Comparison between two-quark, tetra-quark and molecular states of the sigma meson from lattice QCD

SCALAR Collaboration

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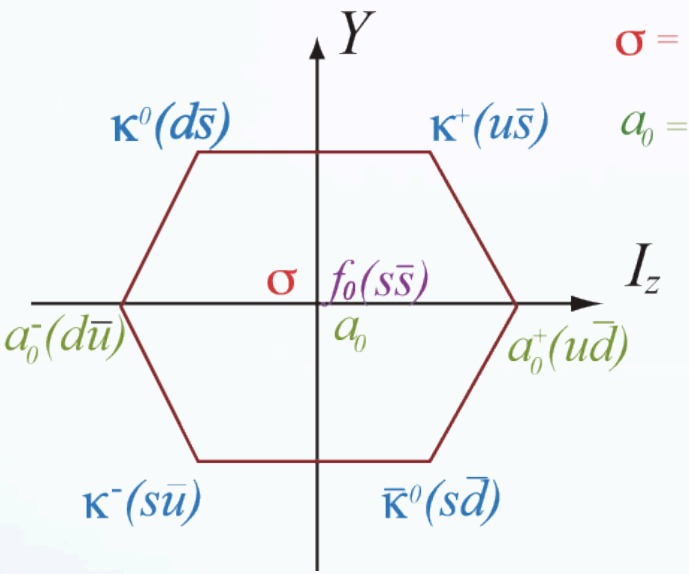
Motivation

- In the conventional $\bar{q}q$ quark model,
  the sigma meson is one of the scalar ($J^P = 0^+$) octet.



$$\sigma = \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$$

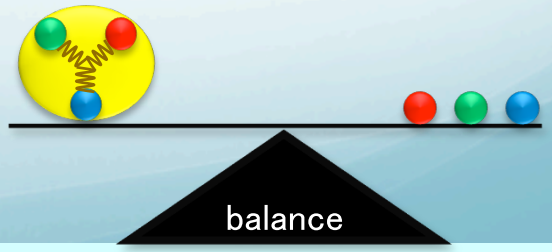
$$a_0 = \frac{u\bar{u}-d\bar{d}}{\sqrt{2}}$$

<u>Particle Data Group (2012)</u>

σ or $f_0(500)$:

I=0, mass = 400 - 550 MeV

- Since the sigma meson is considered as the chiral partner of pion,
  it seems to be related to the mechanism of hadron mass generation.



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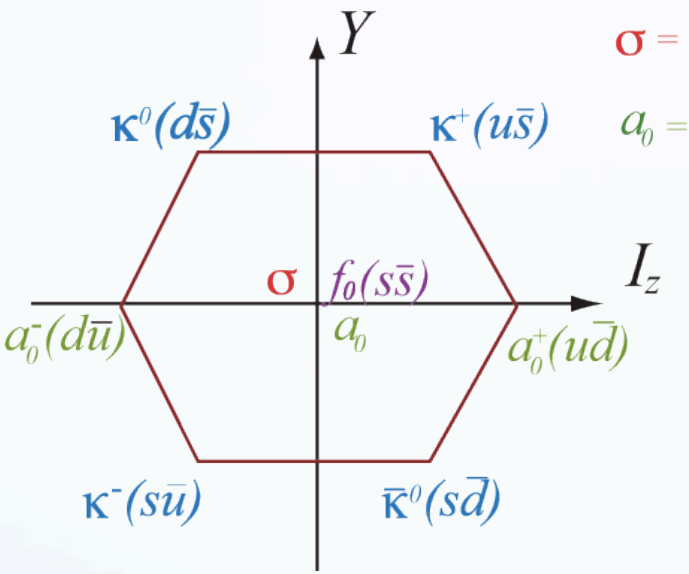

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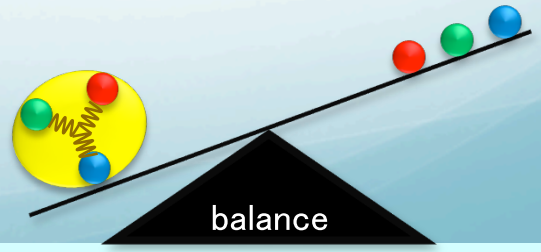
<u>Particle Data Group (2012)</u>

σ or $f_0(500)$:

I=0, mass = 400 - 550 MeV

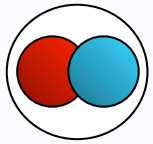
The mass of the sigma meson is too light !

- Since the sigma meson is considered as the chiral partner of pion, it seems to be related to the mechanism of hadron mass generation.



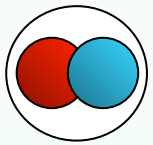
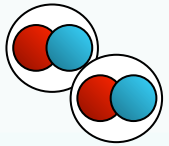
Structure of the sigma meson?

Masses of almost all mesons can be explained by the conventional $\bar{q}q$ quark model, $c_0 \sim 1$.

$\left|\begin{matrix}\text{Physical}\\ \text{state}\end{matrix}\right\rangle \sim c_0 \Big|$  $\Big\rangle$

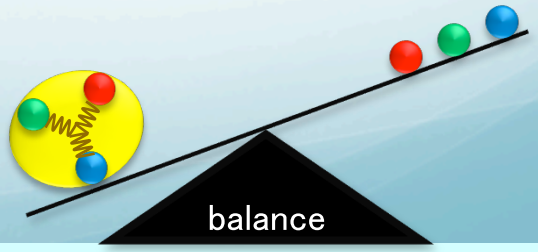
Masses of the scalar mesons can not be explained by the conventional $\bar{q}q$ quark model, $c_0 \nsim 1$. This suggests an importance of the four-quark states and glueball states in the scalar mesons.

$$\left(\sum_i c_i^2 = 1\right)$$

$\left|\begin{matrix}\text{Physical}\\ \text{state}\end{matrix}\right\rangle = c_0 \Big|$  $\Big\rangle + c_1 \Big|$ $\Big\rangle + c_2 \Big|$  $\Big\rangle + c_3 \Big|$

$\Big\rangle + \cdots$

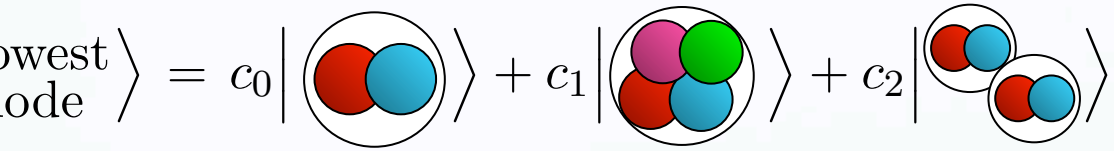
The study for the structure of the sigma mesons is important in understanding the hadron mass generation.



Structure of the sigma meson?

Our goal is to understand the structure of the sigma meson using lattice QCD.

