Non-perturbative renormalization of bilinear operators with Möbius domain-wall fermions in the coordinate space

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1. Introduction — NPR by X-space method

- Renormalization

  $\mathcal{O}^{lat}(1/a) \to \mathcal{O}^{\overline{\rm MS}}(\mu) = Z^{\overline{\rm MS}/lat}(1/a \to \mu)\mathcal{O}^{lat}(1/a)$

- We investigate NPR of quark bilinears by the X-space method (Martinelli et al '97, Giménez et al 2004, Cichy et al 2012)

- Advantages
  - ✧ able to renormalize by gauge invariant quantities
  - ✧ perturbative matching is available up to 4-loop level (Chetyrkin-Maier, 2011)

- Disadvantages
  - ✧ Suffer from the window problem

- Examine the potential of X-space method with the Möbius domain-wall fermions

2. Strategy — sketch of X-space method

- X-space method uses corrlation functions of quark bilinears
  - ✧ $\Pi_{\Gamma\Gamma}$: current-current correlators

$$\Pi_{PP}(x) = \langle P(x)P(0)\rangle, \quad \Pi_{SS}(x) = \langle S(x)S(0)\rangle,$$

$$\Pi_{VV}(x) = \sum_{\mu=1}^{4} \langle V_\mu(x)V_\mu(0)\rangle, \quad \Pi_{AA}(x) = \sum_{\mu=1}^{4} \langle A_\mu(x)A_\mu(0)\rangle$$

- Renormalization condition in the X-space scheme

$$\left(Z_\Gamma^{X/\bullet}(\mu_X = 1/|x|)\right)^2 \Pi_{\Gamma\Gamma}^{\bullet}(x) = \Pi_{\Gamma\Gamma}^{\mathrm{free}}(x)$$

$$\Longrightarrow \quad Z_\Gamma^{X/\bullet}(x) = \sqrt{\frac{\Pi_{\Gamma\Gamma}^{\mathrm{free}}(x)}{\Pi_{\Gamma\Gamma}^{\bullet}(x)}} \qquad Z_\Gamma^{X/\bullet}(x) \equiv Z_\Gamma^{X/\bullet}(\mu_X)$$

- Matching through the X-space scheme

$$Z_\Gamma^{\overline{\mathrm{MS}}/lat}(2\ \mathrm{GeV}) = \frac{Z_\Gamma^{X/lat}(x)}{Z_\Gamma^{X/\overline{\mathrm{MS}}}(x, 2\ \mathrm{GeV})} = \sqrt{\frac{\Pi_{\Gamma\Gamma}^{\overline{\mathrm{MS}}}(x, 2\ \mathrm{GeV})}{\Pi_{\Gamma\Gamma}^{lat}(x)}}$$

2. Strategy — sketch of X-space method

- Renormalization constants

$$\boxed{Z_\Gamma^{\overline{\rm MS}/lat}(2\ {\rm GeV}) = \sqrt{\frac{\Pi_{\Gamma\Gamma}^{\overline{\rm MS}}(x, 2\ {\rm GeV})}{\Pi_{\Gamma\Gamma}^{lat}(x)}}}$$ to be evaluated at $m_q = 0$

- Steps:

① Lattice calculation $\Longrightarrow$ $\Pi_{\Gamma\Gamma}^{lat}(x)$ in chiral limit

② Perturbation $\Longrightarrow$ $\Pi_{\Gamma\Gamma}^{\overline{\rm MS}}(x, 2\ {\rm GeV})$ at $m_q = 0$

