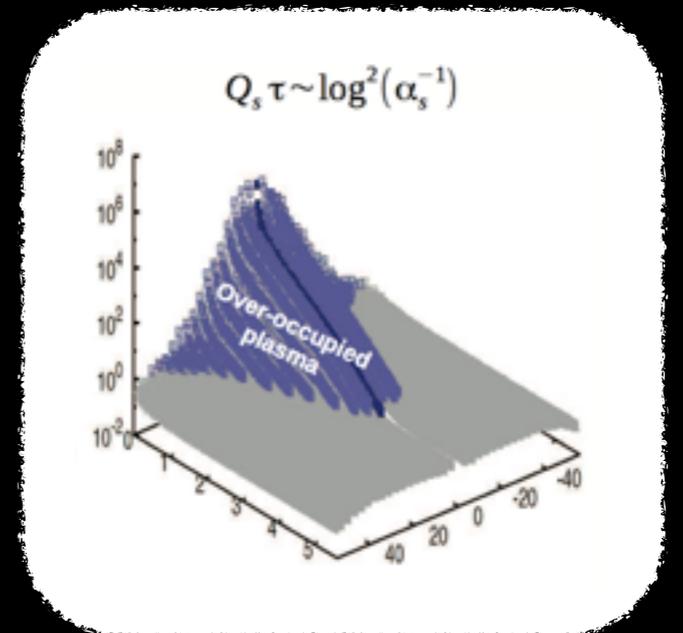
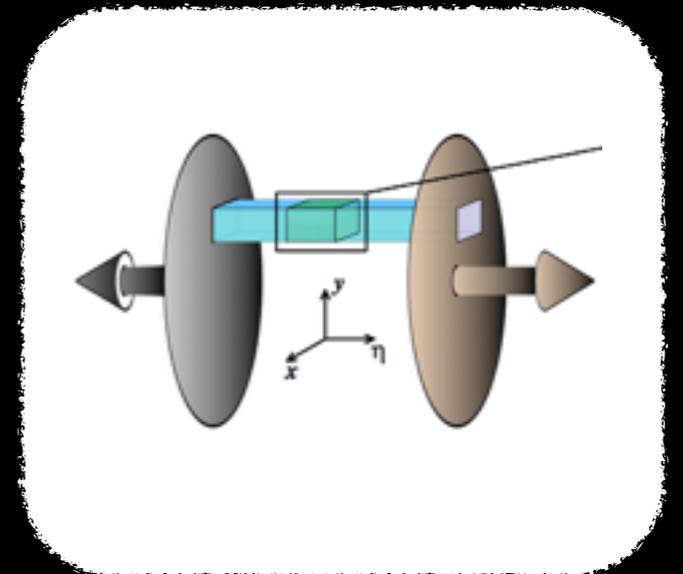
Qualitative description

- We will neglect the transverse expansion of the system and consider a system which is only expanding in the longitudinal direction
- Characterize the initial state at $\tau_0 \sim 1/Q_s$ in terms of the initial gluon distribution

$$f(p_T, p_z, \tau_0) = \frac{n_0}{\alpha_s} \Theta \left(Q - \sqrt{p_T^2 + (\xi_0 p_z)^2} \right)$$

