Electromagnetic currents induced by color fields

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Outline

- Introduction:
  photon puzzle and photon production in glasma

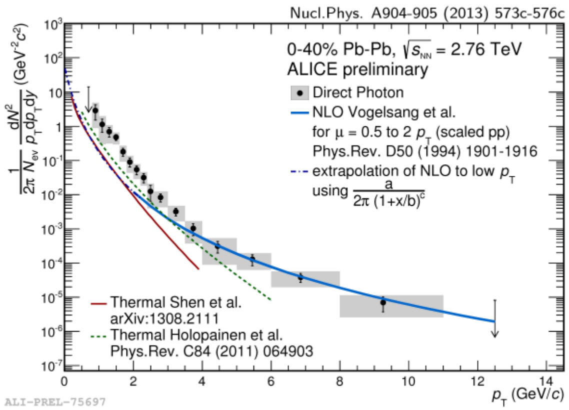
- Electromagnetic current:
  an important building block to compute photon spectra

- EM current in uniform color electric fields
  - Abelianization
  - SU(2) vs. SU(3)
  - Color direction dependence

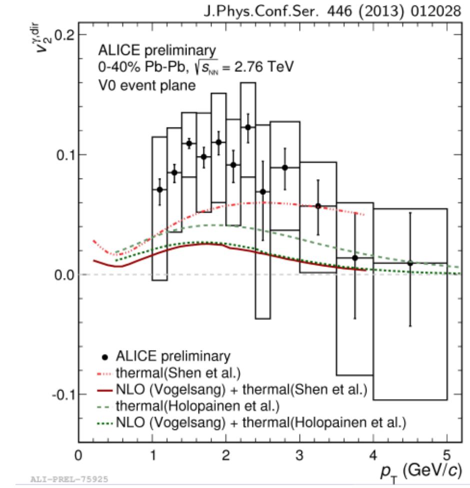
- Inhomogeneous color fields

Photon puzzle

Direct photon excess

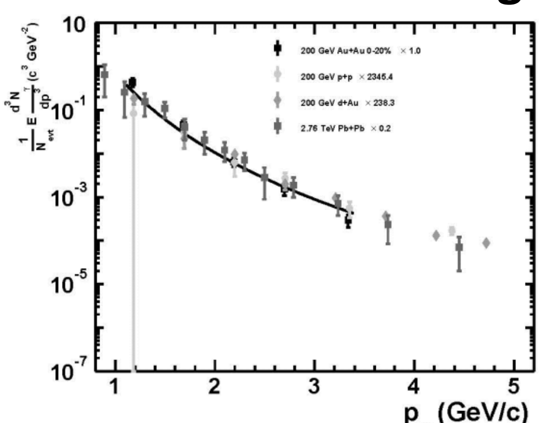


Large photon v2



Hydrodynamic models fail to describe simultaneously photon yield, temperature and v2.

Geometrical scaling



C.K-Boesing, L.McLerran (2014)

photon production in pre-equilibrium?

Quark production in glasma

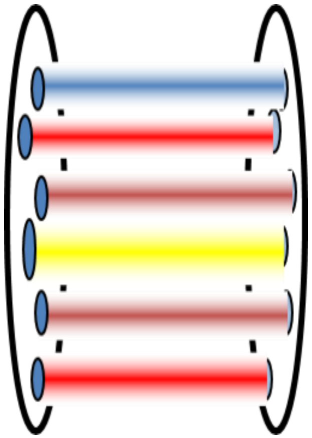
Glasma gauge fields produce quarks

⬇

Quarks are accelerated or kicked by the gauge fields

⬇

Chemical and thermal equilibration?



