Scaling studies of an improved actions on quenched lattices

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motivation

- For precise calculation of heavy-light (D and B) and heavy-heavy (Ψ and Υ) systems, e.g. for determination of the Standard Model parameters, we need good control of discretization effects of $(am_Q)^n$.

- We plant to use the fine-lattice ensembles (a^{-1}=2.4-4.8GeV, JLQCD collaboration) with Möbius domain-wall fermions[see J.I.Noaki's talk]. Still, am_Q is not very small (am_Q~0.54-0.27 for charm). Improved fermion formulations would be necessary.

- We test existing and newly developed formulations
  - as joint effort of the JLQCD collaboration and Southampton.

Plan of this Talk

1. Lattice fermion formulation for heavy quarks
    - generalized Domain-wall fermions
    - $O(a^2)$-improved Brillouin fermion
2. Scaling studies on quenched lattices
    - Dispersion relation,hyperfine splitting of PS meson
    - Decay constant for heavy-heavy systems
3. Summary

1. Fermion formulation for heavy quarks

- generalized domain-wall fermions
- $O(a^2)$-improved Brillouin fermion

Lattice fermions

- Wilson fermions
  - Discretization error is O(am). O(a)-improvement is often considered, then $O(am)^2$
- Domain-wall fermions
  - preserve chiral symmetry. Discretization error is $O(am)^2$. Limitation on the value of am.
- Improved fermions
  - We developed an $O(a^2)$-improved fermion formulation based on the Brillouin fermion. It has good properties (dispersion relation, ...) for heavy quarks.

Generalized domain-wall fermions

[Kaplan, 1992; Shamir, 1993; Furman-Shamir, 1995; Edwards-Heller 2001; Boric¸i, 1999; Chiu, 2003; Brower-Neff-Orginos, 2005; JLQCD 2013]

- live on the 5D space-time, exact chiral symmetry at $L_s \to \infty$(Overlap fermions), but expensive.

- 4D effective action (with Möbius kernel)

$$D^4_{DW} = \frac{1+m}{2} + \frac{1-m}{2}\gamma_5 tanh\left(L_s tanh^{-1}\left(\gamma_5 \frac{bD_w}{2+cD_w}\right)\right)$$

- From a residual-mass study, we employ b=2, c=1 with link smearing (N_{smr}=3) and finite L_s(=8).(JLQCD collaboration)

- m_{res} is at the level of 0.1~0.5 MeV on the dynamical lattices.

- We expect that the good chiral symmetry guarantees small O(a) discretization effect.

Brillouin fermions

[S.Durr,G.Koutsou Phys.RevD83(2011)114512]

[M.Creutz,T.Kimura, T.Misumi JHEP 1012:041,2010]

$$D^{Bri}(n,m) = \sum_{\mu} \gamma_\mu \nabla_\mu^{iso}(n,m) - \frac{a}{2} \triangle^{bri}(n,m) + m_0 \delta_{n,m}$$

Derivative term

$$\nabla_x^{std}\psi_n = \frac{1}{2a}\left(\psi_{n+\hat{x}} - \psi_{n-\hat{x}}\right)$$

$$\simeq \partial_x \psi_n + \frac{a^2}{6}\partial_x^3 \psi_n$$

$$= \left(1 + \frac{a^2}{6}\partial_x^2\right)\partial_x \psi_n \quad \Rightarrow \quad \left(1 + \frac{a^2}{6}\triangle\right)\partial_x \psi_n$$

unisotropic error

isotropic error

isotropic x-derivative

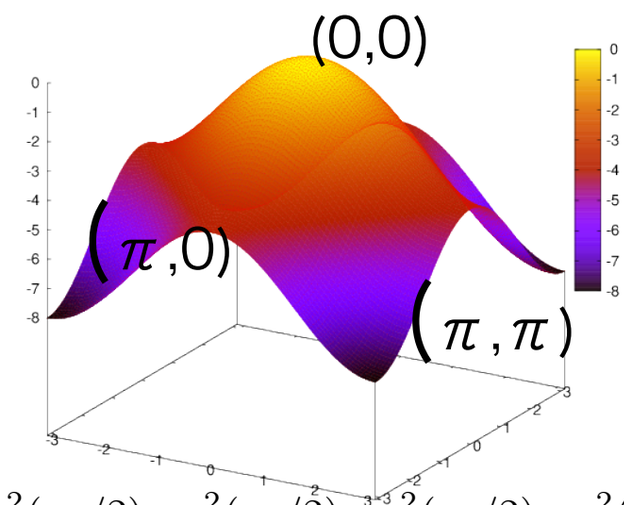
$$\nabla_x^{iso}\psi_n = \left(1 + \frac{a^2}{6}\partial_y^2\right)\left(1 + \frac{a^2}{6}\partial_x^2\right)\partial_x \psi_n$$

$$= \left(1 + \frac{a^2}{6}\partial_y^2\right)\nabla_x^{std}\psi_n$$

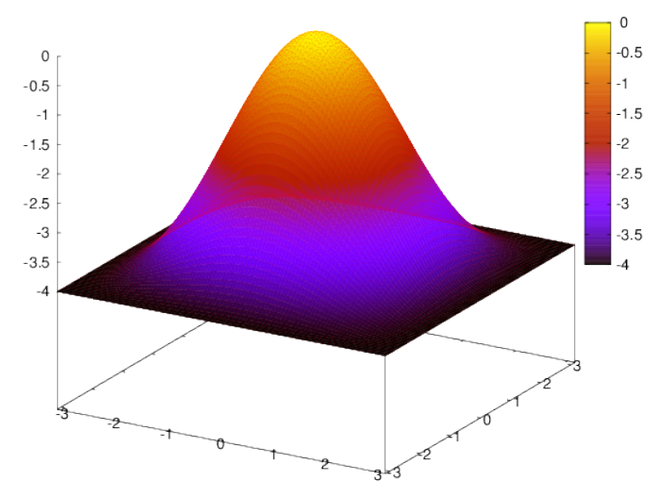
=>add a 2-hop term (in 2D)

Laplacian term

$$\triangle^{std}(p) = 2(cos(p_x) + cos(p_y) + cos(p_z) + cos(p_t) - 4)$$



$$\triangle^{bri}(p) = 4cos^2(p_x/2)cos^2(p_y/2)cos^2(p_z/2)cos^2(p_t/2) - 4$$

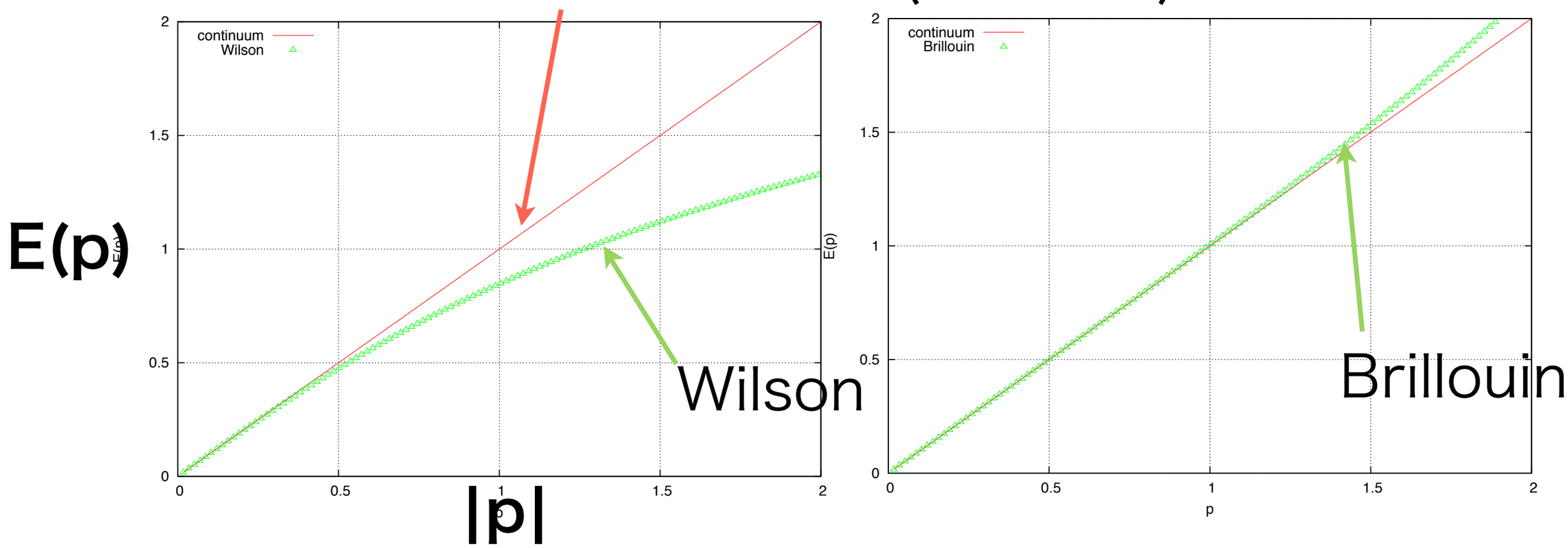


dispersion relation (free)

estimate the energy E(p) from the pole of $D^{-1}(p)$ in the momentum space.

m = 0.0

$$E(\vec{p}, m) = \sqrt{\vec{p}^2 + m^2}$$ (continuum)



Dispersion relation of meson, baryon is good too. Difference of Wilson and Brillouin becomes more significant at heavy quark regions.

[S.Durr, G.Koutsou, T.Lippert Phys.Rev.D86(2012) 114514]

Discretization errors for Brillouin fermions

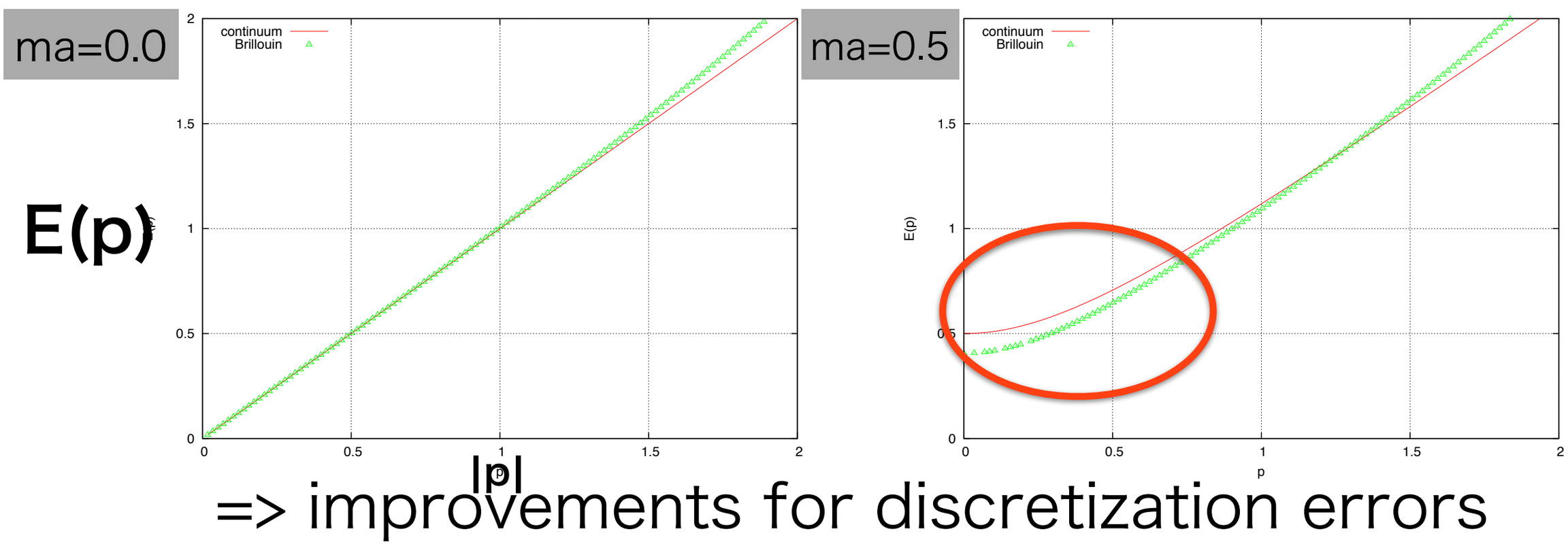
- expand the energy up to O(a^5)

$$E\left(\vec{0}, ma\right)^2 = (ma)^2 - (ma)^3 + \frac{11}{12}(ma)^4 - \frac{5}{6}(ma)^5$$

$$E = log(1 + ma)$$

=> O(a),O(a^2),O(a^3) errors

- dispersion relation for massive quarks



=> improvements for discretization errors

Symanzik improvement for Brillouin fermions

- eliminate O(a) and O(a^2)-errors at tree-level

- Improved Brillouin Dirac operator

$$D^{IB} = \sum_\mu \gamma_\mu (1 - \frac{a^2}{12}\triangle^{bri})\nabla_\mu^{iso}(1 - \frac{a^2}{12}\triangle^{bri}) + c_{IB}a^3\left(\triangle^{bri}\right)^2 + ma \qquad c_{IB} = 1/8$$

