F. Méot, BNL, Sept. 2011

TRANSPORT OF ^{3}He↑ IN BOOSTER AND AGS

Contents

1 Layout **3**

2 Particle data **4**

3 Dynamics parameters in Booster and AGS **5**

4 Booster **6**

4.1 Resonances in Booster, imperfection, $G\gamma - n = 0$ 6

4.2 Resonances in Booster, intrinsic, $G\gamma \pm Q_y - n = 0$ **:** 7

4.3 Comments (1/3) 8

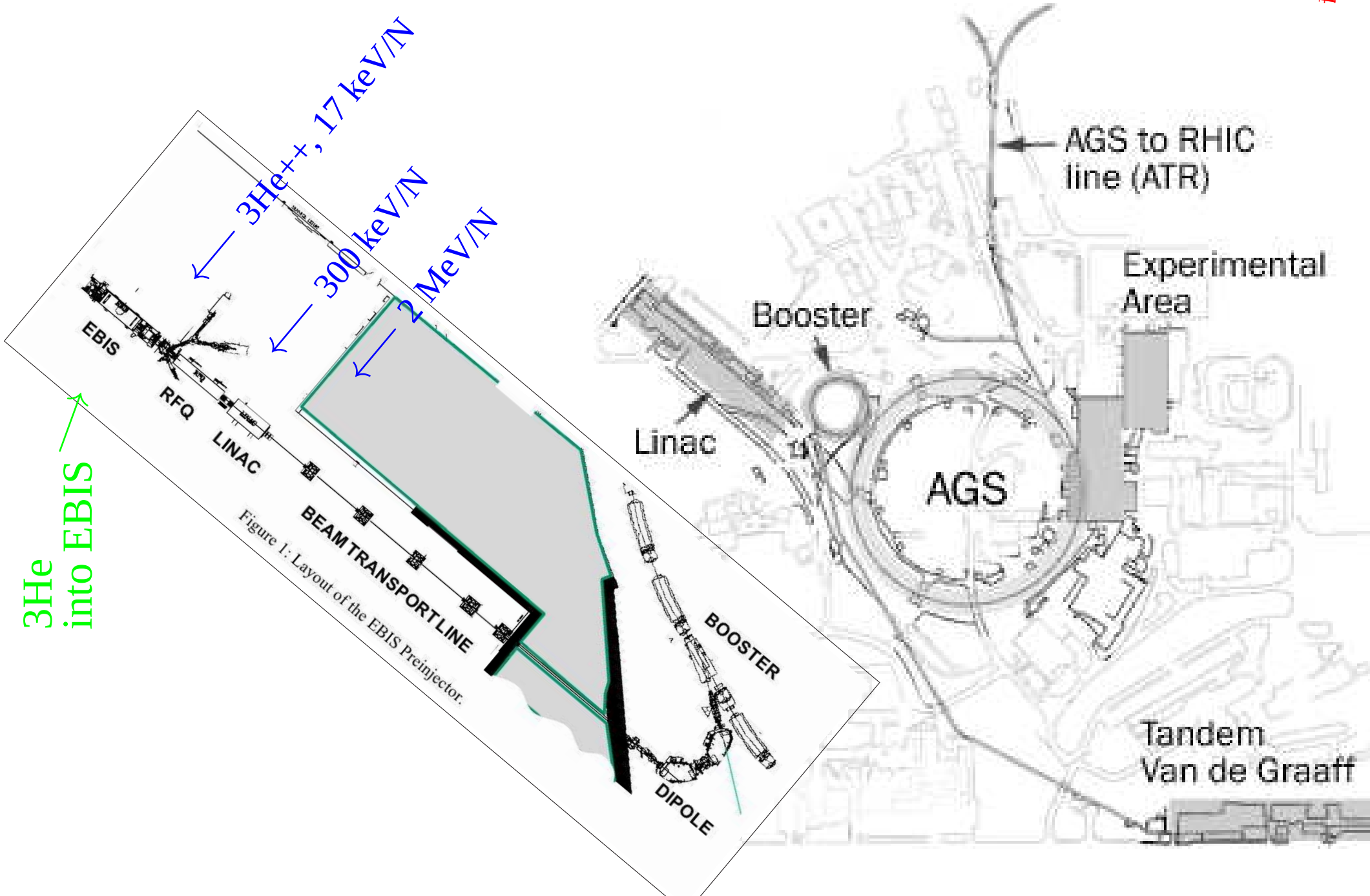
5 AGS 11
5.1 Resonances in AGS, imperfection, $G\gamma - n = 0$ (Qx / Qy = 8.71 / 8.77 optics) : 11
5.2 Snake strength 12
5.3 AGS snakes 13
5.4 Resonances in AGS, intrinsic, $G\gamma \pm Q_y - n = 0$: 14
5.5 $^3He^{2+}$ resonance strengths 15
5.6 Discussion 16
5.7 Tentative tracking : Xing 60-Qy 18

6 AGS snakes 19

7 Possible plans 25

1 Layout

RHIC at coll. rigidity, 833.9 T.m :
proton : p=250 GeV/c, $G\gamma$=477.7
$^3He^{2+}$: p=500 GeV/c, $G\gamma$=-745



2 Particle data

	units	proton (for comparison)	$^2H^+$	$^3He^{2+}$
mass	MeV/c^2	938.272046	1875.6128	2808.39148
	10^{-27}kg	1.672621777	3.34358348	5.00641234
mass/amu	MeV/c^2	1.00727646	2.013553212	3.0149322
charge Q	$\|e\|$	+1	+1	+2
number of nucleons, N		1	2	3
number of protons, Z		1	1	2
spin quantum number		1/2	1	1/2
magnetic moment, μ	μ_N	2.7928474	0.85743823	-2.1276253
gyromagnetic ratio $g = \frac{\mu}{\mu_N} \times 2\frac{m_p \mu_N}{Ze\hbar I}$	μ/μ_N	2	1.99900759	2.99315281
anomalous mag. moment $G = \frac{g-2}{2}$		1.7928474	-0.1429872	-4.18415382
Imp. resonance interval $\|mc^2/G\|$	MeV	523.3	13117	671.2

nuclear magneton unit, $\mu_N = \frac{e\hbar}{2m_p}$	eV/T	3.1524512×10^{-8}
atomic mass unit, amu	MeV/c^2	931.494061
elementary charge, e	C	$1.602176565 \times 10^{-19}$
speed of light in vac.	m/s	2.99792458×10^{8}
$\hbar$	J.s	$1.054571726 \times 10^{-34}$
	eV.s	$6.58211928 \times 10^{-16}$

3 Dynamics parameters in Booster and AGS

Proton, for comparison

		Comments
BOOSTER INJECTION :		
kin-E (GeV)	0.2	Hyp. : Energy out of linac (Ref. CJG)
$B\rho$ (T.m)	2.1496	
γ	1.2132	
$\beta\gamma$	0.6868	
$G\gamma$	2.175	
BOOSTER EXTRACTION / AGS INJECTION :		
kin-E (GeV)	1.4167	
$B\rho$ (T.m)	7.2051	Max. $B\rho$ in Booster is 17 T.m
γ	2.5100	w/ Qx/y=4.73/4.82 optics
$\beta\gamma$	2.3021	
$G\gamma$	4.5	
AGS EXTRACTION :		
kin-E (GeV)	22.8738	
$B\rho$ (T.m)	79.3668	
γ	25.3786	
$\beta\gamma$		
$G\gamma$	45.5	Half-int. for AtR match ; not more due to AtR matching limitations.

$^3He^{2+}$, possible data :

		Comments
BOOSTER INJECTION :		
kinetic E (MeV/N)	2.0146	Hyp. : $\beta = 0.0655$ from EBIS IH DTL
$B\rho$ (T.m)	0.3075	Note : $B\rho_{inj,^3He^{2+}} \ll B\rho_{inj,p}$
γ	1.0022	
$\beta\gamma$	0.0656	
$G\gamma$	-4.1932	
BOOSTER EXTRACTION / AGS INJECTION :		
kinetic E (GeV)	2.2256	0.74186 GeV/N
$B\rho$ (T.m)	6.9678	Consistent w/ p : $B\rho_{inj,^3He^{2+}} \approx B\rho_{inj,p}$.
γ	1.7925	
$\beta\gamma$	1.4876	
$G\gamma$	-7.5	Requires $Q_y < 4.5$ so to escape $12 - Q_y$
Other possibility	-10.5 ?	Interest : injection into AGS beyond 0+. Requires $Q_y > 4.5$ so to avoid $6 + Q_y$
AGS EXTRACTION :		
kinetic E (GeV)	27.7311	9.24369 GeV/N
$B\rho$ (T.m)	50.7185	$B\rho_{xtr,^3He^{2+}} \ll B\rho_{xtr,p}$, whereas $B\rho = 79.36 \rightarrow G\gamma = -71$, this leaves room for extraction beyond 60-.
γ	10.8743	
$\beta\gamma$	10.8282	
$G\gamma$	-45.5	
Other possblties :		
$B\rho$ 54 T.m	-48.5	Better spin match to vertical $\vec{n}_0$ in RHIC.
$B\rho$ 55 T.m	-49.5	Note : plateau is further away from 36+
$B\rho \rightarrow$ 79 T.m	< -51 ?	

