# Spin tracking of e-lens lattice

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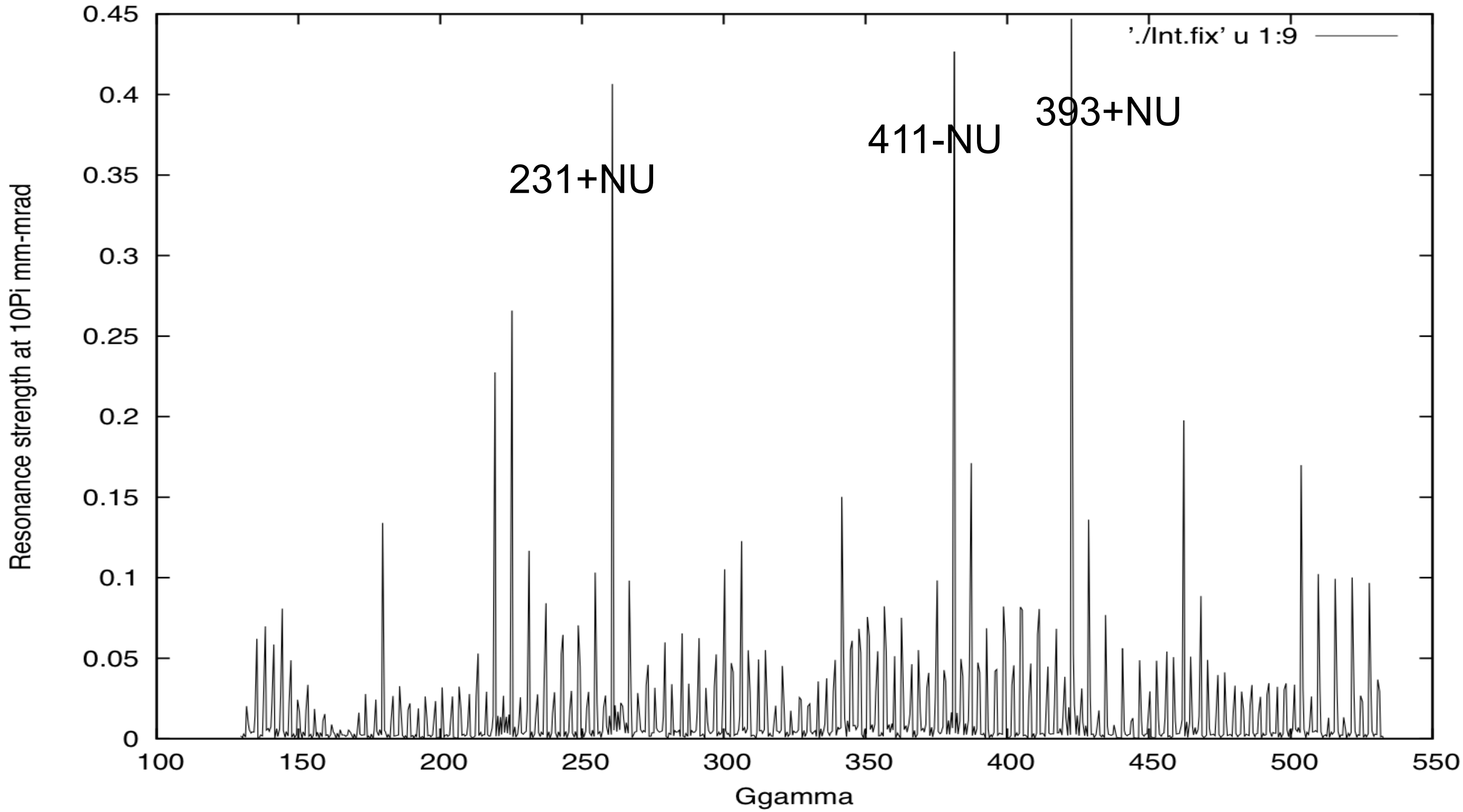
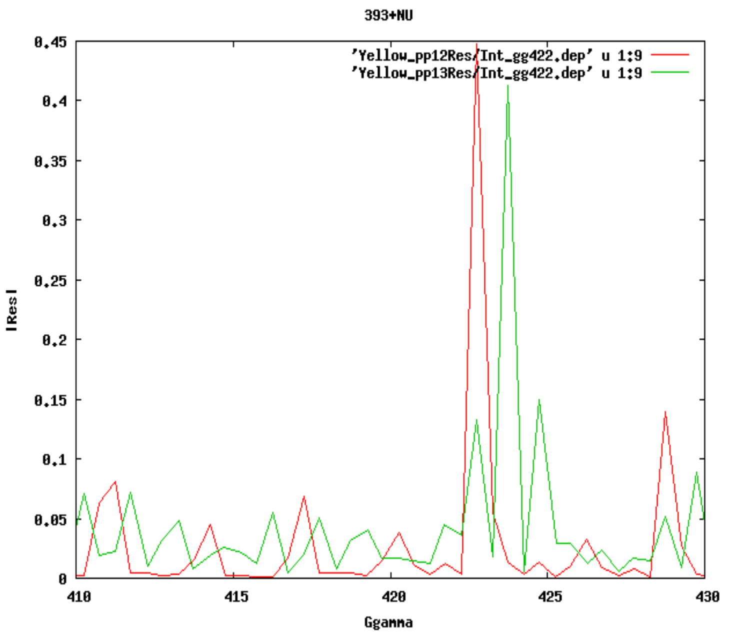
# Overview of Resonances on RHIC ramp

