Process dependence Prompt Photon Production
Sivers and Collins(?)

1 August 2012

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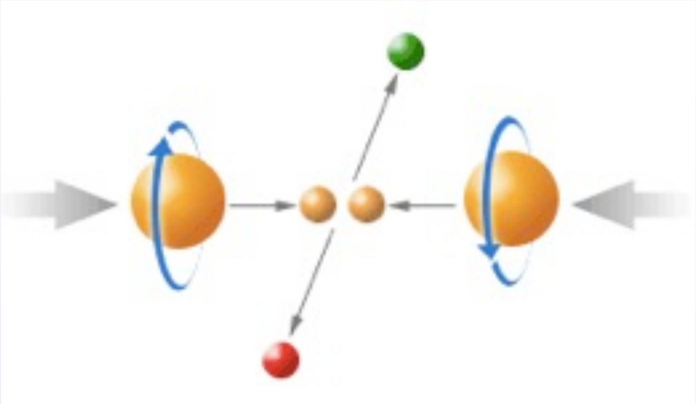
Phys.Lett. B696 2011 w/ Zhongbo Kang ***BNL***
Phys.Lett. B704 2011 w/ U. D'Alesio, LG Zhong-Bo Kang, F. Murgia, C. Pisano
LG & Zhongbo Kang - in Prep

Outline

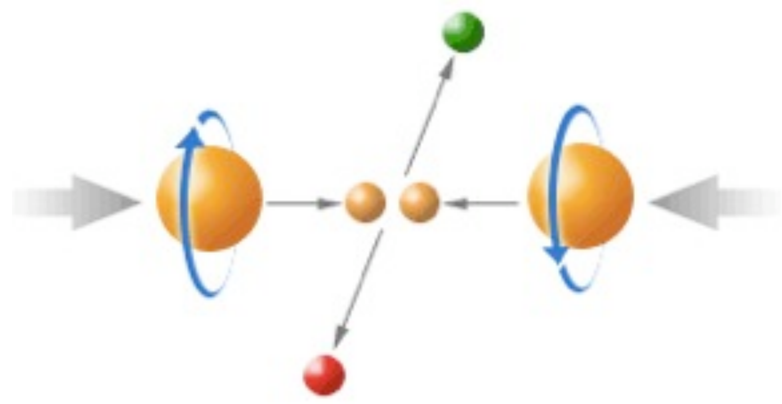
- **Transverse spin Effects - TSSAs**
- **Color Gauge Inv. & Gauge Links- “T-odd” TMDs**
- **T-odd PDFs & moments via ISI/FSIs ... QCD-Phases**
- Connection of twist 2 & twist 3 approach
- Generalizing the Generalized Parton Model (GPM) color gauge invariance CGI-GPM
- Contributions in Prompt γ
  - Direct & Fragmentation Sivers + Collins
- Some pheno results---(Collins is PRELIM!)

Comments Importance of TMDs in studying partonic content of the nucleon



- Single inclusive hadron production in hadronic collisions largest/ oldest observed TSSAs
- From theory view notoriously challenging from partonic picture twist-3 power suppressed in hard scale (vs. w/ SIDIS, DY, e^+e^-)

Comments



- Connection w/ twist 2 "TMD" approach
  - Operator level ETQS fnct 1st moment of Sivers

$$gT_F(x,x) = -\int d^2k_T \frac{|k_T^2|}{M} f_{1T}^{\perp}(x,k_T^2)$$ Boer Piljman Mulders NPB 2003

$$= -2M f_{1T}^{\perp(1)}(x) \quad + \quad \text{"UV"} \ldots$$

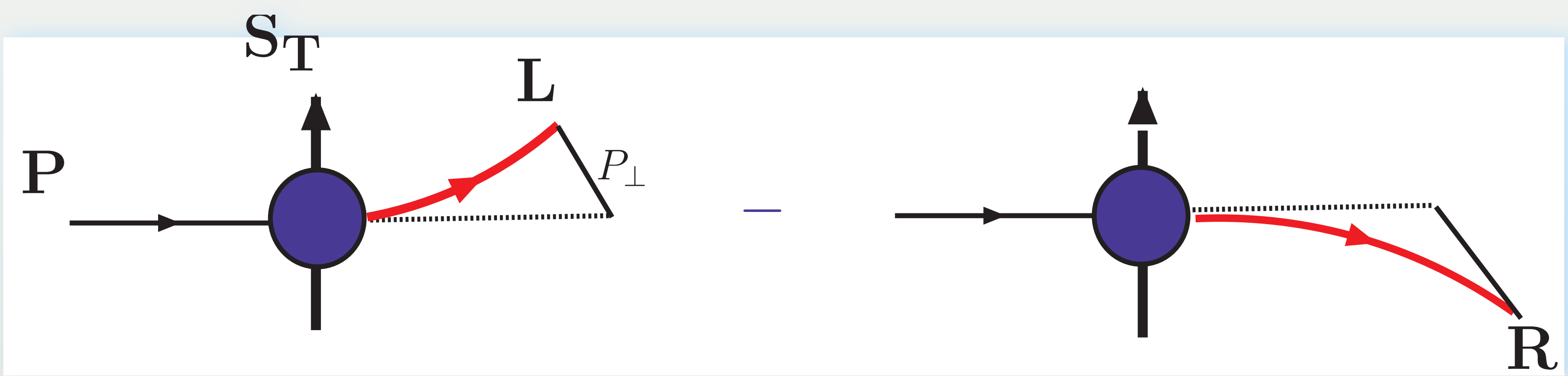
Kang, Qiu, Vogelsang, Yuan prd 2011
Kang & Prokudin prd 2012
"compatibility study"

$$f_{1T}^{\perp}(x,|\boldsymbol{b}_T|) = -2M \int d^2p_T \frac{|p_T|}{|\boldsymbol{b}_T|M} J_1(|\boldsymbol{b}_T||p_T|) f_{1T}^{\perp}(x,p_T^2)$$

Boer, LG, Musch, Prokudin to appear JHEP--arXiv:1107.529

Ingredients transverse SPIN Observable kinematics $P^{\uparrow}P \to \pi X$

- **Single Spin Asymmetry**



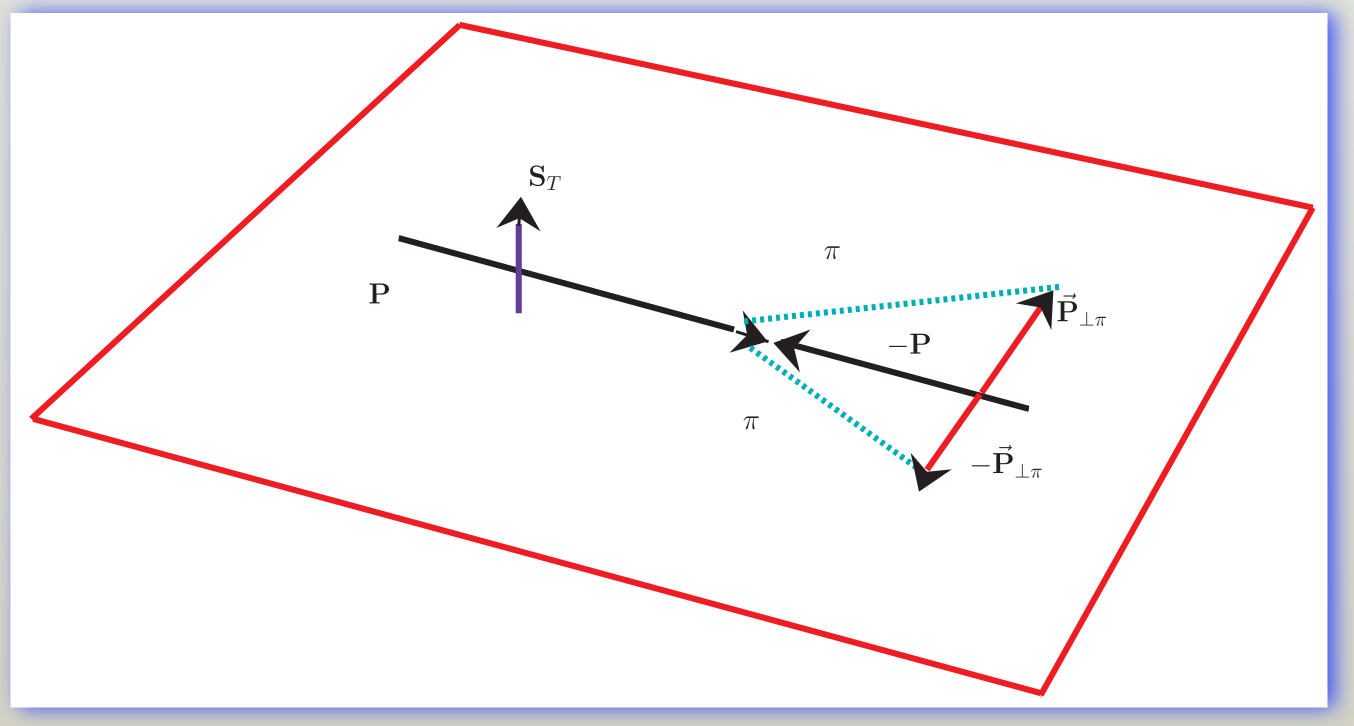
Parity Conserving interactions: SSAs Transverse Scattering plane

$$\Delta\sigma \sim iS_T \cdot (\mathbf{P} \times P_\perp^\pi)$$

- **Rotational invariance** $\sigma^{\downarrow}(x_F, \boldsymbol{p}_\perp) = \sigma^{\uparrow}(x_F, -\boldsymbol{p}_\perp)$

  $\Rightarrow$ ***Left-Right Asymmetry***

$$\boldsymbol{A}_N = \frac{\sigma^{\uparrow}(x_F,\boldsymbol{p}_\perp)-\sigma^{\uparrow}(x_F,-\boldsymbol{p}_\perp)}{\sigma^{\uparrow}(x_F,\boldsymbol{p}_\perp)+\sigma^{\uparrow}(x_F,-\boldsymbol{p}_\perp)} \equiv \Delta\sigma$$



Reaction Mechanism w/ Partonic Description

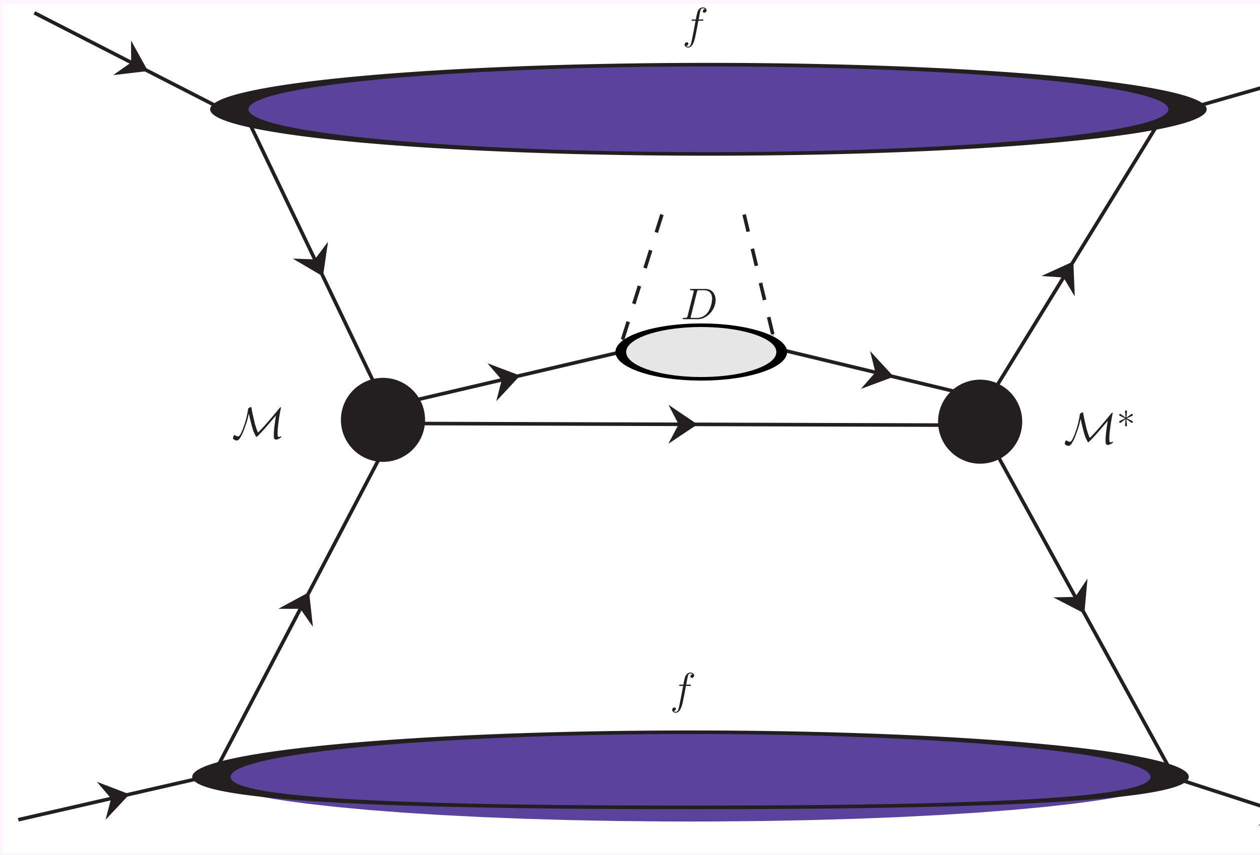
Collinear factorized QCD parton dynamics

$$\Delta\sigma^{pp^{\uparrow}\to\pi X} \sim f_a \otimes f_b \otimes \Delta\hat{\sigma} \otimes D^{q\to\pi}$$

$$\Delta\hat{\sigma} \equiv \hat{\sigma}^{\uparrow} - \hat{\sigma}^{\downarrow}$$

$$|\uparrow / \downarrow\rangle = (|+\rangle \pm i|-\rangle)$$

$$\hat{a}_N = \frac{\hat{\sigma}^{\uparrow} - \hat{\sigma}^{\downarrow}}{\hat{\sigma}^{\uparrow} + \hat{\sigma}^{\downarrow}} \sim \frac{\mathrm{Im}\left(\mathcal{M}^{+*}\mathcal{M}^{-}\right)}{|\mathcal{M}^{+}|^2 + |\mathcal{M}^{-}|^2}$$



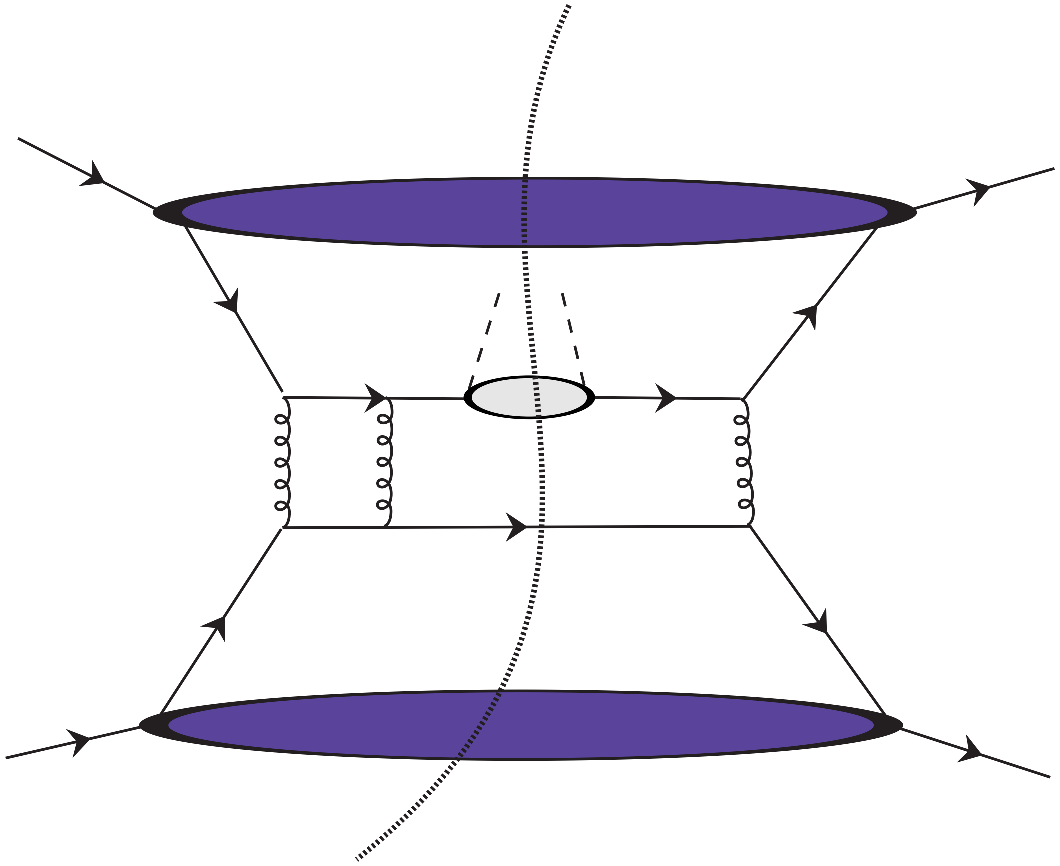
Transv. polarization cross section "interference" of helicity flip and non-flip amps.

Interference of helicity flip and non-flip amps

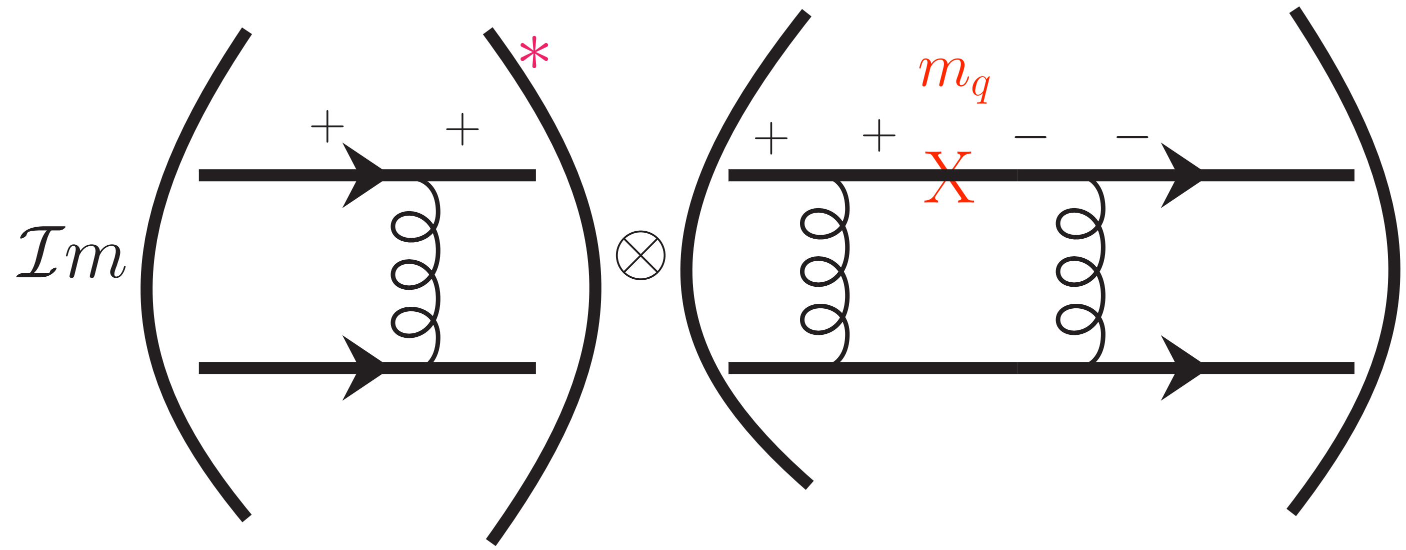
1) requires breaking of chiral symmetry m_q/E

2) relative phases require higher order corrections

Factorization Theorem at Partonic level



$$\Delta\hat{\sigma} \sim \mathcal{I}m[M^{*}{}_{+}M_{-}]$$



- **Born amps are real -- need "loops"----> phases**
- **QCD interactions conserve helicity up to corrections** $\mathcal{O}\left(\frac{m_q}{E_q}\right)$

Twist three and trivial in chiral limit

$$A_N \propto \frac{m_q}{E}\alpha_s$$ at the partonic level   Kane & Repko, PRL: 1978

Early theory in striking contrast exp. TSSAs in Inclusive Reactions

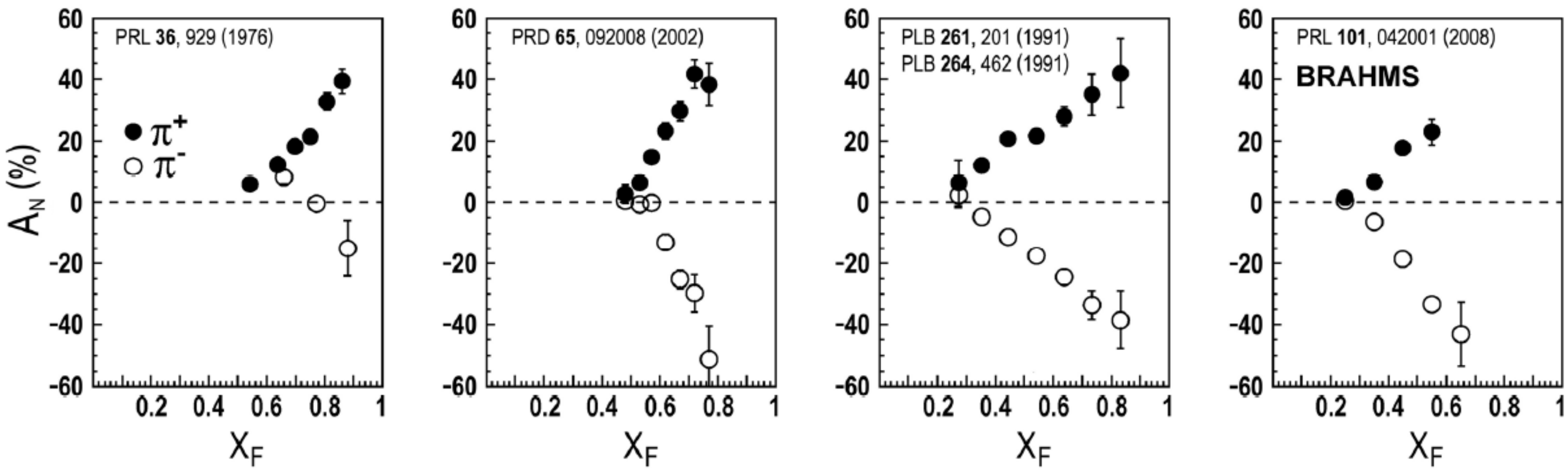
Transverse Single-Spin Asymmetries: From Low to High Energies!

BRAHMS

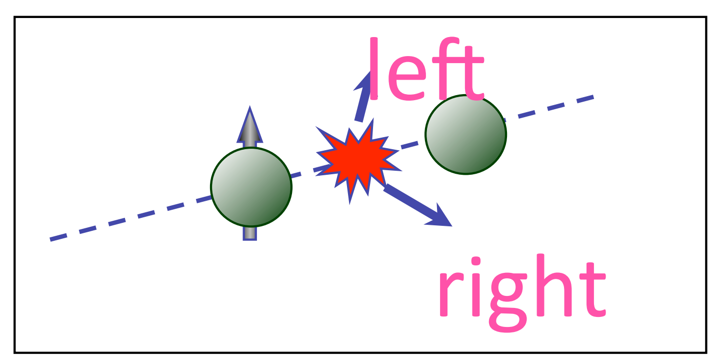
p_{beam}=12 Gev/c	p_{beam}=22 Gev/c	p_{beam}=200 Gev/c	
ANL	BNL	FNAL	RHIC
$\sqrt{s}$=4.9 GeV	$\sqrt{s}$=6.6 GeV	$\sqrt{s}$=19.4 GeV	$\sqrt{s}$=62.4 GeV

