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Collinear parton splittings for early thermal and chemical equilibration

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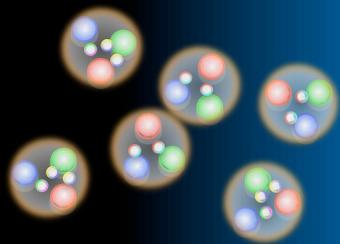
Brain Workshop 2014 “Frontiers of Hadronic Physics: Brains Recirculate Two”

13th March 2014, Brookhaven National Laboratory, Upton NY, USA

Introduction

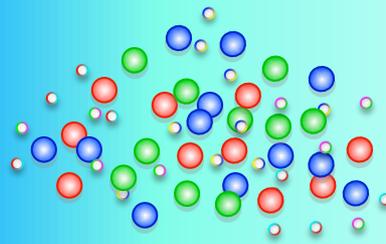
- **Quark-gluon plasma (QGP)**: many-body system of deconfined quarks and gluons

Graphics by AM



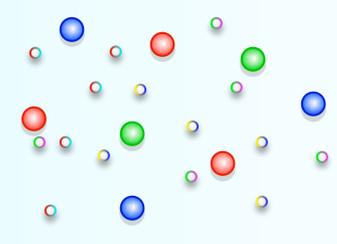
Hadron phase

(crossover)



sQGP

QGP phase



(wQGP?)

The QGP is supposed to have filled the early universe;
It can be produced in **heavy ion experiments** at RHIC & LHC

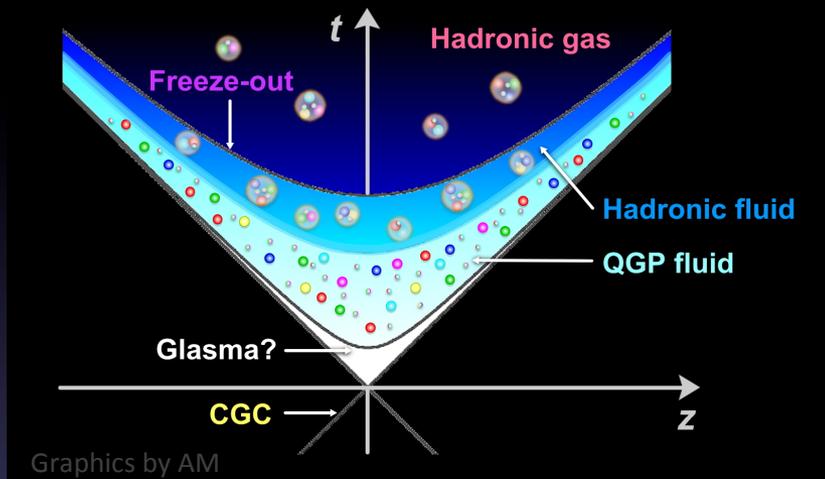
⇒ Heavy ion QGP is found to exhibit near-perfect fluidity

⇒ **Early local equilibration** ($\tau < 0.4\text{-}1\text{ fm}$) is implied



Introduction

■ Modeling a “Little Bang”



- ▶ $\tau > 10 \text{ fm}/c$: **hadronic gas**
(weakly coupled)
- ▶ $\tau \sim 1\text{-}10 \text{ fm}/c$: **QGP/hadronic fluid**
(strongly coupled)
- ▶ $\tau \sim 0\text{-}1 \text{ fm}/c$: CGC \rightarrow QGP ?
(pre-equilibrated)
- ▶ $\tau < 0 \text{ fm}/c$: **color glass condensate**
(saturated gluons)

Pre-equilibrated QCD medium is a high-energy frontier in hadron physics

Dynamical description is required for understanding QCD matter

Introduction

■ Color glass condensate (CGC)

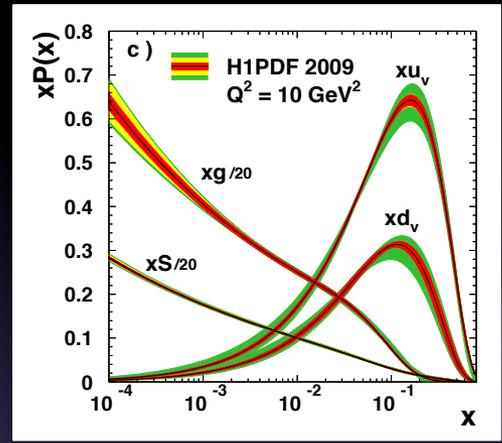
▶ Gluons emitted from gluons emit gluons in a fast-travelling nucleon

⇒ They start to overlap and saturated

gluon gluon
gluon gluon

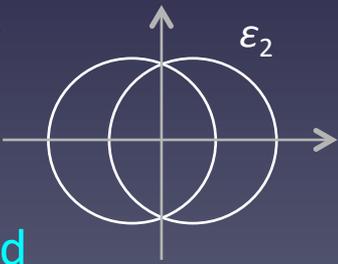
⇒ QCD matter at the initial stage of heavy ion collisions is **dominated by gluons**

H1 Collaboration (2009)