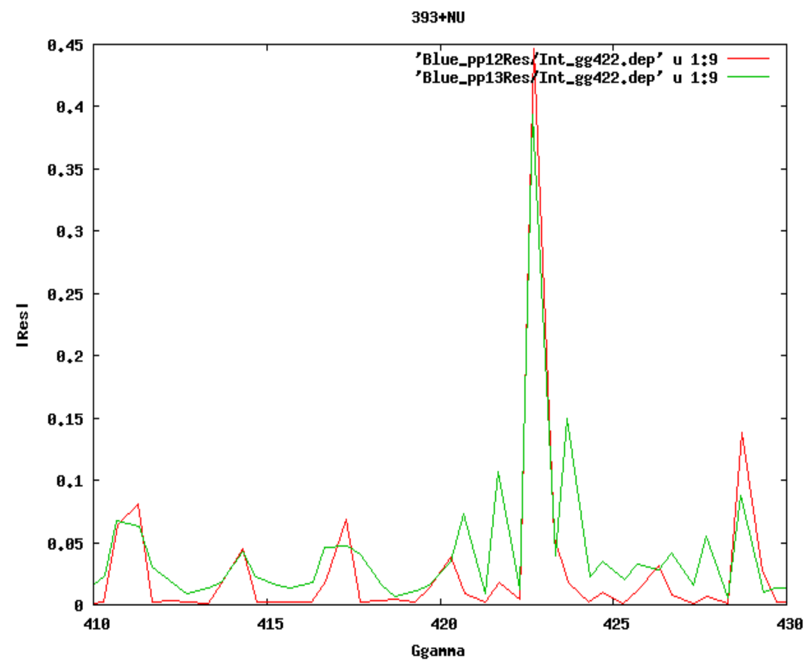


# What is Special about e-lens lattices?

- Understood Differences from Standard lattice:
  - Main 3 intrinsic Resonances weaker in both lattices
    - Blue weakest then Yellow
  - Some Secondary Resonances larger:
    - About x10 larger up to 0.1 in strength
  - The overlap of these secondary resonances can cause problems:
    - Although when considered individually they are weak enough that the snakes should easily handle them
    - However when they overlap with a the strong main resonances we can see depolarization via Parametric resonance