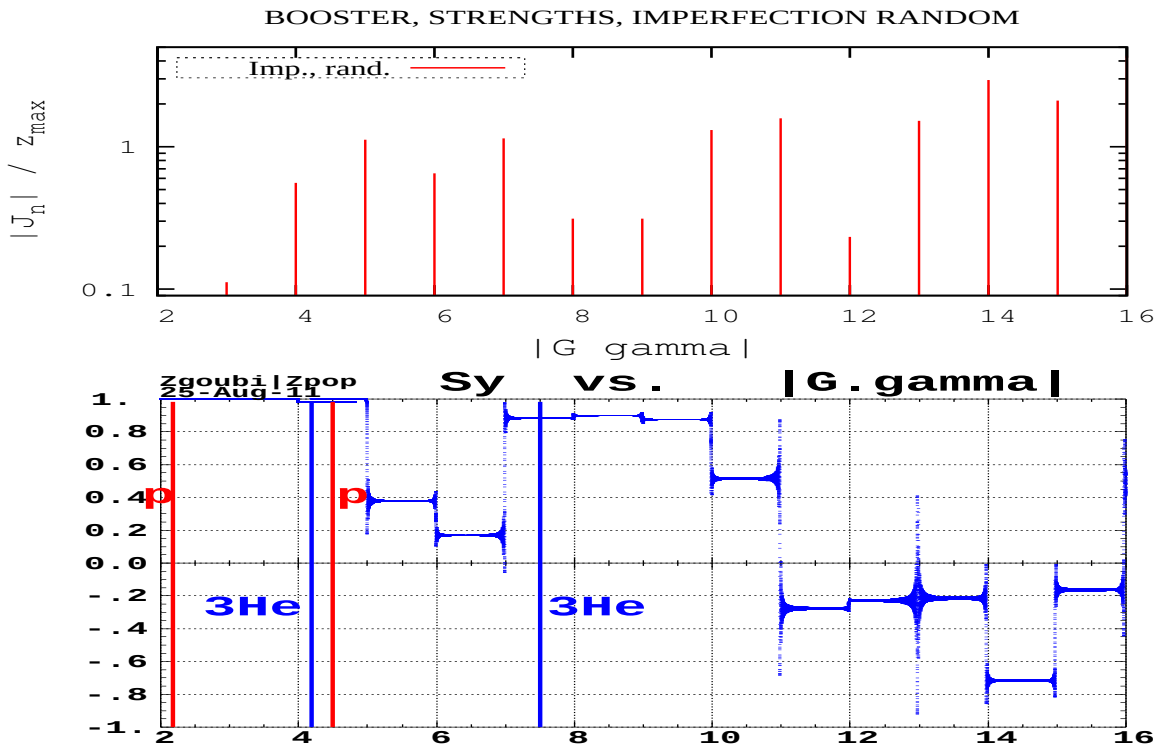
• Note : Transition γ :
Booster : 4.8 ; AGS : 8.4 ; RHIC : 23.6.

4 Booster

4.1 Resonances in Booster, imperfection, $G\gamma - n = 0$

- Acceleration range, proton : $G\gamma \ : 2.175 \rightarrow 4.5$ **: 2 resonances, at** $G\gamma = \begin{cases} 3 & : \text{harm. correction} \\ 4 & : \text{flipped} \end{cases}$

- Possible, $^3He^{2+}$: $G\gamma \ : -4.19 \rightarrow -7.5$ **: 3 resonances, at** $-G\gamma =$**5, 6, 7.**



$$J_n = \frac{1+G\gamma}{2\pi} \Sigma_{Qpoles} \left(\cos(G\gamma\alpha_i) + i\ \sin(G\gamma\alpha_i)\right) (KL)_i z_{co,i}$$

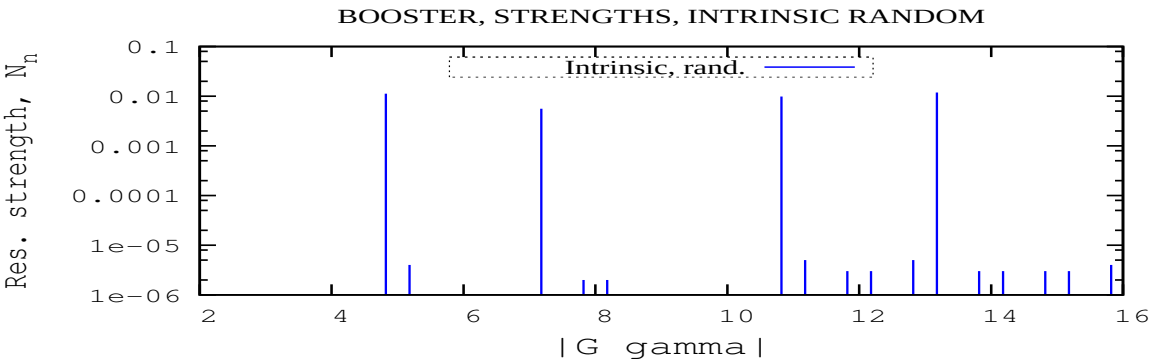
- **Top : Strengths of imperfection resonances in Booster, thin-lens model. Qx / Qy = 4.73 / 4.82. Range shown covers up to max. booster** $B\rho \approx 17$ **T.m.**
- **Bottom :** S_y **motion.** $G\gamma : 2.18 \rightarrow 4.5$ **for proton,** $|G\gamma| : 4.19 \rightarrow 7.5$ **for $^3He^{2+}$ ($\gamma < \gamma_{tr} = 4.8$).**
- **Particle is on** $\hat{y}_{co} = 1$ **mm max. amplitude closed orbit with zero H and V invariants.**
- **Vertical lines show** $G\gamma$ **ranges for p and for $^3He^{2+}$.**

4.2 Resonances in Booster, intrinsic, $G\gamma \pm Q_y - n = 0$:

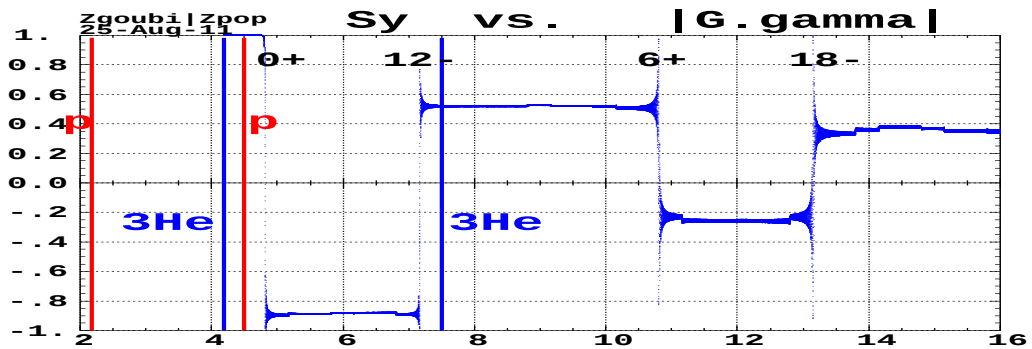
Ranges shown are :

- Proton : $G\gamma \ : 2.175 \rightarrow 4.5$ **: no systematic (n=6×integer) resonance.**

- ^{3}He^{2+}, possible : $G\gamma \ : -4.193 \rightarrow -7.5$ **: 2 systematic resonances, at** $G\gamma = 0 + Q_y,\ 12 - Q_y$**.**



$$N_n^{\pm} = \frac{1+G\gamma}{4\pi}\Sigma_{Qpoles}\left(\cos(G\gamma\alpha_i \pm \psi_i) + i\sin(G\gamma\alpha_i \pm \psi_i)\right)(KL)_i\sqrt{\frac{\beta_{z,i}}{\epsilon_z/\pi}}$$



$\dot{\gamma} = 16$ **here. (Actual : 24.3,** $\alpha 6\,10^{-5}$**,** $\dot{B}4.91$ **T/s.)**

- **Top : Strength (normalized) of intrinsic resonances in Booster, Qy = 4.82.**

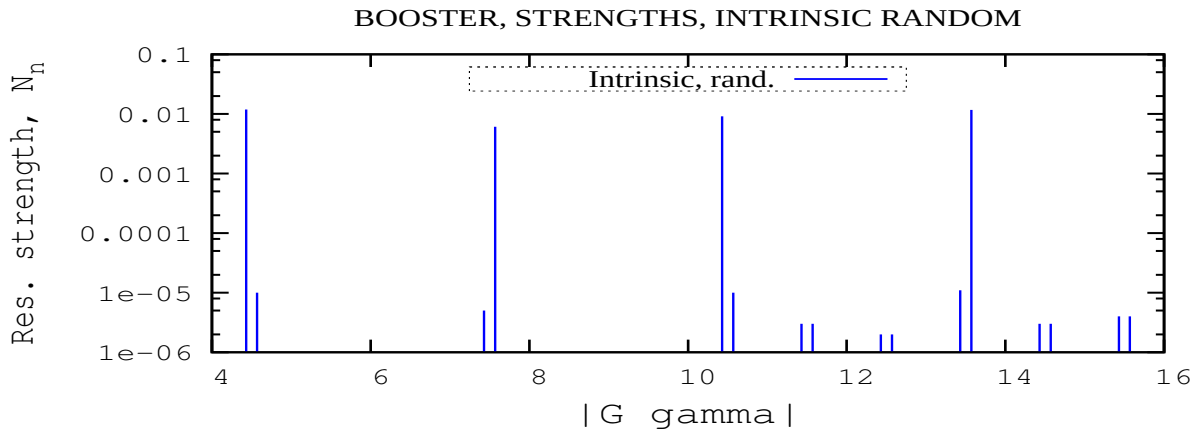
Major lines are at $G\gamma = 0 + Q_y,\ 12-,\ 6+,\ 18-$**.**

