Current status of ε_K calculated on the lattice

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ε_K and $\hat{B}_K$, V_{cb} I

- Definition of ε_K

$$\varepsilon_K = \frac{A[K_L \to (\pi\pi)_{I=0}]}{A[K_S \to (\pi\pi)_{I=0}]}$$

- Relation between ε_K and $\hat{B}_K$ in standard model.

$$\varepsilon_K = \exp(i\phi_\varepsilon)\ \sin(\phi_\varepsilon)\ C_\varepsilon\ \mathrm{Im}\lambda_t\ X\ \hat{B}_K + \xi_0 + \xi_{LD}$$

$$X = \mathrm{Re}\lambda_c[\eta_1 S_0(x_c) - \eta_3 S_0(x_c, x_t)] - \mathrm{Re}\lambda_t \eta_2 S_0(x_t)$$

$$\lambda_i = V_{is}^* V_{id}, \qquad x_i = m_i^2/M_W^2, \qquad C_\varepsilon = \frac{G_F^2 F_K^2 m_K M_W^2}{6\pi^2 \Delta M_K}$$

$$\xi_0 = \exp(i\phi_\varepsilon)\sin(\phi_\varepsilon)\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} \approx 7\%$$

$\xi_{LD} =$ Long Distance Effect $\approx 2\%$ $\longrightarrow$ we neglect it!

ε_K and $\hat{B}_K$, V_{cb} II

- Inami-Lim functions:

$$S_0(x_t) = \frac{4x_t - 11x_t^2 + x_t^3}{4(1-x_t)^2} - \frac{3x_t^3 \ln(x_t)}{2(1-x_t)^3} \qquad \to 55\%$$

$$S_0(x_c, x_t) = x_c \left[\ln(\frac{x_t}{x_c}) - \frac{3x_t}{4(1-x_t)} - \frac{3x_t^2 \ln(x_t)}{4(1-x_t)^2} \right] \qquad \to 34\%$$

$$S_0(x_c) = x_c \qquad \to 11\%$$

- Dominant contribution (≈55%) comes with $|V_{cb}|^4$.

$$\mathrm{Im}\lambda_t \cdot \mathrm{Re}\lambda_t = \bar{\eta}\lambda^2 |V_{cb}|^4 (1-\bar{\rho})$$

$$\mathrm{Re}\lambda_c = -\lambda(1 - \frac{\lambda^2}{2}) + \mathcal{O}(\lambda^5)$$

$$\mathrm{Re}\lambda_t = -(1 - \frac{\lambda^2}{2}) A^2 \lambda^5 (1-\bar{\rho}) + \mathcal{O}(\lambda^7)$$

ε_K and $\hat{B}_K$, V_{cb} III

$$\mathrm{Im}\lambda_t = \eta A^2 \lambda^5 + \mathcal{O}(\lambda^7)$$

- Definition of B_K in standard model.

$$B_K = \frac{\langle \bar{K}_0 | [\bar{s}\gamma_\mu(1-\gamma_5)d][\bar{s}\gamma_\mu(1-\gamma_5)d] | K_0 \rangle}{\frac{8}{3}\langle \bar{K}_0 | \bar{s}\gamma_\mu\gamma_5 d | 0 \rangle \langle 0 | \bar{s}\gamma_\mu\gamma_5 d | K_0 \rangle}$$

$$\hat{B}_K = C(\mu) B_K(\mu), \qquad C(\mu) = \alpha_s(\mu)^{-\frac{\gamma_0}{2b_0}} [1 + \alpha_s(\mu) J_3]$$

- Experiment:

$$\varepsilon_K = (2.228 \pm 0.011) \times 10^{-3} \times e^{i\phi_\varepsilon}$$
$$\phi_\varepsilon = 43.52(5)^\circ$$

Wolfenstein Parameters

Input Parameters for Angle-Only-Fit (AOF)

- ϵ_K, $\hat{B}_K$, and $|V_{cb}|$ are used as inputs to determine the UT angles in the global fit of `UTfit` and `CKMfitter`.
- Instead, we can use angle-only-fit result for the UT apex ($\bar{\rho}$, $\bar{\eta}$).
- Then, we can take λ independently from

$$|V_{us}| = \lambda + \mathcal{O}(\lambda^7)\,,$$

which comes from K_{l3} and $K_{\mu 2}$.