# First Test of modified Intrinsic resonance in RHIC

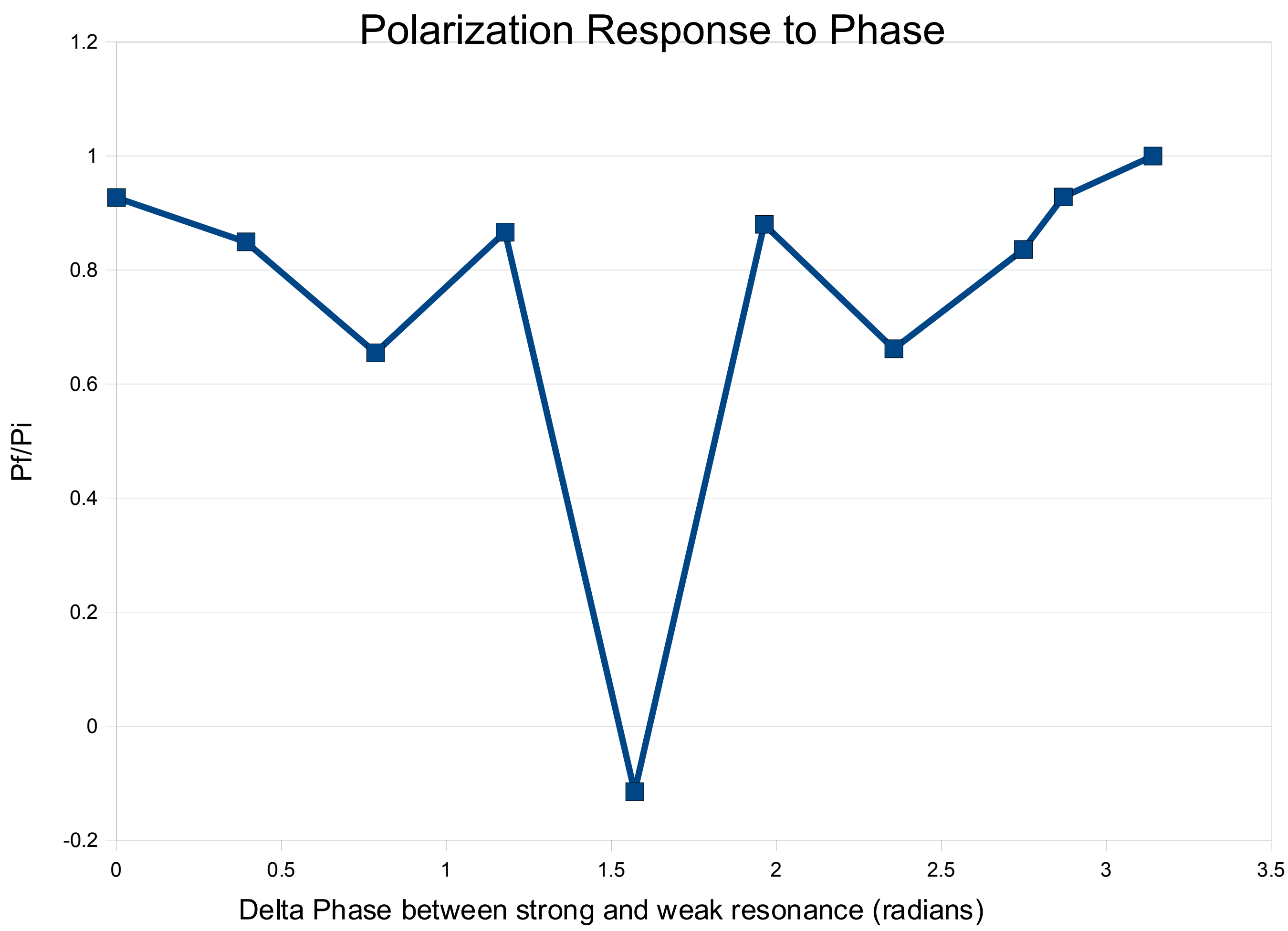



Reduced Resonances by 10 to 14%

| Resonances | Blue (new-old) | Yellow (new-old) |
|---|---|---|
| 231+NU | -0.0387 | -0.0415 |
| 411-NU | -0.06134 | -0.0655 |
| 393+NU | -0.05347 | -0.0347 |

# Blue e-lens strength at 15 pi mm-mrad normal acceleration rate



# Why is Phase an issue?

If you look at the BMT equation for two resonance crossings without snakes you can get an idea of why. Using a parametric transformation you can reduce things to a Hills' like equation:

$$\frac{d^2\Psi_I^+}{d\theta^2} - \left(i f_3(\theta) + \frac{\xi'(\theta)}{\xi(\theta)}\right)\frac{d\Psi_I^+}{d\theta} + \frac{\xi(\theta)\xi(\theta)^*}{4}\Psi_I^+ = 0$$

$$\beta(\theta) = -\left(i f_3(\theta) + \frac{\xi'(\theta)}{\xi(\theta)}\right) \qquad D(\theta) = \frac{1}{2}\int^t d\tau \beta(\tau)$$

$$q(\theta) = e^{D(\theta)}\Psi_I^+(\theta)$$

Introducing the above transformations will eliminate the 1st order part of the differential Equation and get us to a Hill's like equation.