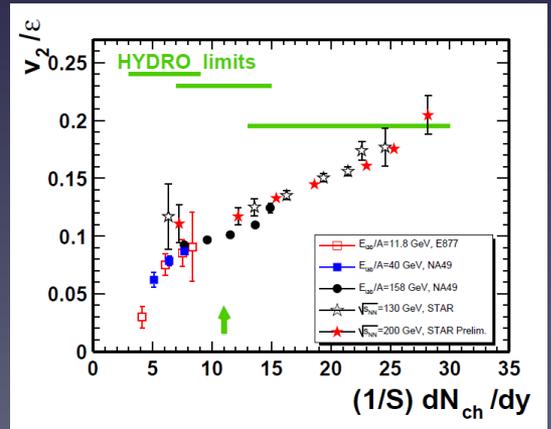
■ QGP fluid

▶ Azimuthal momentum anisotropy v_2 is large compared with spatial one ϵ_2



⇒ QCD matter is **locally equilibrated** at some point and behaves as a fluid

NA49 Collaboration (2003)



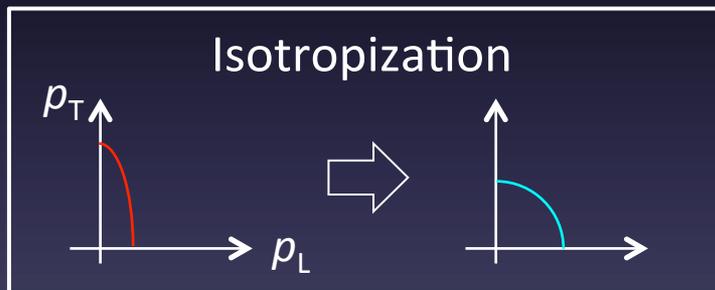
Motivation (I)

■ Early (local) equilibration

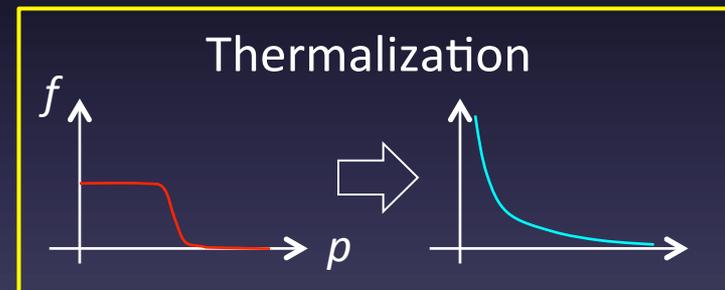
▶ One of the long standing issues in heavy ion physics

- Naïve CGC and glasma lead to negative longitudinal pressure
- CGC has large fraction of high-momentum gluons

▶ Equilibration requires



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⇒ We focus on thermalization and provide a model based on **collinear splitting** to introduce low momentum partons

Cf: parton cascade picture
bottom-up approaches
Bose condensate

K. Geiger and B. Müller (1992)

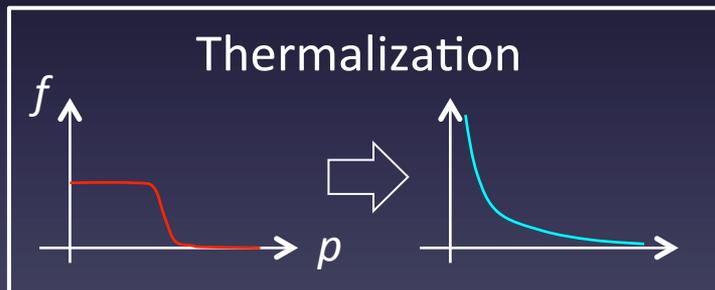
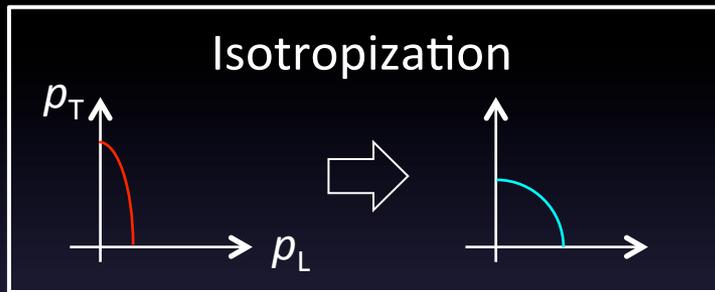
R. Baier et. al. (2001), P. Arnold et al. (2003)

J.-P. Blaizot et al. (2012, 2013), X.-G. Huang and J. Liao (2013)

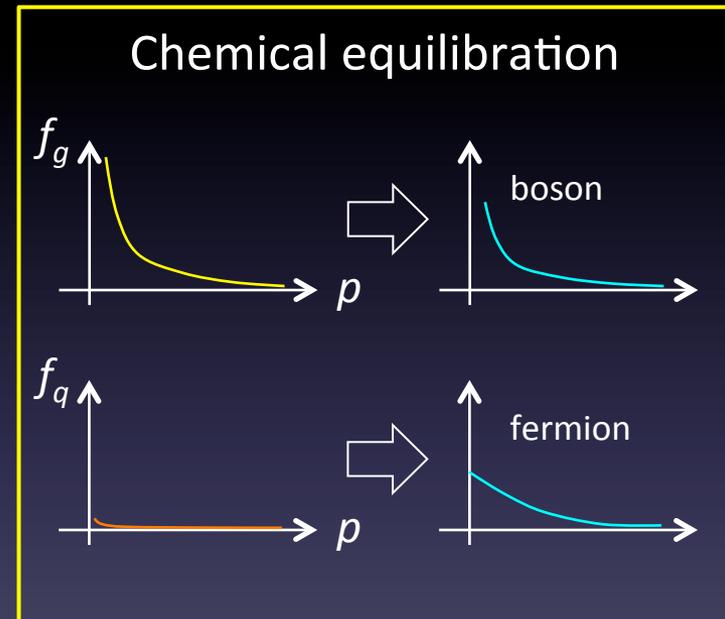
Motivation (II)

■ Early (local) equilibration

▶ Equilibration requires *more*



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⇒ Gluon medium eventually becomes *quark-gluon* plasma
 Is quark thermalization fast enough?
 Is the QGP chemical equilibrated?