$\Big\}$ $Z_\Gamma^{\overline{\rm MS}}(2\ {\rm GeV})$

- Renormalization window

① needs sufficiently large x in order to avoid discretization effects

② needs sufficiently small x where perturbation theory is reliable

➢ $\boxed{a \ll x \ll \Lambda_{\rm QCD}^{-1}}$

2. Strategy — lattice action & ensembles

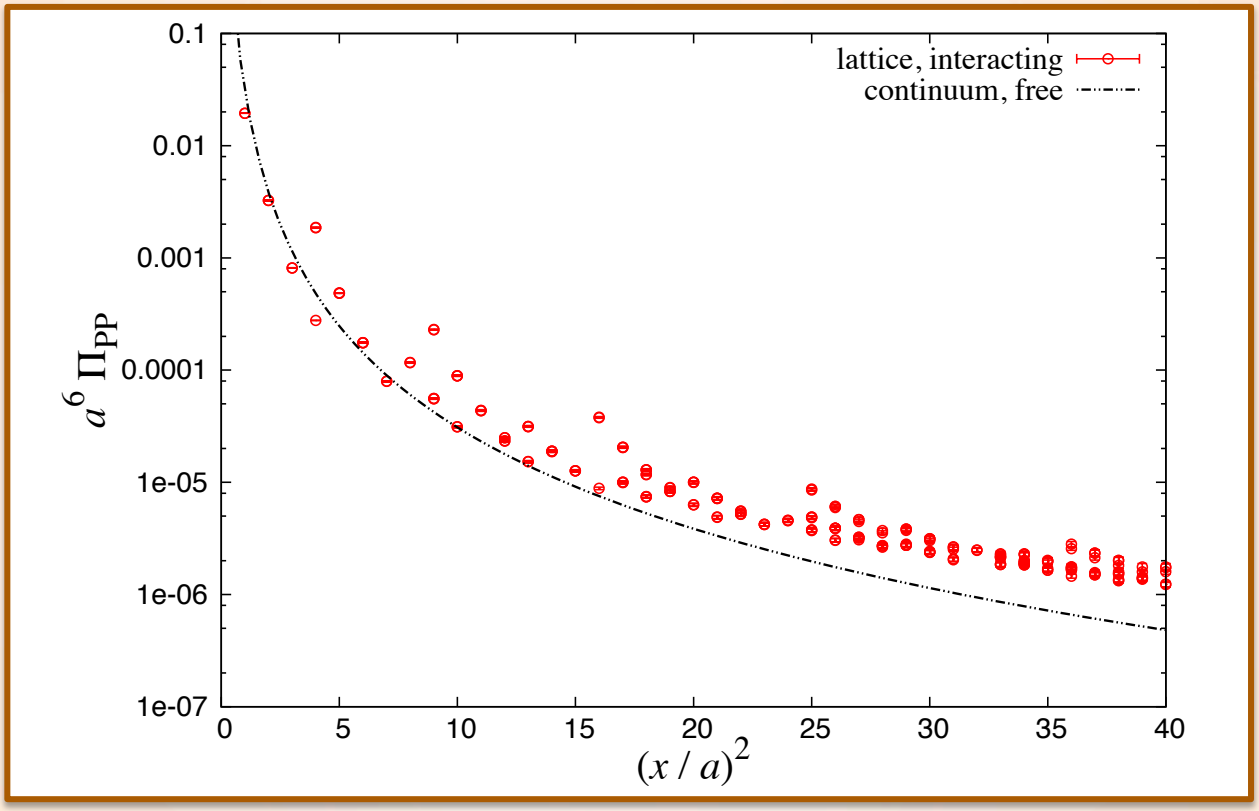
- Lattice action
  - ✧ 2+1 generalized (Möbius) domain-wall fermions with 3-times stout smearing
  - ✧ Symanzik improved gauge action
- Ensembles

β	a^{-1} [GeV]	Volume	am_s	am_{ud} (Confs/Trajectory)
4.17	2.4	32^3x64	0.040	0.0190(20/10000), 0.0120(20/10000), 0.0070(20/10000), 0.0035(20/10000)
			0.030	0.0190(20/10000), 0.0120(20/10000), 0.0070(20/10000)
4.35	3.6	48^3x96	0.025	0.0120(20/10000), 0.0080(20/10000), 0.0042(20/10000)
			0.018	0.0120(20/10000), 0.0080(20/4260), 0.0042(20/10000)
4.47	4.5	32^3x64	0.018	0.0090(10/10000), 0.0060(10/10000), 0.0040(10/10000)
			0.015	0.0090(10/10000), 0.0060(10/10000), 0.0030(10/10000)