initial occupancy

initial anisotropy

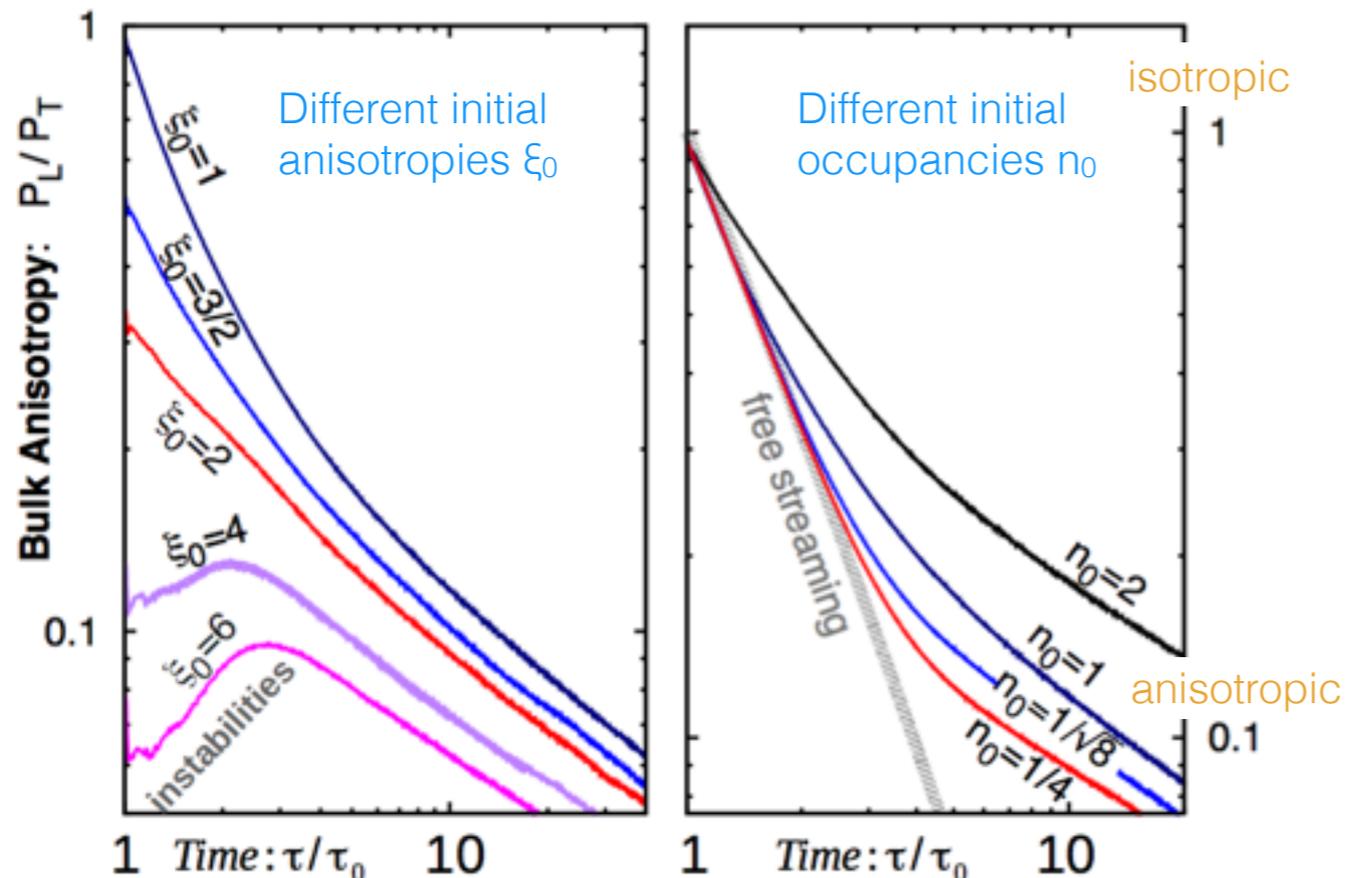


-> High gluon densities allow for an effectively classical description

Hydrodynamic quantities

Classical Yang-Mills simulations for a variety of initial conditions

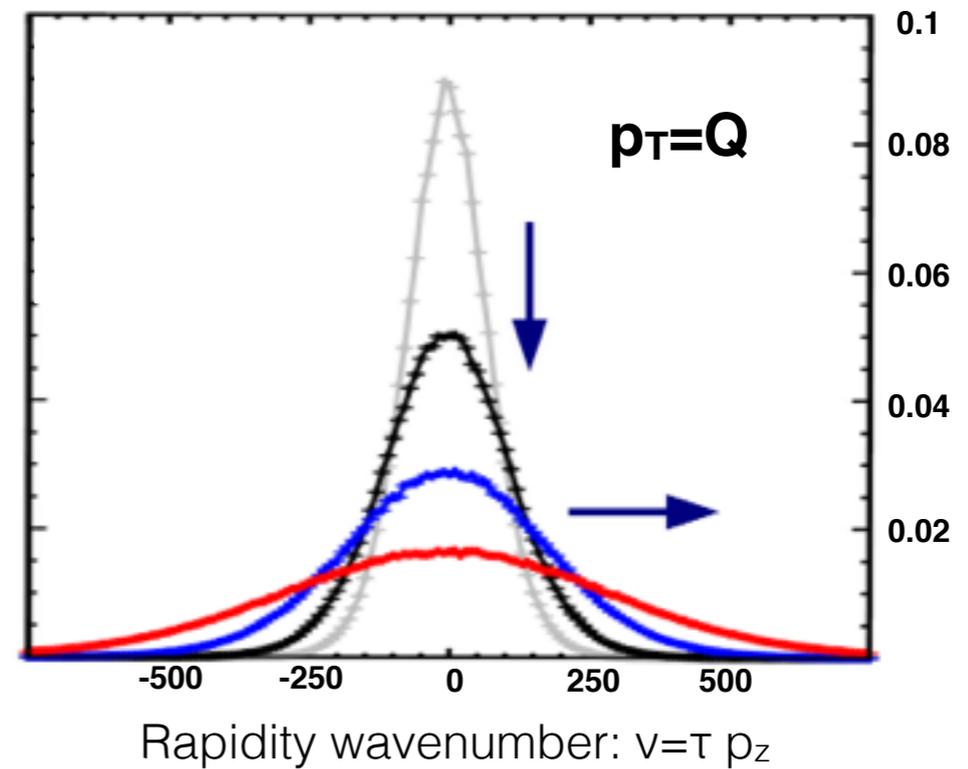
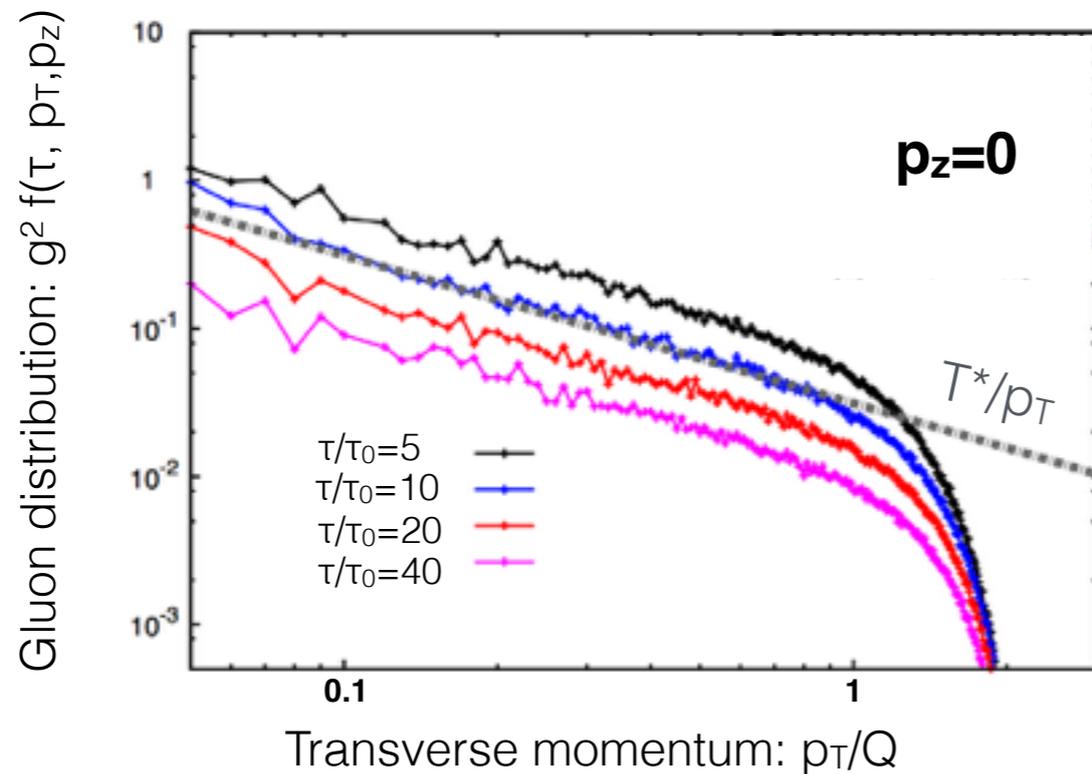
Evolution of the bulk anisotropy