← Fixed target →

Collider

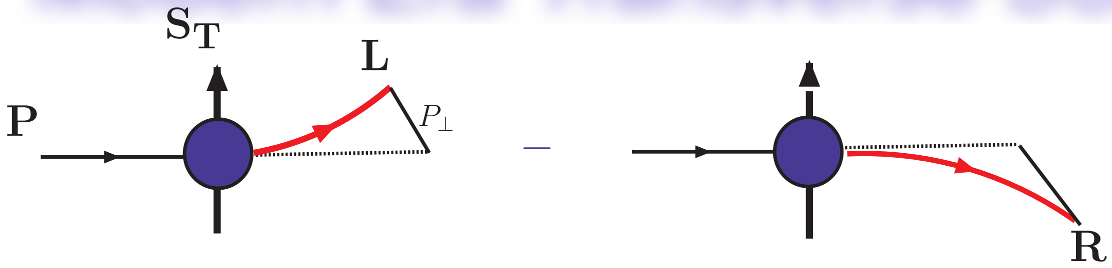


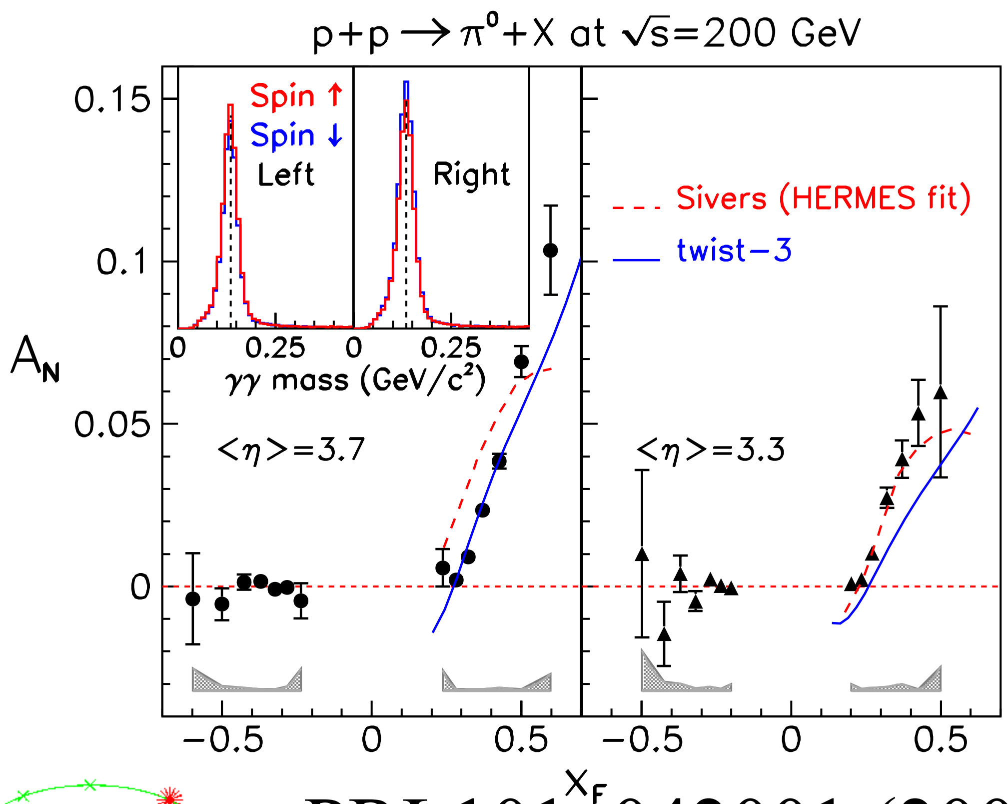
$$x_F = 2 p_{long} / \sqrt{s}$$



see talks of F. Giordono & T. Burton

Modern Era Transverse SSA's at $\sqrt{s}$ = 62.4 & 200 GeV at RHIC

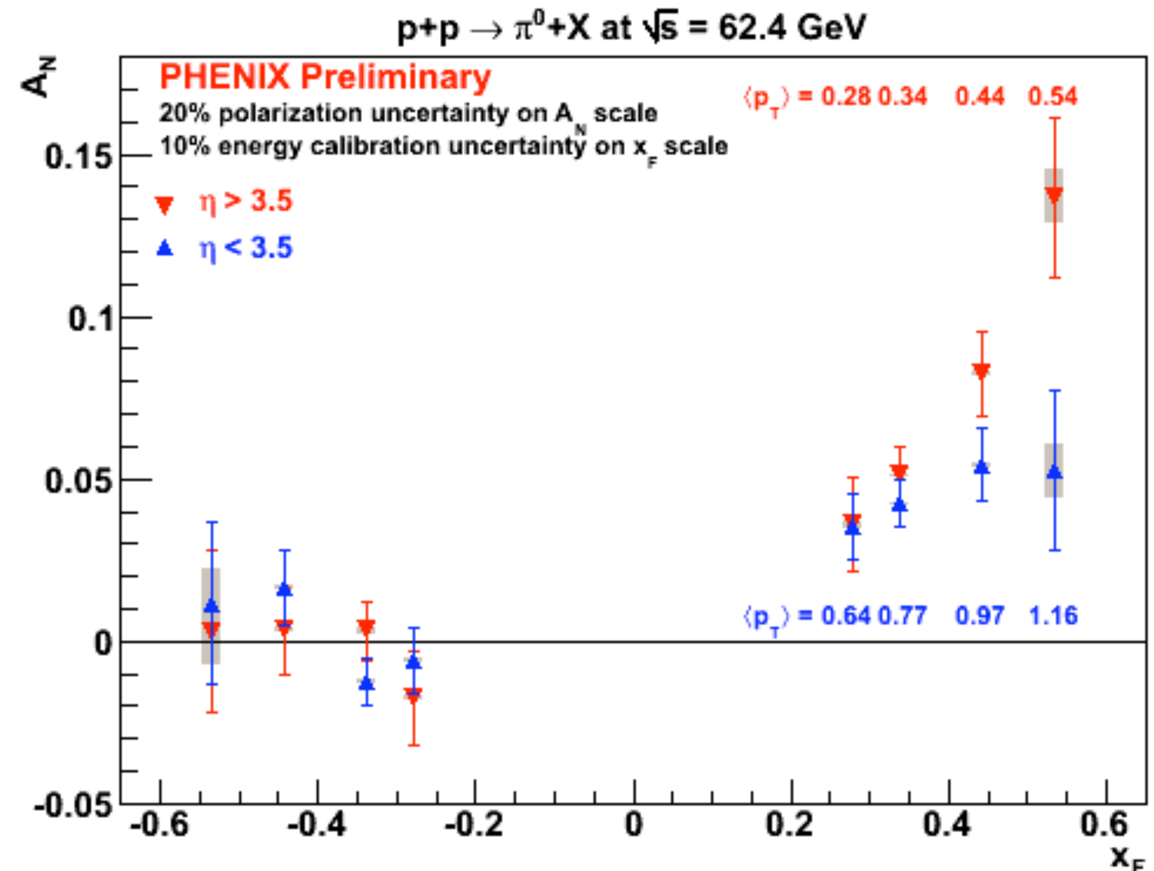




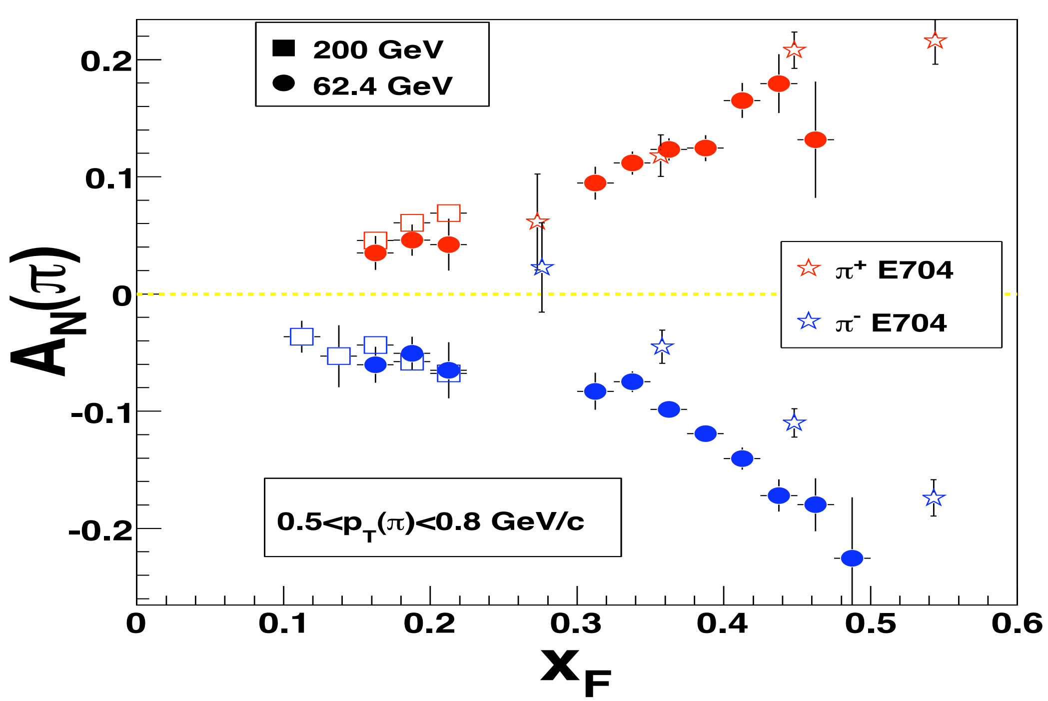
STAR

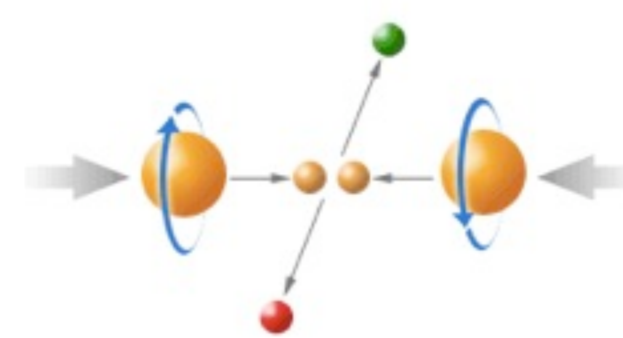
PRL101, 042001 (2008)

PHENIX



BRAHMS

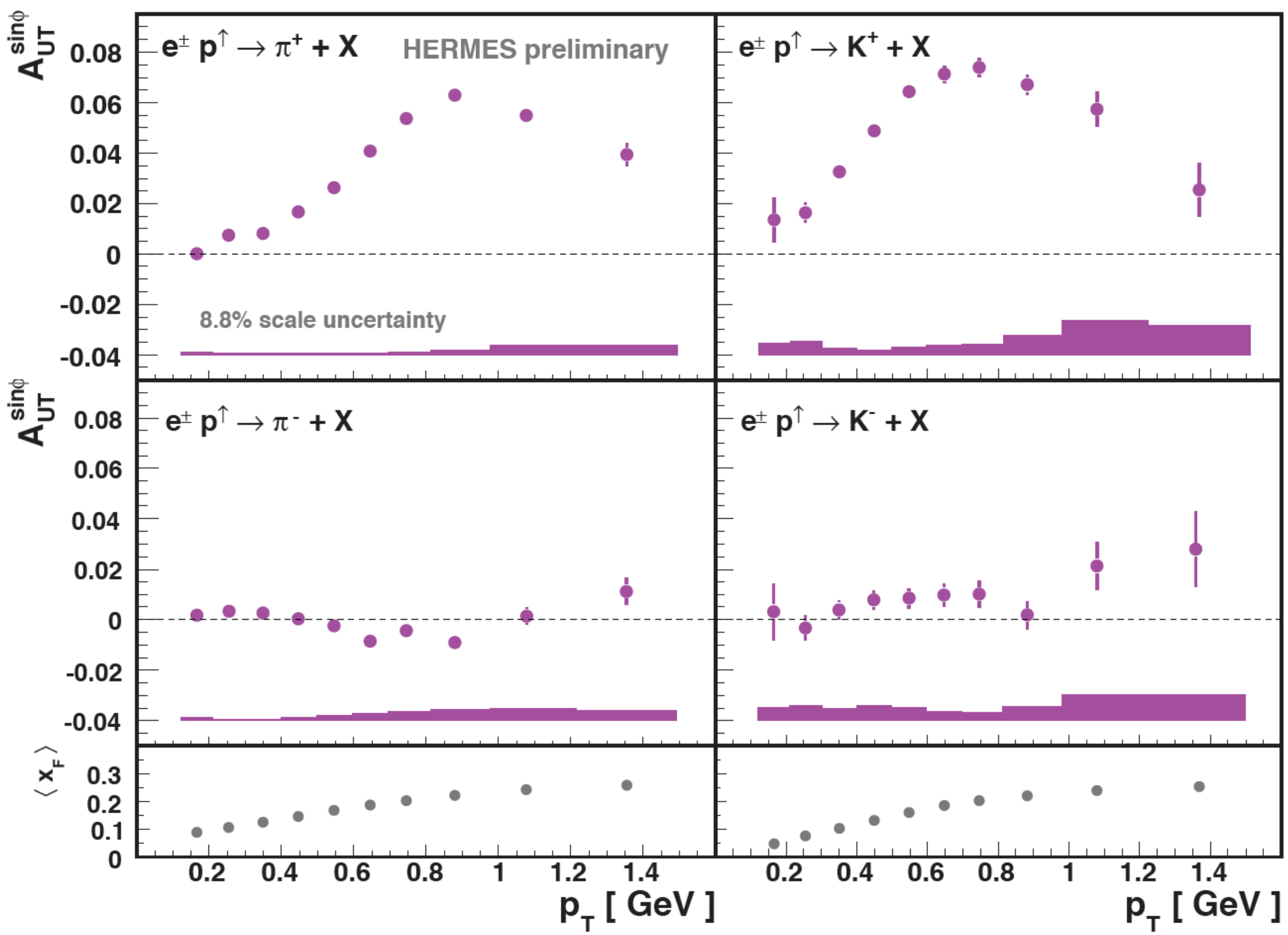




inclusive hadrons: $e\,p^{\uparrow} \to h\,X$

- scattered electron not detected (ignored) → Q2 ≈ 0 → huge statistics

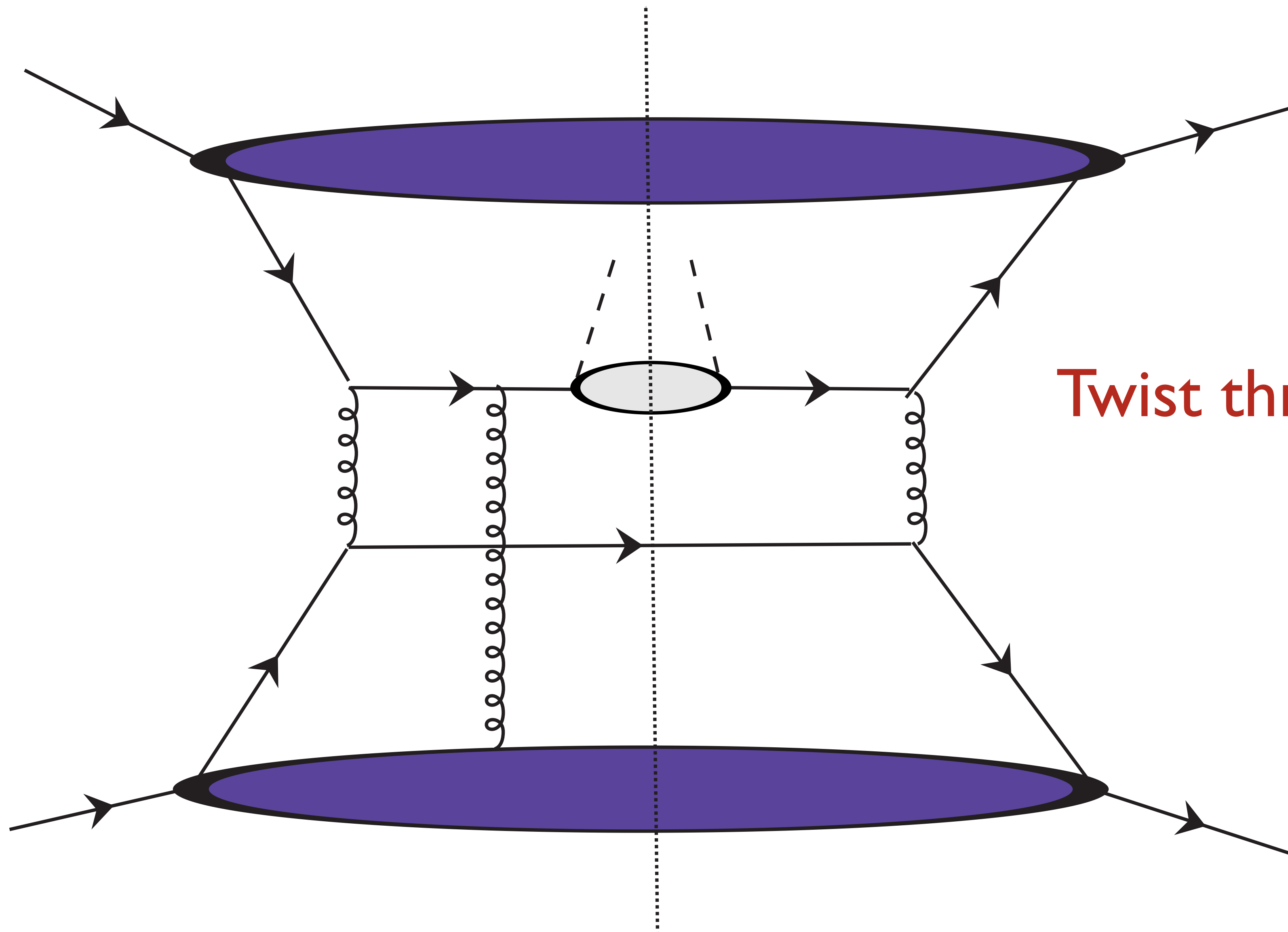
  P_T, x_F w.r.t. **beam direction**



$$A_N = \frac{2}{\pi} A_{UT}^{\sin\varphi}$$

from D. Hasch

Not the full story @ Twist 3 approach ETQS approach



Twist three supressed by hard scale
but non-trival?!

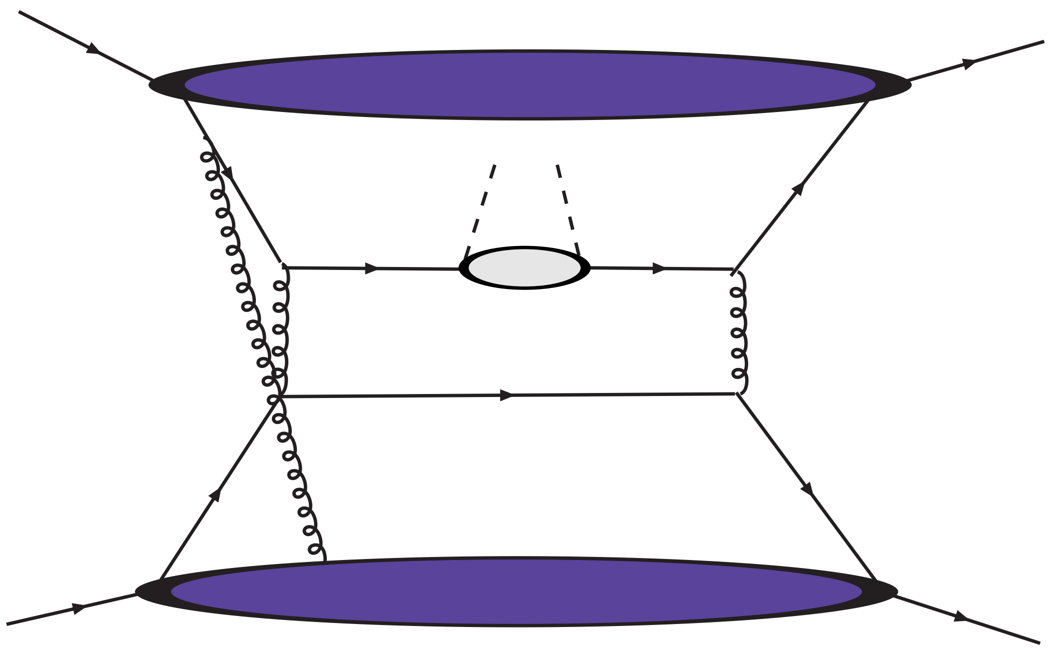
Phases in soft poles of parton propagators in hard sub-process
Efremov & Teryaev PLB 1982 Qiu, Sterman 1991,1999

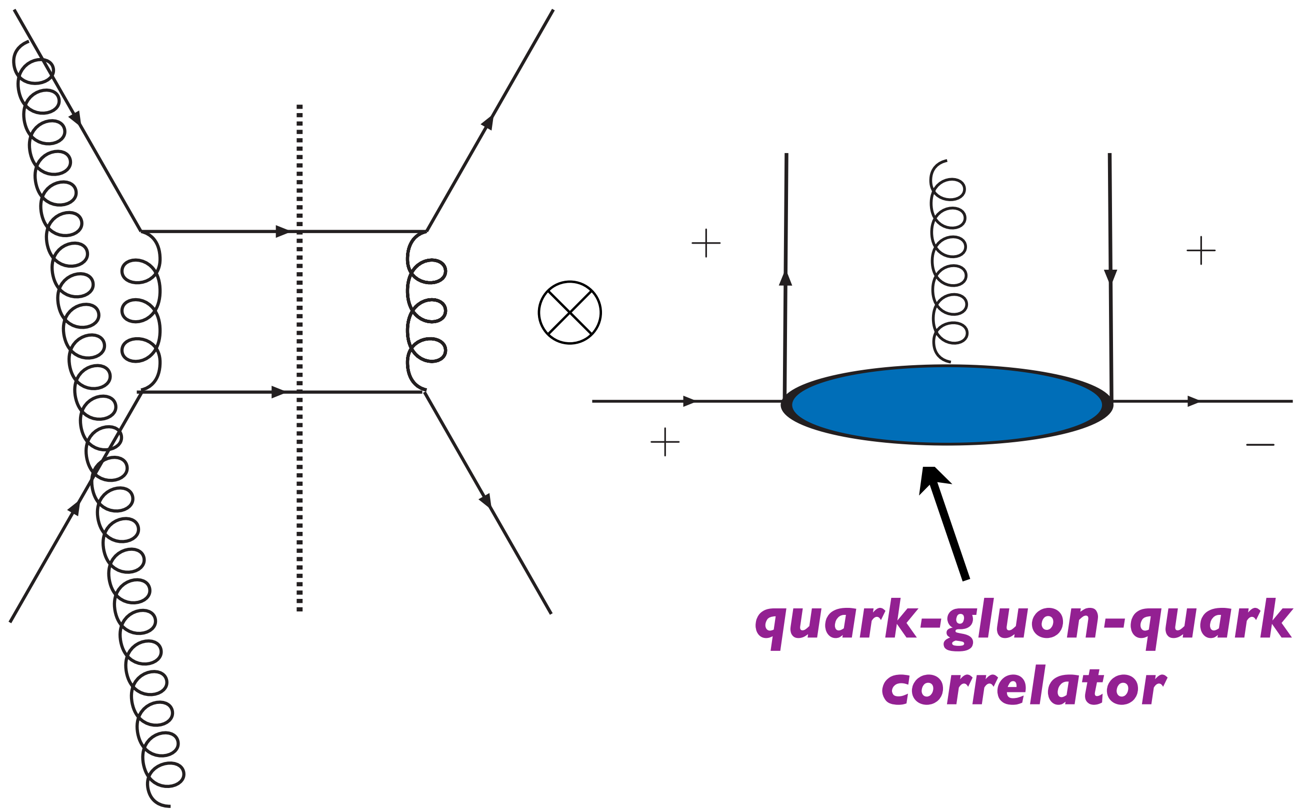
Factorization and Pheno: Qiu, Sterman 1991,1999..., Koike et al, 2000, ... 2011
Ji, Qiu, Vogelsang, Yuan, 2005 ... 2008 , Yuan, Zhou 2008, 2009, Kang, Qiu, 2008, 2009 ...
Vogelsang and Yuan 2007
Pheno studies, Kouvaris Ji, Qiu, Vogelsang! 2006

Twist 3 ETQS approach-"Partonic Picture"

$Q \sim P_T >> \Lambda_{\rm qcd}$ One scale Collinear fact Twist 3

Phases in soft poles of prop hard processes Efremov & Teryaev PLB 1982





$$\Delta\sigma \sim f_a \otimes T_F \otimes H_{ETQS} \otimes D^{q\to h}$$

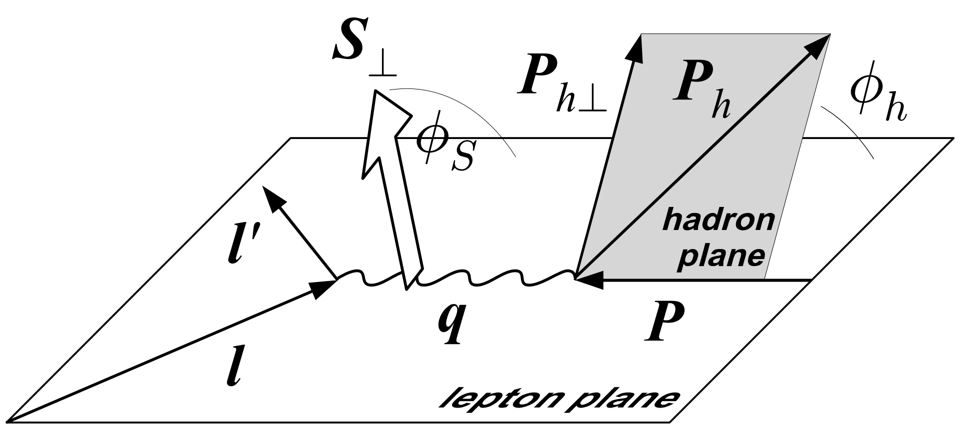
$$\frac{1}{xs + i\epsilon} = \mathcal{P}\left(\frac{1}{xs}\right) \pm i\pi\delta(xs)$$

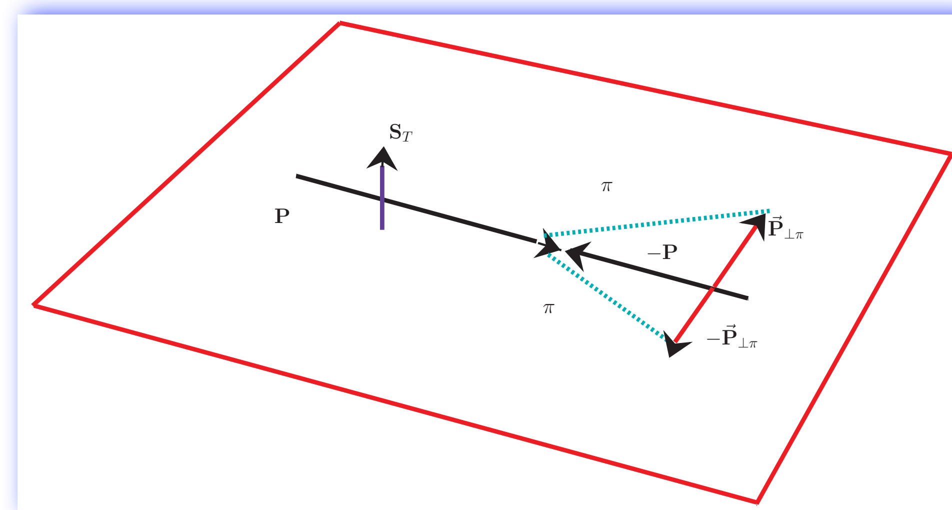
- Phases from interference two parton three parton scattering amplitudes

- Factorization and Pheno: Qiu, Sterman 1991,1999..., Koike et al, 2000, ... 2010, Ji, Qiu, Vogelsang, Yuan, 2005 ... 2008 ..., Yuan, Zhou 2008, 2009, Kang, Qiu, 2008, 2009 ... Kouvaris Ji, Qiu, Vogelsang! 2006, Vogelsang and Yuan PRD 2007

Two methods to generate SSA in QCD

- Depends on momentum of probe $q^2 = -Q^2$ and momentum of produced hadron $\boldsymbol{P}_{h\perp}$ relative to hadronic scale $k_T^2 (\equiv k_\perp^2) \sim \Lambda_{\rm QCD}^2$





- $k_\perp^2 \sim P_{h\perp}^2 \ll Q^2$ two scales-TMDs

$$\Delta\sigma(P_h, S) \sim \Delta f_{a/A}^{\perp}(x, p_\perp) \otimes D_{h/c}(z, K_\perp) \otimes \hat{\sigma}_{\rm parton}$$

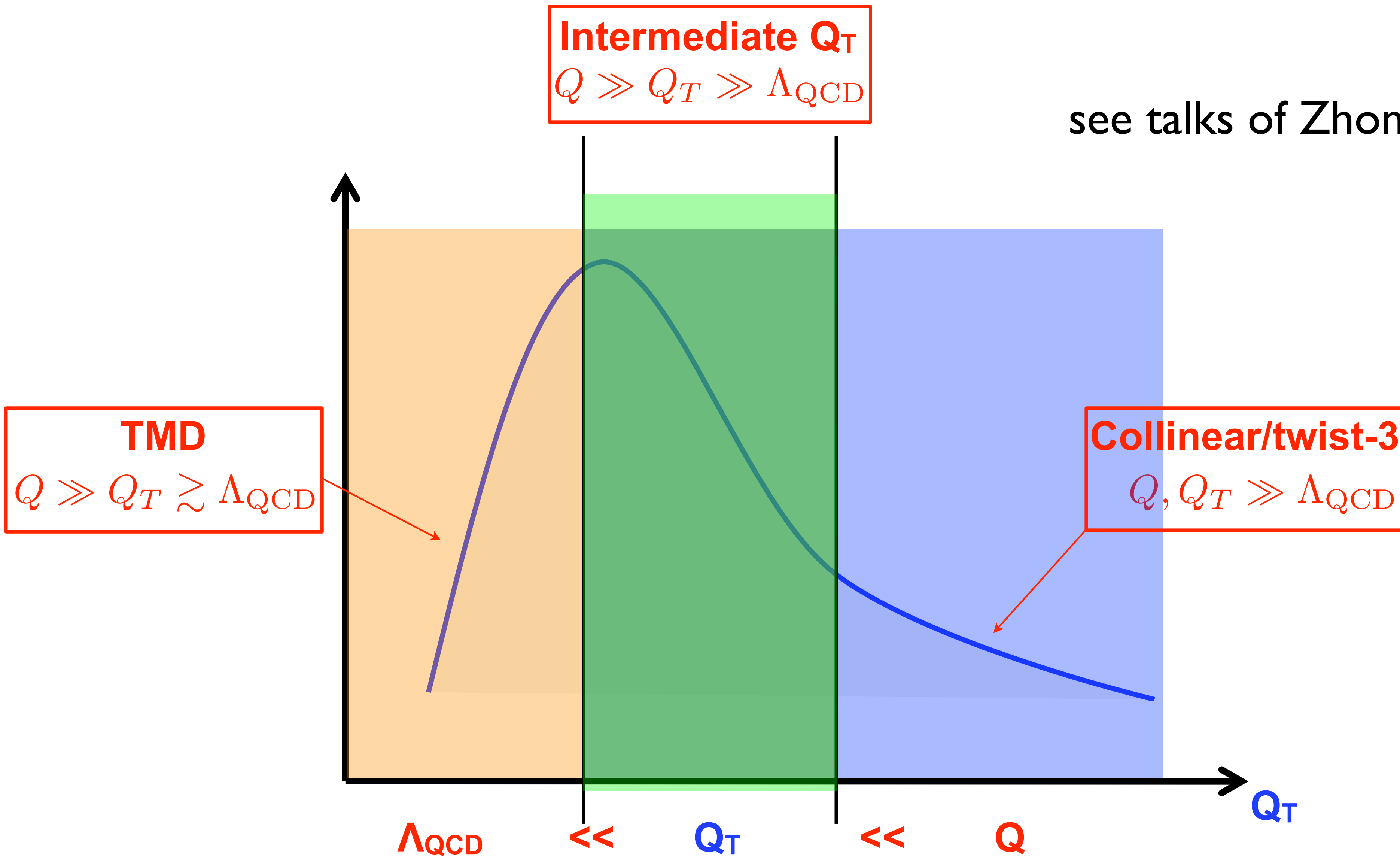
- $k_\perp^2 \ll P_{h\perp}^2 \sim Q^2$ twist 3 factorization-ETQSs

$$\Delta\sigma(P_h, S) \sim \frac{1}{Q} f_{a/A}^{\perp}(x) \otimes f_{b/B}(x) \otimes D_{h/c}(z) \otimes \hat{\sigma}_{\rm parton}$$

Connection of twist 3 and twist 2 approach "overlap regime"

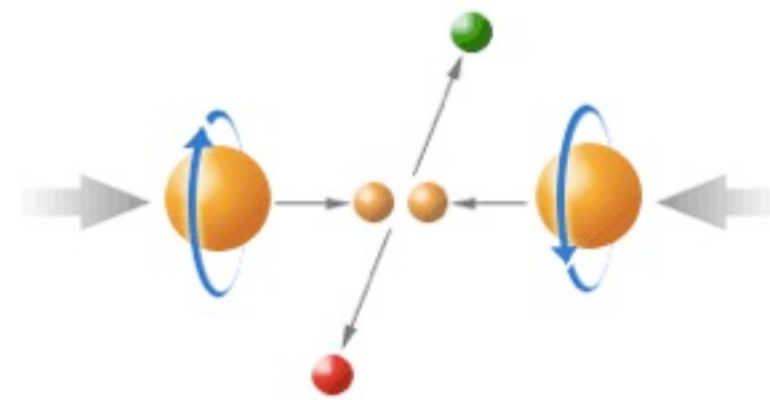
Ji,Qiu,Vogelsang, Yuan PRL 2006 ...
Bacchetta, Boer, Diehl, Mulders JHEP 2008

see talks of Zhongo-Bo and Jian Wei



- Explore role parton model processes in twist-2&3 approaches
  LG & Z. Kang PLB 2011 & for Collins in prep, "exploring impact of Gauge Inv"

Motivation Study Process dependence using inclusive processes



- Using btwn twist 2 "TMD" approach and twist 3 ETQS
  - Attempts to study process dependence in inclusive processes

$$gT_F(x,x) = -\int d^2k_T \frac{|k_T^2|}{M} f_{1T}^{\perp}(x,k_T^2)$$ Boer Piljman Mulders NPB 2003

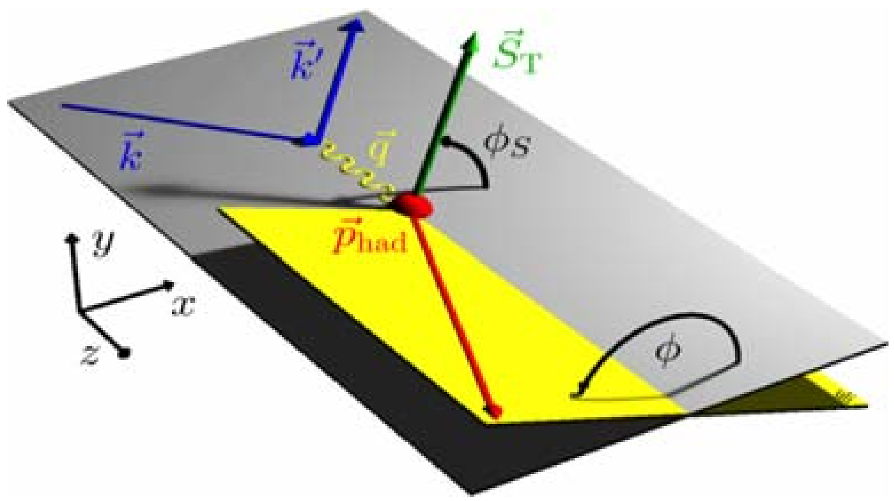
$$= -2M f_{1T}^{\perp(1)}(x)$$ + "UV" ...

