Transverse Spin Physics

Lecture III

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The plan:

- **Lecture I:**

  Transverse spin structure of the nucleon
  Overview of past experiments
  History of interpretation
  Overview of present understanding

- **Lecture II**

  Transverse Momentum Dependent distributions (TMDs)
  Sivers function
  Twist-3

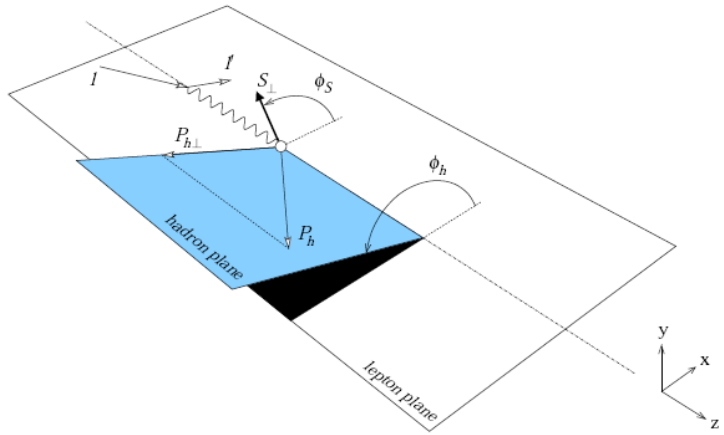
- **Lecture III**

  Transversity
  Collins Fragmentation Function
  Global analysis

- **Lecture IV**

  Evolution of TMDs

Semi Inclusive Deep Inelastic scattering



One can rewrite the cross-section in terms of **18** structure functions

Each structure function encodes parton dynamics via convolutions of TMDs when factorization is applicable

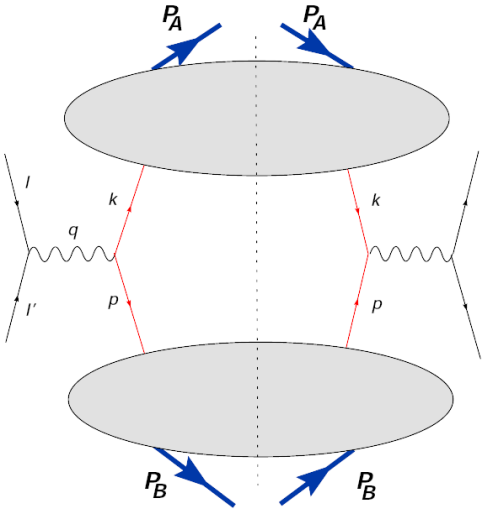
Mulders, Tangerman (1995),
Boer, Mulders (1998)
Bacchetta et al (2007)

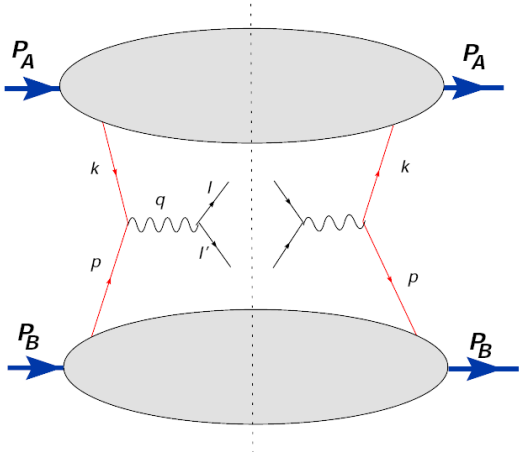
$$\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} =$$

$$\frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Bigg\{F_{UU,T}+\varepsilon F_{UU,L}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h}$$

$$+\,\varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}+\lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h}\;+\ldots$$

e+e- and Drell-Yan





One can rewrite the cross-section of e+e- in terms of **72** structure functions

Boer, Jacob, Mulders (1997),
Pitonyak, Schlegel, Metz (2013)

One can rewrite the cross-section of Drell-Yan in terms of **48** structure functions

Tangerman, Mulders (1995),
Boer (1999),
Arnold, Metz, Schlegel (2009)

TMD distributions

N \ q	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1 $h_{1T}^\perp$

8 functions in total (at leading twist)

Each represents different aspects of partonic structure

Each function is to be studied

Kotzinian (1995), Mulders, Tangerman (1995), Boer, Mulders (1998)

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Unpolarised distribution

transversity

Kotzinian (1995), Mulders, Tangerman (1995), Boer, Mulders (1998)

TMD Fragmentation Functions

N \ q	U	L	T
U	D_1		$H_1^\perp$
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8 functions describing fragmentation of a quark into spin ½ hadron

Mulders, Tangerman (1995), Meissner, Metz, Pitonyak (2010)

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8 functions describing fragmentation of a quark into spin ½ hadron

Collins FF

Unpolarised FF

Mulders, Tangerman (1995), Meissner, Metz, Pitonyak (2010)

Twist-2 collinear PDFs

Quark-quark correlator can be decomposed by means of
3 Parton Distributions Functions (PDF) in collinear (kt integrated) case

$$\Phi(x;P,S) = \frac{1}{2}\left\{ f_1(x)\ \not{P} + S_L g_1(x)\gamma_5\ \not{P} + \frac{1}{2}h_1(x)\gamma_5[\not{S}_T, \not{P}] \right\}$$

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Unpolarised PDF

Helicity distribution

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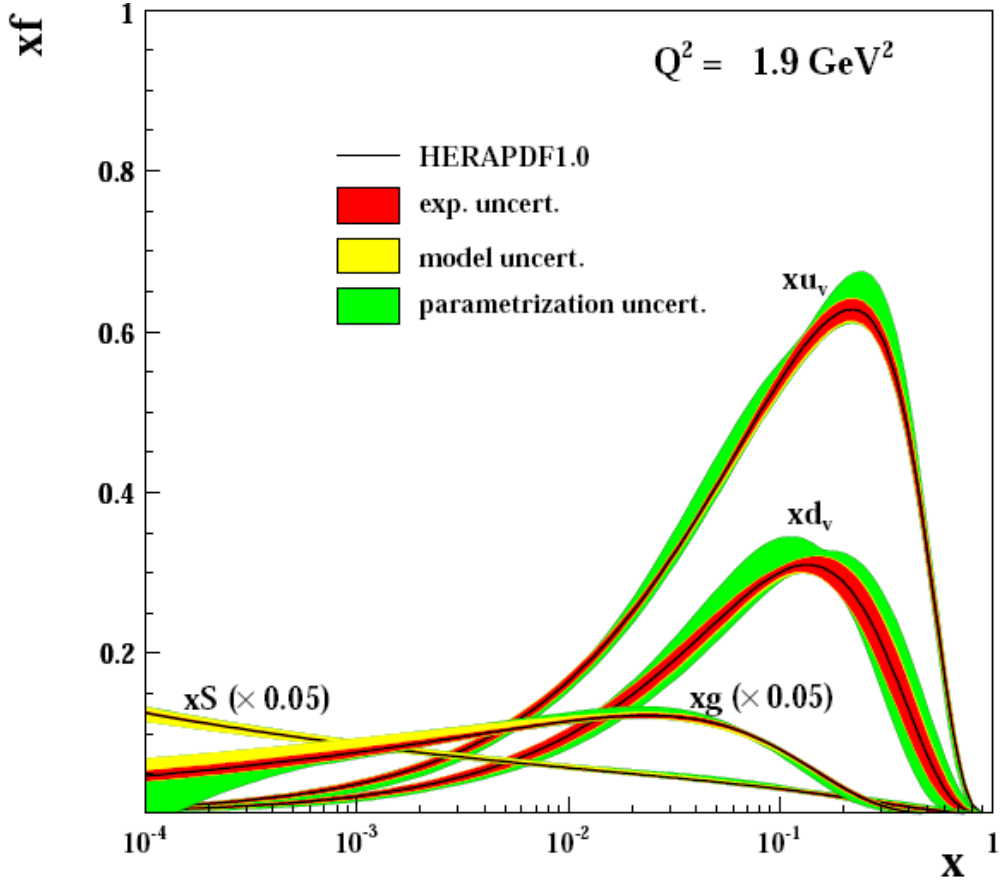
Unpolarised PDF

Helicity distribution

Transversity distribution

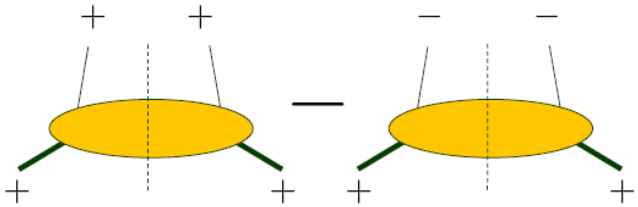
Unpolarised PDFs

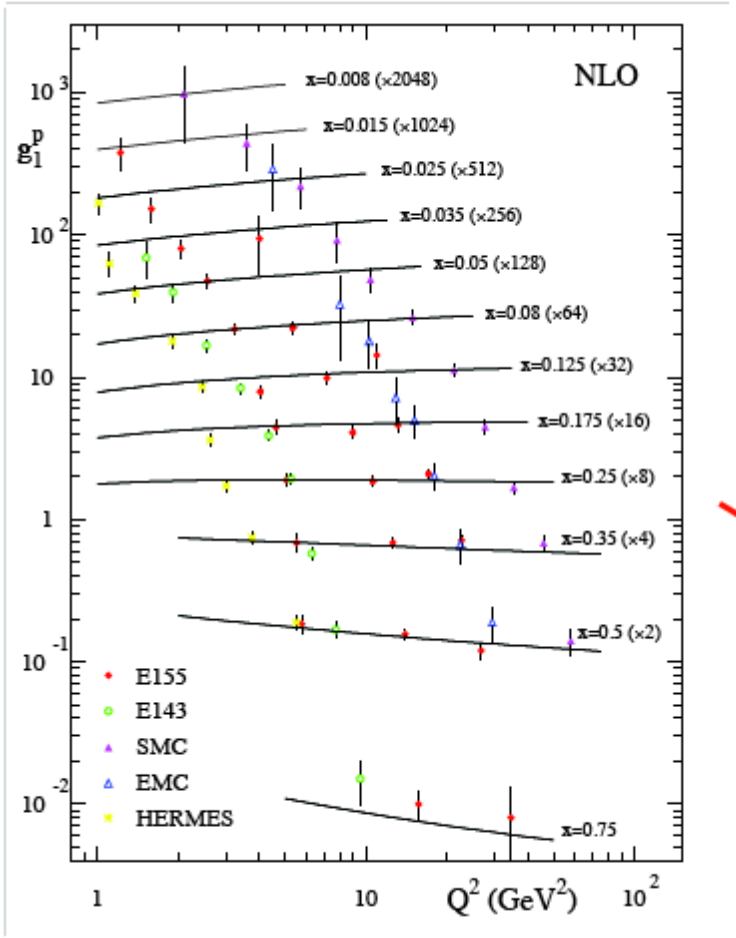
Good knowledge of unpolarised Parton Distribution Functions is acquired

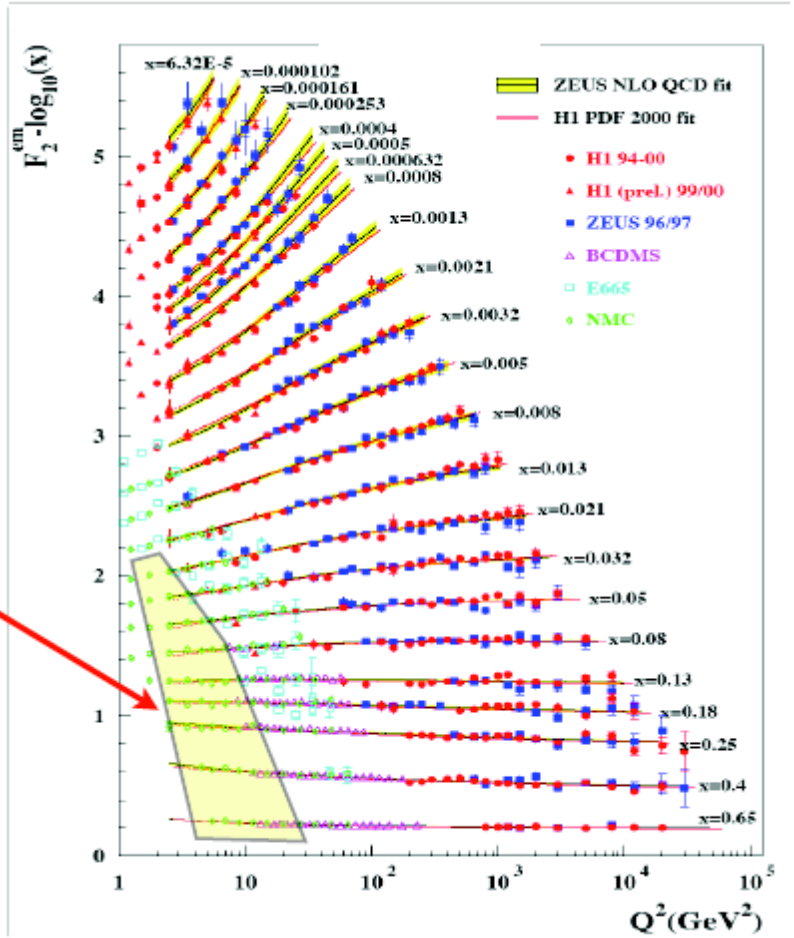


Helicity distributions

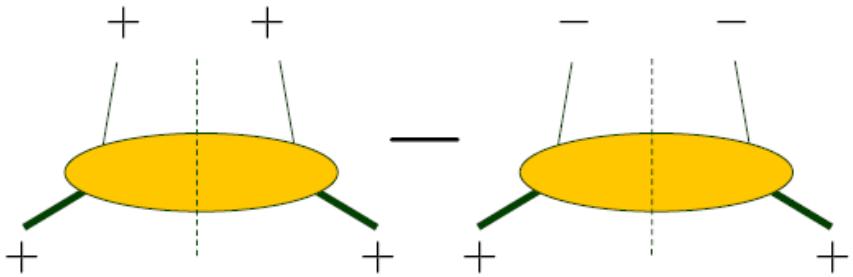
Helicity distributions
are relatively well known

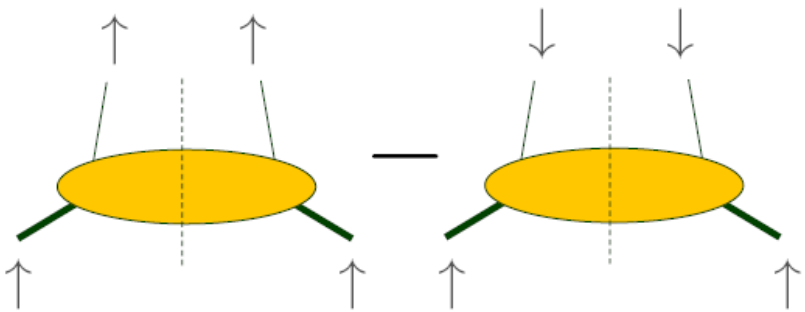






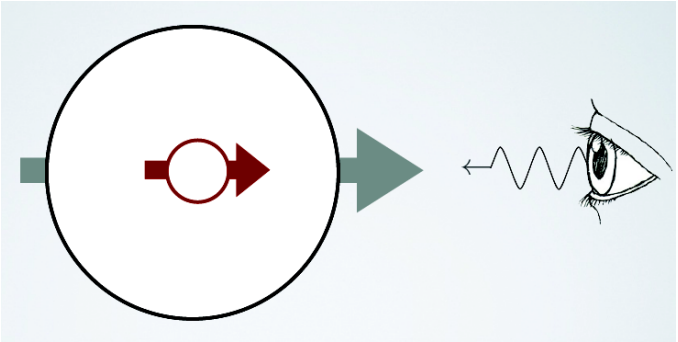
Helicity distribution Transversity distribution