- Small occupancy leads to initial free streaming behavior.
- Large initial anisotropy leads to transient increase of isotropy via plasma instabilities
- The system remains **strongly interacting** throughout the entire evolution.
- At late times, the evolution becomes **insensitive to the details of the initial conditions** and the anisotropy increases.

Microscopic properties

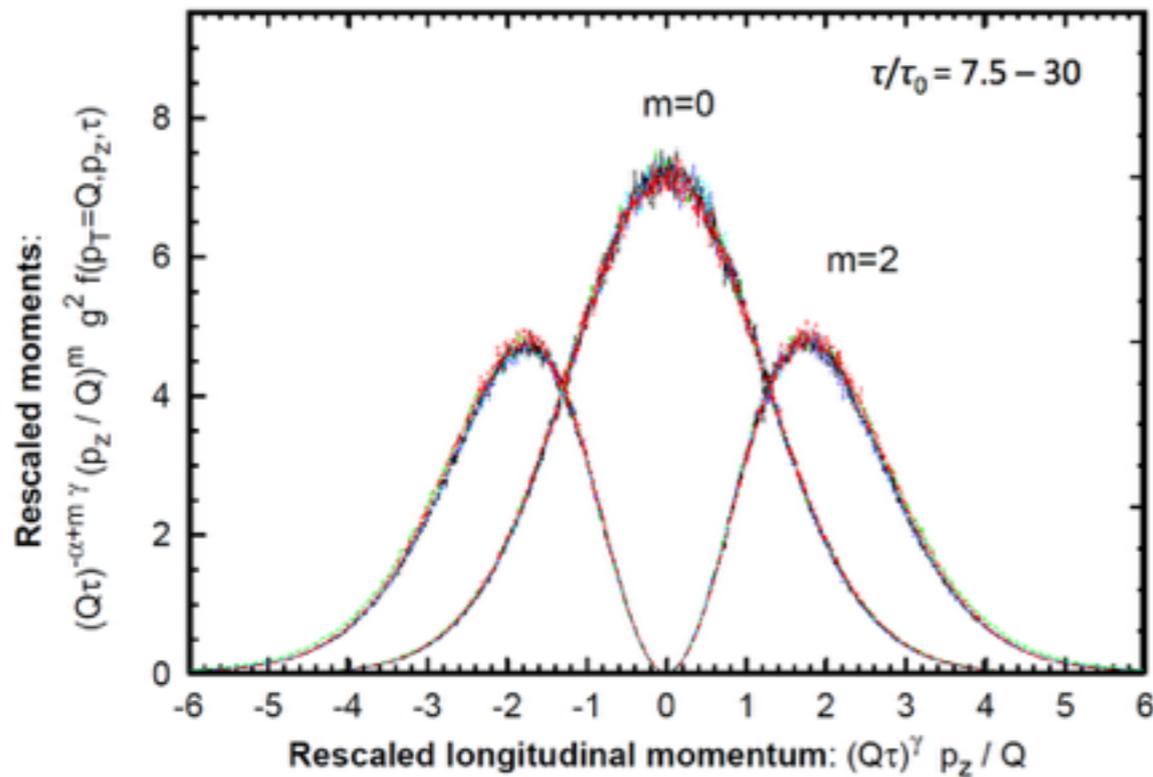
Evolution of the single particle spectrum



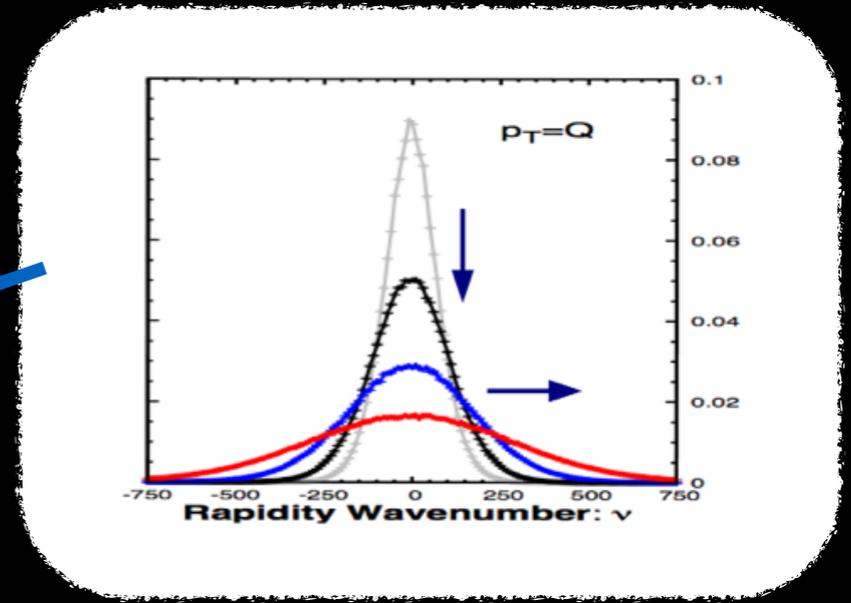
Transverse spectrum shows thermal-like $1/p_T$ behavior up to Q_s .