$E_z^a \ , \ B_z^a$

Glasma flux tubes

can be computed by real-time lattice simulations
with the classical(-statistical) approximation of gauge fields

N.T. and Gelis, in progress
Kasper, Hebenstreit and Berges (2014)

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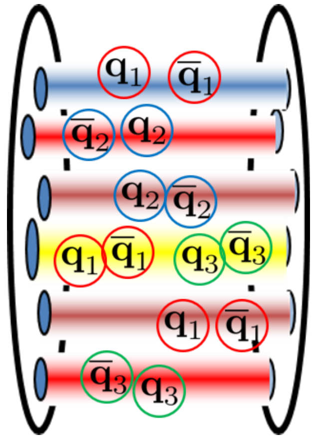

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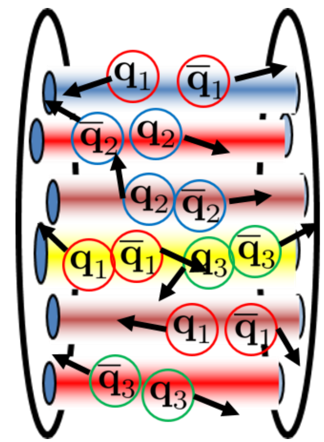

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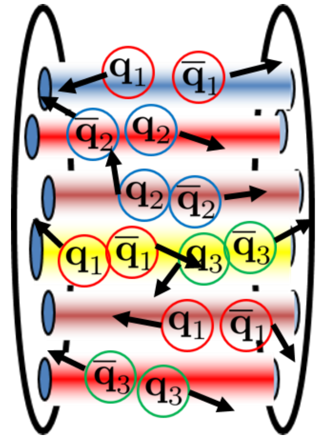
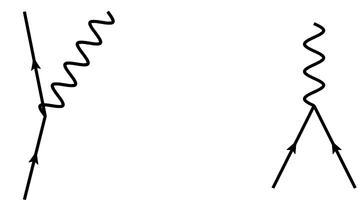


E_z^a , B_z^a

Glasma flux tubes

During these processes, quarks can emit photons.



radiation annihilation

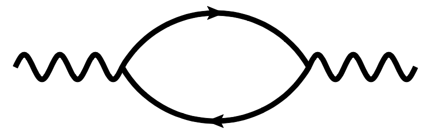
Can we see glasma by photons?

Photon production formula

In thermal equilibrium,

McLerran and Toimela (85),
Weldon (90), Gale and Kapsta(91)

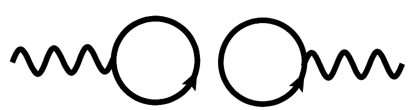
$$\omega \frac{dR}{d^3k} = -\frac{g^{\mu\nu}}{2(2\pi)^3} \int d^4x\, e^{ik\cdot x} \langle J_\mu(0) J_\nu(x) \rangle_\beta$$



Extension to non-equilibrium….

One of characteristic features of a non-equilibrium state is nonzero current expectation.

$$\langle J_\mu(x) \rangle = e \langle \overline{\psi}(x) \gamma_\mu \psi(x) \rangle \neq 0$$



Gives the same order contribution in α as the connected one-loop

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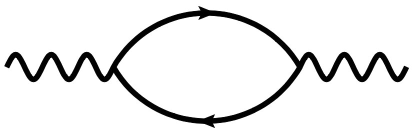

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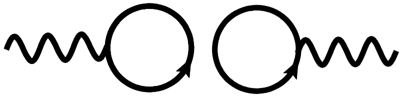

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Abelianization of color fields

G.C.Nayak (2005)
N.T. (2010)

$$F^a_{\mu\nu}(x) = F_{\mu\nu}(x) n^a \quad \Longleftarrow \quad A^a_\mu(x) = A_\mu(x) n^a$$

constant vector in color space

Diagonalize $n^a T^a$

$$D_\mu \psi = [\partial_\mu + igA_\mu n^a T^a]\,\psi \longrightarrow \left[\partial_\mu + igA_\mu \begin{pmatrix} w_1 & & \\ & w_2 & \\ & & w_3 \end{pmatrix}\right] \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}$$

U(1) theory with effective coupling $w_c g$

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U(1) theory with effective coupling $w_c g$

- An important difference between SU(2) and SU(3)