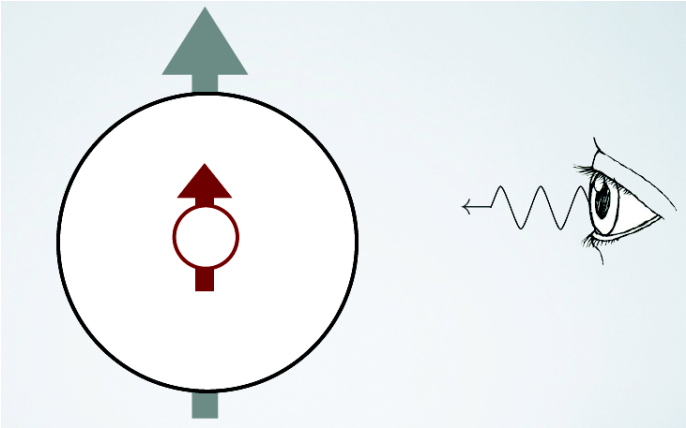


Distribution of transversely polarised quarks inside transversely polarised nucleon

Helicity distribution

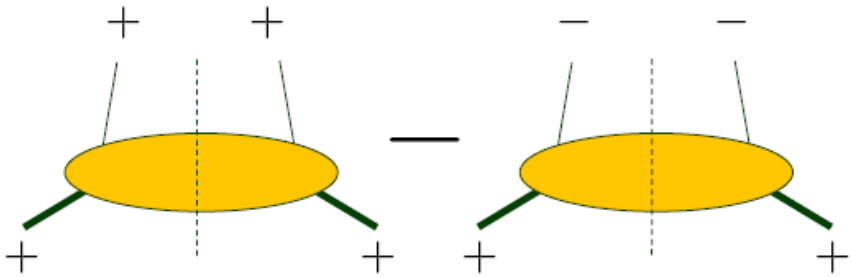


Transversity distribution

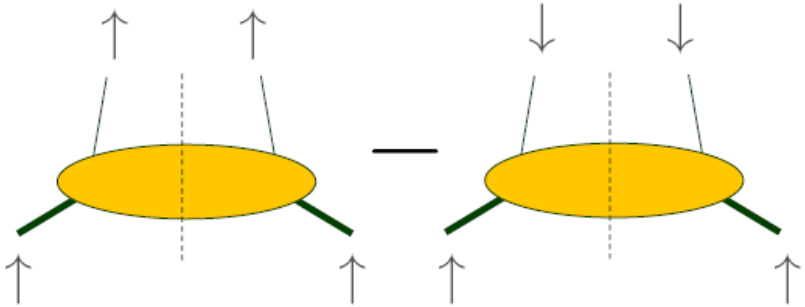


Boost and rotation do not commute → helicity and transversity are different and difference a relativistic effect

Helicity distribution

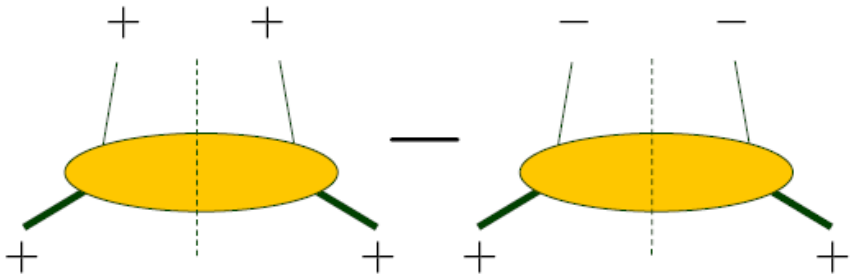


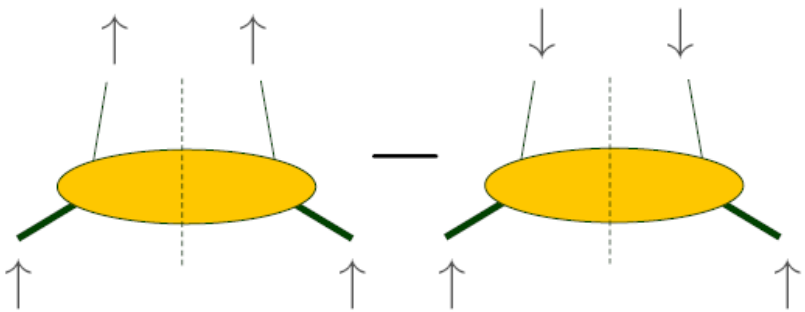
Transversity distribution



2005: first data on transversity

Helicity distribution Transversity distribution



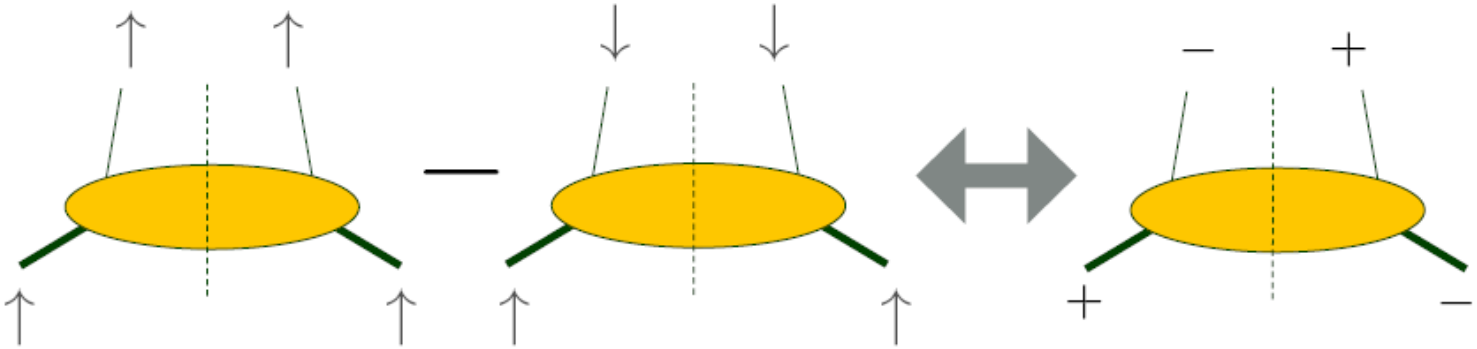


2005: first data on transversity

2012: hundred points from HERMES , COMPASS and JLAB

Why difficult to measure transversity distribution?

Transversity in helicity basis $|\uparrow,\downarrow\rangle = \frac{1}{\sqrt{2}}(|+\rangle \pm i|-\rangle)$



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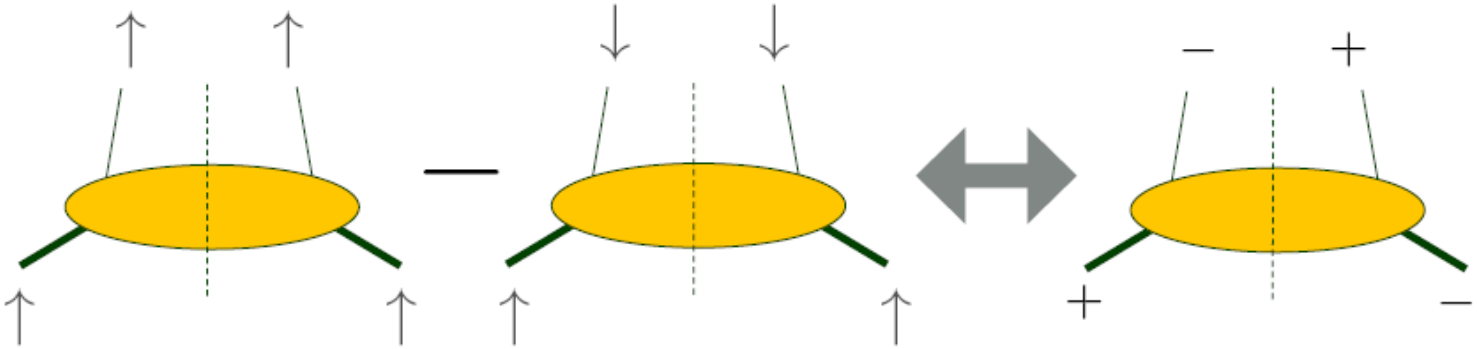
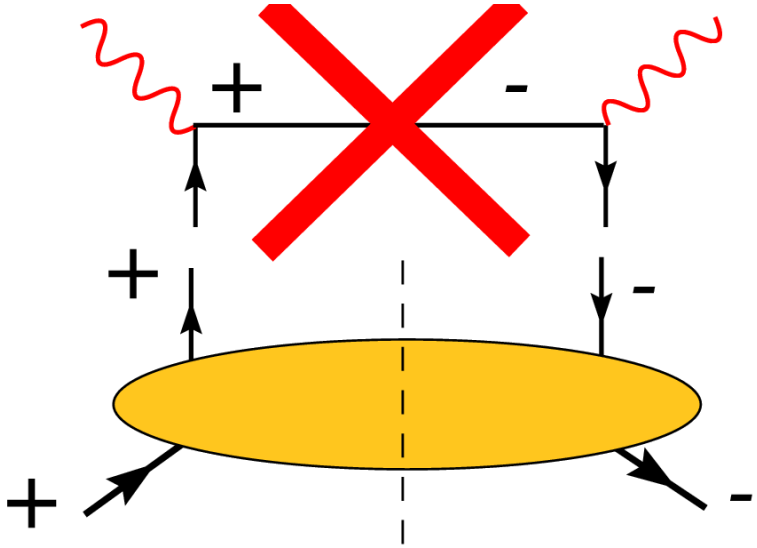


Chiral Odd!

Transversity distribution

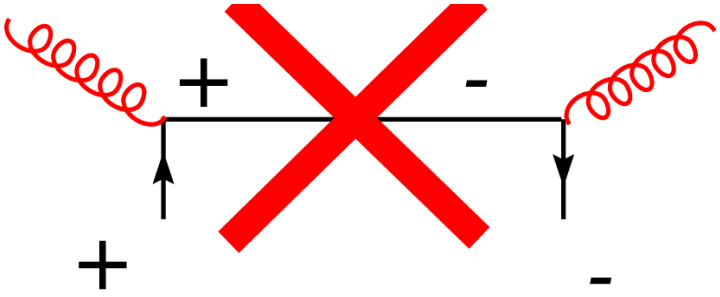
Chiral Odd: it cannot be measured in Deep Inelastic Scattering process



Needs another chiral odd function to be measured

Transversity distribution

QCD Evolution: no gluon contribution in the evolution



$h_1(x, Q^2)$ is suppressed at low x

JLab 12 is an ideal place to measure transversity → as JLab explores high x region

Transversity distribution

Bounds on transversity distribution: The Soffer Bound

$$|h_1(x)| \leq \frac{1}{2}\left(f_1(x) + g_1(x)\right)$$

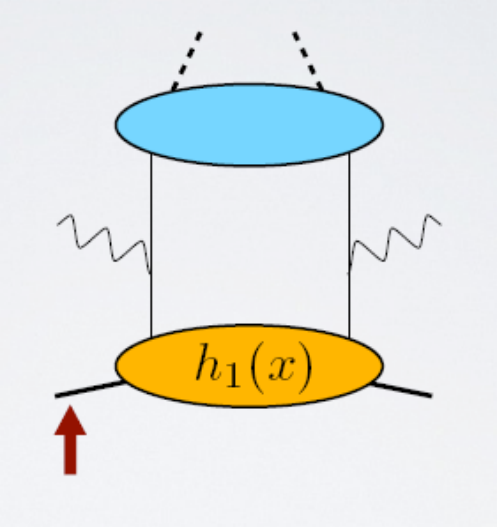
Valid at LO QCD, Barone 97, Bourelly et al 98

Valid at NLO QCD, Vogelsang 98

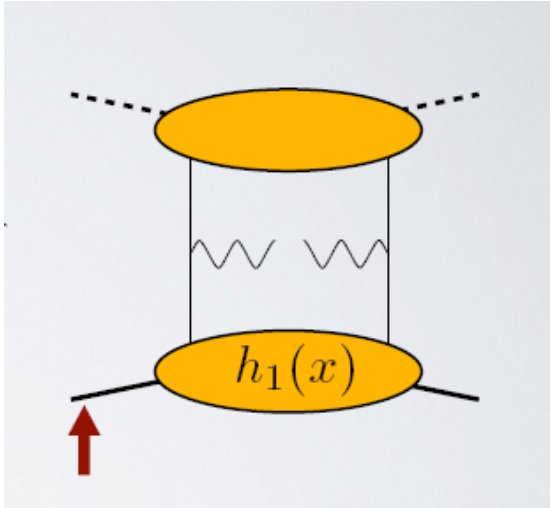
Transversity how to measure?

Transversity needs another chiral odd function to be measured

Semi Inclusive DIS (SIDIS)



Drell-Yan



Transversity how to measure?

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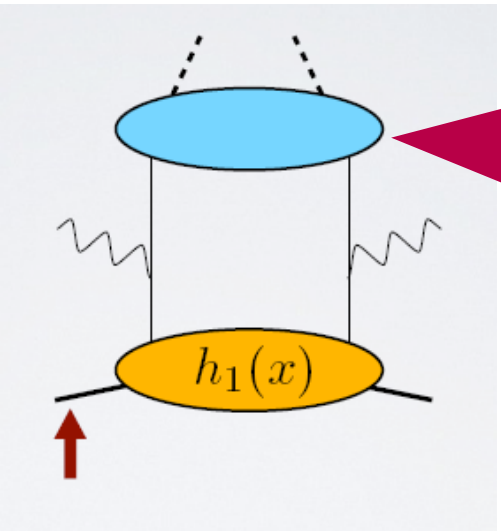
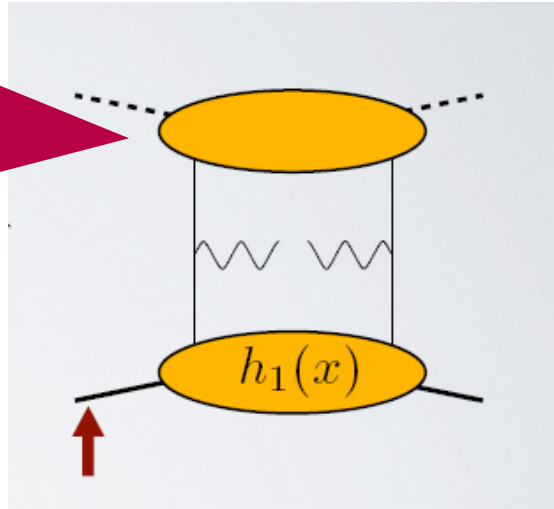
Semi Inclusive DIS (SIDIS) **Drell-Yan**



?