Dynamics in the scaling regime consists of *longitudinal momentum broadening* — not strong enough to completely compensate for red-shift due to longitudinal expansion

Self-similarity



rescaling



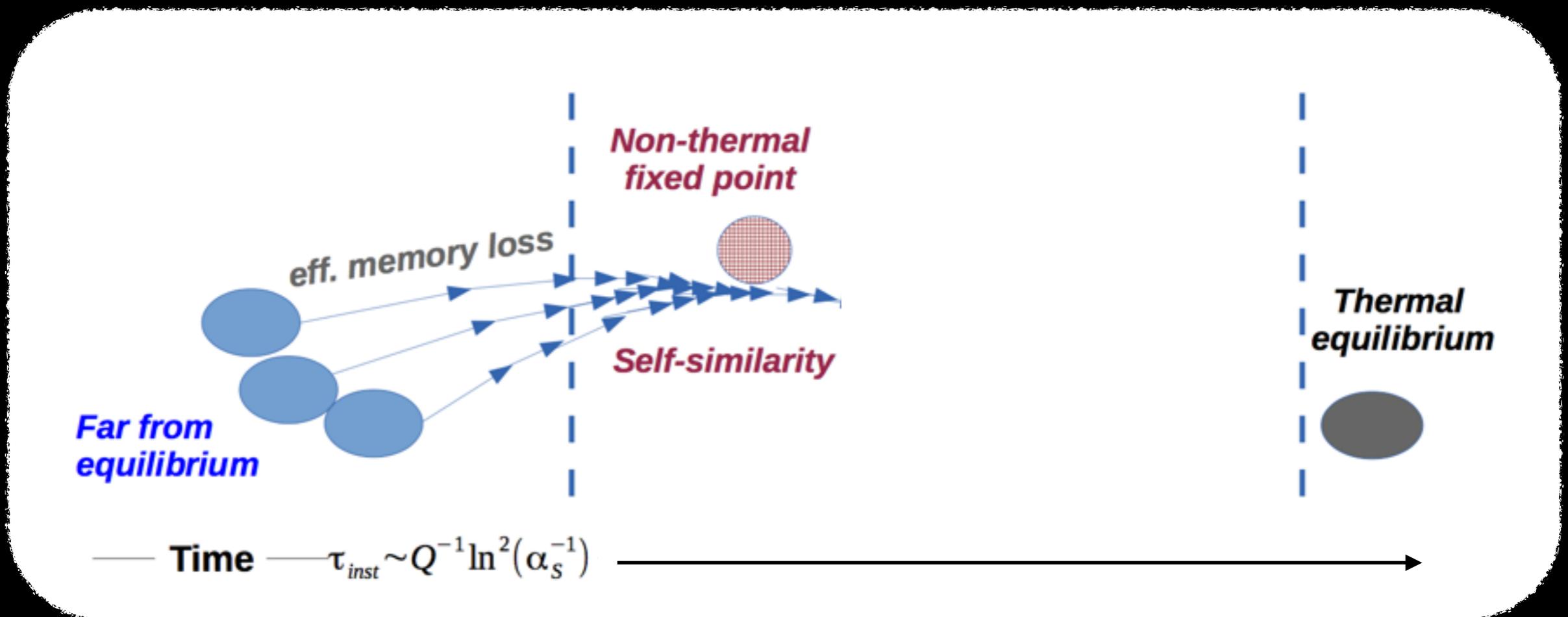
Dynamics can be entirely described in terms of universal scaling exponents $\alpha=-2/3$, $\beta=0$, $\gamma=1/3$ and scaling function f_S extracted from simulations

$$f(p_T, p_z, \tau) = (Q\tau)^\alpha f_S\left((Q\tau)^\beta p_T, (Q\tau)^\gamma p_z\right)$$

(Berges, Boguslavski, SS, Venugopalan PRD 89 (2014) 074011; 89 (2014) 114007)

Non-thermal fixed point

Non-equilibrium evolution leads to a universal attractor solution, where the system exhibits self-similar scaling behavior.



Similar behavior also known in Cosmology and Cold-Atomic gases
-> *Closely related to the phenomenon of wave turbulence*

Kinetic theory interpretation

Consider the Boltzmann equation (c.f. Baier et al. PLB 502 (2001) 51-58)

$$[\partial_\tau - \frac{p_z}{\tau} \partial_{p_z}] f(p_T, p_z, \tau) = C[f](p_T, p_z, \tau)$$

with a self-similar evolution

$$f(p_T, p_z, \tau) = (Q\tau)^\alpha f_s((Q\tau)^\beta p_T, (Q\tau)^\gamma p_z)$$

→ **Non-thermal fixed point solution** ($f \gg 1$)

$$[\alpha + \beta p_T \partial_{p_T} + (\gamma - 1) p_z \partial_{p_z}] f_s(p_T, p_z) = Q^{-1} C[f_s](p_T, p_z)$$

→ **Scaling exponents determined by scaling relations for**

- Small angle elastic scattering $(2\alpha - 2\beta + \gamma = -1)$
- Energy conservation $(\alpha - 3\beta - \gamma = -1)$
- Particle number conservation $(\alpha - 2\beta - \gamma = -1)$

→ $\alpha = -2/3, \beta = 0, \gamma = 1/3$ **in excellent agreement with lattice data!**

Scaling exponents independent of detailed microscopic features
-> Chance to observe identical behavior in different physical systems