Kang, Qiu, Vogelsang, Yuan prd 2011
Kang & Prokudin prd 2012
"compatibility study"

Generalizing the GPM CGI-GPM

- Feynman, Field, Fox (PRD 77 & 78)-incorporate intrinsic $\boldsymbol{k}_T$
- Include Transverse spin pol. w/ intrisic $\boldsymbol{k}_T$--Anselmino, Boglione, Murgia, ... et al. *PLB 95* & 98
- Pheno-Torino Cagliari group .... 1995-2012 inclusive processes
- Inclusive processes studied TW-3 formalism Kouvaris, Qiu, Vogelsang, Yuan PRD 2006, $pp \to \pi X$ & $pp \to \gamma X$
- What happens when you adopt ansatz of GPM including dynamical reaction mechanism of FSI/ISI in inclusive processes
- Take into account ISI/FSI process dependent Sivers function
- Since one scale, process-twist three is GPM connected w/twist 3 ? While we use $\boldsymbol{k}_T$ dependent TMDs we integrate over $\boldsymbol{k}_T$. Guides us to perform collinear expansion from GPM

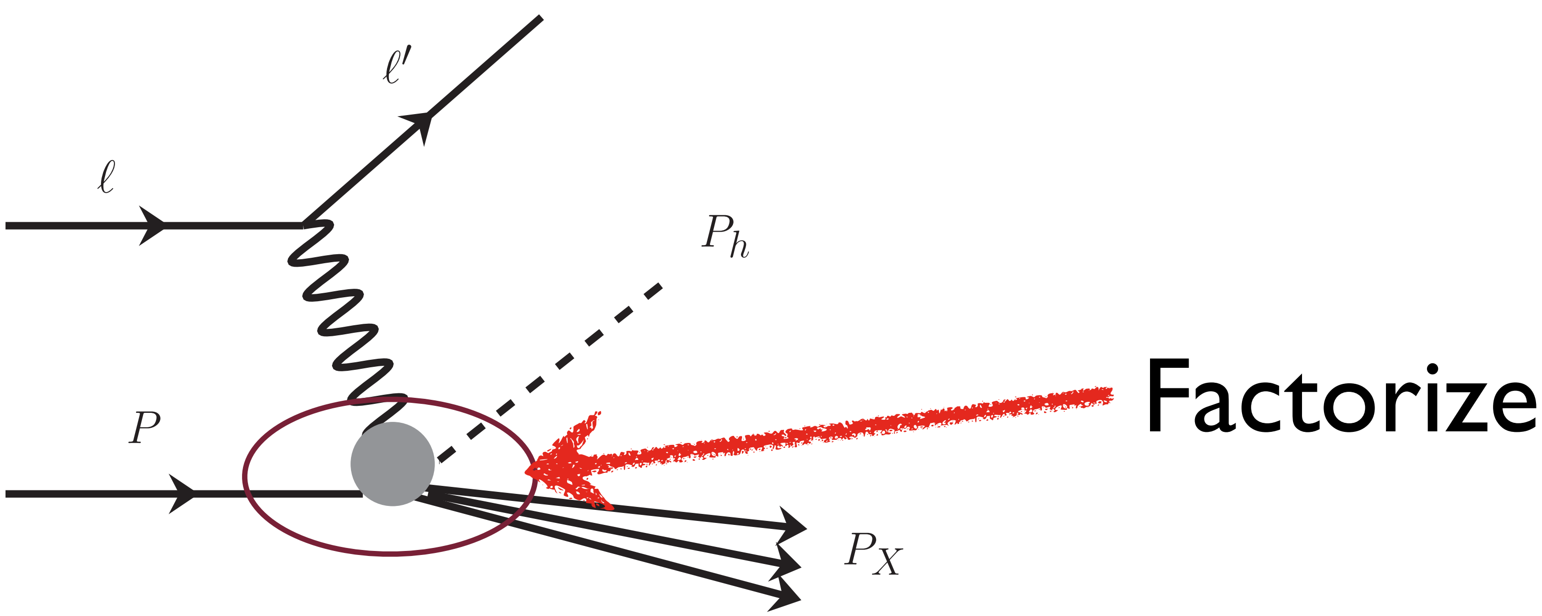
TMD Factorization in Parton Model-"kinematics"

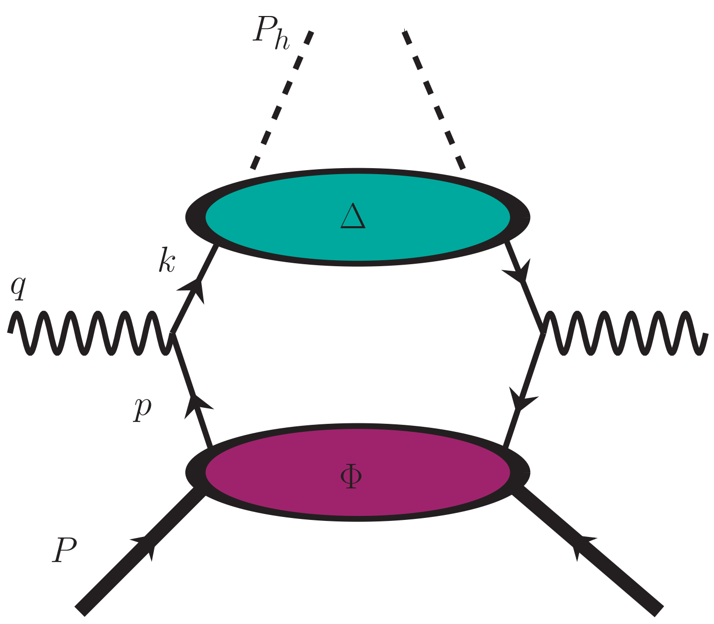


$$x_B = \frac{Q^2}{2P\cdot q}$$

$$z_h = \frac{P\cdot P_h}{P\cdot q} \approx \frac{P_h^-}{q^-}$$

Parton model & DIS kinematics



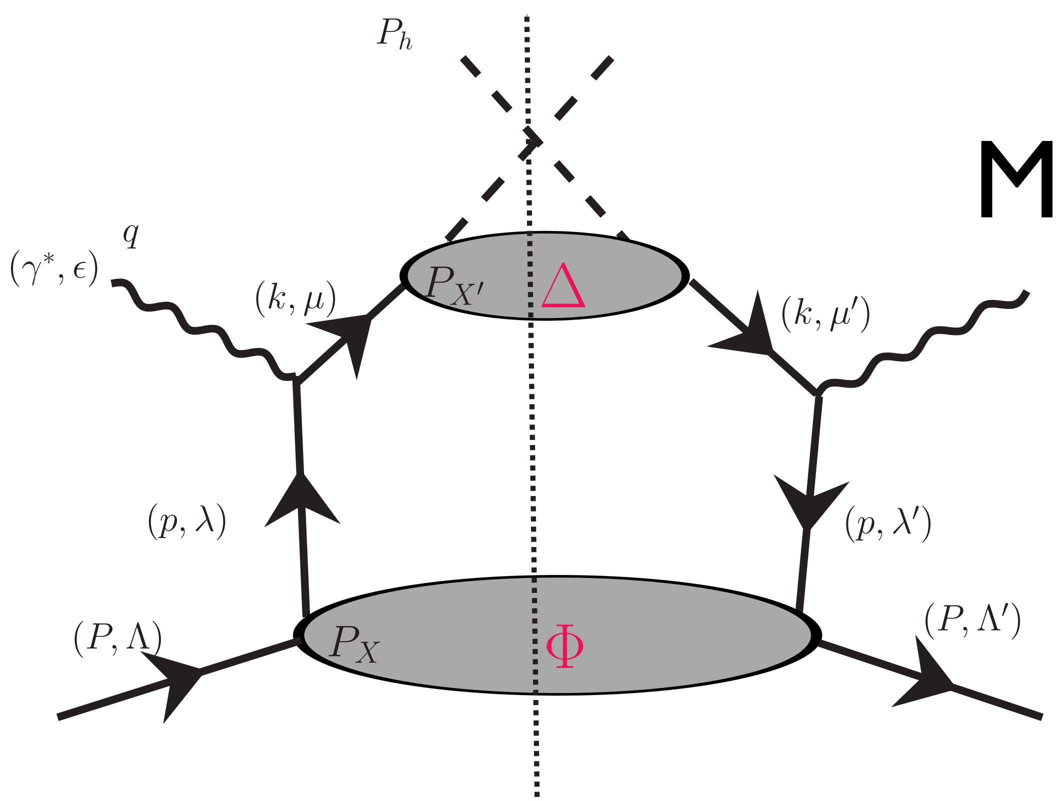


Factorization parton model P_T of hadron small sensitive to intrinsic transv. momentum of partons

$$W^{\mu\nu}(q,P,S,P_h) = \int \frac{d^2\mathbf{p}_T}{(2\pi)^4} \int \frac{d^2\mathbf{k}_T}{(2\pi)^4} \delta^2\left(\mathbf{p}_T - \frac{\mathbf{P}_{h\perp}}{z} - \mathbf{k}_T\right) \mathrm{Tr}\left[\left(\int dp^-\Phi\right)\gamma^\mu \left(\int dk^+\Delta\right)\gamma^\nu\right]$$

Small transverse momentum !!!

$$\Phi(x,\mathbf{p}_T,S) \equiv \int dp^-\Phi(p,P,S)\Big|_{p^+=x_BP^+}, \qquad \Delta(z,\mathbf{k}_T) \equiv \int dk^+\Delta(k,P_h)\Big|_{k^-=\frac{P^-}{z_h}}$$



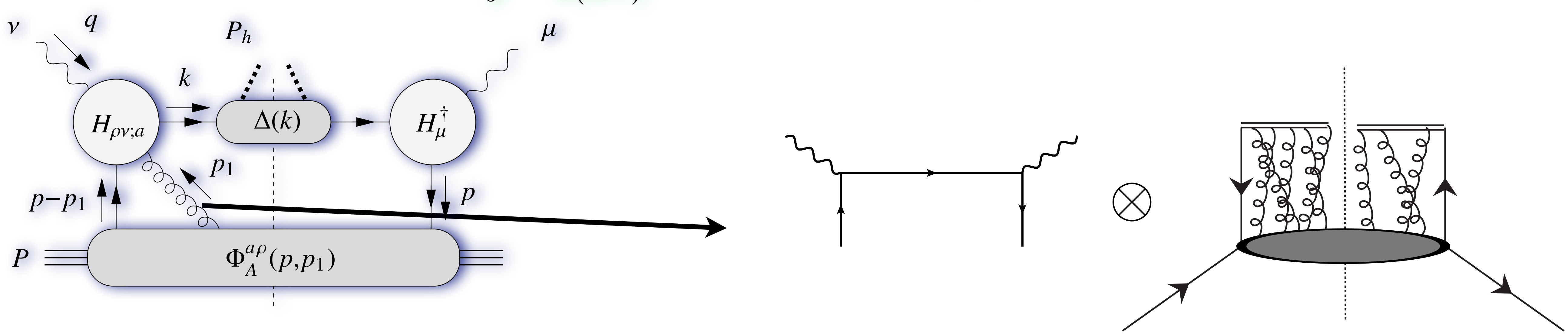
Minimal requirement satisfy color gauge invariance

Minimal Requirement Color Gauge Inv. Gauge links

Gauge link determined re-summing leading gluon interactions btwn soft and hard

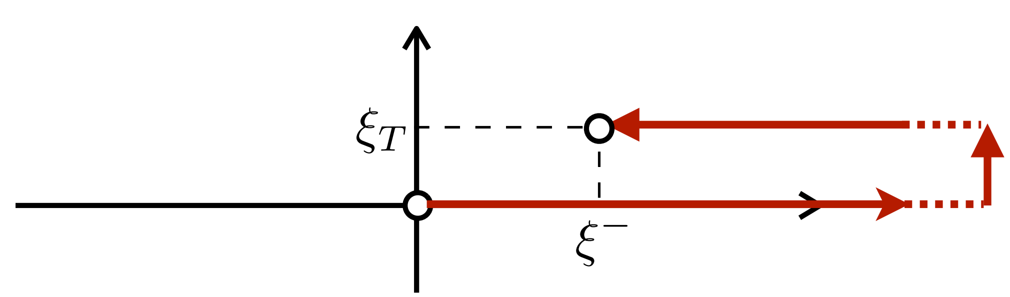
Efremov,Radyushkin Theor. Math. Phys. 1981,Belitsky, Ji, Yuan NPB 2003,
Boer, Bomhof, Mulders Pijlman, et al. 2003 - 2008- NPB, PLB, PRD

$$\Phi^{[\mathcal{U}[\mathcal{C}]]}(x, p_T) = \int \frac{d\xi^- d^2\xi_T}{2(2\pi)^3} e^{ip\cdot\xi} \langle P|\overline{\psi}(0)\mathcal{U}^{[C]}_{[0,\xi]}\psi(\xi^-, \xi_T)|P\rangle|_{\xi^+=0}$$



The path $[C]$ fixed by the hard subprocess in *factorization* procedure

$$W_{\mu\nu}(q, P, S, P_h) = \int d^4p d^4k \delta^4(p+q-k) \mathrm{Tr}\left[\Phi^{[U^C_{[\infty;\xi]}]}(p) H^\dagger_\mu(p,k)\Delta(k)H_\nu(p,k)\right]$$

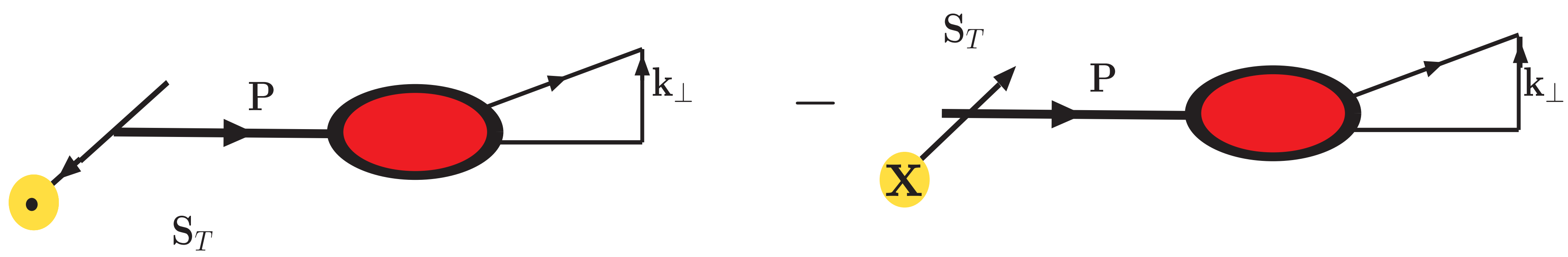


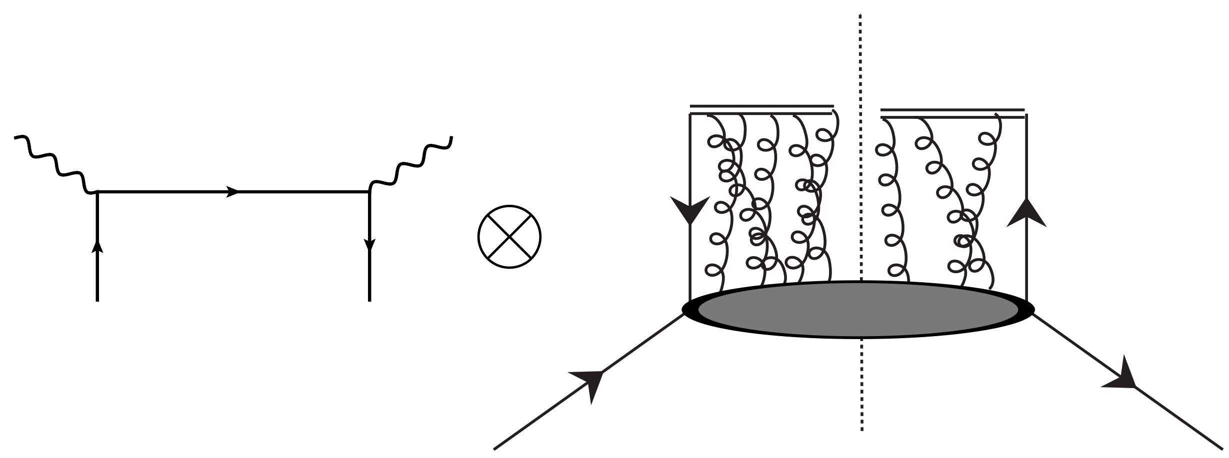
$U_{[+]}$

TSSAs thru "T-odd" non-pertb. spin-orbit correlations....

Sensitivity to $p_T \sim \mathbf{k}_T << \sqrt{Q^2}$

- **Sivers PRD: 1990** TSSA is associated w/ correlation *transverse* spin and momenta in initial state hadron

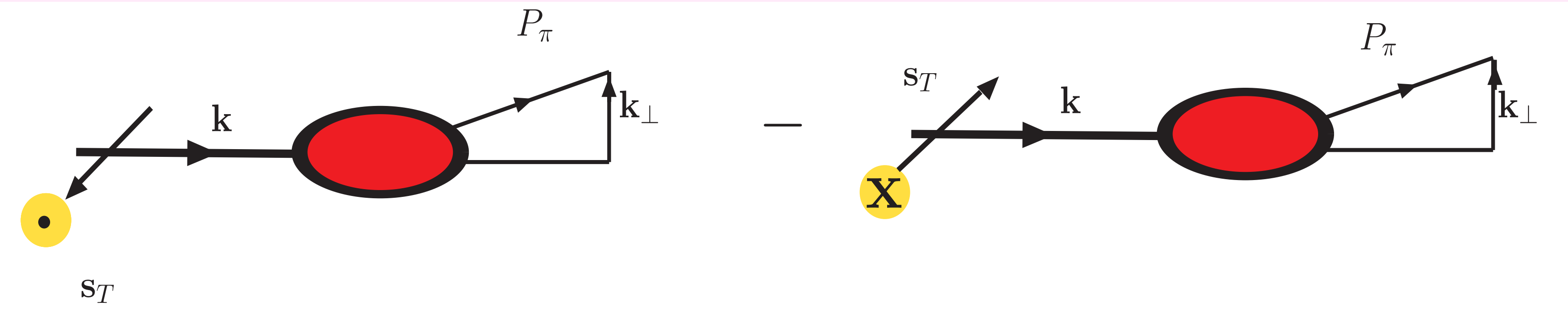




$$\Delta\sigma^{pp^\uparrow \to \pi X} \sim D \otimes f \otimes \Delta f^\perp \otimes \hat{\sigma}_{Born} \Longrightarrow \boldsymbol{\Delta f^\perp(x, k_\perp) = i S_T \cdot (P \times k_\perp)}\, f_{1T}^\perp(x, \boldsymbol{k}_\perp)$$

.... Fragmentation

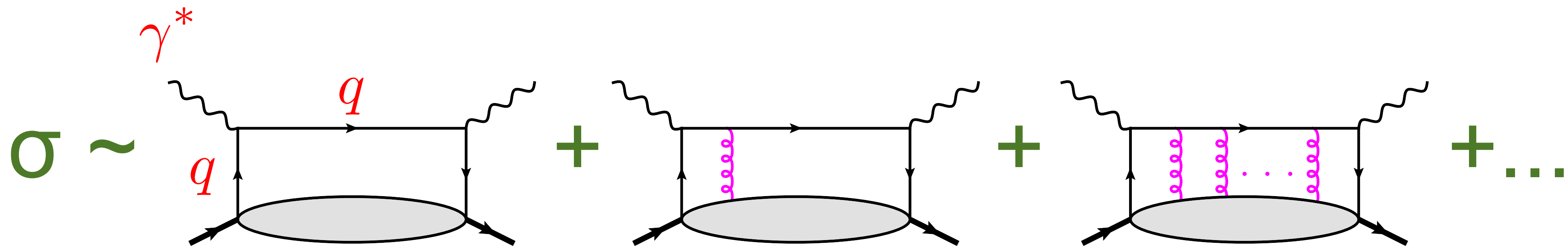
- Collins NPB: 1993 **TSSA is associated with *transverse* spin of fragmenting quark and transverse momentum of final state hadron**

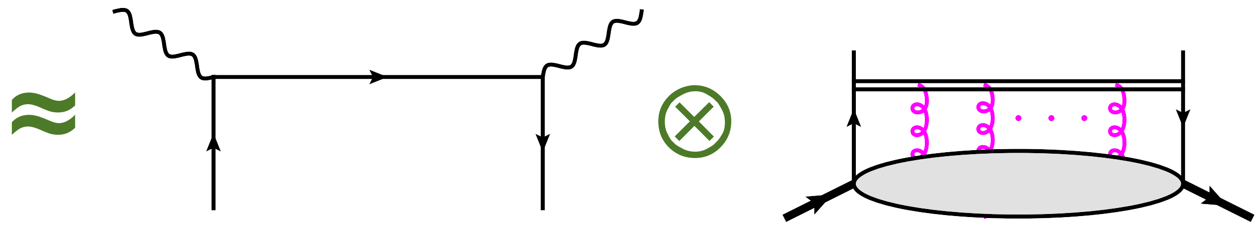


$$\Delta\sigma^{ep^\uparrow \to e\pi X} \sim \Delta D^\perp \otimes f \otimes \hat{\sigma}_{Born}$$

$$\boldsymbol{\Delta D^\perp(x, p_\perp) = i s_T \cdot (P \times p_\perp)} H_1^\perp(x, \boldsymbol{p}_\perp)$$

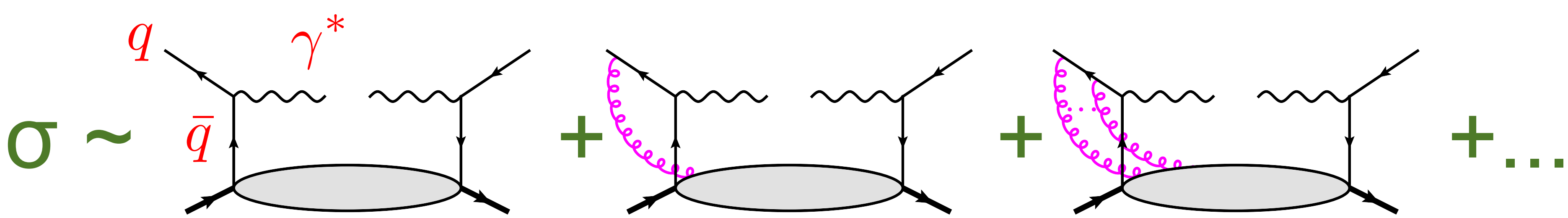
Process Dependence break down of Universality

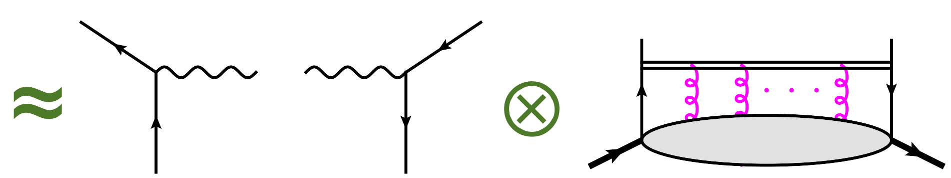




PDFs with SIDIS gauge link

$$\mathcal{P}\, e^{ig\int_y^{\infty} d\lambda\cdot A(\lambda)}$$



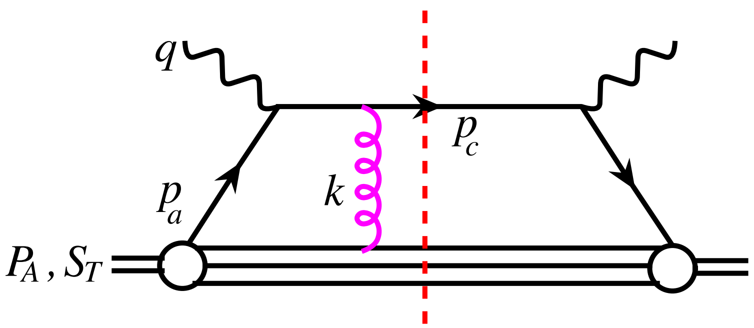


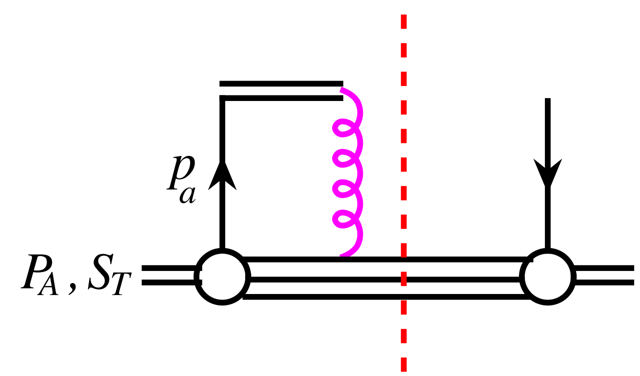
PDFs with DY gauge link

$$\mathcal{P}\, e^{ig\int_y^{-\infty} d\lambda\cdot A(\lambda)}$$

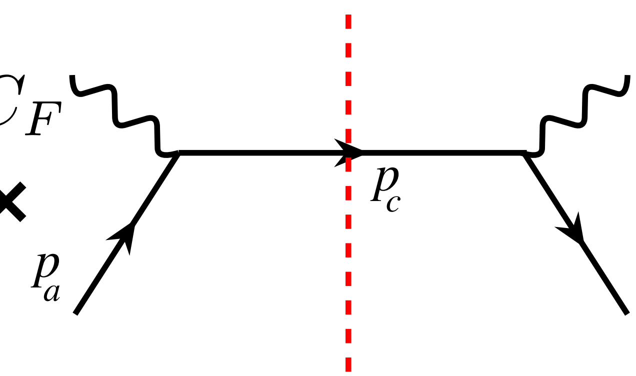
Classic example-same real pts opposite imaginary pts