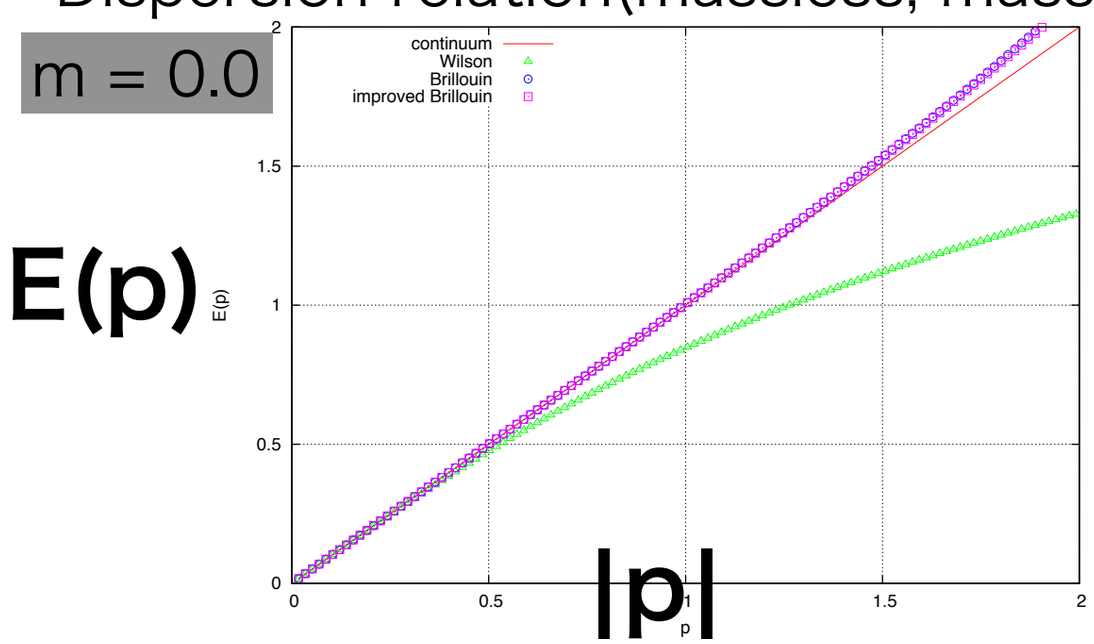
- expansion of energy up to O(a^5)

$$E^2\left(\vec{0}, ma\right) = (ma)^2 + \boxed{\frac{(ma)^5}{4}}$$

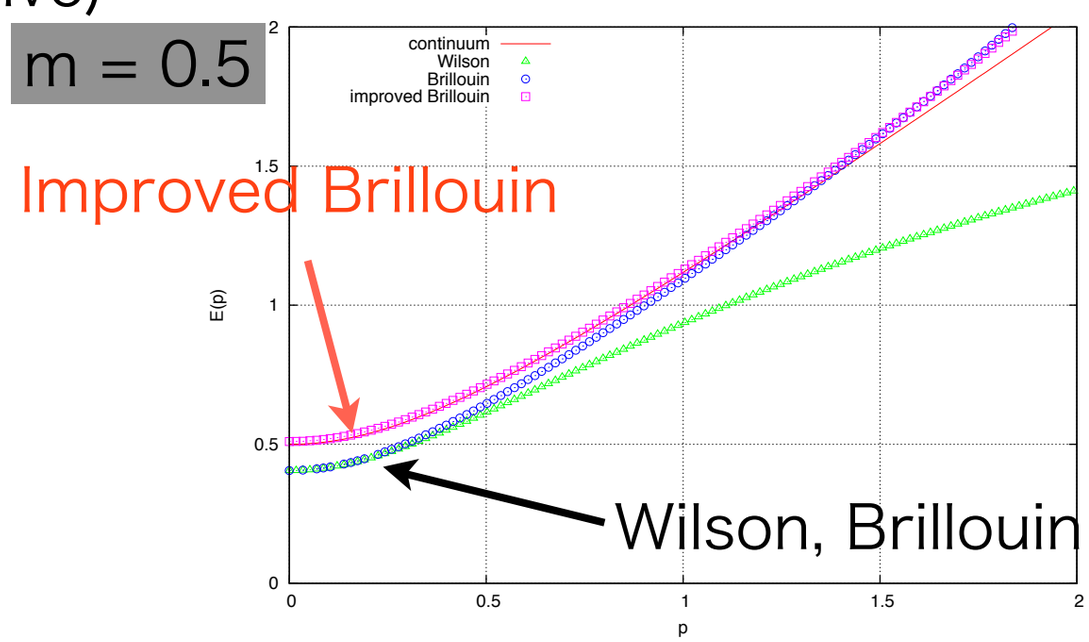
~~O(a),O(a2) errors~~ => start from O(a^3)

- Dispersion relation(massless, massive)

m = 0.0



m = 0.5



2. Scaling studies on the quenched lattices

- Dispersion relation of pseudo-scalar meson, hyperfine splitting

- Decay constant for heavy-heavy systems

Strategy

Test for scaling

- dispersion relation, hyperfine splitting
- decay constant

on quenched configurations of a^{-1}=2.0, 2.8. 3.8 GeV

×

With

- $O(a^2)$-improved Brillouin fermion
- Generalized Domain-wall fermion

against the naive Wilson fermion.

QUENCHED CONFIGURATIONS

[see J.Tsang's talk]

L/a	β	a[fm]	a
16	4.41	0.0987(34)	2.00(07)
24	4.66	0.0702(22)	2.81(09)
32	4.89	0.0350(13)	3.80(12)

- Tree-level Symanzik gauge action
- Generated with CHROMA using Heat bath algorithm (IRIDIS HPC Facility, University of Southampton)
- L is kept fixed to ~1.6 fm. Lattice spacing is determined through the Wilson flow w_0 introduced in [BMW-c, arXiv:1203.4469]

2. Scaling studies on the quenched lattices

- Dispersion relation of pseudo- scalar meson, hyperfine splitting

- Decay constant for heavy-heavy systems

Dispersion relation of PS meson, hyperfine splitting

- For heavy-heavy mesons of

$$m_{ps} = 1.0, 1.5, 2.0, 2.5, 3.0[GeV]$$

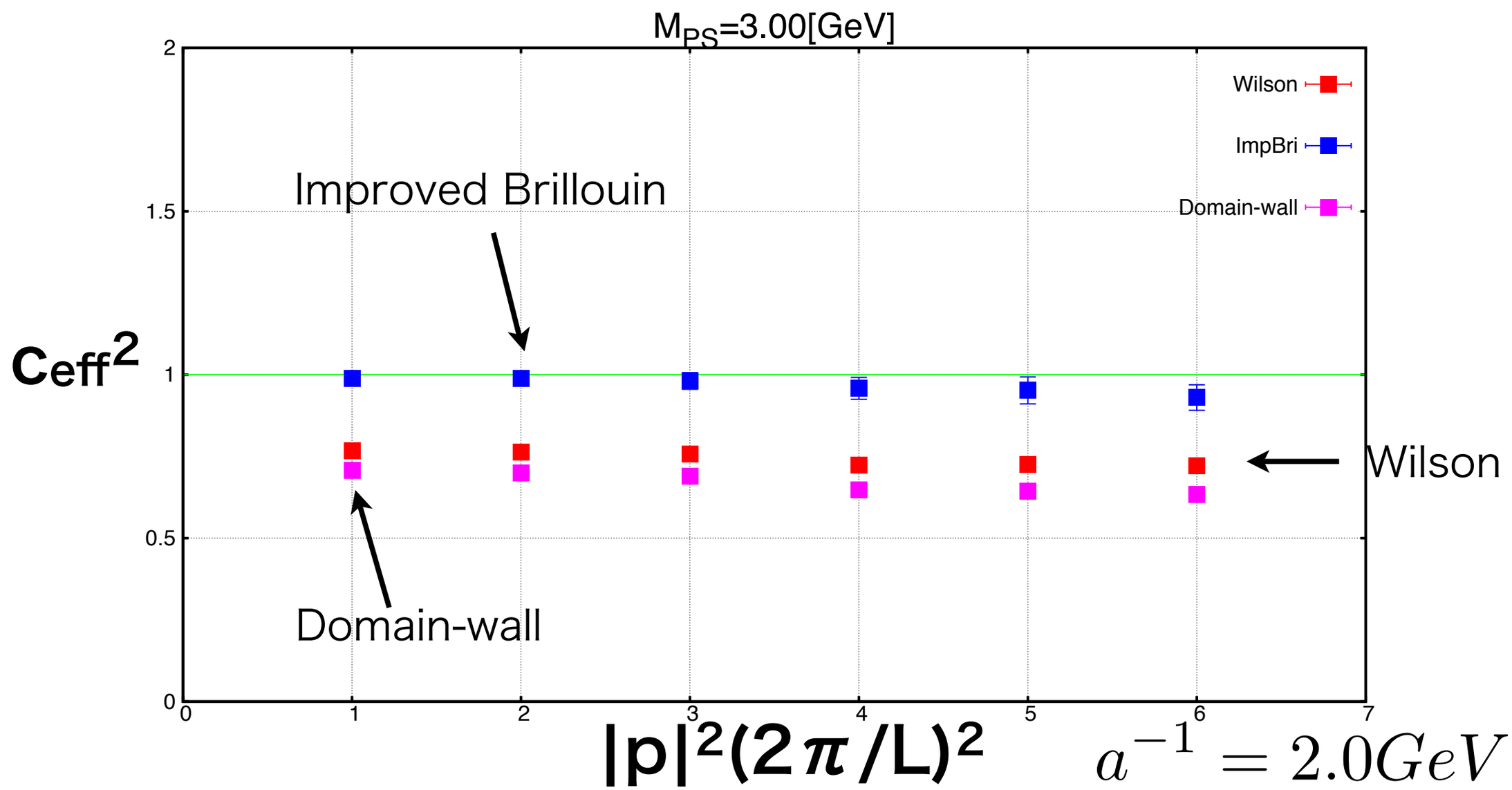
- We calculate effective speed of light for the pseudo-scalar meson

$$c_{eff}^2\left(p^2\right) = \frac{E^2(\overrightarrow{p}) - E^2(\overrightarrow{0})}{\overrightarrow{p}^2}$$