Scalar field theory

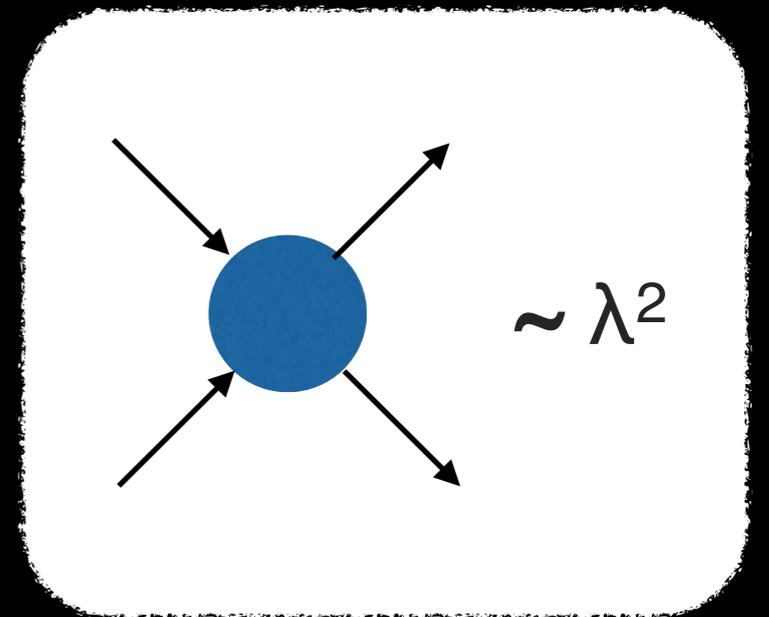
Consider massless N-component scalar field theory with quartic self-interaction

$$S[\varphi] = \int d^4x \sqrt{-g(x)} \left(\frac{1}{2} (\partial_\mu \varphi_a) g^{\mu\nu} (\partial_\nu \varphi_a) - \frac{\lambda}{24N} (\varphi^2)^2 \right)$$

in a longitudinally expanding setup.

Even though perturbatively there is no preference for small angle scattering, one at least expects elastic processes to dominate

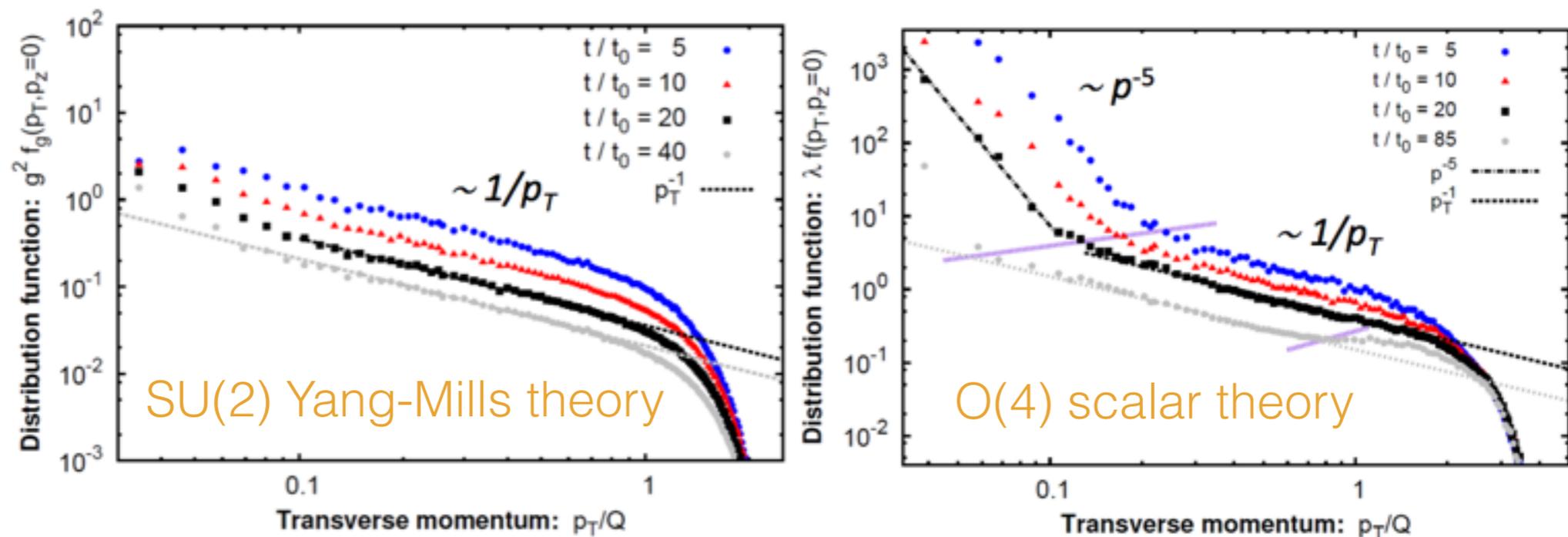
-> *Energy conservation*
& *Particle number conservation*



Comparison of simulations

Comparison of longitudinally expanding Yang-Mills and scalar field theory in the classical regime of high occupancy

Evolution of the single particle spectrum



Scalar theory shows three distinct scaling regimes at soft ($\sim p^{-5}$), intermediate ($\sim 1/p_T$) and hard momenta ($\sim \text{const}$)

