Isospin Effects by Mass Reweighting

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Mass Reweighting

Why ?

- Tuning of quark masses,
  e.g., m_s in a 2+1 simulation, isospin splitting, ...
- Quark mass dependence

$\rightarrow$ for small corrections: applicable and cheaper than new simulation

How ?

- rewrite determinant: using pseudofermion integral
- using stochastic estimation

Observable:

$$\langle O \rangle_W = \langle OW \rangle / \langle W \rangle = \langle O\tilde{W} \rangle$$

with the corrections introduced by the mass reweighting factor of n_f-flavors $W = \prod_{i=1}^{n_f} [\det D(m_{new,i}) / \det D(m_{old,i})]$ and normalized factor $\tilde{W} = W / \langle W \rangle$

One-flavor integral:

$$\frac{1}{\det M} = \int \mathrm{D}[\eta] \exp\{-\eta^\dagger M \eta\} \rightarrow \frac{1}{N_\eta} \sum_{i=1}^{N_\eta} e^{-\eta_i^\dagger (M-1) \eta_i}$$

holds for $\lambda(M + M^\dagger) > 0$

[J.F., Knechtli, Leder (2013)]

Outline

Ensembles : CLS - $n_f = 2$ - Wilson $\mathcal{O}(a)$ improv. fermions with $m_{ud} = m_u = m_d$

Name	a [fm]	m_π [MeV]	N_{cnfg}
A5	0.076	330	202
E4	0.066	580	100
D5	"	440	2012
E5	"	440	99
F7	"	270	350
G8	"	190	37
O7	0.049	270	49

[Fritzsch et al. (2012)]

`https://twiki.cern.ch/twiki/bin/view/CLS/WebHome`

- still improving the statistics
$\rightarrow$ *RESULTS are Preliminary*

Goals:

- scaling of the fluctuations
- extract up- and down-quark mass

Isospin Reweighting

In general: lattice simulations are done in the Isospin symmetric limit in the light quark sector

$$m_u = m_{ud} = m_d$$

Idea: using mass reweighting to introduce Isospin breaking effects with $m_u + m_d = const$

$$m_u = m_{ud} - 0.5 \cdot \Delta m_{ud} \leftarrow \quad m_{ud} \quad \rightarrow m_{ud} + 0.5 \cdot \Delta m_{ud} = m_d$$

Isospin reweighting factor:

$$W_{iso} = \frac{\det D(m_u) \det D(m_d)}{\det D(m_{ud})^2}$$

fluctuations:

stochastic fluctuations (expanding in Δm_{ud}):

$$\sigma_{st}^2(N_{inv}) \sim \frac{\Delta m_{ud}^4}{N_{inv}} \operatorname{Tr} \frac{1}{\left(DD^\dagger\right)^2} + \mathcal{O}\left(\Delta m_{ud}^6\right)$$

ensemble fluctuations (expanding in Δm_{ud}) :

$$\sigma_{ens}^2 = \Delta m_{ud}^4 \operatorname{var}\left(\operatorname{Tr}\left[\frac{1}{D^2}\right]\right) + \mathcal{O}\left(\Delta m_{ud}^6\right)$$

Cost given by :