- Hyperfine splitting $m_{vec} - m_{ps}$

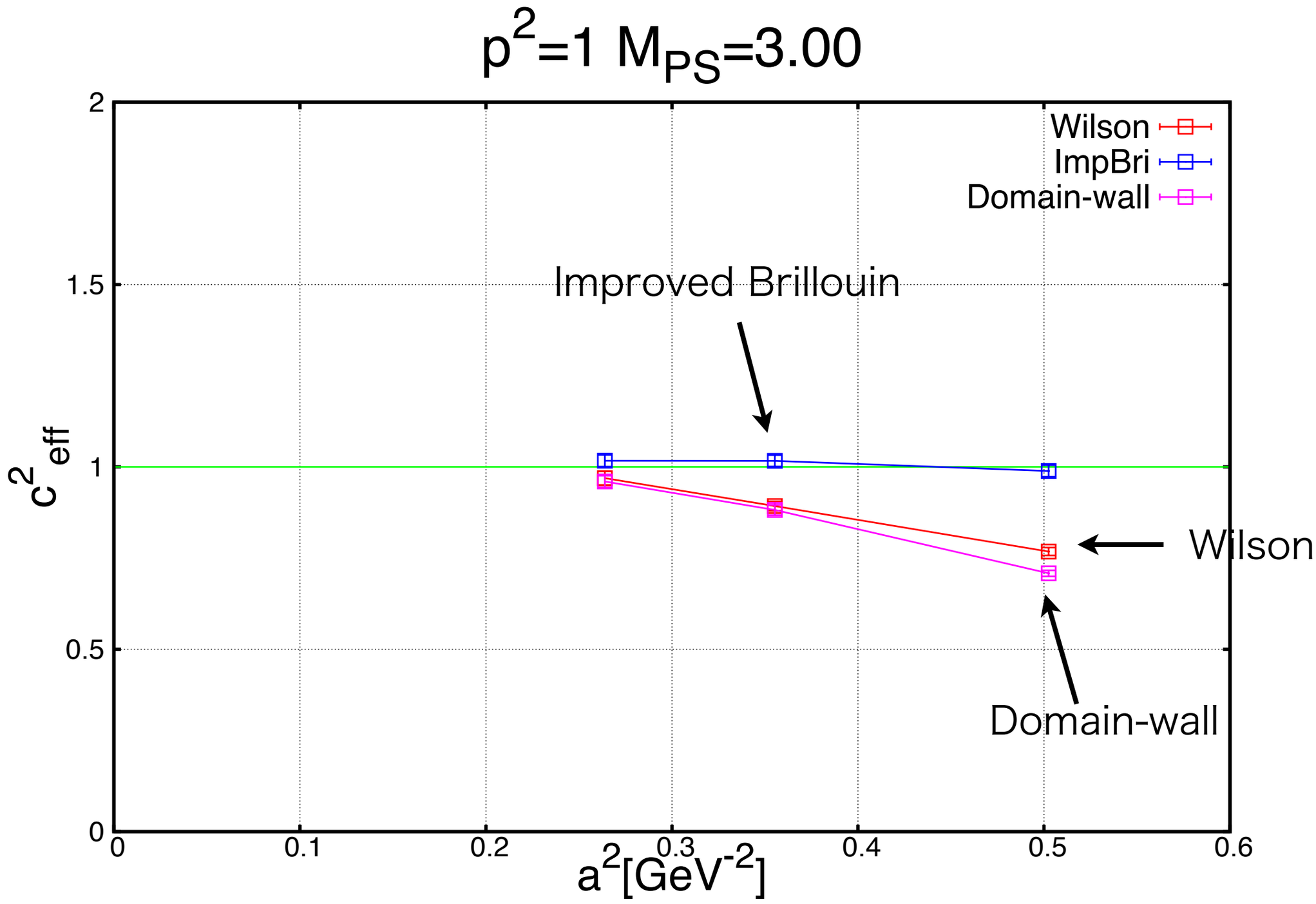
Effective speed-of-light for pseudo-scalar meson

$$C(t,\vec{p}) = \sum_{\vec{x}} \left\langle J(\vec{x},t)\,\bar{J}\left(\vec{0},0\right)\right\rangle e^{i\vec{p}\,\vec{x}} \quad \Rightarrow \quad c_{eff}^2(p^2) = \frac{E^2(\vec{p}) - E^2(\vec{0})}{\vec{p}^{\,2}}$$



$a^{-1} = 2.0GeV$

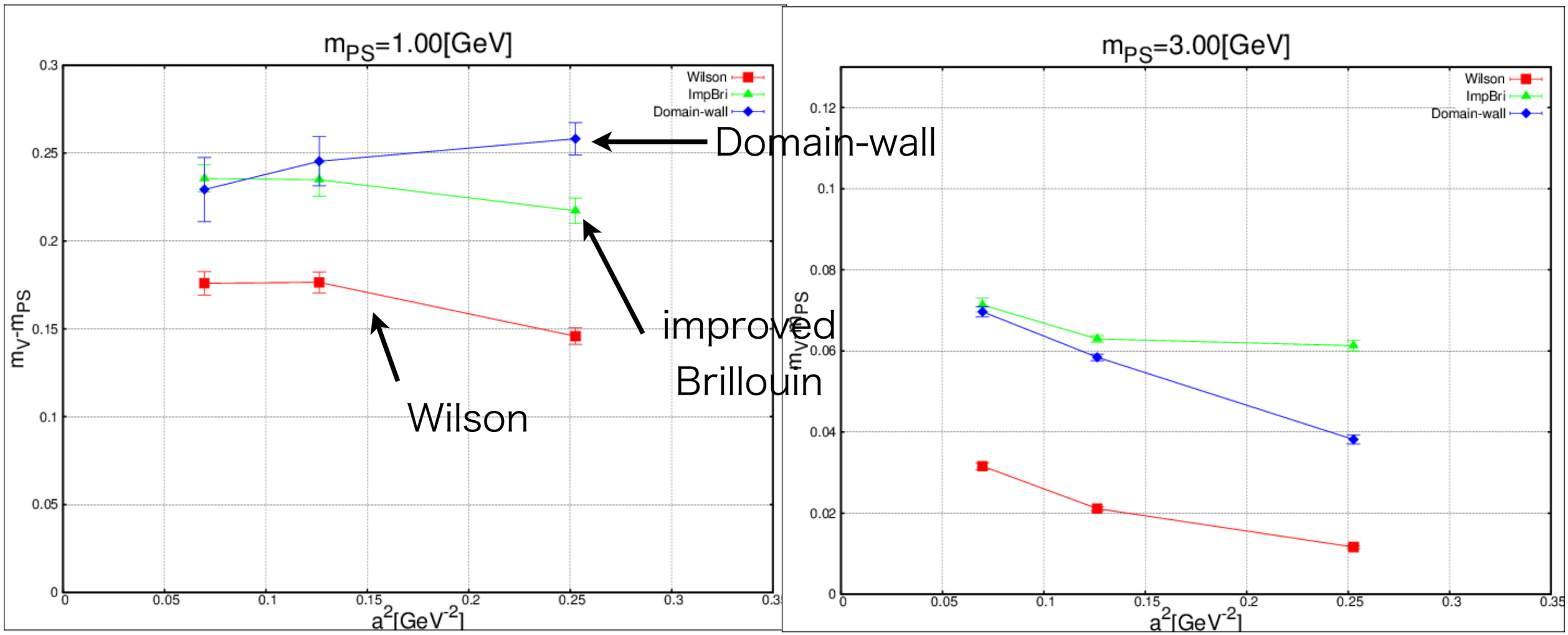
scaling for speed of light



⇒excellent scaling of the improved action

scaling for hyperfine splitting

hyperfine splitting = $m_{vec} - m_{ps}$



⇒Scaling for the improved action is good.

Domain-wall is slightly worse.

2. Scaling studies on the quenched lattices

- Dispersion relation of pseudo-scalar meson, hyperfine splitting
- Decay constant for heavy-heavy systems

heavy-heavy Decay constant

- Calculate ratio of decay constant to avoid undetermined renormalization factor Z_A.

$$\frac{\sqrt{m_{PS}} f_{PS}}{\sqrt{m_{ref}} f_{ref}}$$

- reference value is m_{PS} = 1.0[GeV]

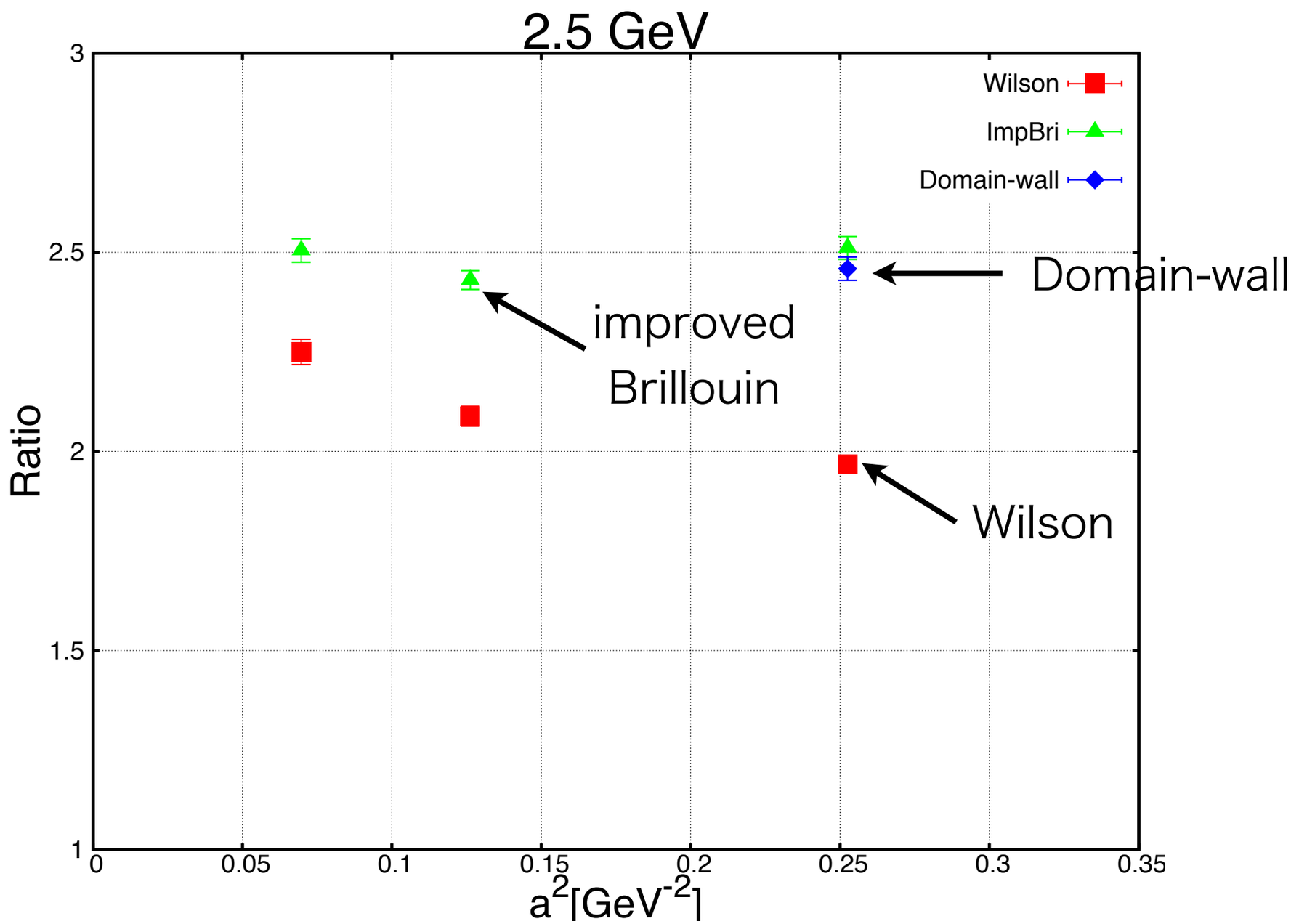
- We used local-local <AA>,<AP> and smeard-local <PP> simultaneously.