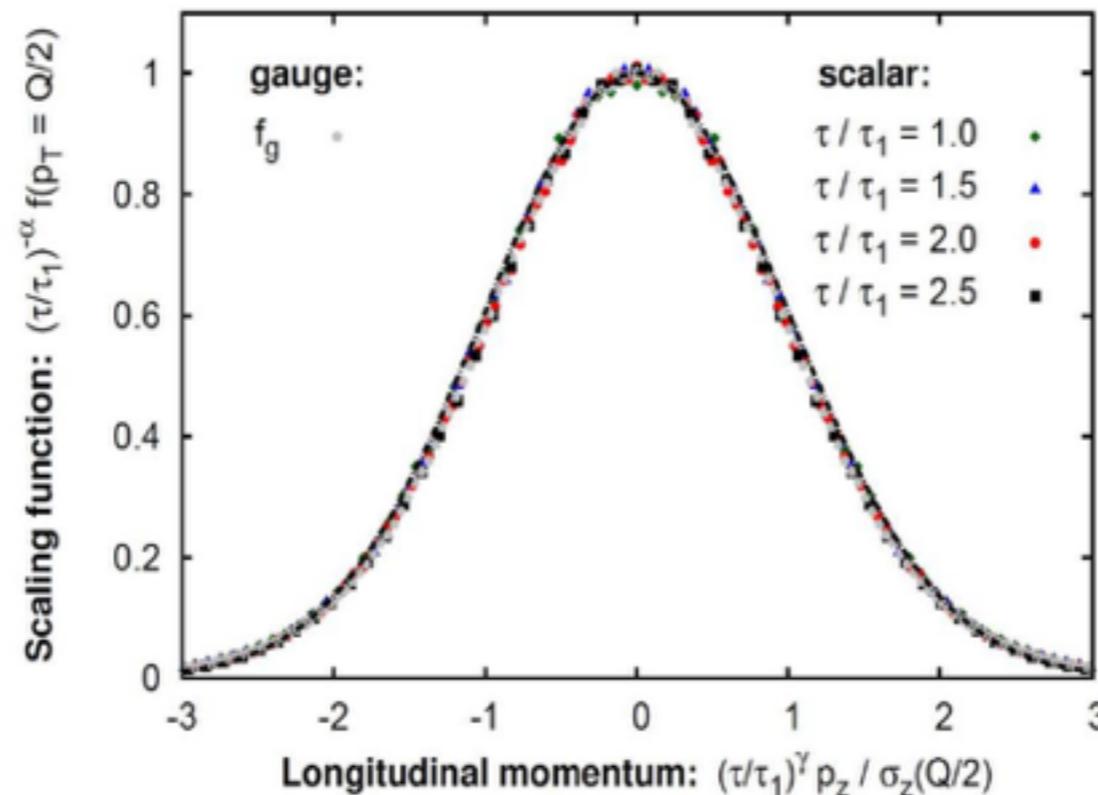
-> Common $\sim 1/p_T$ scaling regime

(Berges, Boguslavski, SS, Venugopalan PRL 114 (2015) 061601)

Universality far from equilibrium

- Scaling exponents and scaling functions agree in the inertial range of momenta, where both theories show $1/p_T$ behavior

Normalized fixed-point distribution

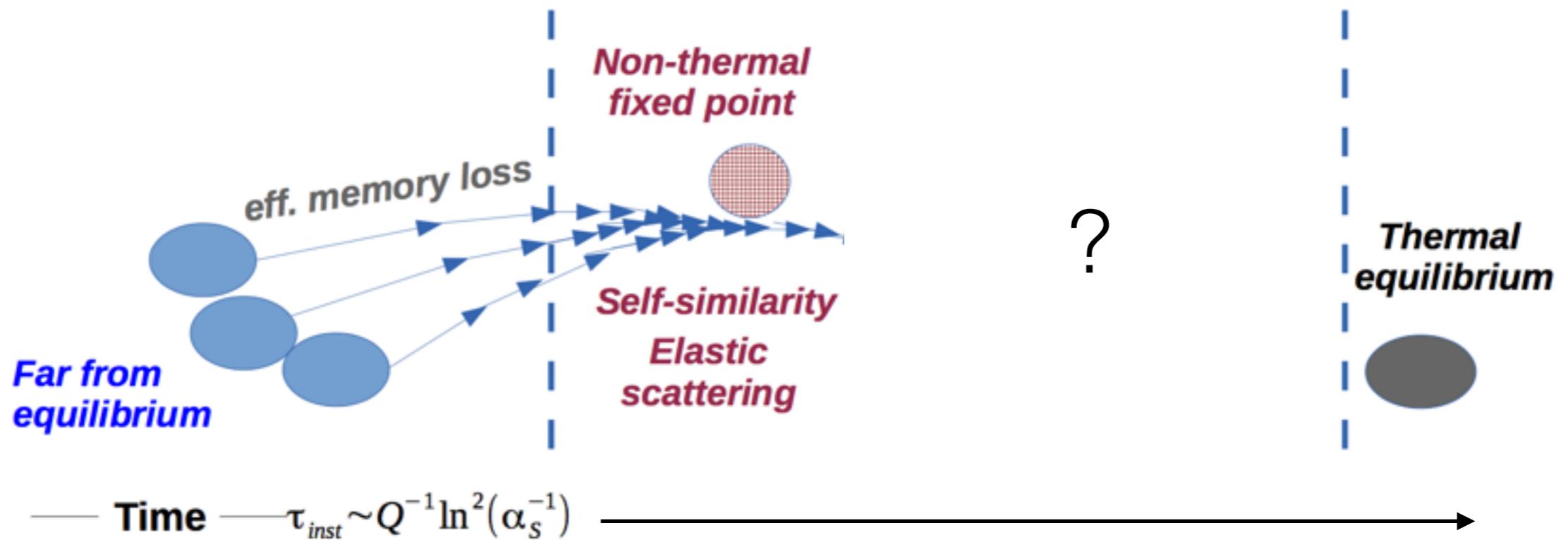


Kinetic interpretation in scalar theory still unclear —
Vertex corrections? More general than small angle scattering?