- Use $|V_{cb}|$ instead of A.

$$|V_{cb}| = A\lambda^2 + \mathcal{O}(\lambda^7)$$

λ	$0.22535(65)$	[1] CKMfitter
	$0.22535(65)$	[1] UTfit
	$0.2252(9)$	[1] $\lvert V_{us}\rvert$ (AOF)
$\bar{\rho}$	$0.131^{+0.026}_{-0.013}$	[1] CKMfitter
	$0.136(18)$	[1] UTfit
	$0.130(27)$	[2] UTfit (AOF)
$\bar{\eta}$	$0.345^{+0.013}_{-0.014}$	[1] CKMfitter
	$0.348(14)$	[1] UTfit
	$0.338(16)$	[2] UTfit (AOF)

Input Parameters of B_K, V_{cb} and others

B_K

$\hat{B}_K$	0.7661(99)	[3] Lat Avg
	0.7379(47)(365)	[4] SWME

V_{cb}

V_{cb} $\times 10^{-3}$	42.42(86)	[1] Incl.
	39.04(49)(53)(19)	[5] Excl.

Others

G_F	$1.1663787(6) \times 10^{-5}$ GeV^{-2}	[1]
M_W	80.385(15) GeV	[1]
$m_c(m_c)$	1.275(25) GeV	[1]
$m_t(m_t)$	163.3(2.7) GeV	[6]
η_1	1.43(23)	[7]
η_2	0.5765(65)	[7]
η_3	0.496(47)	[8]
θ	$43.52(5)^\circ$	[1]
m_{K^0}	497.614(24) MeV	[1]
ΔM_K	$3.484(6) \times 10^{-12}$ MeV	[1]
F_K	156.1(8) MeV	[1]

ξ_0
Input Parameters

$$\xi_0 = \frac{\mathrm{Re}A_0}{\mathrm{Im}A_0}$$

ξ_0	$-1.63(19)(20) \times 10^{-4}$	[9]

- RBC-UKQCD collaboration performs lattice calculation of $\mathrm{Im}A_2$. From this result, ξ_0 can be obtained by the relation

$$\mathrm{Re}\left(\frac{\epsilon'_K}{\epsilon_K}\right) = \frac{1}{\sqrt{2}|\epsilon_K|}\omega\left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \xi_0\right).$$

  Other inputs ω, ϵ_K and ϵ'_K/ϵ_K are taken from the experimental values.
- Here, we choose an approximation of $\cos(\phi_{\epsilon'} - \phi_\epsilon) \approx 1$.
- $\phi_\epsilon = 43.52(5)$, $\phi_{\epsilon'} = 42.3(1.5)$