- Some data points have but signal that we discarded.

$$< A_4(t,x) A_4(0,0) > \quad < A_4(t,x) P(0,0) > \quad < P(t,x) P(0,0) >$$

$$A_\mu(x) = \bar{\psi}(x)\gamma_\mu\gamma_5\psi(x) \quad P(x) = \bar{\psi}(x)\gamma_5\psi(x)$$

Scaling of decay constant ratio



⇒scaling of the improved action is good.

3.Summary

- Scaling of the improved action is excellent for all three quantities.

- Domain-wall fermion is good scaling, except for the dispersion relation. (but for decay constant, data is limited.) [see J.Tsang's talk for unsmeard Domain-wall]

- We apply these fermion formalisms for charm quarks physics(f_D,f_{Ds},$\cdots$) on the dynamical domain-wall configurations(a^{-1}=2.4-4.8GeV).

Thank you!

Back Up

LAPLACIAN TERM:(BRILLOUIN LAPLACIAN)

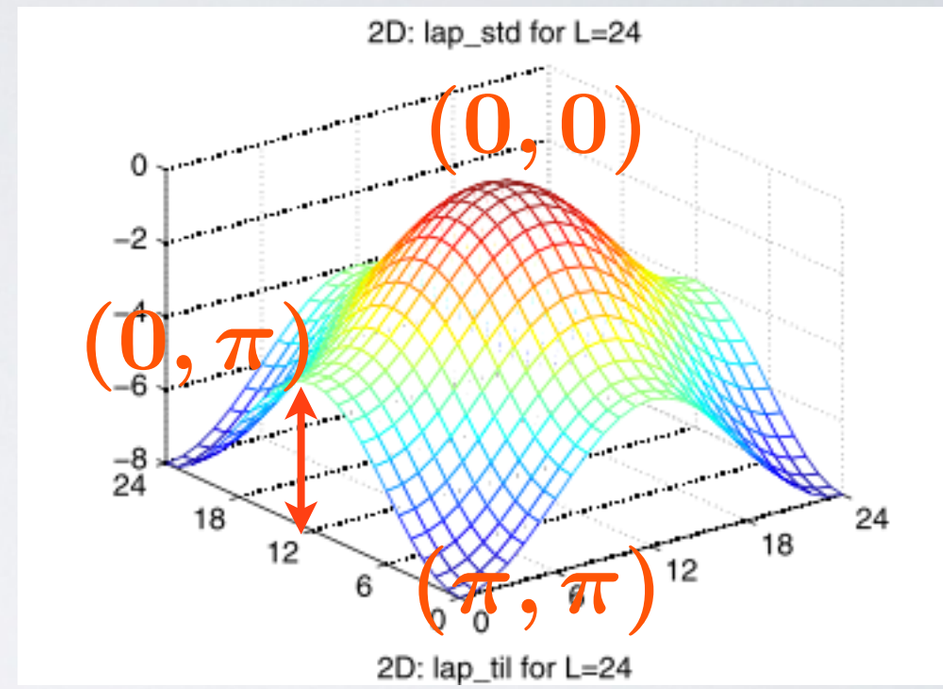
$$\triangle^{std}(p) = 2(cos(p_x) + cos(p_y) + cos(p_z) + cos(p_t) - 4)$$

$$\triangle^{bri}(p) = 4cos^2(p_x/2)cos^2(p_y/2)cos^2(p_z/2)cos^2(p_t/2) - 4$$

$\triangle^{std}$

momentum space(2D)

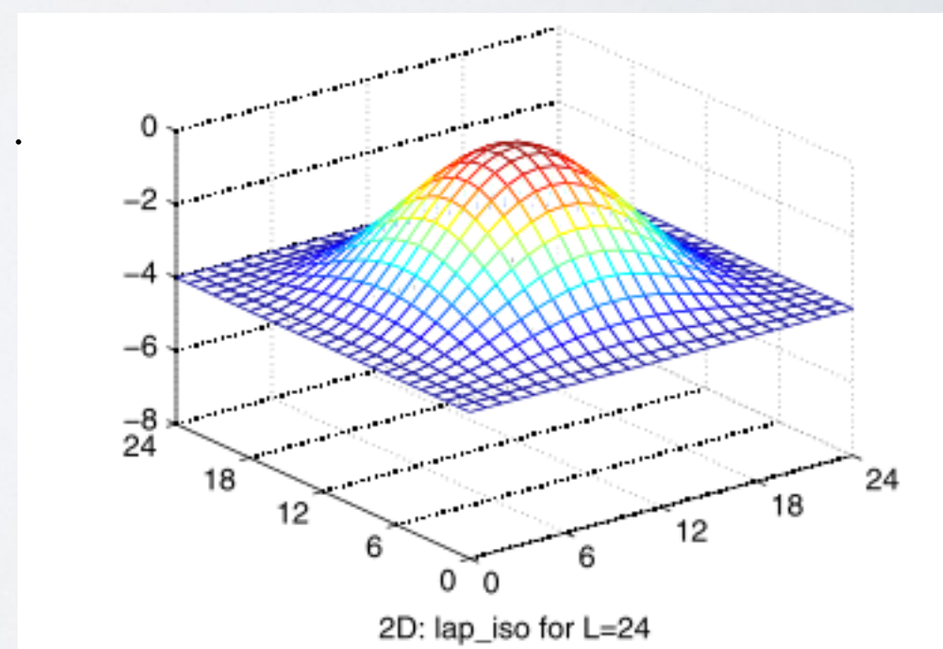
[S.Durr,G.Koutsou Phys.RevD83(2011)114512]



$$M(p) = M - \frac{r}{2}\triangle^{std}(p)$$
$$= M - r(cos(px) + cos(py) + cos(pz) + cos(pt) - 4)$$

$$p_\mu = (0,0,0,0) \;\rightarrow\; M(p) = M \quad (\times 1)$$
$$p_\mu = (\pi,0,0,0),\cdots \rightarrow\; M(p) = M + 2r \;\; (\times 4)$$
$$p_\mu = (\pi,\pi,0,0),\cdots \rightarrow\; M(p) = M + 4r \;\; (\times 6)$$
$$p_\mu = (\pi,\pi,\pi,0),\cdots \rightarrow\; M(p) = M + 6r \;\; (\times 4)$$
$$p_\mu = (\pi,\pi,\pi,\pi),\cdots \rightarrow\; M(p) = M + 8r \;\; (\times 1)$$

$\triangle^{bri}$

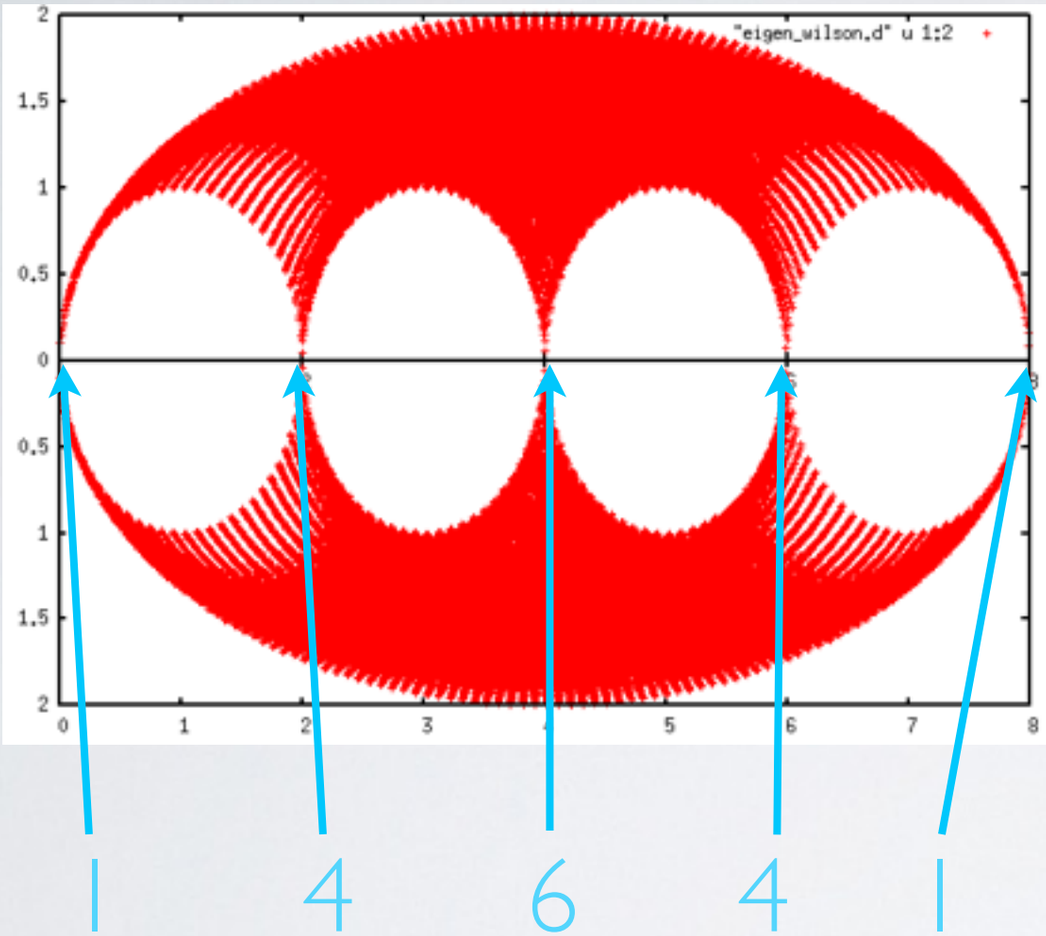


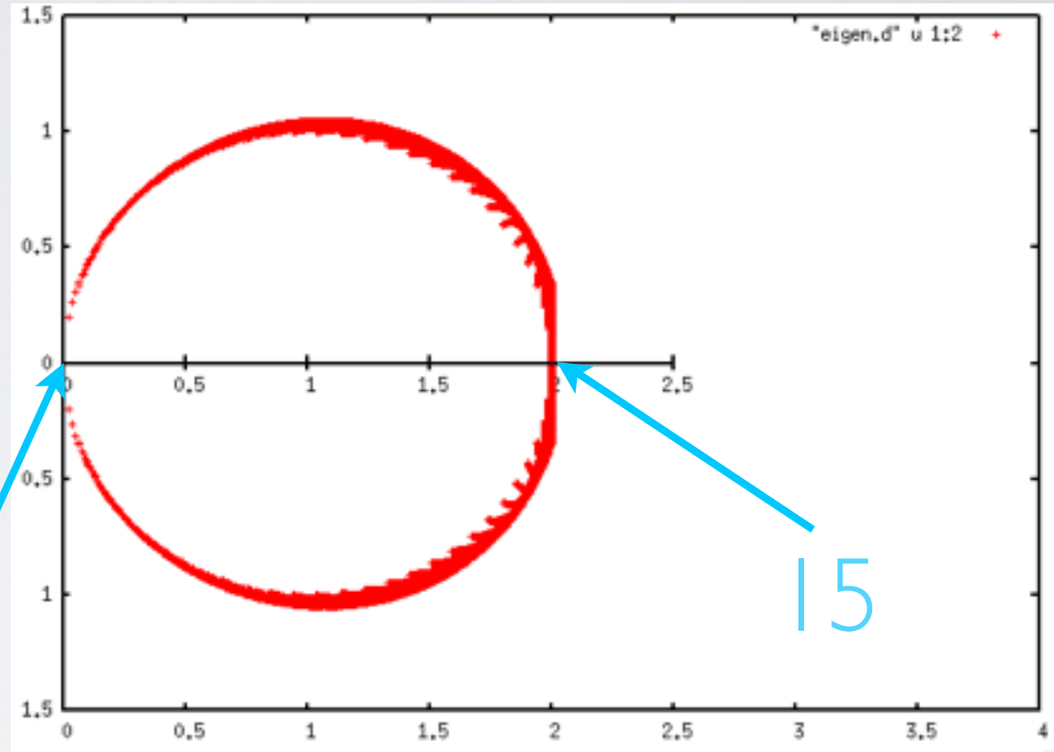
$$M(p) = M - \frac{r}{2}\triangle^{bri}(p)$$
$$= M - 2r\left(cos^2(px/2)\,cos^2(py/2)\,cos^2(pz/2)\,cos^2(pt/2) - 1\right).$$