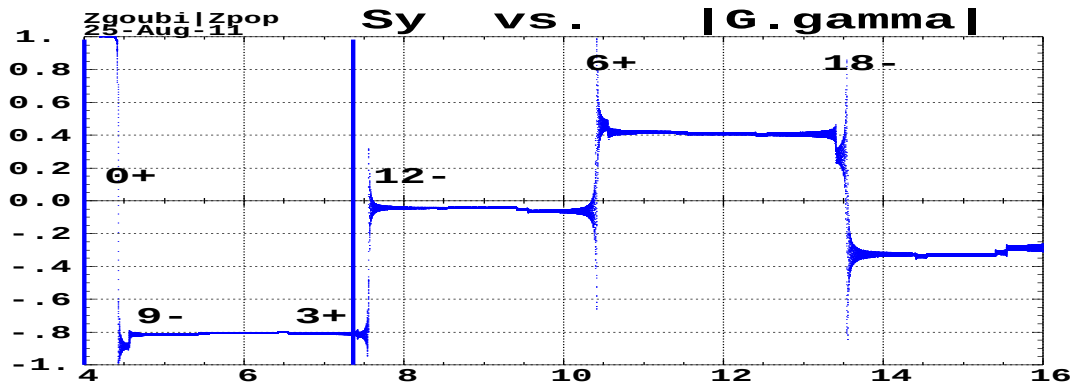
- **Bottom :** S_y **motion for ^{3}He^{2+} in** $|G\gamma| \ : \ 4.19 \rightarrow 7.5$**. Particle on zero vertical orbit with** $\epsilon_y \approx 0.6\,\pi$**mm.mrad invariant.**
- **Random resonances are excited (tunes preserved) with** $\delta K = \pm 1\%$ **defect in QV[D1]/QV[E1].**
- **Vertical lines show** $G\gamma$ **ranges for p and for ^{3}He^{2+}.**

4.3 Comments (1/3)

• **¿ Could push 12-Qy beyond extraction $G\gamma$:**
For downstream polarization >99% upstream one : $|G\gamma+Q_y-12| > 7N_n \rightarrow Q_y \lesssim 4.5-7\times N_n \approx 4.45$ **at extraction.**

If on the other hand $Q_y > |G\gamma|_{injection} \approx 4.19$ **then only 0+Qy to be crossed.**





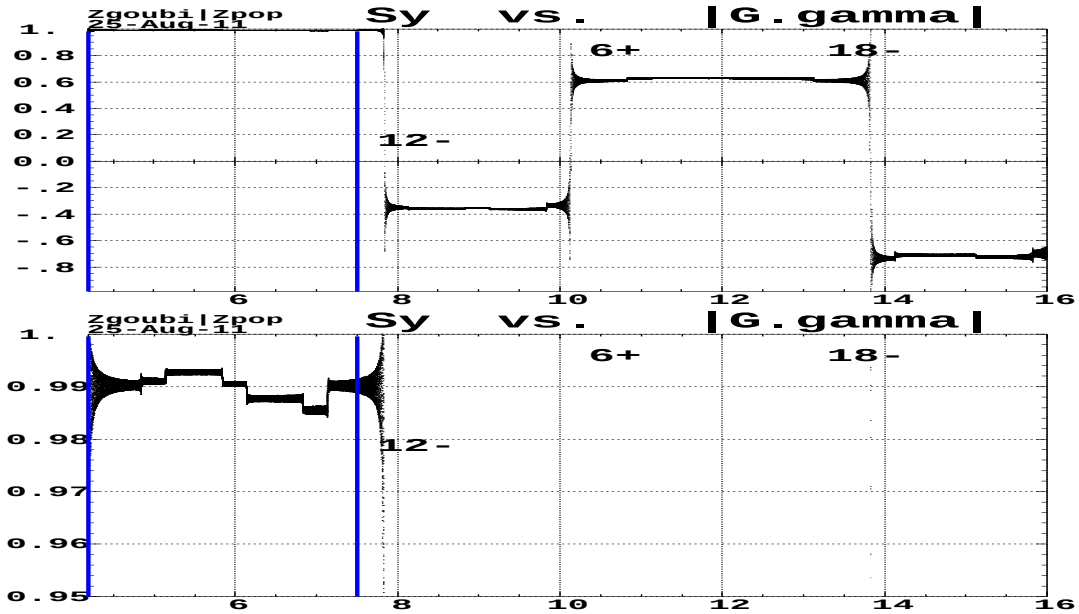
$\dot{\gamma} = 16$

• **Top :** $^3He^{2+}$**, strengths** N_n **of intrinsic resonances,** $\boxed{\textbf{Qy=4.43}}$**,** $\epsilon_y = 2.5\pi$**mm.mrad, norm.**
• **Bottom :** S_y **motion. Particle is on zero vertical orbit.**
• **Vertical lines show** $G\gamma$ **range for** $^3He^{2+}$**.**

Comments (2/3)

- ¿ Could push both $\left\{ \begin{array}{c} 0^+ \\ 12 - Qy \end{array} \right.$ beyond [4.19,7.5] range $\Rightarrow$ no systematic resonance to be crossed.
- Strength of 0^+ for $\epsilon_y = 2.5\pi$mm.mrad, norm., is $N_n \approx 0.012$.

Take distance to the resonance $\Delta = |G\gamma + Q_y - n| > 7 \times N_n$, then
given $n = 0$ here tune at injection should satisfy $Q_y < -G\gamma|_{inj} - 7 \times N_n \approx 4.11$.



- Top : S_y motion. Particle is on zero vertical orbit with non-zero vertical invariant.
- Bottom : Zoom on random intrinsic resonances in acceleration range.
- Vertical lines show $G\gamma$ range for $^3He^{2+}$.

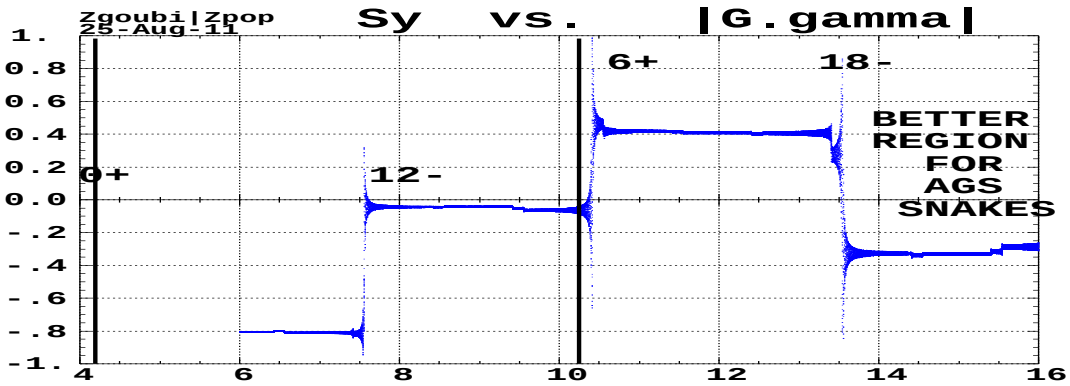
Comments (3/3)

• **Extract even further, in $G\gamma = -10.5$ region or beyond**

- below 6+ (needs $Qy > 4.5$) if -10.5 ($B\rho = 10.8$ T.m).

If $B\rho \to 15$ T.m ($G\gamma \to -14$) then get closer to asymptotic, better behaved region in snake optics.

- Note : still below transition γ ($|G\gamma|_{tr} > 16$), and below max. $B\rho = 17$ T.m.



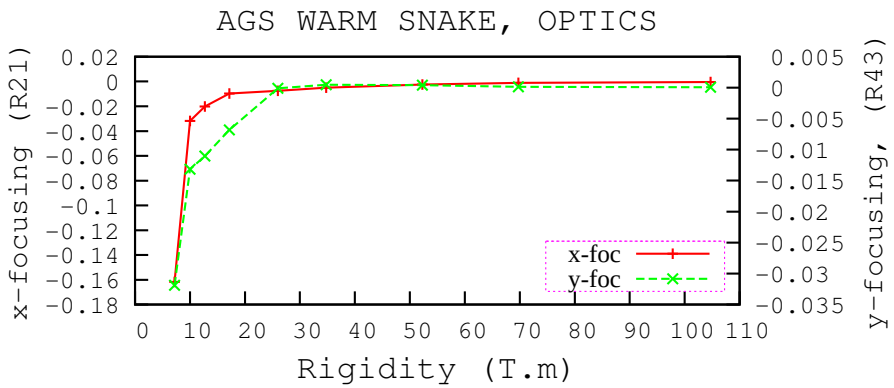
Strengths (Qx/Qy=4.73/4.82) :

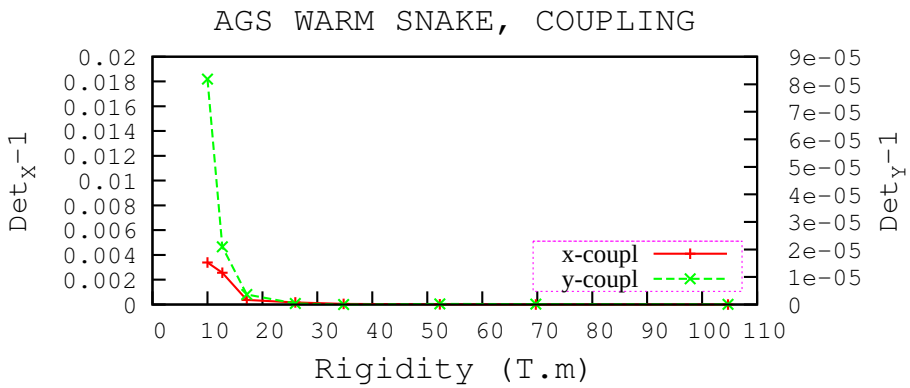
$G\gamma$	0+	12-	6+	18-
N_n at 2.5π	0.011	0.0006	0.010	0.012

• **Interest :**

- $G\gamma = -7.5$ is $B\rho = 6.97$ T.m, even lower than proton $G\gamma = 4.5$, $B\rho = 7.21$;

- this low-rigidity region is less favorable as to snake optics : perturbed ring focusing, coupling.