64^3x128 on the product run

3. Lattice Part — short-distance correlator

- Discretization effect is very large



3. Lattice Part — short-distance correlator

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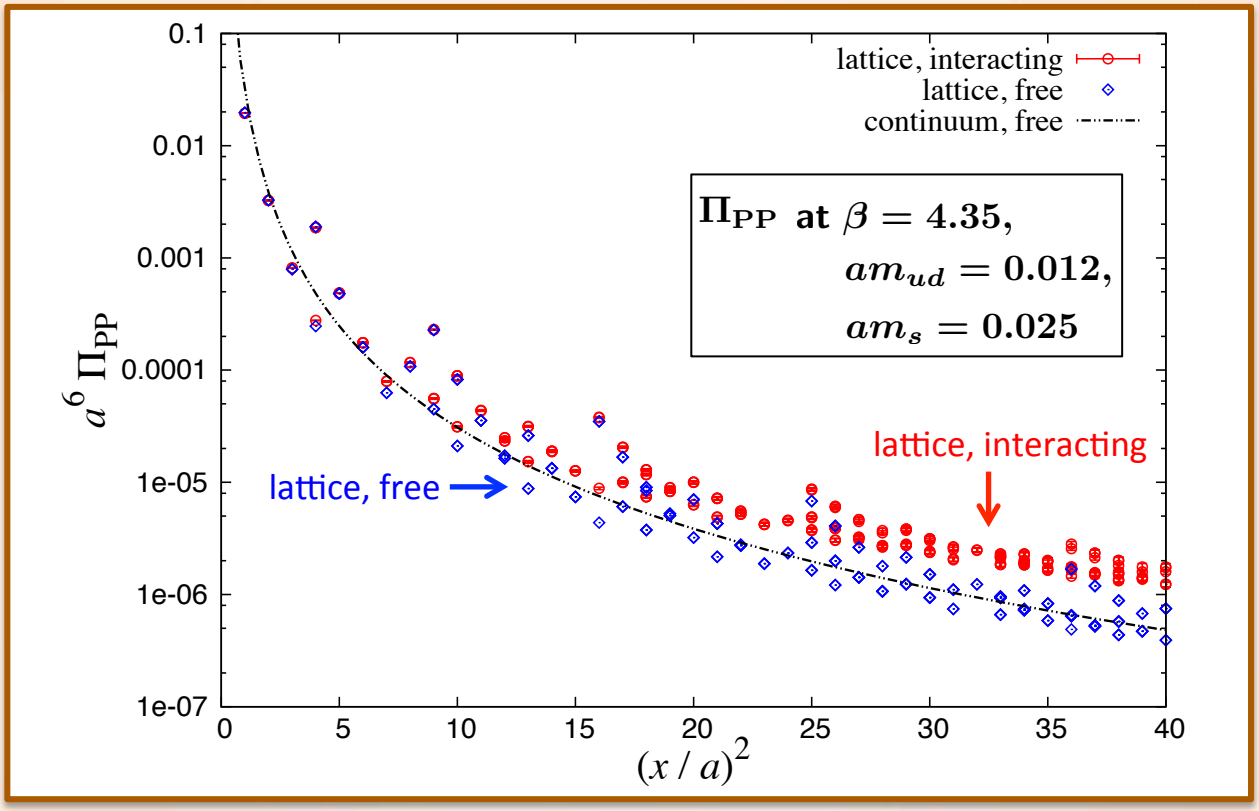
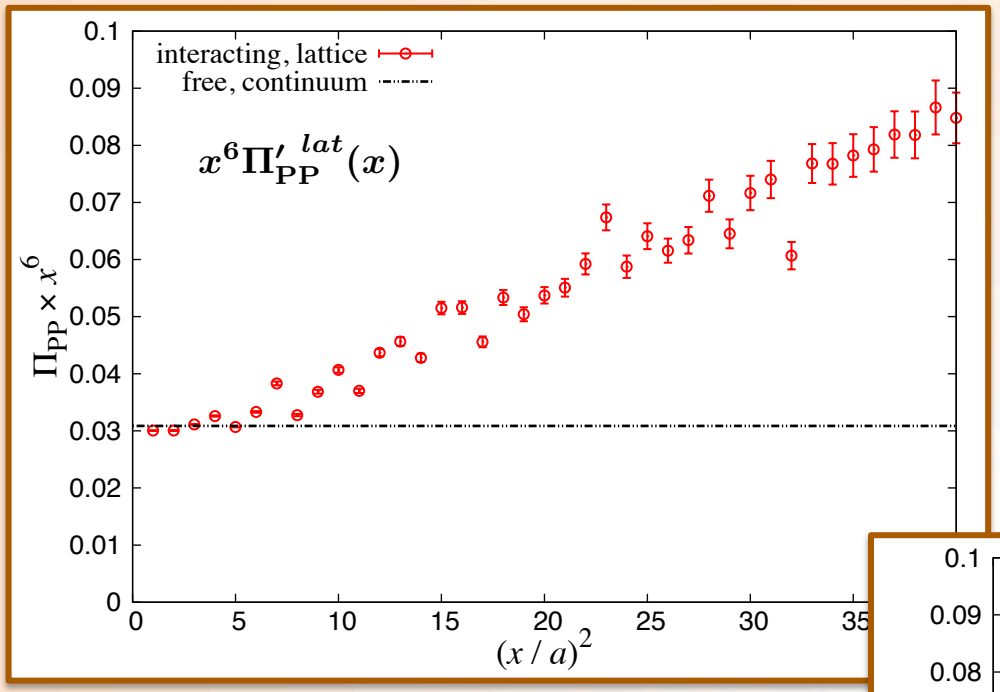


- "lattice, interacting" and "lattice, free" are strongly correlated
  $\Rightarrow$ discretization effect is similar