Final-state interaction in SIDIS



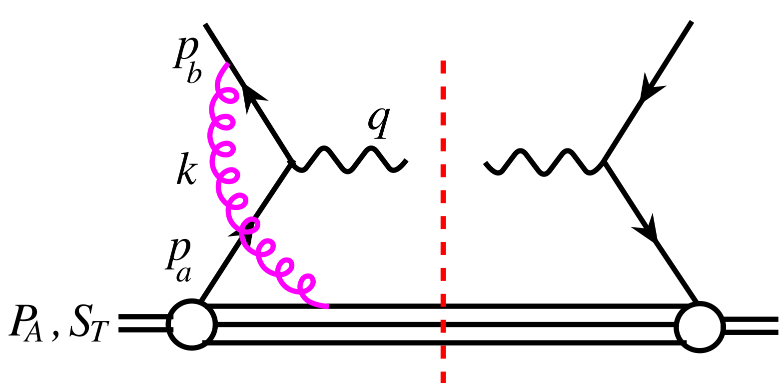


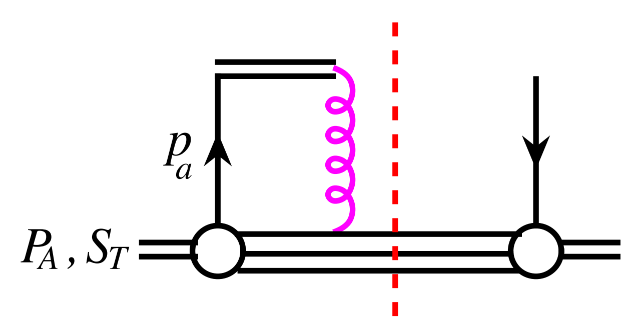
C_F ×



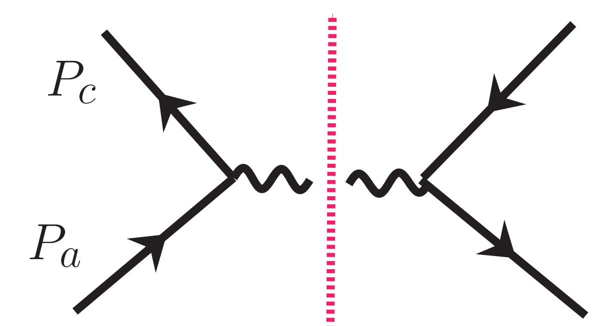
$$\bar{u}(p_c)(-ig)\gamma^- T^a \frac{i(\not{p}_c - \not{k})}{(p_c - k)^2 + i\epsilon} \approx \bar{u}(p_c)\left[\frac{g}{-k^+ + i\epsilon} T^a\right] \qquad \to -\bar{u}(p_c) T^a i\pi\delta(k^+)$$

and initial-state interaction in DY





C_I ×



$$\bar{v}(p_b)(-ig)\gamma^- T^a \frac{-i(\not{p}_b + \not{k})}{(p_b + k)^2 + i\epsilon} \approx \bar{v}(p_b)\left[\frac{g}{-k^+ - i\epsilon} T^a\right], \qquad \to v(p_b) T^a i\pi\delta(k^+)$$

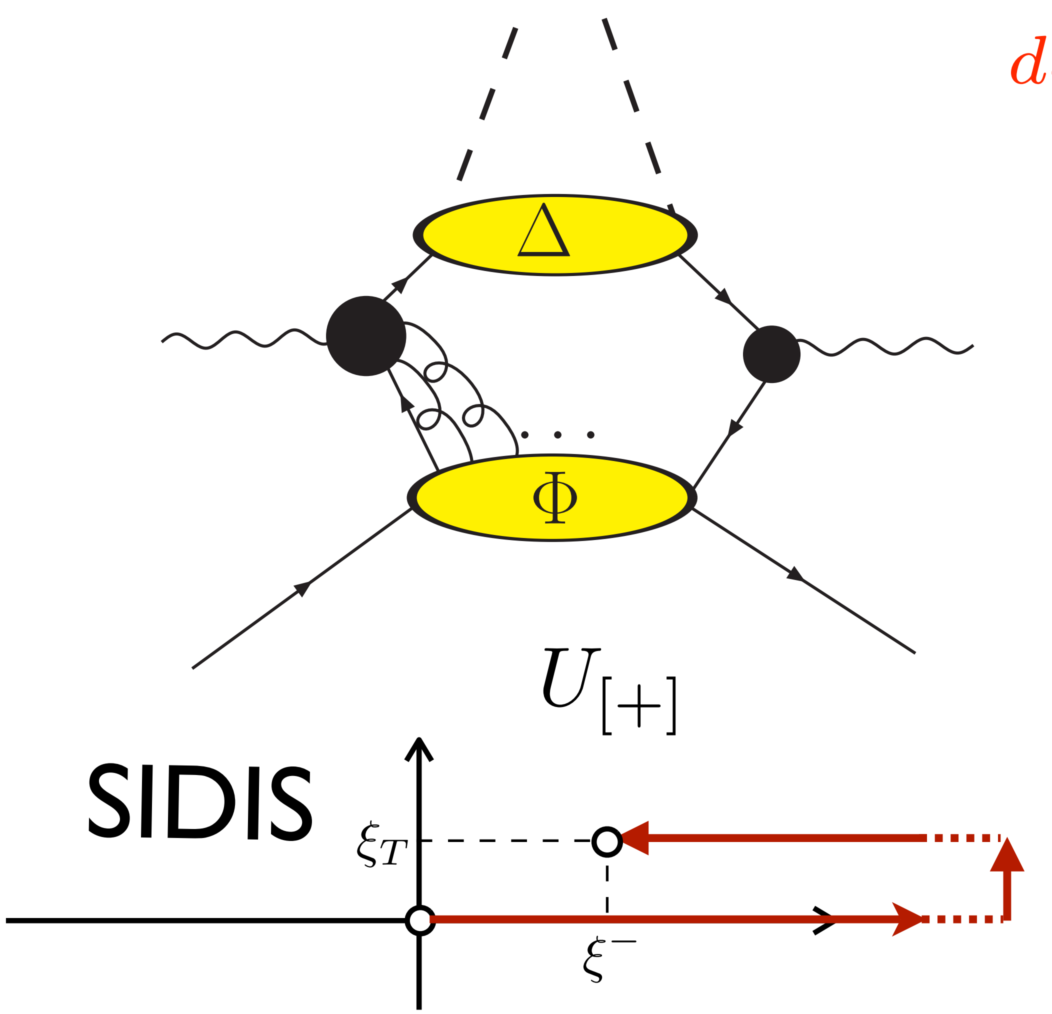
“Generalized Universality” Fund. Prediction of QCD Factorization

$$f_{1T\,sidis}^{\perp}(x,k_T) = -f_{1T\,DY}^{\perp}(x,k_T) \qquad p_T \sim \mathbf{k}_T << \sqrt{Q^2}$$

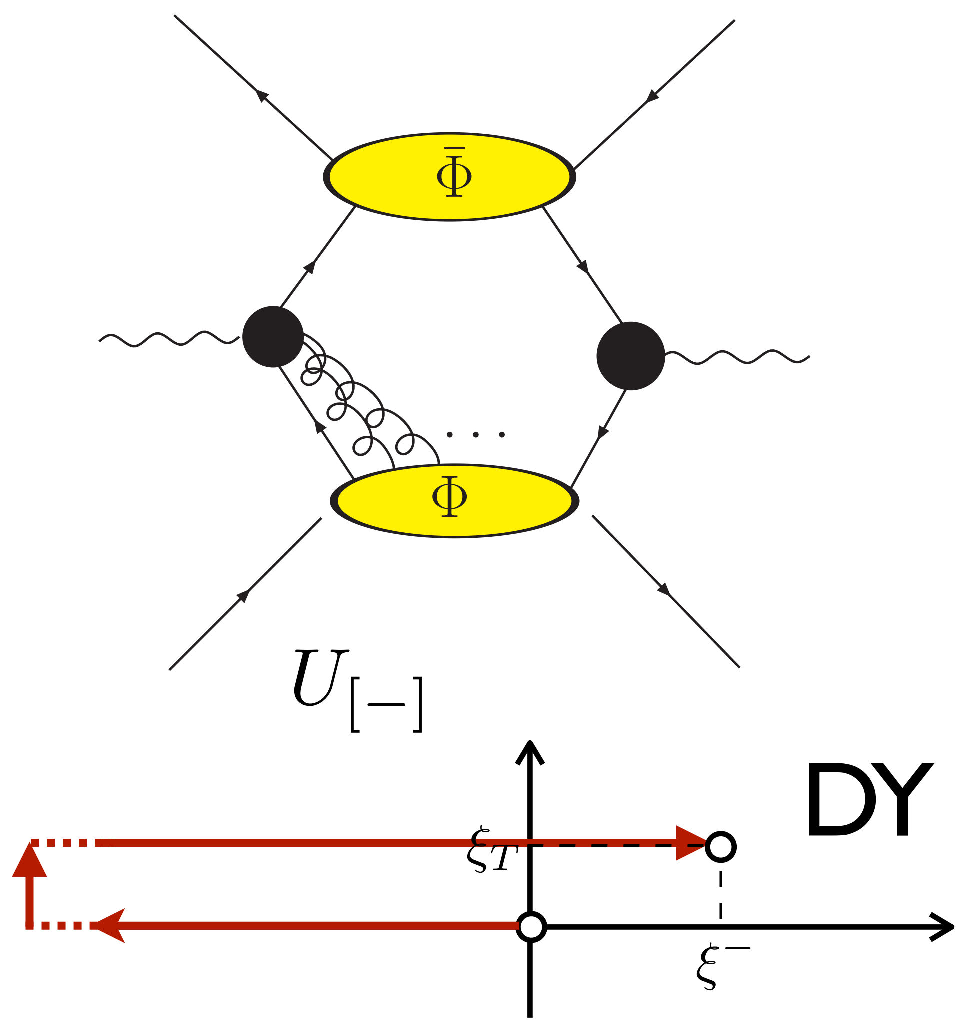
EIC conjunction with DY exp. E906-Fermi, RHIC II, Compass, JPARC

Process Dependence, Collins PLB 02, Brodsky et al. NPB 02, Boer Mulders Pijlman Bomhoff 03, 04 ...

$$d\sigma = L_{\mu\nu}\mathcal{W}^{\mu\nu} \Rightarrow$$



P&T



$$\Phi^{[+]*}(x,p_T) = i\gamma^1\gamma^3\Phi^{[-]}(x,p_T)i\gamma^1\gamma^3$$

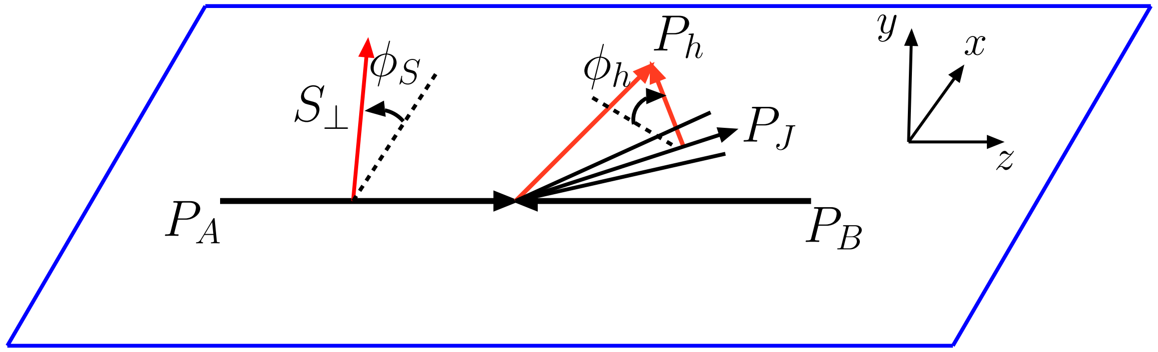
{\em Model Assumptions}

- "WTIM" consider hadronic processes taking into account ISI/FSI in gen. parton model GPM
- Consider impact in three cases
  - Inclusive pion production at forward rapidity- Both Collins and Sivers can contribute
  - Direct photon - Sivers only, can be used to test sign change as in DY
  - Prompt Photon--?? Collins Contribution??

cont ...

Azimuthal asymmetric distribution of hadrons inside a high energy jet in transverse polarized nucleon-nucleon scattering

$$p^{\uparrow}\, p \longrightarrow h_1\, \mathrm{jet}\, X$$



- Collins effect Yuan PRL 2008
- Pion about jet-Can disentangle Collins & Sivers
-

w/o ISI/FSI- D'Alesio, Murgia, Pisano PRD10,
w/ ISI/FSI- D'Alesio, LG, Kang, Murgia, Pisano w/ ISI/FSI-PLB 2011

  - Inclusive jet - Only Sivers, so can be used to test sign change as in DY

Caution !!! Comments

Similar studies performed for weighted k_T and unweighted

- photon Jet $p^{\uparrow}\, p \longrightarrow \gamma\, \mathrm{jet}\, X$ Bacchetta Bomhof, D'Alesio, Mulders, Murgia PRL 09
- 2-particle inclusive hadron production $p^{\uparrow}\, p \longrightarrow h_1\, h_2\, X$

Bacchetta Bomhof, Mulders, Piljman PRD05, Qiu Vogelsang Yuan PRD2007, Vogelsang Yuan PRD 2007

Merits "Pre-Collins Qiu Mulders Rogers" "PCQMR period"

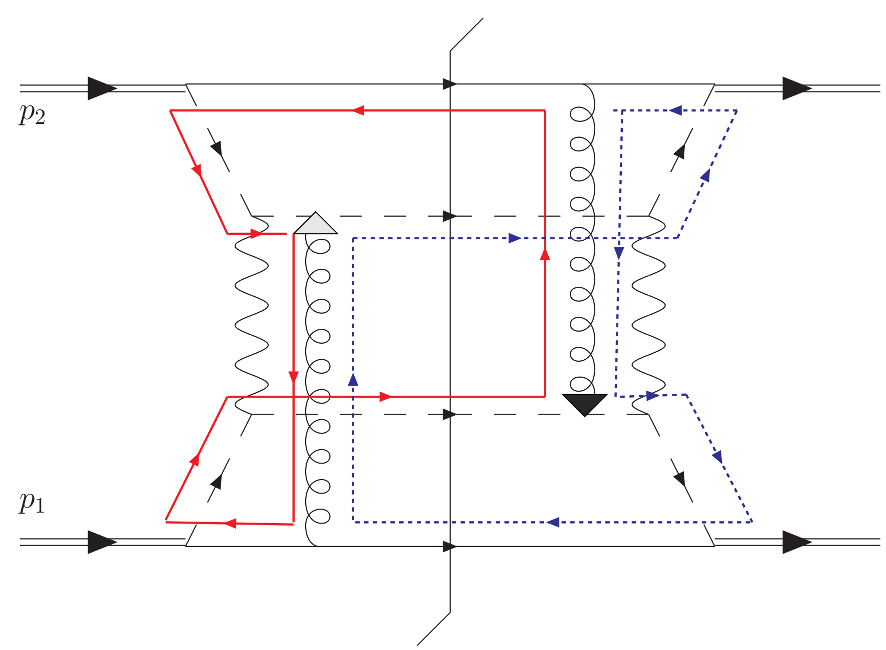
1) two scale problem--TMD factorization

2) weighted submits to transverse moments leads to gluonic pole factors & gluonic pole matrix elements--connection to twist three formalism

Problems/Challenges-"post CQMR period"

Collins Qiu PRD 2007 & Mulders Rogers 2010

*) factorization violated cannot define even a generalized gauge link-color entangled



Generalizing the GPM CGI-GPM

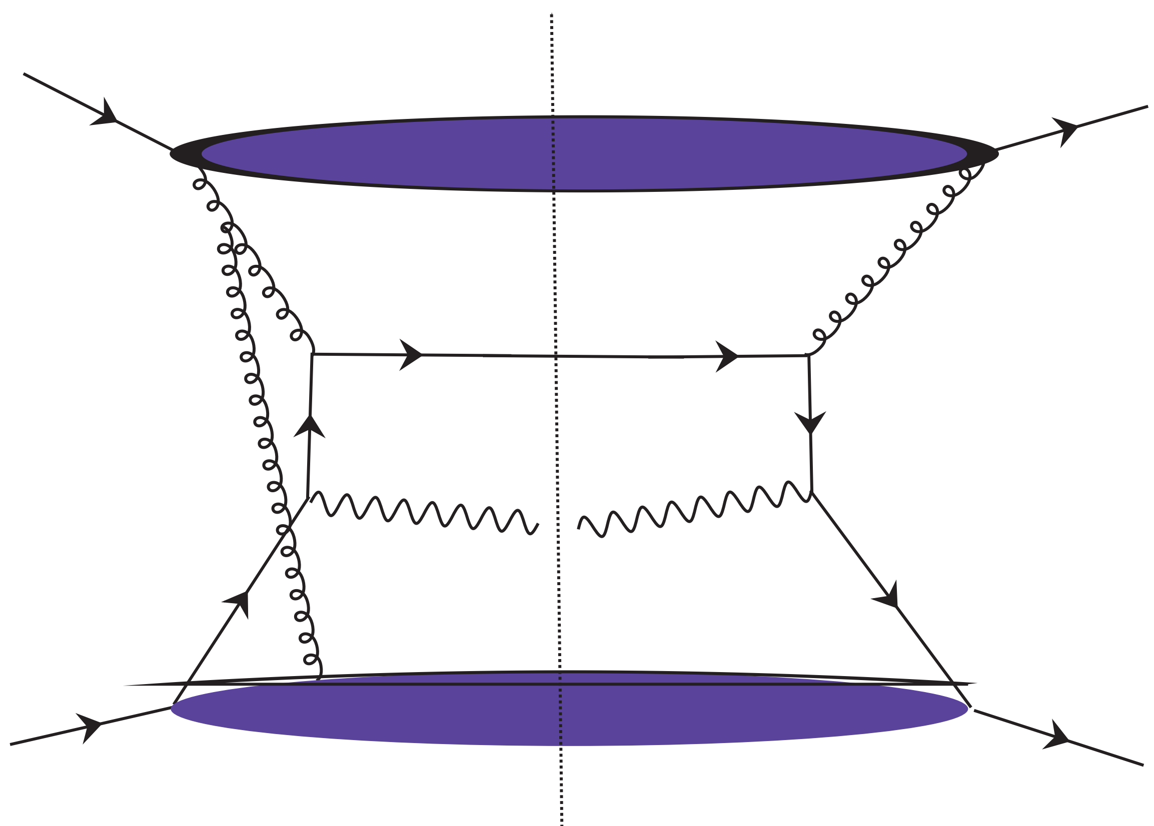
- Feynman, Field, Fox (PRD 77 & 78)-incorporate intrinsic $\boldsymbol{k}_T$
- Include Transverse spin pol. w/ intrisic $\boldsymbol{k}_T$--Anselmino, Boglione, Murgia, ... et al. *PLB 95* & 98
- Pheno-Torino Cagliari group .... 1995-2012 inclusive processes
- Inclusive processes studied TW-3 formalism Kouvaris, Qiu, Vogelsang, Yuan PRD 2006, $pp \to \pi X$ & $pp \to \gamma X$
- What happens when you adopt ansatz of GPM including dynamical reaction mechanism of FSI/ISI in inclusive processes
- Take into account ISI/FSI process dependent Sivers function
- Since one scale, process-twist three is GPM connected w/twist 3 ? While we use $\boldsymbol{k}_T$ dependent TMDs we integrate over $\boldsymbol{k}_T$. Guides us to perform collinear expansion from GPM

Consider direct Photon in GPM

$$\Delta\sigma^{pp^{\uparrow}\to\gamma X} \sim \Delta f_a \otimes f_b \otimes \Delta\hat{\sigma}$$

Factorize w/ leading 1 gluon exchange get color phase

Vogelsang & Yuan PRD 2007 & agrees w/ "color flow" approach Bomhoff, Mulders, Pijlman 2006...



Method

- Use diagramatic rather than helicity approach
  Bacchetta Bomhoff Mulders Pijlman 2005 PRD
- Has advantage of directly connecting to matrix elements of quark and gluon fields
- Allows inclusion of effects of ISI/FSI to determine color structure

Spin Dependent Cross Section in GPM $pp \longrightarrow \gamma X$

GPM Anselmino et al.

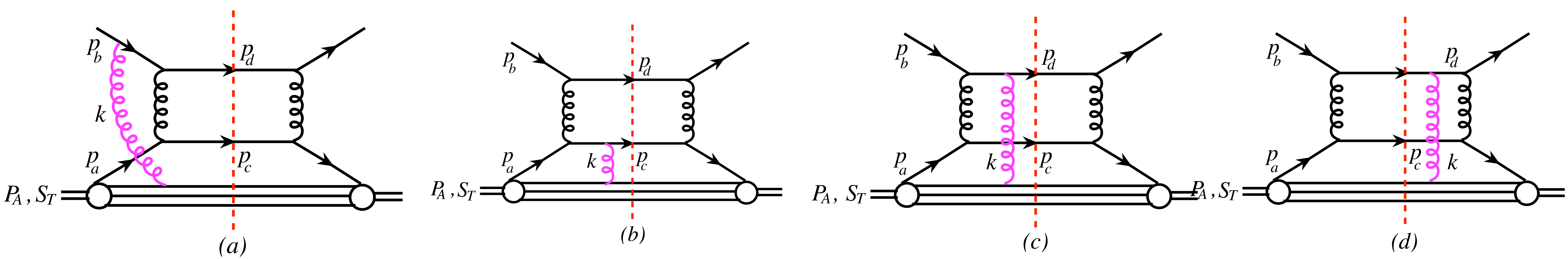
A_N is defined by the ratio $$A_N = E_\gamma \frac{d\Delta\sigma}{d^3P_\gamma} \Big/ E_\gamma \frac{d\sigma}{d^3P_\gamma}.$$

$$E_\gamma \frac{d\Delta\sigma}{d^3P_\gamma} = \frac{\alpha_{em}\alpha_s}{S} \sum_{a,b} \int \frac{dx_a}{x_a} d^2k_{aT} \Delta^N f_{a/A}^{\mathrm{DIS}}(x_a, k_{aT}) \frac{1}{2} S_A \cdot (\hat{P}_A \times \hat{k}_{aT})$$
$$\times \int \frac{dx_b}{x_b} d^2k_{bT} f_{b/B}(x_b, k_{bT}) H^U_{ab\to\gamma}(\hat{s}, \hat{t}, \hat{u}) \delta(\hat{s} + \hat{t} + \hat{u}).$$

$$f_{q/A^\uparrow}(x, \vec{k}_T) = f_{q/A}(x, k_T^2) + \tfrac{1}{2} \Delta^N f_{q/A^\uparrow}(x, k_T^2) \vec{S} \cdot (\hat{P} \times \vec{k}_T)$$

Observation

- Crucial point: Sivers function in inclusive single particle production contains both ISI and FSI
- Color factors entirely due to color structure of the partonic subprocess
- consider channel $qq' \to qq'$



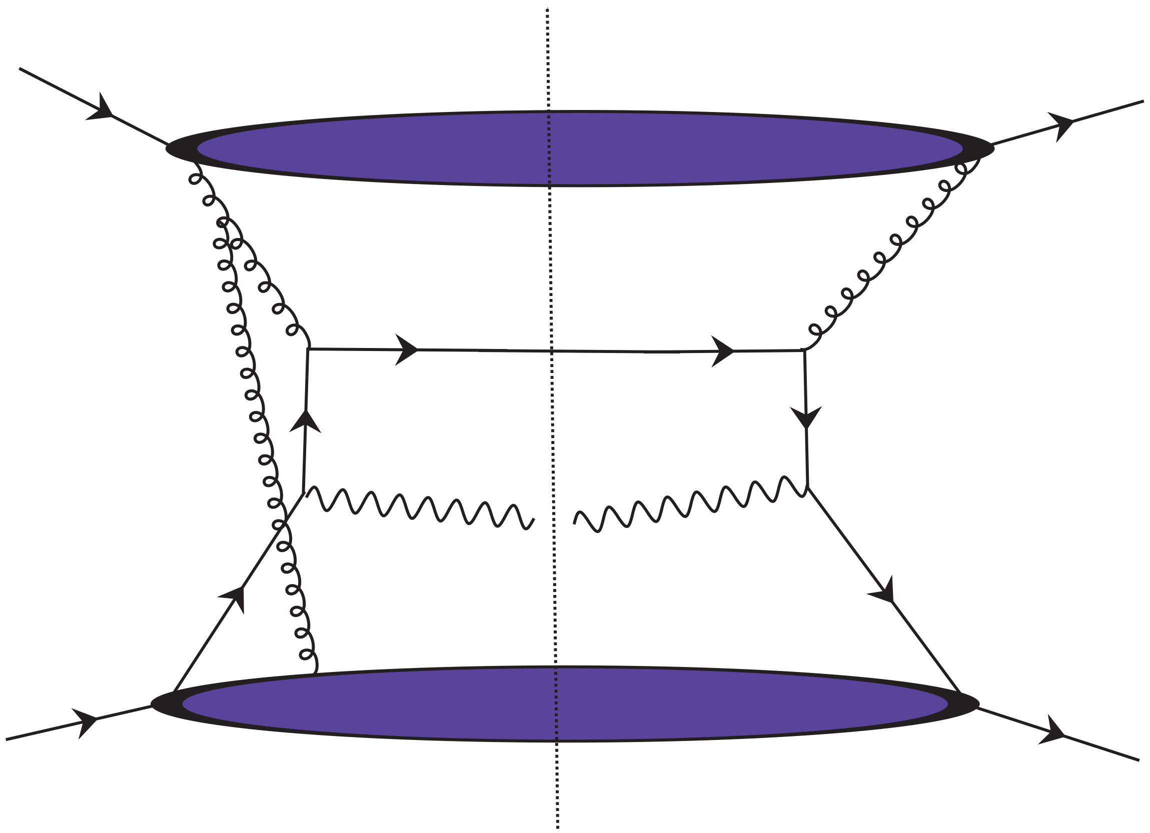
Consider direct Photon in GPM

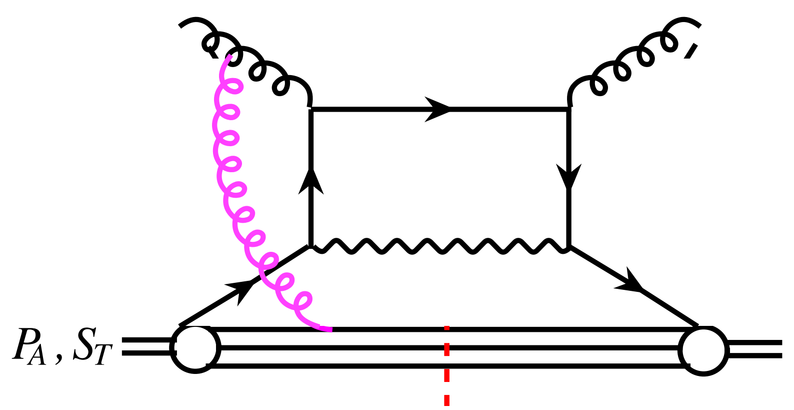
GPM w/color
LG & Z. Kang
Phys.Lett. B696 2011

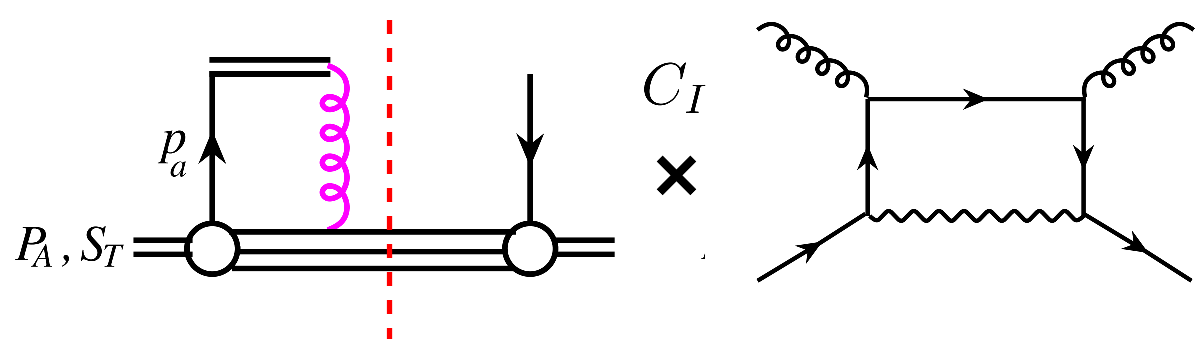
$$\Delta\sigma^{pp^{\uparrow}\to\gamma X} \sim \Delta f_a \otimes f_b \otimes \Delta\hat{\sigma}$$

Factorize w/ leading 1 gluon exchange get color phase

Vogelsang & Yuan PRD 2007 & agrees w/ "color flow" approach Bomhoff, Mulders, Pijlman