5 AGS

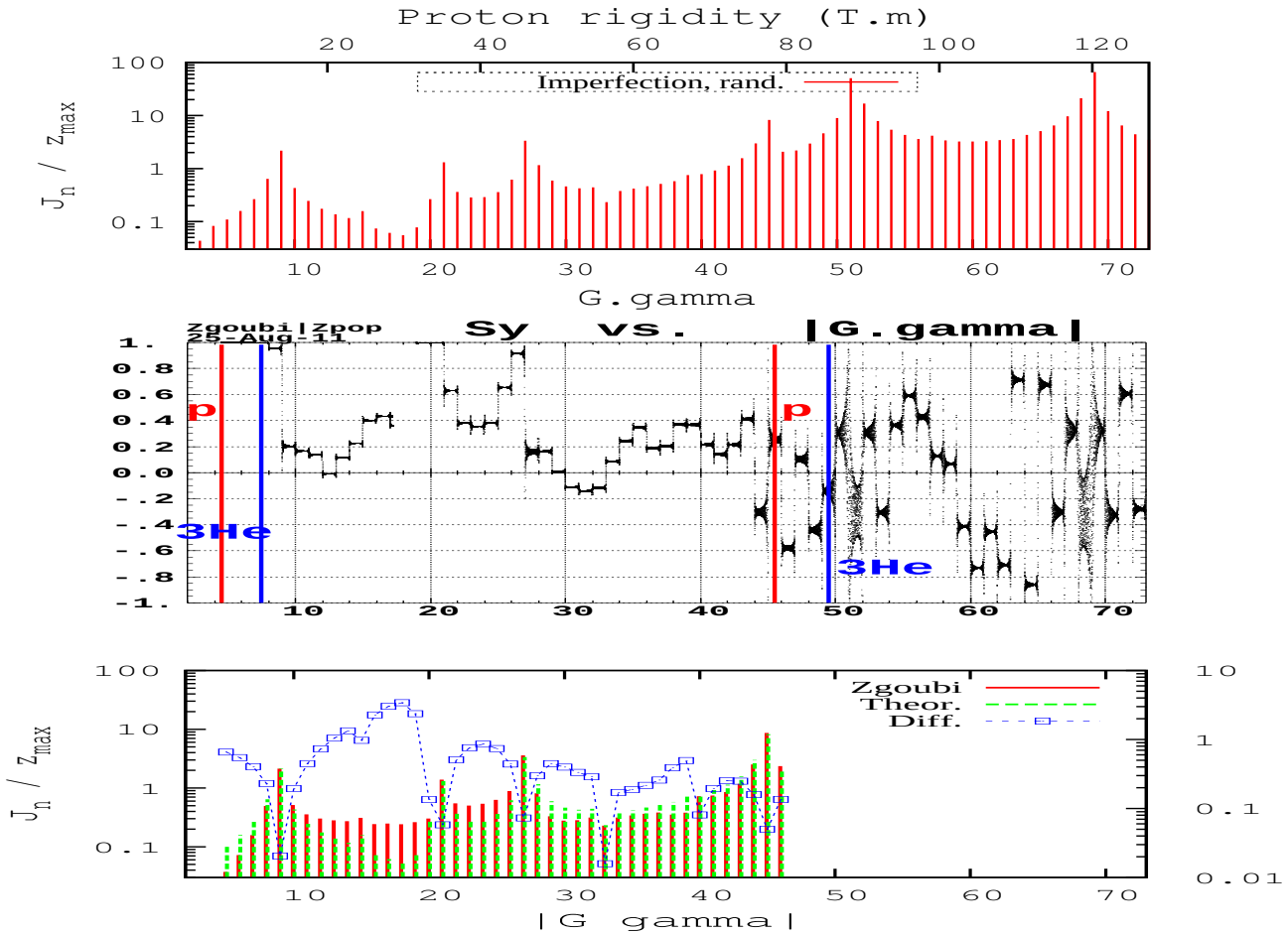
5.1 Resonances in AGS, imperfection, $G\gamma - n = 0$ (Qx / Qy = 8.71 / 8.77 optics) :

- Proton, acceleration range :
$G\gamma \ : 4.5 \rightarrow 45.5$,
41 integer resonances crossed.

- $^3He^{2+}$, possible range :
$G\gamma \ : -7.5 \rightarrow -49.5$,
42 resonances crossed.

Strengths extracted using Froissart-Stora :

$$J_n = \sqrt{\frac{-2\alpha}{\pi} \ln\left(\frac{1+P_f/P_i}{2}\right)} \longrightarrow$$



- **Top : Strengths of imperfection resonances in AGS, thin-lens model, normalized to $\hat{y}_{c.o.}$.**
- **Middle : Proton, S_y motion, $\hat{y}_{co} \approx 1.53$ mm closed orbit, zero H and V invariants.**

Pf/Pi value is correlated to resonance strength by Froissart-Stora formula.
Red, blue lines show injection, extraction $G\gamma$ values for p and $^3He^{2+}$.

- **Bottom : Strengths from tracking & thin-lens, superposed, and (right axis) their relative difference.**

5.2 Snake strength

• **A partial snake forces polarization to flip : at Xing of any integer resonance one gets**

$$\frac{P_{final}}{P_{initial}} = \left(2\exp(-\frac{\pi}{2\alpha}\left|\frac{\phi}{2\pi} + J_n\right|^2) - 1 \right) = -1 \qquad \textbf{\textit{at better than}}\ 10^{-3}$$

as long as snake strength $\phi/2\pi$ dominates over strength J_n of major imperfection resonances :

$$\frac{\phi}{2\pi} \gtrsim |J_n| + 2\sqrt{\alpha}$$

• **Preserving polarisation up to $G \gtrsim 45$ against imperfection resonances, snake strength requirement :**

					Beyond 36+	
Strong resonances, $G\gamma$	**9**	**21**	**27**	**45**	**51**	**Comments**
$J_n/\hat{y}_{co}$	**2.2**	**1.3**	**3.4**	**8.2**	**51**	
J_n, **assuming** $\hat{y}_{co} = 4$**mm**	**0.009**	**0.005**	**0.014**	**0.033**	**0.20**	$J_n \propto \hat{y}_{co}$
Min. snake strength $\frac{\phi}{2\pi}$				**0.045**	**0.21**	$\frac{\phi}{2\pi} \gtrsim \lvert J_n\rvert + 2\sqrt{\alpha}$, $\alpha = 4.5\,10^{-5}$
"% snake", $\frac{\phi}{\pi}$				**9**	**42**	*Xing* $G\gamma = 51$ *requires 42% snake, 76 deg. (4 mm* $\hat{V}$ *orbit).*

5.3 AGS snakes

	Warm snake	Cold snake	Comments
B max (T)		**3**	
B nominal (T)	**1.5**	**2.1**	*Case of tracking below*
snake angle (deg.) { **proton :**	≈ 11	≈ 21	**Discussed further on**
snake angle (deg.) { ^{3}**He**$^{2+}$ **:**	26	44	
"% snake" { **proton :**	6	12	
"% snake" { ^{3}**He**$^{2+}$ **:**	15	25	

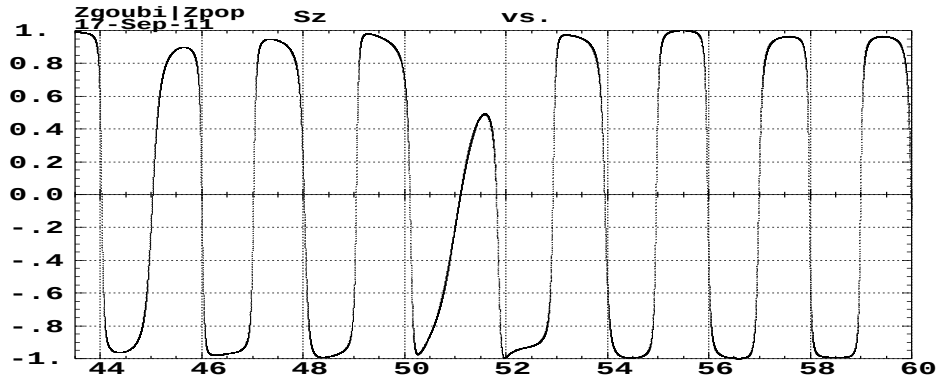
- **Example :**

Flipping across imperfection resonances, $G\gamma$: $43.5 \rightarrow 60$**, rigidities :** { **proton :** $75.9 \rightarrow 104.7$ **T.m**; ^{3}**He**$^{2+}$ **:** $48.47 \rightarrow 67.0$ **T.m**