3. Lattice Part — free theory correction

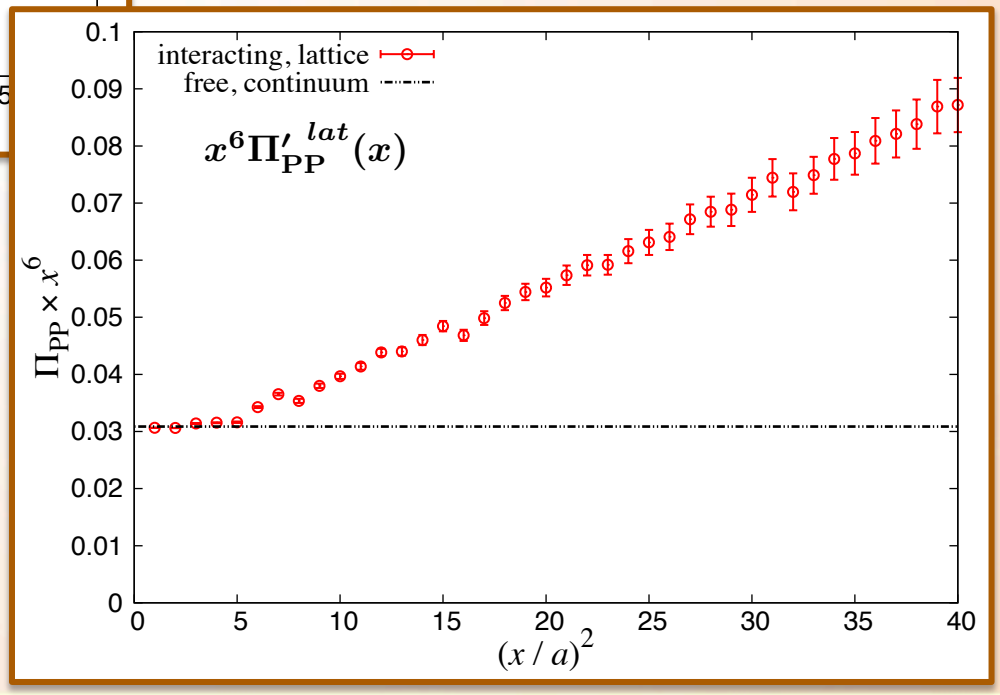


a correction (Gimenez et al 2004)

$$\Pi^{lat}_{\Gamma\Gamma}(x) \to \Pi'^{lat}_{\Gamma\Gamma}(x)$$
$$= \Pi^{lat}_{\Gamma\Gamma}(x) \frac{\Pi^{cont,\mathrm{free}}_{\Gamma\Gamma}(x)}{\Pi^{lat,\mathrm{free}}_{\Gamma\Gamma}(x)}$$

We use

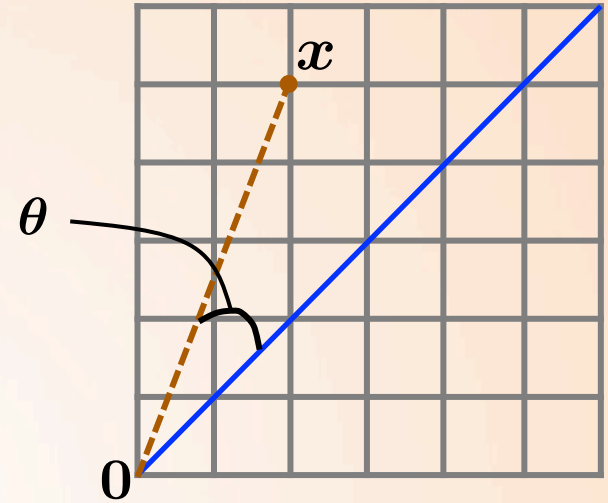
$$\Pi^{lat}_{\Gamma\Gamma}(x) \to \Pi'^{lat}_{\Gamma\Gamma}(x)$$
$$= \Pi^{lat}_{\Gamma\Gamma}(x) + \Pi^{cont,\mathrm{free}}_{\Gamma\Gamma}(x) - \Pi^{lat,\mathrm{free}}_{\Gamma\Gamma}(x)$$