$$\Omega^2(\theta) = \frac{\beta'(\theta)}{2} + \frac{\beta(\theta)^2}{4} - \frac{\xi(\theta)\xi(\theta)^*}{4}$$

$$\frac{d^2 q}{d\theta^2} = \Omega^2(\theta) q$$

# Expansion of Hills Kernel

For the two resonance case the 1/xi terms are what give us the most problem we proceed to expand them assuming there is a dominant and weaker resonance we expand using the Ratio of this parameter. $\epsilon = a_2/a_1$

$$\begin{aligned}\frac{1}{\xi(\theta)} &= \frac{1}{a_1 e^{-i(K_1\theta+\phi_1)} + a_2 e^{-i(K_2\theta+\phi_2)}} \\ &\approx \frac{e^{i(K_1\theta+\phi_1)}}{a_1}(1-(\epsilon e^{i\Delta\phi})e^{i\delta\theta})\end{aligned}$$

Multiplying this expansion out and keeping only first order epsilon terms we get:

$$\Omega^2(\theta) \approx W_0^2 + C_1\theta + C_2\theta^2 + C_3 e^{i\delta\theta} + C_4\theta e^{i\delta\theta} + C_5 e^{-i\delta\theta}$$

With the constant C1-C5 terms defined below and the frequency in terms of difference between The two resonances $\delta = K_1 - K_2$

$$\begin{aligned}C_1 &= \alpha\frac{K_1-\kappa_0}{2} \\ C_2 &= -\frac{\alpha^2}{4} \\ C_3 &= (\frac{3K_1\delta}{2} - \frac{(K_1^2-K_2^2)}{2} - \frac{a_1^2}{4} - \frac{\delta\kappa_0}{2})\epsilon_p \\ C_4 &= -\frac{\alpha\delta}{2}\epsilon_p \\ C_5 &= -\frac{a_1^2}{4}\epsilon_m\end{aligned}$$

$$W_0^2 = -i\frac{\alpha}{2} - \frac{\kappa_0^2}{4} + \frac{K_1\kappa_0}{2} - \frac{K_1^2}{4} - \frac{a_1^2}{4} - \frac{a_2^2}{4}$$

$$\begin{aligned}\epsilon_p &= \epsilon e^{i\Delta\phi} \\ \epsilon_m &= \epsilon e^{-i\Delta\phi}\end{aligned}$$

$$\Delta\phi = \phi_1 - \phi_2$$

# Parametric Resonance Approximation

Following the work done by by Richard Rand and others [1,2], It can be shown that the oscillating pieces only contribute significantly in a parametric resonance tongue region:

$$W_0^2 \approx -\delta^2/4$$

In this region q0 becomes:

$$q_0(t) = A(\eta)e^{i\delta t/2} + B(\eta)e^{-i\delta t/2}$$

The q1 equations generate secular terms which can be now canceled using A() and B() for Which we get two coupled first order differential equations which can be solved:

$$i\delta\frac{dA}{d\eta} = -B \qquad A = e^{\sqrt{\epsilon_0}\eta/\delta} \qquad \epsilon_0 = C_5/C_3$$