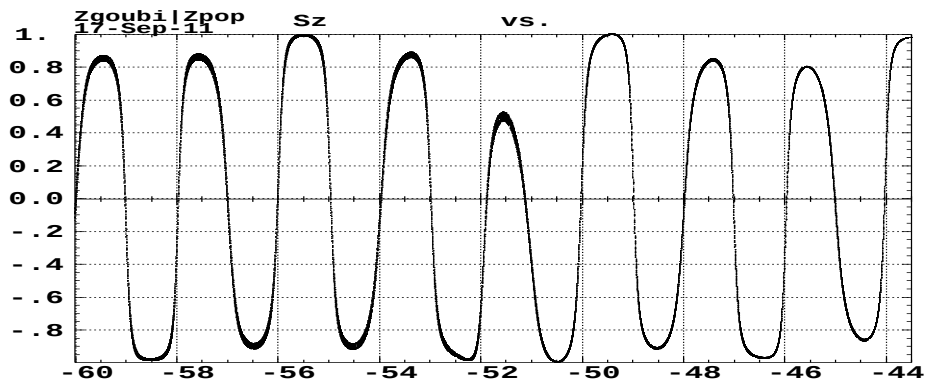
- Single particle on vertical orbit : { **proton :** $\hat{y}_{co} \approx$ **8 mm**; ^{3}**He**$^{2+}$ **:** $\hat{y}_{co} \approx$ **5 mm**

- H orbit < 1 **mm, H and V invariants ≈zero,** $\delta p/p|_{synch.} < 10^{-4}$**.**

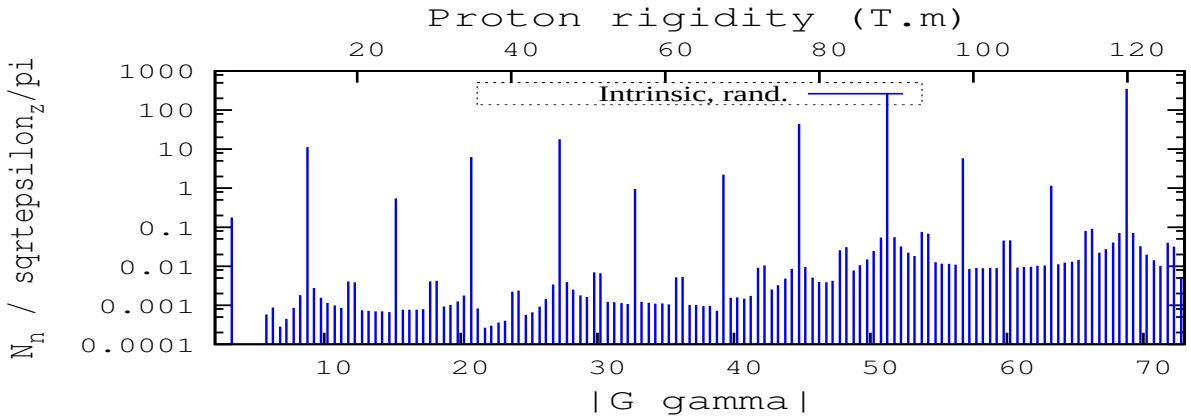
Proton

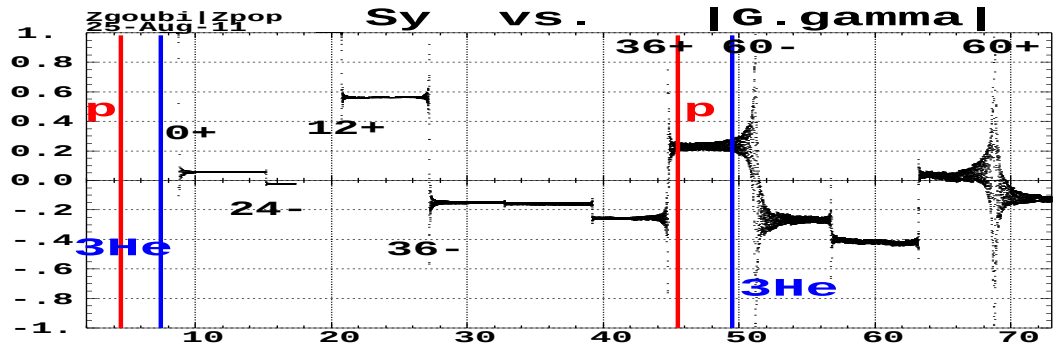


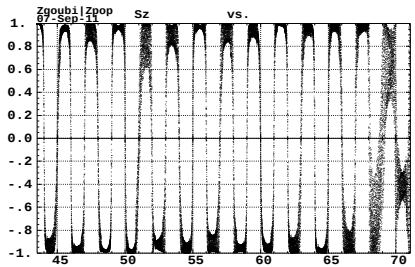
^{3}**He**$^{2+}$**, tentative, snakes unchanged : 15%+25%**



5.4 Resonances in AGS, intrinsic, $G\gamma \pm Q_y - n = 0$:







- Proton, acceleration range :
$G\gamma$: $4.5 \rightarrow 45.5$,
4 strong resonances crossed.

- $^3He^{2+}$, possible range :
$G\gamma$: $-7.5 \rightarrow -49.5$,
4 strong resonances crossed.

- **Top : Strengths of intrinsic resonances in AGS, thin-lens model.**
- **Middle : S_y motion, proton on OCO with $\beta\gamma\epsilon_y \approx 2\,\pi$ mm.mrad.**

Vertical lines show $G\gamma$ ranges for p and $^3He^{2+}$.

- **Bottom : $Q_y \approx 8.97$ is in forbidden Q_s band, hence avoiding intrinsic resonances.**

$\epsilon_x \approx 15\,\pi$**mm.mrad norm. (to excite $G\gamma \pm Q_x - n = 0$), $\epsilon_y \approx 2\,\pi$mm.mrad norm., $dp/p = 1.5\,10^{-4}$ peak.**

5.5 $^3He^{2+}$ resonance strengths

• **All stronger for $^3He^{2+}$ compared to protons :**

At given $G\gamma$ and given optics, identical *normalized* vertical emittance, taking $v/c = 1$, then intrinsic resonances are stronger in the ratio

$$\frac{|N_{n,^3He^{2+}}|}{|N_{n,p}|} = \sqrt{\frac{|G_{^3He^{2+}}|}{G_p}} \approx \sqrt{2.334} \approx 1.5$$

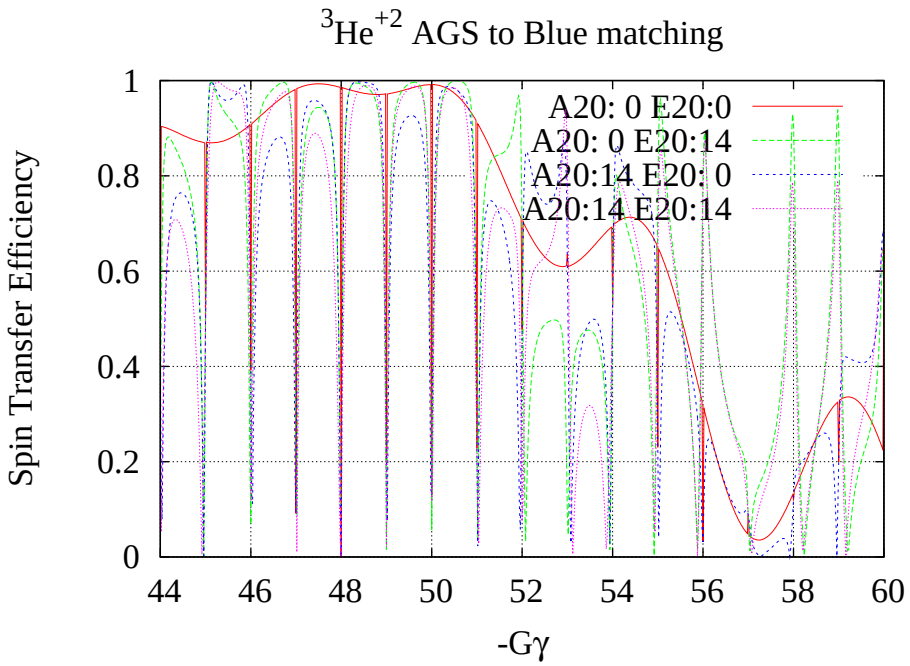
• **Strengths of strongest intrinsic resonances :**