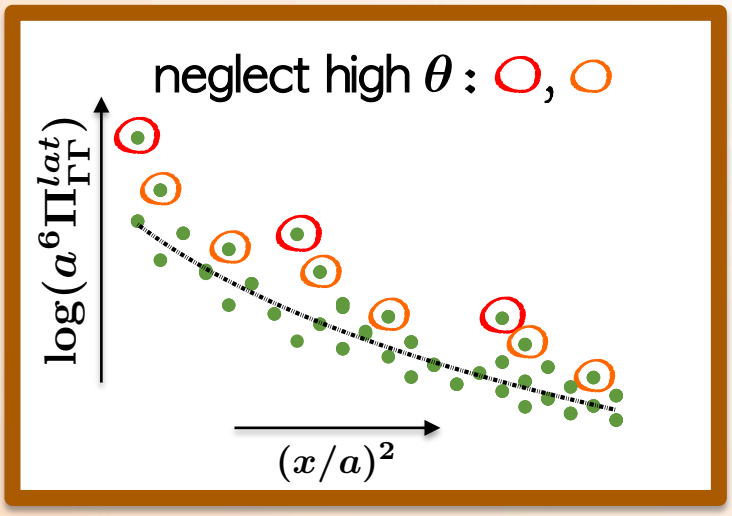


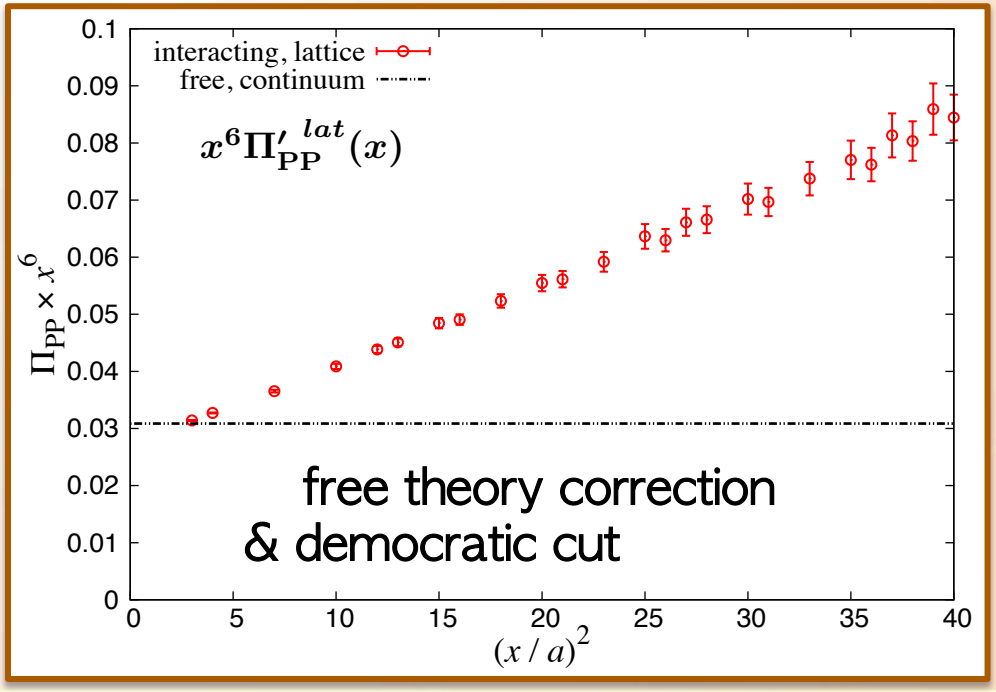
3. Lattice Part — democratic cut

Ref: *Cichy et al, Nucl.Phys.B864(2012)268*

- θ: angle between x and (1,1,1,1)
- Correlators at large θ are more distorted
- Free correlators at small θ are closer to those in continuum theory
- We neglect correlators at $\theta > 30°$







4. Continuum Part — procedure

① Perturbative expansion of correlation functions up to 4-loop order (Chetyrkin-Maier, 2011)

$$\Pi^{\overline{\rm MS}}_{\rm PP,SS}(x,\mu) = \Pi^{\widetilde{\rm MS}}_{\rm PP,SS}(x,\tilde{\mu}) = \frac{3}{\pi^4 x^6}\left(1+\sum_{n=1}^{\infty}\tilde{C}^{\rm S}_n \tilde{a}_s^n\right)$$

$$\Pi^{\overline{\rm MS}}_{\rm VV,AA}(x) = \Pi^{\widetilde{\rm MS}}_{\rm VV,AA}(x) = \frac{6}{\pi^4 x^6}\left(1+\sum_{n=1}^{\infty}\tilde{C}^{\rm V}_n \tilde{a}_s^n\right)$$

$$\tilde{a}_s = \frac{\alpha_s^{\widetilde{\rm MS}}(\tilde{\mu}=1/x)}{\pi} = \frac{\alpha_s^{\overline{\rm MS}}(\mu = 2e^{-\gamma_E}/x)}{\pi}$$

② Scale evolution

$$\Pi^{\widetilde{\rm MS}}_{\rm SS,PP}(x,\tilde{\mu}') = \frac{c(a_s(\tilde{\mu}'))}{c(a_s(\tilde{\mu}))}\Pi^{\widetilde{\rm MS}}_{\rm SS,PP}(x,\tilde{\mu}) \leftarrow \text{RG equation}$$

$c(x)$: known up to 4-loop order (Chetyrkin '97, Vermasren et al '97)