$$p_\mu = (0,0,0,0) \;\rightarrow\; M(p) = M \quad (\times 1)$$
$$p_\mu = (\pi,0,0,0),\cdots \rightarrow\; M(p) = M + 2r \;\; (\times 4)$$
$$p_\mu = (\pi,\pi,0,0),\cdots \rightarrow\; M(p) = M + 2r \;\; (\times 6)$$
$$p_\mu = (\pi,\pi,\pi,0),\cdots \rightarrow\; M(p) = M + 2r \;\; (\times 4)$$
$$p_\mu = (\pi,\pi,\pi,\pi),\cdots \rightarrow\; M(p) = M + 2r \;\; (\times 1)$$

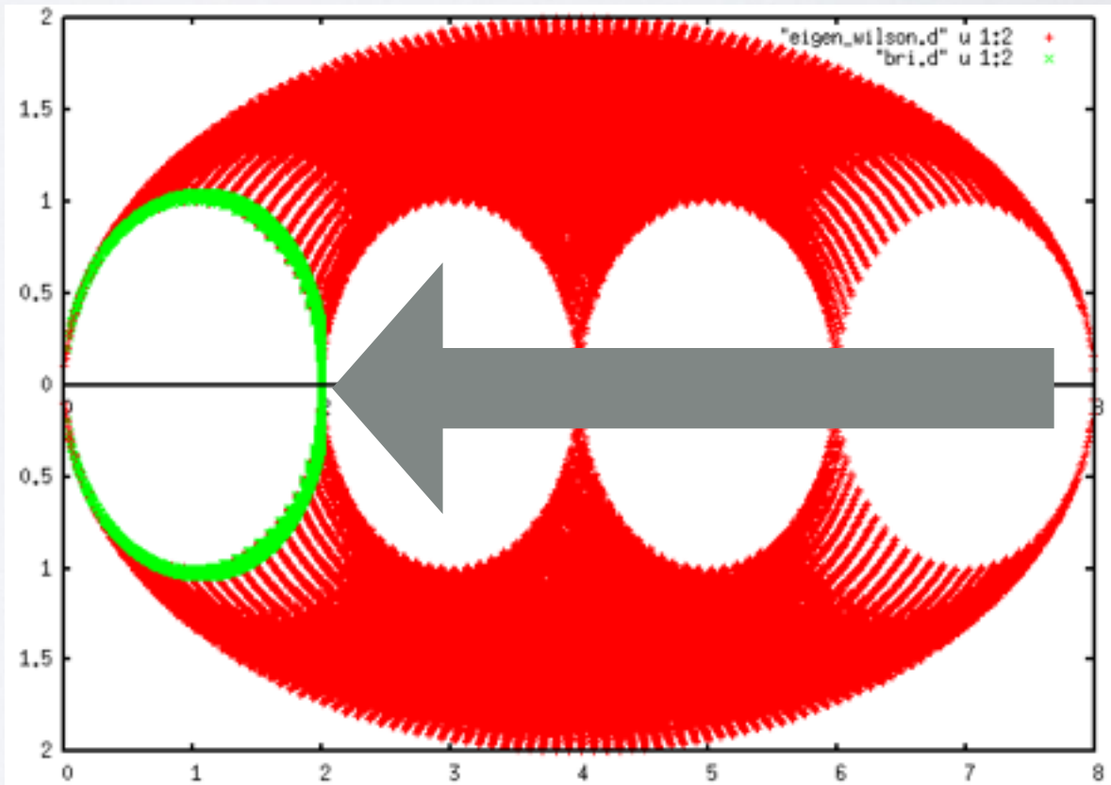
$\Rightarrow$ all doublers have a same mass.

EIGENVALUE SPECTRA (FREE)

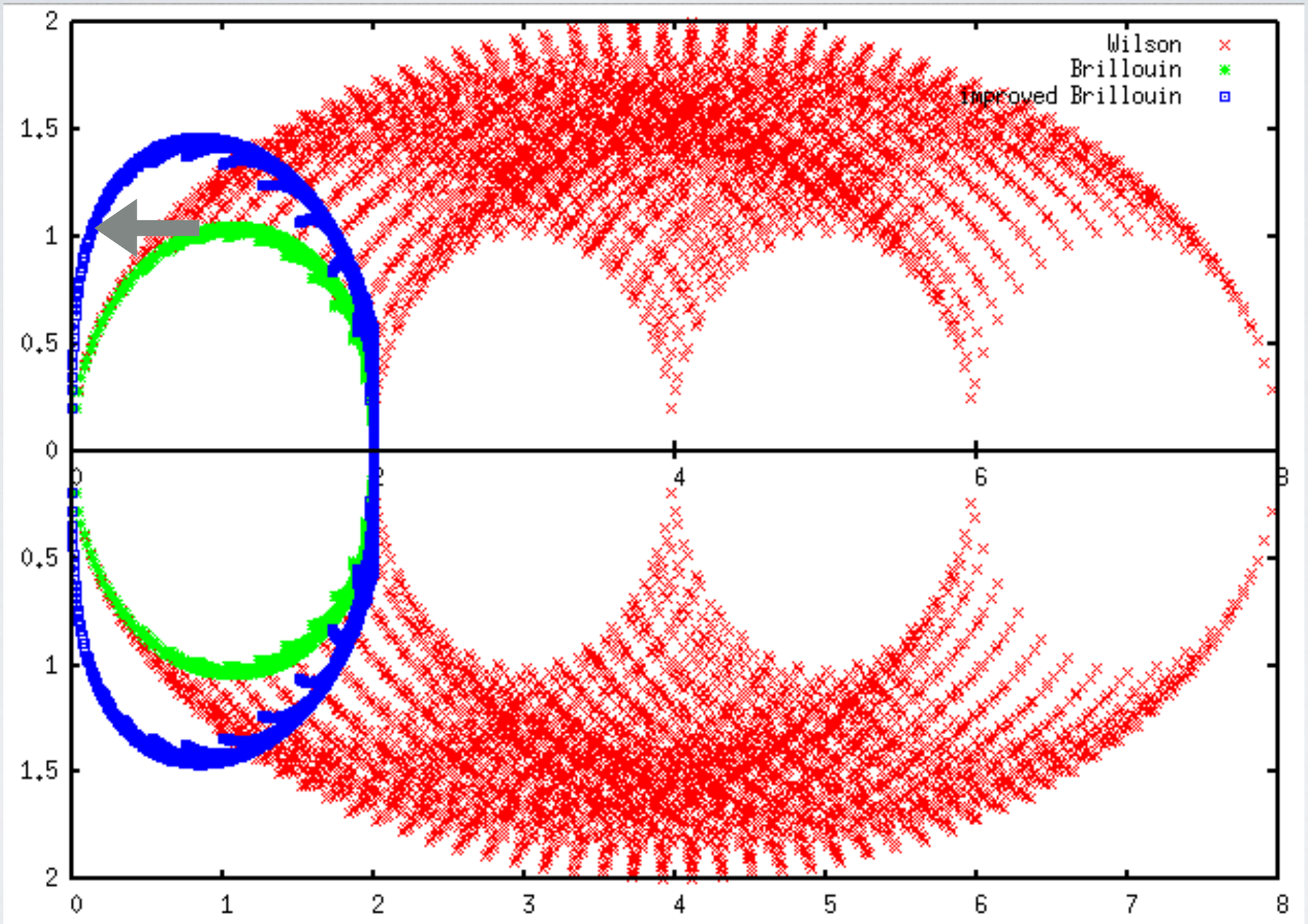




Ginsparg-Wilson like<=



EIGENVALUE SPECTRA(FREE)



Eigenvalue spectra of Improved Brillouin operator get close to the imaginary axis. (more continuum like)

ISOTROPIC DERIVATIVE

position space

$$
\begin{aligned}
a\nabla^{iso}_{\mu=\hat{x}}(n,m) = \frac{1}{432}[& -\delta_{n-\hat{x}-\hat{y}-\hat{z}-\hat{t},m} + \delta_{n+\hat{x}-\hat{y}-\hat{z}-\hat{t},m} - 4\delta_{n-\hat{x}-\hat{z}-\hat{t},m} + 4\delta_{n+\hat{x}-\hat{z}-\hat{t},m} \\
& -\delta_{n-\hat{x}+\hat{y}-\hat{z}-\hat{t},m} + \delta_{n+\hat{x}+\hat{y}-\hat{z}-\hat{t},m} - 4\delta_{n-\hat{x}-\hat{y}-\hat{t},m} + 4\delta_{n+\hat{x}-\hat{y}-\hat{t},m} \\
& -16\delta_{n-\hat{x}-\hat{t},m} + 16\delta_{n+\hat{x}-\hat{t},m} - 4\delta_{n-\hat{x}+\hat{y}-\hat{t},m} + 4\delta_{n+\hat{x}+\hat{y}-\hat{t},m} \\
& -\delta_{n-\hat{x}-\hat{y}+\hat{z}-\hat{t},m} + \delta_{n+\hat{x}-\hat{y}+\hat{z}-\hat{t},m} - 4\delta_{n-\hat{x}+\hat{z}-\hat{t},m} + 4\delta_{n+\hat{x}+\hat{z}-\hat{t},m} \\
& -\delta_{n-\hat{x}+\hat{y}+\hat{z}-\hat{t},m} + \delta_{n+\hat{x}+\hat{y}+\hat{z}-\hat{t},m} - 4\delta_{n-\hat{x}-\hat{y}-\hat{z},m} + 4\delta_{n+\hat{x}-\hat{y}-\hat{z},m} \\
& -16\delta_{n-\hat{x}-\hat{z},m} + 16\delta_{n+\hat{x}-\hat{z},m} - 4\delta_{n-\hat{x}+\hat{y}-\hat{z},m} + 4\delta_{n+\hat{x}+\hat{y}-\hat{z},m} \\
& -16\delta_{n-\hat{x}-\hat{y},m} + 16\delta_{n+\hat{x}-\hat{y},m} - 64\delta_{n-\hat{x},m} + 64\delta_{n+\hat{x},m} \\
& -16\delta_{n-\hat{x}+\hat{y},m} + 16\delta_{n+\hat{x}+\hat{y},m} - 4\delta_{n-\hat{x}-\hat{y}+\hat{z},m} + 4\delta_{n+\hat{x}-\hat{y}+\hat{z},m} \\
& -16\delta_{n-\hat{x}+\hat{z},m} + 16\delta_{n+\hat{x}+\hat{z},m} - 4\delta_{n-\hat{x}+\hat{y}+\hat{z},m} + 4\delta_{n+\hat{x}+\hat{y}+\hat{z},m} \\
& -\delta_{n-\hat{x}-\hat{y}-\hat{z}+\hat{t},m} + \delta_{n+\hat{x}-\hat{y}-\hat{z}+\hat{t},m} - 4\delta_{n-\hat{x}-\hat{z}+\hat{t},m} + 4\delta_{n+\hat{x}-\hat{z}+\hat{t},m} \\
& -\delta_{n-\hat{x}+\hat{y}-\hat{z}+\hat{t},m} + \delta_{n+\hat{x}+\hat{y}-\hat{z}+\hat{t},m} - 4\delta_{n-\hat{x}-\hat{y}+\hat{t},m} + 4\delta_{n+\hat{x}-\hat{y}+\hat{t},m} \\
& -16\delta_{n-\hat{x}+\hat{t},m} + 16\delta_{n+\hat{x}+\hat{t},m} - 4\delta_{n-\hat{x}+\hat{y}+\hat{t},m} + 4\delta_{n+\hat{x}+\hat{y}+\hat{t},m} \\
& -\delta_{n-\hat{x}-\hat{y}+\hat{z}+\hat{t},m} + \delta_{n+\hat{x}-\hat{y}+\hat{z}+\hat{t},m} - 4\delta_{n-\hat{x}+\hat{z}+\hat{t},m} + 4\delta_{n+\hat{x}+\hat{z}+\hat{t},m} \\
& -\delta_{n-\hat{x}+\hat{y}+\hat{z}+\hat{t},m} + \delta_{n+\hat{x}+\hat{y}+\hat{z}+\hat{t},m}]
\end{aligned}
$$

momentum space

$$\nabla^{iso}_{\mu=\hat{x}}(p) = isinp_x(cosp_y + 2)(cosp_z + 2)(cosp_t + 2)/27$$