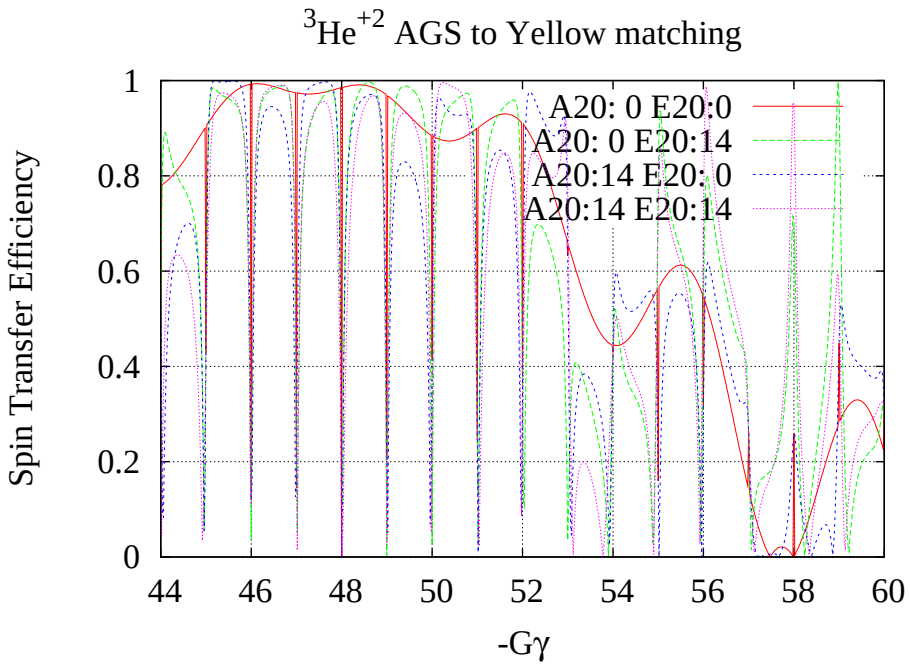
					Beyond 36+		
$G\gamma$	0+Q_y	12+Q_y	36-	36+	60-	48+	
$N_n^2/\epsilon_y/\pi$ (Fig. ??)	127	39	321	1937	73379	34	
proton :							
N_n at $\epsilon_{y,norm.} = 2.5\,\pi$mm.mrad	0.008	0.003	0.007	0.014	0.080	0.002	$N_n \propto \sqrt{\epsilon_y}$
N_n at $\epsilon_{y,norm.} = 10\,\pi$mm.mrad	0.016	0.0058	0.015	0.028	0.16	0.003	
$^3He^{2+}$:							
N_n at $\epsilon_{y,norm.} = 2.5\,\pi$mm.mrad	0.013	0.005	0.011	0.021	0.123	0.003	
N_n at $\epsilon_{y,norm.} = 10\,\pi$mm.mrad	0.025	0.009	0.023	0.042	0.25	0.006	

• **All are harmful -** $N_n \approx 0.004$ **results in** $\frac{P_f}{P_i} = \frac{1-\pi|N_n|^2/\alpha}{1+\pi|N_n|^2/\alpha} \longrightarrow 0$

5.6 Discussion

- **AtR : Hypotheses in Ref. W.MacKay, CA/AP/296 :**
  (i) Hyp. #1 : Convenient spin match at injection into RHIC obtained at $G\gamma = 48.5,\ 49.5$:





- **Can be improved ?**
  - check effect of varying warm-snake, on extraction plateau, used as a matching variable.

  - Use AGS solenoid snake as matching variable ? [Ref. TUPLS125, EPAC2006].

(ii) Hyp. #2 : 51=60-Qy is strong, hence recommending extracting below,

- **To be investigated in case of ^{3}He^{2+} given that snakes are stronger**
- **Further : (i) cold snake now is 70%, can be pushed up, (ii) investigate whether warm snake can be increased (from viewpoints of power supply, saturation, as well as particle and spin optics)**
- **Inserting a rotator in AtR, e.g.,**

  - **Horizontal bump encompassing first vertical dipole (N. Tsoupas)**

  - **Solenoid or helix**

might allow thus mandatory substantial change in matching conditions.

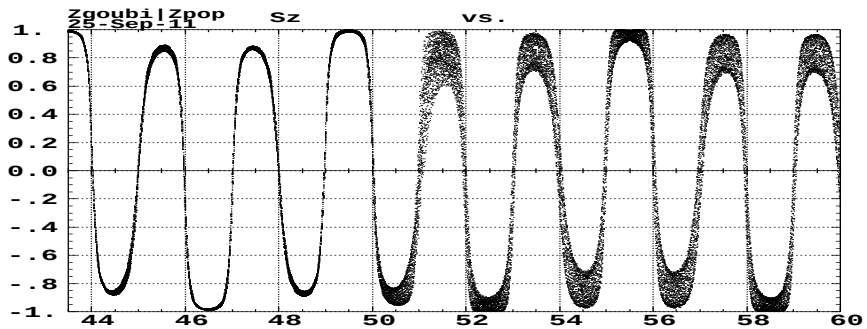
(iii) Hyp. #3 : Low-rigidity injection in RHIC (≈50 T.m, compared to 79, $G\gamma$ =45.5 proton) doable.

RHIC γ_{tr} ≈23.6,
- versus proton $\gamma = 25.378$ at $G\gamma = 45.5$,
- whereas ^{3}He^{2+} $\gamma = 11.71 < \gamma_{tr}$ at $G\gamma = 49$.

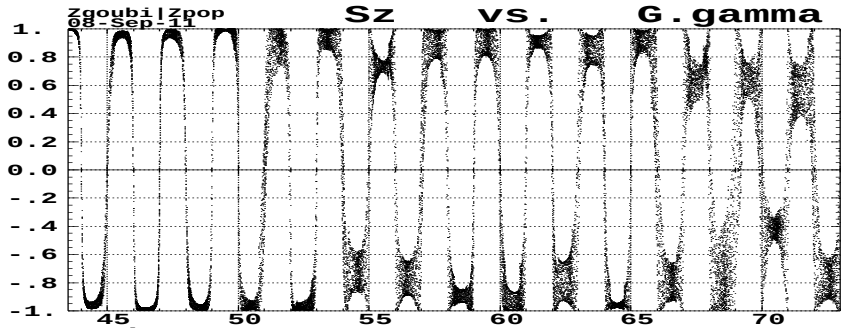
- **Could be increased if $G\gamma = 60 - Q_y$ can be Xed.**

5.7 Tentative tracking : Xing 60-Qy

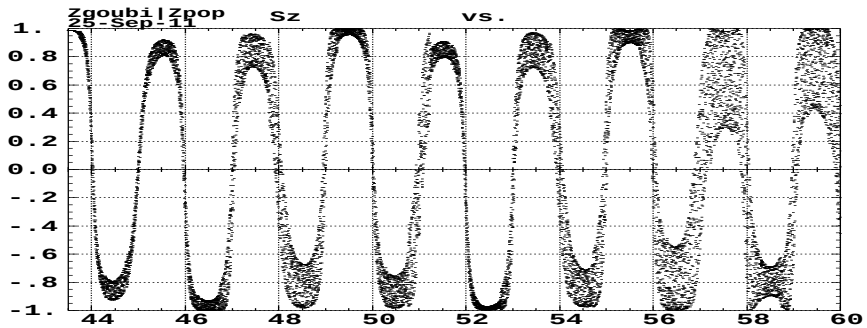
- Snakes have their nominal field, 2.1 T and 1.5 T,
- Optics : Qx/Qy 8.72 / 8.97. Orbits are $H \ll 1$ mm, $V \approx 1$ mm peak. Jump quads off,
- For reference : zero emittances case is that shown on slide 13.



^{3}He^{2+}. $\epsilon_x \approx 0$, $\epsilon_y \approx 1.25\,\pi$mm.mrad norm., $dp/p \in [-10^{-4}, +10^{-4}]$.



Proton, for comparison. $\epsilon_x \approx 15\,\pi$ norm., $\epsilon_y \approx 2\,\pi$ norm., $dp/p \in [-10^{-3}, +10^{-3}]$.



^{3}He^{2+}. $\epsilon_x \approx 1.25\pi$, ϵ_y very small, $dp/p \in [-10^{-4}, +10^{-4}]$.

- Depolarization may result from betatron coupling [Ref.H.Huang et al, PAC97]. Need investigate with simulations including coupling correction.
- Worth exploring increased snake strength : cold snake is 70%, warm snake is 1.5 T with 2600 Amp in 0.5 Tesla/kAmp working region.

6 AGS snakes

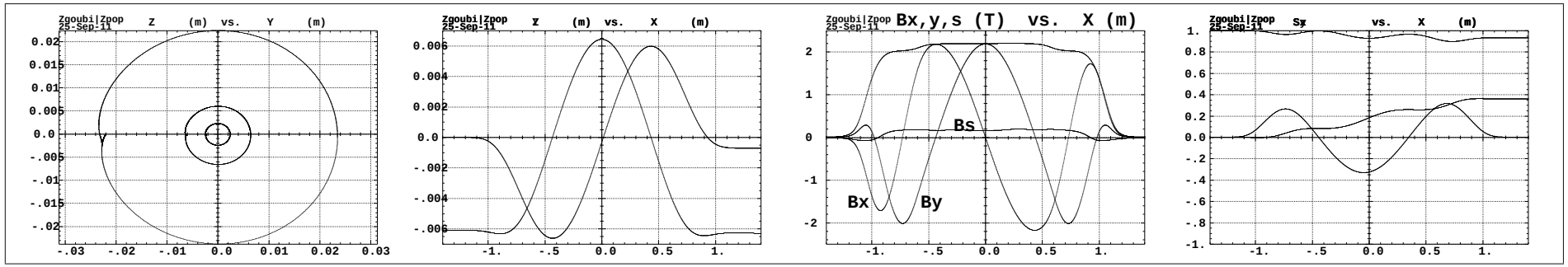
- **A symmetric sequence of 3 sections with 2 different pitches, at ends and in body**

	Cold snake	Warm snake	
mag. length (m)	2	2	
pitches, (deg/m)	420 - 208 - 420	400 - 195 - 400	
max field (T)	3	?	
operating field (T)	2.1	1.5	
spin rotation (deg)	15 - 11	27	proton
% snake, proton	≈ 6	≈ 15	
coupling compensation	solenoid	none	$\sim 0.2\,T$, *weak effect on spin*
compensation quads	A17,A19 + B1,B3	E17,E19 + F1,F3	
operated since	2005	2004	