Collins fragmentation function

$$H_1^{\perp}(z, p_{\perp})$$

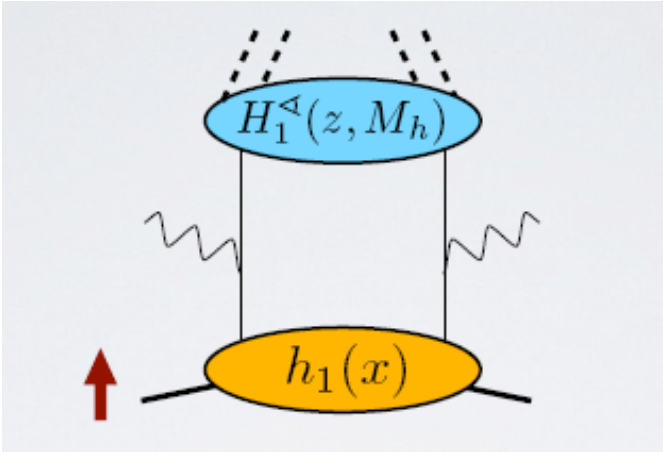
Transversity or Boer-Mulders function

$$h_1(x) \qquad h_1^{\perp}(x, k_{\perp})$$

Transversity how to measure?

Another way to measure transversity is via dihadron fragmentation functions

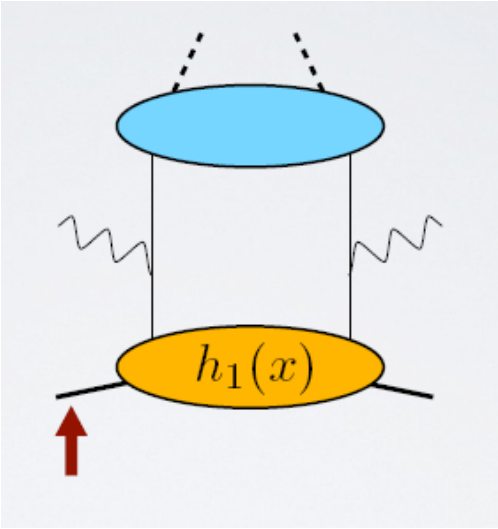
SIDIS



Dihadron fragmentation function

$$H_1^{\sphericalangle}(z, M_h)$$

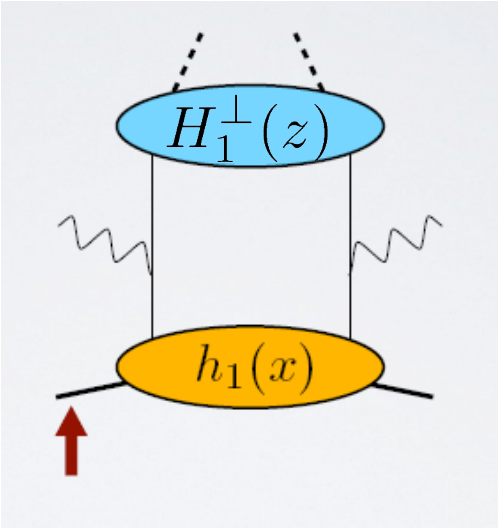
Transversity from SIDIS



First extraction in 2007, Anselmino et al 07

$$A_{UT}^{\sin(\Phi_h+\Phi_S)} \propto \frac{\sum e_q^2 h_1^q \otimes H_1^{\perp q}}{\sum e_q^2 f_1^q \otimes D_1^q}$$

Transversity from SIDIS



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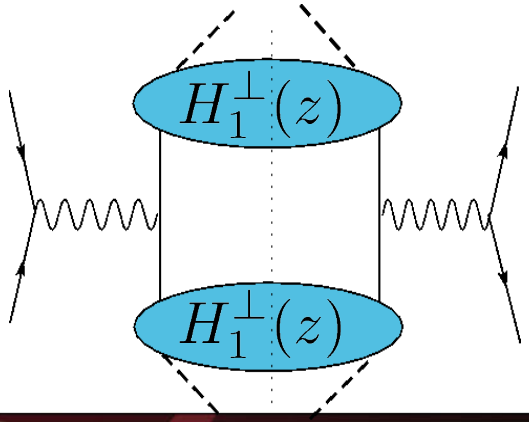
Two unknowns, transversity $h_1(x)$
Collins Fragmentation Function $H_1^\perp(z)$

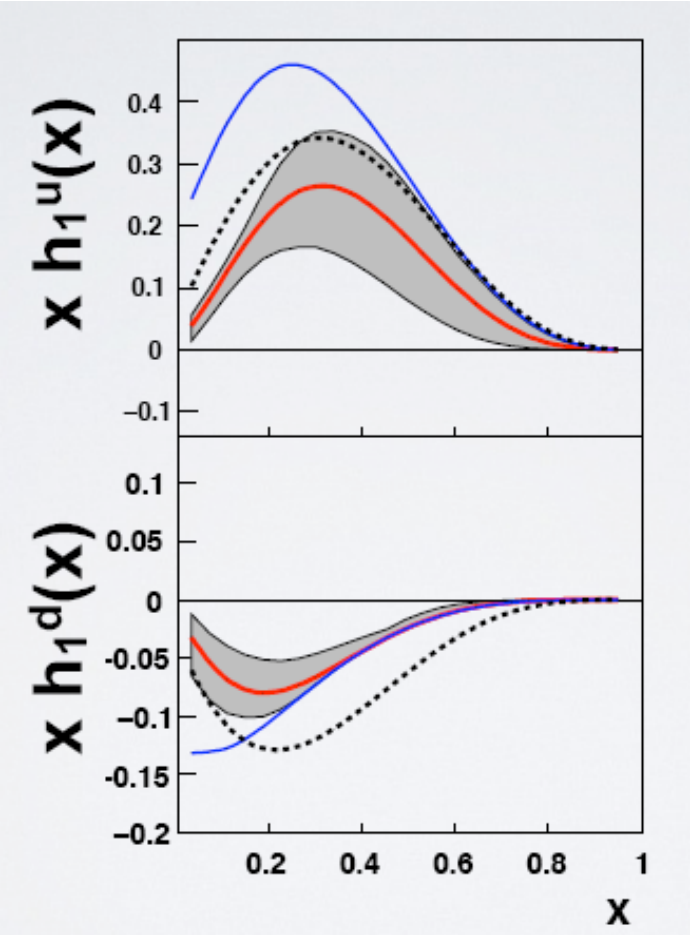
Fortunately information on $H_1^\perp(z)$ is available from e^+e^-



$$A_{e^+e^-} \propto \frac{\sum e_q^2 H_1^{\perp q} \otimes H_1^{\perp \bar{q}}}{\sum e_q^2 D_1^q \otimes D_1^{\bar{q}}}$$

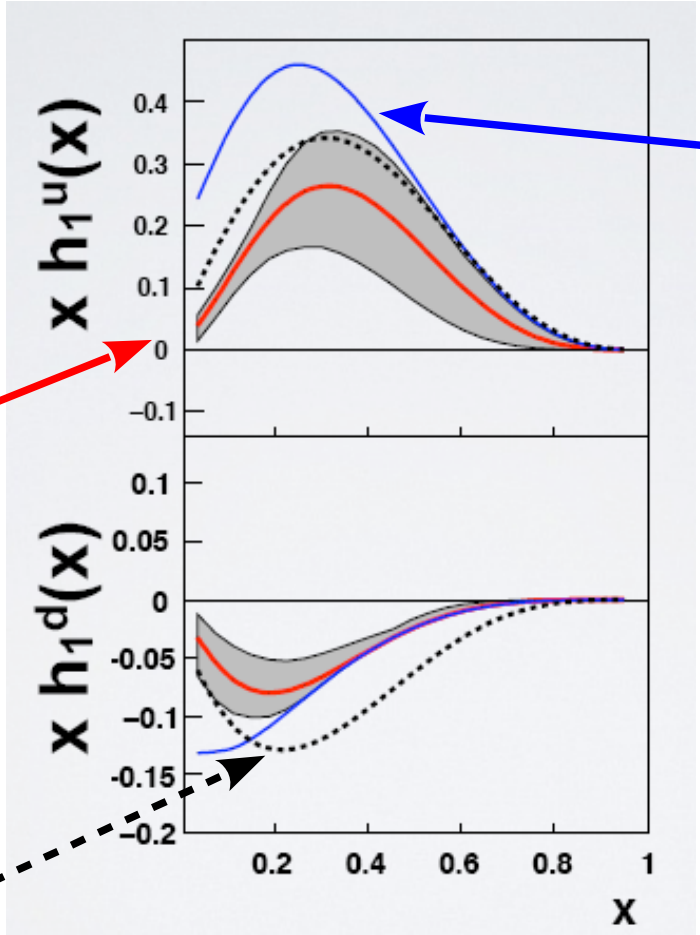
JSA Jefferson Lab

Transversity



Anselmino et al 09

Transversity



Soffer Bound

Transversity

Helicity

Anselmino et al 09

Comparison with models

1 - Barone et al., (1997)
2 - Soffer et al., (2002)
3 - Korotkov et al., (2001)
4 - Schweitzer et al., (2001)
5 - Wakamatzu, (2007)
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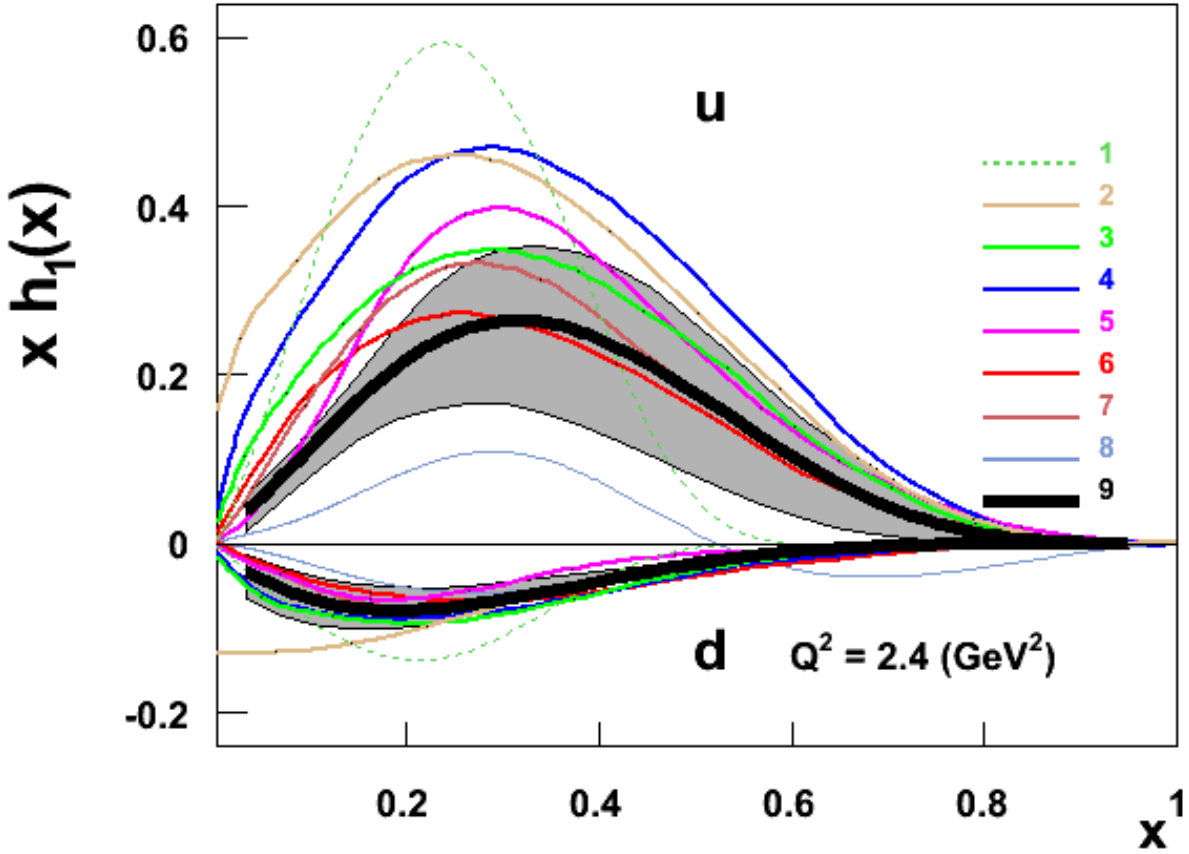

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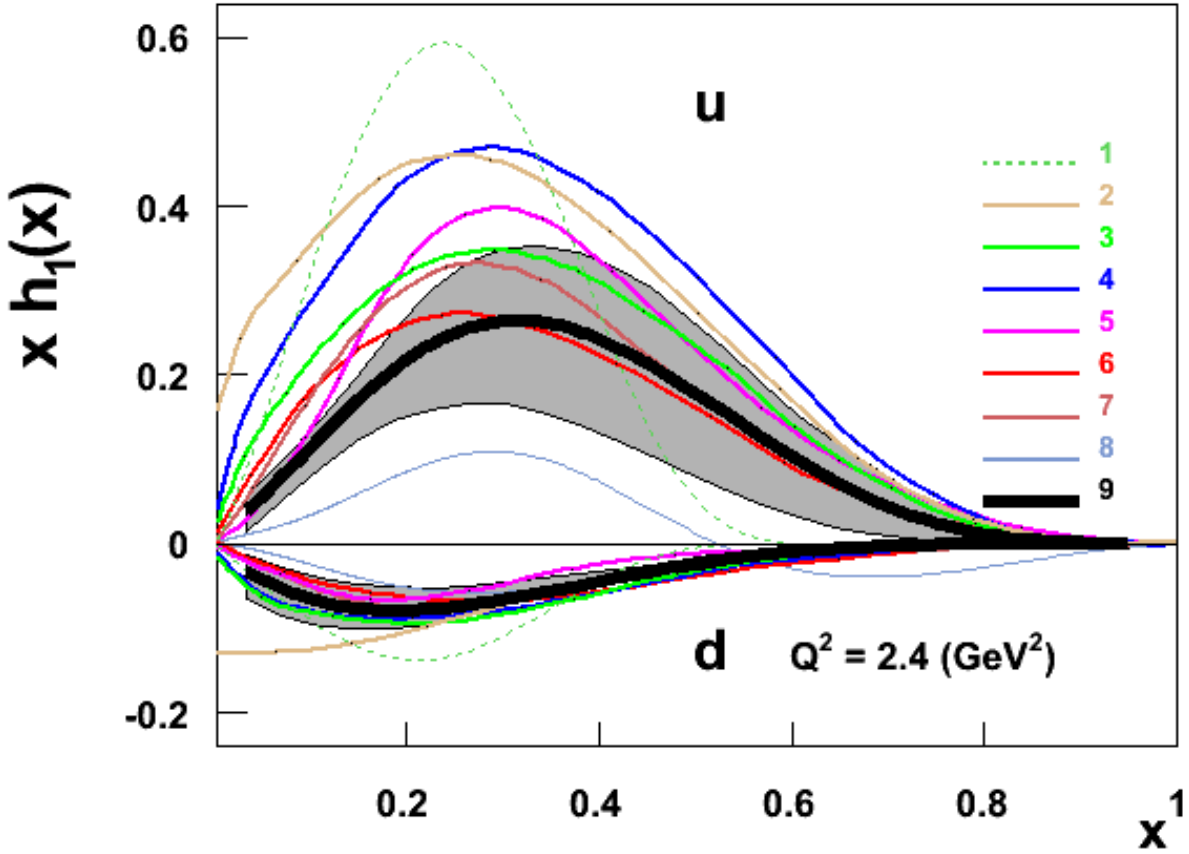


Good agreement with models in sign and size of transversity.