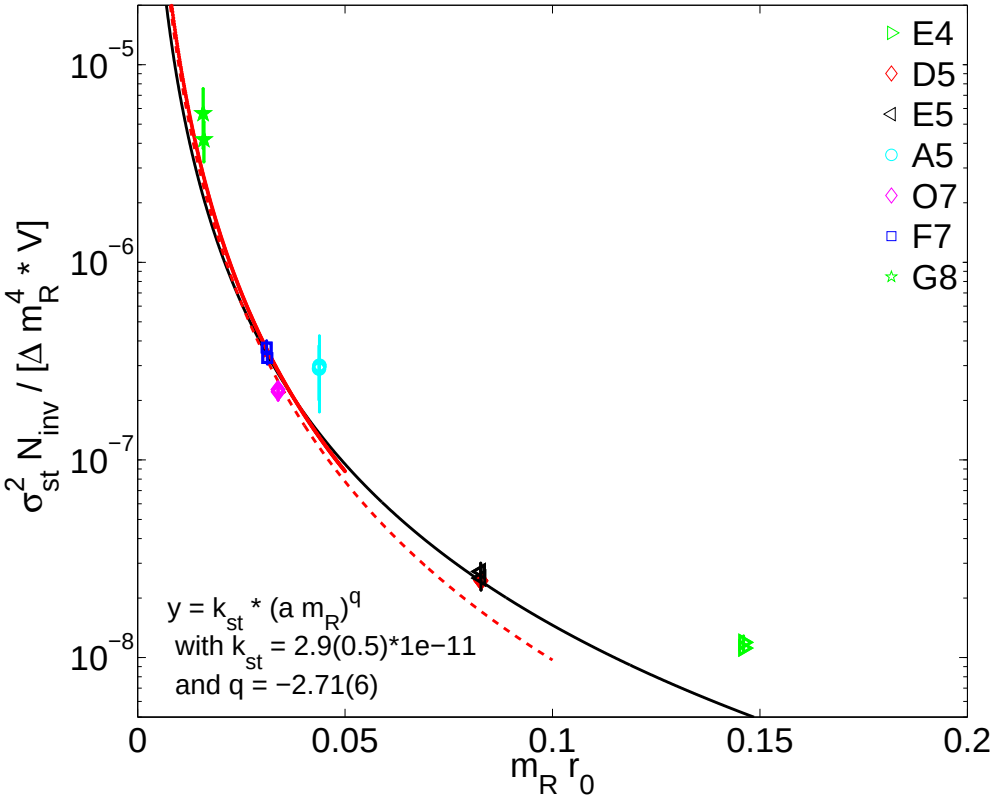
$$\sigma_{st}^2(N_{inv})/\sigma_{ens}^2 \overset{!}{\sim} 10\%$$

Stochatic fluctuations

Stochastic fluctuations (expanding in Δm_{ud}):

$$\sigma_{st}^2(N_{inv}) \sim \frac{\Delta m_{ud}^4}{N_{inv}} \, \mathrm{Tr} \frac{1}{\left(DD^\dagger\right)^2} + \mathcal{O}\left(\Delta m_{ud}^6\right)$$

$\Rightarrow$ chiral perturbation theory: $\mathrm{Tr} \frac{1}{\left(DD^\dagger\right)^2} \propto \frac{\Sigma V}{m_R^3}$



$$\sigma_{st}^2 \approx k_{st} \frac{\Delta m_R^4 V}{N_{inv} m_R^3} \frac{1}{r_0^3}$$

Ensemble fluctuations

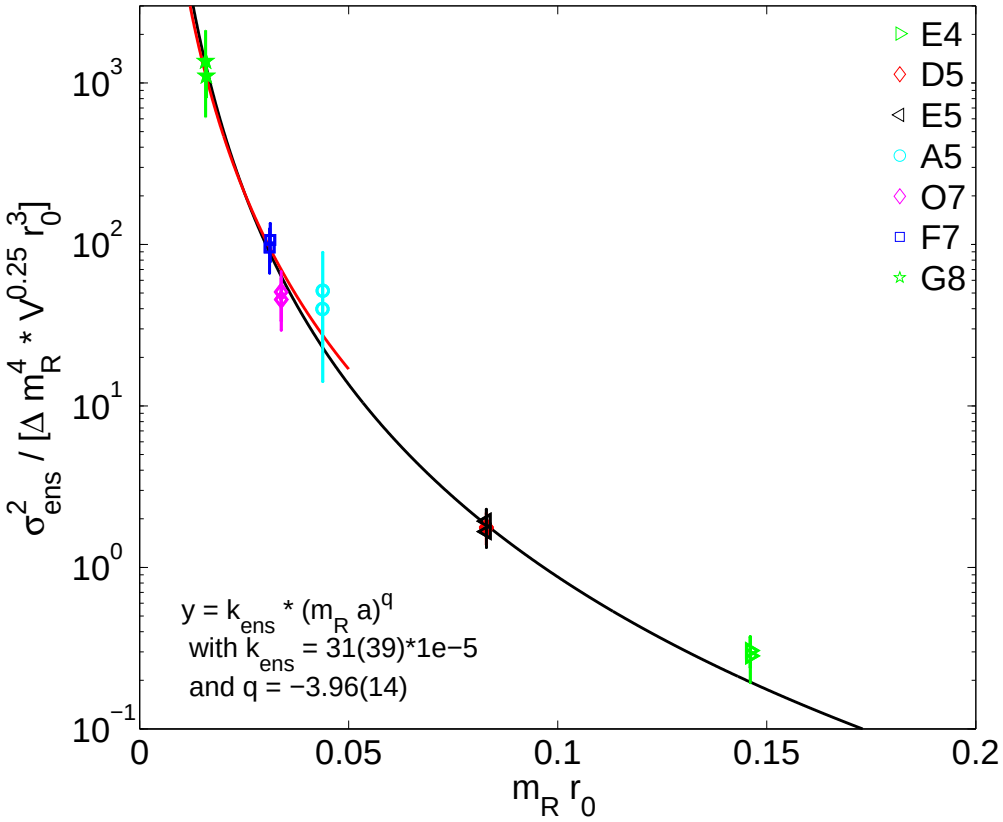
Ensemble fluctuations (expanding in Δm_{ud}) :

