Renormalization group flow of linear sigma model with $U_A(1)$ anomaly

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Chiral phase transition of Nf=2 QCD

Chiral symmetry $SU_L(2) \otimes SU_R(2) \otimes U_V(1) \otimes \cancel{U_A(1)}$

Broken by anomaly

Effective theory

U(2) x U(2) linear sigma model (LSM) + $U_A(1)$ breaking term

Assumption:

$U_A(1)$ breaking is infinitely large at Tc

O(4) LSM

R.D.Pisarski, F.Wilczek (1984)

Estimation : Phase transition is 2nd order with O(4) universality

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Assumption:

$U_A(1)$ breaking is infinitely large at Tc ? ? ?

Hot QCD(2012), JLQCD(2013), TWQCD(2013), LLNL/RBC(2014)
S. Aoki, H. Fukaya and Y. Taniguchi (2012)

O(4) LSM

Estimation : Phase transition is 2nd order with O(4) universality ?

Chiral phase transition of Nf=2 QCD

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$U_A(1)$ breaking is infinitely large at Tc

O(4) LSM

$U_A(1)$ restores at Tc

U(2)xU(2) LSM

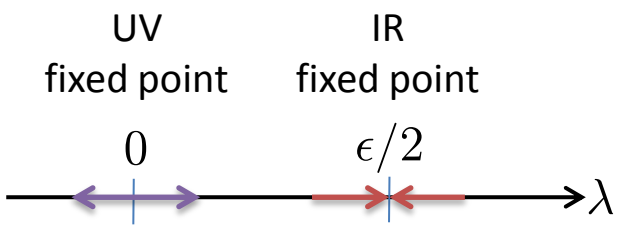
Difference of RG flow

2nd order phase transition ⟺ IR fixed point

RG flow at 1-loop level in d=4-ε

Large $U_A(1)$ breaking

$O(4)$



There is an IR fixed point

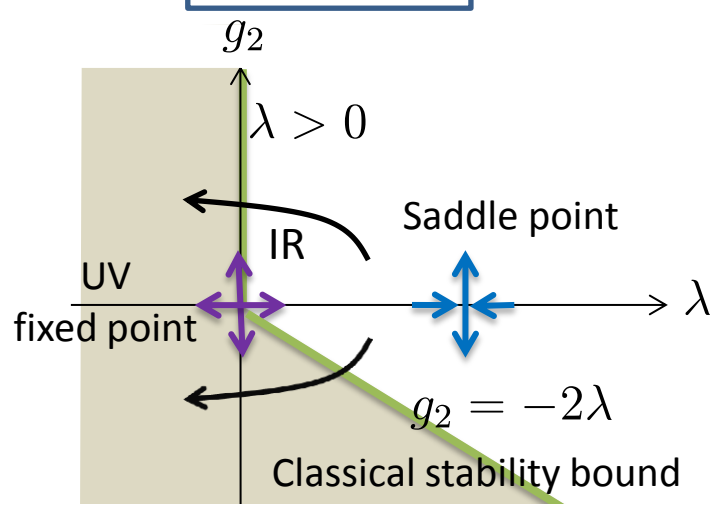
$\lambda = \epsilon/2$

II

2nd order phase transition

$U_A(1)$ restoration

$U(2) \otimes U(2)$



No IR fixed point at 1-loop order

※) Existence of IR fixed point of U(2)xU(2) LSM in higher order is still under discussion

A. Pelissetto and E. Vicari (2013) B. Delamotte and D. Mouhanna (2003)

Chiral phase transition of Nf=2 QCD

Chiral symmetry $SU_L(2) \otimes SU_R(2) \otimes U_V(1) \otimes \cancel{U_A(1)}$

Broken by anomaly

Effective theory

U(2) x U(2) linear sigma model (LSM) + UA(1) breaking term

UA(1) breaking is infinitely large at Tc

O(4) LSM

UA(1) breaking is small but nonzero

? ? ? ?

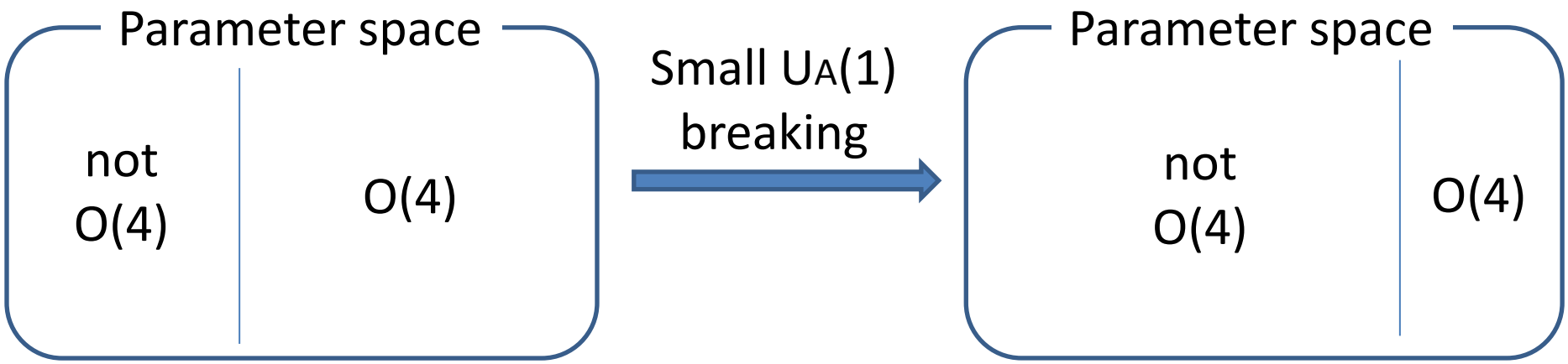
UA(1) restores at Tc

U(2)xU(2) LSM

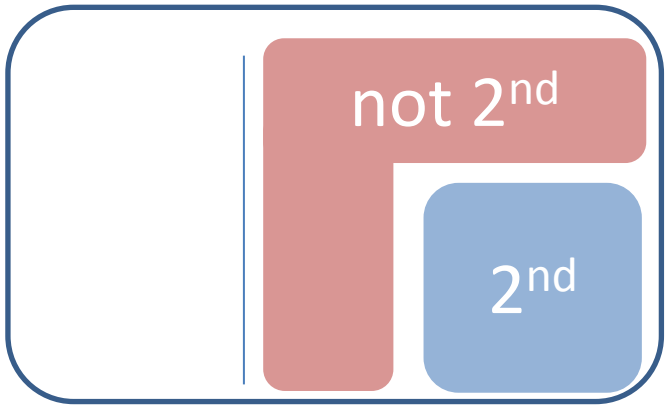
RG flow and effective potential with finite breaking