(Berges, Boguslavski, SS, Venugopalan PRL 114 (2015) 061601)

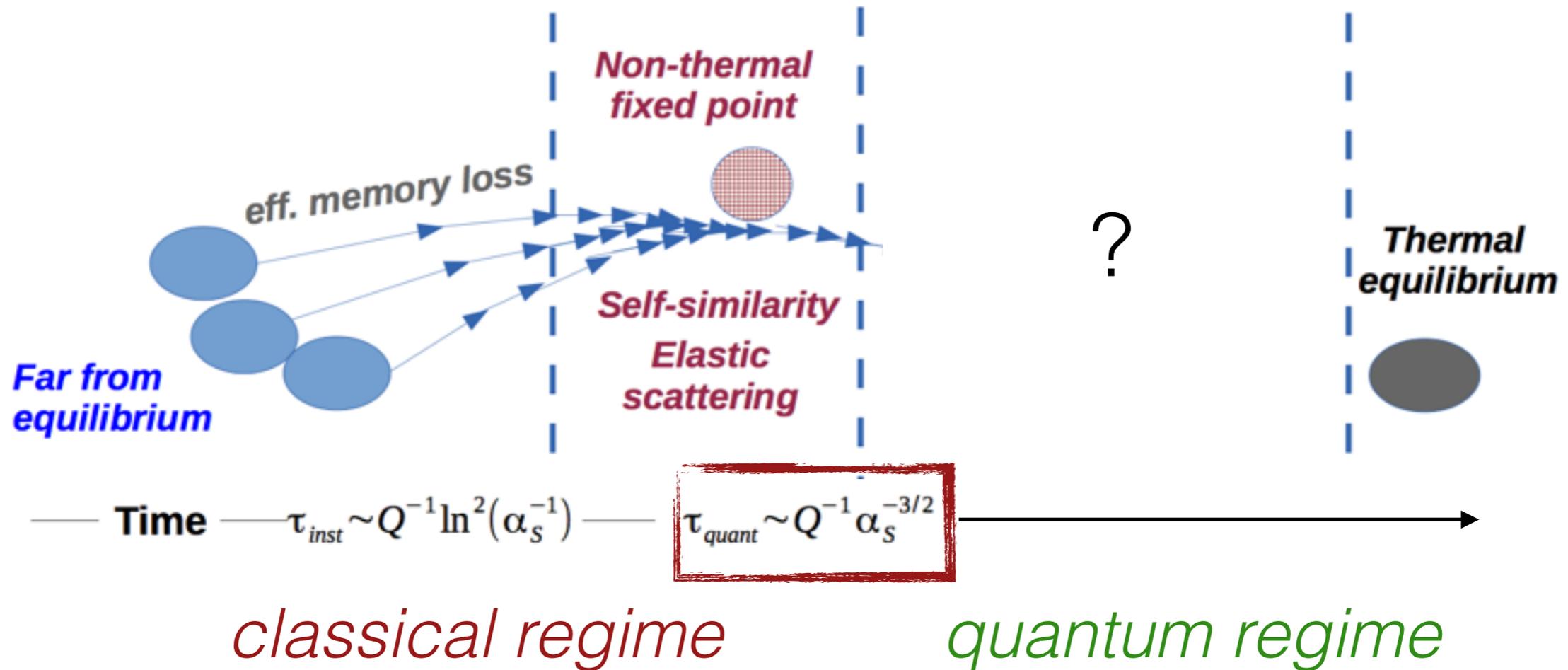
Thermalization process

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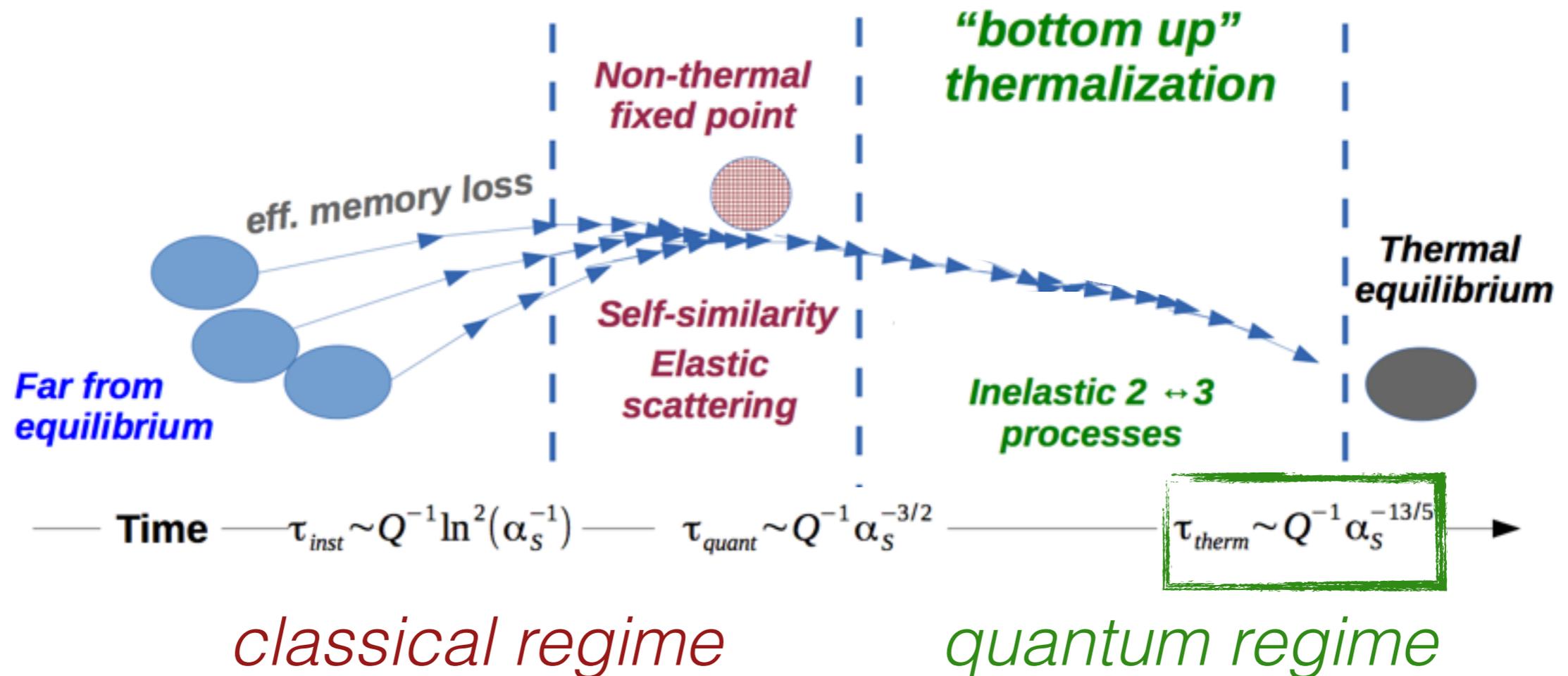
Thermalization process