BRILLOUIN LAPLACIAN

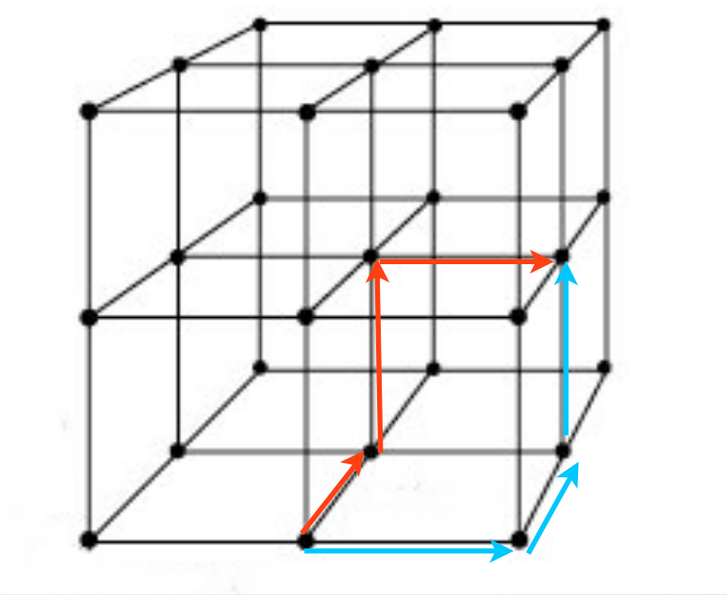
position space

$$
\begin{aligned}
a^2 \triangle^{bri}(n,m) = \frac{1}{64}\,[\;& \delta_{n-\hat{x}-\hat{y}-\hat{z}-\hat{t},m} + 2\delta_{n-\hat{y}-\hat{z}-\hat{t},m} + \delta_{n+\hat{x}-\hat{y}-\hat{z}-\hat{t},m} + 2\delta_{n-\hat{x}-\hat{z}-\hat{t},m} + 4\delta_{n-\hat{z}-\hat{t},m} + 2\delta_{n+\hat{x}-\hat{z}-\hat{t},m} \\
& + \delta_{n-\hat{x}+\hat{y}-\hat{z}-\hat{t},m} + 2\delta_{n+\hat{y}-\hat{z}-\hat{t},m} + \delta_{n+\hat{x}+\hat{y}-\hat{z}-\hat{t},m} + 2\delta_{n-\hat{x}-\hat{y}-\hat{t},m} + 4\delta_{n-\hat{y}-\hat{t},m} + 2\delta_{n+\hat{x}-\hat{y}-\hat{t}} \\
& + 4\delta_{n-\hat{x}-\hat{t},m} + 8\delta_{n-\hat{t},m} + 4\delta_{n+\hat{x}-\hat{t},m} + 2\delta_{n-\hat{x}+\hat{y}-\hat{t},m} + 4\delta_{n+\hat{y}-\hat{t},m} + 2\delta_{n+\hat{x}+\hat{y}-\hat{t},m} \\
& + \delta_{n-\hat{x}-\hat{y}+\hat{z}-\hat{t},m} + 2\delta_{n-\hat{y}+\hat{z}-\hat{t},m} + \delta_{n+\hat{x}-\hat{y}+\hat{z}-\hat{t},m} + 2\delta_{n-\hat{x}+\hat{z}-\hat{t},m} + 4\delta_{n+\hat{z}-\hat{t},m} + 2\delta_{n+\hat{x}+\hat{z}-\hat{t}} \\
& + \delta_{n-\hat{x}+\hat{y}+\hat{z}-\hat{t},m} + 2\delta_{n+\hat{y}+\hat{z}-\hat{t},m} + \delta_{n+\hat{x}+\hat{y}+\hat{z}-\hat{t},m} + 2\delta_{n-\hat{x}-\hat{y}-\hat{z},m} + 4\delta_{n-\hat{y}-\hat{z},m} + 2\delta_{n+\hat{x}-\hat{y}-} \\
& + 4\delta_{n-\hat{x}-\hat{z},m} + 8\delta_{n-\hat{z},m} + 4\delta_{n+\hat{x}-\hat{z},m} + 2\delta_{n-\hat{x}+\hat{y}-\hat{z},m} + 4\delta_{n+\hat{y}-\hat{z},m} + 2\delta_{n+\hat{x}+\hat{y}-\hat{z},m} \\
& + 4\delta_{n-\hat{x}-\hat{y},m} + 8\delta_{n-\hat{y},m} + 4\delta_{n+\hat{x}-\hat{y},m} + 8\delta_{n-\hat{x},m} - 240\delta_{n,m} + 8\delta_{n+\hat{x},m} \\
& + 4\delta_{n-\hat{x}+\hat{y},m} + 8\delta_{n+\hat{y},m} + 4\delta_{n+\hat{x}+\hat{y},m} + 2\delta_{n-\hat{x}-\hat{y}+\hat{z},m} + 4\delta_{n-\hat{y}+\hat{z},m} + 2\delta_{n+\hat{x}-\hat{y}+\hat{z},m} \\
& + 4\delta_{n-\hat{x}+\hat{z},m} + 8\delta_{n+\hat{z},m} + 4\delta_{n+\hat{x}+\hat{z},m} + 2\delta_{n-\hat{x}+\hat{y}+\hat{z},m} + 4\delta_{n+\hat{y}+\hat{z},m} + 2\delta_{n+\hat{x}+\hat{y}+\hat{z},m} \\
& + \delta_{n-\hat{x}-\hat{y}-\hat{z}+\hat{t},m} + 2\delta_{n-\hat{y}-\hat{z}+\hat{t},m} + \delta_{n+\hat{x}-\hat{y}-\hat{z}+\hat{t},m} + 2\delta_{n-\hat{x}-\hat{z}+\hat{t},m} + 4\delta_{n-\hat{z}+\hat{t},m} + 2\delta_{n+\hat{x}-\hat{z}+\hat{t}} \\
& + \delta_{n-\hat{x}+\hat{y}-\hat{z}+\hat{t},m} + 2\delta_{n+\hat{y}-\hat{z}+\hat{t},m} + \delta_{n+\hat{x}+\hat{y}-\hat{z}+\hat{t},m} + 2\delta_{n-\hat{x}-\hat{y}+\hat{t},m} + 4\delta_{n-\hat{y}+\hat{t},m} + 2\delta_{n+\hat{x}-\hat{y}+\hat{t}} \\
& + 4\delta_{n-\hat{x}+\hat{t},m} + 8\delta_{n+\hat{t},m} + 4\delta_{n+\hat{x}+\hat{t},m} + 2\delta_{n-\hat{x}+\hat{y}+\hat{t},m} + 4\delta_{n+\hat{y}+\hat{t},m} + 2\delta_{n+\hat{x}+\hat{y}+\hat{t},m} \\
& + \delta_{n-\hat{x}-\hat{y}+\hat{z}+\hat{t},m} + 2\delta_{n-\hat{y}+\hat{z}+\hat{t},m} + \delta_{n+\hat{x}-\hat{y}+\hat{z}+\hat{t},m} + 2\delta_{n-\hat{x}+\hat{z}+\hat{t},m} + 4\delta_{n+\hat{z}+\hat{t},m} + 2\delta_{n+\hat{x}+\hat{z}+\hat{t}} \\
& + \delta_{n-\hat{x}+\hat{y}+\hat{z}+\hat{t},m} + 2\delta_{n+\hat{y}+\hat{z}+\hat{t},m} + \delta_{n+\hat{x}+\hat{y}+\hat{z}+\hat{t},m}\;]
\end{aligned}
$$

momentum space

$$
\triangle^{bri}(p) = 4cos^2\left(p_x/2\right) cos^2\left(p_y/2\right) cos^2\left(p_z/2\right) cos^2\left(p_t/2\right) - 4
$$

THE BRILLOUIN OPERATOR WITH GAUGE FIELDS



- take a average of all paths for every hopping term
- recursion algorithm of standard derivative and laplacian

$$a\triangle^{bri}(n,m)\,\psi_m = \frac{1}{64}\sum_{\mu} D_\mu^+ \psi_n''' - \frac{15}{4}\psi_n$$

$$\psi_n''' \equiv 8\psi_n + \frac{1}{2}\sum_{\nu\neq\mu} D_\nu^+ \psi_n''$$

$$\psi_n'' \equiv 4\psi_n + \frac{1}{3}\sum_{\rho\neq\mu,\nu} D_\rho^+ \psi_n'$$

$$\psi_n' \equiv 2\psi_n + \frac{1}{4}\sum_{\sigma\neq\mu,\nu,\rho} D_\sigma^+ \psi_n$$

$$\nabla_x^{iso}(n,m)\,\psi_m = \frac{1}{432}\left(D_x^- \xi_n''' + \frac{1}{2}\sum_{\nu\neq x} D_\nu^+ \eta_n'''\right)$$

$$\xi_n''' \equiv 64\psi_n + \frac{1}{2}\sum_{\nu\neq x} D_\nu^+ \xi_n'' \qquad \eta_n''' \equiv D_x^- \xi_n'' + \frac{1}{3}\sum_{\rho\neq x,\nu} D_\rho^+ \eta_n''$$