Collinear splitting model

■ Gluon splitting

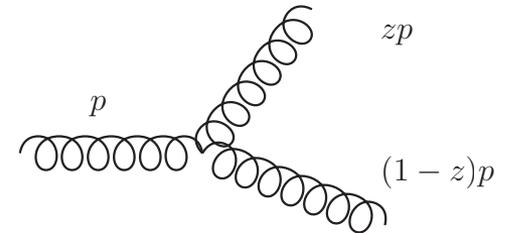
- ▶ A splitting modifies the parton distribution f as

$$f(p_i) \rightarrow z^{-d} f\left(\frac{p_i}{z}\right) + (1-z)^{-d} f\left(\frac{p_i}{1-z}\right)$$

as number is **doubled** while momentum is **conserved**:

$$2 \int \frac{dp^d}{(2\pi)^d} f(p_i) = \int \frac{dp^d}{(2\pi)^d} \left[z^{-d} f\left(\frac{p_i}{z}\right) + (1-z)^{-d} f\left(\frac{p_i}{1-z}\right) \right]$$

$$\int \frac{dp^d}{(2\pi)^d} p_j f(p_i) = \int \frac{dp^d}{(2\pi)^d} p_j \left[z^{-d} f\left(\frac{p_i}{z}\right) + (1-z)^{-d} f\left(\frac{p_i}{1-z}\right) \right]$$



- ▶ Time evolution of gluon distribution by collinear splitting

$$\left. \frac{\partial f(p_i)}{\partial t} \right|_{\text{sp}} = \frac{1}{2} \int_0^1 dz r(z) [f^z(p_i) + f^{1-z}(p_i) - f(p_i)] \equiv \mathcal{C}_{\text{sp}}(p_i)$$

where $f^z(p_i) \equiv z^{-d} f\left(\frac{p_i}{z}\right)$, $f^{1-z}(p_i) = (1-z)^{-d} f\left(\frac{p_i}{1-z}\right)$ and

$$r(z) = \Gamma R_{gg}(z) = 6\Gamma \frac{[1-z(1-z)]^2}{z(1-z)} \quad \text{is the splitting rate}$$

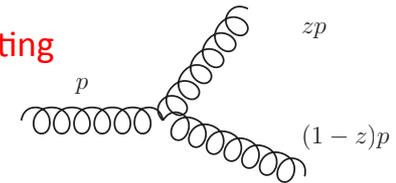
Collinear splitting model

■ Gluon + quark splittings

▶ Time evolution of parton distributions by collinear processes

$$\frac{\partial f_g(p_i)}{\partial t} \Big|_{\text{sp}} = \frac{1}{2} \int_0^1 dz r_{gg}(z) [f_g^z(p_i) + f_g^{1-z}(p_i) - f_g(p_i)]$$

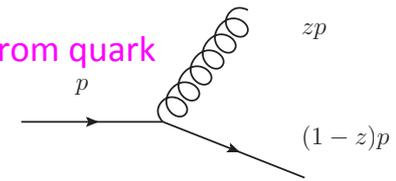
Gluon splitting



$$+ \int_0^1 dz r_{gq}(z) f_q^z(p_i) - \int_0^1 dz r_{qg}(z) f_g(p_i) \equiv C_{\text{sp}}^g(p_i)$$

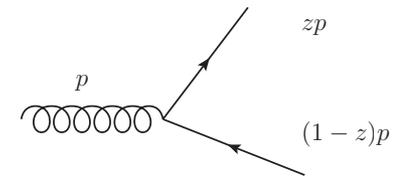
$$\frac{\partial f_q(p_i)}{\partial t} \Big|_{\text{sp}} = \int_0^1 dz r_{gq}(z) [f_q^{1-z}(p_i) - f_q(p_i)]$$

Gluon emission from quark



$$+ \int_0^1 dz r_{qg}(z) [f_g^z(p_i) + f_g^{1-z}(p_i)] \equiv C_{\text{sp}}^q(p_i),$$

Quark pair production



Splitting functions

$$r_{gg}(z) = \Gamma R_{gg}(z) = 6 \frac{[1 - z(1 - z)]^2}{z(1 - z)} \Gamma$$

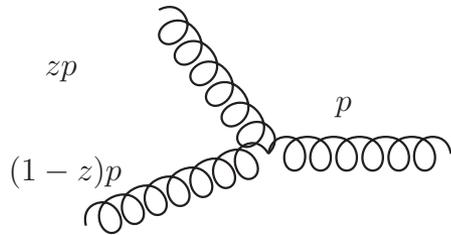
$$r_{gq}(z) = \Gamma R_{gq}(z) = \frac{4}{3} \frac{1 + (1 - z)^2}{z} \Gamma$$

$$r_{qg}(z) = 2N_f \Gamma R_{qg}(z) = 3[z^2 + (1 - z)^2] \Gamma \quad (N_f = 3)$$