Conclusions

1. O(4) attractive basin depends on size of $U_A(1)$ breaking



2. Even inside O(4) attractive basin, there is the region where 2nd order phase transition doesn't occur



METHOD

Strategy

1. Calculate RG flow of effective theory

→ Use ε expansion and mass dependent scheme

Mass independent scheme (e.g. $\overline{\mathrm{MS}}$) can not deal with finite $U_A(1)$ breaking

2. Estimate effective potential

→RG flow analysis is not sufficient to identify the order of phase transition

U(2) x U(2)+~~UA(1)~~ LSM

U(2) x U(2) LSM with UA(1) breaking term $\mathcal{L} = \mathcal{L}_{U(2)\otimes U(2)} + \mathcal{L}_A$

U(2)xU(2) symmetric

$$\mathcal{L}_{U(2)\otimes U(2)} = \frac{1}{2}\mathrm{tr}\left[\partial_\mu \Phi \partial^\mu \Phi^\dagger\right] + \frac{1}{2}m^2 \mathrm{tr}\left[\Phi\Phi^\dagger\right] + \frac{\pi^2}{3} g_1 \left(\mathrm{tr}[\Phi\Phi^\dagger]\right)^2 + \frac{\pi^2}{3} g_2 \mathrm{tr}\left[(\Phi\Phi^\dagger)^2\right]$$

Φ : 2 x 2 complex scalar matrix (8 degrees of freedom)

Φ transforms as $\Phi \to e^{2i\theta_A} L^\dagger \Phi R \; (L \in SU_L(2),\; R \in SU_R(2))$

2 couplings : g_1, g_2

UA(1) breaking

$$\mathcal{L}_A = -\frac{c_A}{4}(\det\Phi + \det\Phi^\dagger) + \frac{\pi^2}{3} x\, \mathrm{Tr}[\Phi\Phi^\dagger](\det\Phi + \det\Phi^\dagger) + \frac{\pi^2}{3} y\, (\det\Phi + \det\Phi^\dagger)^2$$

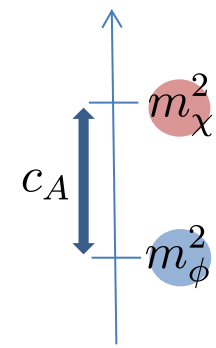
Extra mass parameter : c_A (mass dimension 2)

Extra 2 couplings : x, y

U(2) x U(2)+~~UA(1)~~ LSM

Rewriting Φ in terms of scalars ϕ_a and χ_a (a=0,1,2,3)

$$\Phi = (\phi_0 - i\chi_0)T^0 + (\chi_i + i\phi_i)T^i$$



$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi_a)^2 + \frac{1}{2}(\partial_\mu\chi_a)^2 + \frac{1}{2}\underbrace{(m^2 - c_A/2)}_{m_\phi^2}{\phi_a}^2 + \frac{1}{2}\underbrace{(m^2 + c_A/2)}_{m_\chi^2}{\chi_a}^2$$

$$+\frac{\pi^2}{3}\left[\lambda({\phi_a}^2)^2 + (\lambda - 2x)({\chi_a}^2)^2 + 2(\lambda + g_2 - z){\phi_a}^2{\chi_b}^2 - 2g_2(\phi_a\chi_a)^2\right]$$

$$(a = 0, 1, 2, 3,\ \lambda \equiv g_1 + g_2/2 + x + y,\ z \equiv x + 2y)$$

Assumption : phase transition is 2nd order

and critical point is defined by $m_\phi^2 = 0,\ \rightarrow\ m_\chi^2 = c_A$

c_A=0 & $c_A \to \infty$ limit

- $U_A(1)$ restored limit

$$c_A = 0, \quad x = 0, \quad z = 0 \quad \Rightarrow \quad \mathcal{L}_A = 0, \quad \mathcal{L} = \mathcal{L}_{U(2)\otimes U(2)}$$

U(2)xU(2) LSM

- Large $U_A(1)$ breaking limit

$$c_A \to \infty \quad \Rightarrow \quad m_\chi^2 \to \infty$$

χ decouples

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi_a)^2 + \cancel{\frac{1}{2}(\partial_\mu \chi_a)^2} + \cancel{\frac{1}{2} m_\chi^2 {\chi_a}^2}$$

$$+ \frac{\pi^2}{3}\left[\lambda({\phi_a}^2)^2 + \cancel{(\lambda - 2x)({\chi_a}^2)^2} + \cancel{2(\lambda + g_2 - z){\phi_a}^2{\chi_b}^2} - \cancel{2g_2(\phi_a \chi_a)^2}\right]$$

$$\to \mathcal{L}_{O(4)} = \frac{1}{2}(\partial_\mu \phi_a)^2 + \frac{\pi^2}{3}\lambda({\phi_a}^2)^2$$

O(4) LSM

RG FLOW

β functions

1-loop β functions in d=4-ε

$$\beta_\lambda \equiv \mu\frac{d\lambda}{d\mu} = -\epsilon\lambda + 2\lambda^2 + \frac{1}{6}f(\hat{\mu})(4\lambda^2 + 6\lambda g_2 + 3g_2^2 - 8\lambda z - 6g_2 z + 4z^2)$$

$$\beta_{g_2} \equiv \mu\frac{dg_2}{d\mu} = \cdots$$

$$\vdots$$

$$\hat{\mu} = \frac{\mu}{\sqrt{c_A}}, \quad f(x) = 1 - \frac{4}{x\sqrt{4+x^2}}\arctan\sqrt{\frac{x^2}{4+x^2}}$$