  SU(2) is rank 1: $U^\dagger n^aT^aU = T^3$

  SU(3) is rank 2: $U^\dagger n^aT^aU = T^3\cos\theta - T^8\sin\theta$

color direction parameter

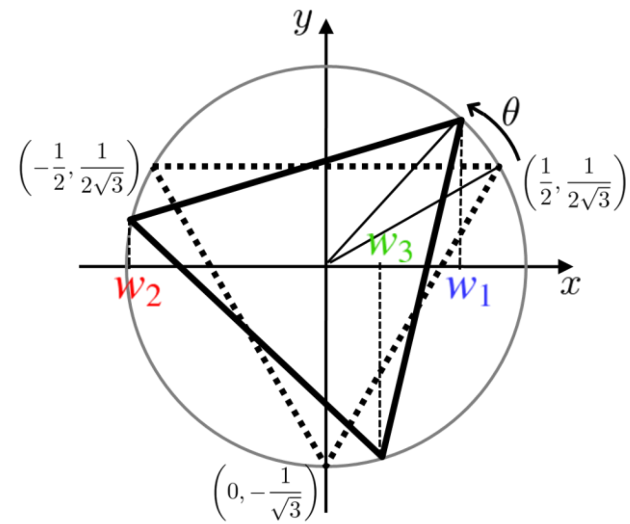
Abelianization of SU(3) fields

$$U^\dagger n^a T^a U = T^3 \cos\theta - T^8 \sin\theta = \frac{1}{\sqrt{3}} \begin{pmatrix} \cos(\theta+\frac{\pi}{6}) & & \\ & \cos(\theta+\frac{5\pi}{6}) & \\ & & \sin\theta \end{pmatrix}$$

Relation between θ and n^a

$$\sin^2 3\theta = 3(d^{abc} n^a n^b n^c)^2$$

gauge invariant quantity (Casimir invariant)
characterizing the color direction



rotated weight diagram

The color direction can be parametrized in a gauge-invariant way.

Physical observables can depend on it.

Quark production in SU(2) uniform electric fields

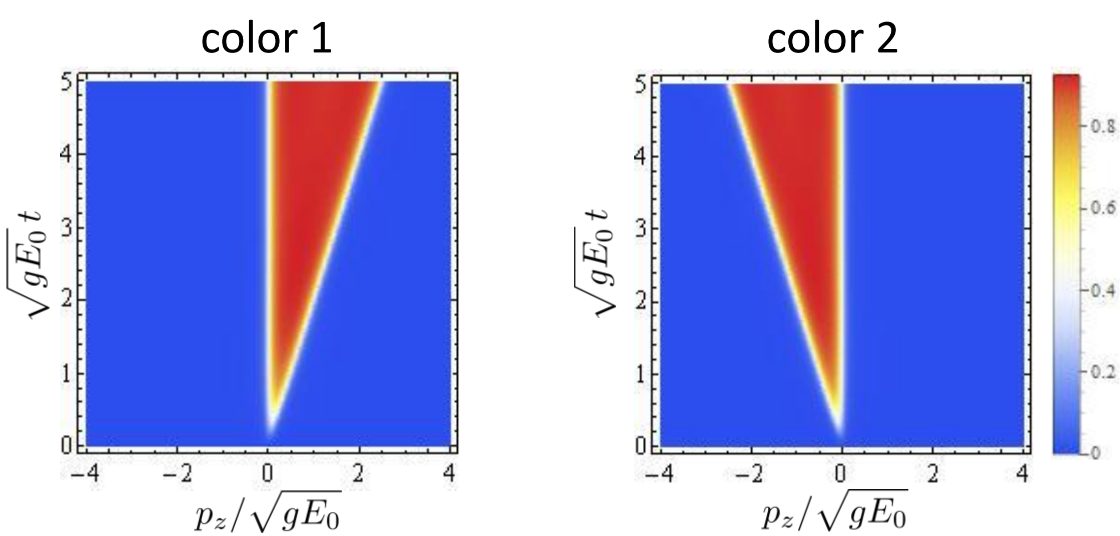
$$D_\mu \psi = [\partial_\mu + igA_\mu n^a T^a]\,\psi \longrightarrow \left[\partial_\mu + igA_\mu \begin{pmatrix} 1/2 & \\ & -1/2 \end{pmatrix}\right] \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

The diagonalized effective couplings are always 1/2 and -1/2.