ϵ_K: Lat. Avg. $\hat{B}_K$, AOF of $(\bar{\rho}, \bar{\eta})$, V_{us}

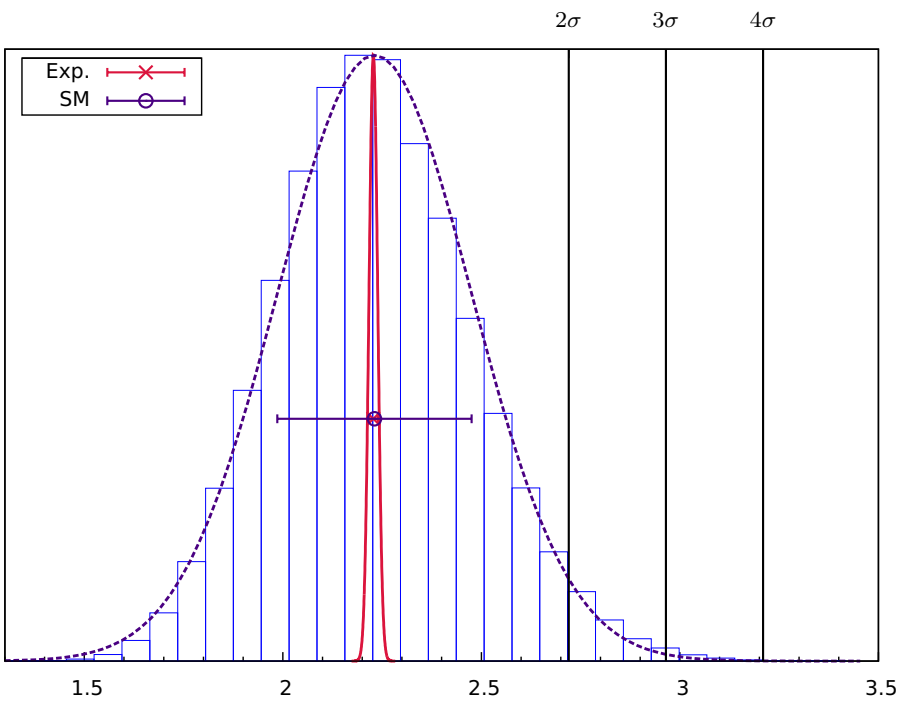

Figure: Inclusive V_{cb}

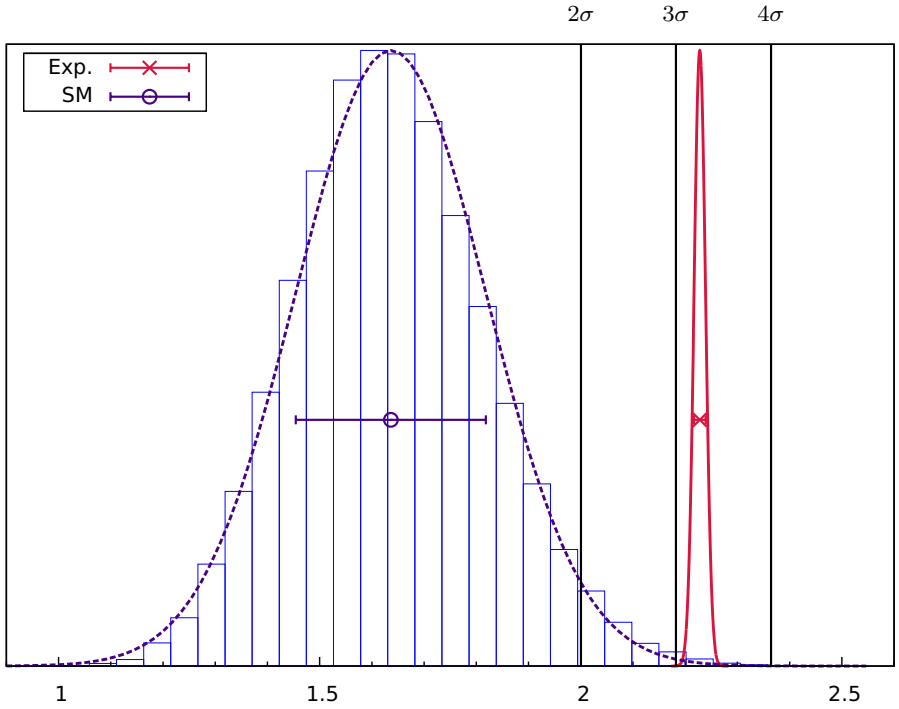

Figure: Exclusive V_{cb}

- With exclusive V_{cb}, it shows 3.3σ tension.

$$\epsilon_K^{Exp} = 2.228(11) \times 10^{-3}$$
$$\epsilon_K^{SM} = 1.636(182) \times 10^{-3}$$