Neglect the glueball state. There is a suggestion from lattice QCD that the glueball state is heavy. PRD73, 014516(2006)

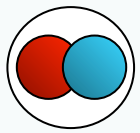


$$\left|\begin{matrix}\text{Lowest}\\ \text{mode}\end{matrix}\right\rangle = c_0\left|\quad\right\rangle + c_1\left|\quad\right\rangle + c_2\left|\quad\right\rangle \qquad \left(\sum_{i=0}^{2} c_i^2 = 1\right)$$

(※ two-flavor)

two-quark state ?

$$\mathbf{1}_{\bar{q}\otimes q} = \mathbf{1}$$

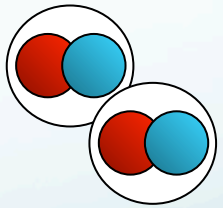


tetra-quark state ?

$$\bar{\mathbf{3}}_{q\otimes q} \otimes \mathbf{3}_{\bar{q}\otimes\bar{q}} = \mathbf{1} \oplus \mathbf{8}$$

molecular state ?

$$\mathbf{1}_{\bar{q}\otimes q} \otimes \mathbf{1}_{\bar{q}\otimes q} = \mathbf{1}$$



Previous works for the scalar mesons from full lattice QCD

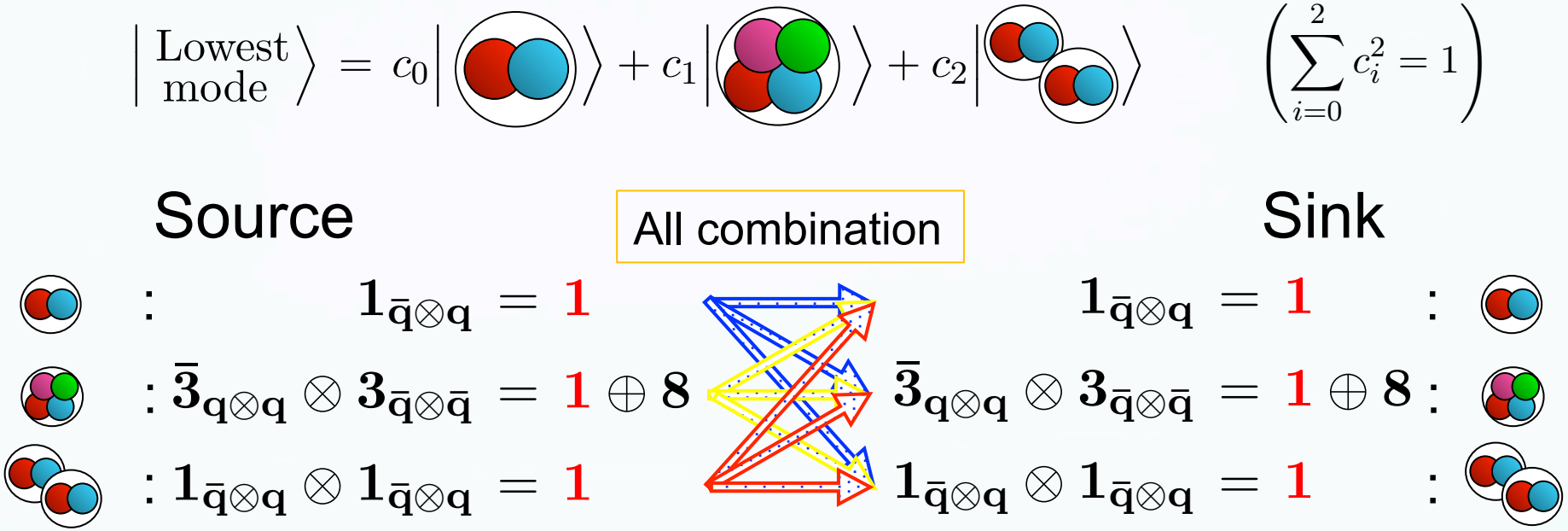
$\left|\begin{matrix}\text{Physical}\\ \text{state}\end{matrix}\right\rangle = c_0\left|\right\rangle + c_1\left|\right\rangle + c_2\left|\right\rangle + c_3\left|\text{Glue ball}\right\rangle + \cdots$

- for σ ..... SCALAR Collaboration, PRD70 (2004) 034504
- for κ ..... SCALAR Collaboration, PLB652 (2007) 250
- for σ, κ, a_0 ..... UKQCD Collaboration, PRD74 (2006) 114504; UKQCD Collaboration, PRD74 (2006) 014508
- for σ, κ, a_0 ..... S. Prelovsek *et al*, PRD79 (2009) 014503; S. Prelovsek *et al*, PRD82 (2010) 094507
- for κ, a_0 ..... BGR Collaboration, PRD85 (2012) 034508
- and for κ, a_0 ... ETM Collaboration, JHEP 1304 (2013) 137

There are no study considered , , and these mixing states at the same time.

Analysis method

$$\left|\begin{matrix}\text{Lowest}\\ \text{mode}\end{matrix}\right\rangle = c_0 \left|\right\rangle + c_1 \left|\right\rangle + c_2 \left|\right\rangle \qquad \left(\sum_{i=0}^{2} c_i^2 = 1\right)$$



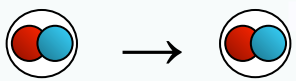
Sigma meson propagators

$$G(t) = \begin{pmatrix} G^{\text{two-two}}(t) & G^{\text{two-tetra}}(t) & G^{\text{two-molec}}(t) \\ G^{\text{tetra-two}}(t) & G^{\text{tetra-tetra}}(t) & G^{\text{tetra-molec}}(t) \\ G^{\text{molec-two}}(t) & G^{\text{molec-tetra}}(t) & G^{\text{molec-molec}}(t) \end{pmatrix}$$

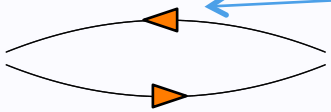
We evaluate the coefficients c_0, c_1 and c_2
for the lowest mode with **the variational method**.

Diagonal elements

$$G^{\text{two-two}}(t) = -G^{\text{conn}}(t) + 2G^{\text{disc}}(t)$$



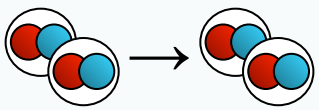
$G^{\text{conn}}(t)$



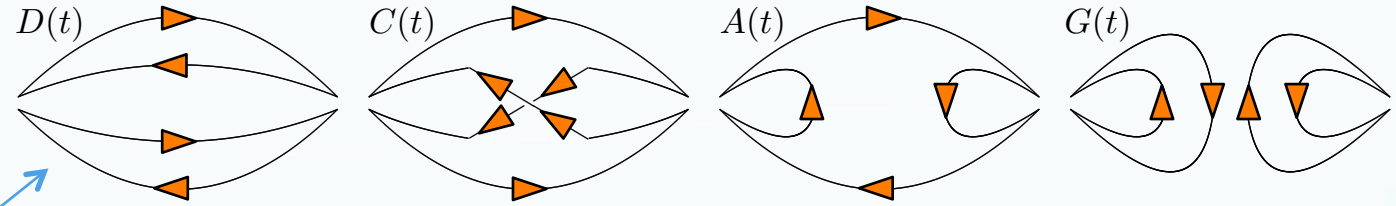
The arrows mean the quark lines.