$$\lim_{x\to 0} f(x) = 0, \quad \lim_{x\to\infty} f(x) = 1$$

f(μ) represents effect of finite c_A

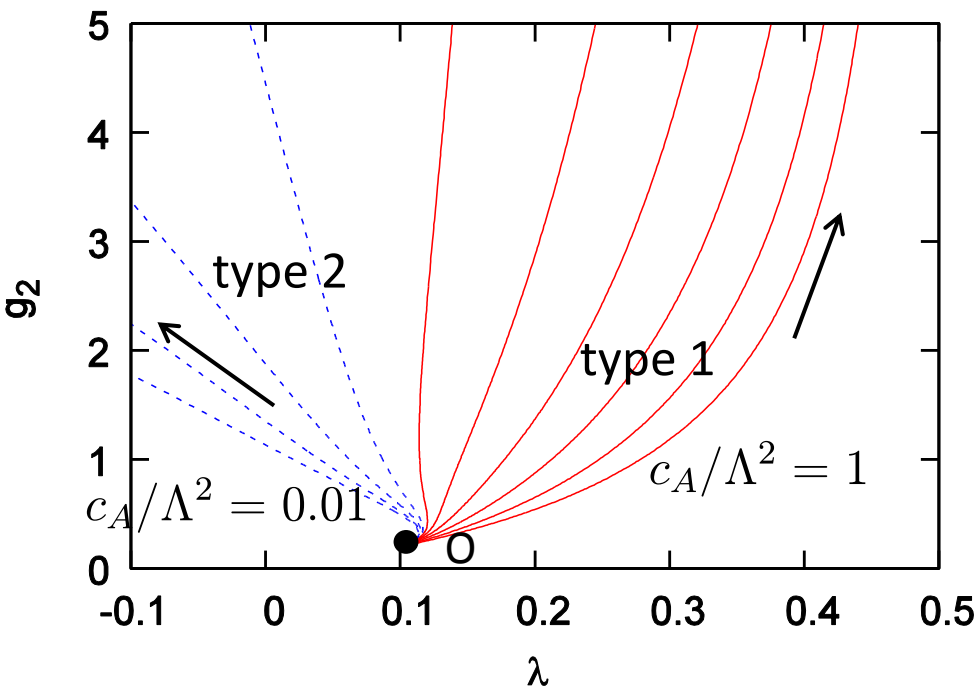
$c_A \to \infty$ ⇒ $f(\hat{\mu}) \to 0$

$c_A \to 0$ ⇒ $f(\hat{\mu}) \to 1$ (Coincides with $\overline{\text{MS}}$)

RG flow

There are two types of RG flow depending on c_A

$O:\ (\lambda(\Lambda), g_2(\Lambda), x(\Lambda), z(\Lambda)) = (0.1, 0.2, 0, 0),\ \epsilon = 1$



Λ: Initial value of renormalization scale

type 1 (larger c_A)

λ→1/2 in IR limit

➡ **Fixed point of O(4) LSM**

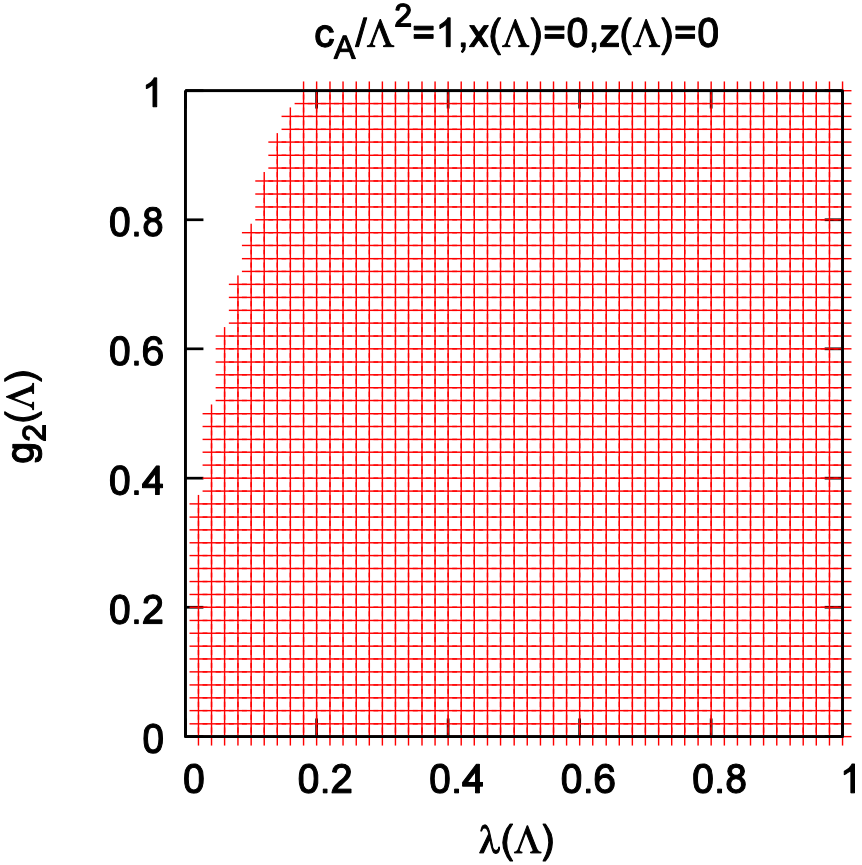
type 2 (smaller c_A)