High uncertainty especially in high-x region

Tensor charge

Transversity is the source of information on tensor charge

$$\delta q = \int_0^1 dx (h_1^q(x) - h_1^{\bar{q}}(x))$$

Fundamental quantity

Caveat: no sum rules

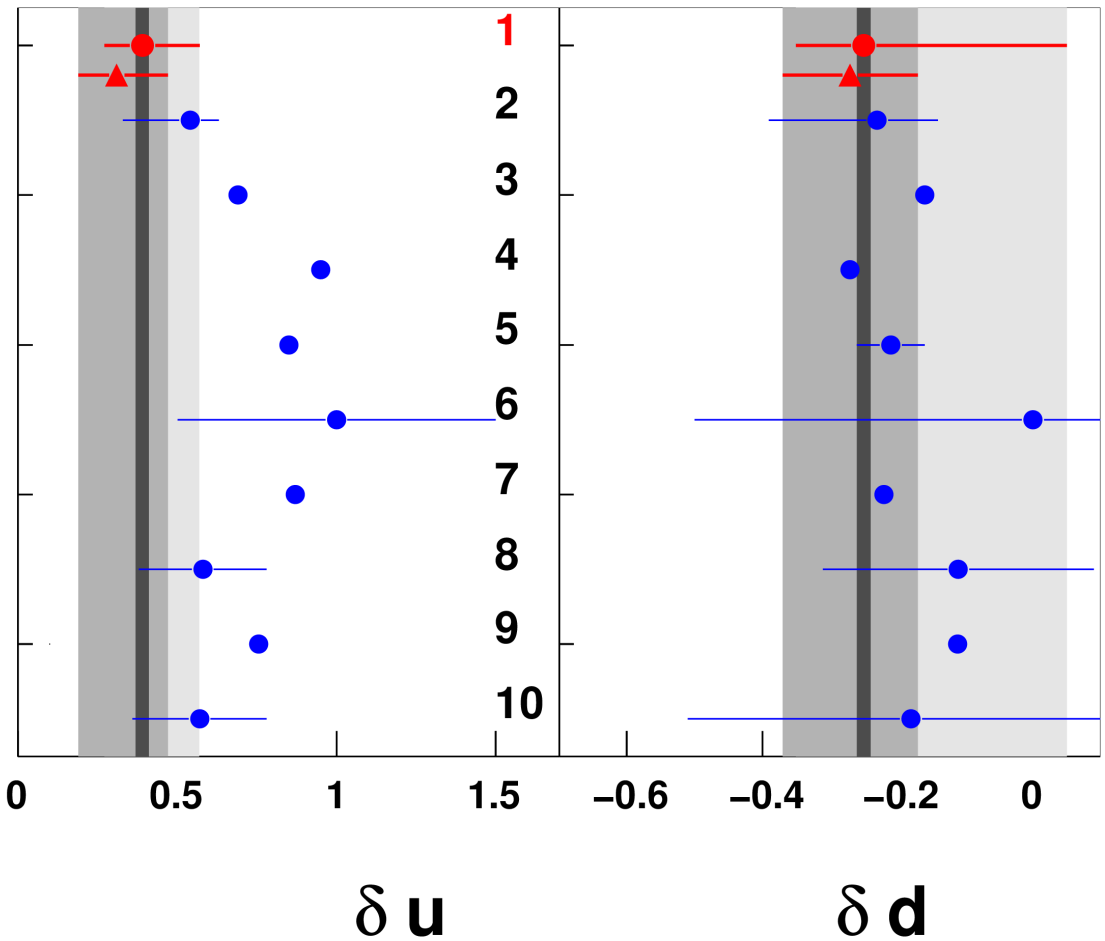
Tensor charge

1 – Anselmino et al (2013)
2 – Anselmino et al., Nucl. Phys. Proc. Suppl. (2009)
3 – Cloet, Bentz and Thomas, Phys. Lett. B (2008)
4 – Wakamatsu, Phys. Lett. B (2007)
5 – Gockeler et al., Phys. Lett. B (2005)
6 – He and Ji, Phys. Rev. D (1995)
7 – Pasquini et al, Phys. Rev. D (2007)
8 – Gamberg and Goldstein, Phys. Rev. Lett. (2001)
9 – Hecht, Roberts and Schmidt, Phys. Rev. C (2001)
10 – Bacchetta, Courtoy, Radici, arXiv:1212.3568

$$\delta q = \int_0^1 dx \, (h_1^q(x) - h_1^q(x))$$

● $\delta u = 0.39^{+0.18}_{-0.12}$, $\delta d = -0.25^{+0.3}_{-0.1}$

▲ $\delta u = 0.31^{+0.16}_{-0.12}$, $\delta d = -0.27^{+0.1}_{-0.1}$



JLab 12 data is going to improve our knowledge of tensor charge

Tensor vs axial charges

$$\Delta q = \int_0^1 dx(g_1^q(x) + g_1^{\bar{q}}(x)) \qquad \delta q = \int_0^1 dx(h_1^q(x) - h_1^{\bar{q}}(x))$$

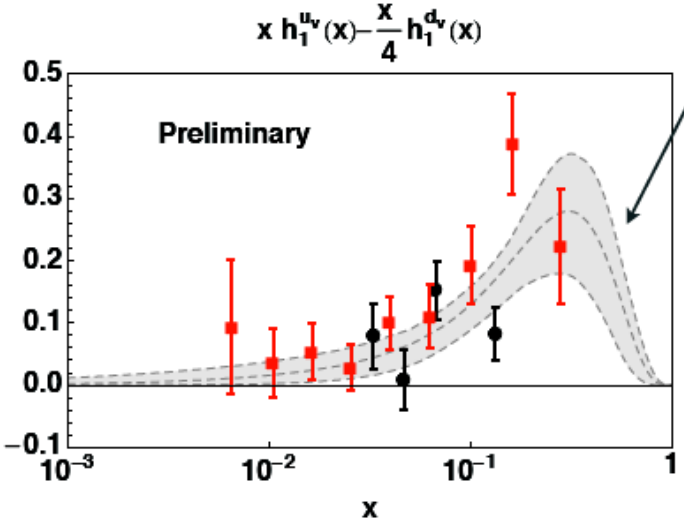
	Axial, DSSV	Tensor, Anselmino
u	0.82	0.54
d	-0.45	-0.23
s	-0.11	0
sum	0.26	0.39

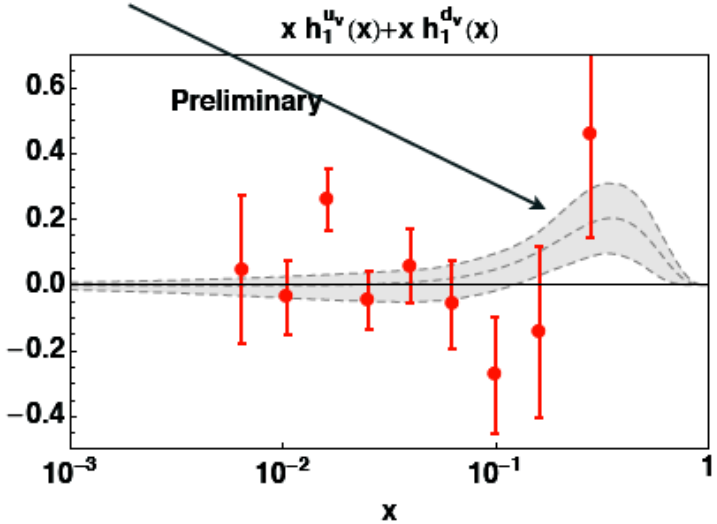
DSSV: De Florian, Sassot, Stratmann, Vogelsang (2008)

Jefferson Lab

Transversity from dihadron fragmentation

Torino's fit Anselmino et al 09



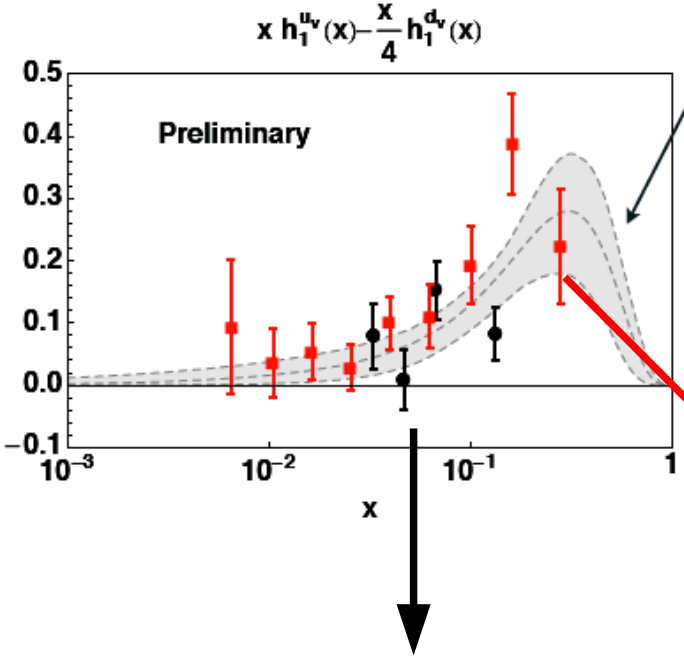


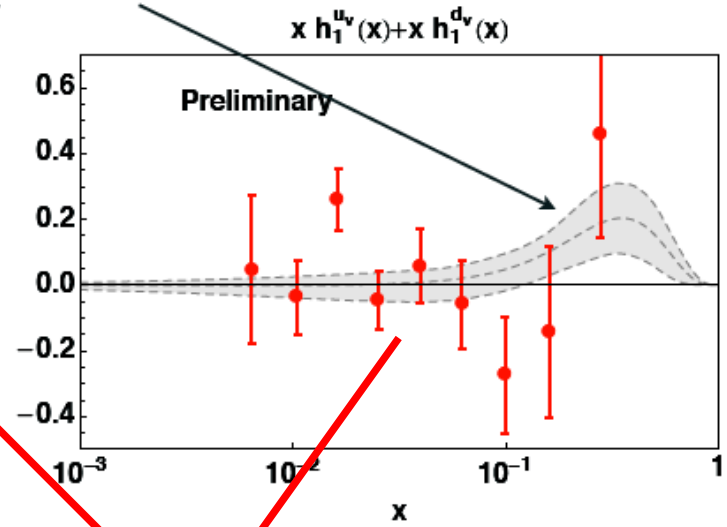
Bacchetta et al. PRL 107,012001 (2011)

Bacchetta et al.,arXiv:1206.1836

Transversity from dihadron fragmentation

Torino's fit Anselmino et al 09





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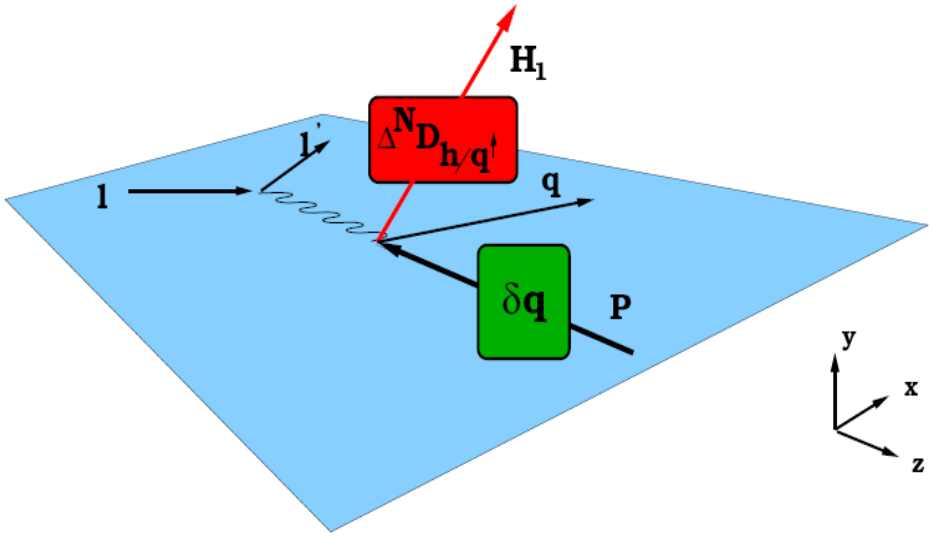
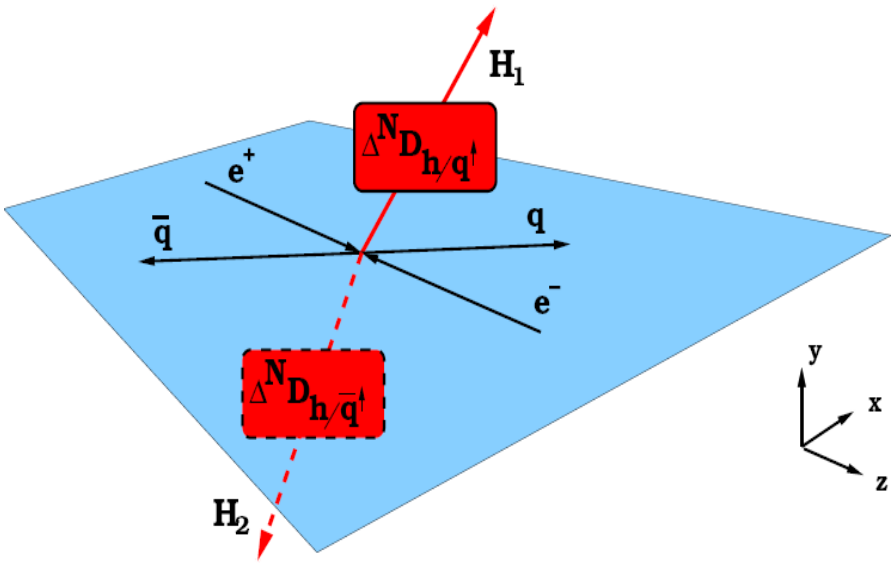
Good qualitative agreement of two methods of extraction

How to measure Collins FF?

SIDIS



e+ e-



Transversity $\otimes$ Collins FF

Collins FF $\otimes$ Collins FF

Mulders, Tangerman (1995), Boer, Jacob, Mulders (1997)

SIDIS and e+e-

It is important to prove that the same TMDs enter in the convolution.

TMDs are known for process dependence, or “generalized universality”. Sivers function that changes sign from SIDIS to Drell-Yan is a notorious example.