$G^{\text{disc}}(t)$

$$G^{\text{molec-molec}}(t) = 2\left[D(t) + \frac{1}{2}C(t) - 3A(t) + \frac{3}{2}G(t)\right]$$



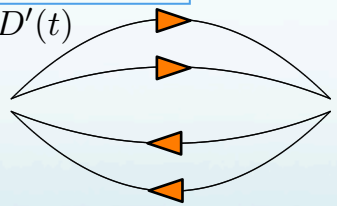
$D(t)$ $C(t)$ $A(t)$ $G(t)$



$$G^{\text{tetra-tetra}}(t) = 2\left(D_1'(t) + D_2'(t)\right) - 2\left(A_1'(t) + A_2'(t) + A_3'(t) + A_4'(t)\right) + \left(G_1'(t) + G_2'(t) + G_3'(t) + G_4'(t)\right)$$

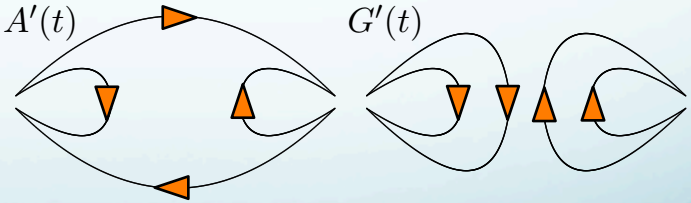
D and D' diagrams are difference of quark line contraction.

$D'(t)$



Indices mean the different of color combination.

$A'(t)$ $G'(t)$



Diagonal elements

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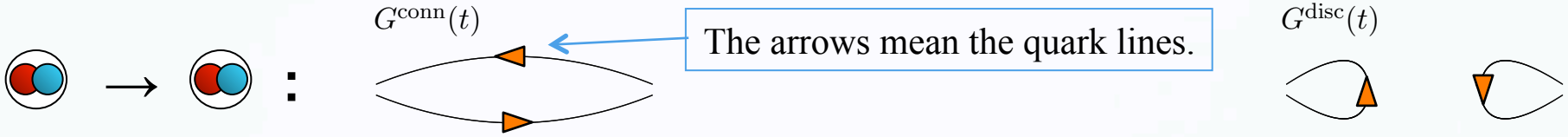

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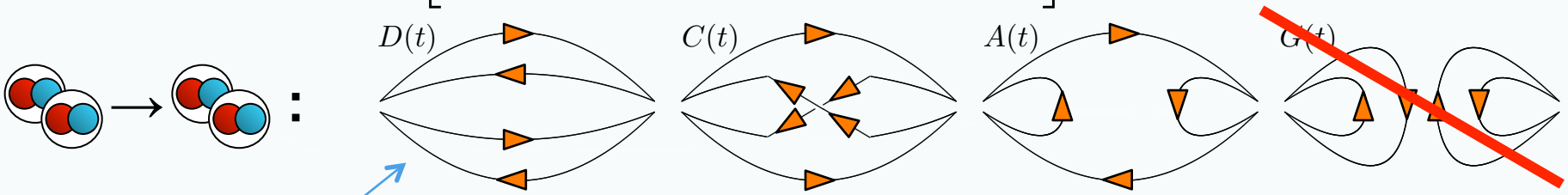
$$G^{\text{molec-molec}}(t) = 2\left[D(t) + \frac{1}{2}C(t) - 3A(t) + \cancel{\frac{3}{2}G(t)}\right]$$



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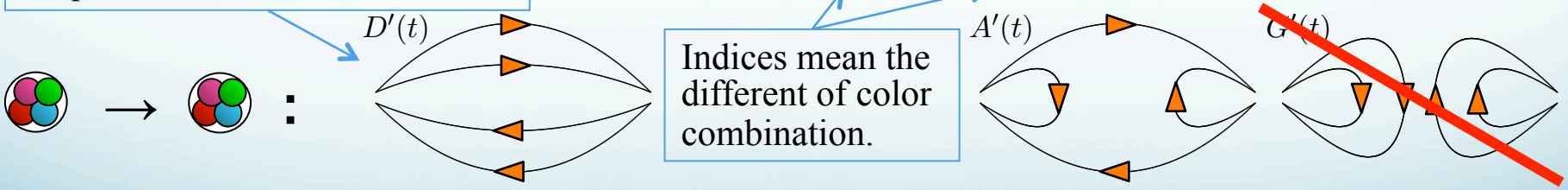

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We neglect the doubly disconnected diagrams, because in PRD 88, 074506 (2013), they suggest that the contribution of the doubly disconnected diagrams are N_c order smaller than one of the singly disconnected diagrams.

Order estimation in large N_c limit

One loop of the color line contributes by the power of N_c.

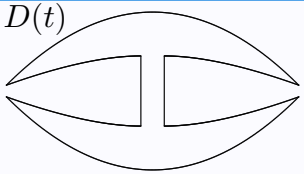
One gluon coupling contributes by the power of $1/\sqrt{N_c}$.

Replacement with color lines

One quark line
↓
One color line

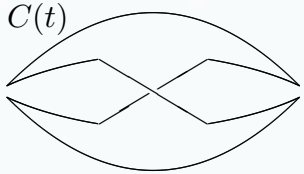
One gluon line
↓
Two color lines

$D(t)$



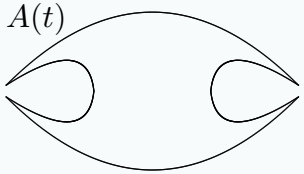
$$\sim \mathcal{O}\left((N_c)^{\mathbf{1}} \times \left(\frac{1}{\sqrt{N_c}}\right)^{\mathbf{2}}\right) \sim \mathcal{O}(1)$$

$C(t)$



$$\sim \mathcal{O}\left((N_c)^{\mathbf{1}} \times \left(\frac{1}{\sqrt{N_c}}\right)^{\mathbf{0}}\right) \sim \mathcal{O}(N_c)$$

$A(t)$



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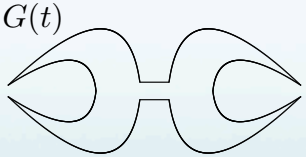
$G(t)$



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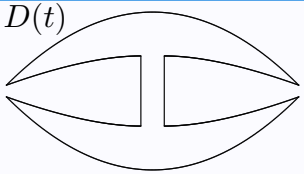
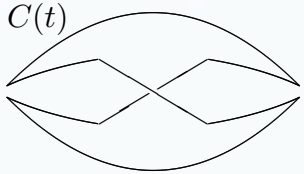


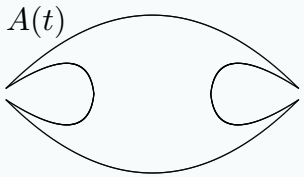
$$\sim \mathcal{O}\left((N_c)^1 \times \left(\frac{1}{\sqrt{N_c}}\right)^2\right) \sim \mathcal{O}(1)$$

$C(t)$



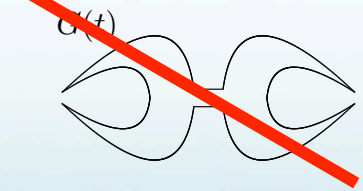
$$\sim \mathcal{O}\left((N_c)^1 \times \left(\frac{1}{\sqrt{N_c}}\right)^0\right) \sim \mathcal{O}(N_c)$$

$A(t)$



$$\sim \mathcal{O}\left((N_c)^1 \times \left(\frac{1}{\sqrt{N_c}}\right)^0\right) \sim \mathcal{O}(N_c)$$