Classical description breaks down once the typical occupancies become of order one -> *Departure from classical fixed point*



Thermalization process

Quantum regime can be studied within effective kinetic description
(Baier, Mueller, Schiff & Son PLB 502 (2001) 51-58)



-> Inelastic processes lead to formation of soft thermal bath
Hard gluons loose energy to the soft bath

Quantum regime

Effective kinetic description used to study the dynamics of the quantum regime up to complete equilibration

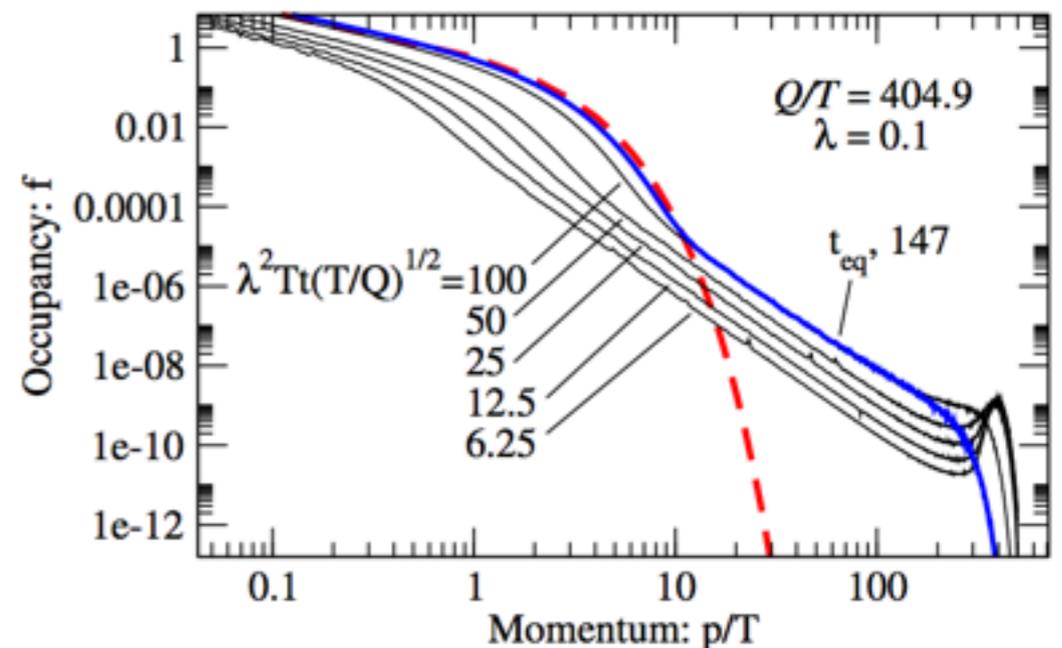
$$\partial_t f(p, t) = -\mathcal{C}_{2 \leftrightarrow 2}[f](p) - \mathcal{C}_{1 \leftrightarrow 2}[f](p).$$

Quantitative estimate of the thermalization time

$$t_{\text{eq}}^{\text{under occ.}} \approx \frac{34. + 21. \ln(Q/T)}{1 + 0.037 \log \lambda^{-1}} \left(\frac{Q}{T}\right)^{1/2} \frac{1}{\lambda^2 T}$$

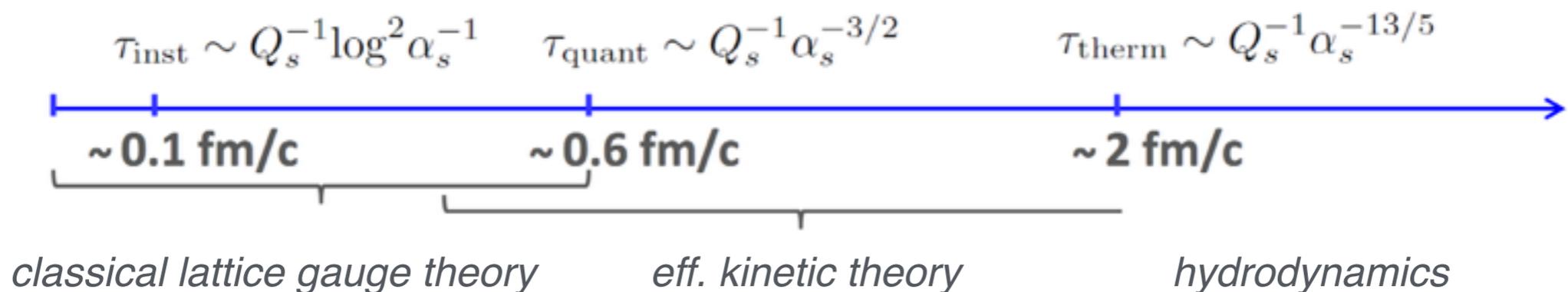
-> $\tau_{\text{eq}} \sim 0.2 - 2 \text{ fm}/c$

Evolution of the single particle spectrum



Summary & Conclusions

- Qualitative understanding of the dynamics of the thermalization process at weak coupling. Different methods agree within the overlapping range of validity.
- Can now compute entire thermalization process in A+A from an interplay of different weak coupling methods



- Striking universality observed between expanding scalar and gauge theory in the early time classical regime. Even though precise origin is still unclear there is an exciting possibility to learn about the thermalization process from different physical systems.