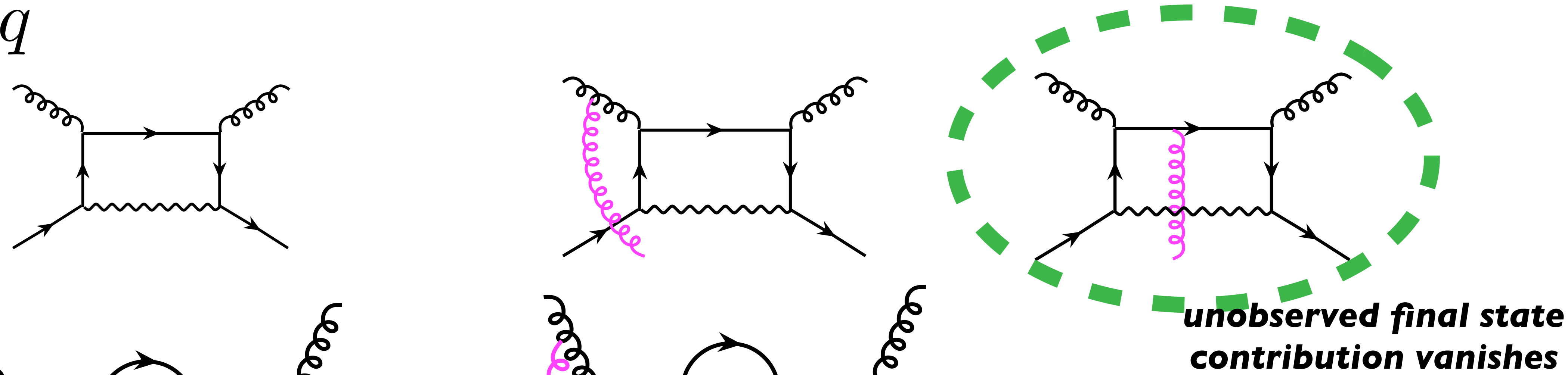


Get Sivers function for this process to use in GPM

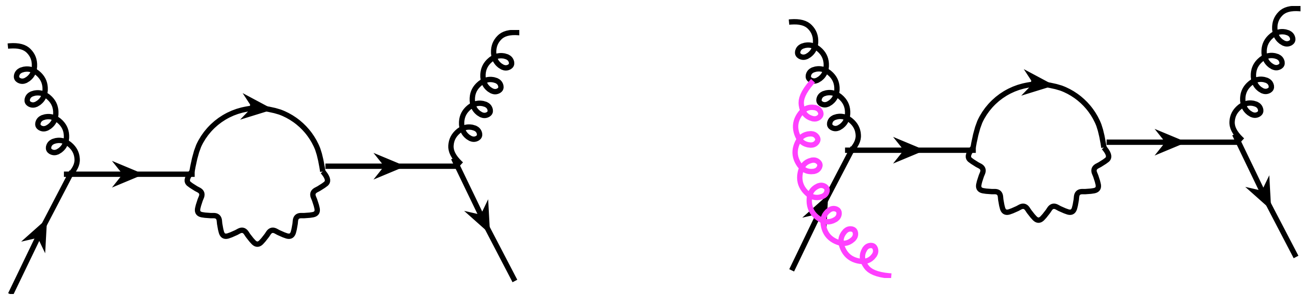
Color modification of hard cross sections due to "phases"

$qg \to \gamma q$

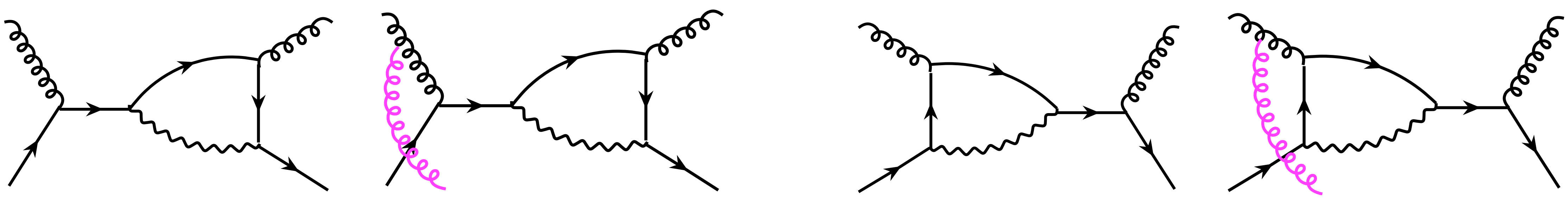
t-channel



s-channel

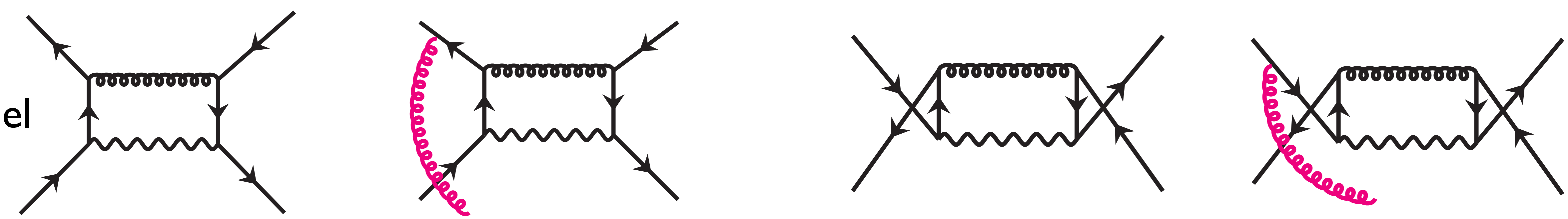


s-t interference



$\bar{q}q \to \gamma g$

t & u-channel



t-u interference etc ....

Spin Dependent Cross Section in GPM $pp \to \gamma X$

$$f_{q/A^\uparrow}(x, \vec{k}_T) = f_{q/A}(x, k_T^2) + \frac{1}{2}\Delta^N f_{q/A^\uparrow}(x, k_T^2)\vec{S}\cdot(\hat{P}\times\vec{k}_T)$$

A_N is defined by the ratio

$$A_N = E_\gamma \frac{d\Delta\sigma}{d^3P_\gamma}\Big/ E_\gamma \frac{d\sigma}{d^3P_\gamma}.$$

$$E_\gamma \frac{d\Delta\sigma}{d^3P_\gamma} = \frac{\alpha_{em}\alpha_s}{S}\sum_{a,b}\int \frac{dx_a}{x_a} d^2k_{aT} \Delta^N f_{a/A}^{\text{DIS}}(x_a, k_{aT}) \frac{1}{2} S_A \cdot (\hat{P}_A \times \hat{k}_{aT})$$
$$\times \int \frac{dx_b}{x_b} d^2k_{bT} f_{b/B}(x_b, k_{bT}) H^U_{ab\to\gamma}(\hat{s}, \hat{t}, \hat{u})\delta(\hat{s}+\hat{t}+\hat{u}).$$

GPM Anselmino et al.

$$E_\gamma \frac{d\Delta\sigma}{d^3P_\gamma} = \frac{\alpha_{em}\alpha_s}{S}\sum_{a,b}\int \frac{dx_a}{x_a} d^2k_{aT} \Delta^N f_{a/A}^{ab\to\gamma}(x_a, k_{aT}) \frac{1}{2} S_A \cdot (\hat{P}_A \times \hat{k}_{aT})$$
$$\times \int \frac{dx_b}{x_b} d^2k_{bT} f_{b/B}(x_b, k_{bT}) H_{ab\to\gamma}(\hat{s}, \hat{t}, \hat{u})\delta(\hat{s}+\hat{t}+\hat{u}),$$

GPM w/color
LG & Z. Kang
Phys.Lett. B696 2011

process-dependent Sivers function denoted as $\Delta^N f_{a/A}^{ab\to c}(x_a, k_{aT})$

Spin Dependent Cross Section in GPM $pp \to \pi X$

A_N is defined by the ratio $A_N \equiv E_h \frac{d\Delta\sigma}{d^3P_h} \bigg/ E_h \frac{d\sigma}{d^3P_h}.$

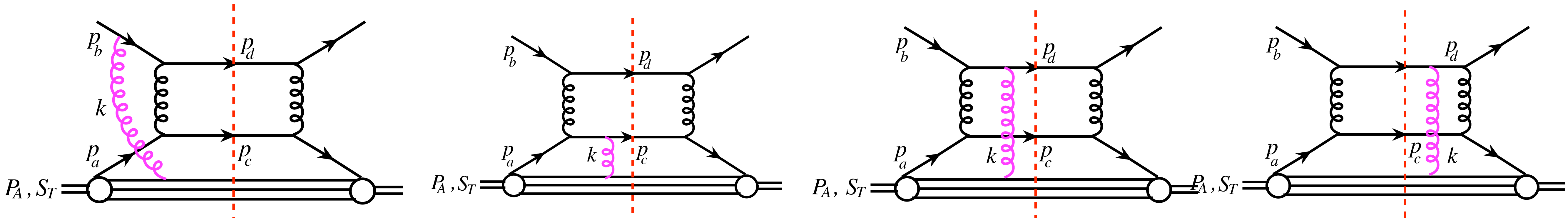
$$E_h \frac{d\Delta\sigma}{d^3P_h} = \frac{\alpha_s^2}{S} \sum_{a,b,c} \int \frac{dx_a}{x_a} d^2k_{aT} \Delta^N f_{a/A}(x_a, k_{aT}) \frac{1}{2} S_A \cdot (\hat{P}_A \times \hat{k}_{aT}) \int \frac{dx_b}{x_b} d^2k_{bT} f_{b/B}(x_b, k_{bT}) \times \int \frac{dz_c}{z_c^2} D_{h/c}(z_c) H^U_{ab\to c}(\hat{s}, \hat{t}, \hat{u}) \delta(\hat{s} + \hat{t} + \hat{u}),$$

GPM Anselmino et al.

$$E_h \frac{d\Delta\sigma}{d^3P_h} = \frac{\alpha_s^2}{S} \sum_{a,b,c} \int \frac{dx_a}{x_a} d^2k_{aT} \Delta^N f_{a/A}^{ab\to c}(x_a, k_{aT}) \frac{1}{2} S_A \cdot (\hat{P}_A \times \hat{k}_{aT}) \int \frac{dx_b}{x_b} d^2k_{bT} f_{b/B}(x_b, k_{bT}) \times \int \frac{dz_c}{z_c^2} D_{h/c}(z_c) H^U_{ab\to c}(\hat{s}, \hat{t}, \hat{u}) \delta(\hat{s} + \hat{t} + \hat{u}),$$

process-dependent Sivers function denoted as $\Delta^N f_{a/A}^{ab\to c}(x_a, k_{aT})$

One gluon exchange approx for ISI and FSI



(a) (b) (c) (d)

$$\left[\frac{-g}{-k^+ - i\epsilon} T^a\right]$$

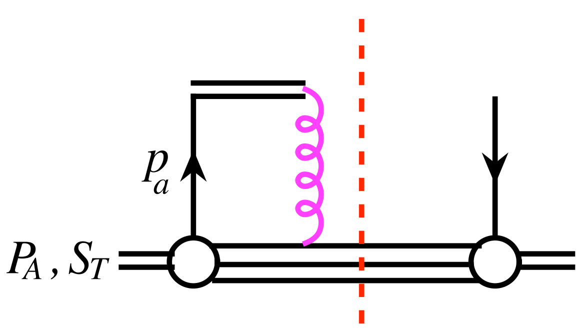
$$\rightarrow\ C_I = -\frac{1}{2N_c^2},$$

$$\left[\frac{g}{-k^+ + i\epsilon} T^a\right]$$

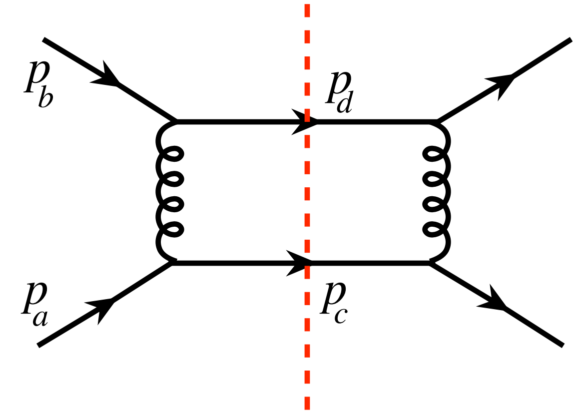
$$\rightarrow\ C_{F_c} = -\frac{1}{4N_c^2},$$

interaction w/unobserved particle "d" vanishes after summing over both cuts

calculate color factors



× C_I or C_{F_c} ×



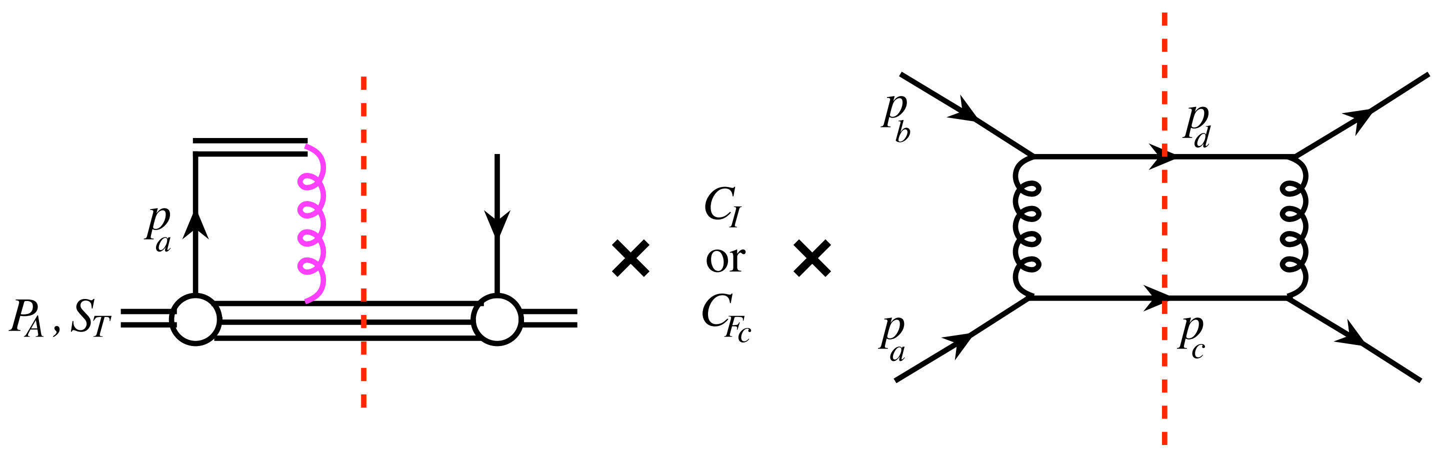
(a) (b)

Note unpolarized color factor

$$C_u = \frac{N_c^2 - 1}{4N_c^2}$$

Comparing imag. pt of eikonal propagators for subprocess in SIDIS and inclusive single particle production

Sivers function probed in $qq' \to qq'$ process is related to those in SIDIS



(a) (b)

$$\Delta^N f_{a/A}^{qq' \to qq'} = \frac{C_I + C_{F_c}}{C_u} \Delta^N f_{a/A}^{\rm SIDIS}.$$

Alternatively one can move color factors ”process dependence” to hard parts

$$E_h \frac{d\Delta\sigma}{d^3 P_h} = \frac{\alpha_s^2}{S} \sum_{a,b,c} \int \frac{dx_a}{x_a} d^2 k_{aT} \Delta^N f_{a/A}^{ab\to c}(x_a, k_{aT}) \frac{1}{2} S_A \cdot (\hat{P}_A \times \hat{k}_{aT}) \int \frac{dx_b}{x_b} d^2 k_{bT} f_{b/B}(x_b, k_{bT})$$
$$\times \int \frac{dz_c}{z_c^2} D_{h/c}(z_c) H_{ab\to c}^{U}(\hat{s}, \hat{t}, \hat{u}) \delta(\hat{s} + \hat{t} + \hat{u}),$$

$$E_h \frac{d\Delta\sigma}{d^3 P_h} = \frac{\alpha_s^2}{S} \sum_{a,b,c} \int \frac{dx_a}{x_a} d^2 k_{aT} \Delta^N f_{a/A}^{\mathrm{SIDIS}}(x_a, k_{aT}) \frac{1}{2} S_A \cdot (\hat{P}_A \times \hat{k}_{aT}) \int \frac{dx_b}{x_b} d^2 k_{bT} f_{b/B}(x_b, k_{bT})$$
$$\times \int \frac{dz_c}{z_c^2} D_{h/c}(z_c) H_{ab\to c}^{\mathrm{Inc}}(\hat{s}, \hat{t}, \hat{u}) \delta(\hat{s} + \hat{t} + \hat{u}),$$

In spirit of twist 3 approach, color factors from hard part

In spirit of twist 3 approach

That is rearrange

$$\Delta^N f_{a/A}^{qq'\to qq'} H^U_{qq'\to qq'} = \frac{C_I + C_{F_c}}{C_u} \Delta^N f_{a/A}^{\rm SIDIS} H^U_{qq'\to qq'} = \Delta^N f_{a/A}^{\rm SIDIS} \left[C_I h_{qq'\to qq'} + C_{F_c} h_{qq'\to qq'} \right],$$

where hard partonic c.s. w/o color factors

$$h_{qq'\to qq'} = 2\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}.$$

Then “modified” GPM is

$$E_h \frac{d\Delta\sigma}{d^3P_h} = \frac{\alpha_s^2}{S} \sum_{a,b,c} \int \frac{dx_a}{x_a} d^2k_{aT} \Delta^N f_{a/A}^{\rm SIDIS}(x_a, k_{aT}) \frac{1}{2} S_A \cdot (\hat{P}_A \times \hat{k}_{aT}) \int \frac{dx_b}{x_b} d^2k_{bT} f_{b/B}(x_b, k_{bT})$$

$$\times \int \frac{dz_c}{z_c^2} D_{h/c}(z_c) H_{ab\to c}^{\rm Inc}(\hat{s}, \hat{t}, \hat{u}) \delta(\hat{s} + \hat{t} + \hat{u}),$$

$$H_{qq'\to qq'}^{\rm Inc} \equiv H_{qq'\to qq'}^{\rm Inc-I} + H_{qq'\to qq'}^{\rm Inc-F},$$

where,

$$H_{qq'\to qq'}^{\rm Inc-I} = C_I h_{qq'\to qq'}, \qquad H_{qq'\to qq'}^{\rm Inc-F} = C_{F_c} h_{qq'\to qq'}$$

The contributions for $pp \to \pi X$

the various contributing partonic subprocesses are given by

$$H^{\mathrm{Inc-I}}_{qq' \to qq'} = -H^{\mathrm{Inc-I}}_{\bar{q}\bar{q}' \to \bar{q}\bar{q}'} = -\frac{1}{N_c^2}\left[\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}\right]$$

$$H^{\mathrm{Inc-F}}_{qq' \to qq'} = -H^{\mathrm{Inc-F}}_{\bar{q}\bar{q}' \to \bar{q}\bar{q}'} = -\frac{1}{2N_c^2}\left[\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}\right]$$

$$H^{\mathrm{Inc-I}}_{q\bar{q}' \to q\bar{q}'} = -H^{\mathrm{Inc-I}}_{\bar{q}q' \to \bar{q}q'} = -\frac{N_c^2 - 2}{2N_c^2}\left[\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}\right]$$

$$H^{\mathrm{Inc-F}}_{q\bar{q}' \to q\bar{q}'} = -H^{\mathrm{Inc-F}}_{\bar{q}q' \to \bar{q}q'} = -\frac{1}{2N_c^2}\left[\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}\right]$$

$$H^{\mathrm{Inc-I}}_{qq' \to q'q} = -H^{\mathrm{Inc-I}}_{\bar{q}\bar{q}' \to \bar{q}'\bar{q}} = -\frac{1}{N_c^2}\left[\frac{\hat{s}^2 + \hat{t}^2}{\hat{u}^2}\right]$$

$$H^{\mathrm{Inc-F}}_{qq' \to q'q} = -H^{\mathrm{Inc-F}}_{\bar{q}\bar{q}' \to \bar{q}'\bar{q}} = \frac{N_c^2 - 2}{2N_c^2}\left[\frac{\hat{s}^2 + \hat{t}^2}{\hat{u}^2}\right]$$

$$H^{\mathrm{Inc-I}}_{q\bar{q}' \to \bar{q}'q} = -H^{\mathrm{Inc-I}}_{\bar{q}q' \to q'\bar{q}} = -\frac{N_c^2 - 2}{2N_c^2}\left[\frac{\hat{s}^2 + \hat{t}^2}{\hat{u}^2}\right]$$

$$H^{\mathrm{Inc-F}}_{q\bar{q}' \to \bar{q}'q} = = -H^{\mathrm{Inc-F}}_{\bar{q}q' \to q'\bar{q}} = \frac{1}{N_c^2}\left[\frac{\hat{s}^2 + \hat{t}^2}{\hat{u}^2}\right]$$

$$H^{\mathrm{Inc-I}}_{qq \to qq} = -H^{\mathrm{Inc-I}}_{\bar{q}\bar{q} \to \bar{q}\bar{q}} = -\frac{1}{N_c^2}\left[\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2} + \frac{\hat{s}^2 + \hat{t}^2}{\hat{u}^2}\right] + \frac{N_c^2 + 1}{N_c^3}\frac{\hat{s}^2}{\hat{t}\hat{u}}$$

$$H^{\mathrm{Inc-F}}_{qq \to qq} = -H^{\mathrm{Inc-F}}_{\bar{q}\bar{q} \to \bar{q}\bar{q}} = -\frac{1}{2N_c^2}\left[\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}\right] + \frac{N_c^2 - 2}{2N_c^2}\left[\frac{\hat{s}^2 + \hat{t}^2}{\hat{u}^2}\right] + \frac{1}{N_c^3}\frac{\hat{s}^2}{\hat{t}\hat{u}}$$

$$H^{\mathrm{Inc-I}}_{q\bar{q} \to q'\bar{q}'} = -H^{\mathrm{Inc-I}}_{\bar{q}q \to \bar{q}'q'} = \frac{1}{2N_c^2}\left[\frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2}\right]$$

$$H^{\mathrm{Inc-F}}_{q\bar{q} \to q'\bar{q}'} = -H^{\mathrm{Inc-F}}_{\bar{q}q \to \bar{q}'q'} = \frac{N_c^2 - 2}{2N_c^2}\left[\frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2}\right]$$

$$H^{\mathrm{Inc-I}}_{q\bar{q} \to \bar{q}'q'} = -H^{\mathrm{Inc-I}}_{\bar{q}q \to q'\bar{q}'} = \frac{1}{2N_c^2}\left[\frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2}\right]$$

$$H^{\mathrm{Inc-F}}_{q\bar{q} \to \bar{q}'q'} = -H^{\mathrm{Inc-F}}_{\bar{q}q \to q'\bar{q}'} = \frac{1}{N_c^2}\left[\frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2}\right]$$

$$H^{\mathrm{Inc-I}}_{q\bar{q} \to q\bar{q}} = -H^{\mathrm{Inc-I}}_{\bar{q}q \to \bar{q}q} = -\frac{N_c^2 - 2}{2N_c^2}\left[\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}\right] + \frac{1}{2N_c^2}\left[\frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2}\right] - \frac{1}{N_c^3}\frac{\hat{u}^2}{\hat{s}\hat{t}}$$

$$H^{\mathrm{Inc-F}}_{q\bar{q} \to q\bar{q}} = -H^{\mathrm{Inc-F}}_{\bar{q}q \to \bar{q}q} = -\frac{1}{2N_c^2}\left[\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}\right] + \frac{N_c^2 - 2}{2N_c^2}\left[\frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2}\right] + \frac{1}{N_c^3}\frac{\hat{u}^2}{\hat{s}\hat{t}}$$

$$H^{\mathrm{Inc-I}}_{q\bar{q} \to \bar{q}q} = -H^{\mathrm{Inc-I}}_{\bar{q}q \to q\bar{q}} = -\frac{N_c^2 - 2}{2N_c^2}\left[\frac{\hat{s}^2 + \hat{t}^2}{\hat{u}^2}\right] + \frac{1}{2N_c^2}\left[\frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2}\right] - \frac{1}{N_c^3}\frac{\hat{t}^2}{\hat{s}\hat{u}}$$

$$H^{\mathrm{Inc-F}}_{q\bar{q} \to \bar{q}q} = -H^{\mathrm{Inc-F}}_{\bar{q}q \to q\bar{q}} = \frac{1}{N_c^2}\left[\frac{\hat{s}^2 + \hat{t}^2}{\hat{u}^2} + \frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2}\right] - \frac{N_c^2 + 1}{N_c^3}\frac{\hat{t}^2}{\hat{s}\hat{u}}$$

$$H^{\mathrm{Inc-I}}_{qg \to qg} = -H^{\mathrm{Inc-I}}_{\bar{q}g \to \bar{q}g} = \frac{1}{2(N_c^2 - 1)}\left[-\frac{\hat{s}}{\hat{u}} - \frac{\hat{u}}{\hat{s}}\right] + \frac{N_c^2}{2(N_c^2 - 1)}\left[\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}\frac{\hat{u}}{\hat{s}}\right]$$

$$H^{\mathrm{Inc-F}}_{qg \to qg} = -H^{\mathrm{Inc-F}}_{\bar{q}g \to \bar{q}g} = \frac{1}{2N_c^2(N_c^2 - 1)}\left[-\frac{\hat{s}}{\hat{u}} - \frac{\hat{u}}{\hat{s}}\right] - \frac{1}{N_c^2 - 1}\left[\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}\right]$$

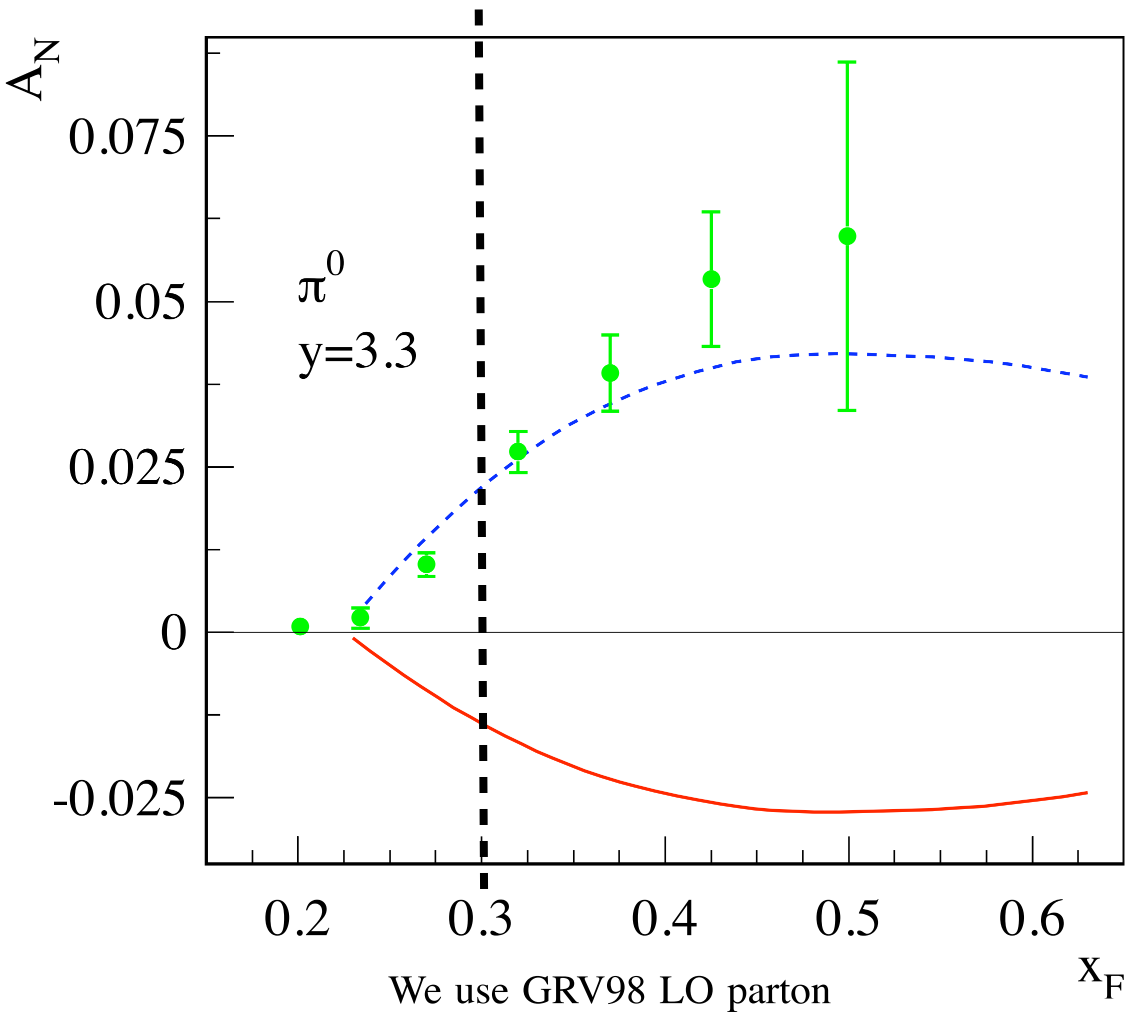
$$H^{\mathrm{Inc-I}}_{qg \to gq} = -H^{\mathrm{Inc-I}}_{\bar{q}g \to g\bar{q}} = \frac{1}{2(N_c^2 - 1)}\left[-\frac{\hat{s}}{\hat{t}} - \frac{\hat{t}}{\hat{s}}\right] + \frac{N_c^2}{2(N_c^2 - 1)}\left[\frac{\hat{s}^2 + \hat{t}^2}{\hat{u}^2}\frac{\hat{t}}{\hat{s}}\right]$$

$$H^{\mathrm{Inc-F}}_{qg \to gq} = -H^{\mathrm{Inc-F}}_{\bar{q}g \to g\bar{q}} = -\frac{1}{2(N_c^2 - 1)}\left[-\frac{\hat{s}}{\hat{t}} - \frac{\hat{t}}{\hat{s}}\right] - \frac{N_c^2}{2(N_c^2 - 1)}\left[\frac{\hat{s}^2 + \hat{t}^2}{\hat{u}^2}\frac{\hat{s}}{\hat{t}}\right]$$

$$H^{\mathrm{Inc-I}}_{q\bar{q} \to gg} = -H^{\mathrm{Inc-I}}_{\bar{q}q \to gg} = -\frac{1}{2N_c^3}\left[\frac{\hat{u}}{\hat{t}} + \frac{\hat{t}}{\hat{u}}\right] - \frac{1}{N_c}\left[\frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2}\right]$$