$$\sigma^2_{ens} = \Delta m^4_{ud}\ \mathrm{var}\left(\mathrm{Tr}\left[\frac{1}{D^2}\right]\right) + \mathcal{O}\left(\Delta m^6_{ud}\right)$$

$\Rightarrow$ behavior of $\mathrm{var}\left(\mathrm{Tr}\left[\frac{1}{D^2}\right]\right)$ unknown

numerical observe: $\rightarrow$ weak volume dependence V^q with $q < 1$

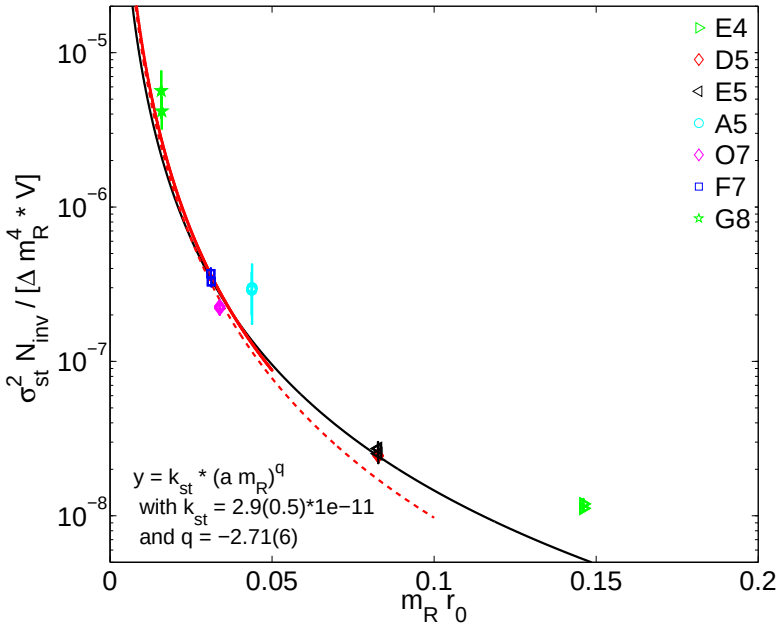
$\Rightarrow$ a good fit : $q = 0.25$



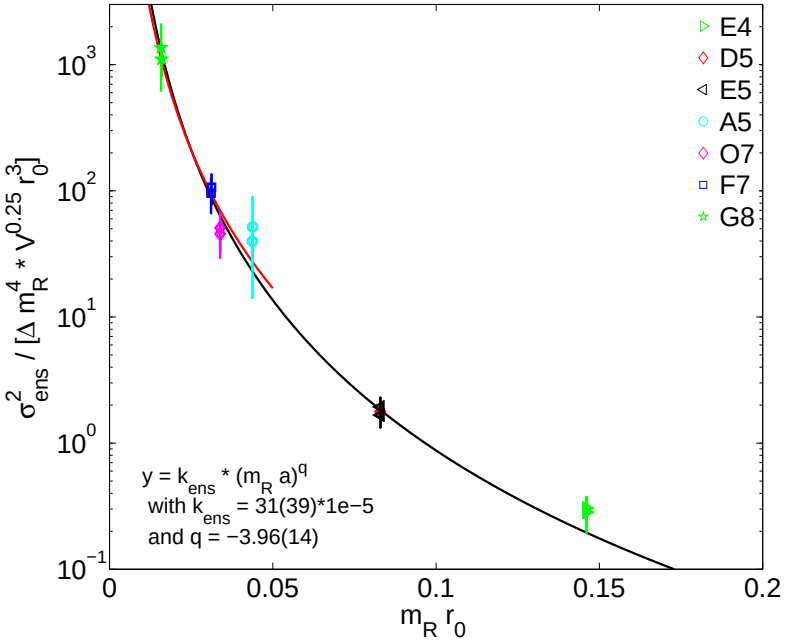
$$\sigma^2_{ens} \approx k_{ens}\,\frac{\Delta m^4_R\,\sqrt[4]{V}}{m^4_R}\,\frac{1}{r_0}$$