Quark-antiquark symmetry assumed

Collinear splitting model

■ Gluon recombination



Time evolution should stop at local equilibrium (otherwise **2nd law of thermodynamics** is violated)

⇒ **Recombination** should occur when density is large (similar but different from Balitsky-Kovshegov equation)

▶ Parameterization of recombination process

$$\frac{\partial f(p_i)}{\partial t} \Big|_{\text{rc}} = -\frac{1}{2} \int_0^1 dz \tilde{r}(p_i, z) \frac{f_{\text{eq}}^z(p_i) + f_{\text{eq}}^{1-z}(p_i) - f_{\text{eq}}(p_i)}{f_{\text{eq}}(zp_i) f_{\text{eq}}((1-z)p_i)} f(zp_i) f((1-z)p_i)$$

$$\equiv \mathcal{C}_{\text{rc}}(p_i)$$

where $f_{\text{eq}}(p) = \frac{d_g}{\exp(\sqrt{p^2 + m_{\text{th}}^2}/T) - 1}$ is equilibrium distribution in medium

Degeneracy $d_g = 16$

Effective thermal mass $m_{\text{th}} \sim \alpha_s^{1/2} T$

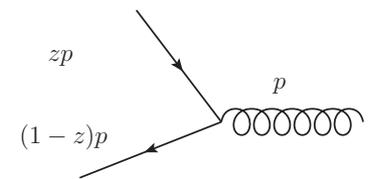
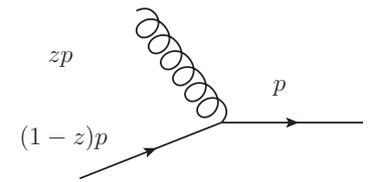
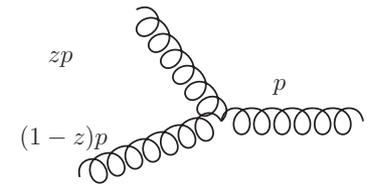
Collinear splitting model

■ Gluon + quark recombination

► For quark-gluon mixture system

$$\begin{aligned} \frac{\partial f_g(p_i)}{\partial t} \Big|_{\text{rc}} = & -\frac{1}{2} \int_0^1 dz \tilde{r}_{gg}(p, z) \frac{f_{g\text{eq}}^z(p_i) + f_{g\text{eq}}^{1-z}(p_i) - f_{g\text{eq}}(p_i)}{f_{g\text{eq}}(zp_i) f_{g\text{eq}}((1-z)p_i)} f_g(zp_i) f_g((1-z)p_i) \\ & - \int_0^1 dz \tilde{r}_{gq}(p, z) \frac{f_{q\text{eq}}^z(p_i)}{f_{g\text{eq}}(zp_i) f_{q\text{eq}}((1-z)p_i)} f_g(zp_i) f_q((1-z)p_i) \\ & + \int_0^1 dz \tilde{r}_{qg}(p, z) \frac{f_{g\text{eq}}(p_i)}{f_{q\text{eq}}(zp_i) f_{q\text{eq}}((1-z)p_i)} f_q(zp_i) f_q((1-z)p_i) \equiv \mathcal{C}_{\text{rc}}^g(p_i) \end{aligned}$$

$$\begin{aligned} \frac{\partial f_q(p_i)}{\partial t} \Big|_{\text{rc}} = & - \int_0^1 dz \tilde{r}_{gq}(p_i, z) \frac{f_{q\text{eq}}^{1-z}(p_i) - f_{q\text{eq}}(p_i)}{f_{g\text{eq}}(zp_i) f_{q\text{eq}}((1-z)p_i)} f_g(zp_i) f_q((1-z)p_i) \\ & - \int_0^1 dz \tilde{r}_{qg}(p_i, z) \frac{f_{g\text{eq}}^z(p_i) + f_{g\text{eq}}^{1-z}(p_i)}{f_{q\text{eq}}(zp_i) f_{g\text{eq}}((1-z)p_i)} f_q(zp_i) f_q((1-z)p_i) \equiv \mathcal{C}_{\text{rc}}^q(p_i) \end{aligned}$$



Equilibrium distributions

$$f_{g\text{eq}}(p) = \frac{d_g}{\exp(\sqrt{p^2 + m_{\text{th}}^2}/T) - 1}$$

$$f_{q\text{eq}}(p) = \frac{d_q}{\exp(\sqrt{p^2 + m_{\text{th}}^2}/T) + 1}$$



$$d_g = 16$$

$$d_q = 31.5 \quad (N_f = 3)$$

$$m_{\text{th}} \sim \alpha_s^{1/2} T$$

T : fixed from momentum conservation

Collinear splitting model

■ Momentum smearing effect

- ▶ Splitting occurs when a parton becomes **off-shell** by interacting with medium