$G(t)$



$$\sim \mathcal{O}\left((N_c)^1 \times \left(\frac{1}{\sqrt{N_c}}\right)^2\right) \sim \mathcal{O}(1)$$

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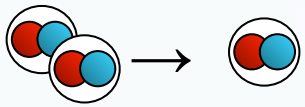
Off-diagonal elements

$$G^{\mathrm{molec-two}}(t) = \sqrt{6}\left[-\hat{A}(t) + \hat{G}(t)\right]$$



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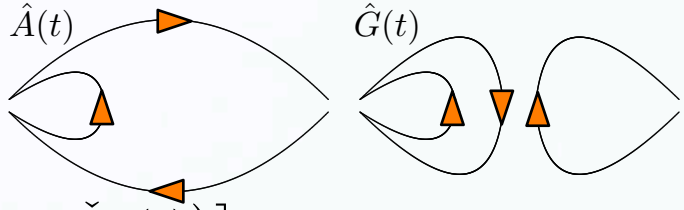
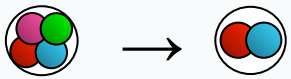


$$G^{\mathrm{tetra-two}}(t) = \frac{4}{\sqrt{6}}\left[-\left(\check{A}_1(t) + \check{A}_2(t)\right) + \left(\check{G}_1(t) + \check{G}_2(t)\right)\right]$$



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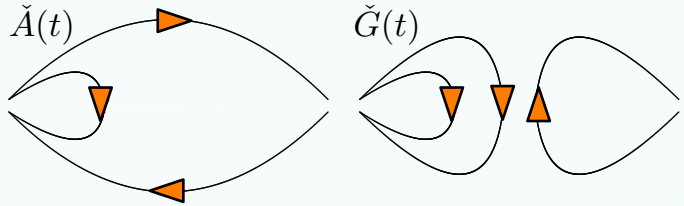
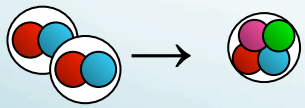


$$G^{\mathrm{molec-tetra}}(t) = \frac{1}{\sqrt{3}}\left[-3\left(\tilde{C}_1(t) + \tilde{C}_2(t)\right) + 5\left(\tilde{A}_1(t) + \tilde{A}_2(t)\right) - 3\left(\tilde{G}_1(t) + \tilde{G}_2(t)\right)\right]$$



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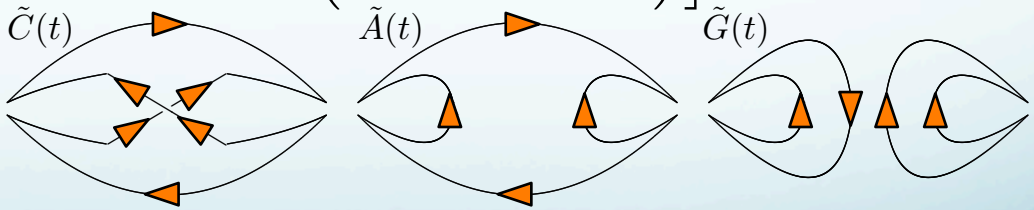

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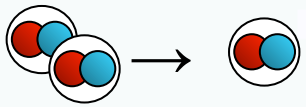

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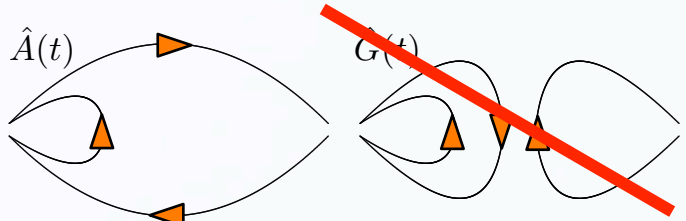
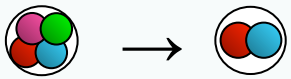


$$G^{\mathrm{tetra-two}}(t) = \frac{4}{\sqrt{6}}\left[-\left(\check{A}_1(t) + \check{A}_2(t)\right) + \cancel{\left(\check{G}_1(t) + \check{G}_2(t)\right)}\right]$$



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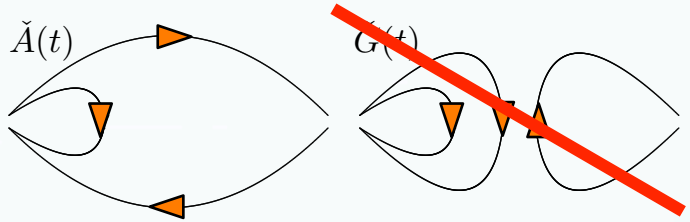
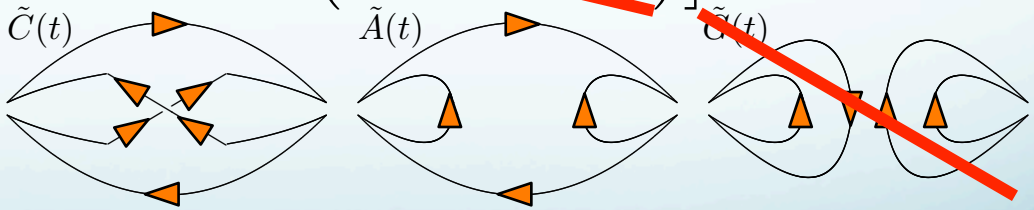


$$G^{\mathrm{molec-tetra}}(t) = \frac{1}{\sqrt{3}}\left[-3\left(\tilde{C}_1(t) + \tilde{C}_2(t)\right) + 5\left(\tilde{A}_1(t) + \tilde{A}_2(t)\right) - 3\cancel{\left(\tilde{G}_1(t) + \tilde{G}_2(t)\right)}\right]$$

:



We neglect the doubly disconnected diagrams, because in PRD 88, 074506 (2013), they suggest that the contribution of the doubly disconnected diagrams are N_c order smaller than one of the singly disconnected diagrams.

Simulation parameters

Gauge configuration

Two-flavor full QCD configurations

- Wilson gauge action
- Clover fermion action

$\text{Lattice size} = 8^3 \times 16$ $\quad \beta = 1.7 \quad C_{sw} = 1.68$

$\kappa = 0.146, 0.147, 0.148$ $\quad a = 0.253(3)$ [fm]

$m_\pi = 791(11),\ 725(9),\ 646(10)$ [MeV] Heavy quark mass

$m_\rho = 1116(17), 1059(17), 1017(20)$ [MeV]