Uniform and constant electric field $E_z^a = E_0 n^a$

strong field classical limit $gE_0 = \text{const},\ g \to 0$

no gauge field fluctuations
no backreaction



$m/\sqrt{gE_0} = 0.1$

The distribution functions of produced quarks

The distributions of anti-particle is given by $p \leftrightarrow -p$

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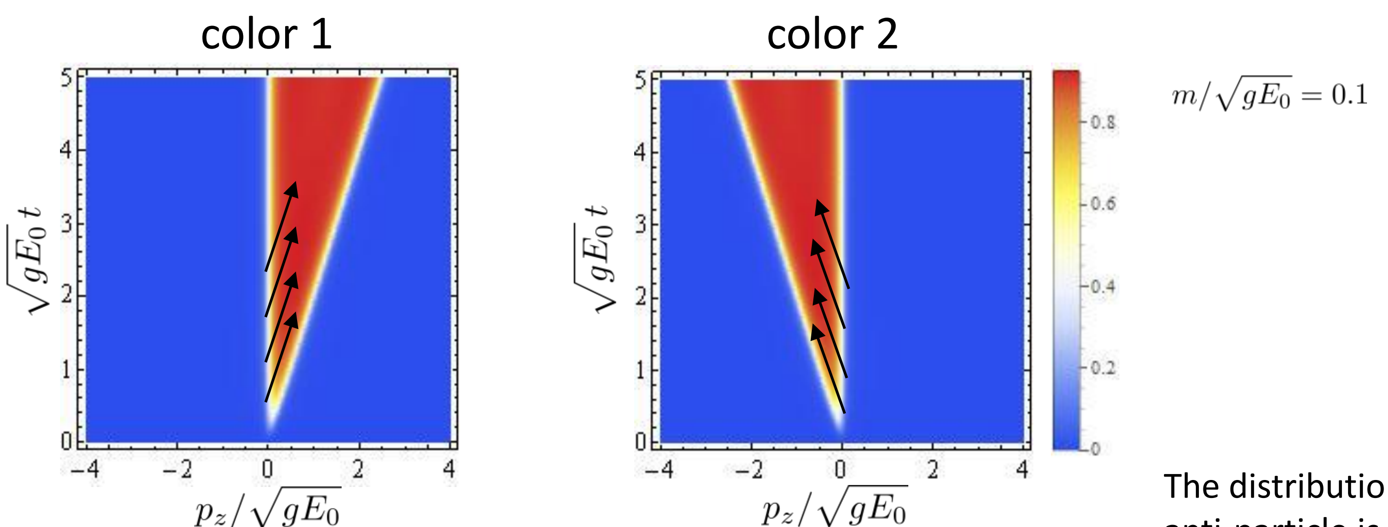