$\Pi^{\widetilde{\rm MS}}_{\rm VV,AA}(x)$: scale independent ← WTI

- ① + ② → $\Pi^{\overline{\rm MS}}_{\Gamma\Gamma}(x, 2\ {\rm GeV})$

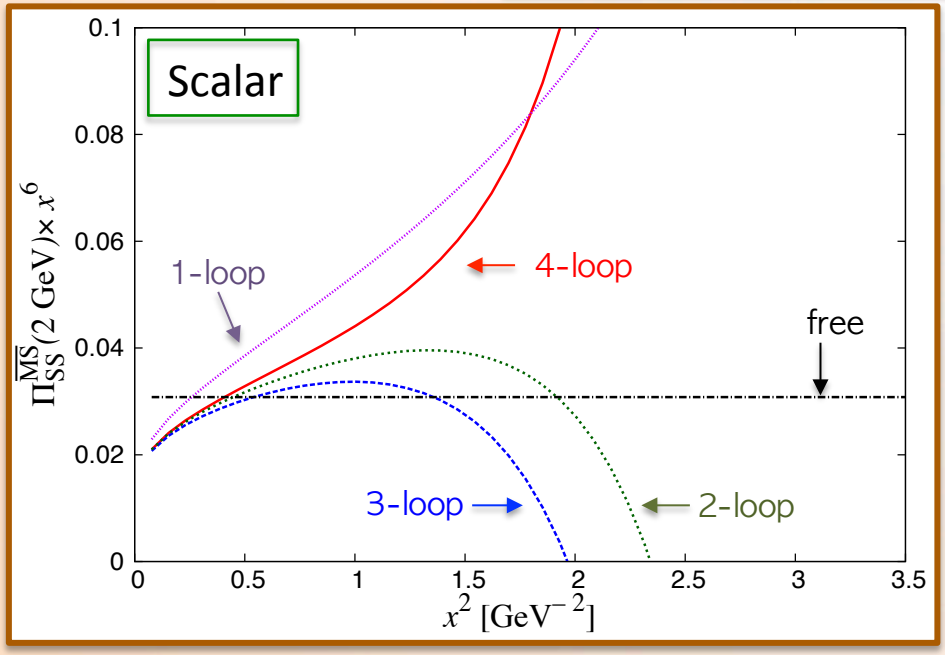
4. Continuum Part — convergence

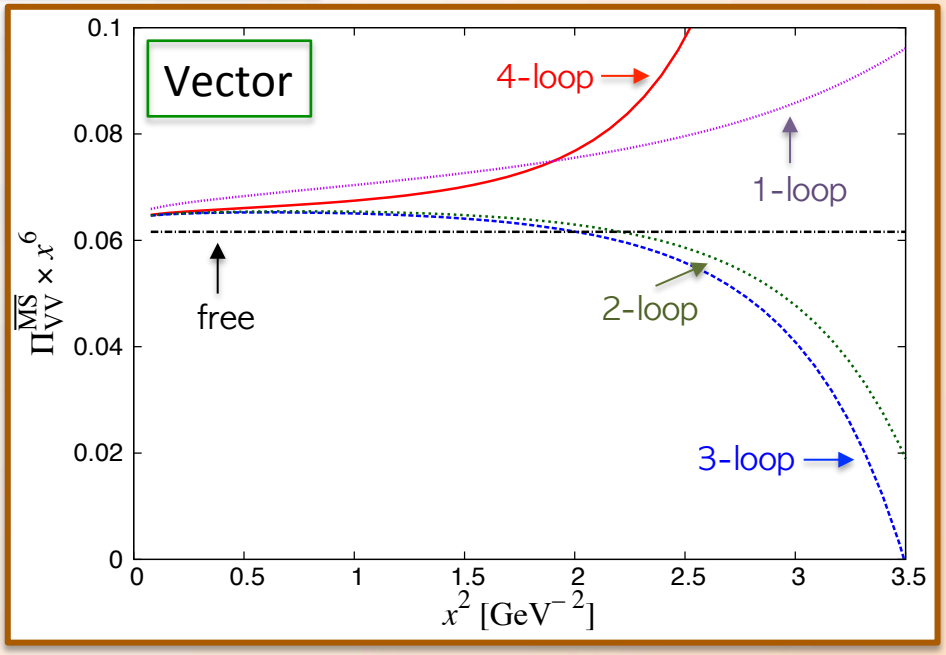
- Convergence of the perturbative series
- Numerical coefficients of $\tilde{a}_s^4$ are relatively large

$$\Pi_{SS}^{\widetilde{MS}}(x,\tilde{\mu}) = \frac{3}{\pi^4 x^6}(1 + 0.67\tilde{a}_s - 16.3\tilde{a}_s^2 - 31\tilde{a}_s^3 + 497\tilde{a}_s^4)$$

$$\Pi_{VV}^{\widetilde{MS}}(x) = \frac{6}{\pi^4 x^6}(1 + \tilde{a}_s - 4\tilde{a}_s^2 - 1.9\tilde{a}_s^3 + 94\tilde{a}_s^4)$$

$$\alpha_s(\mu) \longleftarrow N_f = 3, \quad \Lambda_{QCD} = 340 \text{ MeV}$$





4. Continuum Part — improve convergence

- Scale evolution of a_s $\Longrightarrow$ a new perturbative series
  - ✧ $a_s(\tilde{\mu}) = a_s(\tilde{\mu}')\left(1 + \sum_{n=1}^{\infty} \delta_n(l) a_s^n(\tilde{\mu}')\right)$
  - ✧ We know $\delta_1, \delta_2, \delta_3, \delta_4$ as functions of $l = 2\ln(\tilde{\mu}'/\tilde{\mu})$

    $\Longrightarrow \widetilde{\Pi}^{\overline{\mathrm{MS}}}_{\Gamma\Gamma}(x, \tilde{\mu}) = \dfrac{3 \text{ or } 6}{\pi^4 x^6}\left(1 + \sum_{n=1}^{\infty} C'^{\Gamma}_n a_s^n(\tilde{\mu}')\right)$ up to $n = 4$
- Scale evolution of correlation function (for scalar channel)
  - ✧ $\widetilde{\Pi}^{\overline{\mathrm{MS}}}_{\mathrm{SS}}(x, \tilde{\mu}') = \dfrac{c(a_s(\tilde{\mu}'))}{c(a_s(\tilde{\mu}))} \widetilde{\Pi}^{\overline{\mathrm{MS}}}_{\mathrm{SS}}(x, \tilde{\mu})$ as a polynomial of $a_s(\tilde{\mu}')$
  - ✧ Perform the scale evolution : $(\widetilde{\mathrm{MS}}, \tilde{\mu}') \to (\overline{\mathrm{MS}}, 2\ \mathrm{GeV})$
- We use BLM scale for vector channel (Brodsky-Lepage-Mackenzie '83)

$$\boxed{\tilde{\mu}' = \tilde{\mu}^* = \tilde{\mu} \exp\left(-11/6 + 2\zeta(3)\right) \simeq 1.8\tilde{\mu}} \qquad \tilde{a}_s^* \equiv a_s(\tilde{\mu}^*)$$