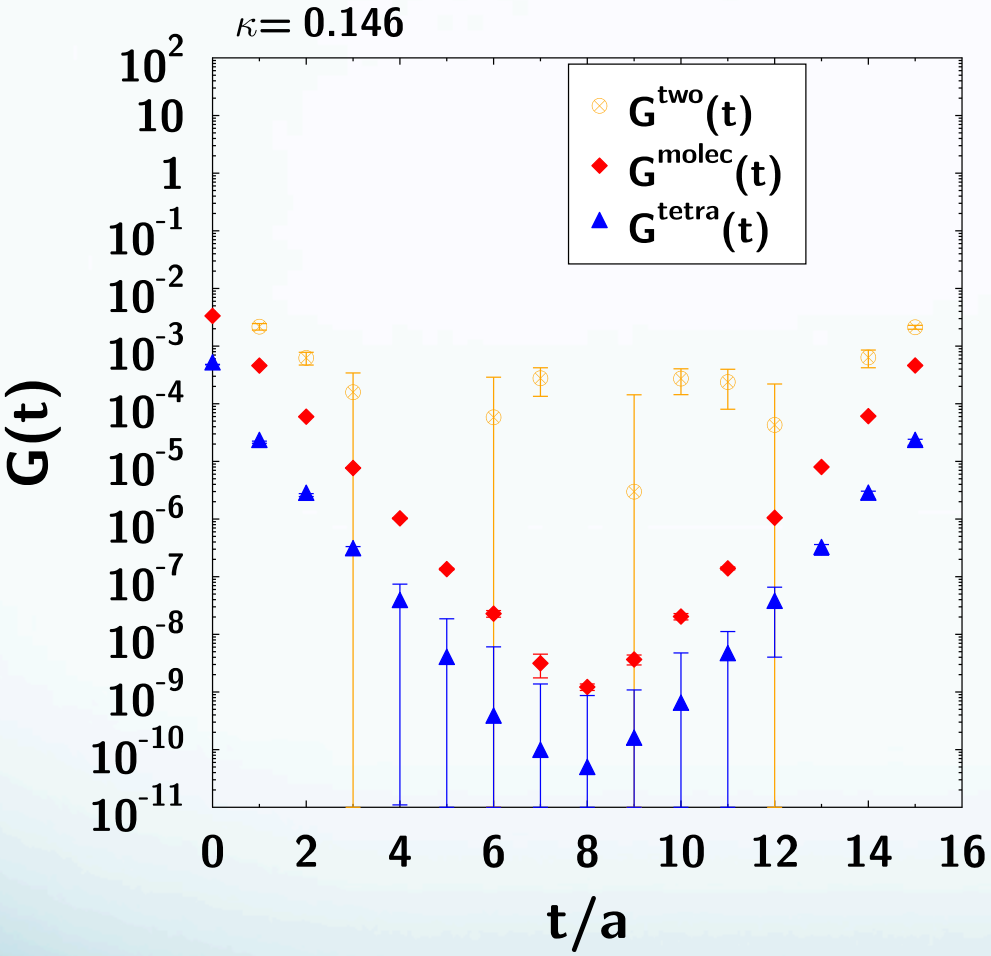
The calculation has enormous significance in establishment of a technique, heavy quark physics of search for four-quark states and contribution to quark mass dependence for the scalar mesons.

Quark Propagator

- Clover fermion action
- Z_2 noise method with truncated eigenmode approach for disconnected diagrams

$\#$ of Noise $= 120 \times 16$ $\quad \#$ of Eigenvector $= 12$

Propagator results before variational method



Preliminary!

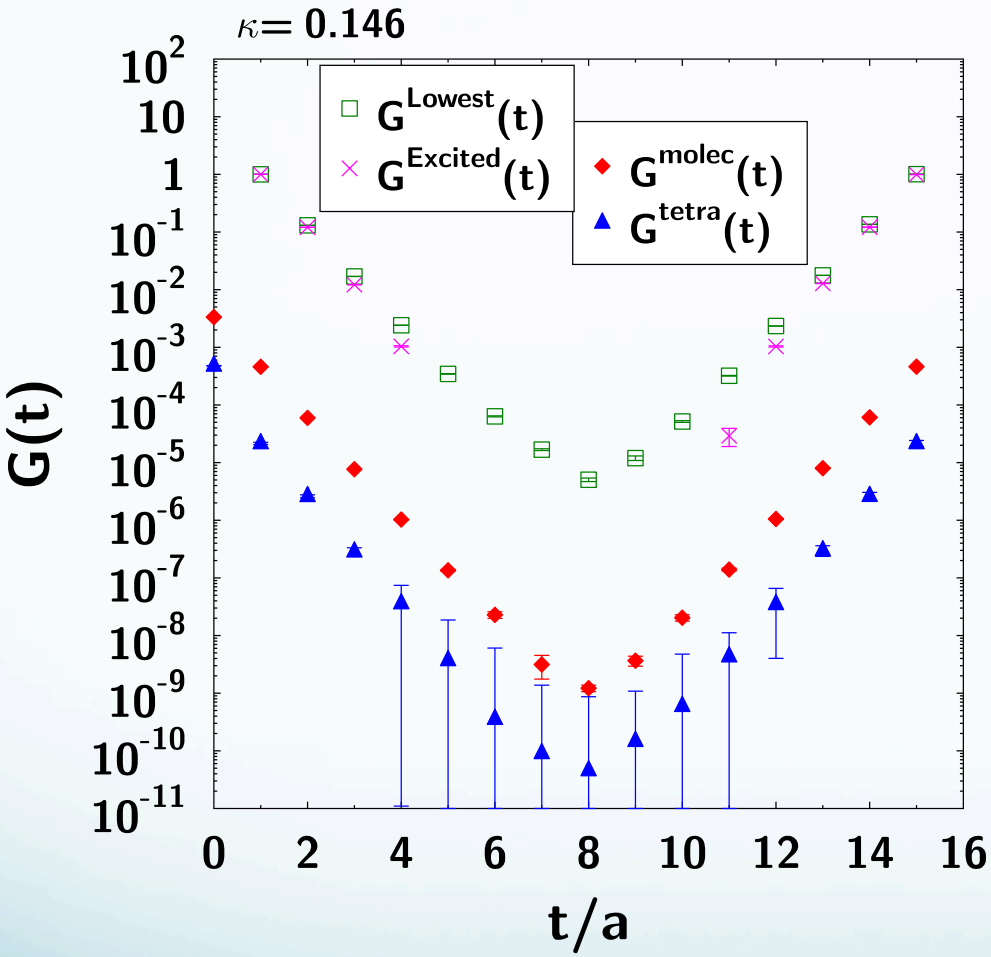
# of Conf. = 12888

We obtain the clear signals for the molecular and tetra-quark propagators in the heavy quark sector.

The two-quark propagator has large error bars which come from the disconnected diagram.

We perform the variational method for the molecular and tetra-quark propagators.

Propagator results after variational method



Preliminary!