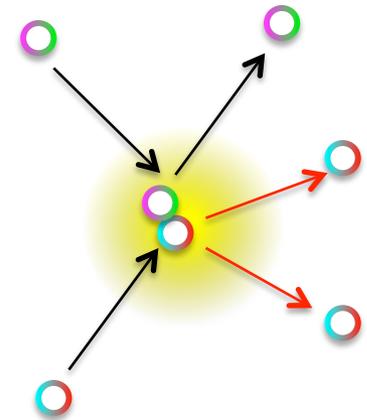
⇒ Parton-medium interaction is taken into account by relativistic **Fokker-Planck equation**

$$\frac{\partial f}{\partial t} \Big|_{\text{FP}} = \frac{\partial}{\partial p_i} \left[A^i f + \frac{\partial}{\partial p_j} (B^{ij} f) \right] \equiv \mathcal{C}_{\text{FP}}$$

$$f = \{f_g, f_q\}$$

where $A^i \sim \nu p^i$ and $B^{ij} \sim D \delta^{ij}$ characterizes drag and diffusion effects

Relativistic Einstein relation $D = \nu ET$ holds



Numerical analyses

■ Initial conditions (CGC-like)

▶ Gluon distribution

$$f_g(p < Q_s) \sim 1/\alpha_s \text{ and } f_g(p > Q_s) \sim 0 \quad Q_s \sim 2 \text{ GeV}$$

▶ Quark distribution

$$f_q(p, t_0) = 0$$

▶ We consider time evolution in a transverse direction for non-expanding systems for $N_f = 0$ and 3

■ Parameter for splitting process

▶ Rough estimation of parton emission rate Γ

In static frame $\Gamma \sim \alpha_s Q$

In boosted frame $\Gamma \sim \alpha_s Q(Q/p) \sim \alpha_s \hat{q}L/p$

⇒ $\Gamma \sim \alpha_s^{1/2}(\hat{q}/p)^{1/2}$ is implied (In thermal system $\Gamma_{\text{th}} \sim \alpha_s^{3/2}T$)

Numerical analyses

■ Parameter for momentum smearing

▶ Rough estimation of drag and diffusion coefficients

Analytic solution of a diffusion equation for a delta function $\delta(p)$

$$\frac{1}{\sqrt{4\pi Dt}} e^{-p^2/4Dt} \quad \Rightarrow \quad \text{Standard deviation } \sigma = \sqrt{2Dt}$$

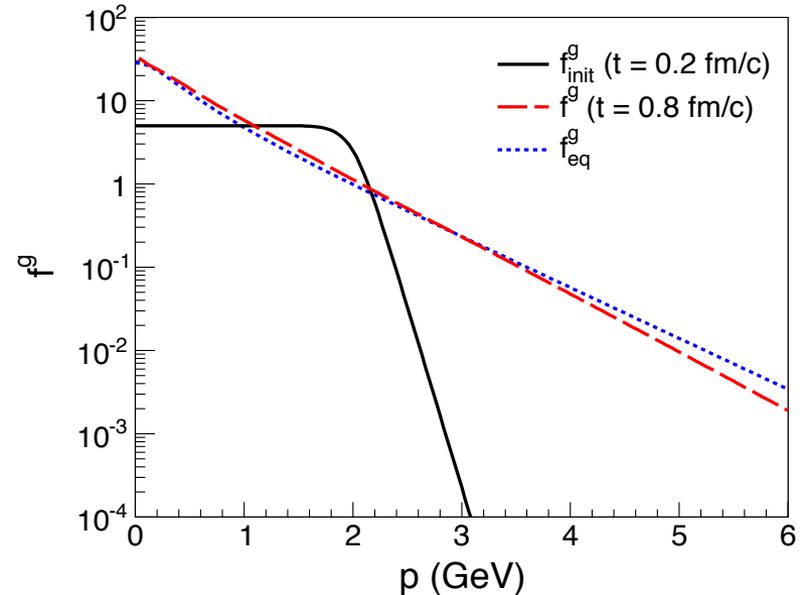
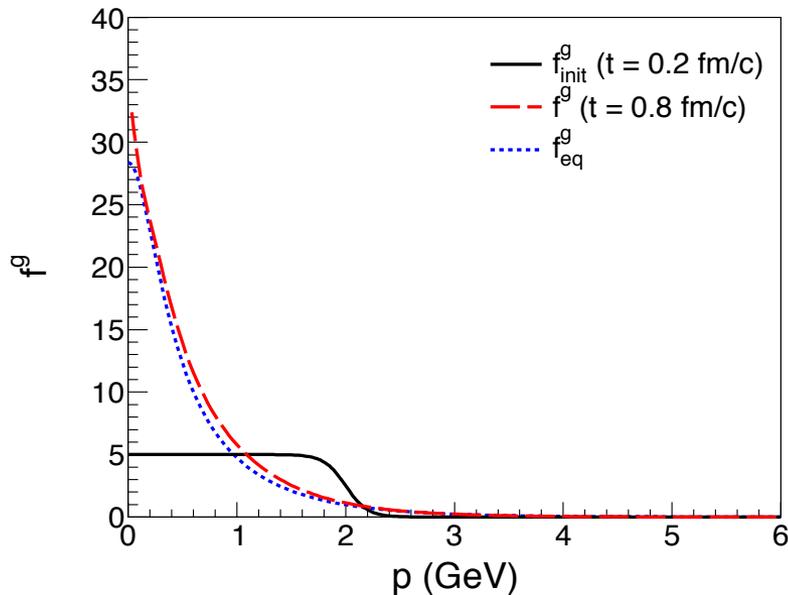
$$\quad \Leftrightarrow \quad \text{Longitudinal momentum modification } \sigma \sim Q^2/2p$$

$$\Rightarrow D \sim \hat{q}^2/\Gamma p^2 \text{ and } D_{\text{th}} \sim \alpha_s^{5/2} T^3 \text{ are implied}$$