Scaling of Fluctuations

Stochatic fluctuations



Ensemble fluctuations



$$\sigma_{st}^2 \approx k_{st} \frac{\Delta m_R^4 V}{N_{inv} m_R^3} \frac{1}{r_0^3}$$

$$\sigma_{ens}^2 \approx k_{ens} \frac{\Delta m_R^4 \sqrt[4]{V}}{m_R^4} \frac{1}{r_0}$$

Cost given by :

$$\sigma_{st}^2(N_{inv})/\sigma_{ens}^2 \sim \frac{k'_{st}}{k'_{ens}} \frac{(LM_{PS})^2 L}{N_{inv} \cdot r_0^2} \qquad \text{with} \quad \frac{k'_{st}}{k'_{ens}} = 1e-3$$

10% @G8 ($m_\pi = 190$ MeV , $a = 0.066$ fm) : $N_{inv} \approx 200$

Fixing the bare mass parameter

Using physical ratios built from meson masses and decay constants to fix the bare mass paramters κ_s, κ_d and κ_u :

$$R_1 = \frac{m_{K^0}^2 + m_{K^\pm}^2}{(f_{K^0} + f_{K^\pm})^2} \quad , \qquad R_2 = \frac{m_{K^0}^2 - m_{K^\pm}^2}{m_{K^0}^2 + m_{K^\pm}^2}$$

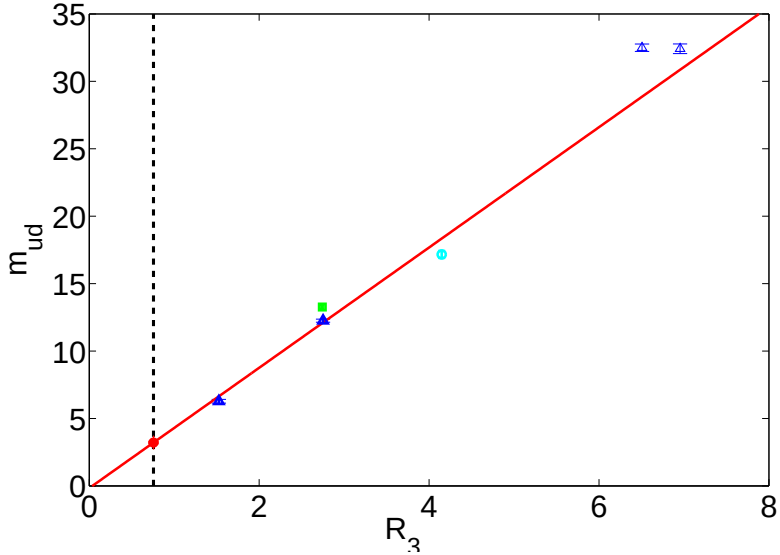
and

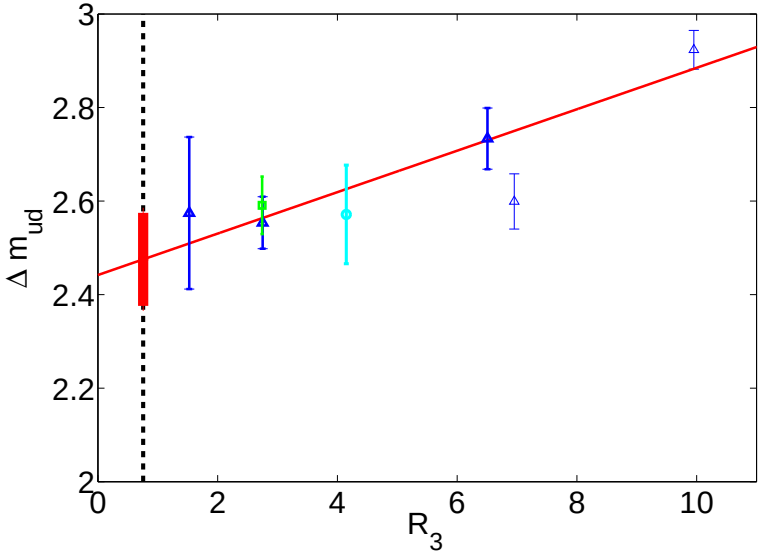
$$R_3 = \frac{m_{\pi^\pm}^2}{(f_{K^0} + f_{K^\pm})^2}$$