ϵ_K: SWME $\hat{B}_K$, AOF of $(\bar{\rho}, \bar{\eta})$, V_{us}

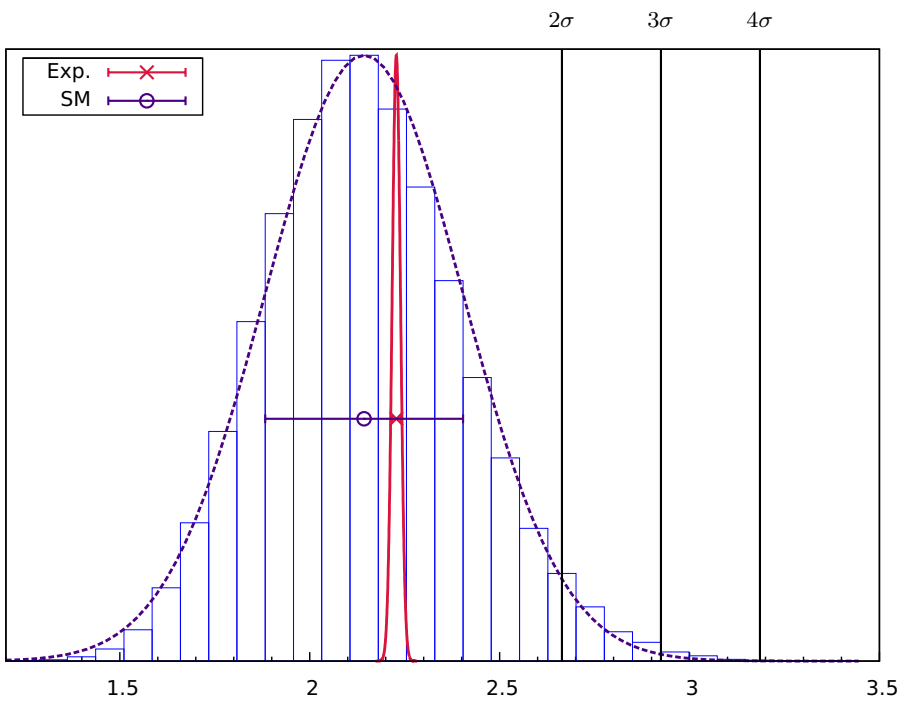

Figure: Inclusive V_{cb}

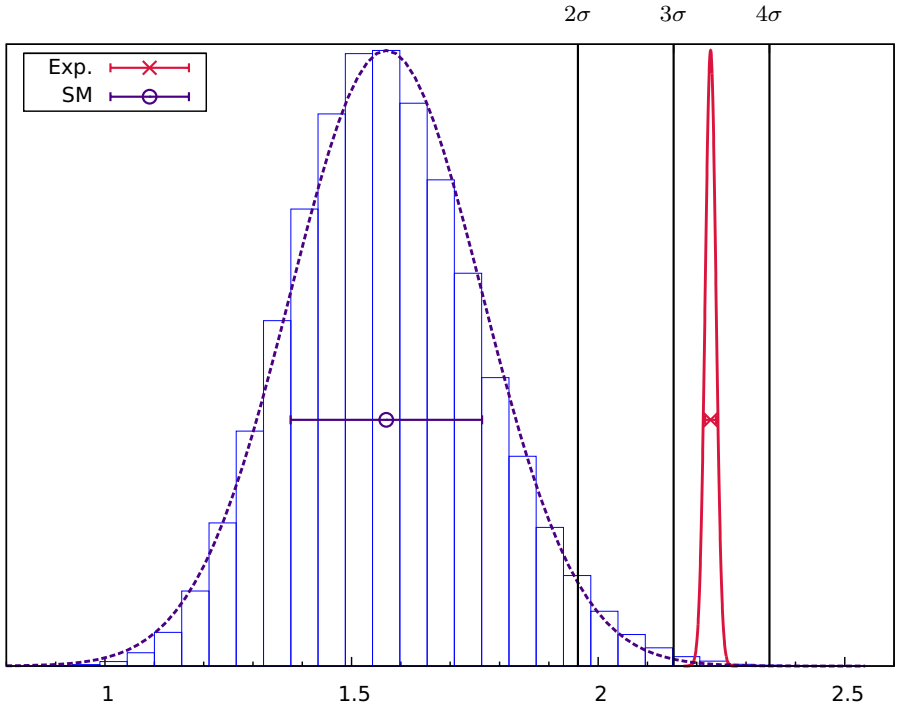

Figure: Exclusive V_{cb}

- With exclusive V_{cb}, it shows 3.4σ tension.