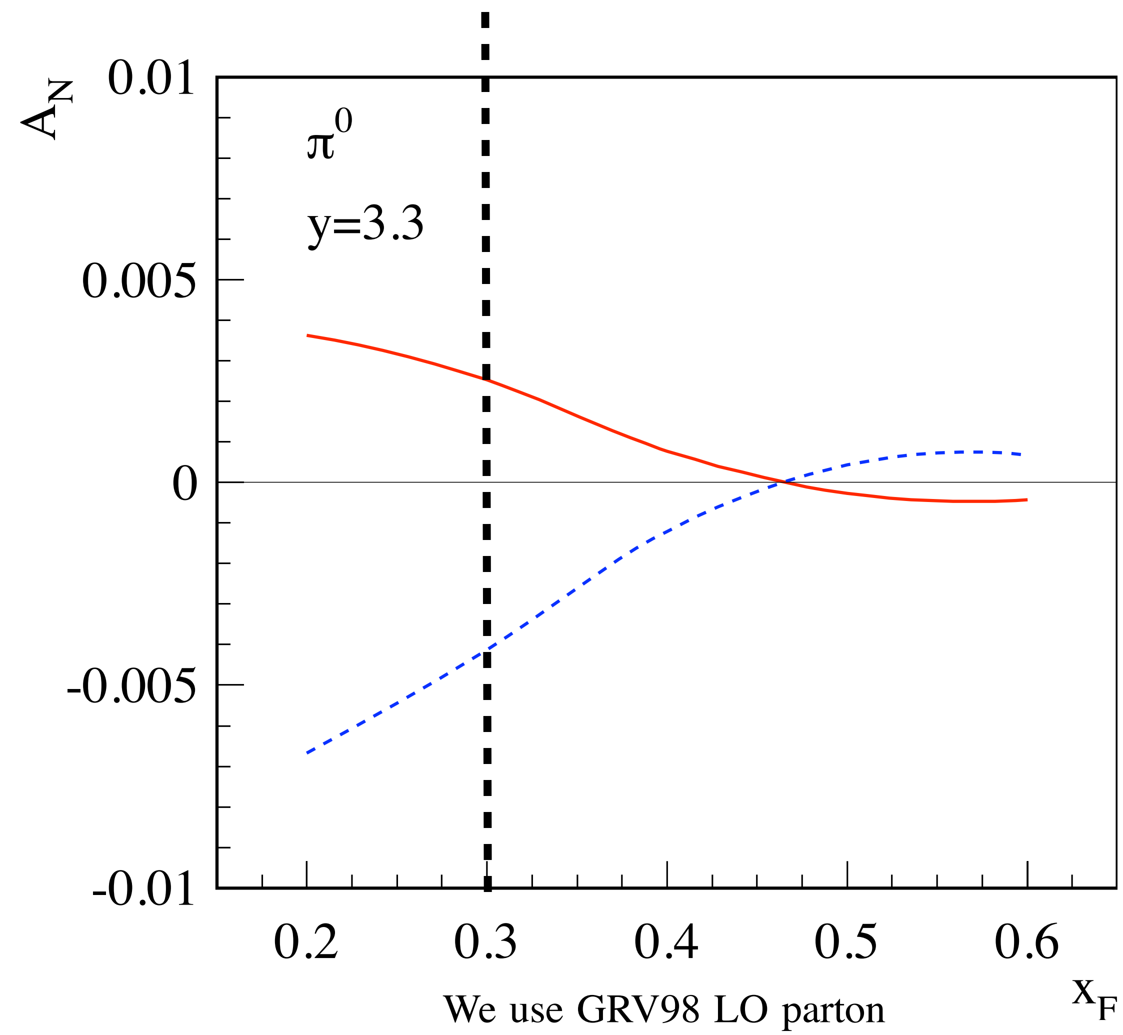
$$H^{\mathrm{Inc-F}}_{q\bar{q} \to gg} = -H^{\mathrm{Inc-F}}_{\bar{q}q \to gg} = -\frac{1}{2N_c}\left[\frac{\hat{u}}{\hat{t}} + \frac{\hat{t}}{\hat{u}}\right] + \frac{N_c}{2}\left[\frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2}\frac{\hat{u}}{\hat{t}}\right]$$

Based on old parameterization



Sivers from Anselmino et al PRD72 (2005)
Fragmentation from Kretzer PRD62 (2000)

Based on new parameterization



Sivers from Anselmino et al EPJA (2009)
Fragmentation from DSS PRD75 (2007)

$$H^{\text{Inc}}_{qg\to qg} = H^{\text{Inc-I}}_{qg\to qg} + H^{\text{Inc-F}}_{qg\to qg} \quad = -\frac{N_c^2+2}{N_c^2-1}\frac{\hat{s}^2}{\hat{t}^2},$$

forward direction, $\hat{t}$ is small, while $\hat{u} \sim -\hat{s}$,

$$H^{U}_{qg\to qg} = \frac{2\hat{s}^2}{\hat{t}^2}.$$

vs.

Model for PDFs

- unpolarized PDFs: $f_1^q(x,k_\perp^2) = f_1^q(x) g(k_\perp)$

- Sivers function: $\Delta^N f_{q/h^\uparrow}(x,k_\perp) = 2\mathcal{N}_q(x) f_1^q(x) h(k_\perp) g(k_\perp)$

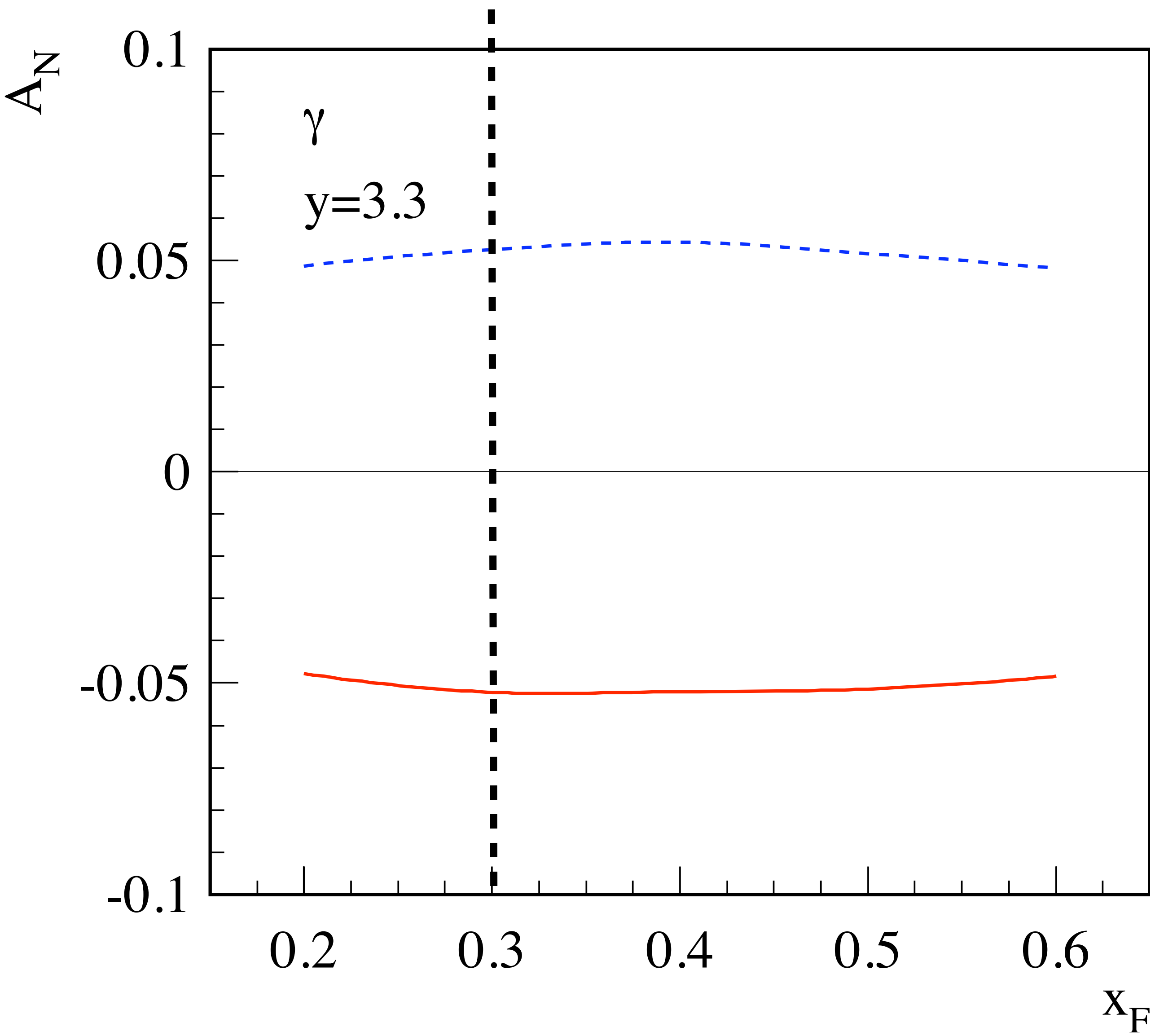
  $\mathcal{N}_q(x)$ is a fitted function

  $$g(k_\perp) = \frac{1}{\pi\langle k_\perp^2\rangle} e^{-k_\perp^2/\langle k_\perp^2\rangle}$$

  old Sivers: $$h(k_\perp) = \frac{2k_\perp M_0}{k_\perp^2 + M_0^2}$$ Anselmino, et.al, 2005

  new Sivers: $$h(k_\perp) = \sqrt{2e}\,\frac{k_\perp}{M_1} e^{-k_\perp^2/M_1^2}$$ Anselmino, et.al, 2009

Based on old parameterization

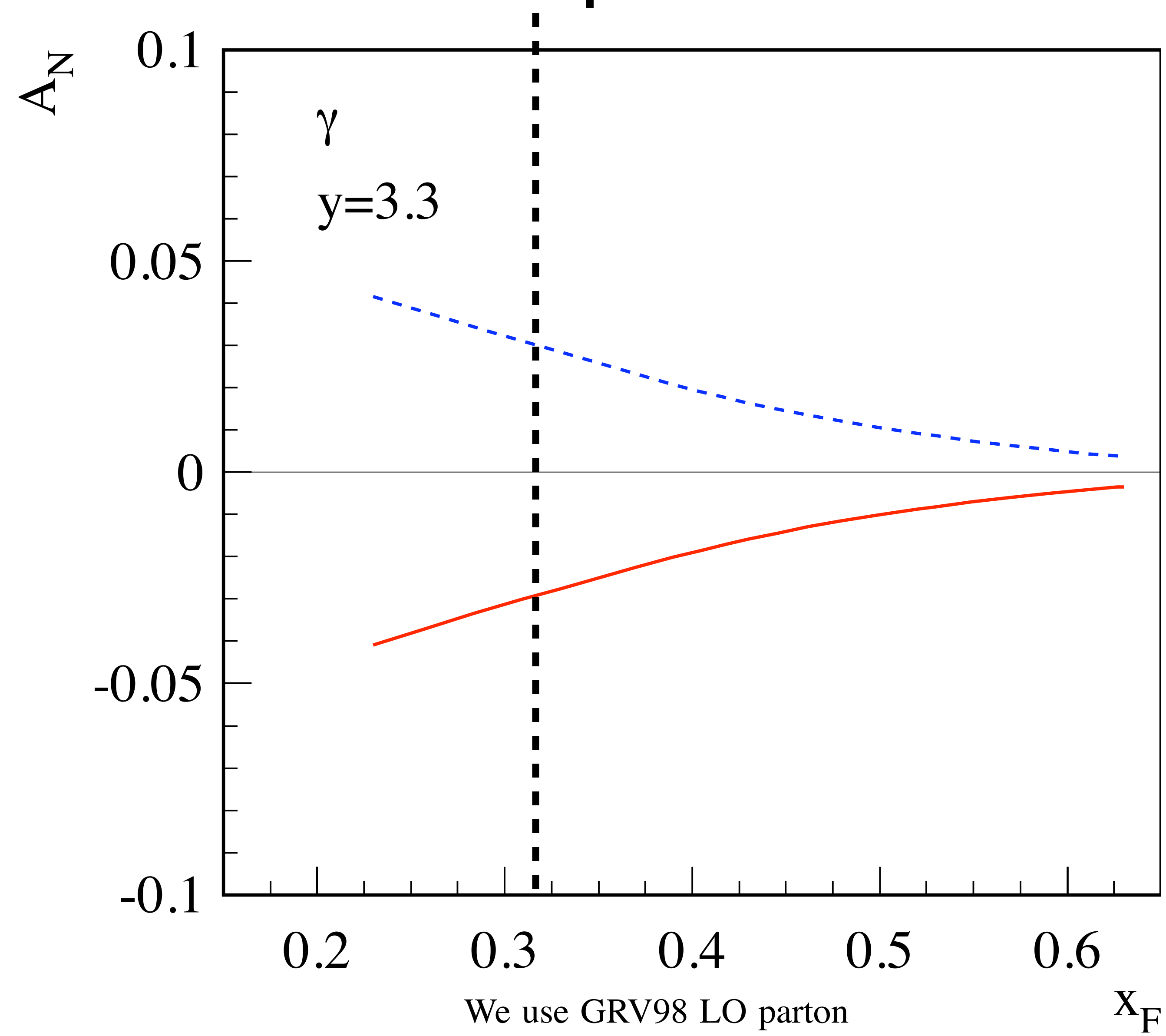


We use GRV98 LO parton

Sivers from Anselmino et al PRD72 (2005)

Fragmentation from Kretzer PRD62 (2000)

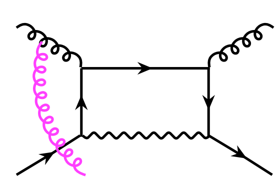
Based on new parameterization



We use GRV98 LO parton

Sivers from Anselmino et al EPJA (2009)

Fragmentation from DSS PRD75 (2007)



ISI drives result

$$H^{U}_{qg\to\gamma q} = \frac{1}{N_c} e_q^2 \left[-\frac{\hat{t}}{\hat{s}} - \frac{\hat{s}}{\hat{t}} \right]$$

$$H^{\rm Inc}_{qg\to\gamma q} = -\frac{N_c}{N_c^2-1} e_q^2 \left[-\frac{\hat{t}}{\hat{s}} - \frac{\hat{s}}{\hat{t}} \right]$$

Observations

- Hard amplitudes squared have same form in Mandelstam variables as twist-3 $\hat{s},\ \hat{t},\ \hat{u}$

see Kouvaris,Qiu ,Vogelsang, and Yuan PRD 2006

- However $\hat{s},\ \hat{t},\ \hat{u}$ depend on k_T in GPM whereas in twist-3 approach there has been collinear expansion on hard and soft factors

- We have shown that GPM expanded with respect to k_T results in twist-3 result {\em almost }

Collinear Expansion in GPM

- Here $\hat{s},\, \hat{t},\, \hat{u}$ depend on k_{aT}
- Delta function $\delta(\hat{s}+\hat{t}+\hat{u}) = \frac{1}{x_b S + T/z_c}\delta\left(x_a - x - \frac{2P_{hT}\cdot k_{aT}}{z_c x_b S + T}\right)$
- Expand in k_{aT} and study contribution from Sivers function and hard cross section

$$E_\gamma \frac{d\Delta\sigma}{d^3P_\gamma} = \frac{\alpha_{em}\alpha_s}{s}\sum_{abc}\int d^2k_{aT}\frac{\epsilon^{\alpha S_T n \hat{n}}}{M}k_{aT\alpha}\frac{1}{x_a}f_{1T}^{\perp\,\mathrm{SIDIS}}(x_a, k_{aT}^2)$$
$$\times \int \frac{dx_b}{x_b}f_{b/B}(x_b)\int\frac{dz_c}{z_c^2}H^{\mathrm{Inc}}_{ab\to c}(\hat{s},\hat{t},\hat{u})\frac{1}{x_b s + T/z_c}\Bigg|_{x_a = x + \frac{2P_{hT}\cdot k_{aT}}{z_c x_b S + T}}$$

Details: "collinear expansion" in GPM and keep linear in k_{aT}

$$\hat{s} = (p_a + p_b)^2 = x_a x_b S + \quad \mathcal{O}(k_T^2)$$

$$\hat{t} = (x_a P_A + k_{aT} - \frac{P_h}{z})^2 = \frac{x_a}{z} T - \frac{2 P_{hT} \cdot k_{aT}}{z}$$

$$\hat{u} = (p_b - p_c)^2 = (x_b P_B - \frac{P_h}{z})^2 = \frac{x_b}{z} U$$

$$\delta(\hat{s} + \hat{t} + \hat{u}) = \frac{1}{x_b s + \frac{T}{z_c}} \delta(x_a - x - \frac{2 P_{hT} \cdot k_{aT}}{x_b s + \frac{T}{z_c}})$$

$$x = -x_b U / (z_c x_b S + T)$$

Collinear twist three

$$E_h \frac{d\Delta\sigma^{(a)}}{d^3P_h} = \frac{\alpha_s^2}{S} \sum_{a,b,c} \int \frac{dz_c}{z_c^2} D_{h/c}(z_c) \frac{\epsilon^{P_{hT} S_A n \bar{n}}}{z_c \tilde{u}} \frac{1}{x} \left[T_{a,F}(x,x) - x \frac{d}{dx} T_{a,F}(x,x) \right] \int \frac{dx_b}{x_b} f_{b/B}(x_b) H^{\mathrm{Inc}}_{ab\to c}(\tilde{s},\tilde{t},\tilde{u}) \frac{1}{x_b S + T/z_c}$$

Almost same as Kouvaris, Qiu, Vogelsang, and Yuan PRD 2006

$$H^{\mathrm{Inc}}_{ab\to c}(\hat{s},\hat{t},\hat{u}) = H^{\mathrm{Inc\text{-}I}}_{ab\to c}(\hat{s},\hat{t},\hat{u}) + H^{\mathrm{Inc\text{-}F}}_{ab\to c}(\hat{s},\hat{t},\hat{u}),$$ CGI GPM

$$H^{\mathrm{twist\text{-}3}}_{ab\to c}(\hat{s},\hat{t},\hat{u}) = H^{\mathrm{twist\text{-}3\text{-}I}}_{ab\to c}(\hat{s},\hat{t},\hat{u}) + H^{\mathrm{twist\text{-}3\text{-}F}}_{ab\to c}(\hat{s},\hat{t},\hat{u})\left(1 + \frac{\hat{u}}{\hat{t}}\right)$$ Kouvaris et al.

N.B. *Difference here due to using eikonal approx. on for both ISI and FSI*

- Another term from k_{aT}-dependence from $H^{\rm Inc}_{ab\to c}(\hat{s},\hat{t},\hat{u})$

Thus to the leading order (linear in k_{aT} terms).

small

$$E_h\frac{d\Delta\sigma}{d^3P_h}=E_h\frac{d\Delta\sigma^{(a)}}{d^3P_h}+E_h\frac{d\Delta\sigma^{(b)}}{d^3P_h},$$

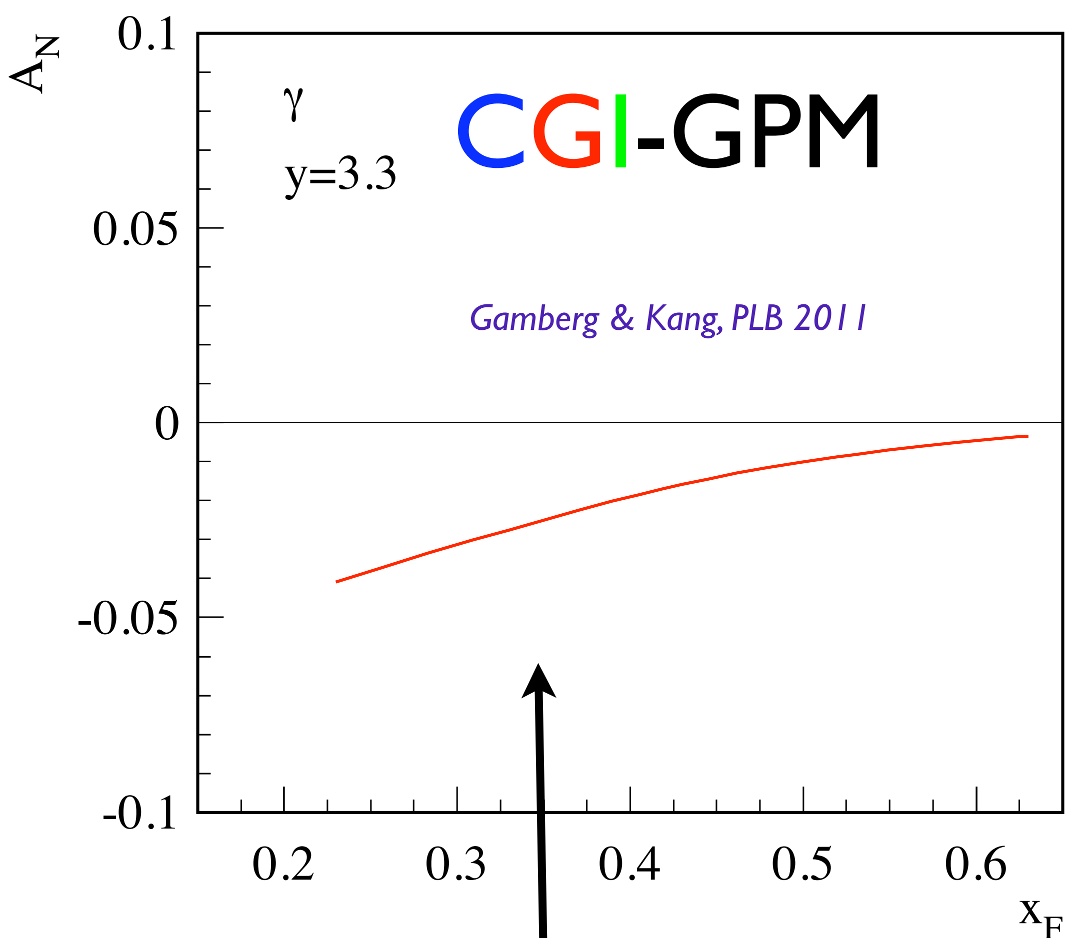
small

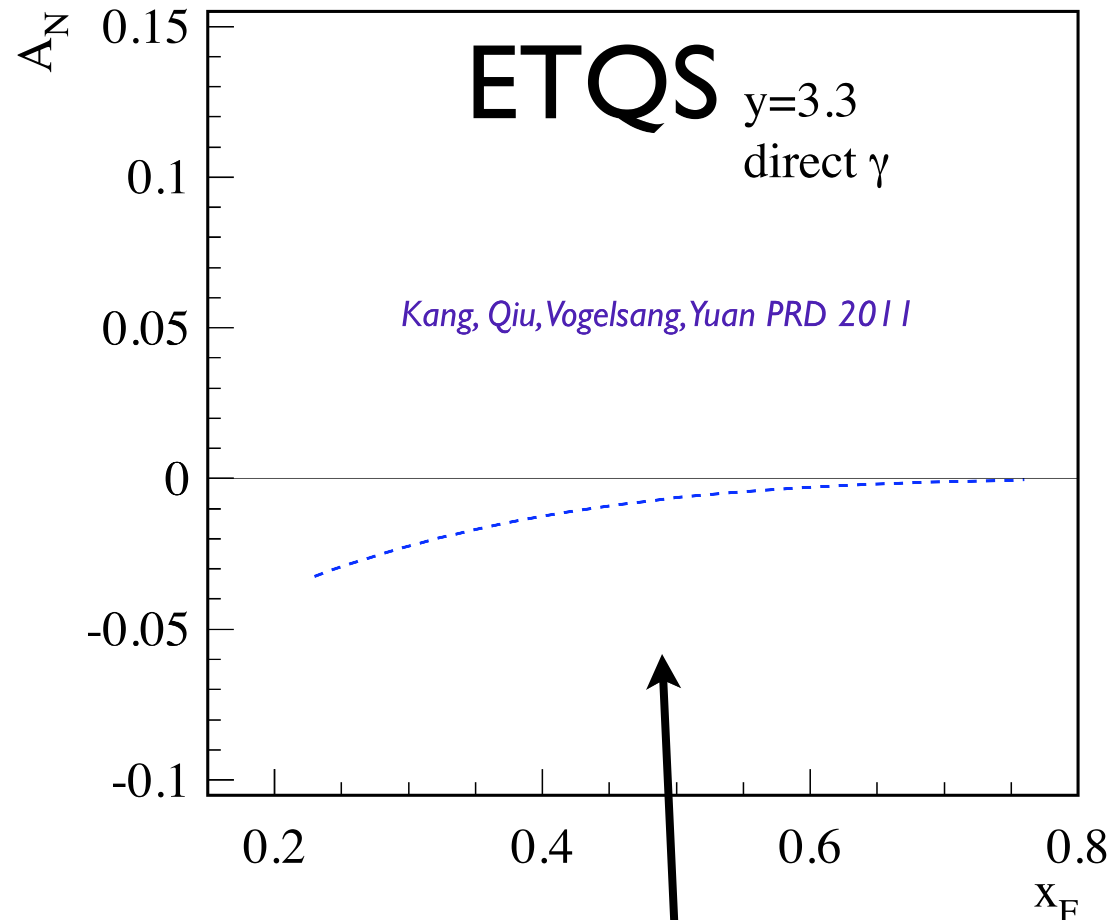
$$E_h\frac{d\Delta\sigma^{(a)}}{d^3P_h}=\frac{\alpha_s^2}{S}\sum_{a,b,c}\int\frac{dz_c}{z_c^2}D_{h/c}(z_c)\frac{\epsilon^{P_{hT}S_A n\bar{n}}}{z_c\tilde{u}}\frac{1}{x}\left[T_{a,F}(x,x)-x\frac{d}{dx}T_{a,F}(x,x)\right]\int\frac{dx_b}{x_b}f_{b/B}(x_b)H^{\rm Inc}_{ab\to c}(\tilde{s},\tilde{t},\tilde{u})\frac{1}{x_bS+T/z_c}$$

small

$$E_h\frac{d\Delta\sigma^{(b)}}{d^3P_h}=\frac{\alpha_s^2}{S}\sum_{a,b,c}\int\frac{dz_c}{z_c^2}D_{h/c}(z_c)\frac{\epsilon^{P_{hT}S_A n\bar{n}}}{z_c\tilde{u}}\frac{1}{x}T_{a,F}(x,x)\int\frac{dx_b}{x_b}f_{b/B}(x_b)\left[-\tilde{s}\frac{\partial}{\partial\tilde{s}}H^{\rm Inc}_{ab\to c}(\tilde{s},-\tilde{s}-\tilde{u},\tilde{u})\right]\frac{1}{x_bS+T/z_c}$$