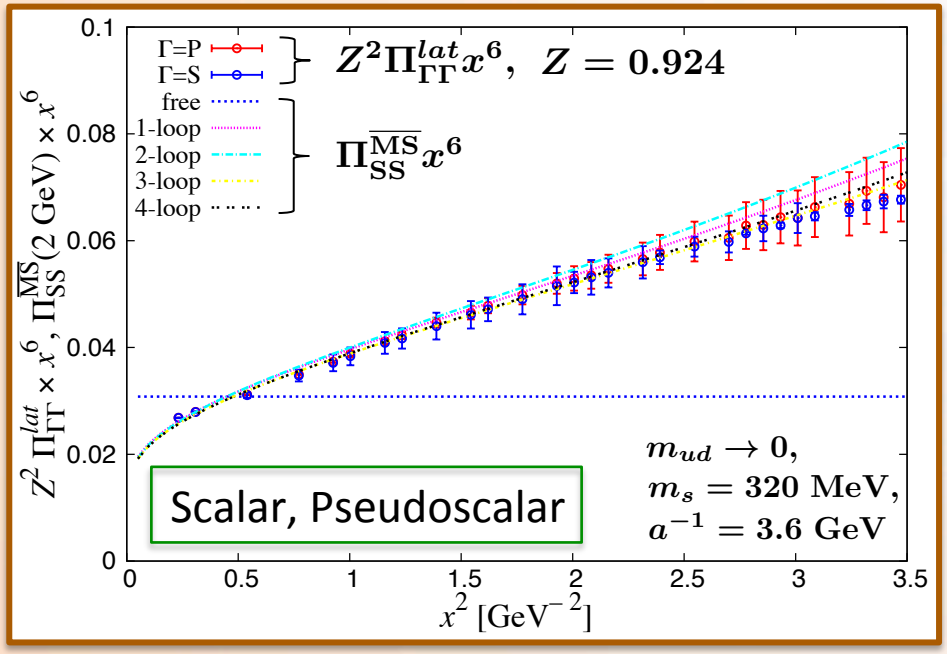
- Improved perturbative series

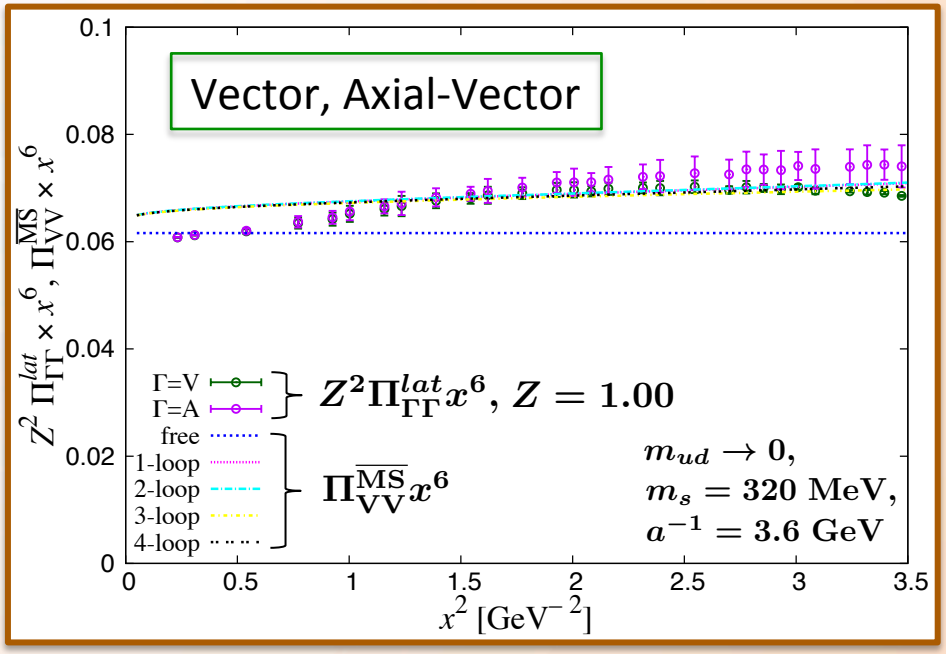
$$\widetilde{\Pi}^{\overline{\mathrm{MS}}}_{\mathrm{SS}}(x, \tilde{\mu}^*) = \frac{3}{\pi^4 x^6}\left(1 + 2.9\tilde{a}_s^* + 1.1\tilde{a}_s^{*2} - 42\tilde{a}_s^{*3} + 24\tilde{a}_s^{*4}\right)$$

$$\widetilde{\Pi}^{\overline{\mathrm{MS}}}_{\mathrm{VV}}(x) = \frac{6}{\pi^4 x^6}\left(1 + \tilde{a}_s^* + 0.083\tilde{a}_s^{*2} - 6\tilde{a}_s^{*3} + 18\tilde{a}_s^{*4}\right)$$

4. Continuum Part — consistency between lattice & PT

- Perturbative series become much better
- Correlators on lattice & PT are roughly same valued