$$\epsilon_K^{Exp} = 2.228(11) \times 10^{-3}$$
$$\epsilon_K^{SM} = 1.570(195) \times 10^{-3}$$

ϵ_K: SWME $\hat{B}_K$, CKMfitter $(\bar{\rho}, \bar{\eta}, \lambda)$

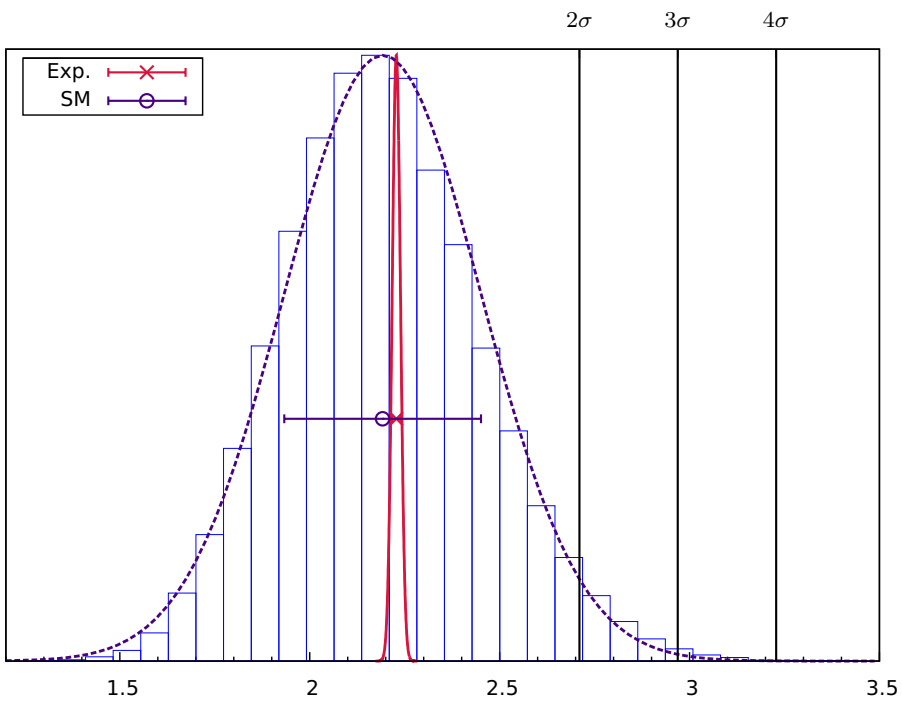

Figure: Inclusive V_{cb}

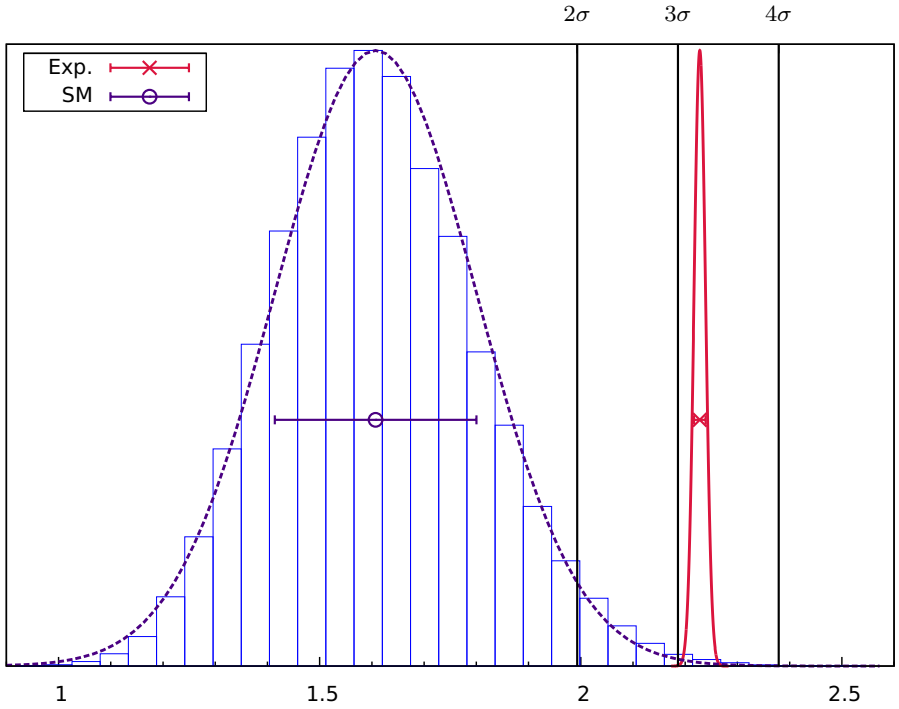

Figure: Exclusive V_{cb}

- With exclusive V_{cb}, it shows 3.2σ tension.

$$\epsilon_K^{Exp} = 2.228(11) \times 10^{-3}$$
$$\epsilon_K^{SM} = 1.607(193) \times 10^{-3}$$