# of Conf. = 12888

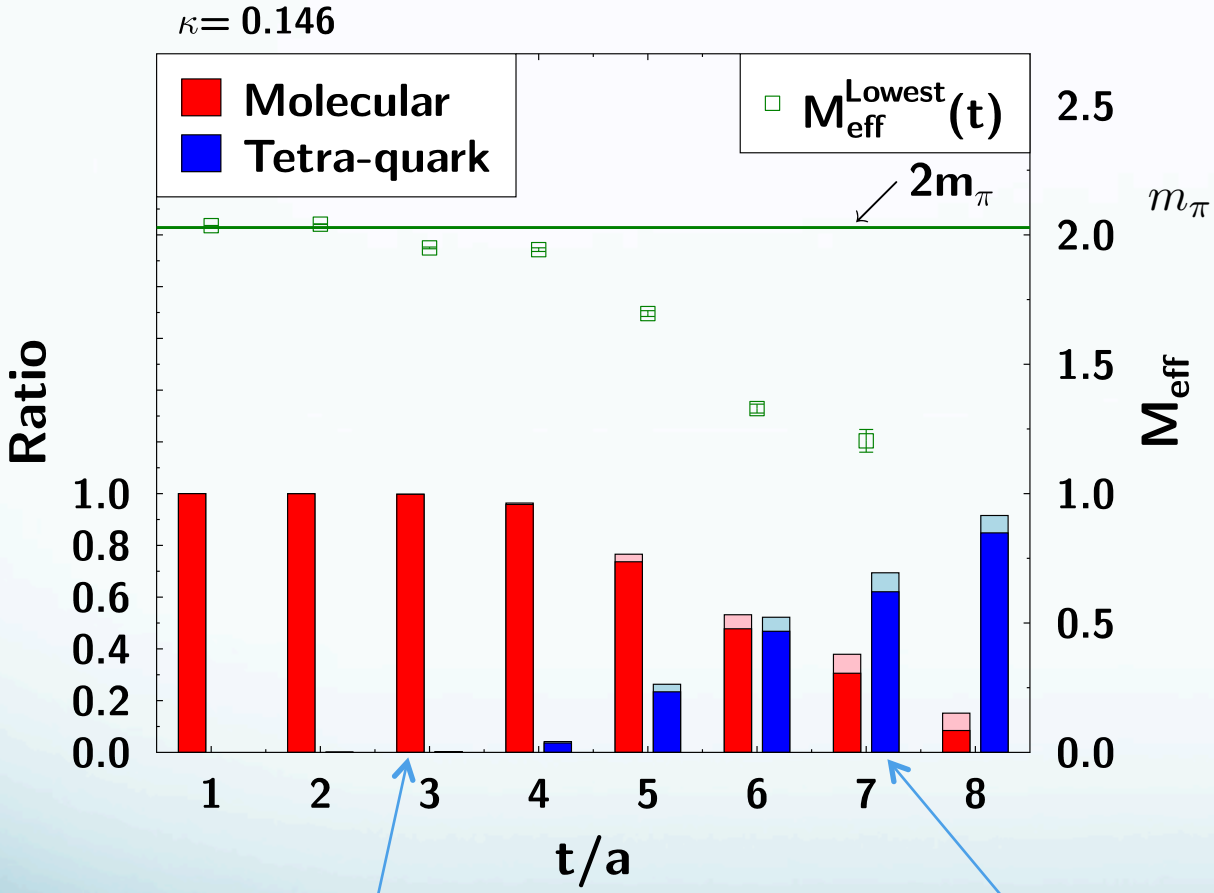
We obtain the clear lowest mode after the variational method.

Ratio between molecular and tetra-quark states and effective mass of the lowest mode

Preliminary!

# of Conf. = 12888

$m_\pi = 791(11)$ [MeV]



In the initial time region, the molecular state is dominant in the lowest mode.

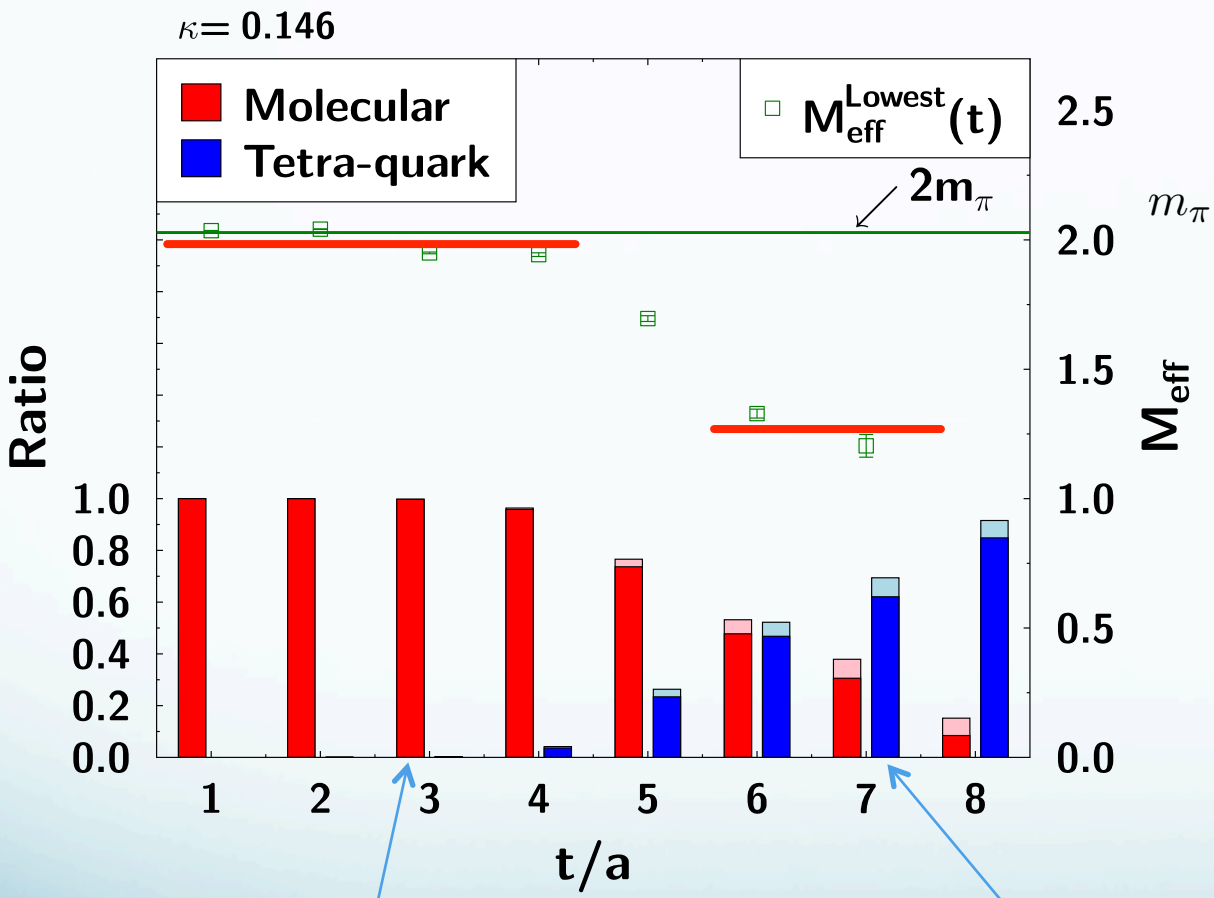
The ratio of the tetra-quark state increases with the increasing time.

Ratio between molecular and tetra-quark states and effective mass of the lowest mode

Preliminary!

# of Conf. = 12888

$m_\pi = 791(11)$ [MeV]



The effective mass of the lowest mode might have two plateaus, the first one consists of the molecular state and the second one is dominant of the tetra-quark state.

In the initial time region, the molecular state is dominant in the lowest mode.