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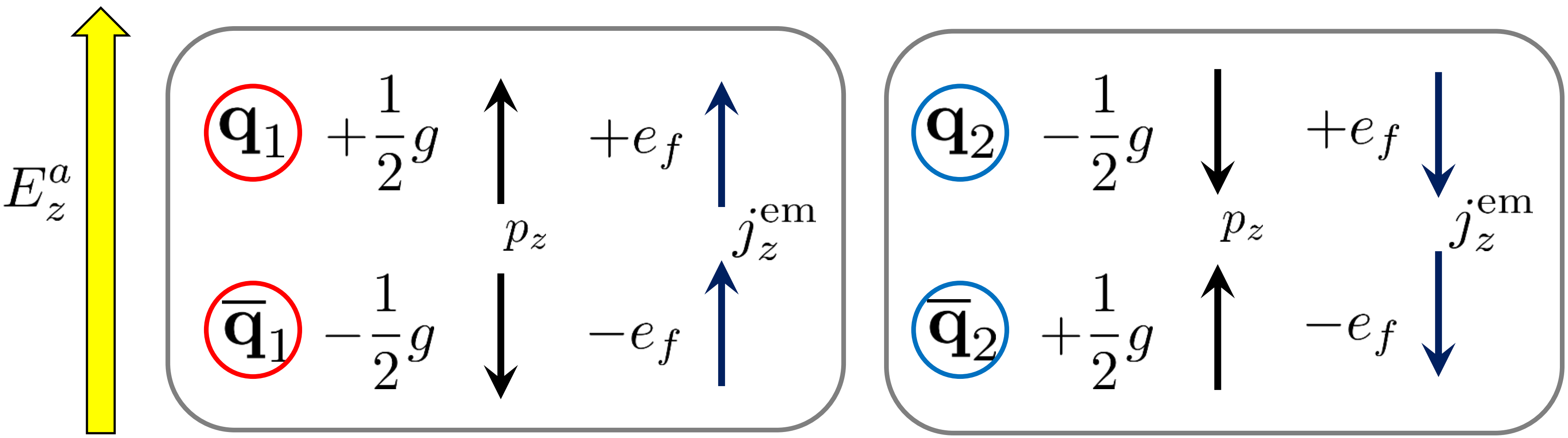
Cancellation of EM current in SU(2) fields



The contributions from color 1 and 2 are cancelled out.

$$J_z^{\mathrm{EM}} \simeq 4e_f \sum_c \int \frac{d^3p}{(2\pi)^3} \frac{p_z}{\omega_p} f_c(t,p)$$