Numerical Comparison of CGI-GPM and ETQS--Direct Photon

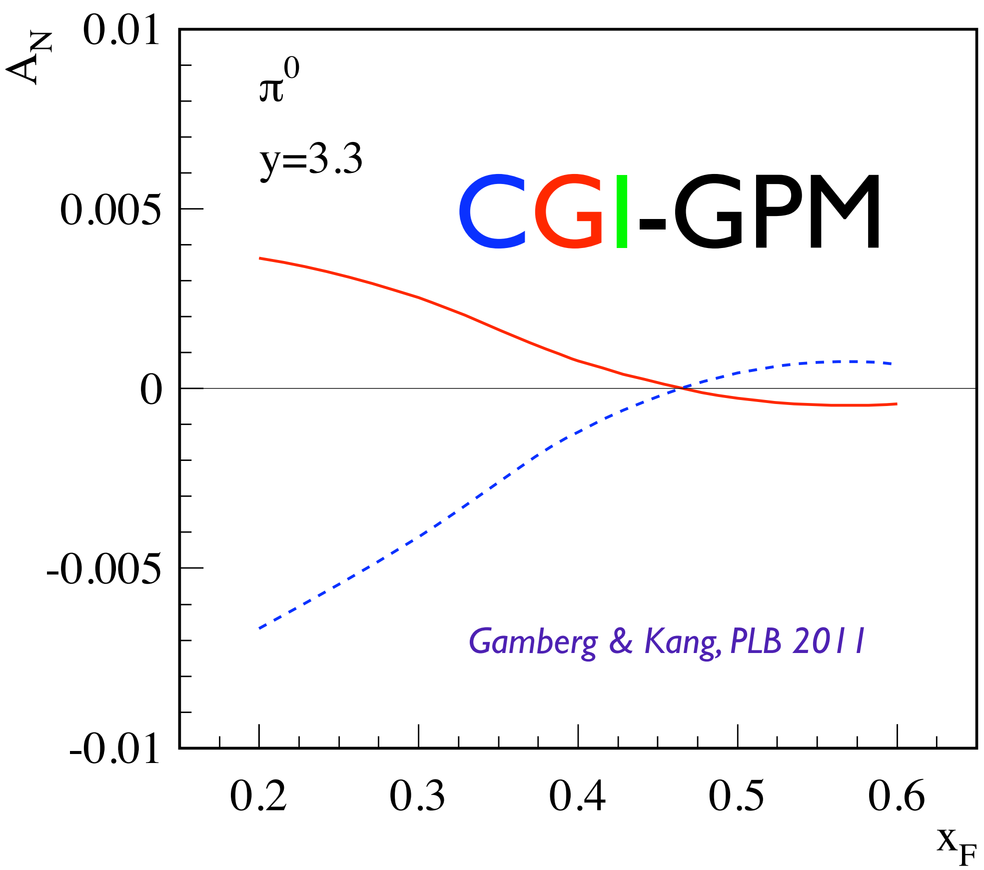


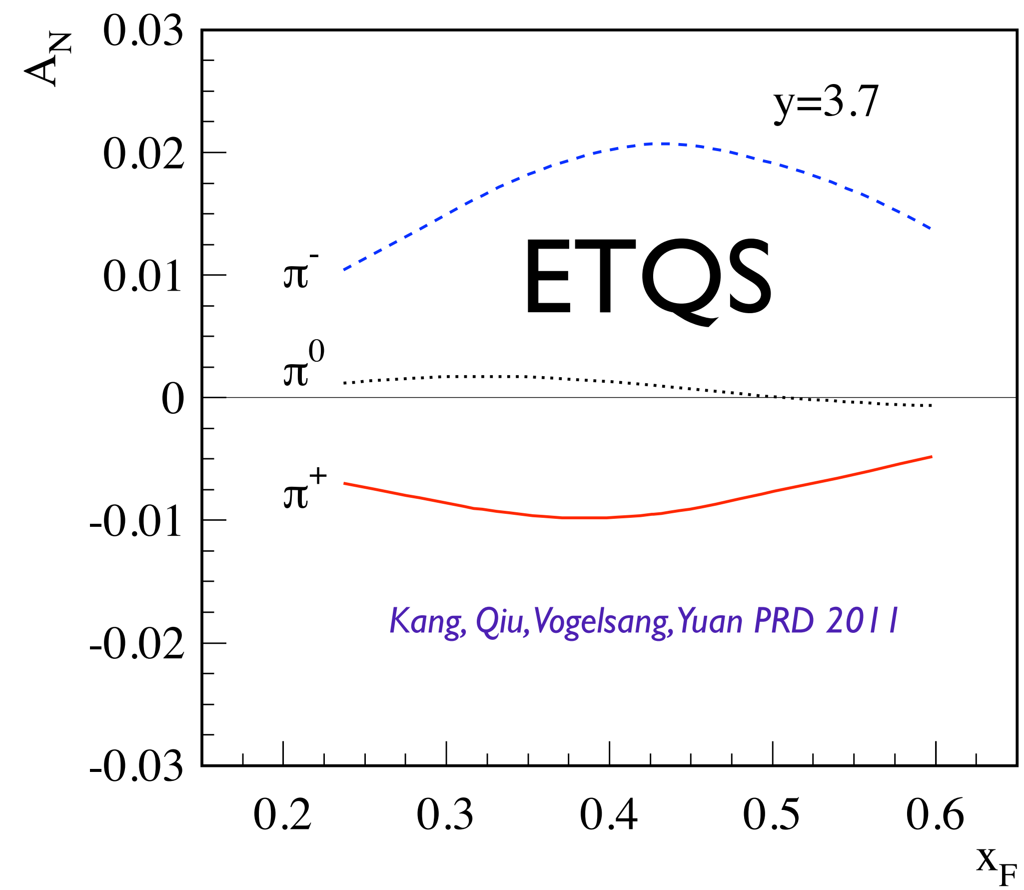


$$E_h \frac{d\Delta\sigma}{d^3P_h} = \frac{\alpha_s^2}{S}\sum_{a,b,c}\int \frac{dx_a}{x_a} d^2k_{aT} f_{1T}^{\perp a,\mathrm{SIDIS}}(x_a, k_{aT}^2)\frac{\epsilon^{k_{aT} S_A n \bar{n}}}{M}\int \frac{dx_b}{x_b} d^2k_{bT} f_{b/B}(x_b, k_{bT}^2) \times \int \frac{dz_c}{z_c^2} D_{h/c}(z_c) H^{\mathrm{Inc}}_{ab\to c}(\hat{s},\hat{t},\hat{u})\delta(\hat{s}+\hat{t}+\hat{u}),$$

$$E_h \frac{d\Delta\sigma(s_\perp)}{d^3P_h} = \frac{\alpha_s^2}{S}\sum_{a,b,c}\int \frac{dz}{z^2} D_{c\to h}(z)\int \frac{dx'}{x'} f_{b/B}(x')\int \frac{dx}{x}\sqrt{4\pi\alpha_s}\left(\frac{\epsilon^{P_{h\perp} s_\perp n \bar{n}}}{z\hat{u}}\right) \times \left[T_{a,F}(x,x) - x\frac{d}{dx}T_{a,F}(x,x)\right] H_{ab\to c}(\hat{s},\hat{t},\hat{u})\delta\left(\hat{s}+\hat{t}+\hat{u}\right),$$

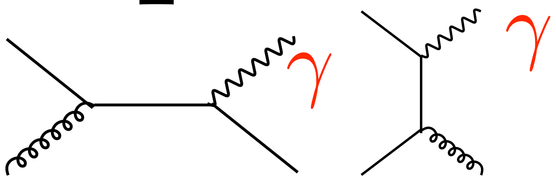
Numerical Comparison of CGI-GPM and ETQS--Inclusive Pion





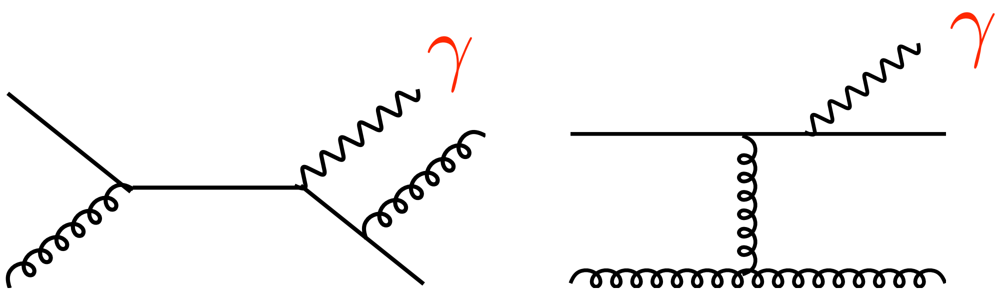
Compare red left w/ black dotted on right

Prompt photon production at RHIC



Leading order photon production, purely “direct” contribution.

A next-to-leading-order (NLO) calculation of prompt photon production is summarized by Vogelsang and Gordon : from which one might know what is called direct and



Photon production at NLO, which contains both “direct” and “fragmentation” contribution.

At NLO, we will have radiative corrections from 2 to 3 processes, such as qq to γ q g, see some sample diagrams .

These 2 to 3 processes have collinear divergence, one of which comes from the situation when the final-state quark is collinear to the outgoing photon.

Using factorization procedure, this part of collinear divergence is absorbed into the so-called photon fragmentation function in other words, when the photon is very close to the final-state quark, it is absorbed into quark-to-photon fragmentation function, this part is what we call ``fragmentation contribution"; while when they are well separated, this part belongs to the ``direct" contribution at NLO. Exactly how much goes to ``direct", how much goes to ``fragmentation" depends on the factorization scale.

II. SPIN ASYMMETRY OF PROMPT PHOTON PRODUCTION

Similarly for the *spin-dependent* cross section, we could write

$$E\frac{d\Delta\sigma}{d^3P} = E\frac{d\Delta\sigma^{\rm dir}}{d^3P} + E\frac{d\Delta\sigma^{\rm frag}}{d^3P}.$$

Eventually the single transverse spin asymmetry is given by

$$A_N = E\frac{d\Delta\sigma}{d^3P}\bigg/ E\frac{d\sigma}{d^3P}.$$

$$E\frac{d\Delta\sigma^{\rm frag}}{d^3P} = E\frac{d\Delta\sigma^{\rm frag}}{d^3P}\bigg|_{\rm Sivers} + E\frac{d\Delta\sigma^{\rm frag}}{d^3P}\bigg|_{\rm Collins},$$

ETQS formalism

$$T_{q,F}(x,x) = -\int d^2k_\perp \frac{|k_\perp|^2}{M} f_{1T}^{\perp q}(x,k_\perp^2)|_{\text{SIDIS}}. \qquad \hat{H}_q(z) = -\frac{z^3}{M_h}\int d^2k_\perp |k_\perp|^2 H_1^{\perp q}(z, z^2k_\perp^2)$$

$$E\frac{d\Delta\sigma^{\text{frag}}}{d^3P} = E\frac{d\Delta\sigma^{\text{frag}}}{d^3P}\bigg|_{\text{Sivers}} + E\frac{d\Delta\sigma^{\text{frag}}}{d^3P}\bigg|_{\text{Collins}},$$

where the first term is given by

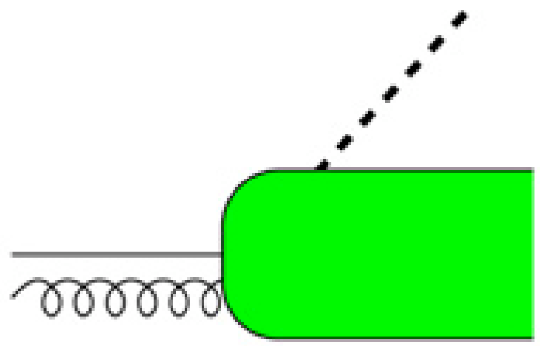
$$E\frac{d\Delta\sigma^{\text{frag}}}{d^3P}\bigg|_{\text{Sivers}} = \epsilon_{\alpha\beta}s_\perp^\alpha P_{h\perp}^\beta \frac{\alpha_s^2}{S}\sum_{a,b,c}\int\frac{dz}{z^2}D_{c\to\gamma}(z)\int\frac{dx'}{x'}f_{b/B}(x')\int\frac{dx}{x}\left[\frac{1}{z\hat{u}}\right]$$
$$\times\left[T_{a,F}(x,x) - x\frac{d}{dx}T_{a,F}(x,x)\right]H_{ab\to c}^{\text{Sivers}}(\hat{s},\hat{t},\hat{u})\delta\left(\hat{s}+\hat{t}+\hat{u}\right),$$

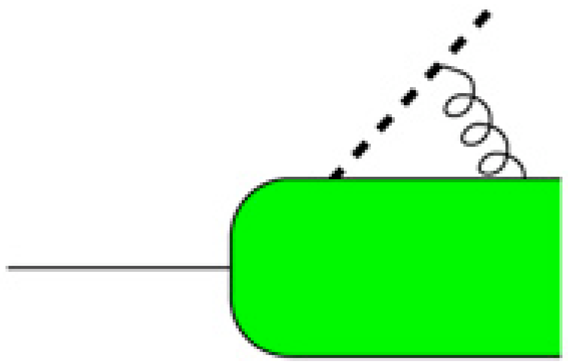
and the second term is given by

$$E\frac{d\Delta\sigma^{\text{frag}}}{d^3P}\bigg|_{\text{Collins}} = \epsilon_{\alpha\beta}s_\perp^\alpha P_{h\perp}^\beta \frac{\alpha_s^2}{S}\sum_{a,b,c}\int\frac{dx}{x}h_a(x)\int\frac{dx'}{x'}f_b(x')\int\frac{dz}{z}\left[-z\frac{\partial}{\partial z}\left(\frac{\hat{H}_c(z)}{z^2}\right)\right]$$
$$\times\left[\frac{1}{z}\frac{x-x'}{x(-\hat{u})+x'(-\hat{t})}\right]H_{ab\to c}^{\text{Collins}}(\hat{s},\hat{t},\hat{u})\delta\left(\hat{s}+\hat{t}+\hat{u}\right)$$

Fragmentation

- In fragmentation the discussion slightly more complicated, since the gauge-links are not the only potential source of *T-odd effects*. As pointed out by Collins NPB93, also the *internal* final state interactions of the observed outgoing hadron with its accompanying jet, in matrix elements appearing as the one-particle inclusive out-state $|P_h\, X\rangle$ can produce T-odd





- Thus due to the explicit appearance of outstates, time-reversal symmetry does not constrain the parametrization of the fragmentation correlators (as does for pdfs)

- Hence *T-odd* fragmentation effects could arise from both FSI and gauge-links

Similarly unintegrated fragmentation there are in principle “two” types of gauge links
However more subtle!!! -Two types of T-odd effects

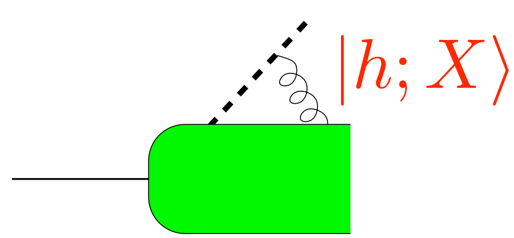
Reliability of Transversity Extraction Universality of Collins Fragmentation Function

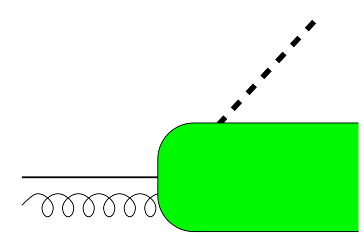
$$\Delta_{\partial}^{\alpha\,[\mathcal{U}]}(z) \quad = \quad \int d^2k_T\; k_T^{\alpha}\Delta^{[\mathcal{U}]}(z,k_T) = \tilde{\Delta}_{\partial}^{\alpha}\left(\tfrac{1}{z}\right) + C_G^{[\mathcal{U}]}\,\pi\Delta_G^{\alpha}\left(\tfrac{1}{z},\tfrac{1}{z}\right)$$

$$\Delta^{[-]}(z,k_T) = \int \frac{d\xi^+ d^2\xi_T}{4z(2\pi)^3}\; e^{ik\cdot\xi}\,\langle 0\;|U^{[-]}_{[-\infty,0]}\psi(0)|x;P_h\rangle\langle x;P_h|\overline{\psi}(\xi^+,\xi_T)U^{[-]}_{[\xi,-\infty]}|0\rangle|_{\xi^-=0}$$

But no such constraint under time reversal

$$\cancel{\Delta^{[+]*}(x,p_T) \neq i\gamma^1\gamma^3\Delta^{[-]}(x,p_T)i\gamma^1\gamma^3}$$





$$\Delta_{\partial}^{\alpha\,[C]}(z) \quad = \quad \tilde{\Delta}_{\partial}^{\alpha[\not C]}\left(\tfrac{1}{z}\right) + C_G^{[U(C)]}\,\pi\Delta_G^{\alpha[\not C]}\left(\tfrac{1}{z},\tfrac{1}{z}\right)$$

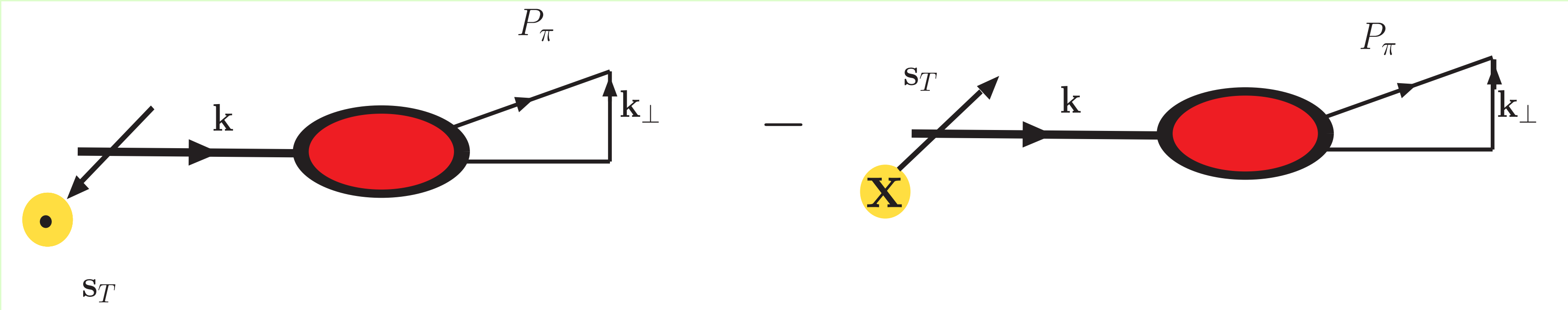
T-odd-FSI **T-odd-Gauge link**

Reliability of Transversity Extraction Universality of Collins Fragmentation Function

- Collins NPB: 1993 **TSSA is associated with** $transverse$ **spin of fragmenting quark and transverse momentum of final state hadron**

$$\Delta_{\partial}^{\alpha\,[\mathcal{U}]}(z) \quad = \quad \int d^2k_T\; k_T^{\alpha}\Delta^{[\mathcal{U}]}(z,k_T) = \tilde{\Delta}_{\partial}^{\alpha}\left(\tfrac{1}{z}\right) + C_G^{[\mathcal{U}]}\,\pi\Delta_G^{\alpha}\left(\tfrac{1}{z},\tfrac{1}{z}\right)$$

$$D_{h/q^{\uparrow}}(z,K_T^2) = D_1^q(z,K_T^2) + H_1^{\perp q}(z,K_T^2)\frac{(\hat{\boldsymbol{k}}\times\boldsymbol{K}_T)\cdot\boldsymbol{s}_q}{zM_h},$$

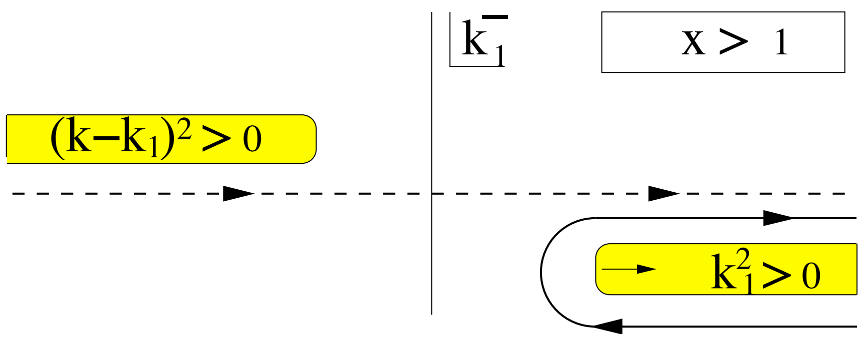


$$\frac{\epsilon_T^{ij}k_{Tj}}{M_h}H_1^{\perp}(z,k_T^2) = \frac{1}{2}\mathrm{Tr}\left[\Delta(z,k_T)i\sigma^{i-}\gamma_5\right].$$

Gluonic pole and higher moments Vanishes Collins Universal

LG, A. Mukherjee, P. Mulders PRD 2011

$$\lim_{x_1 \to 0} \Delta_G(x, x - x_1) = \Delta_G(x, x) \to 0$$



$$\Delta_G(x, x - x_1)$$

$$k_1^- = \frac{k_1^2 + k_{1T}^2 + i\epsilon}{2x_1}$$

For the case $x > 1$ the k_1^- integration can be wrapped around the cut k_1^2 which smoothly vanishes for $x_1 \to 0$ describes the by the arrow inside branch cut indicates that it harmlessly recedes to infinity

Agrees with earlier model analysis Collins, Metz PRL 2004
Agrees with earlier model analysis LG, A. Mukherjee, P. Mulders PRD 2008
Agrees with model independent spectral analysis A. Metz, S. Meissner PRL 2009
Agrees with 1 and 2 gluon exchange calculation from GL in hadron inside jet F. Yuan PRD 2009
Recent ppr. by Boer,Kang,Vogelsang,Yuan-predictions on Lambda polarization in SIDIS &

$e^+ e^-$

Collins *photon* fragmentation contribution

$$H_1^{\perp q}(z, z^2 k_\perp^2) = e_q^2 \frac{\alpha_{\rm em}}{2\pi^2} \alpha_s m_q C_F \left[\text{fig.a} + \text{fig.b} + \text{fig.c} + \text{fig.d}\right]$$

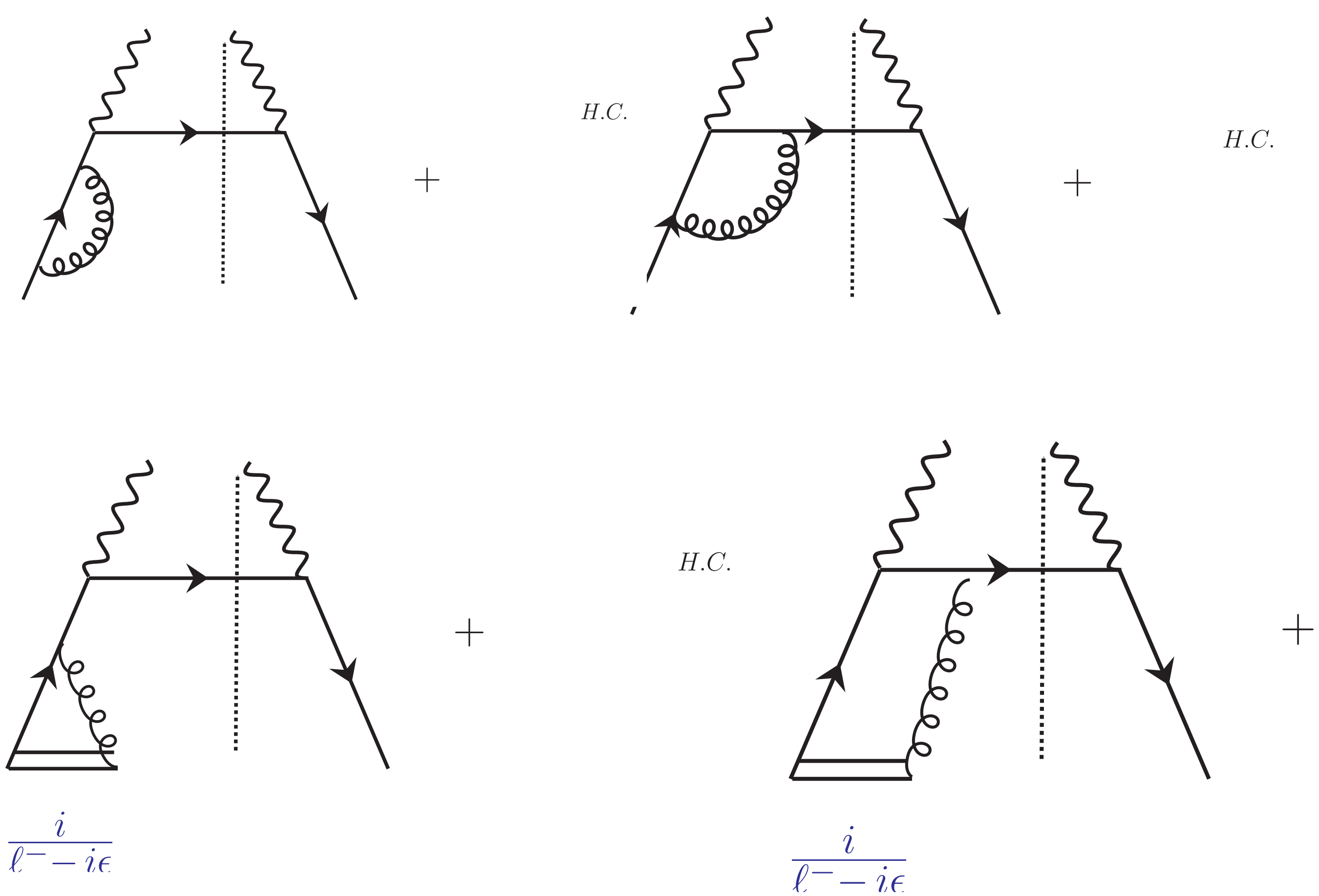
where

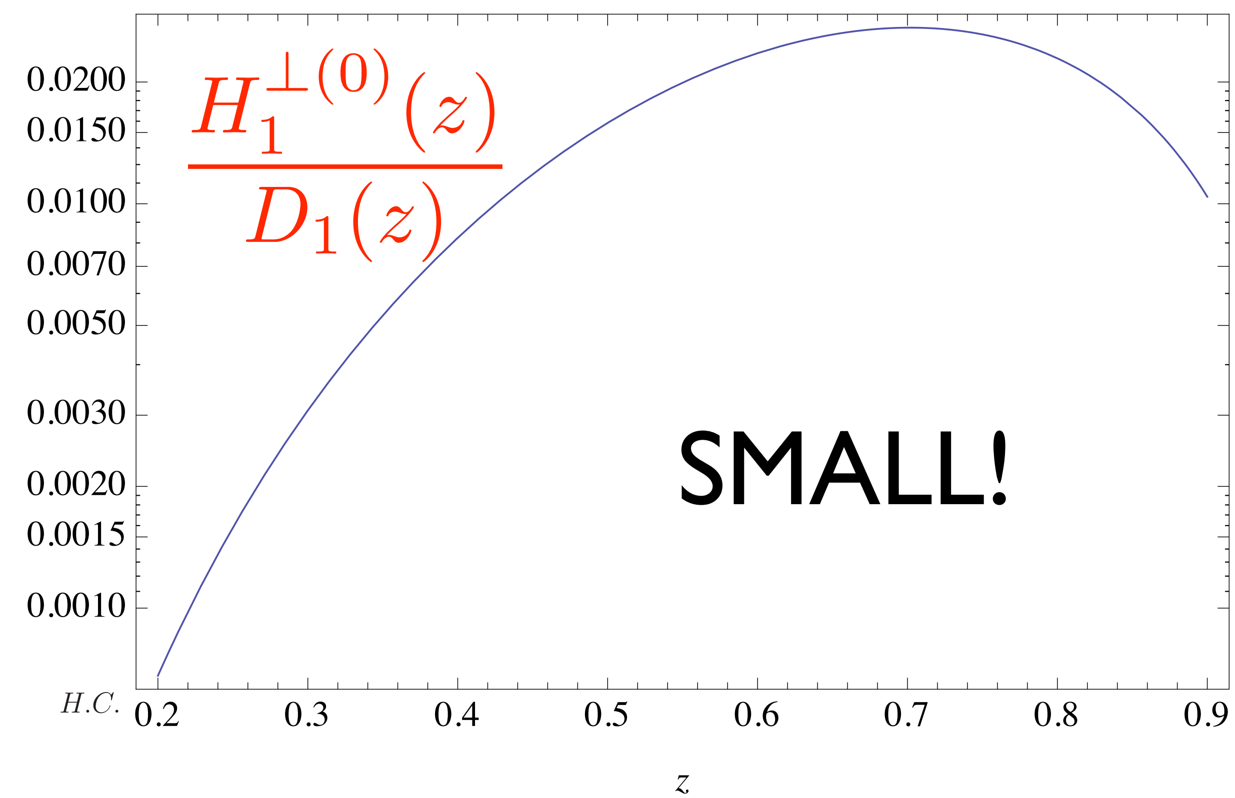
$$\text{fig.a} = \frac{1}{2zk^2(k^2 - m_q^2)} \left(3 - \frac{m_q^2}{k^2}\right)$$

$$\text{fig.b} = \frac{1}{(1-z)(k^2 - m_q^2)^2} \left[\frac{m_q^2}{k^2 - m_q^2} \ln\left(\frac{k^2}{m_q^2}\right) - \frac{1}{2z}\left(4 - 5z + 3(z-2)\frac{m_q^2}{k^2} + 2\left(\frac{m_q^2}{k^2}\right)^2\right)\right]$$

$$\text{fig.c} = 0$$

$$\text{fig.d} = \frac{1}{((1-z)k^2(k^2 - m^2)} \left[1 + \frac{(1-z)k^2}{(1-z)k^2 - m_q^2} \ln\left(\frac{(1-z)k^2}{m_q^2}\right)\right]$$