Strategie:
Use R_1 and R_2 to fix κ_s and $(\Delta m_{ud})_{bare}$
and extrapolate in R_3 towards physical light quark masses

Quark masses

By fixing R_1 and R_2 and
using the m_{PCAC} mass:





with $m_{ud} = 3.21(14)$ MeV and $\Delta m_{ud} = 2.48(10)$ MeV
(at finite lattice $a = 0.066$ [fm])

by assuming:
$m_{ud}(R_3) \approx a_1 R_3$ and $\Delta m_{ud}(R_3) \approx b_0 + b_1 R_3$
with $R_3 \propto m_{ud}$ in χpt at LO

result:

$\Rightarrow$ relative high precision with low statistics

future:

$\Rightarrow$ increasing statistics and including other ensembles

Conclusion

Results:

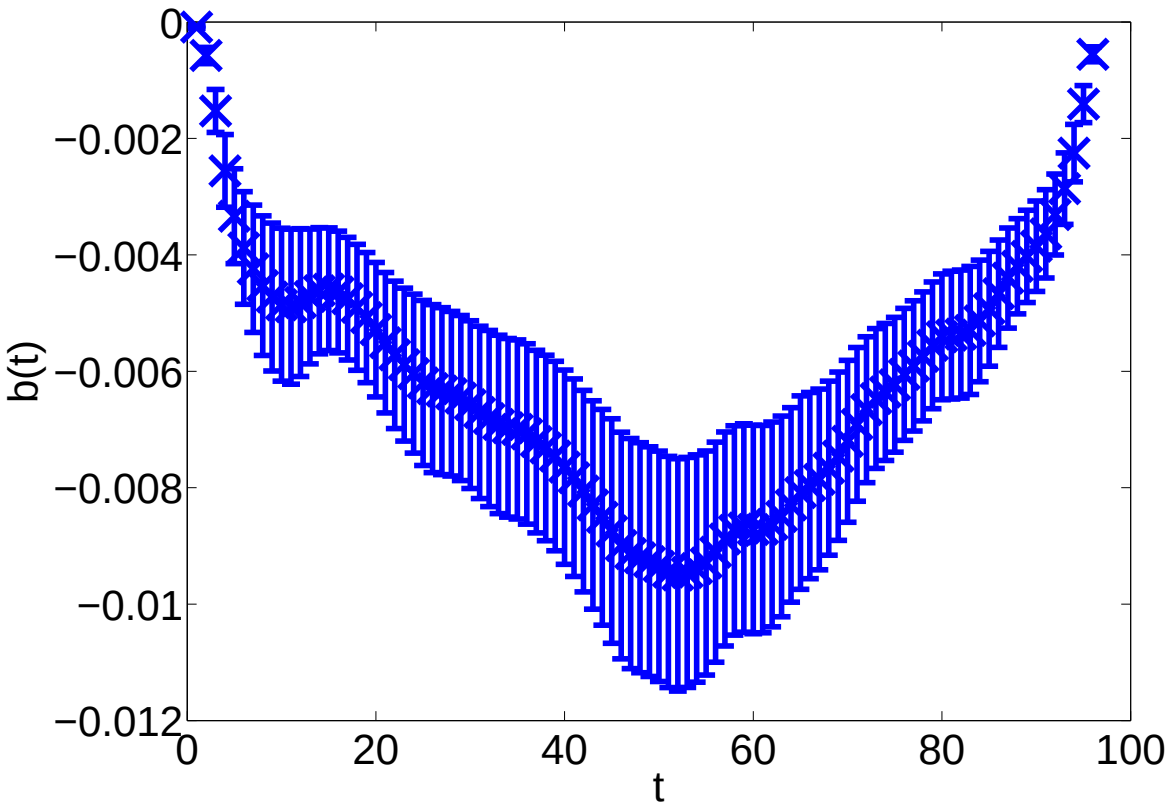
- costs: increases with L and is arround 200 inversions for G8
- fixing condition suitable to extract light quark masses
- relative good results with small statistics

Prospects:

- improving statistics
- including QED-effects

Covariance of $\tilde{W}$ with f_{PP}

$b(t)$ measured on F7:



$$b(t) = \mathrm{cov}(C(t), \tilde{W})/\langle C(t)\rangle$$

with $\langle C(t)\rangle = f_{PP}(t)$