Relativistic Einstein relation $D = \nu ET$ yields the drag coefficient

Glue system ($N_f = 0$)

■ Time evolution ($t = 0.8 \text{ fm}/c$)

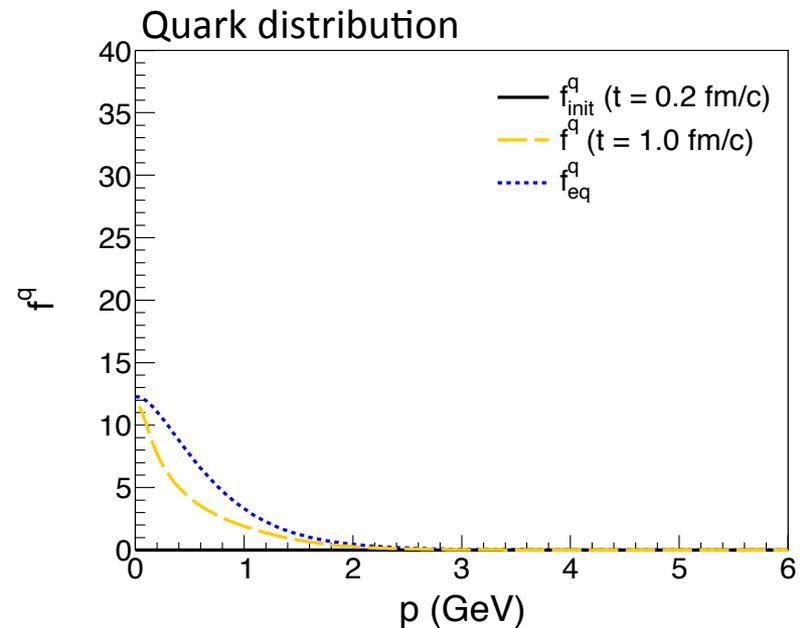
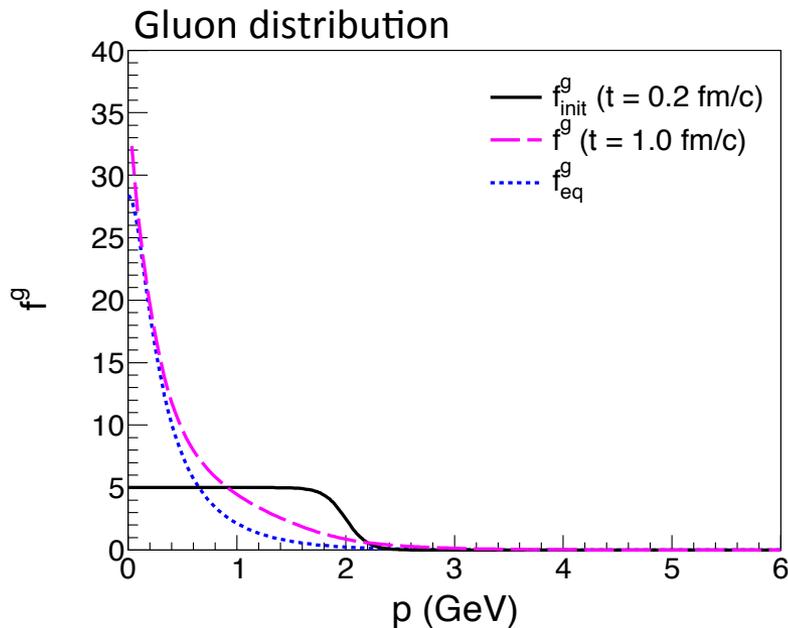


- Collinear gluon splitting contributes visibly to **quick thermalization**
- Entropy production is confirmed positive

▪ Recombination is important; Is the dynamics in dense region strong enough in 3D to enforce isotropization?

Quark-gluon system ($N_f = 3$)