Snake properties (1/2), cold snake

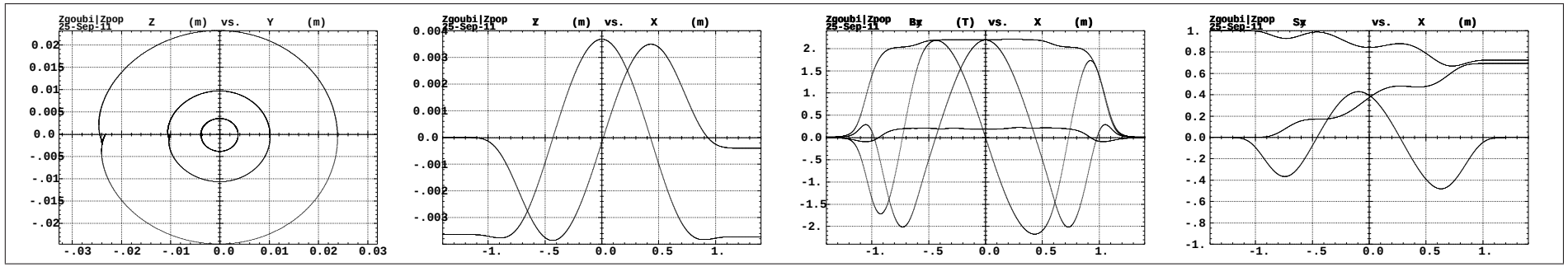
- The optical axis of the snake helix coincides with the reference orbit in the ring ("OCO").
- The beam horizontal orbit follows a bump with energy-dependent amplitude that centers the beam helix on snake axis. Shift and kick at traversal are small.

Cold snake, proton



From left to right : Projection of proton motion on transverse plane ($G\gamma = 4.5, 15, 40$), and ($G\gamma = 15$) on (x,s), (y,s) planes ; field components along trajectory ; motion of spin components.

Cold snake, $^3He^{2+}$

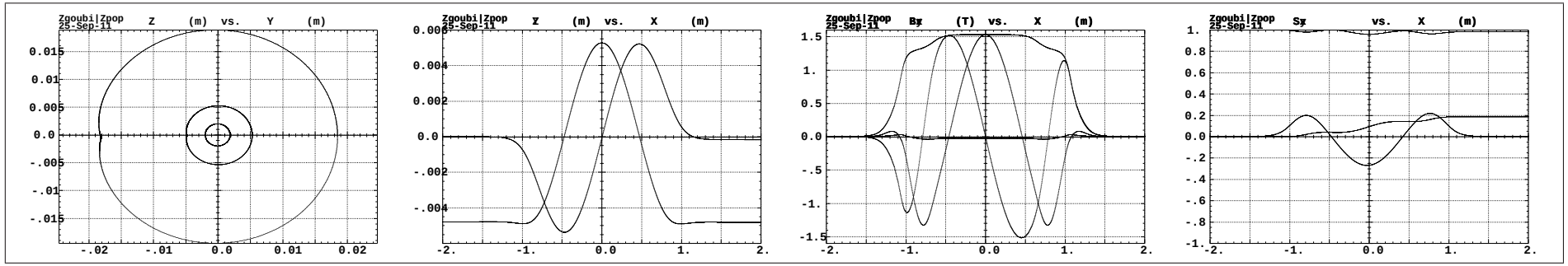


From left to right : Projection of $^3He^{2+}$ motion on transverse plane ($G\gamma = -7.5, -15, -40$), and ($G\gamma = 40$) on (x,s), (y,s) planes ; field components along trajectory ; motion of spin components.

Snake properties (2/2), warm snake

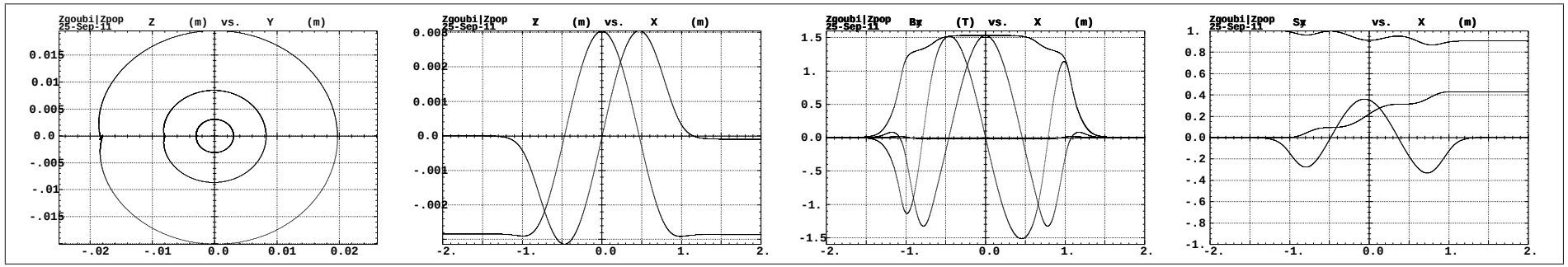
- **Snakes' axes are on OCO.**
- **The horizontal orbit is bumped so to center beam helix on snake axis, shift and kick at traversal are small.**

Warm snake, proton



From left to right : Projection of proton motion on transverse plane ($G\gamma = 4.5, 15, 40$), and ($G\gamma = 15$) on (x,s), (y,s) planes ; field components along trajectory ; motion of spin components.

Warm snake, $^3He^{2+}$



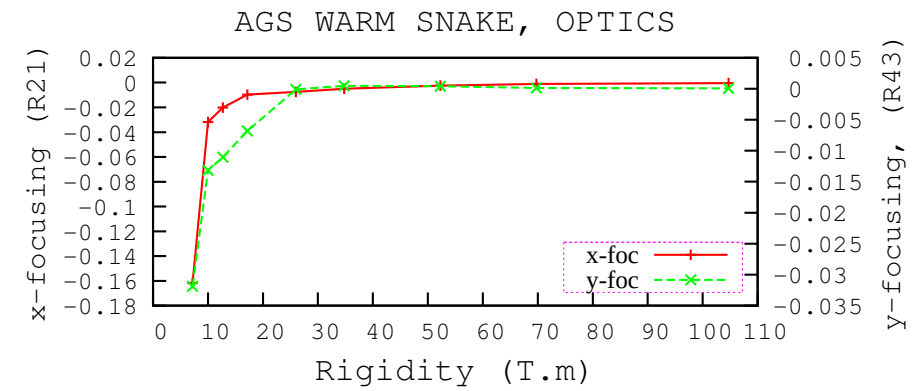
From left to right : Projection of $^3He^{2+}$ motion on transverse plane ($G\gamma = -7.5, -15, -40$), and ($G\gamma = 40$) on (x,s), (y,s) planes ; field components along trajectory ; motion of spin components.

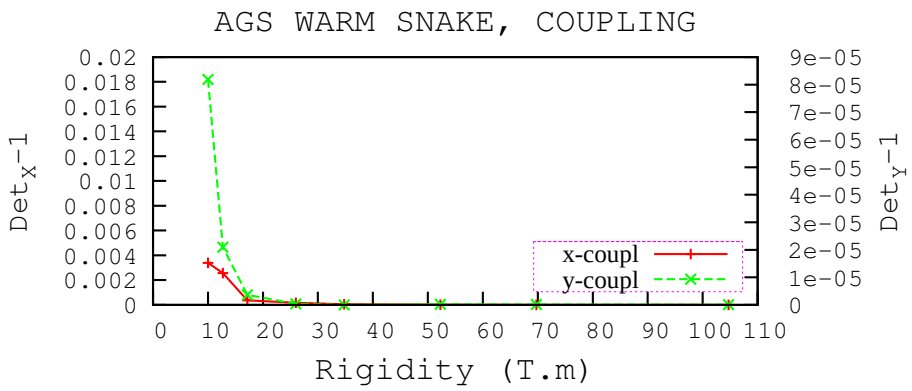
• Optical properties

- Both snakes introduce focusing and coupling at low energy.

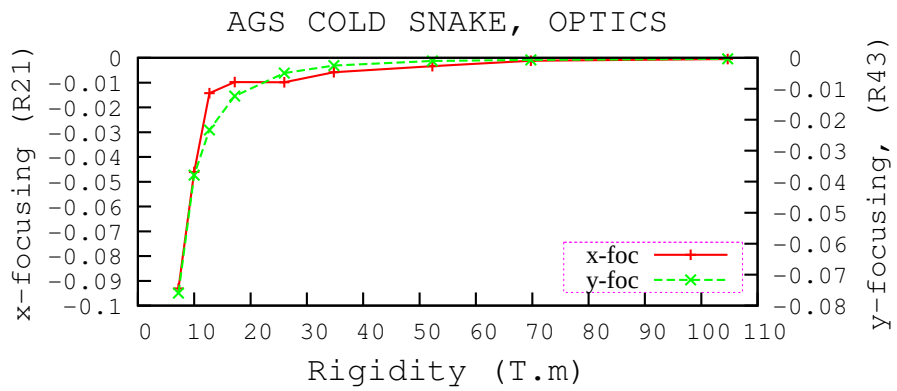
- $^3He^{2+}$ acceleration range covers less favorable region : $\begin{cases} \textbf{proton,} & B\rho\ : 7.2 \rightarrow 79\ \textbf{T.m} \quad (G\gamma = 45.5) \\ \textbf{$^3He^{2+}$,} & B\rho\ : 6.9 \rightarrow 55\ \textbf{T.m} \quad (G\gamma = 49.5) \end{cases}$