In the case of the Schwinger mechanism,

$$f_c(t,p) \simeq e^{-\pi \frac{m^2+p_\perp^2}{|w_c g E|}} \theta(0 < p_z < w_c g E t)$$

Quark production in SU(3) uniform electric fields

N.T. (2010)

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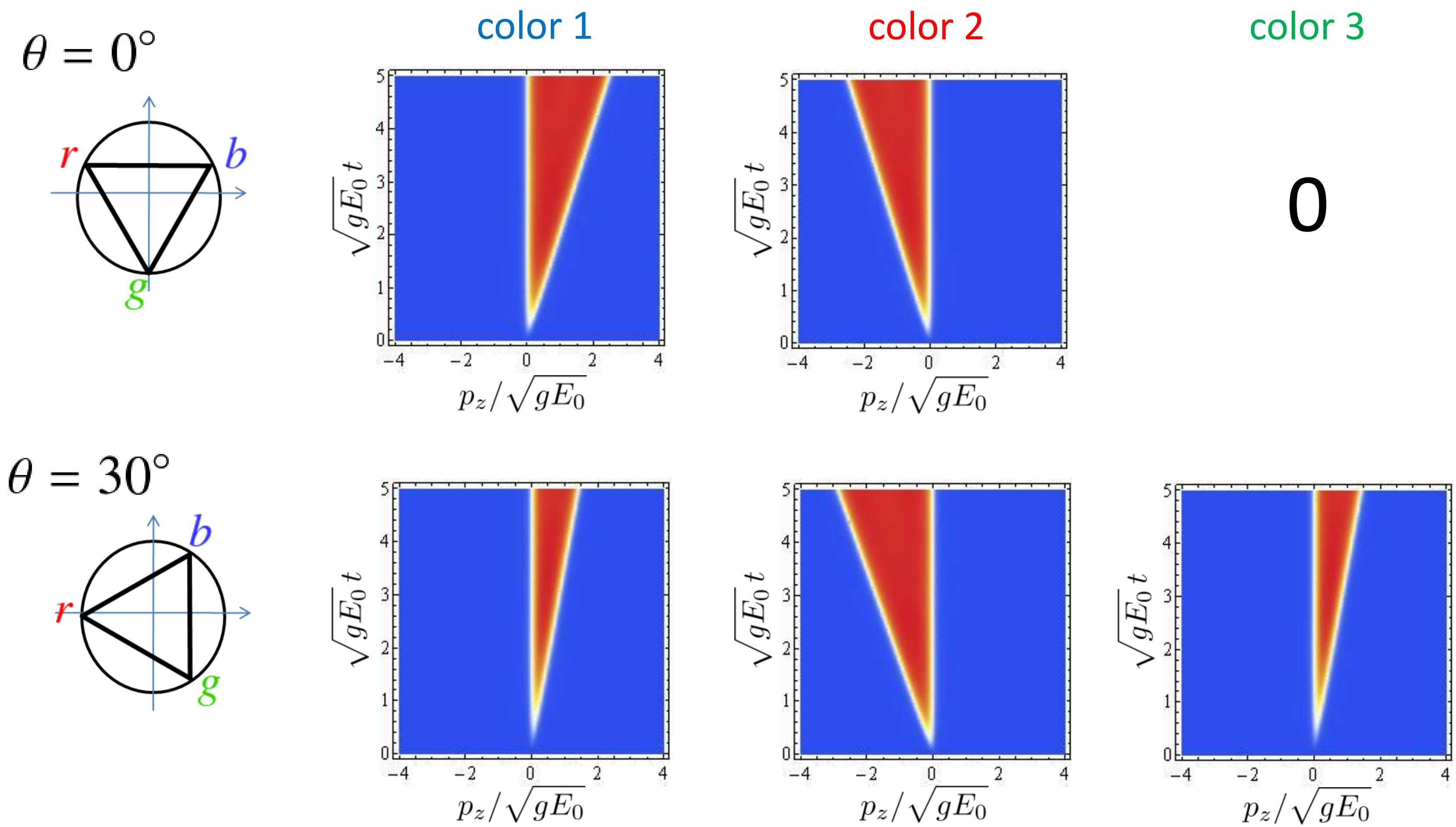