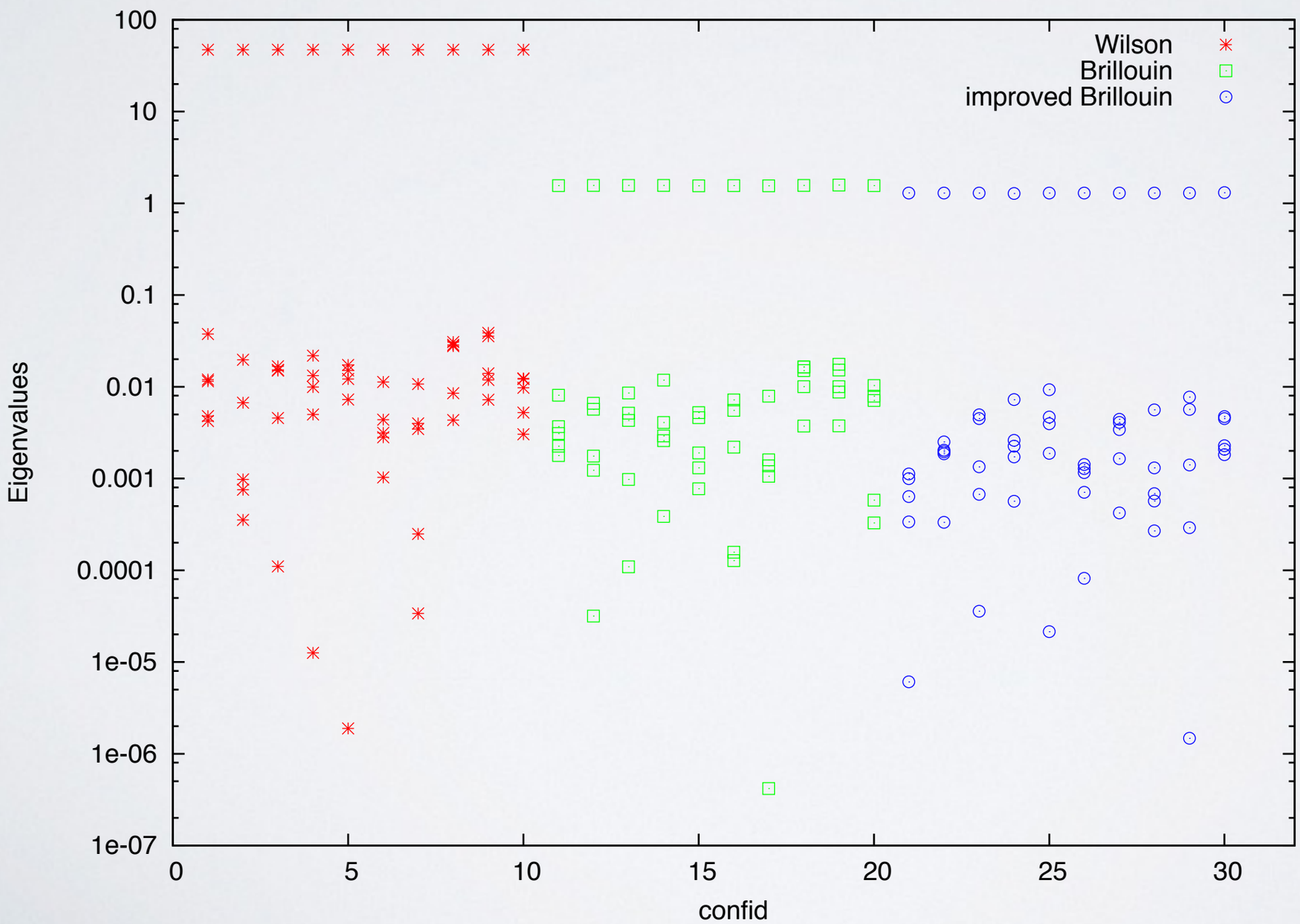
$$\xi_n'' \equiv 16\psi_n + \frac{1}{3}\sum_{\rho\neq x,\nu} D_\rho^+ \xi_n' \qquad \eta_n'' \equiv D_x^- \xi_n' + \frac{1}{4}\sum_{\sigma\neq x,\nu,\rho} D_\sigma^+ \eta_n'$$

$$\xi_n' \equiv 4\psi_n + \frac{1}{4}\sum_{\sigma\neq x,\nu,\rho} D_\sigma^+ \psi_n \qquad \eta_n' \equiv D_x^- \psi_n$$

$$D_\mu^\pm = U_\mu(n)\,\psi_{n+\hat{\mu}}''' \pm U_\mu^\dagger(n-\hat{\mu})\,\psi_{n-\hat{\mu}}'''$$

EIGENVALUES ON NON-TRIVIAL GAUGE CONFIGURATIONS

=>highest mode and lowest modes(5) of $D^\dagger D$



Nsmear : 3

mass : -1.0

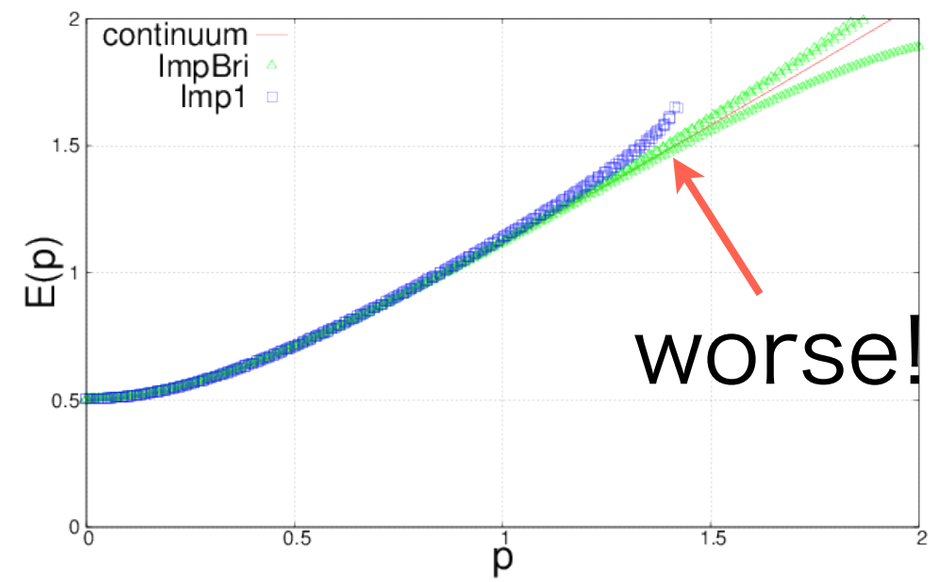
a^{-1} : 2.038(GeV)

Volume=16^3x32

Another possible way?

- reduce numerical costs for the improved action

$$D^{imp} = \sum_\mu \gamma_\mu (1 - \frac{1}{12} a^2 \triangle^{std}) \nabla_\mu^{iso} (1 - \frac{1}{12} a^2 \triangle^{std}) + c_{imp} a^3 (\triangle^{std})^2$$



- introduce an additional parameter

[T.Misumi,2014]

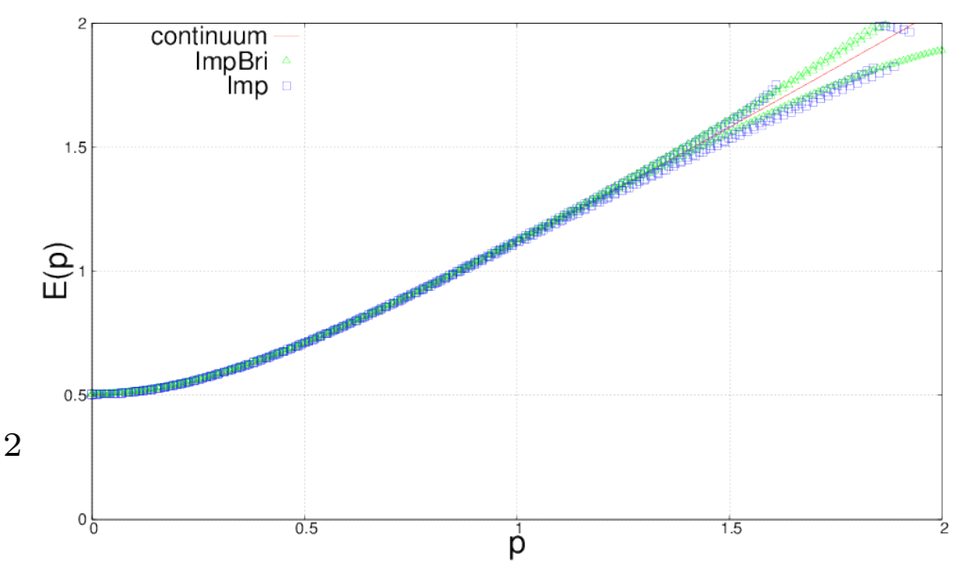
$$\nabla_\mu^{iso}(\delta) = \nabla_\mu^{std} \prod_{i \neq \mu} (1 + \delta a^2 \partial_i^2)$$

$$\delta = 1/6 \; for \; the \; Brillouin \; fermion$$

- improvement with tuned δ

$$D^{imp} = \sum_\mu \gamma_\mu \left(1 - a^2 \triangle_\mu^{imp}\right) a \nabla_\mu^{iso}(\delta) \left(1 - a^2 \triangle_\mu^{imp}\right) + c_{imp} a^3 (\triangle^{std})^2$$

$$\triangle_\mu^{imp} = \frac{1}{12} \triangle_\mu^{std} + \frac{\delta}{2} \sum_{\nu \neq \mu} \triangle_\nu^{std} \qquad \delta = 1/4$$



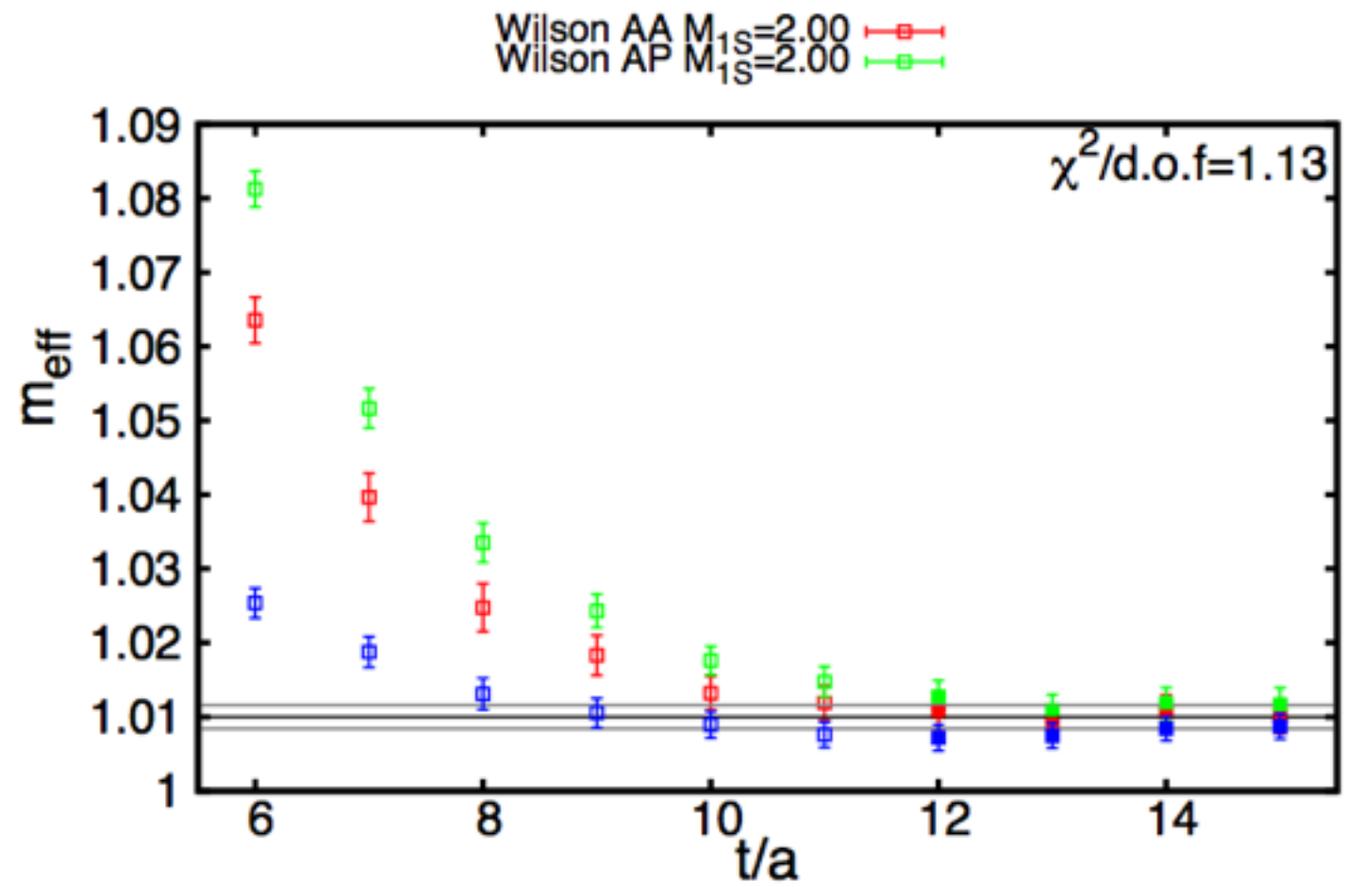
similar dispersion relation!