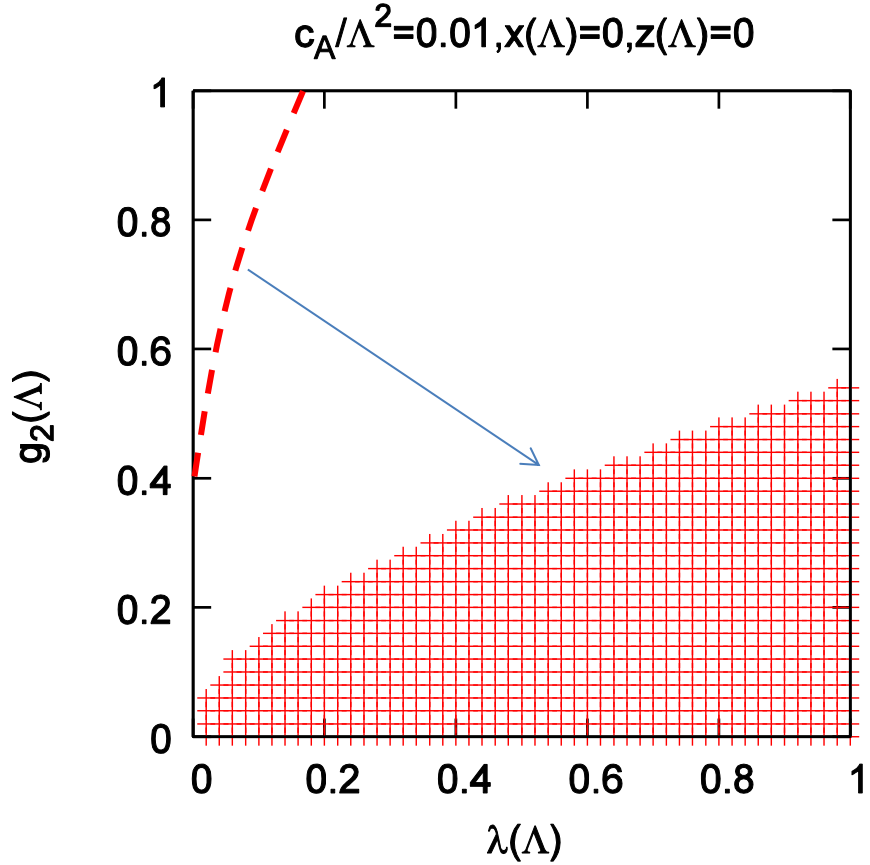
λ ↛1/2 in IR limit

➡ **Doesn't go to O(4)**

O(4) attractive basin

Region of initial values λ(Λ), g2(Λ) flowing into O(4) fixed point





O(4) attractive basin shrinks as c_A vanishes

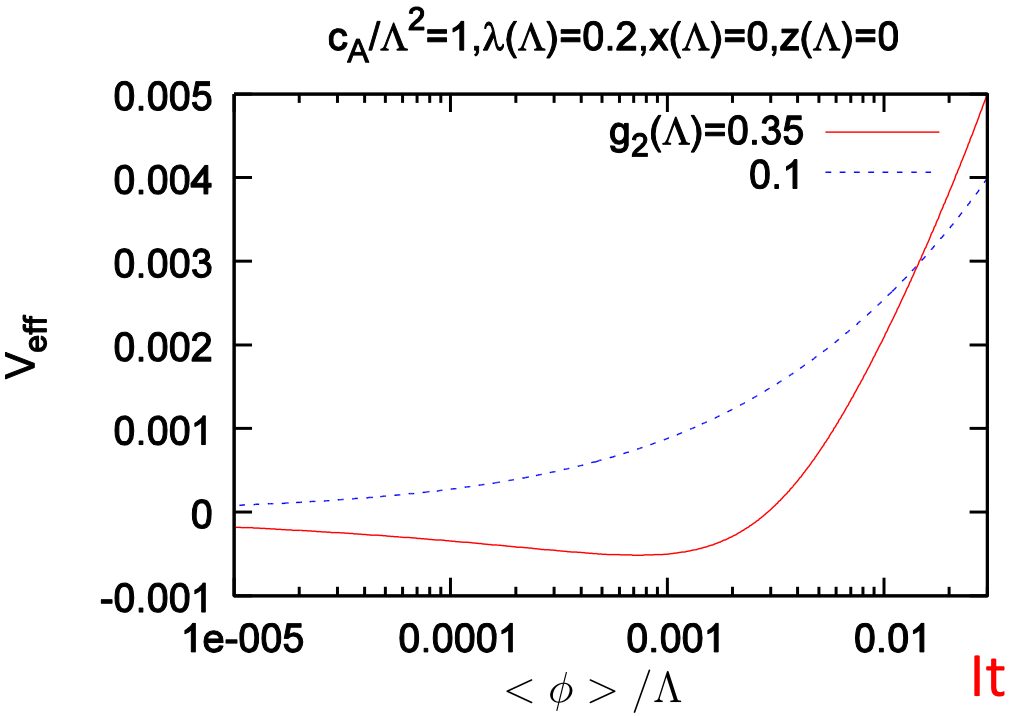
EFFECTIVE POTENTIAL

Effective potential

Even when λ flows into O(4) fixed point,

the system doesn't always show O(4) 2nd order

<χ>=0 slice of the effective potential



Blue: Minimum is at origin

➡ consistent with assumption

Red: Minimum at non-trivial <ϕ>

➡ inconsistent with assumption

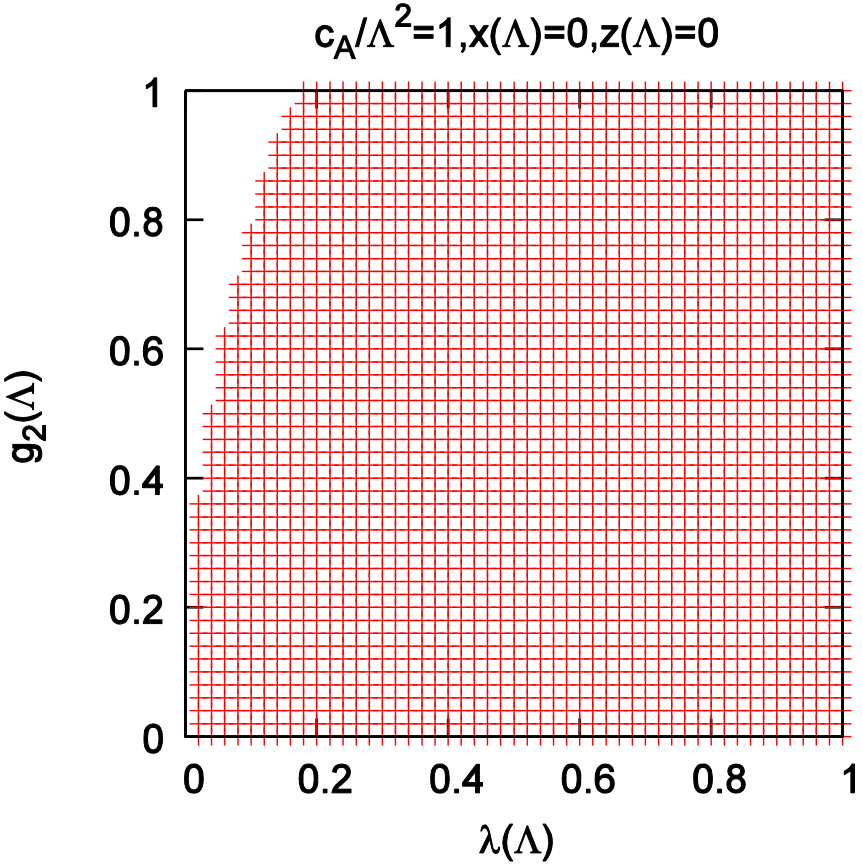