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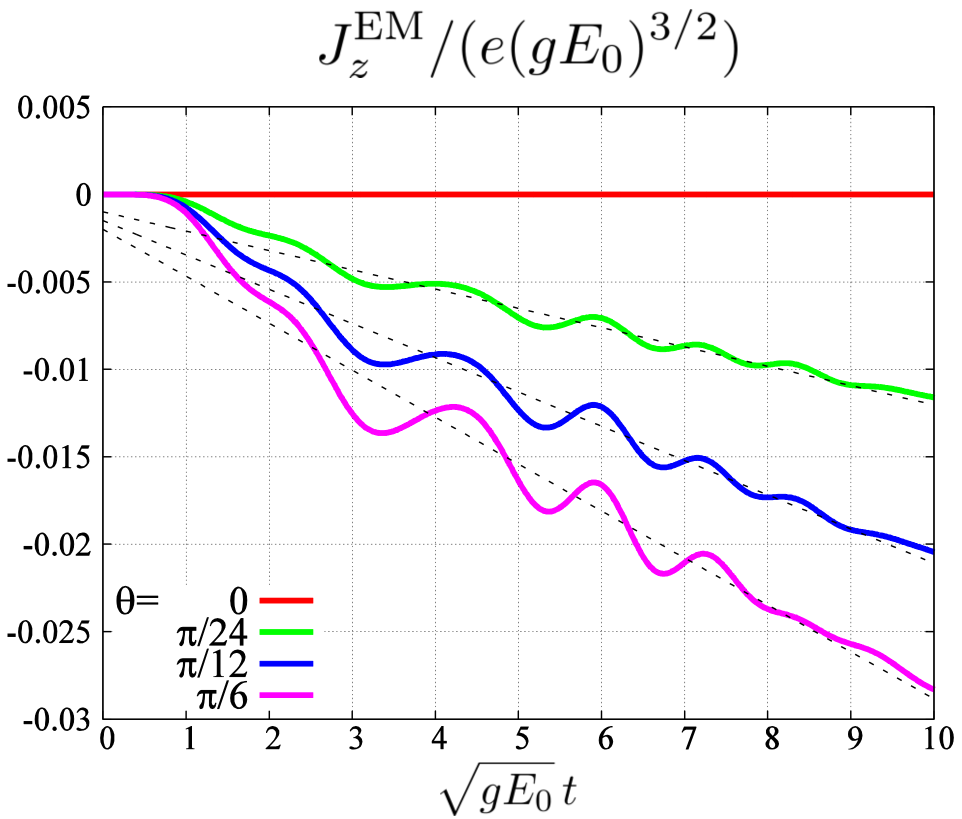
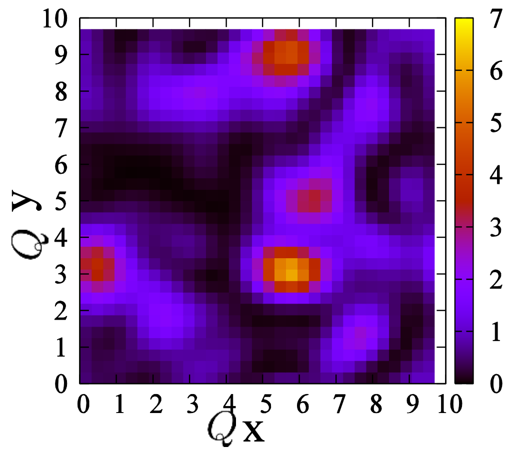
Non-cancellation of EM current in SU(3) fields