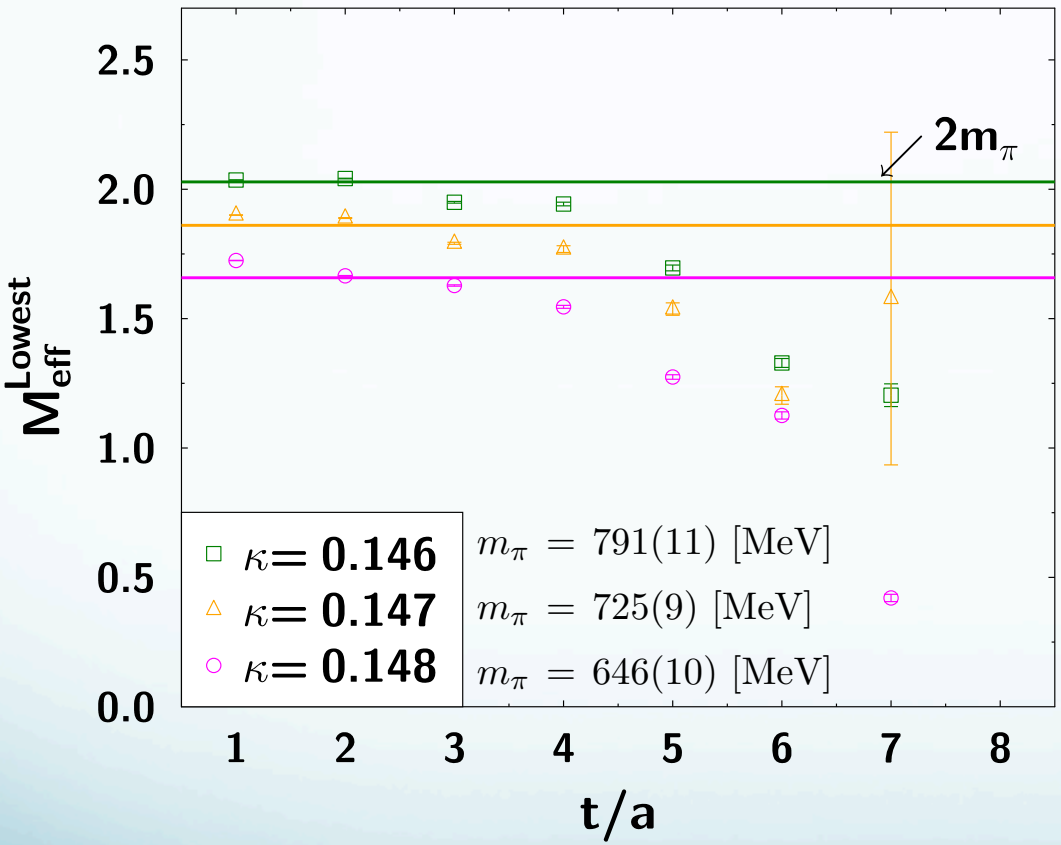
The ratio of the tetra-quark state increases with the increasing time.

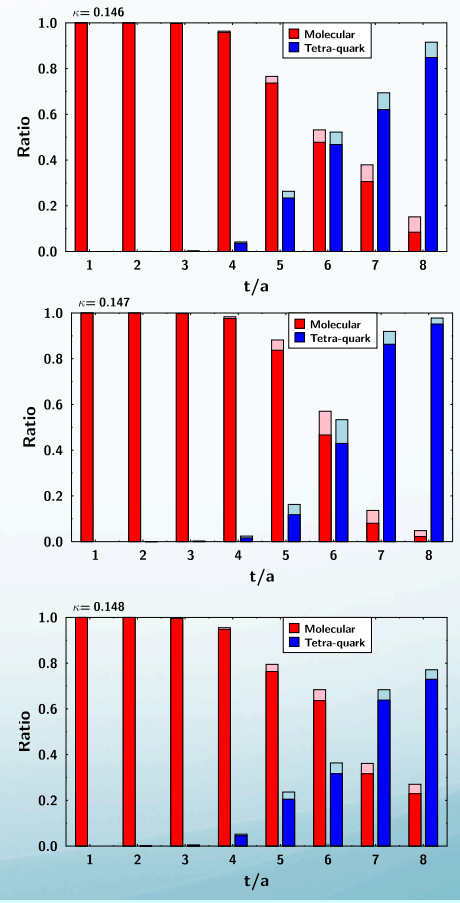
Effective masses of the lowest modes

The results are the effective masses for the three different quark masses.

Preliminary!

# of Conf. = 12888
# of Conf. = 10840
# of Conf. = 7234





We can obtain the first plateaus for three systems although the second ones are ambiguous.

Summary

We study the structure of the scalar meson;
two-quark, molecular, tetra-quark or these mixing state?

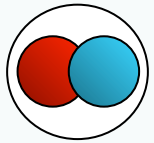
- We calculate two-quark, molecular and tetra-quark propagators in heavy quark systems.
- We perform the variational method for molecular and tetra-quark propagators.

- The ratio of the molecular and the tetra-quark states in the lowest mode changes with time-dependence.
- The effective mass of the lowest mode might have two plateaus for the molecular state and the tetra-quark states.

- The two plateaus are needed to more check in a lager lattice size.
- The calculation is applicable to heavy quark (charm or bottom) physics of search for four-quark states by tuning parameters.

Two-quark operator

$$\mathbf{1}_{\bar{\mathbf{q}}\otimes\mathbf{q}} = 1$$



- two-quark operator for σ meson

$$\mathcal{O}_S^{\mathrm{two}}(t) = \frac{1}{\sqrt{2}} \sum_{\mathbf{x},\mathbf{y}\, a,b\, \alpha} \Big[\bar{u}_\alpha^a(t,\mathbf{x}) S_t^{ab}(\mathbf{x},\mathbf{y}) u_\alpha^b(t,\mathbf{y}) + \bar{d}_\alpha^a(t,\mathbf{x}) S_t^{ab}(\mathbf{x},\mathbf{y}) d_\alpha^b(t,\mathbf{y}) \Big]$$

$S_t^{ab}(\mathbf{x},\mathbf{y})$: Smearing function at timeslice t

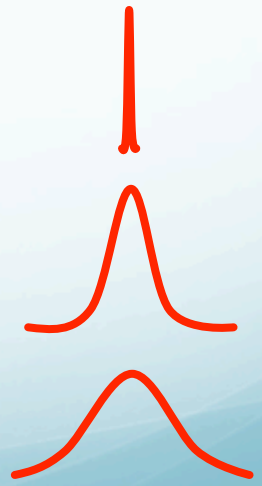
T.Burch *et al*, Phys. Rev. D73 (2006) 094505

$S = 1 \Rightarrow$ Point Source

$S = 2 \Rightarrow$ Narrow Source

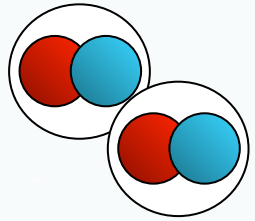
$S = 3 \Rightarrow$ Wide Source

(Gaussian shaped source)



Molecular operator

$$\mathbf{1}_{\bar{\mathbf{q}}\otimes\mathbf{q}} \otimes \mathbf{1}_{\bar{\mathbf{q}}\otimes\mathbf{q}} = \mathbf{1}$$