Fitting of correlators with the local source

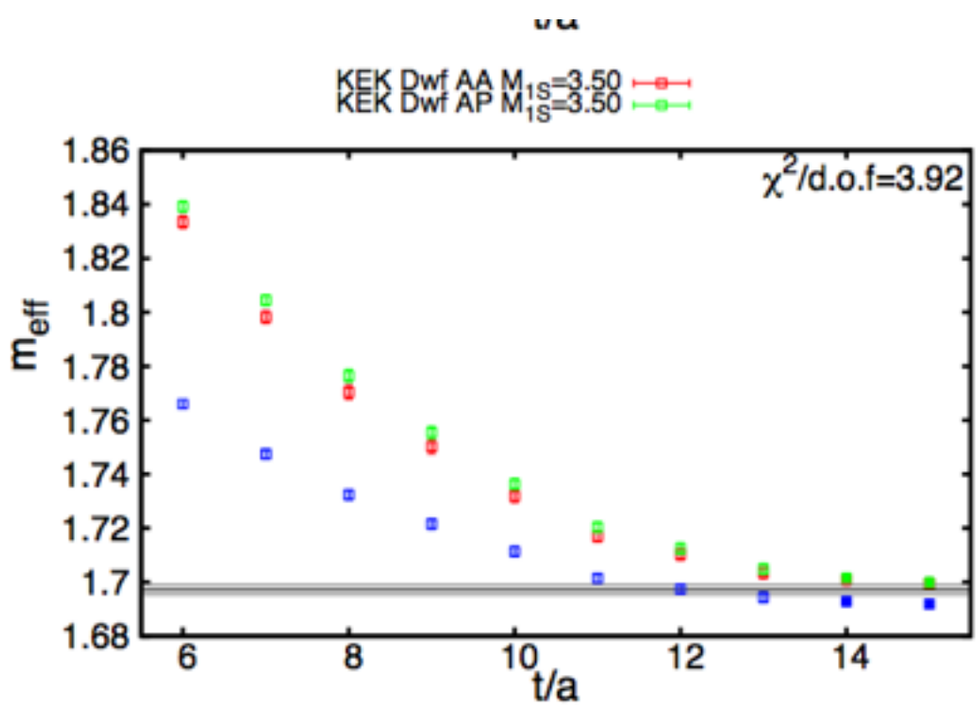
$$C_{AA}^{local-local}(t) = \frac{\left(Z_A^{local}\right)^2}{2m}\left(e^{-mt} + e^{-m(N_t-t)}\right)$$

$$C_{AP}^{local-local}(t) = \frac{Z_A^{local} Z_P^{local}}{2m}\left(e^{-mt} + e^{-m(N_t-t)}\right)$$

$$C_{PP}^{smr-local}(t) = \frac{Z_P^{local} Z_P^{smr}}{2m}\left(e^{-mt} + e^{-m(N_t-t)}\right)$$

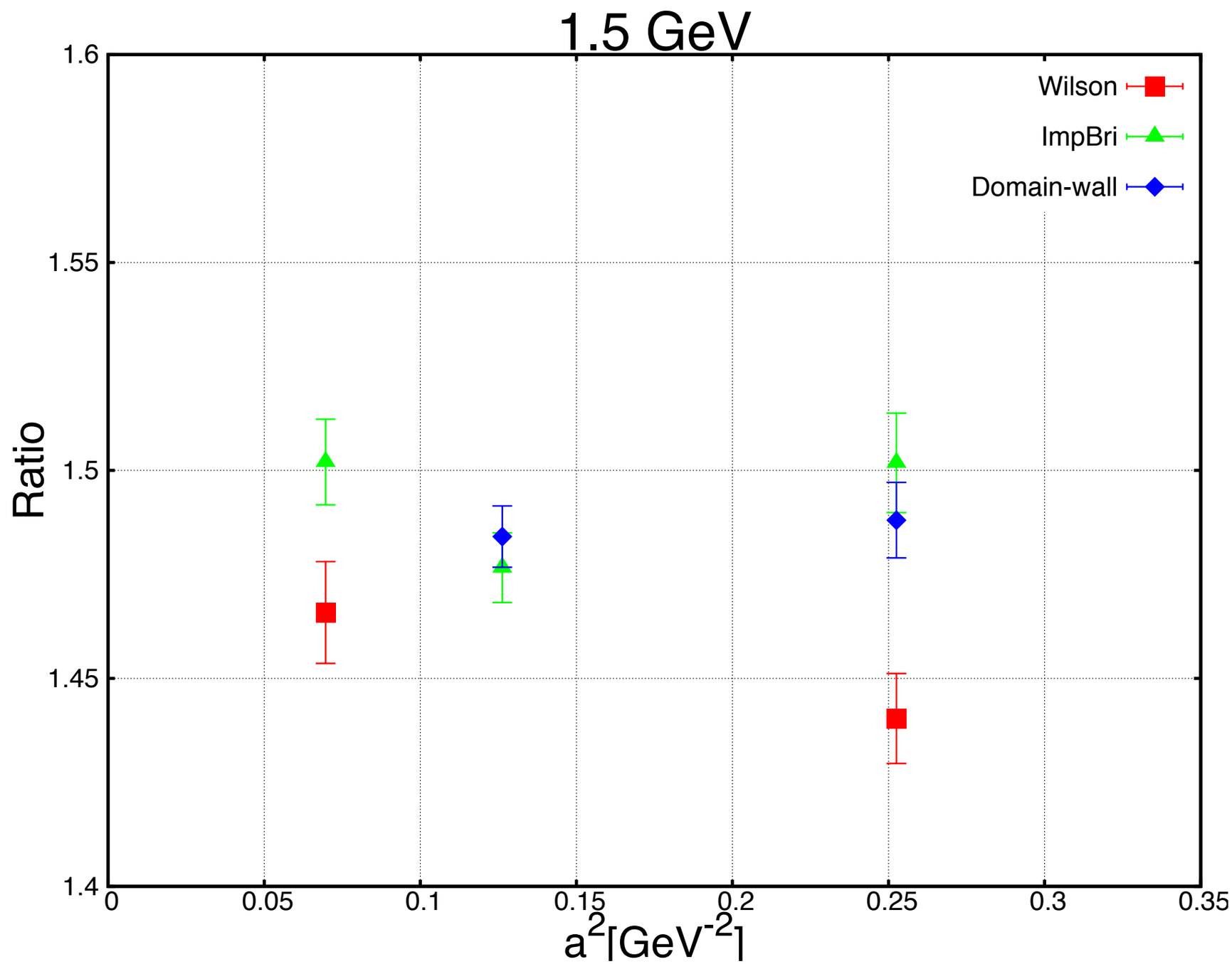
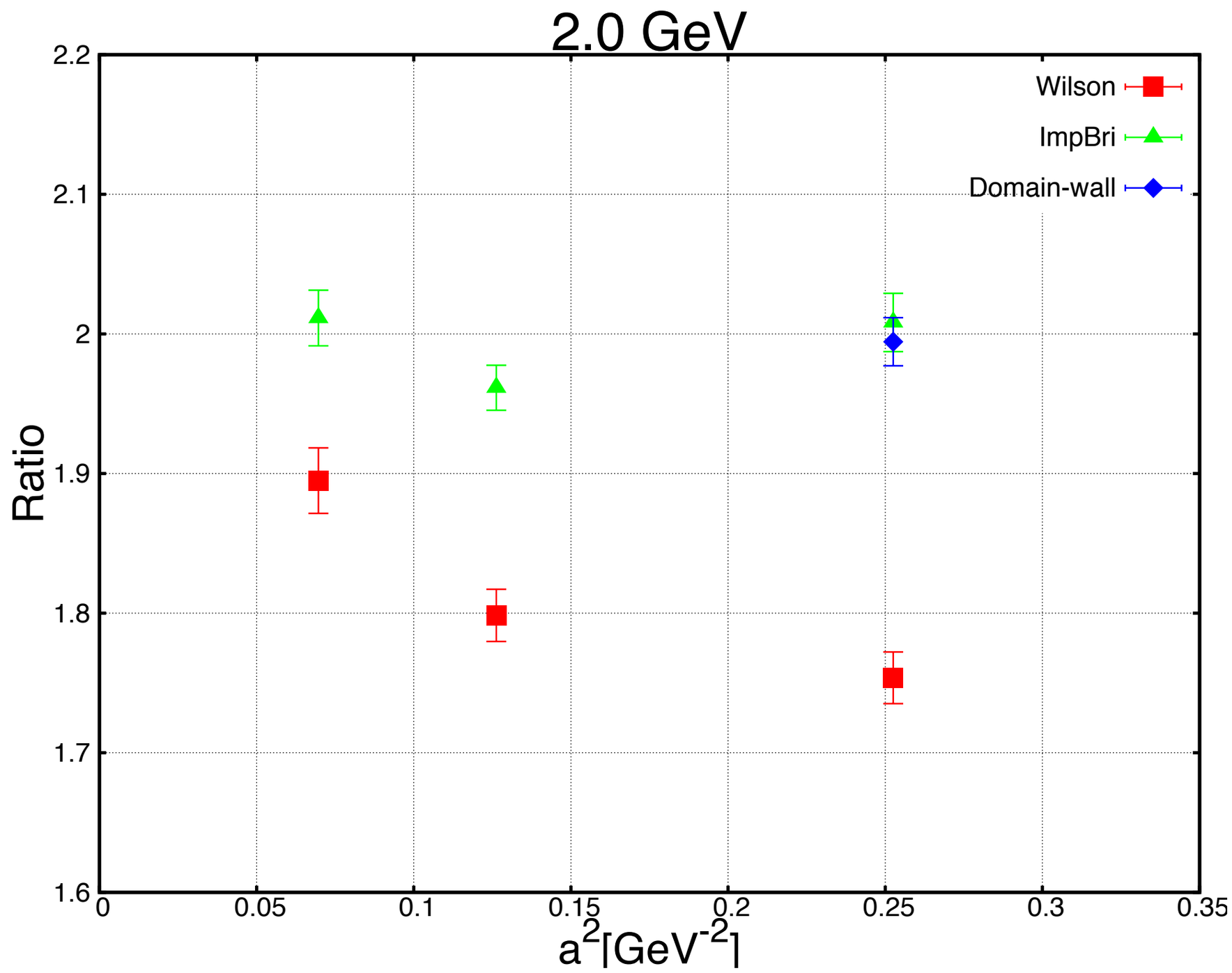


Fitting is often difficult.



⇒ Local source's effective mass is **NOT** consistent with smeard source's one.

scaling of decay constant for m_{PS} = 1.5GeV



scaling of decay constant for m_{PS} =2.0GeV