⬇

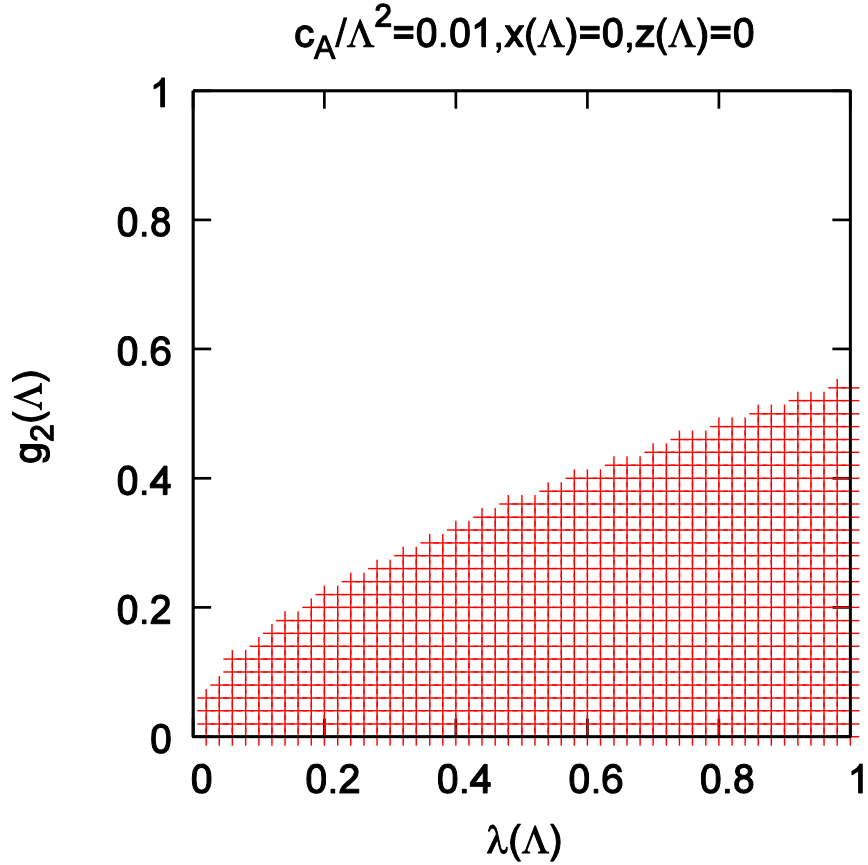
It doesn't 2nd order phase transition

※)There may be other minimum at <χ>≠0

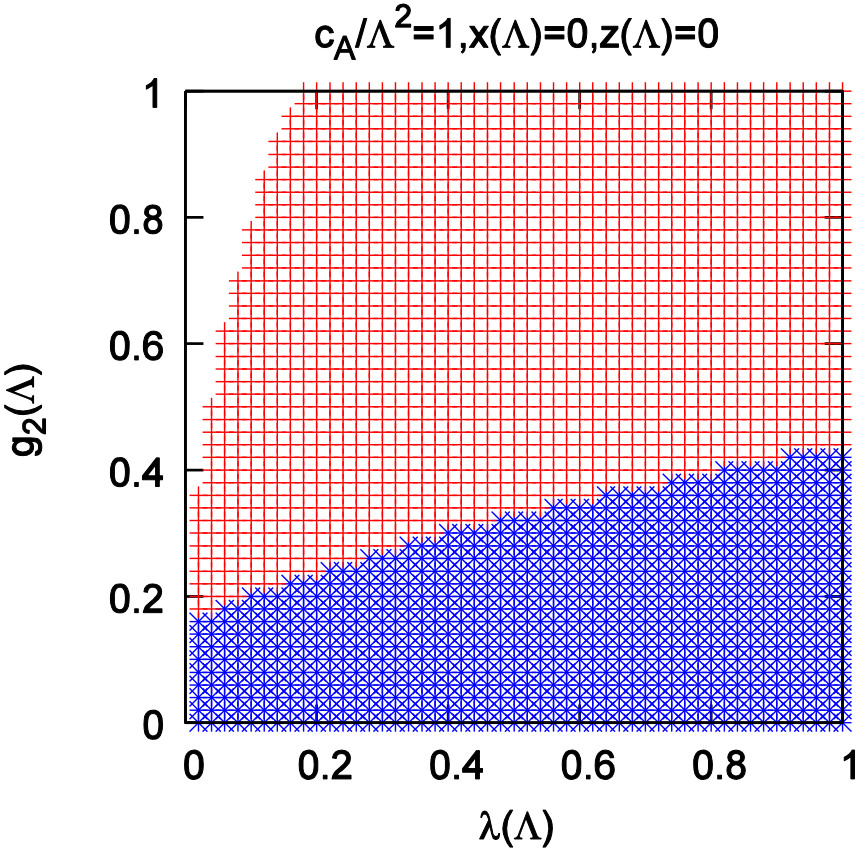
Restriction from effective potential

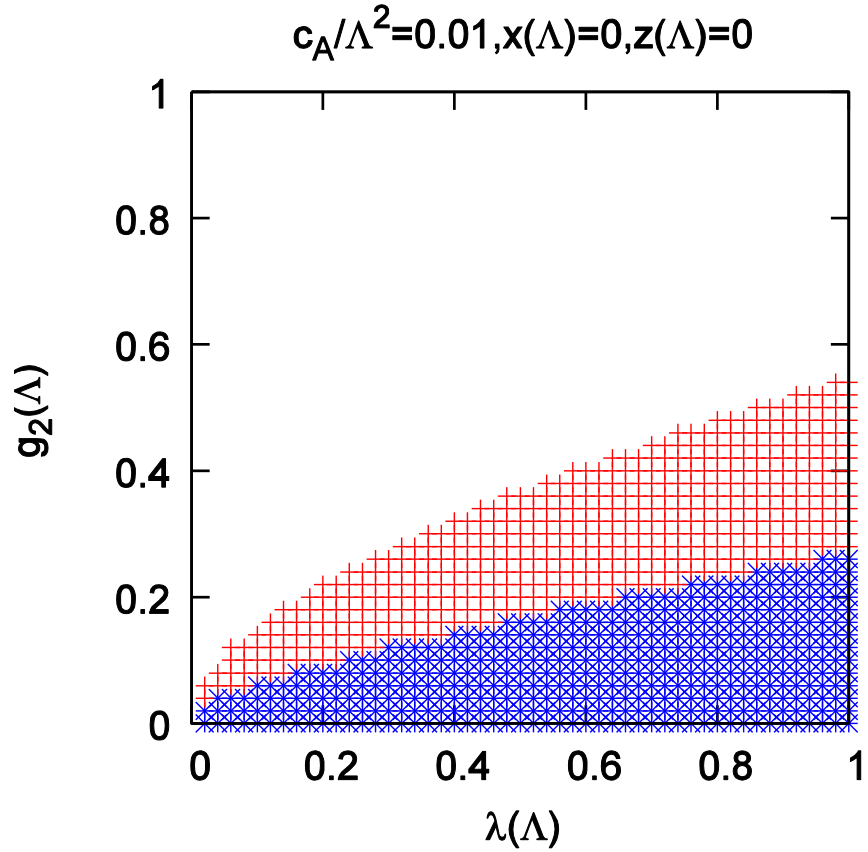
O(4) attractive basin estimated from RG flow





Restriction from effective potential





Blue: There is no minimum point other than origin at least in <χ>=0 slice of effective potential

Conclusions

- We determine the attractive basin flowing into O(4) in coupling space of U(2)xU(2) LSM with finite $U_A(1)$ breaking. It shrinks as the breaking vanishes.