Brodsky, Hwang, Schmidt (2002), Collins (2002), Kang, Qiu (2009)

Collins function (factorization functions in general) was shown to be universal

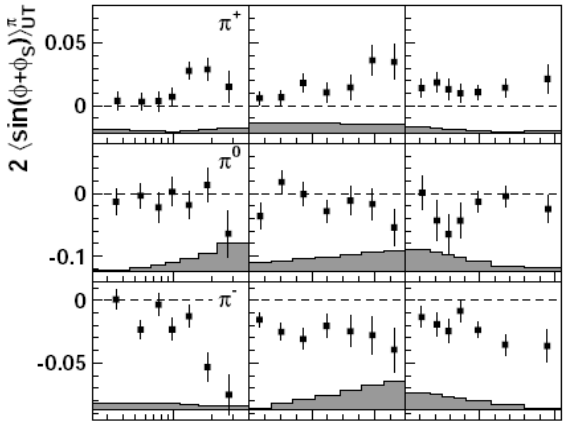
Collins, Metz (2004)

Gamberg, Mukherjee, Mulders (2008)

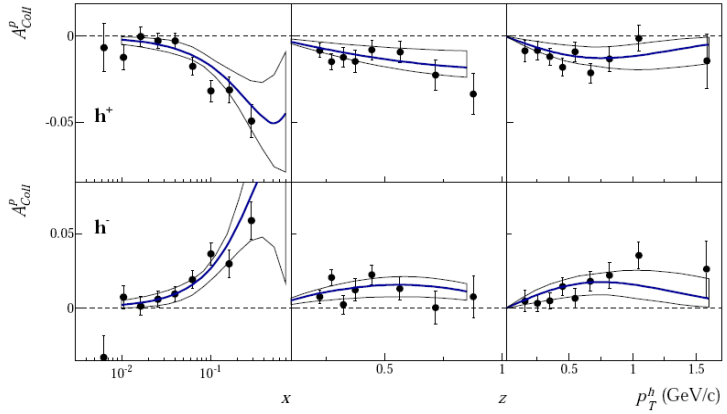
Experimental data

SIDIS

HERMES, PLB 2010

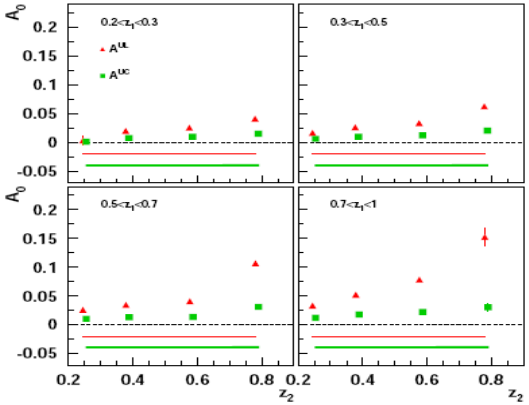


COMPASS, PLB 2012

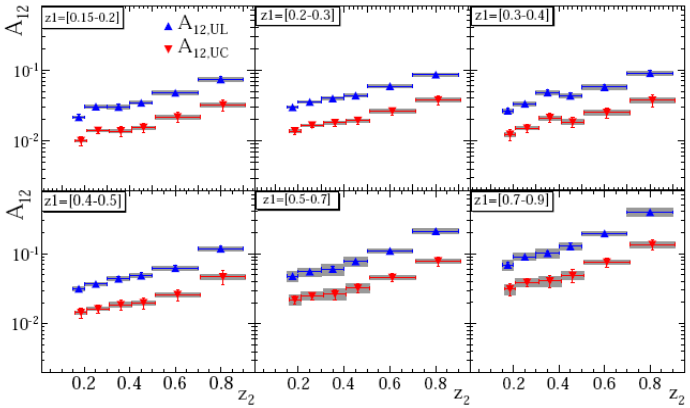


e+ e-

BELLE, PRD 2012



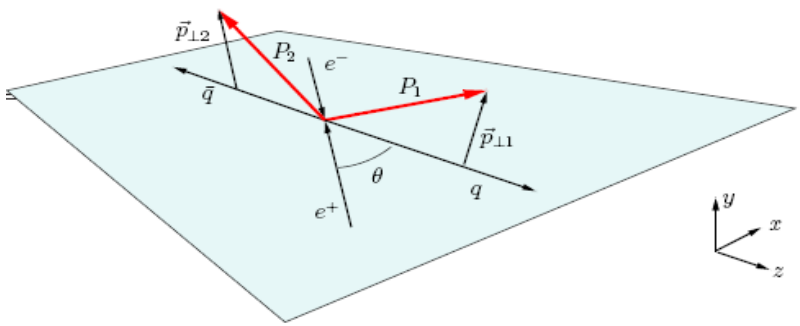
BaBar, 2013



e+e- two methods and two observables

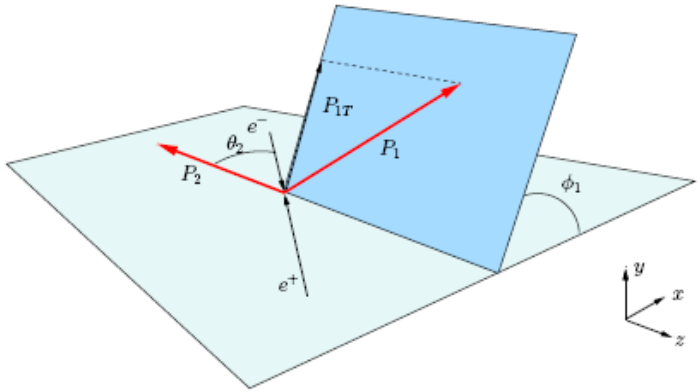
A_{12} thrust axis method

$$A_{12} \propto \cos(\phi_1 + \phi_2)$$



A_0 hadronic plane method

$$A_0 \propto \cos(2\phi_1)$$



e+e- two methods and two observables

To cancel acceptance effects and radiative corrections the following ratios are introduced

$$A^{UL} \propto \frac{\text{Unlike sign} : \pi^+\pi^- + \pi^-\pi^+}{\text{Like sign} : \pi^+\pi^+ + \pi^-\pi^-}$$

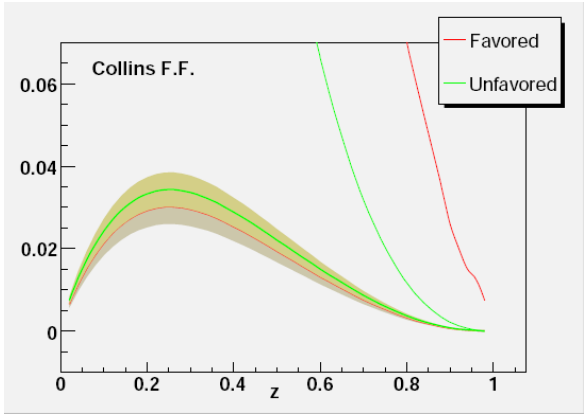
$$A^{UC} \propto \frac{\text{Unlike sign} : \pi^+\pi^- + \pi^-\pi^+}{\text{Charged} : \pi^+\pi^+ + \pi^-\pi^- + \pi^+\pi^- + \pi^-\pi^+}$$

4 sets of asymmetries

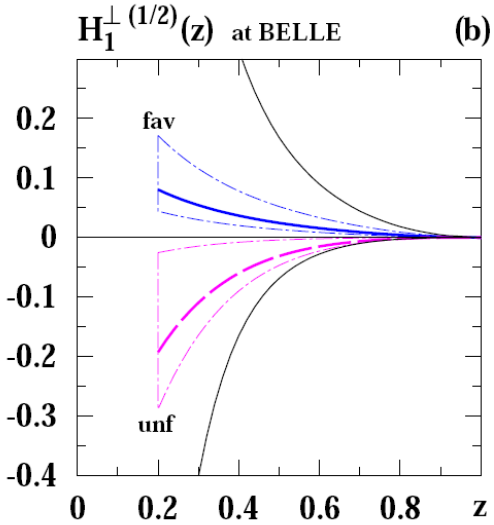
$$A_{12}^{UL}, A_0^{UL}, A_{12}^{UC}, A_0^{UC}$$

Extractions: history

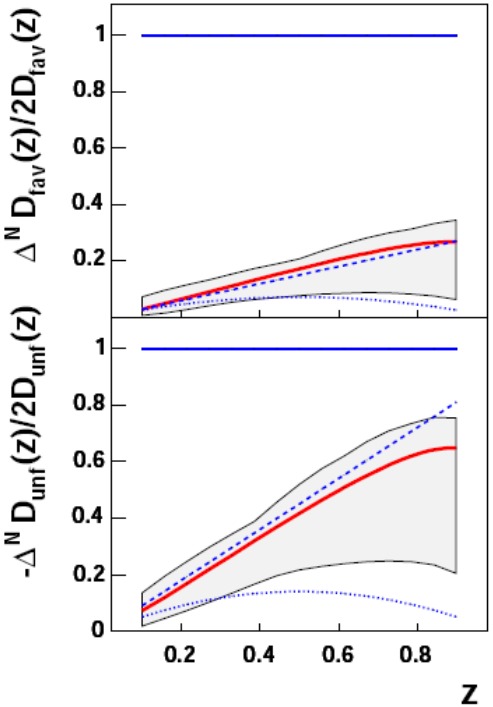
Yuan, Vogelsang (2005)



Efremov, Goeke, Schweitzer (2006)



Anselmino et al (2007)

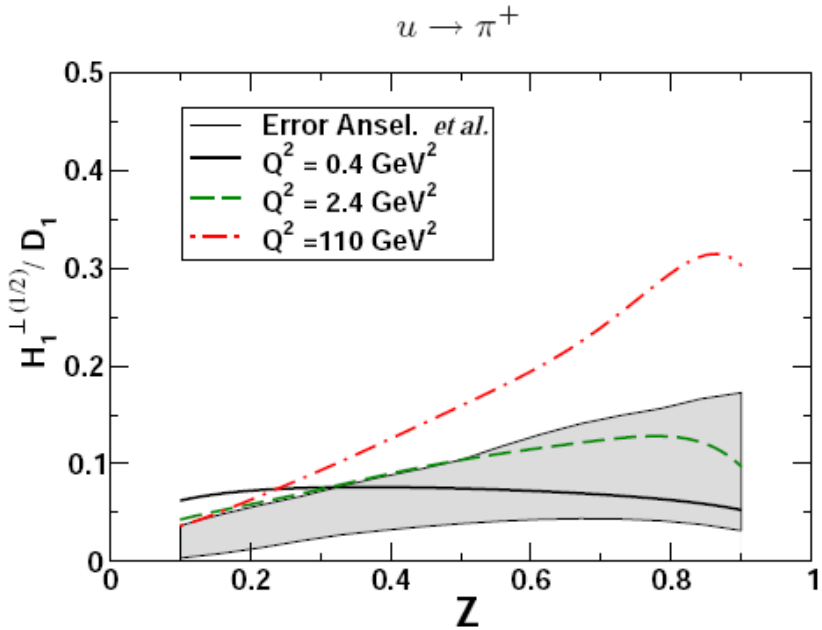


Anselmino et al (2008)
Anselmino et al (2013) update of the previous extractions without TMD evolution

Sun, Yuan (2013) with TMD evolution

Collins FF: models and expectations

Bacchetta et al (2008)



Same sign and magnitude

String fragmentation
Artru, Czyzewsky (1998)

Many more models:
Bacchetta, Kundu, Metz, Mulders (2001)
Amrath, Bacchetta, Metz (2005)
Bacchetta, Gamberg, Goldstein, Mukherjee (2007)
Matevosyan, Thomas, Bentz (2012)
etc

Schafer-Teryaev sum rule
Schafer, Teryaev (2001), Meissner, Metz, Pitonyak (2010)

$$\sum_h \sum_{S_h} \int_0^1 dz z M_h H_1^{\perp(1)q\to h}(z) = 0$$

Anselmino et al (2013)

- HERMES data 2009, π^+, π^-
- COMASS data deuteron 2004, π^+, π^-
- COMPASS data proton 2013, π^+, π^-
- BELLE data 2012, A_0, A_{12}

Anselmino et al (2013)

- HERMES data 2009, π^+, π^-
- COMASS data deuteron 2004, π^+, π^-
- COMPASS data proton 2013, π^+, π^-

146 points

- BELLE data 2012, A_0, A_{12}

16x4 points

Anselmino et al (2013)

- HERMES data 2009, π^+, π^-
- COMASS data deuteron 2004, π^+, π^-
- COMPASS data proton 2013, π^+, π^-

146 points

- BELLE data 2012, A_0, A_{12}

16x4 points

We fit:

- U and D transversity
- Favoured and Unfavoured Collins FF

$$H_1^{\perp fav} \equiv H_1^{\perp u \to \pi^+}$$

$$H_1^{\perp unf} \equiv H_1^{\perp d \to \pi^+}$$

Parametrizations

Anselmino et al (2013)

Unpolarised distributions

$$f_1(x, k_\perp) = f_1(x) \frac{1}{\pi \langle k_\perp^2 \rangle} \exp\left(-\frac{k_\perp^2}{\langle k_\perp^2 \rangle}\right)$$

GRV98LO

$$D_1(z, k_\perp) = D_1(z) \frac{1}{\pi \langle p_\perp^2 \rangle} \exp\left(-\frac{p_\perp^2}{\langle p_\perp^2 \rangle}\right)$$

DSS LO

$$\langle k_\perp^2 \rangle = 0.25 \ \ (GeV^2)$$
$$\langle p_\perp^2 \rangle = 0.2 \ \ (GeV^2)$$

Anselmino et al (2006)

Parametrizations

Anselmino et al (2013)

Unpolarised distributions

$$f_1(x,k_\perp) = f_1(x)\frac{1}{\pi\langle k_\perp^2\rangle}\exp\left(-\frac{k_\perp^2}{\langle k_\perp^2\rangle}\right)$$

GRV98LO

$$D_1(z,k_\perp) = D_1(z)\frac{1}{\pi\langle p_\perp^2\rangle}\exp\left(-\frac{p_\perp^2}{\langle p_\perp^2\rangle}\right)$$

DSS LO

$$\langle k_\perp^2\rangle = 0.25 \;\; (GeV^2)$$
$$\langle p_\perp^2\rangle = 0.2 \;\; (GeV^2)$$

Anselmino et al (2006)

Other choices are possible

Signori, Bacchetta, Radici, Schnell (2013)
Anselmino et al (2013)

Parametrizations

Anselmino et al (2013)

Transversity

$$\Delta_T q(x, k_\perp) = \frac{1}{2}\mathcal{N}_q^T(x)[f_{q/p}(x) + \Delta q(x)]\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle_T}}{\pi\langle k_\perp^2\rangle_T},$$

GRV98LO
GRSV

$$\mathcal{N}_q^T(x) = N_q^T x^\alpha (1-x)^\beta \frac{(\alpha+\beta)^{(\alpha+\beta)}}{\alpha^\alpha \beta^\beta}$$

Collins FF

$$\Delta^N D_{h/q^\uparrow}(z, p_\perp) = 2\mathcal{N}_q^C(z) D_{h/q}(z) h(p_\perp)\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle}}{\pi\langle p_\perp^2\rangle},$$

DSS LO

$$\mathcal{N}_q^C(z) = N_q^C z^\gamma (1-z)^\delta \frac{(\gamma+\delta)^{(\gamma+\delta)}}{\gamma^\gamma \delta^\delta}$$

$$h(p_\perp) = \sqrt{2e}\frac{p_\perp}{M_h} e^{-p_\perp^2/M_h^2}$$

Parametrizations

Anselmino et al (2013)

Transversity

$$\Delta_T q(x, k_\perp) = \frac{1}{2}\mathcal{N}_q^T(x)[f_{q/p}(x) + \Delta q(x)]\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle_T}}{\pi\langle k_\perp^2\rangle_T},$$

$$\mathcal{N}_q^T(x) = N_q^T x^\alpha (1-x)^\beta \frac{(\alpha+\beta)^{(\alpha+\beta)}}{\alpha^\alpha \beta^\beta}$$