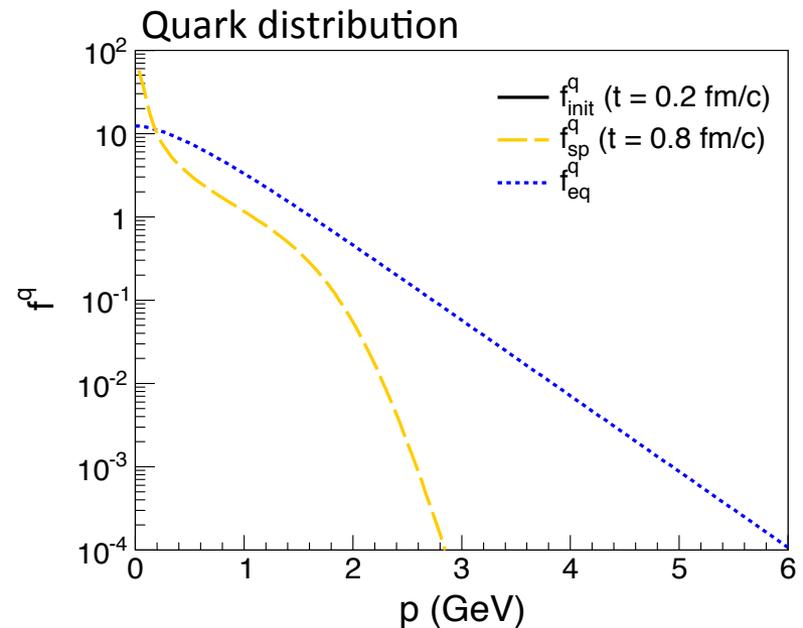
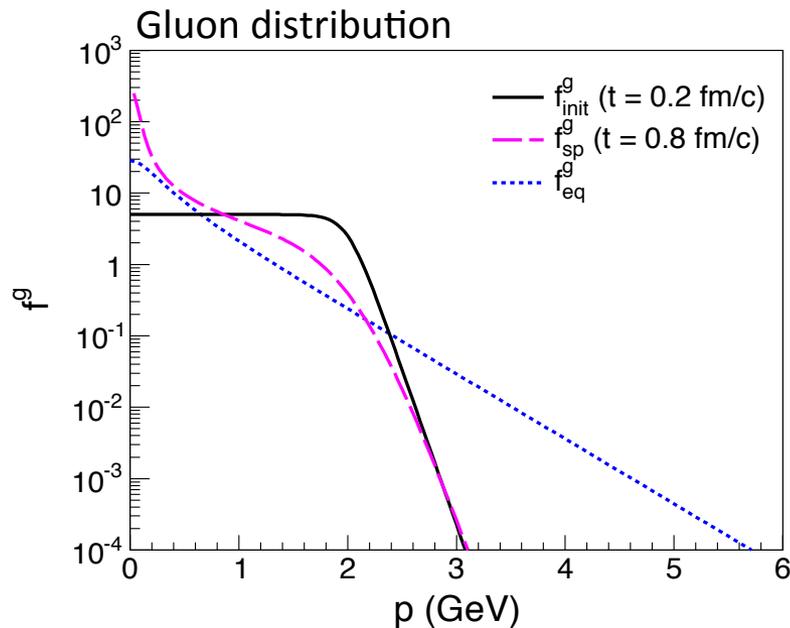
■ Time evolution ($t = 1.0 \text{ fm}/c$)



- Distributions approach the thermal ones; chemical equilibration is **relatively fast** but would be **slower** than thermalization ($\sim 1.5\text{-}2.0 \text{ fm}$)
- Shape of quark distribution reflects that of gluon distribution as quarks are pair-created from gluons
- “Fermi pressure” would not develop as # of quarks are not enough

Quark-gluon system ($N_f = 3$)

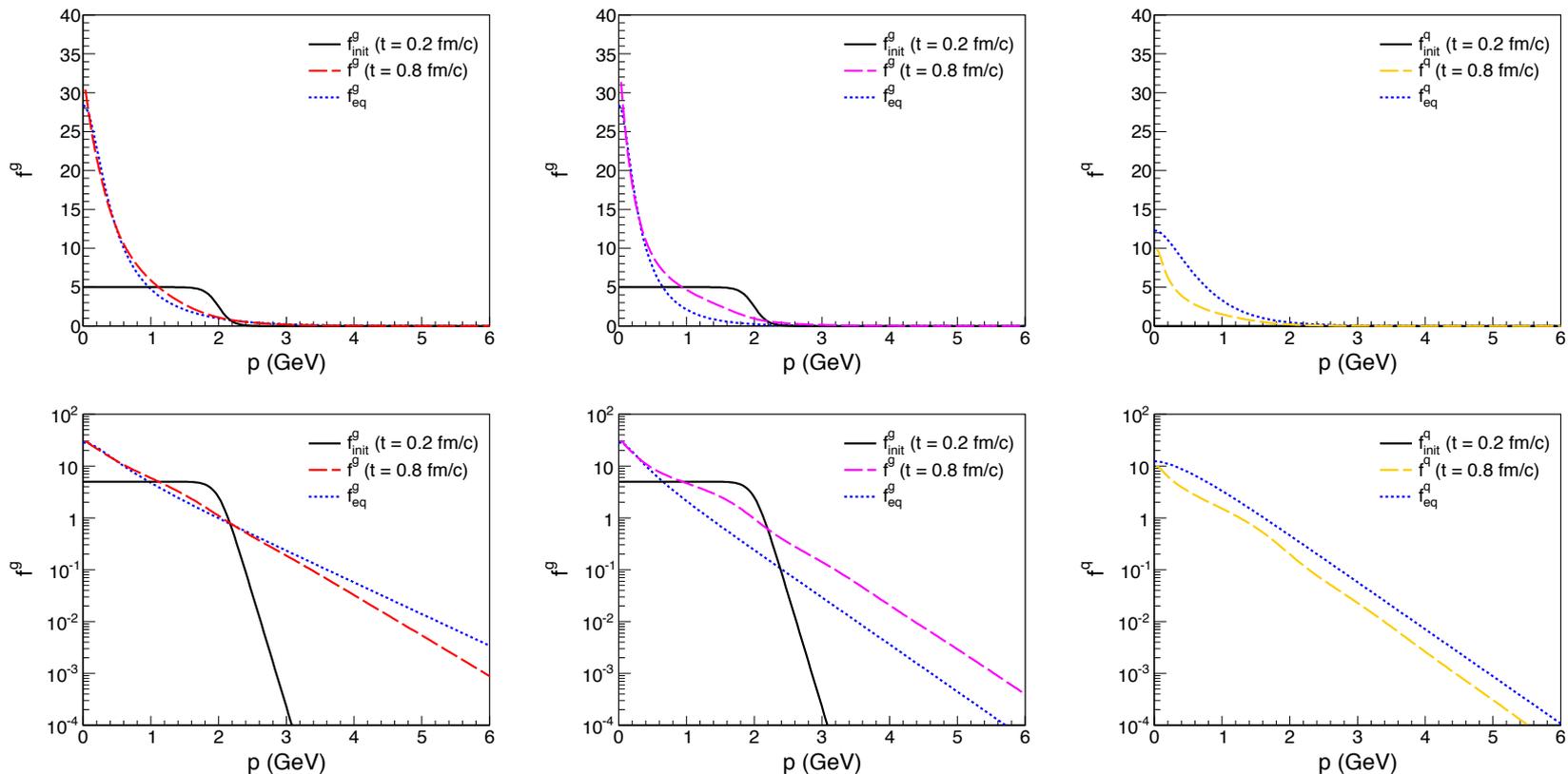
■ Splitting with no recombination (logarithmic scale)