- Effective potential analysis tells that, even in naïve O(4) attractive basin, there is a region in which the system does not show 2nd order phase transition with O(4) universality.

BACKUP

Renormalization conditions

For four-point functions,

$$G_4(\phi_1(p_1), \phi_1(p_2); \phi_2(p_3)\phi_2(p_4))|_{s=t=u=-\mu^2} = -i\frac{8}{3}\pi^2\lambda \prod_i \frac{i}{p_i^2 - m_i^2},$$

$$G_4(\eta_1(p_1), \eta_1(p_2); \eta_2(p_3)\eta_2(p_4))|_{s=t=u=-\mu^2} = -i\frac{8}{3}\pi^2(\lambda - 2x) \prod_i \frac{i}{p_i^2 - m_i^2},$$

$$G_4(\phi_1(p_1), \eta_2(p_2); \phi_1(p_3)\eta_2(p_4))|_{s=t=u=-\mu^2} = -i\frac{8}{3}\pi^2(\lambda + g_2 - z) \prod_i \frac{i}{p_i^2 - m_i^2},$$

$$G_4(\phi_1(p_1), \eta_2(p_2); \phi_2(p_3)\eta_1(p_4))|_{s=t=u=-\mu^2} = i\frac{4}{3}\pi^2 g_2 \prod_i \frac{i}{p_i^2 - m_i^2}.$$

For two-point functions, we take the on-shell scheme

$\Rightarrow$ mass doesn't run at 1-loop level

Asymptotic behavior of type 1

Does decoupling occur in flow of type 1? ➡ Behavior of ϕ's self coupling

- β function of λ (self coupling of ϕ)

Goes to 0 Diverging

$$\beta_\lambda \equiv \mu\frac{d\lambda}{d\mu} = -\lambda + 2\lambda^2 + \frac{1}{6}f(\hat{\mu})(4\lambda^2 + 6\lambda g_2 + 3g_2^2 - 8\lambda z - 6g_2 z + 4z^2)$$

Contribution from ϕ χ

How g2 and z diverge (in the case of $\lambda(\mu \to 0) \sim 1/2$)

$$\beta_{g_2}(\mu \to 0) \sim -\frac{5}{6}g_2 + O\left(\frac{\mu}{\sqrt{c_A}}\right),$$

$$\beta_z(\mu \to 0) \sim -\frac{1}{2}z - \frac{1}{4}g_2 + \frac{1}{4} + O\left(\frac{\mu}{\sqrt{c_A}}\right)$$

➡

$$g_2(\mu) \sim g_2(\Lambda)\left(\frac{\mu}{\Lambda}\right)^{-5/6},$$

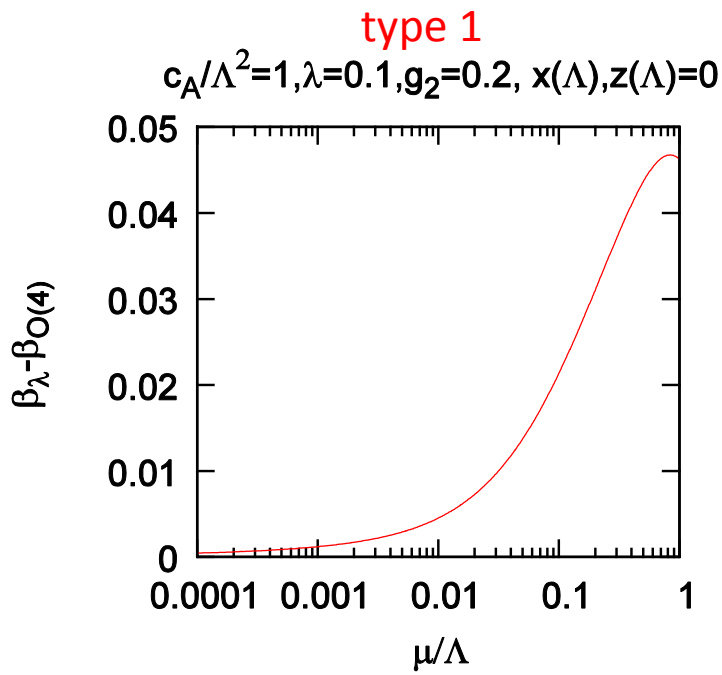
$$z(\mu) \sim \frac{3}{4}g_2(\mu)$$

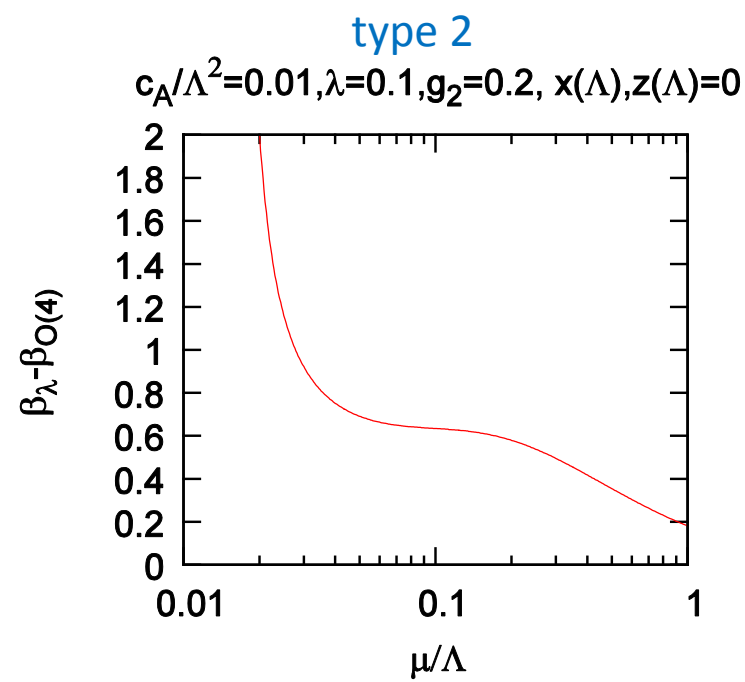
$$\lim_{\mu\to 0} f(\hat{\mu}) \sim \frac{\mu^2}{c_A}$$ ➡ χ's contribution $\sim f(\hat{\mu}){g_2}^2 \sim \mu^{2-\frac{5}{3}} \to 0$

χ's contribution for ϕ's physics vanishes in IR limit = Decoupling !!