Collins FF

$$\Delta^N D_{h/q^\uparrow}(z, p_\perp) = 2\mathcal{N}_q^C(z) D_{h/q}(z) h(p_\perp)\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle}}{\pi\langle p_\perp^2\rangle},$$

$$\mathcal{N}_q^C(z) = N_q^C z^\gamma (1-z)^\delta \frac{(\gamma+\delta)^{(\gamma+\delta)}}{\gamma^\gamma \delta^\delta}$$

$$h(p_\perp) = \sqrt{2e}\frac{p_\perp}{M_h} e^{-p_\perp^2/M_h^2}$$

Bounds

$$\Delta_T q(x) \leq \frac{1}{2}(f(x) + \Delta q(x))$$

Fulfilled by construction

$$\Delta^N D_{\pi/q^\uparrow}(z, p_\perp) \leq 2\, D_{\pi/q}(z, k_\perp)$$

Fulfilled by construction

Torino – Amsterdam

$$\Delta^N D_{\pi/q^\uparrow}(z, p_\perp) = \frac{2p_\perp}{zM} H_1^\perp(z, p_\perp)$$

Parametrizations

Anselmino et al (2013)

Transversity

$$\Delta_T q(x, k_\perp) = \frac{1}{2}\mathcal{N}_q^T(x)[f_{q/p}(x) + \Delta q(x)]\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle_T}}{\pi\langle k_\perp^2\rangle_T},$$

$$\mathcal{N}_q^T(x) = N_q^T x^\alpha (1-x)^\beta \frac{(\alpha+\beta)^{(\alpha+\beta)}}{\alpha^\alpha \beta^\beta}$$

Collins FF

$$\Delta^N D_{h/q^\uparrow}(z, p_\perp) = 2\mathcal{N}_q^C(z) D_{h/q}(z) h(p_\perp)\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle}}{\pi\langle p_\perp^2\rangle},$$

$$\mathcal{N}_q^C(z) = N_q^C z^\gamma (1-z)^\delta \frac{(\gamma+\delta)^{(\gamma+\delta)}}{\gamma^\gamma \delta^\delta}$$

$$h(p_\perp) = \sqrt{2e}\frac{p_\perp}{M_h} e^{-p_\perp^2/M_h^2}$$

Parameters

$N_u^T \quad \alpha$

$N_d^T \quad \beta$

} **4**

$\langle k_\perp^2\rangle_T = 0.25 \ \ (GeV^2)$

$\mathcal{N}_{fav}^C \ \gamma$

$\mathcal{N}_{unf}^C \ \delta$

M_h^2

} **5**

FITS

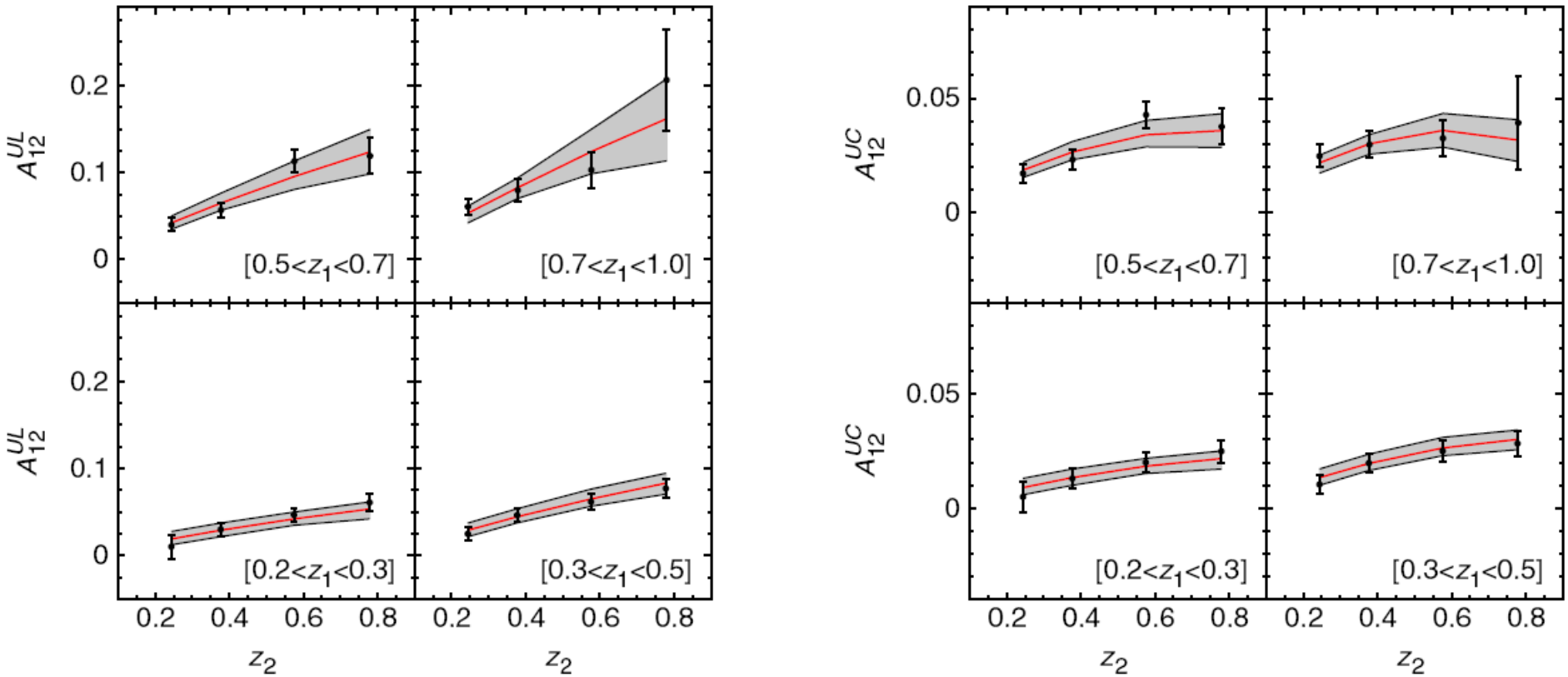
Anselmino et al (2013)

FIT I: BELLE A_{12}, HERMES and COMPASS

FIT II: BELLE A_0 , HERMES and COMPASS

FIT I: BELLE A_{12} , HERMES and COMPASS

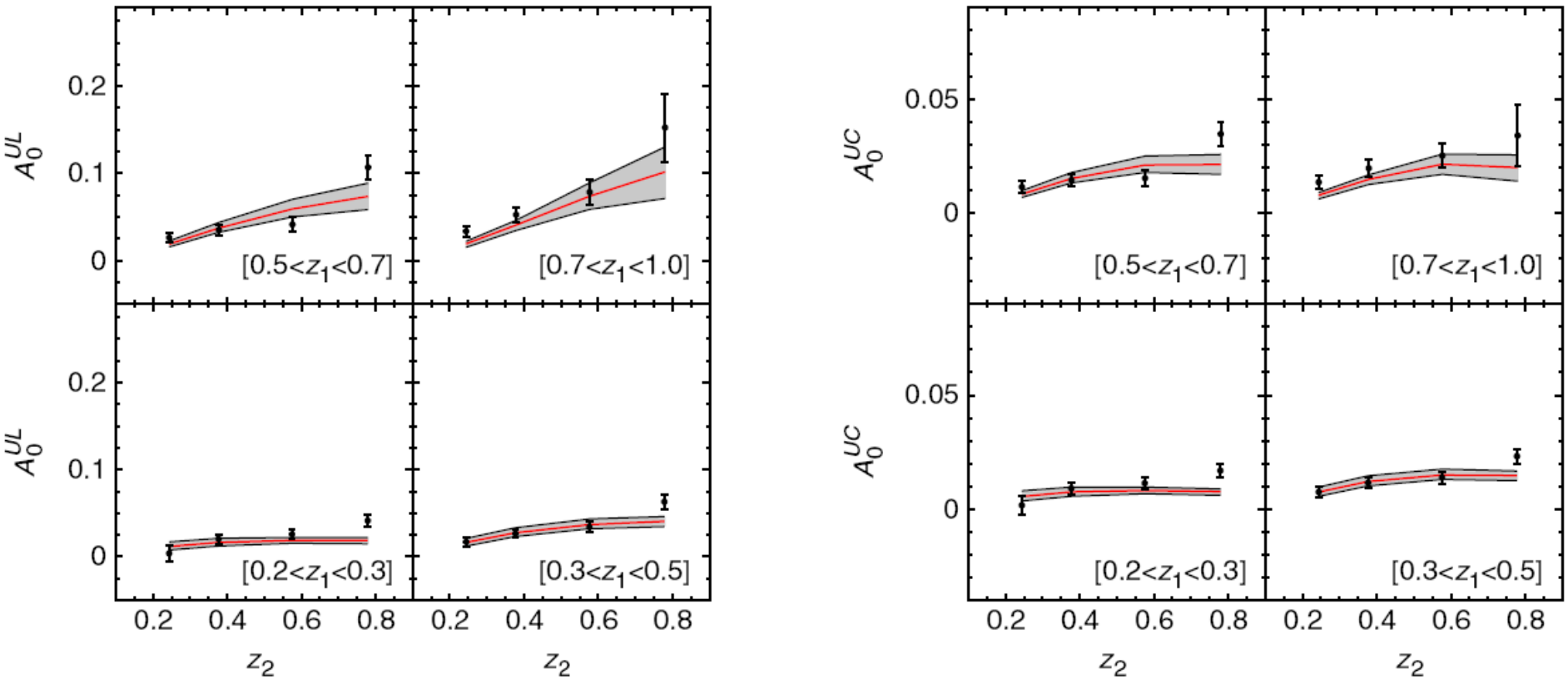
Anselmino et al (2013)



A_{12}^{UL} and A_{12}^{UC} are fully compatible

FIT I: BELLE A_{12} , HERMES and COMPASS

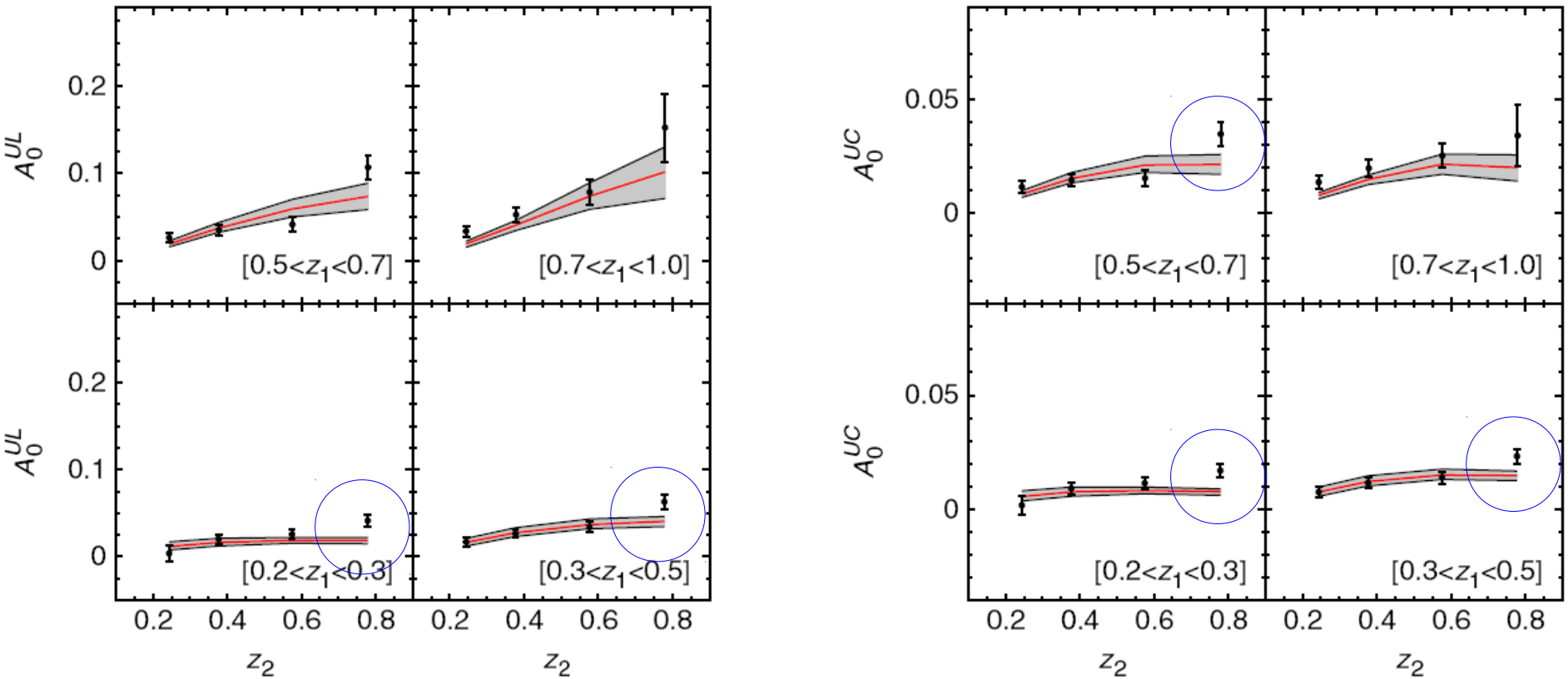
Anselmino et al (2013)



A_0^{UL} and A_0^{UC} are not included in the fit

FIT I: BELLE A_{12} , HERMES and COMPASS

Anselmino et al (2013)

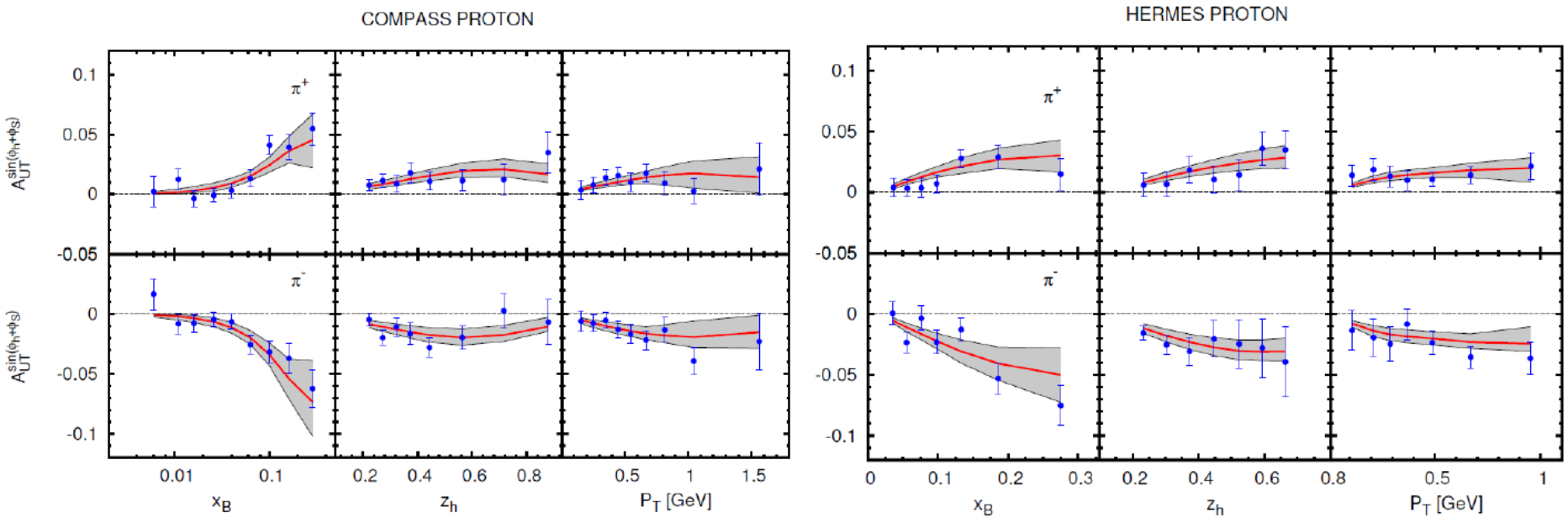


A_0^{UL} and A_0^{UC} are not included in the fit

Still some tension in large-z region!