- The shape of quark distribution follows that of gluon distribution
- Quark number becomes **too large** for fermions near $p = 0$; splitting should be suppressed, leading to slower chemical equilibration

Summary

■ Comparison of f_g (pure gauge), f_g and f_q ($N_f = 3$)



- Quark-gluon equilibration may be less “efficient” but more realistic

Summary and outlook

- Collinear quark and gluon splitting in early thermalization
 - ▶ Low momentum gluons are quickly produced
 - ▶ Quark production is reasonable fast; recombination would be important for fermions
 - ⇒ Describes transition from CGC to QGP
 - ▶ Thermalization might be faster than chemical equilibration
- Future prospects include
 - ▶ Three dimensional modeling for analyses on effects of expansion and isotropization
 - ▶ Non-thermalized partons lead to off-equilibrium energy-momentum tensor at the initial time of hydrodynamic stage

Chemically-equilibrating QGP

■ Enhancement of photon v_2

AM, in preparation (2014)

Energy-momentum conservation

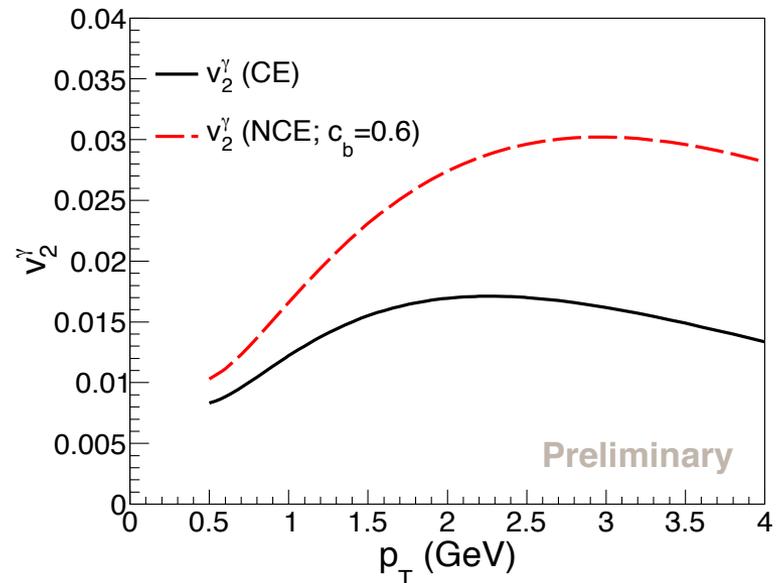
$$\partial_\mu T_g^{\mu\nu} + \partial_\mu T_q^{\mu\nu} = 0$$

Particle number evolution

$$\partial_\mu N_g^\mu = (r_a - r_b)n_g - r_a \frac{1}{n_g^{\text{eq}}} n_g^2 + r_b \frac{n_g^{\text{eq}}}{(n_q^{\text{eq}})^2} n_q^2$$

$$+ r_c n_q - r_c \frac{1}{n_g^{\text{eq}}} n_q n_g$$

$$\partial_\mu N_q^\mu = 2r_b n_g - 2r_b \frac{n_g^{\text{eq}}}{(n_q^{\text{eq}})^2} n_q^2$$



- Slow chemical equilibration = late production of quarks



Photons are emitted from quarks *after* sizable elliptic flow has been developed



Increased photon v_2 ; reduces the “large photon v_2 ” problem

In expanding geometry

■ Expansion effects

- ▶ Effects of expanding systems is implemented in drift term

$$\frac{\partial f}{\partial t} + \frac{p_z}{E} \frac{\partial f}{\partial z} = \frac{\partial f}{\partial t} - \frac{p_z}{t} \frac{\partial f}{\partial p_z} = \mathcal{C}_{\text{sp}} + \mathcal{C}_{\text{rc}} + \mathcal{C}_{\text{FP}}$$

in boost-invariant geometry

The distribution would be indirectly affected even at $p_z = 0$ because the equilibrium distribution changes

The end

- Thank you for listening!
- Website: <http://tkynt2.phys.s.u-tokyo.ac.jp/~monnai/>