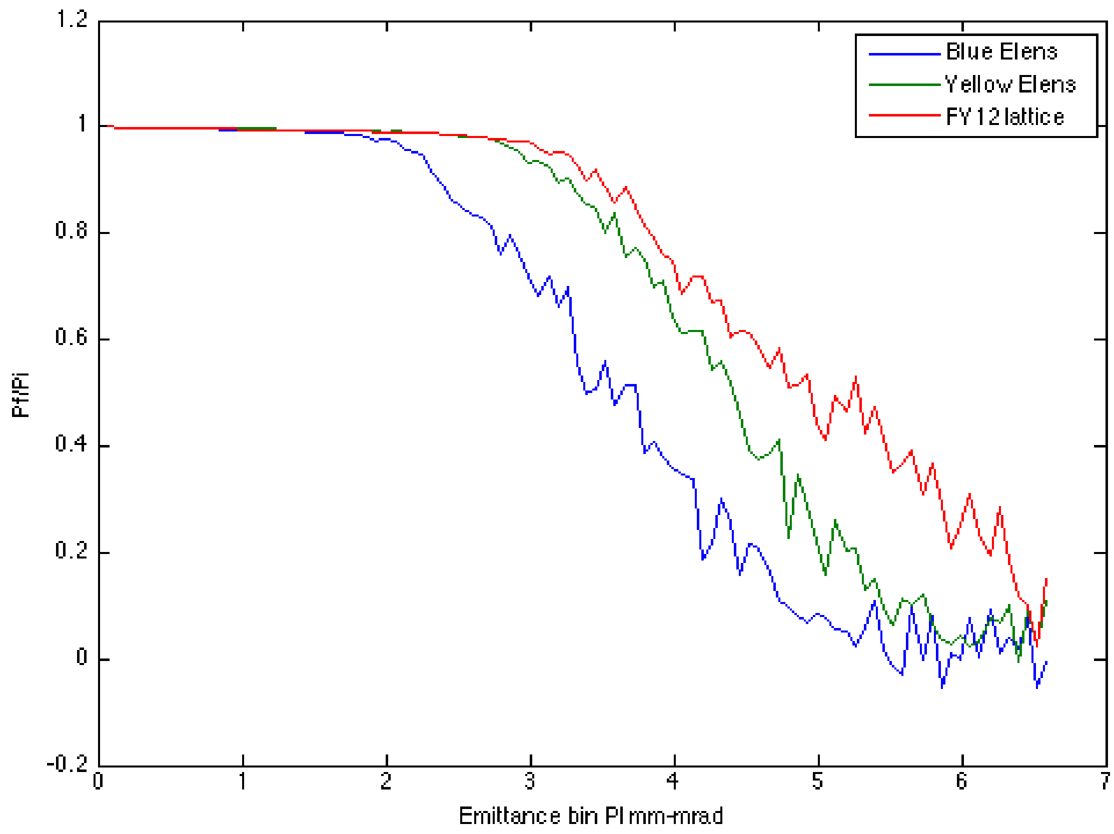
$$i\delta\frac{dB}{d\eta} = \epsilon_0 A \qquad B = -i\sqrt{\epsilon_0}e^{\sqrt{\epsilon_0}\eta/\delta}$$

Putting A and B back into q0 gives a decent approximation valid over the resonance tongue Region; which is plus or minus the |Ca| the maximum amplitude of C3 or C5. We get growing Or damping solution when square root of epsilon zero is real which occurs when **phase= PI/2.**

$$q(\theta) = A_1 e^{\sqrt{\epsilon_0}C_3\theta/\delta}(e^{i\delta\theta/2} - i\sqrt{\epsilon_0}e^{-i\delta\theta/2}) + A_2 e^{-\sqrt{\epsilon_0}C_3\theta/\delta}(e^{i\delta\theta/2} + i\sqrt{\epsilon_0}e^{-i\delta\theta/2})$$

$$W_0^2 \approx \delta^2/4 \pm |C_a|/2;$$

# 6D Tracking Results crossing 393+NU resonance



# How did we do?

| Lattice (before LLRF fix) | Avg Jet Pol. * | Avg. CNI Ramp Eff. ** | Avg R ratio ** |
|---|---|---|---|
| Blue e-lens | 47.7± 0.7% | 0.8202+- 0.0059 | 0.2381 |
| Blue FY12 | 42.7% ± 0.8% | 0.7805+- 0.0089 | 0.3129 |
| Yellow e-lens | 44.1% ± 0.8% | 0.8324+- 0.0064 | 0.2447 |
| Yellow FY12 | 50.0% ± 0.9% | 0.8469+- 0.0105 | 0.2452 |

| Lattice (after LLRF fix) | Avg Jet Pol. * | Avg. CNI Ramp Eff. ** | Avg R ratio ** |
|---|---|---|---|
| Blue FY12 | 51.7 %± 0.3% | 0.8842+- 0.0057 | 0.1287 |
| Yellow FY12 | 55.1%± 0.4% | 0.8834+- 0.006 | 0.1403 |

* Jet Number Courtesy H. Huang  ** CNI Ramp Eff. Courtesy D. Smirnov

# Predictions for e-lens lattice based on Integrating over different emittances

| Lattice | 12PI | 15PI | 20PI |
|---|---|---|---|
| Blue e-lens | 0.88 | 0.79 | 0.656 |
| Yellow e-lens | 0.96 | 0.892 | 0.745 |
| FY12 lattice | 0.98 | 0.977 | 0.90 |

Question why FY12 lattice seemed to under perform tracking expectations by
About 10%.