FIT I: BELLE A_{12} , HERMES and COMPASS

Anselmino et al (2013)



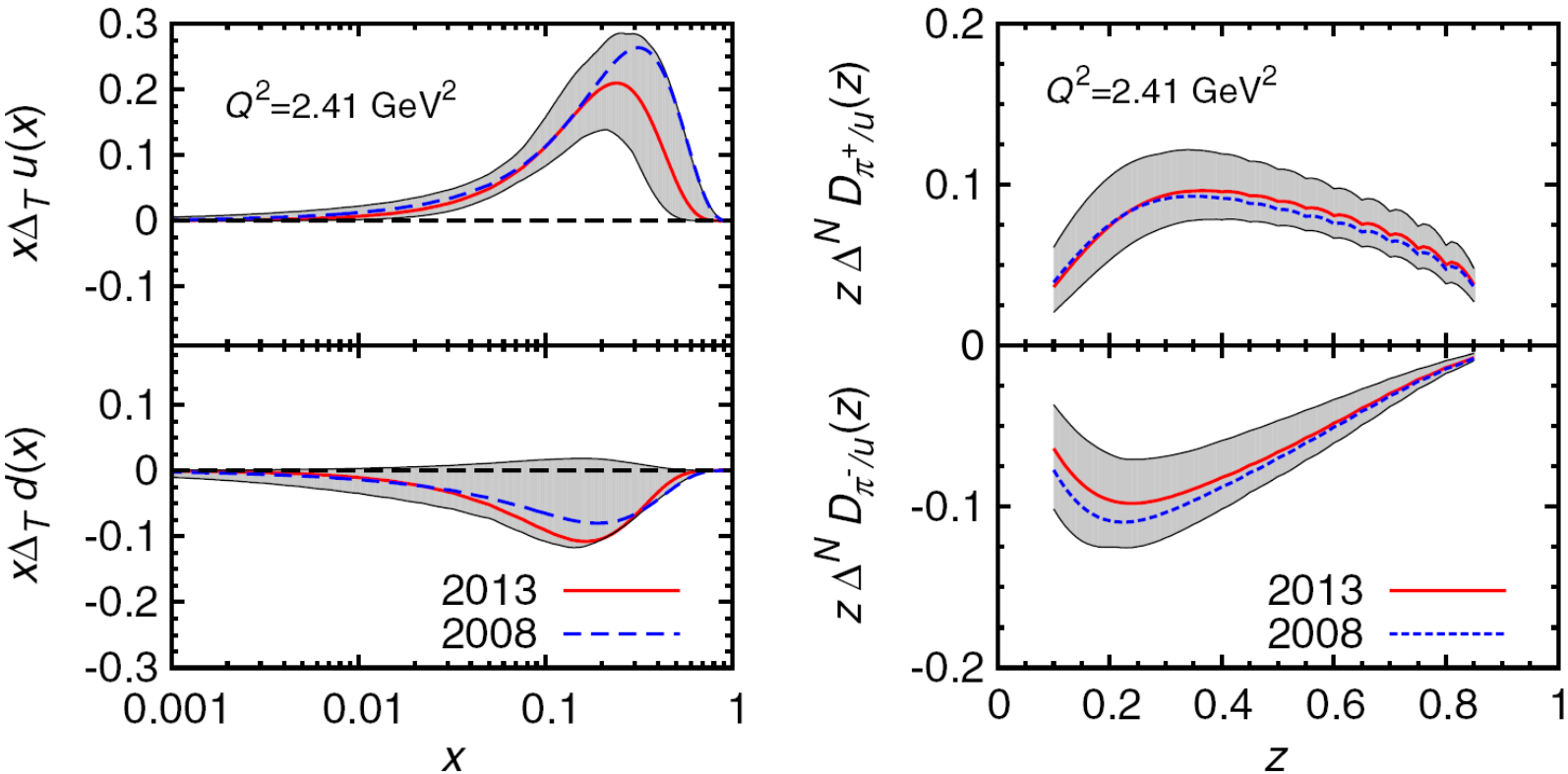
Good description of SIDIS data

FIT I: BELLE A_{12} , HERMES and COMPASS

Anselmino et al (2013)

Results are similar to 2008

$N_u^T = 0.46^{+0.20}_{-0.14}$	$N_d^T = -1.00^{+1.17}_{-0.00}$
$\alpha = 1.11^{+0.89}_{-0.66}$	$\beta = 3.64^{+5.80}_{-3.37}$
$N_{\rm fav}^C = 0.49^{+0.20}_{-0.18}$	$N_{\rm dis}^C = -1.00^{+0.38}_{-0.00}$
$\gamma = 1.06^{+0.45}_{-0.32}$	$\delta = 0.07^{+0.42}_{-0.07}$
$M_h^2 = 1.50^{+2.00}_{-1.12}$ GeV^2	



FIT II: BELLE A_0 , HERMES and COMPASS

Anselmino et al (2013)

	FIT DATA 178 points	SIDIS 146 points	A_{12}^{UL} 16 points	A_{12}^{UC} 16 points	A_0^{UL} 16 points	A_0^{UC} 16 points
Standard parametrization $\chi^2_{\text{d.o.f.}} = 0.80$	$\chi^2_{\text{tot}} = 135$	$\chi^2 = 123$	$\chi^2 = 7$	$\chi^2 = 5$	$\chi^2 = 44$ NO FIT	$\chi^2 = 39$ NO FIT
Standard parametrization $\chi^2_{\text{d.o.f.}} = 1.12$	$\chi^2_{\text{tot}} = 190$	$\chi^2 = 125$	$\chi^2 = 20$ NO FIT	$\chi^2 = 12$ NO FIT	$\chi^2 = 35$	$\chi^2 = 30$

Almost no improvement even if A_0 is fitted

FIT II: BELLE A_0 , HERMES and COMPASS

Anselmino et al (2013)

	FIT DATA 178 points	SIDIS 146 points	A_{12}^{UL} 16 points	A_{12}^{UC} 16 points	A_0^{UL} 16 points	A_0^{UC} 16 points
Standard parametrization $\chi^2_{\text{d.o.f.}} = 0.80$	$\chi^2_{\text{tot}} = 135$	$\chi^2 = 123$	$\chi^2 = 7$	$\chi^2 = 5$	$\chi^2 = 44$ NO FIT	$\chi^2 = 39$ NO FIT
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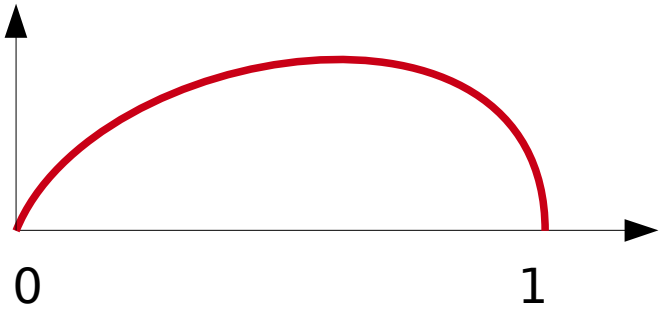
Almost no improvement even if fitted

Probably we should try a different parametrization?

$$z^\alpha (1-z)^\beta \to 0 \quad \text{if} \quad z \to 1, \beta > 0$$

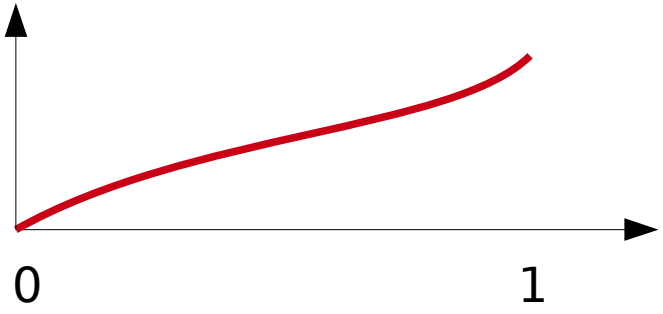


Polynomial parametrization

Anselmino et al (2013)

New polynomial parametrization:

$$z((1-a-b)+az+bz^2) \to 1 \qquad z \to 1$$



(many other choices were considered)

Among other advantages new parametrization can give us idea on "theoretical" error due to parametrization choice.

Results, FIT III and FIT IV

Anselmino et al (2013)

$N_u^T = 0.36^{+0.19}_{-0.12}$	$N_d^T = -1.00^{+0.40}_{0.00}$
$\alpha = 1.06^{+0.87}_{-0.56}$	$\beta = 3.66^{+5.87}_{-2.78}$
$N_{\rm fav}^C = 1.00^{+0.00}_{-0.36}$	$N_{\rm dis}^C = -1.00^{+0.19}_{-0.00}$
$a = -2.36^{+1.24}_{-0.98}$	$b = 2.12^{+0.61}_{-1.12}$
$M_h^2 = 0.67^{+1.09}_{-0.36}$ GeV^2	

	FIT DATA 178 points	SIDIS 146 points	A_{12}^{UL} 16 points	A_{12}^{UC} 16 points	A_0^{UL} 16 points	A_0^{UC} 16 points
Standard Parameterization $\chi^2_{\rm d.o.f} = 0.80$	$\chi^2_{\rm tot} = 135$	$\chi^2 = 123$	$\chi^2 = 7$	$\chi^2 = 5$	$\chi^2 = 44$ NO FIT	$\chi^2 = 39$ NO FIT
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Polynomial Parameterization $\chi^2_{\rm d.o.f} = 0.81$	$\chi^2_{\rm tot} = 136$	$\chi^2 = 123$	$\chi^2 = 8$	$\chi^2 = 5$	$\chi^2 = 45$ NO FIT	$\chi^2 = 39$ NO FIT
Polynomial Parameterization $\chi^2_{\rm d.o.f} = 1.01$	$\chi^2_{\rm tot} = 171$	$\chi^2 = 141$	$\chi^2 = 44$ NO FIT	$\chi^2 = 27$ NO FIT	$\chi^2 = 15$	$\chi^2 = 15$

Results, FIT III and FIT IV

Anselmino et al (2013)

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Polynomial Parameterization $\chi^2_{\rm d.o.f} = 1.01$	$\chi^2_{\rm tot} = 171$	$\chi^2 = 141$	$\chi^2 = 44$ NO FIT	$\chi^2 = 27$ NO FIT	$\chi^2 = 15$	$\chi^2 = 15$

Almost identical results if A_{12} is used in the fit

Results, FIT III and FIT IV

Anselmino et al (2013)

$N_u^T = 0.36^{+0.19}_{-0.12}$	$N_d^T = -1.00^{+0.40}_{0.00}$
$\alpha = 1.06^{+0.87}_{-0.56}$	$\beta = 3.66^{+5.87}_{-2.78}$
$N_{\rm fav}^C = 1.00^{+0.00}_{-0.36}$	$N_{\rm dis}^C = -1.00^{+0.19}_{-0.00}$
$a = -2.36^{+1.24}_{-0.98}$	$b = 2.12^{+0.61}_{-1.12}$
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	FIT DATA 178 points	SIDIS 146 points	A_{12}^{UL} 16 points	A_{12}^{UC} 16 points	A_0^{UL} 16 points	A_0^{UC} 16 points
Standard Parameterization $\chi^2_{\rm d.o.f} = 0.80$	$\chi^2_{\rm tot} = 135$	$\chi^2 = 123$	$\chi^2 = 7$	$\chi^2 = 5$	$\chi^2 = 44$ NO FIT	$\chi^2 = 39$ NO FIT
Standard Parameterization $\chi^2_{\rm d.o.f} = 1.12$	$\chi^2_{\rm tot} = 190$	$\chi^2 = 125$	$\chi^2 = 20$ NO FIT	$\chi^2 = 12$ NO FIT	$\chi^2 = 35$	$\chi^2 = 30$
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Polynomial Parameterization $\chi^2_{\rm d.o.f} = 1.01$	$\chi^2_{\rm tot} = 171$	$\chi^2 = 141$	$\chi^2 = 44$ NO FIT	$\chi^2 = 27$ NO FIT	$\chi^2 = 15$	$\chi^2 = 15$

Description of A_0 is improved with polynomial parametrization

Results, FIT III and FIT IV

Anselmino et al (2013)

$N_u^T = 0.36^{+0.19}_{-0.12}$	$N_d^T = -1.00^{+0.40}_{0.00}$
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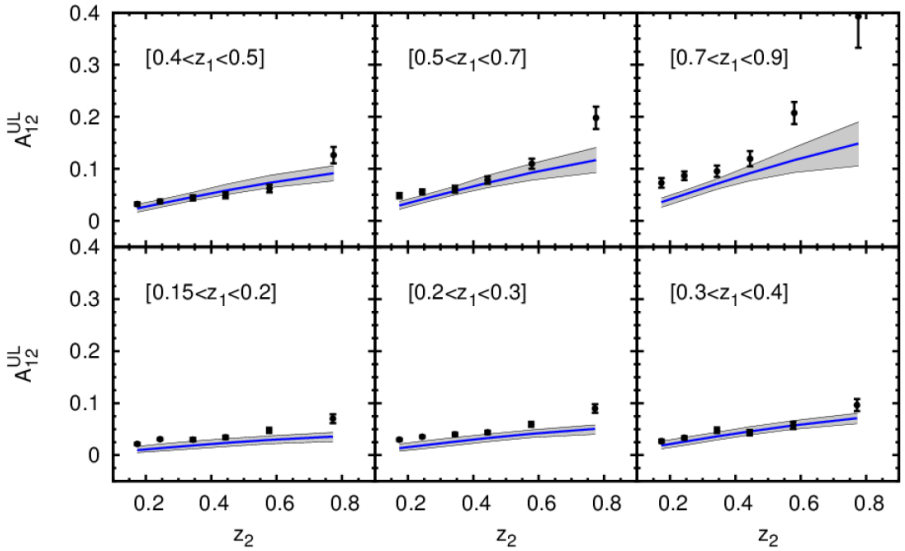
$N_{\rm fav}^C = 1.00^{+0.00}_{-0.36}$	$N_{\rm dis}^C = -1.00^{+0.19}_{-0.00}$
$a = -2.36^{+1.24}_{-0.98}$	$b = 2.12^{+0.61}_{-1.12}$
$M_h^2 = 0.67^{+1.09}_{-0.36}$ GeV^2	