Direct & Fragmentation

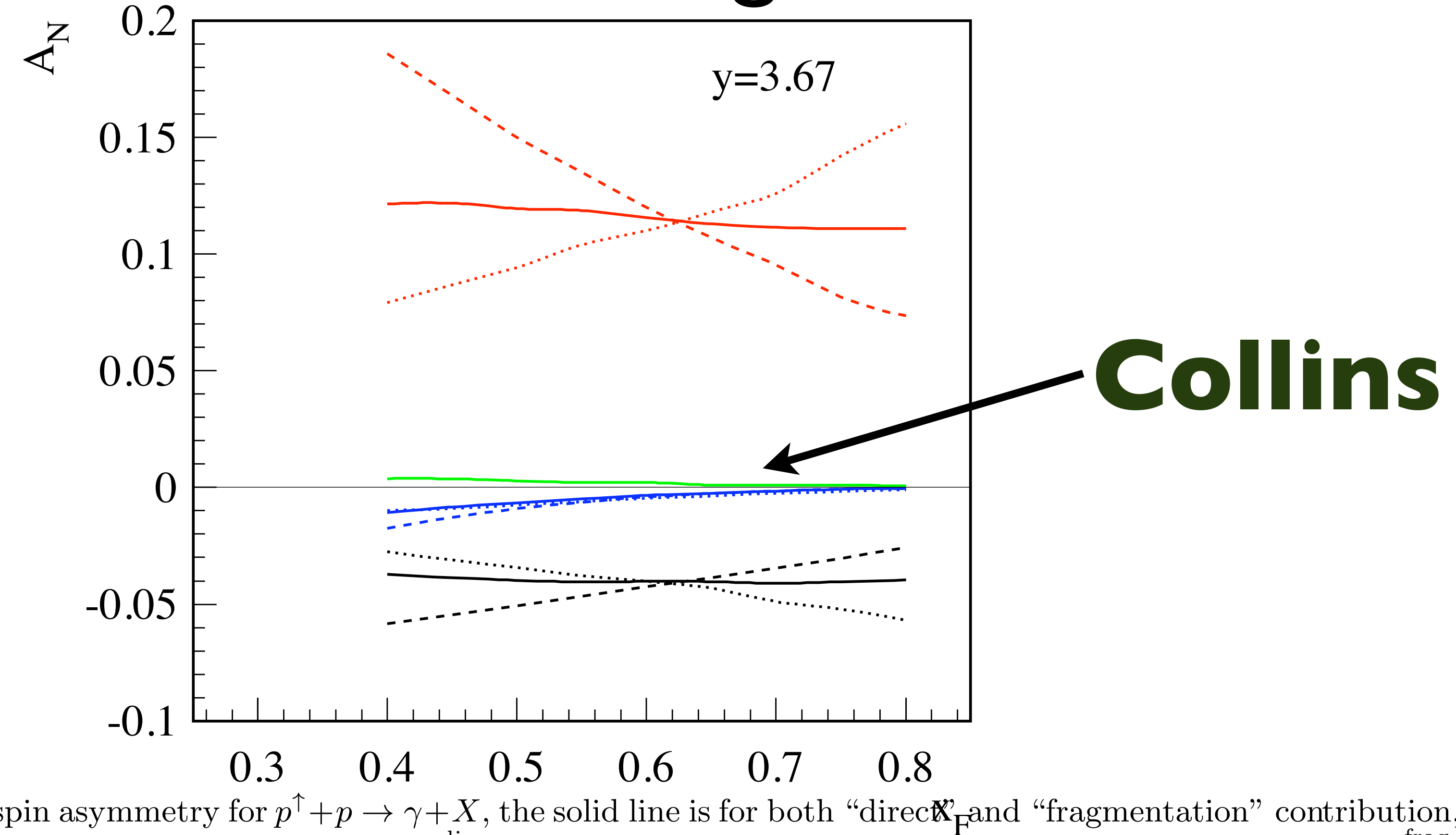
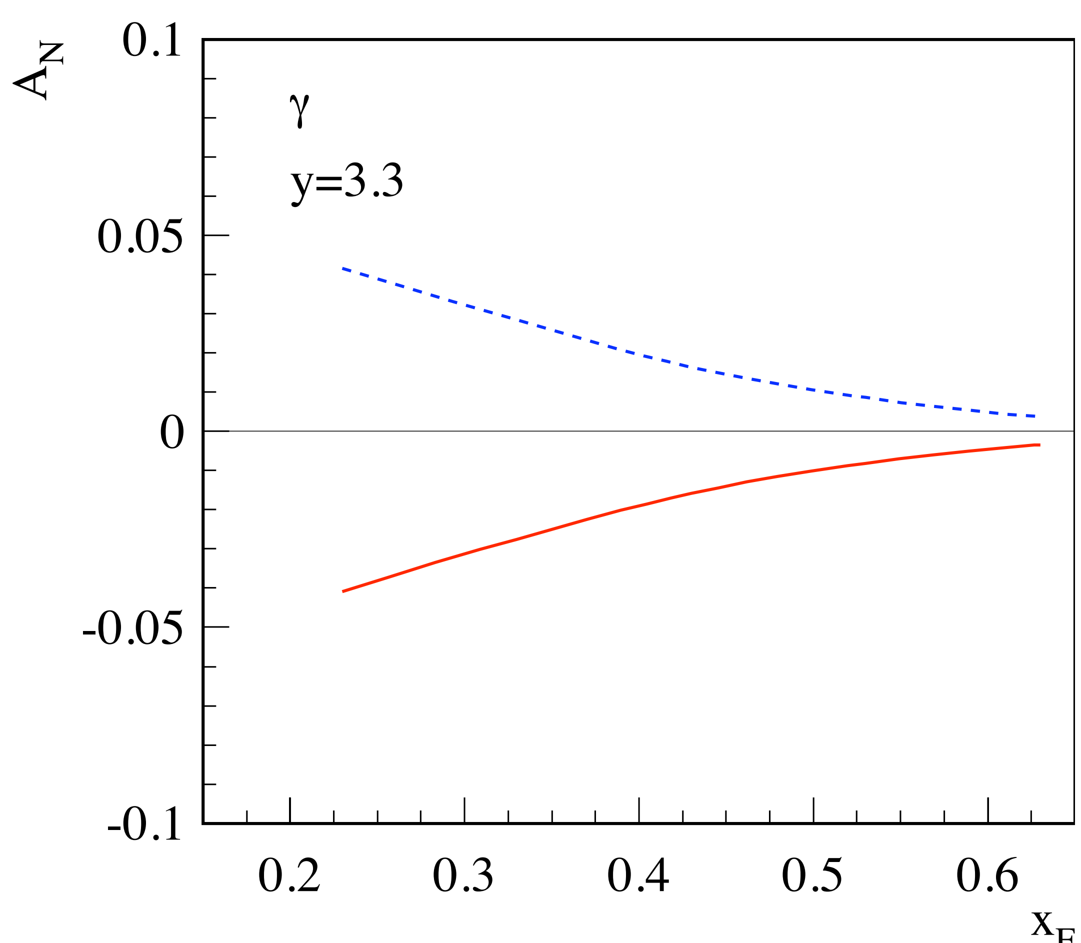


FIG. 4. Single transverse spin asymmetry for $p^{\uparrow}+p \to \gamma+X$, the solid line is for both "direct" and "fragmentation" contribution, while the dashed one is for "direct" contribution only $A_N^{\rm dir}$, and the dotted line is for "fragmentation" contribution only $A_N^{\rm frag}$. Green solid line is the Collins "fragmentation" asymmetry.



Sivers from Anselmino et al EPJA (2009)
Fragmentation from DSS PRD75 (2007)

Pion Collins--Bacchetta, Gamberg, Goldstein, Mukherjee PLB 08

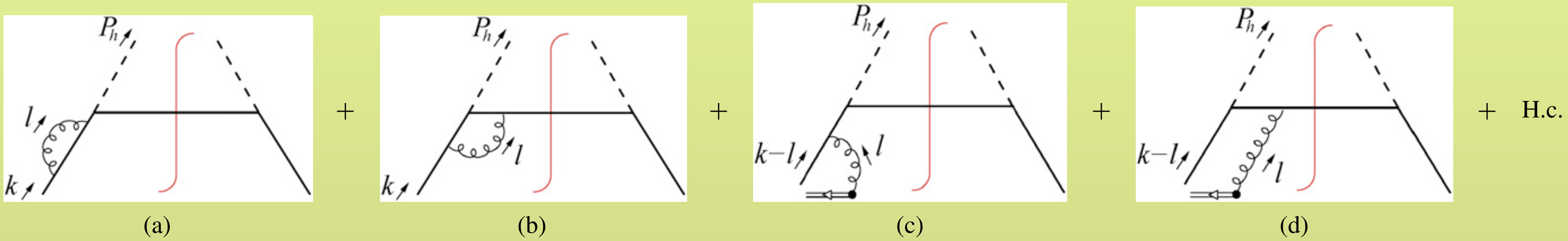


Fig. 3. Single gluon-loop corrections to the fragmentation of a quark into a pion contributing to the Collins function in the eikonal approximation. "H.c." stands for the Hermitian conjugate diagrams which are not shown.

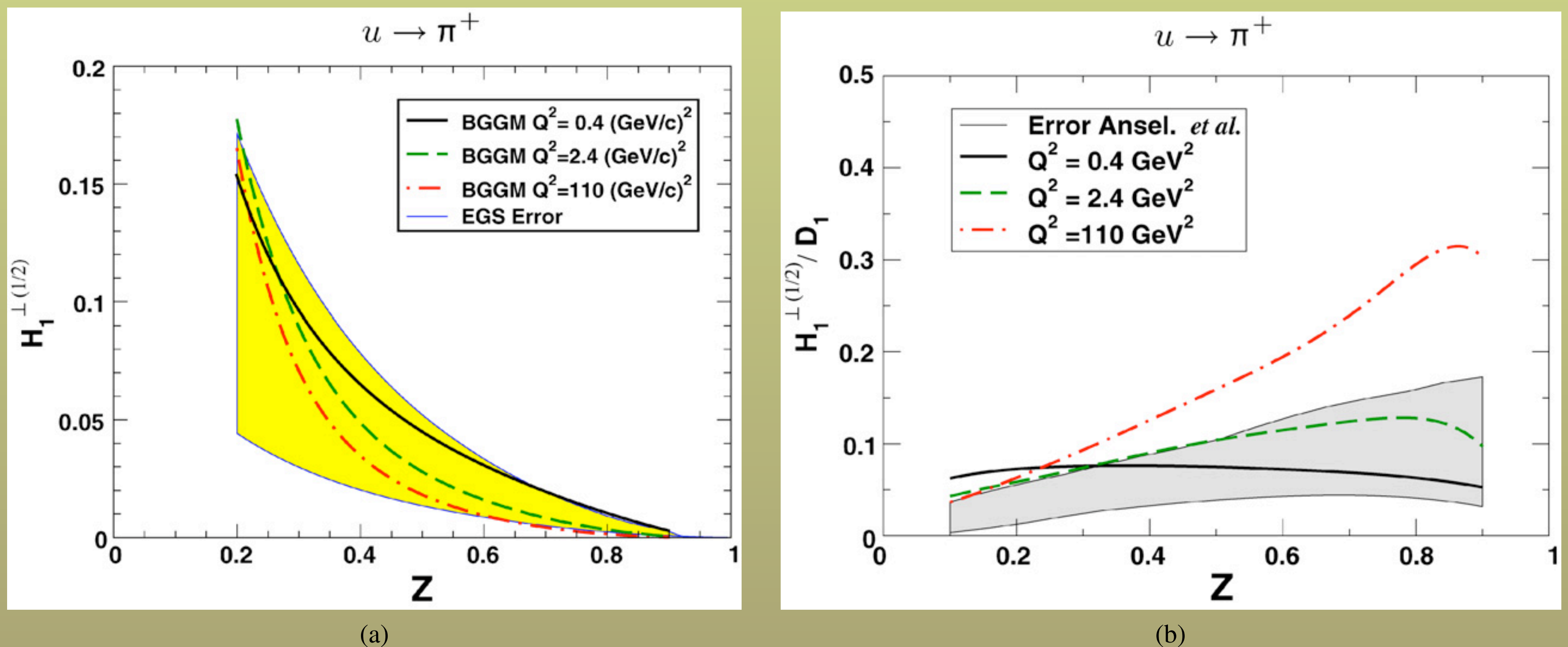


Fig. 4. Half moment of the Collins function for $u \to \pi^+$ in our model. (a) $H_1^{\perp(1/2)}$ at the model scale (solid line) and at a different scale under the assumption in Eq. (37) (dot-dashed line), compared with the error band from the extraction of Ref. [6], (b) $H_1^{\perp(1/2)}/D_1$ at the model scale (solid line) and at two other scales (dashed and dot-dashed lines) under the assumption in Eq. (38). The error band from the extraction of Ref. [7] is shown for comparison.

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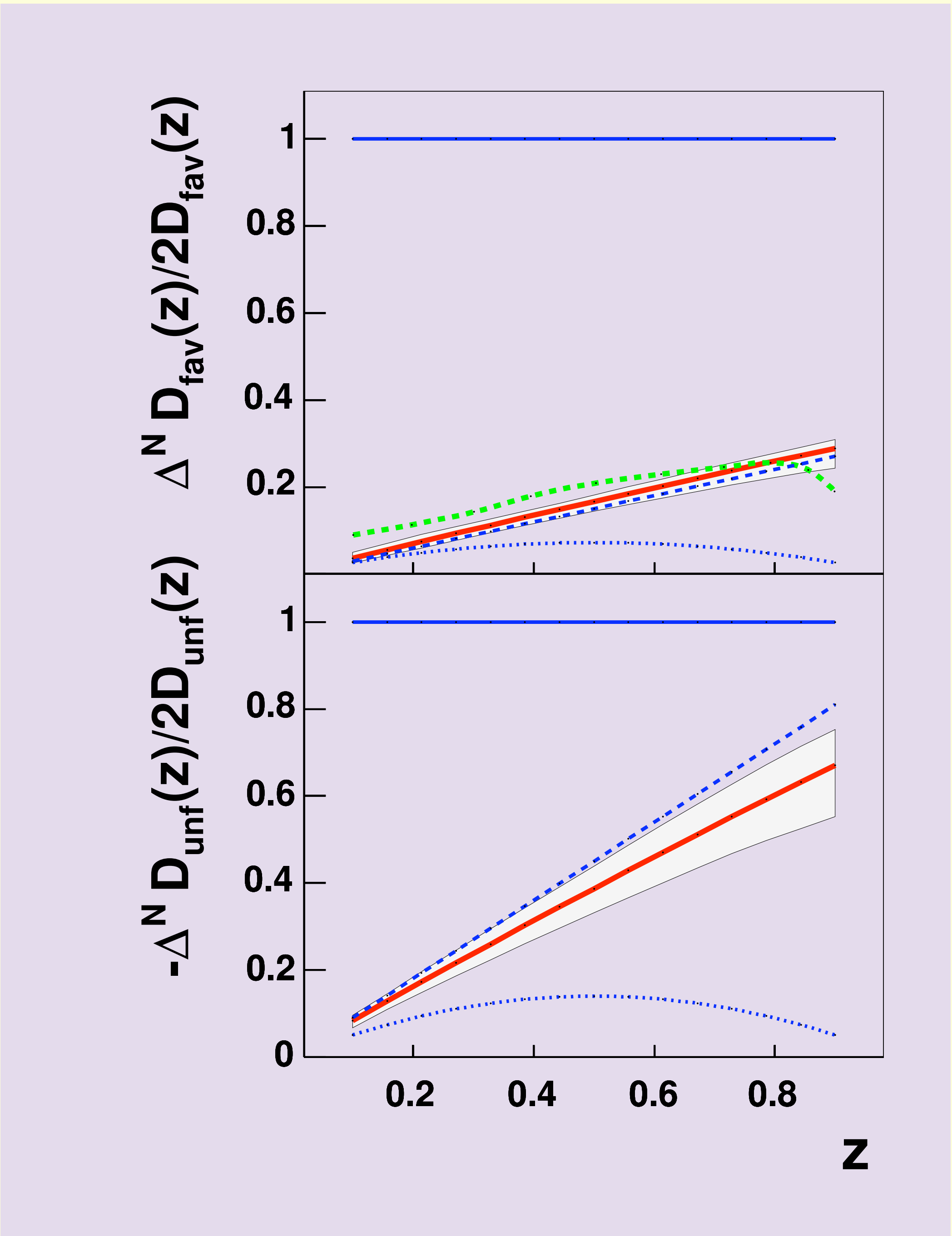

compared to Ref. [1] (dashed line), Ref. [2] (dotted line), and Ref. [3] (dashed green line)

[1] A. V. Efremov, K. Goeke, and P. Schweitzer, Phys. Rev. **D73**, 094025 (2006).
[2] W. Vogelsang and F. Yuan, Phys. Rev. **D72**, 054028 (2005).
[3] A. Bacchetta, L. Gamberg, G. R. Goldstein, A. Mukherjee PLB659:234-243,2008.

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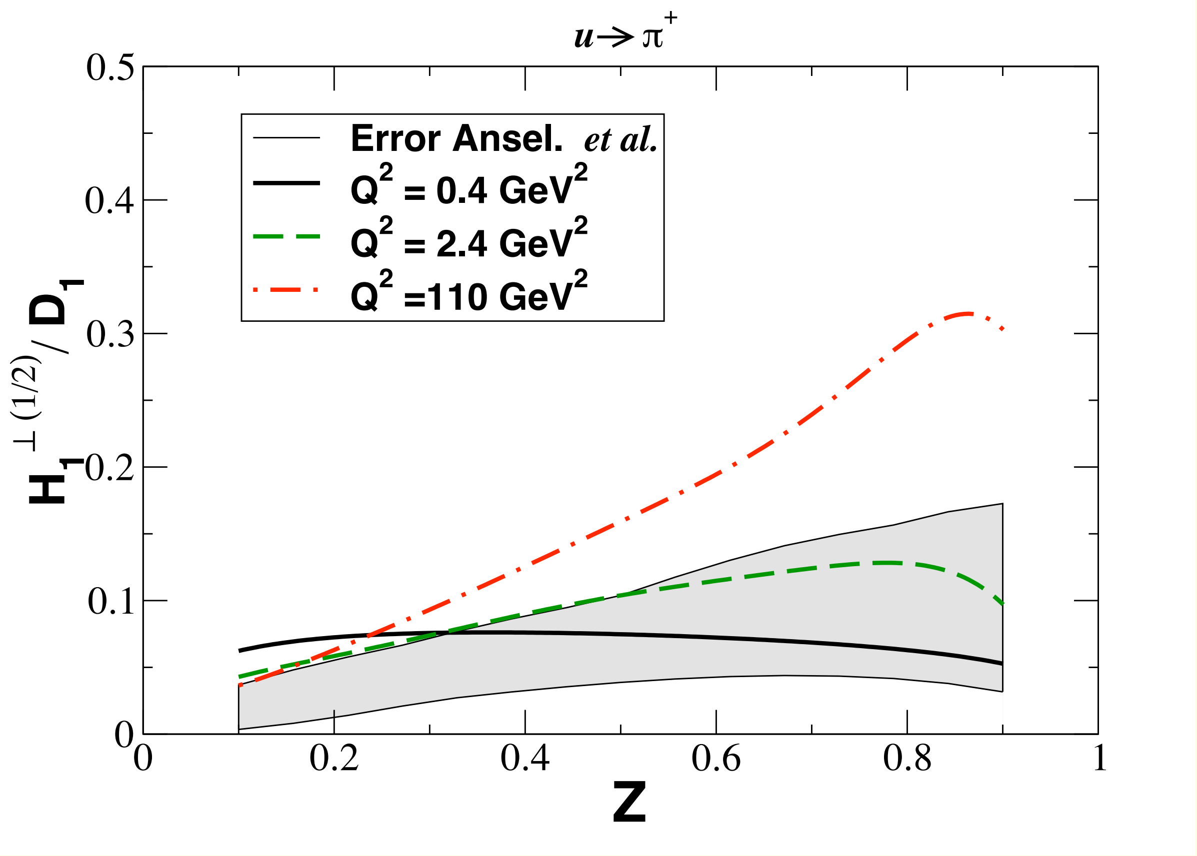

Conclusions

- Generalize GPM w/ color--can then perform global analysis
- Elephant in the room is break down of factorization for these processes
- Appears to be connection between generalized parton model at twist 3 and twist 3 approach
- Estimate mismatch-investigating LG Z. Kang
- TMD fact. is assumed in both GPM and GGPM is this a reasonable pheno. approximation?
- Direct photon driven by same ISI factor as in DY Collins is small!!!

Reliability of Transversity Extraction Universality of Collins Fragmentation Function

Belle KEKB measurement Collins Frag. Function PRL 2006 & PRD 2008

From talks of Ralf Seidl

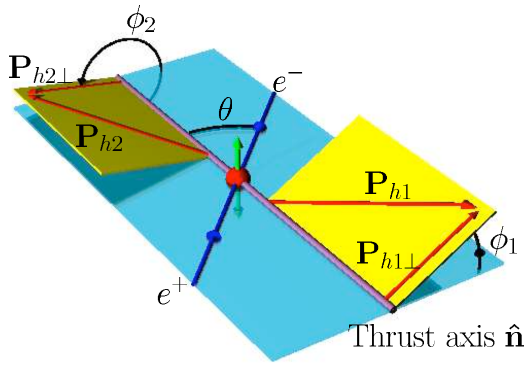

FIG. 2: Definition of the azimuthal angles ϕ_1 and ϕ_2 of the two hadrons, between the scattering plane and their transverse momenta $\mathbf{P}_{hi\perp}$ around the thrust axis $\hat{n}$. The angle θ is defined as the angle between the lepton axis and the thrust axis.

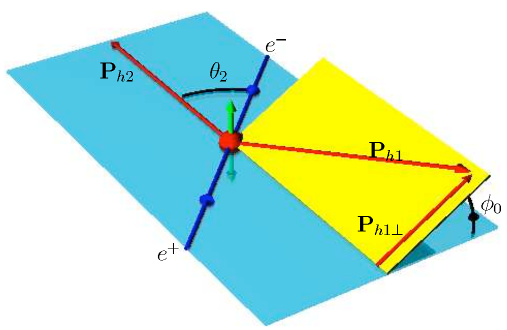

FIG. 3: Definition of the azimuthal angle ϕ_0 formed between the planes defined by the lepton momenta and that of one hadron and the second hadron's transverse momentum $P'_{h1\perp}$ relative to the first hadron.

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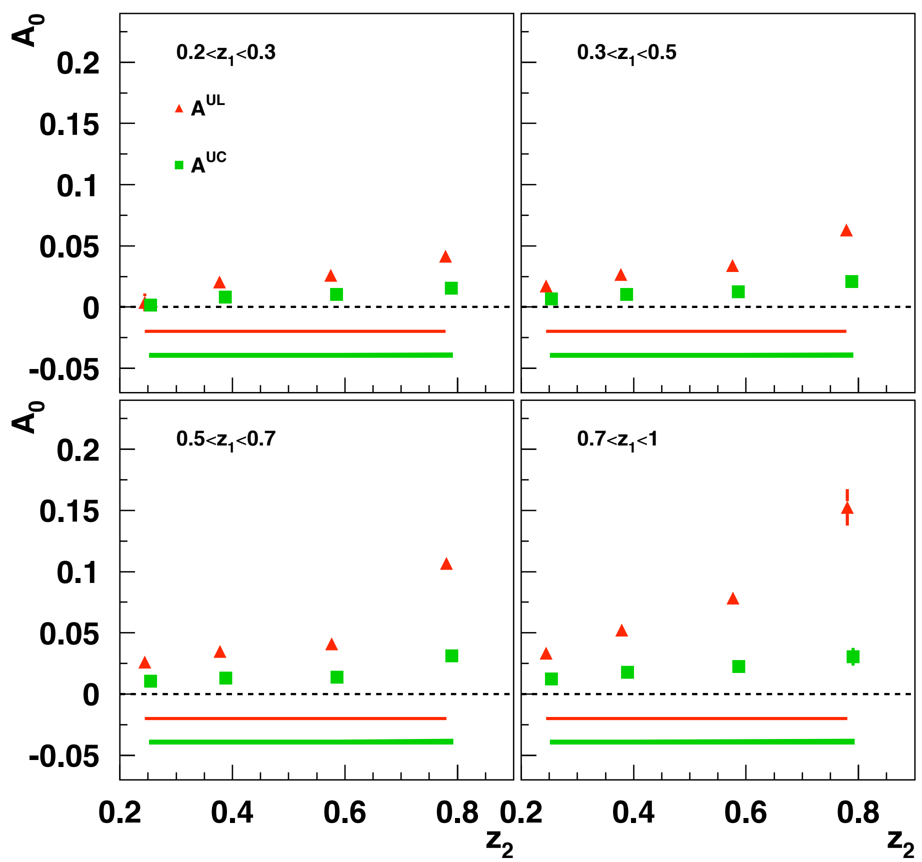

FIG. 17: Light quark (uds) A_0 asymmetry parameters as a function of z_2 for 4 z_1 bins. The UL data are represented by triangles and the systematic error by the upper error band. The UC data are described by the squares and their systematic uncertainty by the lower error band.

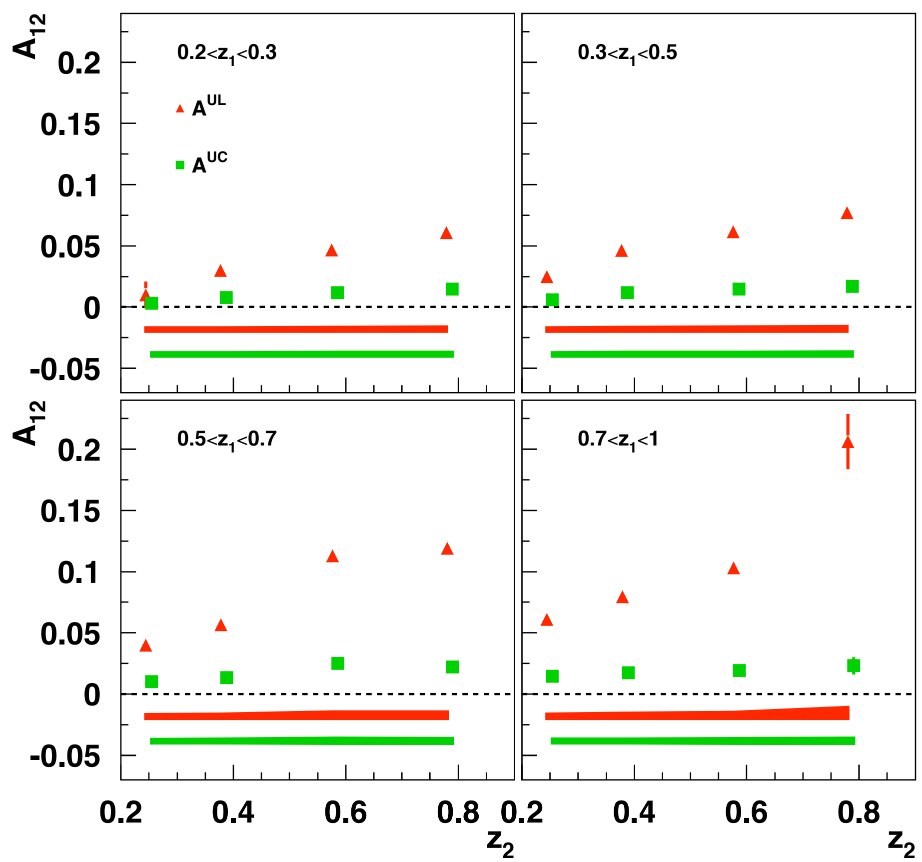

FIG. 18: Light quark (uds) A_{12} asymmetry parameters as a function of z_2 for 4 z_1 bins. The UL data are represented by triangles and the systematic error by the upper error band. The UC data are described by the squares and their systematic uncertainty by the lower error band.