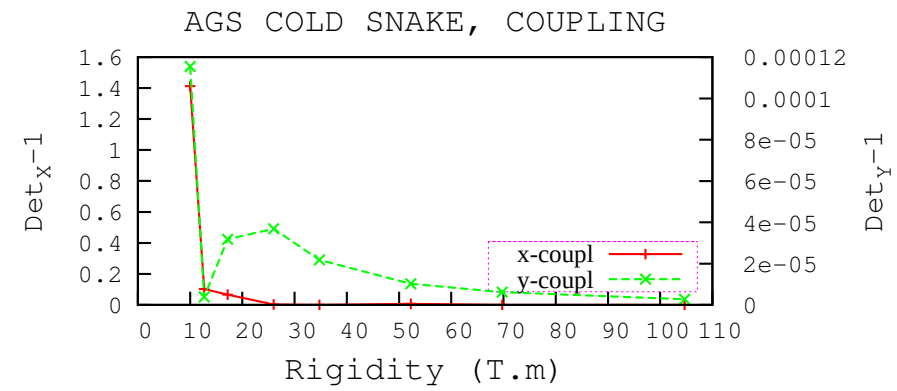
WARM SNAKE





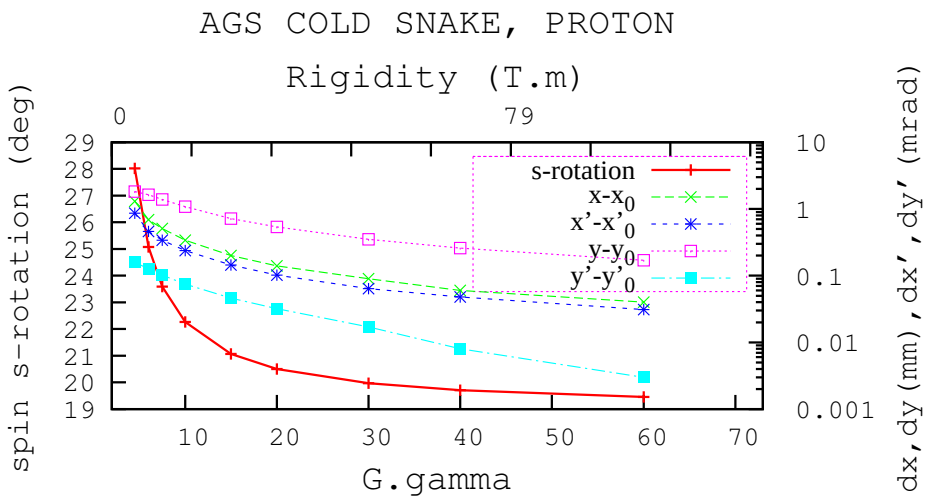
COLD SNAKE

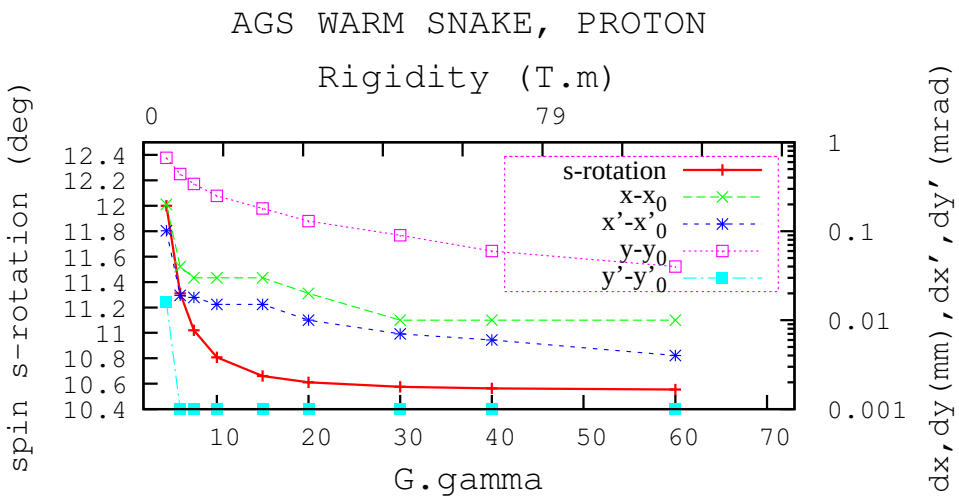




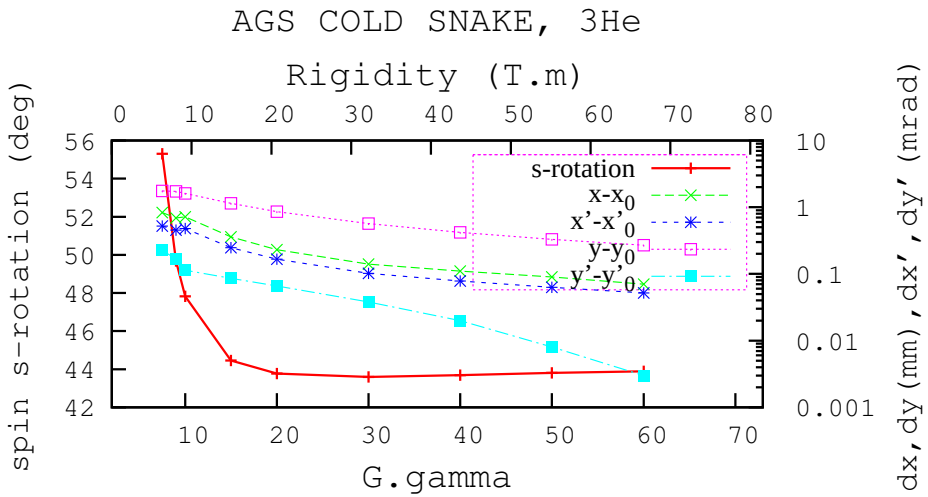
• Orbit effects, spin rotation, versus $G\gamma$

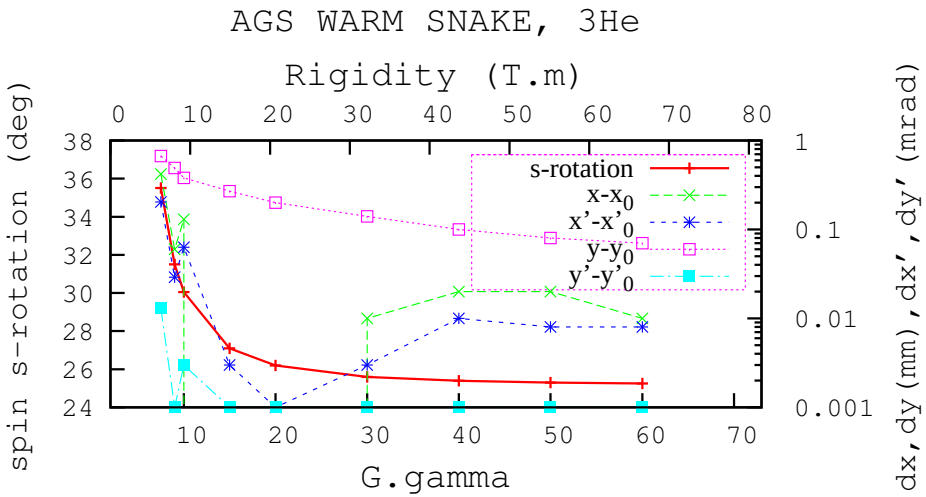
Proton





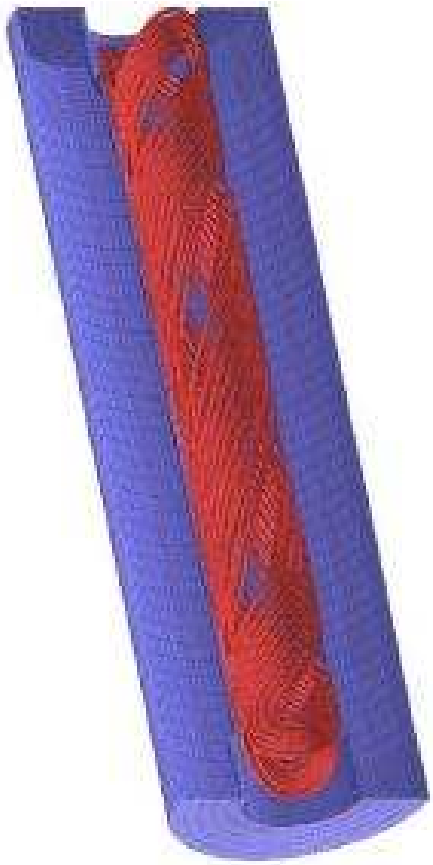
$^3He^{2+}$





• **Note : Warm snake is operated at $\sim$2600 Amp. [Ref.J.Takano,EPAC2004], a region of 0.5 Tesla/kA. Increasing the angle $\rightarrow$ matter of (i) power supply, (ii) saturation effects.**

THANK YOU



7 Possible plans

A non-exhaustive list...

- **BOOSTER**

(i) BD studies relating to avoiding 0+, 12-, with protons, e.g. run Booster with Q close to $4\pm$

(ii) Higher $B\rho$**. in view of higher** $G\gamma$ **extraction : investigate possibility of** $B\rho \rightarrow$ ***max.***$B\rho$

(iii) Investigate resonance Xing : slow acceleration through 0+Qy, 12-Qy

(iv) Investigate AC dipole

- **AGS**

(i) Investigate possibility/interest of ramping warm snake ($\pm$**few kG) in 1.5 T region for AtR matching**

(ii) Investigate using warm snake at higher field

- **BtA matching, AtR matching...**