- pion operator

$$\mathcal{O}_S^{\pi^+}(t) = -\sum_{\mathbf{x},\mathbf{y}\,a,b} \bar{d}^a(t,\mathbf{x})\gamma_5 S_t^{ab}(\mathbf{x},\mathbf{y})u^b(t,\mathbf{y})$$

$$\mathcal{O}_S^{\pi^-}(t) = \sum_{\mathbf{x},\mathbf{y}\,a,b} \bar{u}^a(t,\mathbf{x})\gamma_5 S_t^{ab}(\mathbf{x},\mathbf{y})d^b(t,\mathbf{y})$$

$$\mathcal{O}_S^{\pi^0}(t) = \frac{1}{\sqrt{2}}\sum_{\mathbf{x},\mathbf{y}\,a,b} \Big[\bar{u}^a(t,\mathbf{x})\gamma_5 S_t^{ab}(\mathbf{x},\mathbf{y})u^b(t,\mathbf{y}) - \bar{d}^a(t,\mathbf{x})\gamma_5 S_t^{ab}(\mathbf{x},\mathbf{y})d^b(t,\mathbf{y})\Big]$$

- molecular operator for σ meson

$$\mathcal{O}_S^{\mathrm{molec}}(t) = \frac{1}{\sqrt{3}}\Big[\mathcal{O}_S^{\pi^+}(t)\mathcal{O}_S^{\pi^-}(t) - \mathcal{O}_S^{\pi^0}(t)\mathcal{O}_S^{\pi^0}(t) + \mathcal{O}_S^{\pi^-}(t)\mathcal{O}_S^{\pi^+}(t)\Big]$$

Tetra-quark operator

$$\bar{\mathbf{3}}_{\mathbf{q}\otimes\mathbf{q}} \otimes \mathbf{3}_{\bar{\mathbf{q}}\otimes\bar{\mathbf{q}}} = \mathbf{1} \oplus \mathbf{8}$$

- diquark operator

$$[ud]^a_S(t) = \frac{1}{2}\sum_{\mathbf{x},\mathbf{y}\, b,c,d} \epsilon^{abc}\Big[u^{Tb}(t,\mathbf{x})\; C\gamma_5 S_t^{cd}(\mathbf{x},\mathbf{y})\; d^d(t,\mathbf{y}) - d^{Tb}(t,\mathbf{x})\; C\gamma_5 S_t^{cd}(\mathbf{x},\mathbf{y})\; u^d(t,\mathbf{y})\Big]$$

C : Charge conjugate matrix

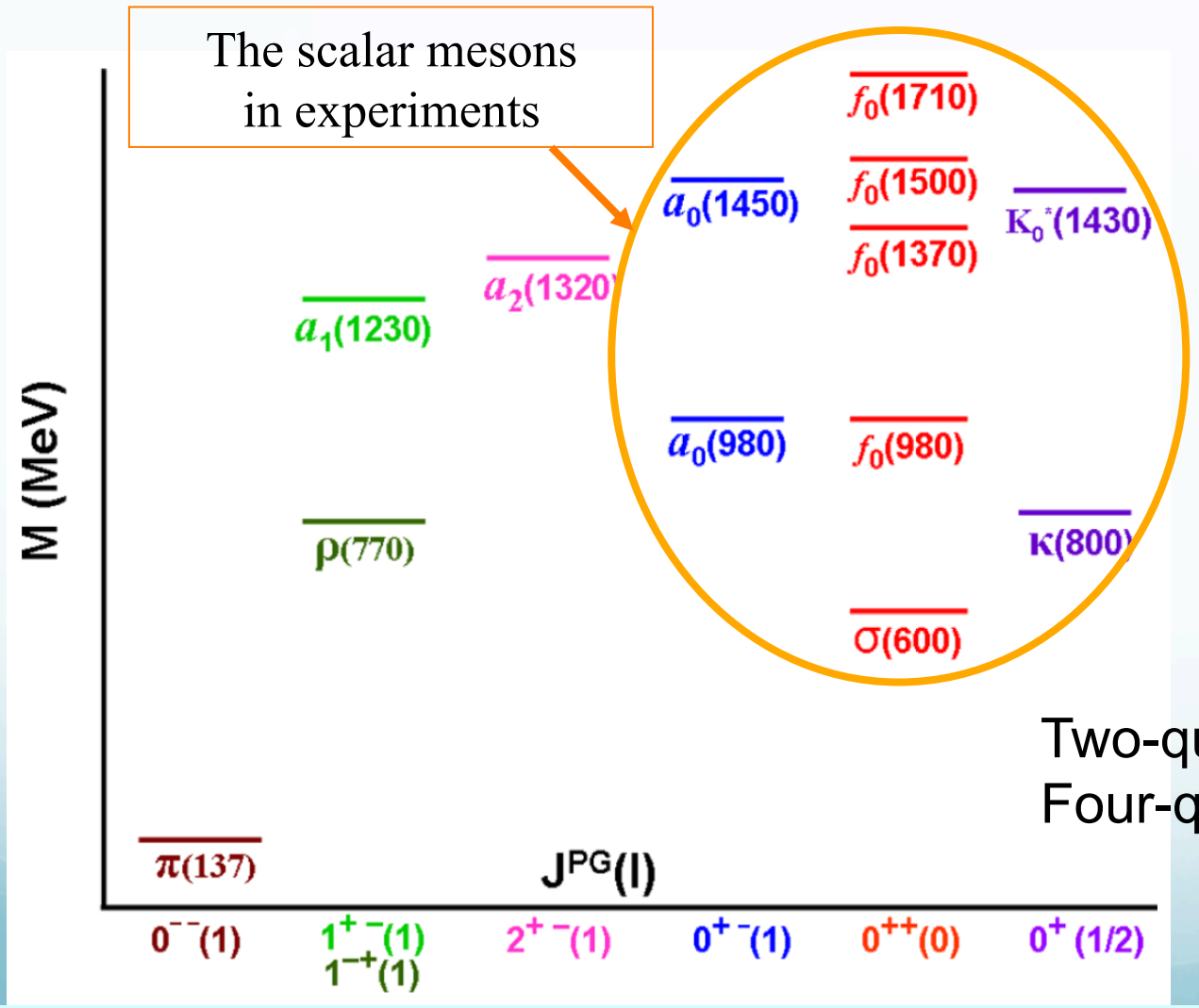
- anti-diquark operator

$$[\bar{u}\bar{d}]^a_S(t) = \frac{1}{2}\sum_{\mathbf{x},\mathbf{y}\, b,c,d} \epsilon^{abc}\Big[\bar{u}^b(t,\mathbf{x})\; C\gamma_5 S_t^{cd}(\mathbf{x},\mathbf{y})\; \bar{d}^{Td}(t,\mathbf{y}) - \bar{d}^b(t,\mathbf{x})\; C\gamma_5 S_t^{cd}(\mathbf{x},\mathbf{y})\; \bar{u}^{Td}(t,\mathbf{y})\Big]$$

- tetra-quark operator for σ meson

$$\mathcal{O}_S^{\mathrm{tetra}}(t) = \sum_a [ud]^a_S(t)[\bar{u}\bar{d}]^a_S(t)$$

Experiment data



J : Total angular momentum

P : Parity

C : C-Parity

G : G-Parity

I : Isospin

Two-quark : $P=(+1)(-1)(-1)^L$

Four-quark : $P=(+1)^2(-1)^2(-1)^L$