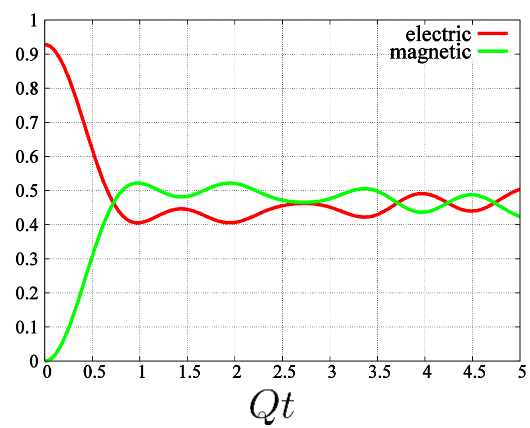
$$J_z^{\rm EM} \simeq \frac{4e_f}{(2\pi)^3}\sum_c w_c|w_c|e^{-\pi\frac{m^2}{|w_c|gE_0}}(gE_0)^2 t$$

Inhomogeneous color electric fields: SU(2)

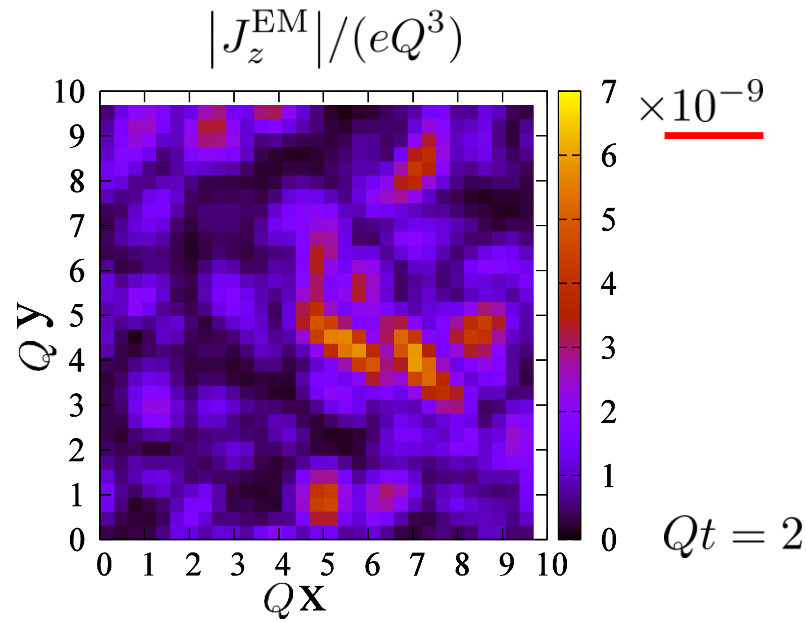


initial energy density of gauge field

uniform in z
randomly distributed in the transverse plane
a scale Q

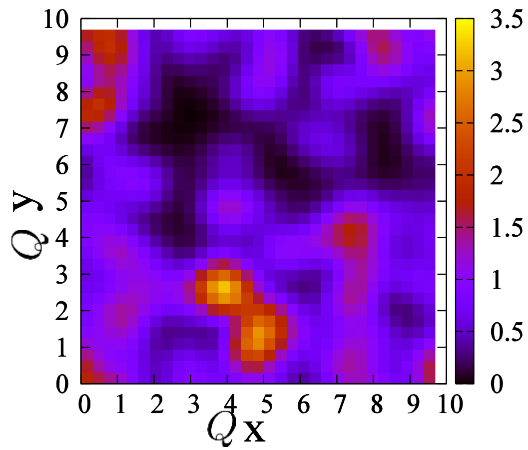


time-evolution of space-averaged energy density



a snapshot of the EM current

Inhomogeneous color electric fields: SU(3)



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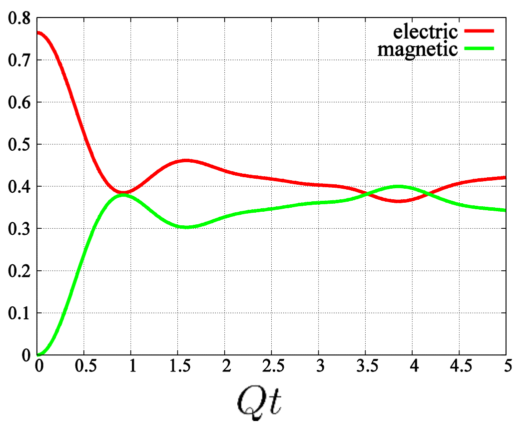

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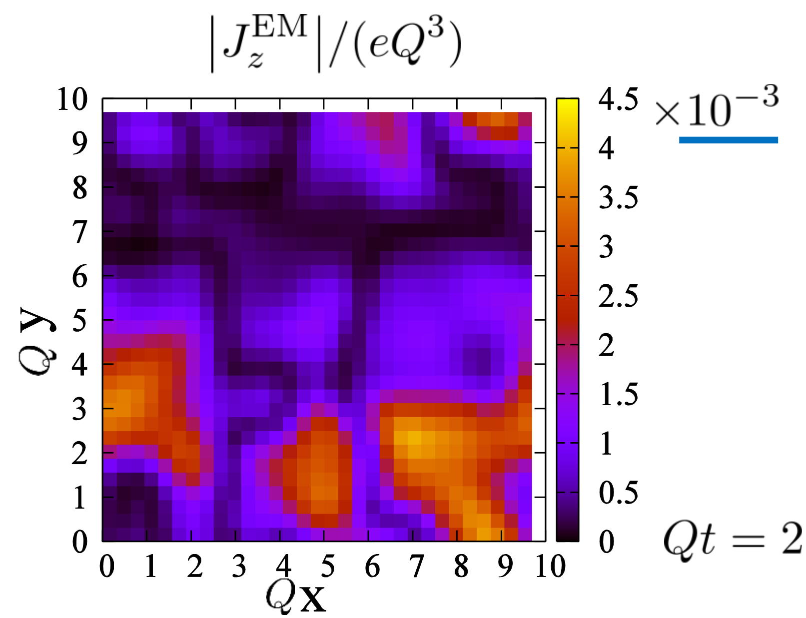

a snapshot of the EM current

Summary and Outlook

- Investigated EM currents induced by color fields as a first step to study photon production in glasma.
- In SU(2) uniform fields, EM currents are not at all induced because of the cancellation between two color components.
- In SU(3) uniform fields, the cancellation is not perfect. The EM currents exist depending on the color direction of the background field.
- In inhomogeneous color fields, SU(2) and SU(3) give quantitatively different results.

  - [ ] Quark production in glasma
  - [ ] Effects of gauge field fluctuations and backreaction
  - [ ] Photon production