5. Result — plot for β = 4.35, am_s = 0.025

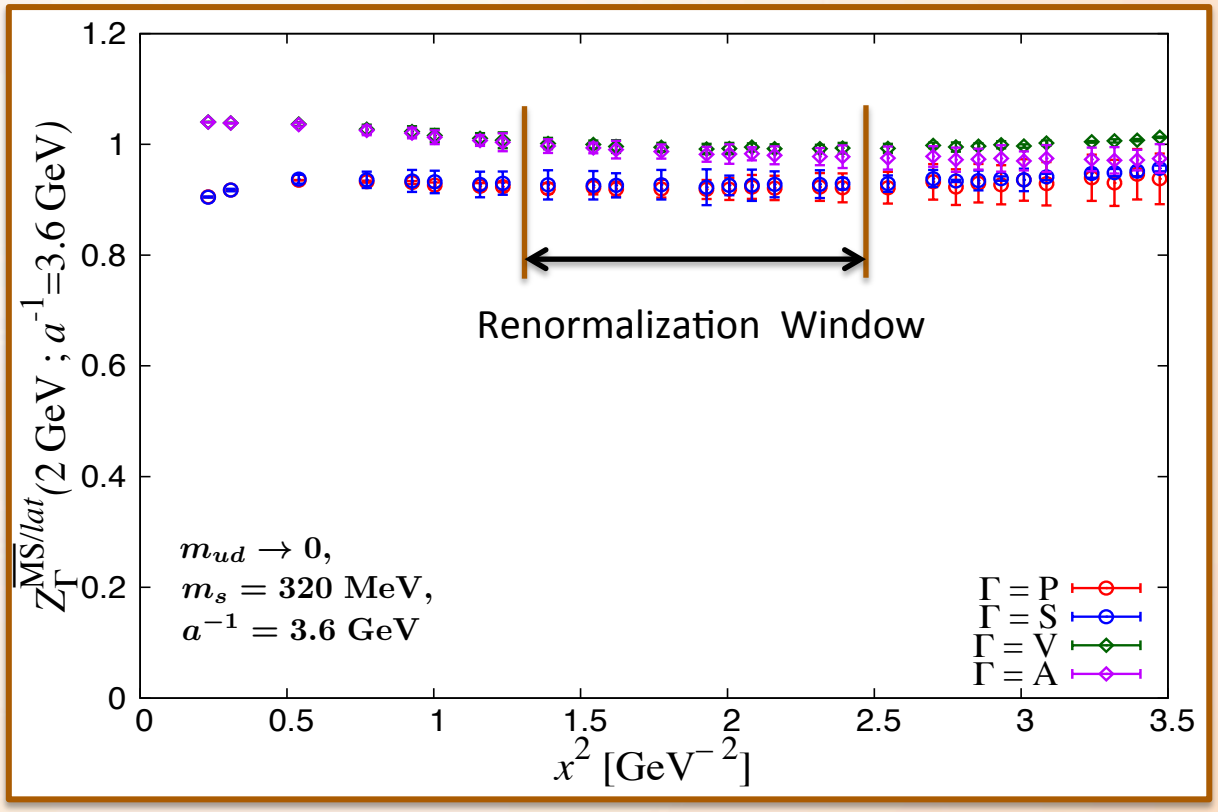
- $Z_\Gamma^{\overline{\rm MS}/lat}(2\ {\rm GeV}) = \sqrt{\dfrac{\Pi_{\Gamma\Gamma}^{\overline{\rm MS}}(x, 2\ {\rm GeV})}{\Pi_{\Gamma\Gamma}^{lat}(x)}}$ are ideally independent of x
- Extract RCs from renormalization window where "x-dependence" of $Z_\Gamma^{\overline{\rm MS}}(2\ {\rm GeV})$ is roughly absent (i.e. plateau)

statistical error

$$Z_{\rm S}^{\overline{\rm MS}/lat}(2\ {\rm GeV}) = 0.924 \pm 0.021 \pm 0.003$$

systematic error

$$Z_{\rm V}^{\overline{\rm MS}/lat} = 0.990 \pm 0.011 \pm 0.007$$



5. Result — preliminary values

- Systematic error $\lesssim$ 1 %
- 20 confs → 200 confs: statistical error → < 1 %

β	a^{-1} [GeV]	am_s	$Z_S^{\overline{MS}/lat}(2\ \mathrm{GeV})$	$Z_V^{\overline{MS}/lat}$
4.17	2.4	0.040	1.144(12)(8)	1.033(3)(11)
		0.030	1.073(28)(13)	1.003(9)(5)
4.35	3.6	0.025	0.924(21)(3)	0.990(11)(7)
		0.018	0.975(15)(13)	1.009(4)(3)
4.47	4.5	0.018	on going	on going
		0.015	on going	on going

systematic error

statistical error

Summary & Future Works

- We investigate NPR of quark bilinears using X-space method
  - ✧ Discretization effect is reduced by applying the free theory correction and democratic cut
  - ✧ Convergence of perturbative series of correlation functions become better by expanding correlators in a polynomials of coupling $\tilde{a}_s^*$ at an appropriate $\tilde{\mu}^*$

- Goal is to obtain RCs within 1% precision
  - ✧ Statistical error is currently larger than systematic error
  - ✧ 20 confs $\rightarrow$ 200 confs: statistical error would become $<$ 1 %

- Furthermore we will try to
  - ✧ extract α_s, $\langle \bar{q}q \rangle$, $\cdots$ from short distance correlators
  - ✧ apply to heavy quark physics