	FIT DATA 178 points	SIDIS 146 points	A_{12}^{UL} 16 points	A_{12}^{UC} 16 points	A_0^{UL} 16 points	A_0^{UC} 16 points
Standard Parameterization $\chi^2_{\rm d.o.f} = 0.80$	$\chi^2_{\rm tot} = 135$	$\chi^2 = 123$	$\chi^2 = 7$	$\chi^2 = 5$	$\chi^2 = 44$ NO FIT	$\chi^2 = 39$ NO FIT
Standard Parameterization $\chi^2_{\rm d.o.f} = 1.12$	$\chi^2_{\rm tot} = 190$	$\chi^2 = 125$	$\chi^2 = 20$ NO FIT	$\chi^2 = 12$ NO FIT	$\chi^2 = 35$	$\chi^2 = 30$
Polynomial Parameterization $\chi^2_{\rm d.o.f} = 0.81$	$\chi^2_{\rm tot} = 136$	$\chi^2 = 123$	$\chi^2 = 8$	$\chi^2 = 5$	$\chi^2 = 45$ NO FIT	$\chi^2 = 39$ NO FIT
Polynomial Parameterization $\chi^2_{\rm d.o.f} = 1.01$	$\chi^2_{\rm tot} = 171$	$\chi^2 = 141$	$\chi^2 = 44$ NO FIT	$\chi^2 = 27$ NO FIT	$\chi^2 = 15$	$\chi^2 = 15$

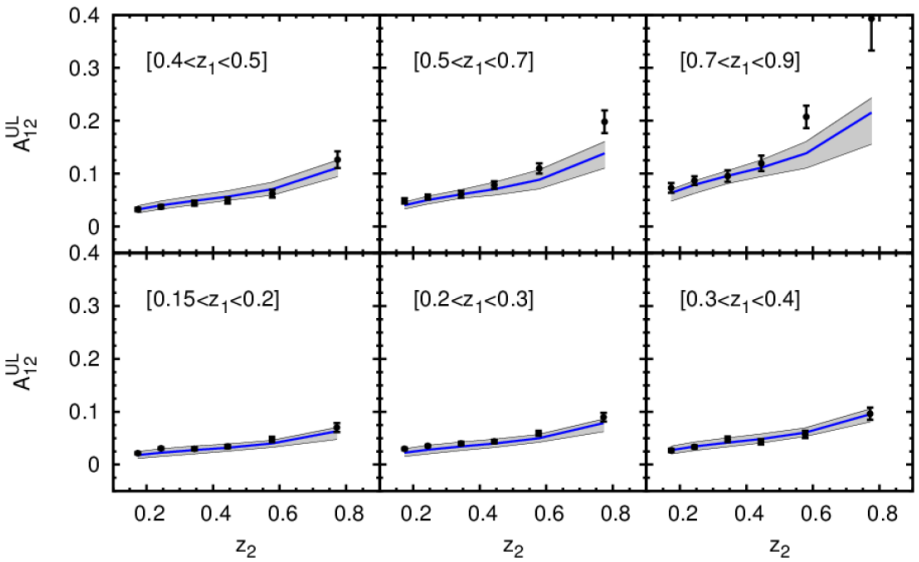
Still tension between A_0 and A_{12}

BABAR PREDICTIONS VS DATA

FIT I, standard

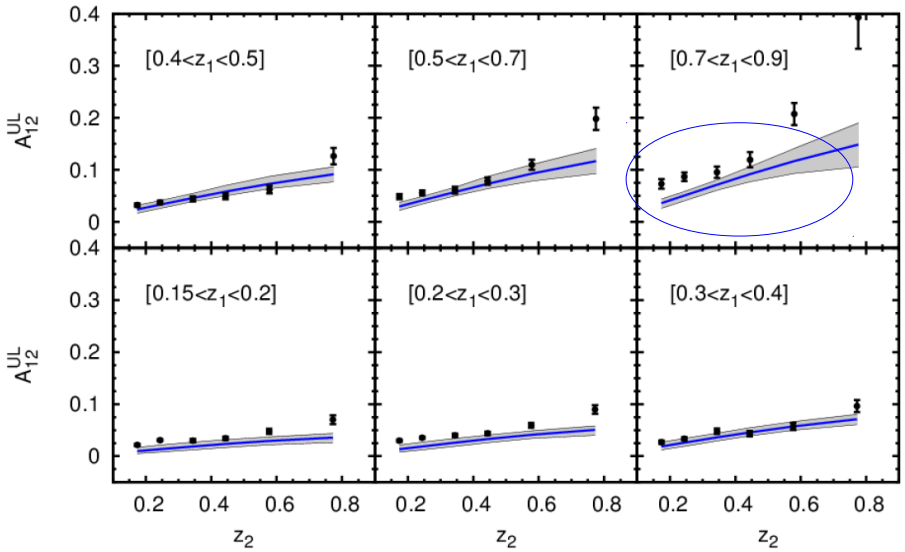


FIT IV, polynomial

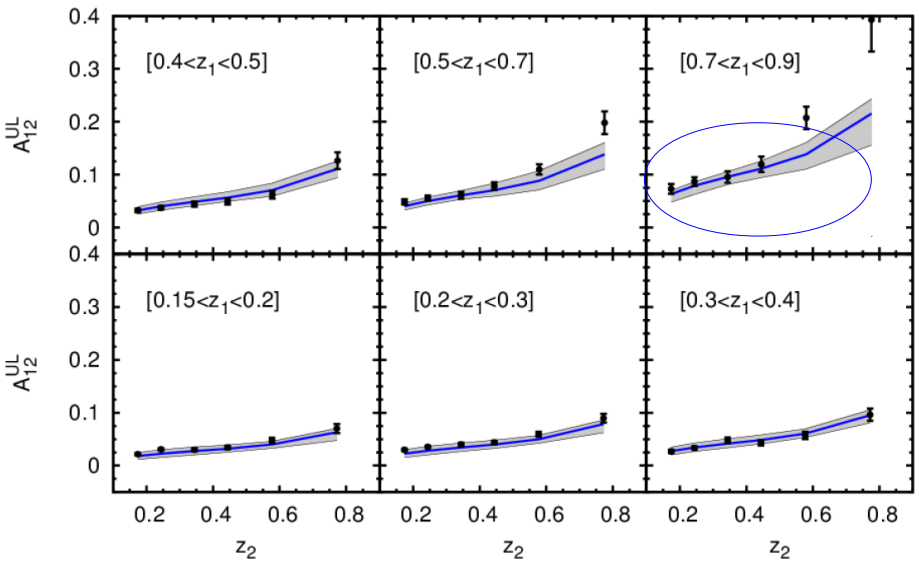


BABAR PREDICTIONS VS DATA

FIT I, standard



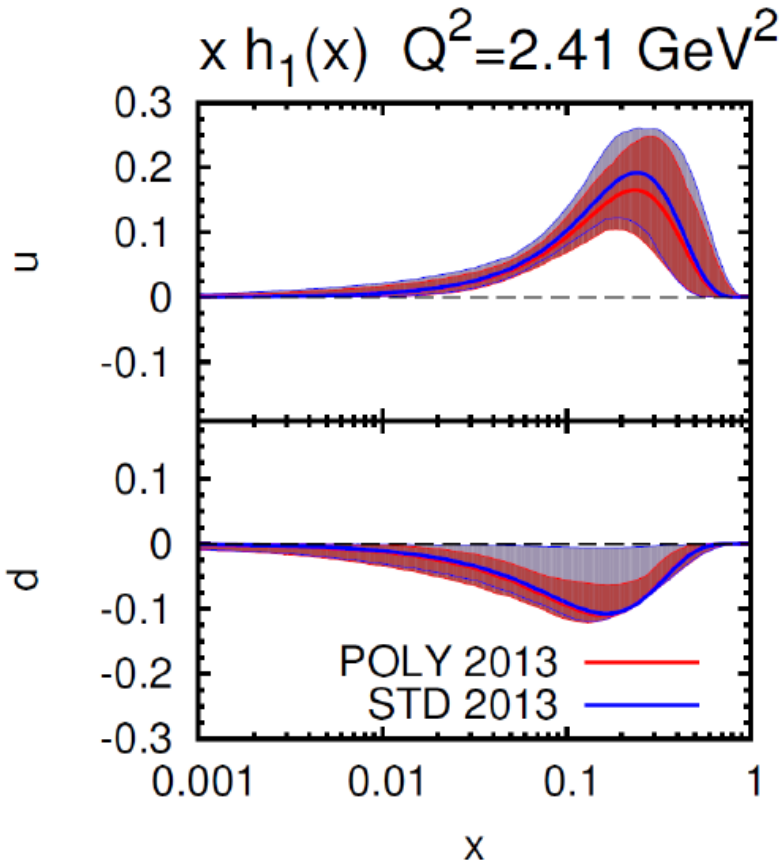
FIT IV, polynomial

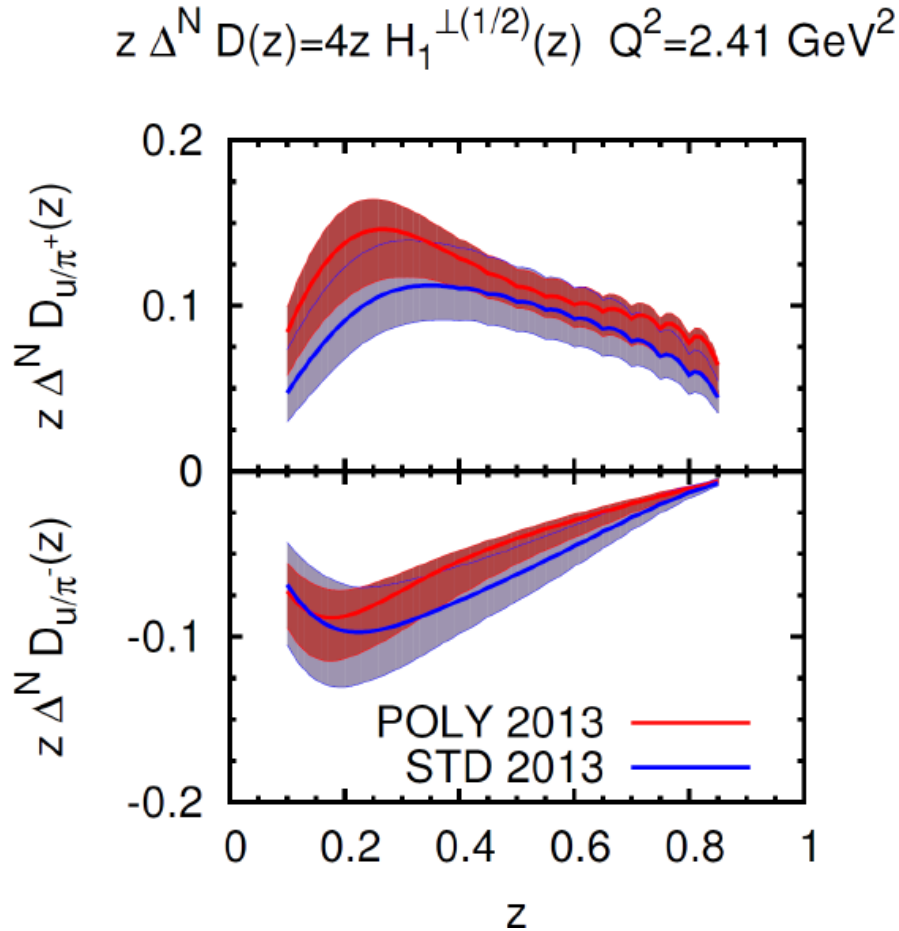


Better description of BaBar (no fit) with polynomial parametrization

RESULTS

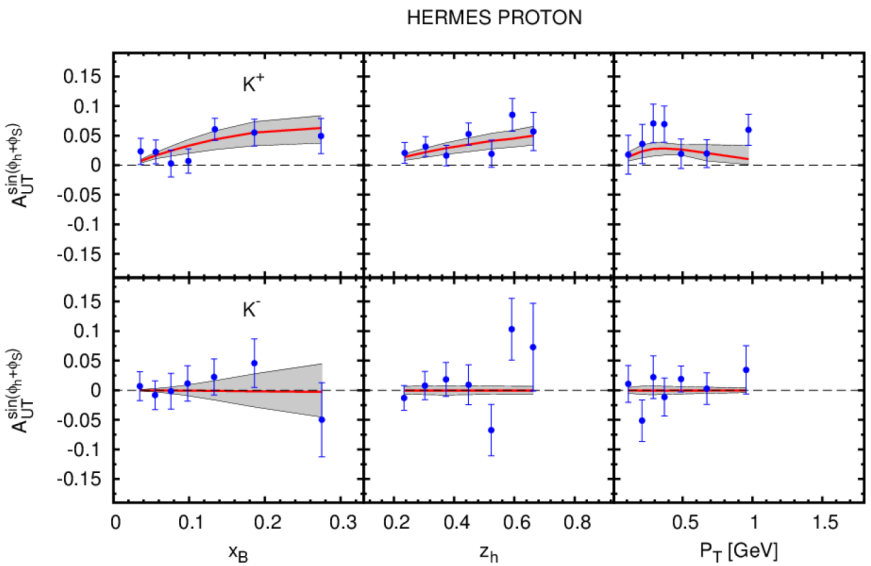
Anselmino et al (2013)

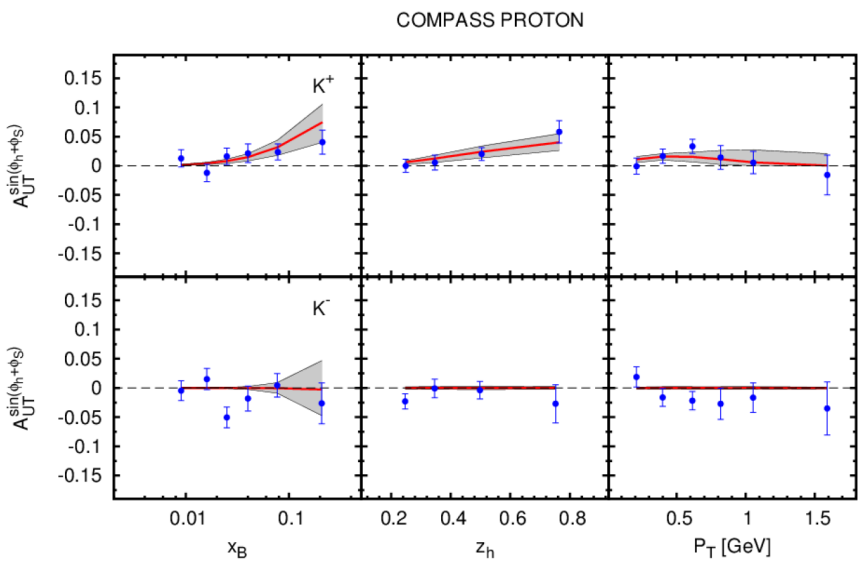




Transversity is very similar
Collins FF is slightly different

What about Kaons?





Disfavoured fragmentation is not determined by SIDIS
Positive favoured fragmentation

Beyond LO

Our extraction is based on LO (tree level) TMD factorization without evolution

Hard scales of SIDIS and e+e- are quite different

$$Q^2_{SIDIS} \sim 1 \div 10 \;\; (GeV^2)$$

$$Q^2_{e^+e^-} \sim 100 \;\; (GeV^2)$$

So far we used only DGLAP evolution in corresponding collinear input functions

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Evolution is needed → next lecture!