Difference between type 1 and 2

Contribution of χ on each RG flow

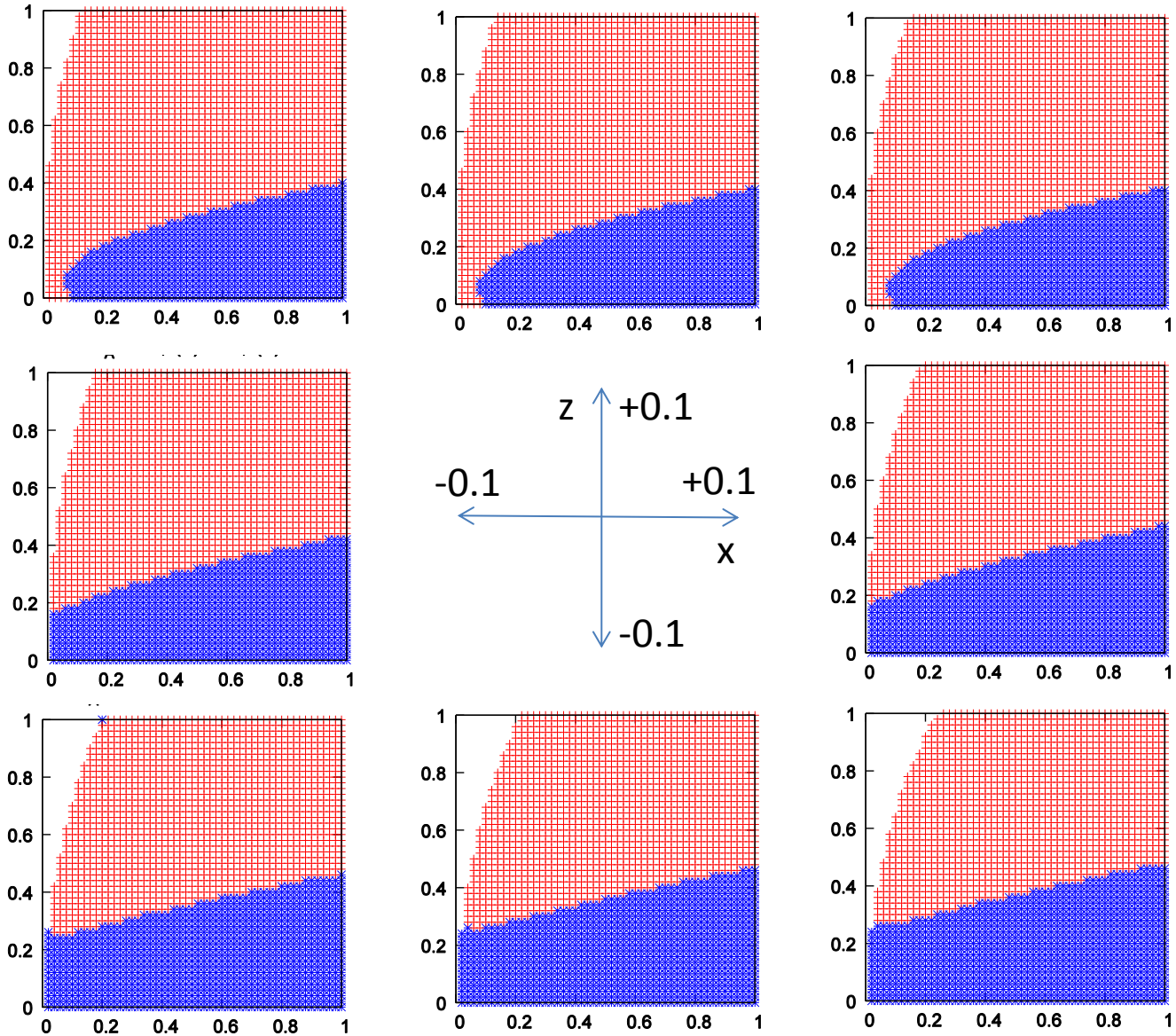




$$\beta_\lambda \equiv \mu\frac{d\lambda}{d\mu} = \underbrace{-\lambda + 2\lambda^2}_{\beta_{O(4)}} + \underbrace{\frac{1}{6}f(\hat{\mu})(4\lambda^2 + 6\lambda g_2 + 3g_2^2 - 8\lambda z - 6g_2 z + 4z^2)}_{\beta_\lambda - \beta_{O(4)}}$$