ϵ_K: SWME $\hat{B}_K$, UTfit $(\bar{\rho}, \bar{\eta}, \lambda)$

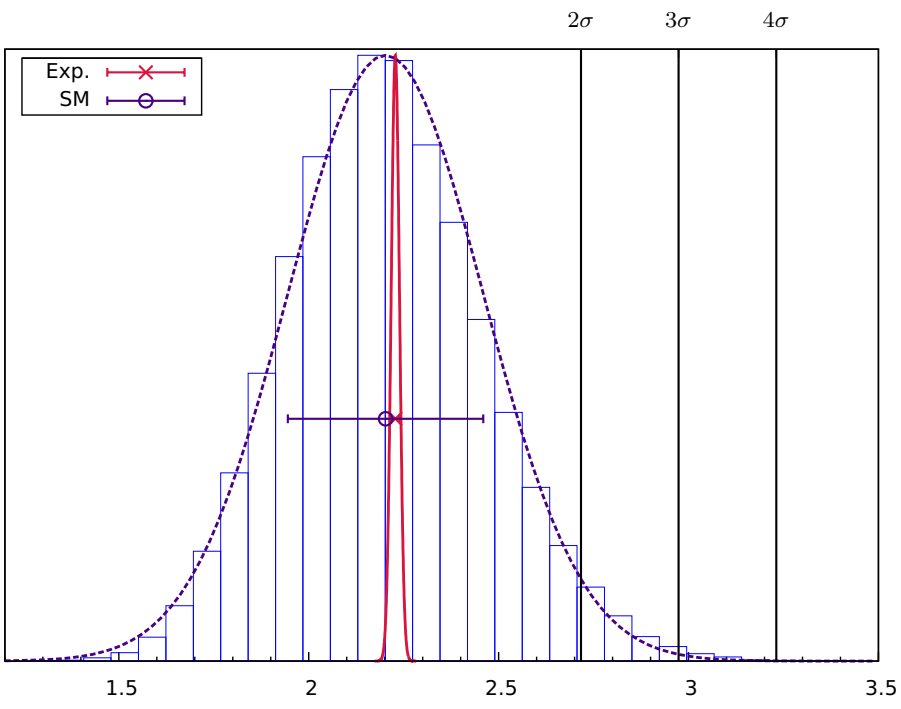

Figure: Inclusive V_{cb}

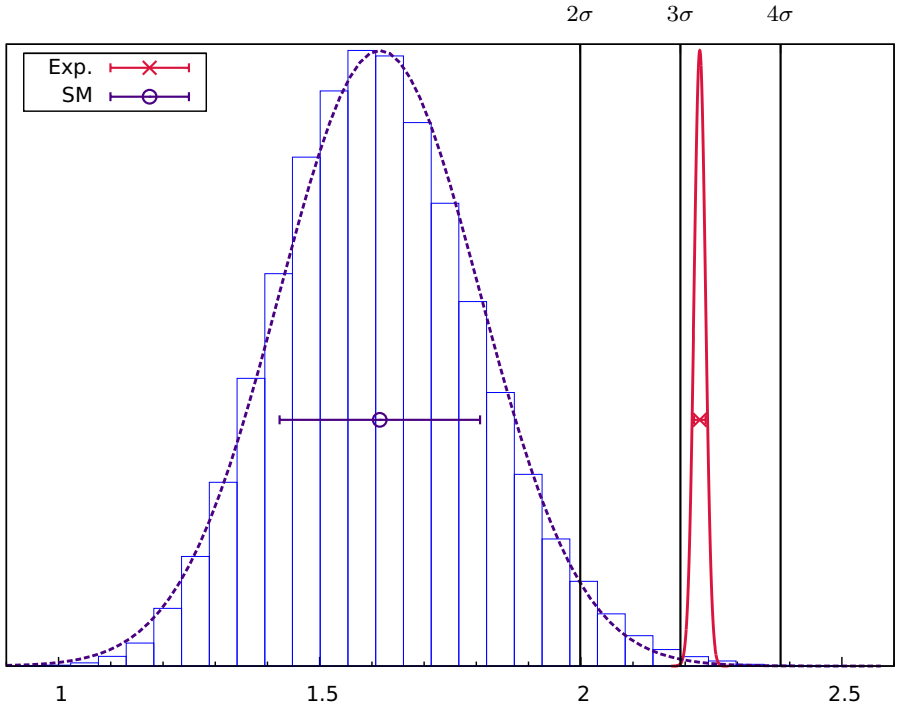

Figure: Exclusive V_{cb}

- With exclusive V_{cb}, it shows 3.2σ tension.

$$\epsilon_K^{Exp} = 2.228(11) \times 10^{-3}$$
$$\epsilon_K^{SM} = 1.615(192) \times 10^{-3}$$

Current Status of ε_K

- SWME 2014: (in units of 1.0×10^{-3}, AOF)

$$\varepsilon_K = 1.57 \pm 0.19 \qquad \text{for Exclusive } V_{cb} \text{ (Lattice QCD)}$$
$$\varepsilon_K = 2.14 \pm 0.26 \qquad \text{for Inclusive } V_{cb} \text{ (QCD Sum Rule)}$$

- Experiments:

$$\varepsilon_K = 2.228 \pm 0.011$$

- Hence, we observe 3.4(3) σ difference between the SM theory (Lattice QCD) and experiments.

- What does this mean? $\longrightarrow$ Breakdown of SM ?

Error Budget of Exclusive ε_K

cause	error (%)	memo
V_{cb}	33.7	Exclusive (FNAL/MILC)
B_K	19.7	SWME
$\bar{\eta}$	17.6	Wolfenstein parameter
η_3	13.8	η_{ct}
η_1	4.1	η_{cc}
$\bar{\rho}$	3.7	Wolfenstein parameter
ξ_0	1.9	Im(A_0)/Re(A_0)
m_c	0.8	Charm quark mass
$\vdots$	$\vdots$	$\vdots$

Conclusion and Future Outlook

1. Lattice determination of ε_K from the standard model with the exclusive V_{cb} channel shows 3.4(3) σ tension compared with the experiment.
2. However, in the inclusive V_{cb} channel determined from the QCD sum rules, we do not observe the same kind of tension.
3. The dominant systematic error in ε_K comes from V_{cb} in the exclusive channel.
4. Hence, it is very crucial to reduce the theoretical error of V_{cb} down to the $\leq 0.5\%$ level: $\rightarrow$ the OK action.
5. Thank God very much for your help!!!

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