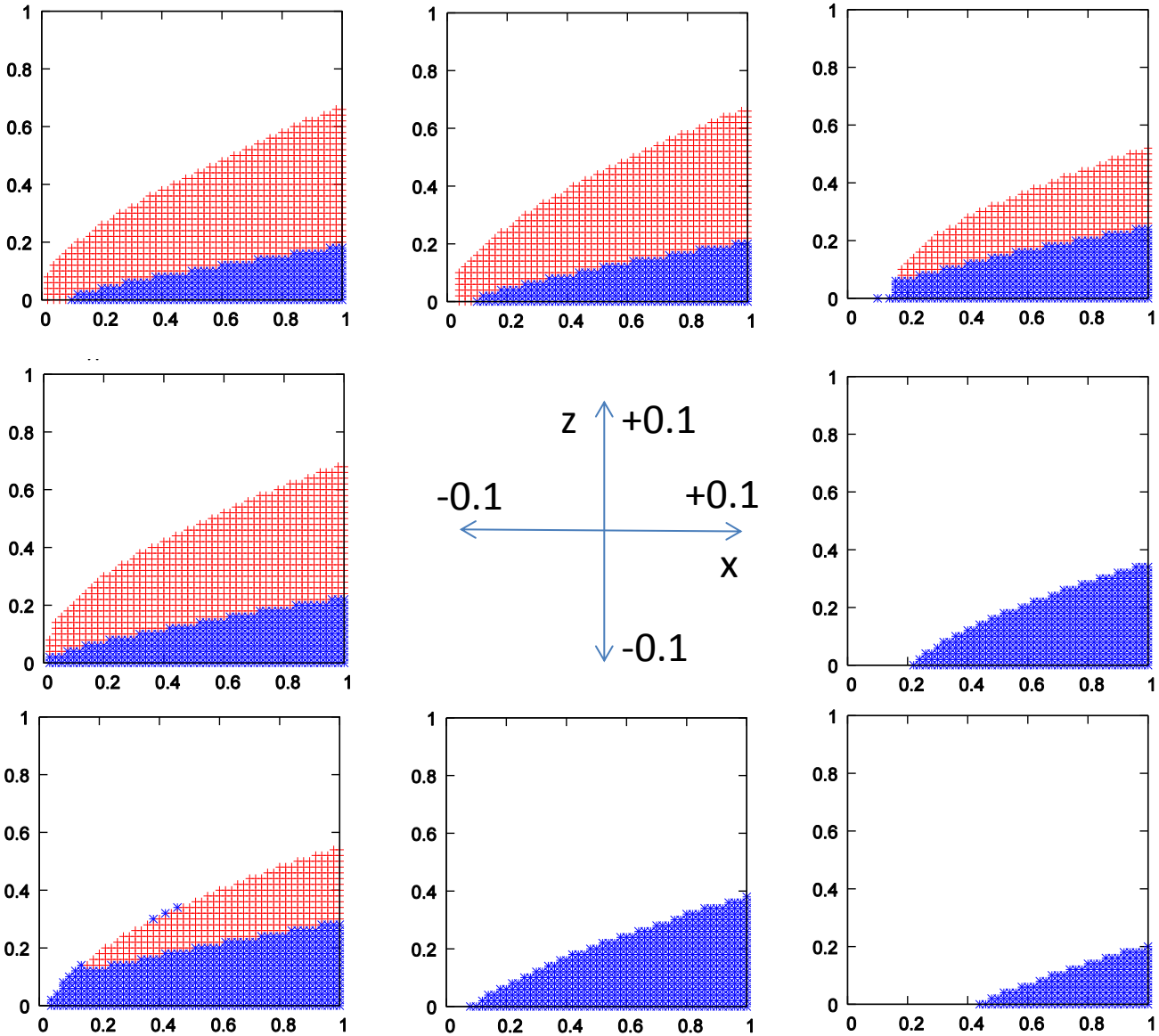
Type 1: χ's contribution vanishes, β_λ coincides $\beta_{O(4)}$

Type 2: χ's contribution doesn't vanishes, β_λ goes away from $\beta_{O(4)}$



c_A is fixed
as $c_A/\Lambda=1$

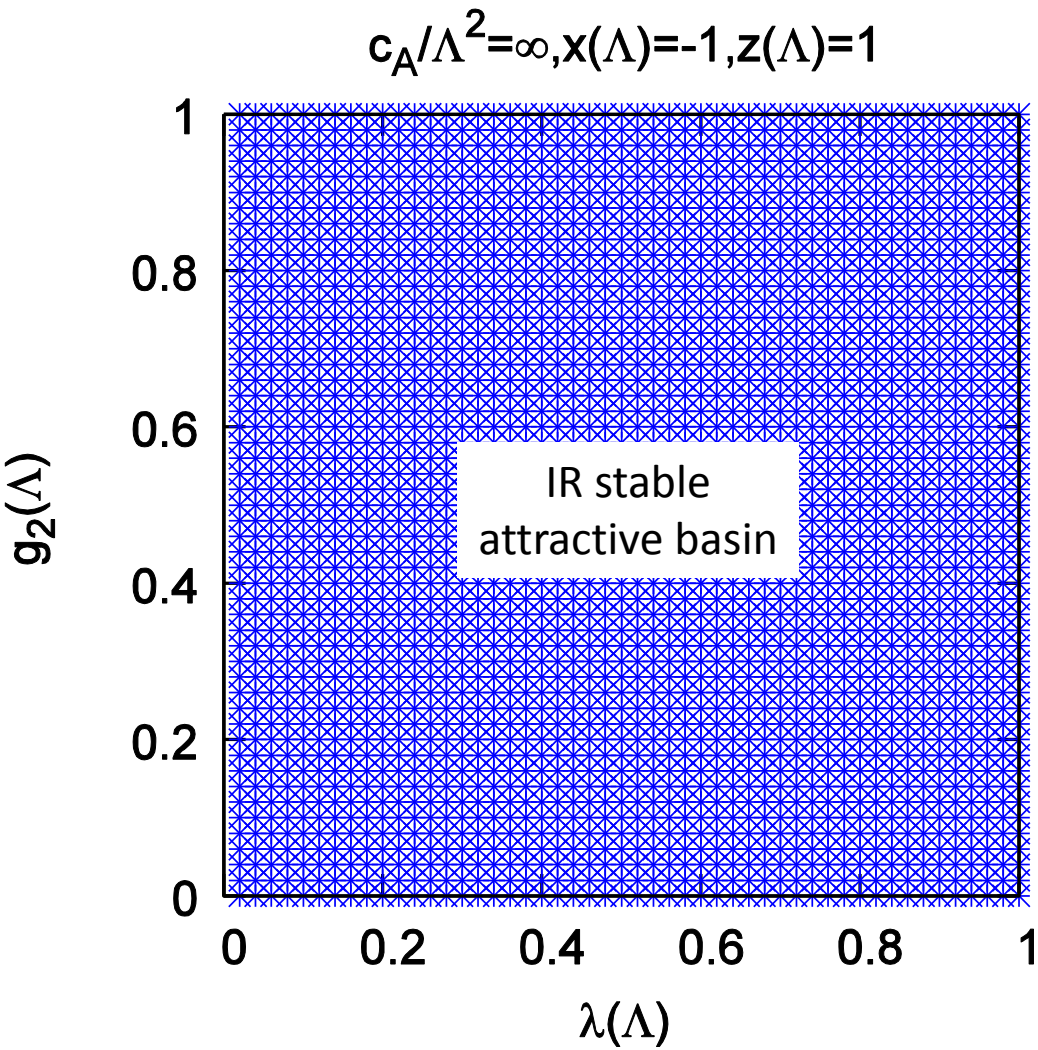
Attractive basin for non-zero x(Λ),z(Λ)



c_A is fixed as $c_A/\Lambda=0.01$

x(Λ) and z(Λ) are changed between -0.1 to 0.1

Large breaking limit



In the limit of cA→∞,
and x(Λ)=-1, z(Λ)=1

All flow drop into
O(4) fixed point